

# Multi-differential Higgs theory predictions: vetos and double-differential resummation

Luca Rottoli

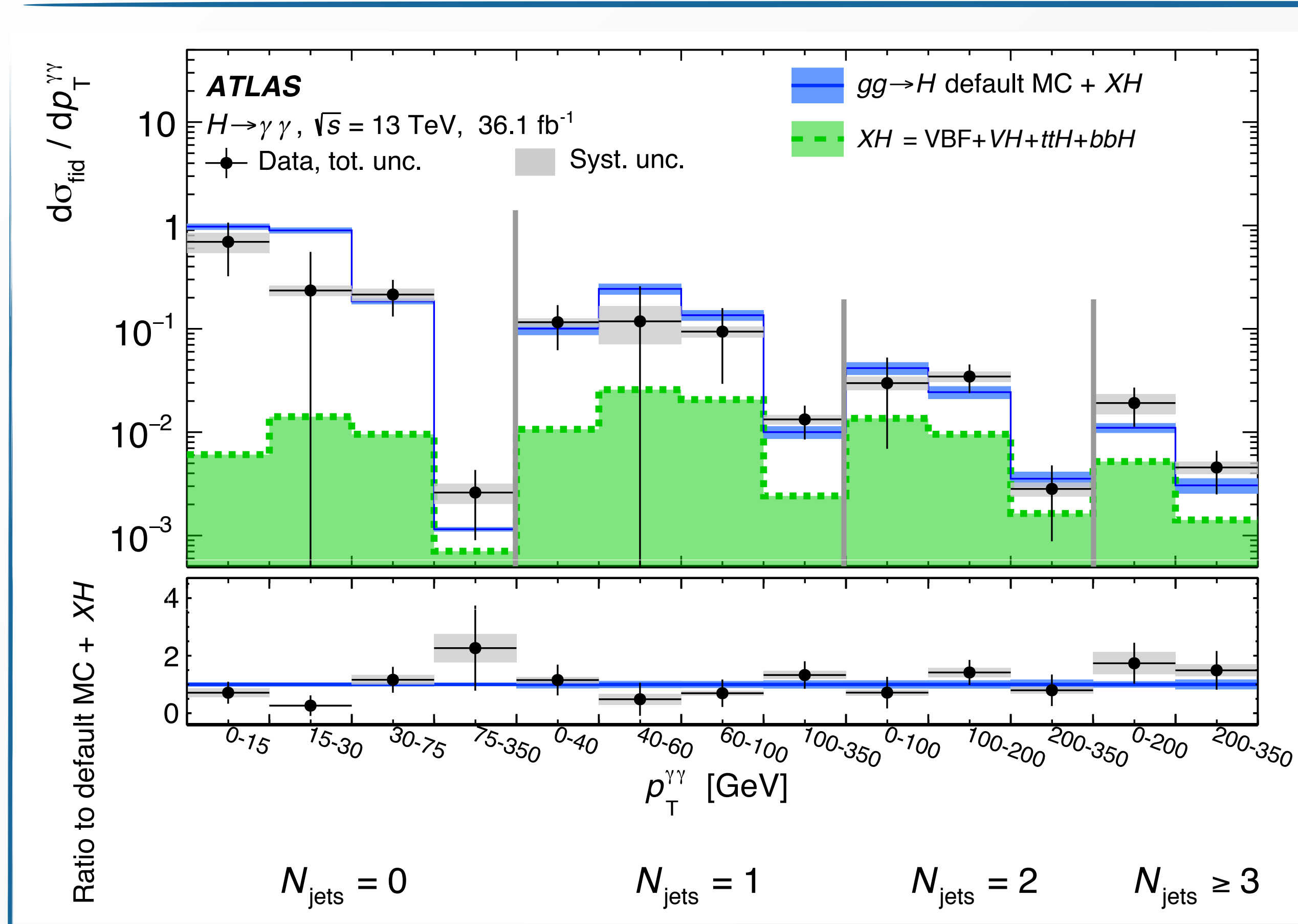
University of Milan-Bicocca & LBNL



*Based on 1909.04704 with P. Monni and P. Torrielli*



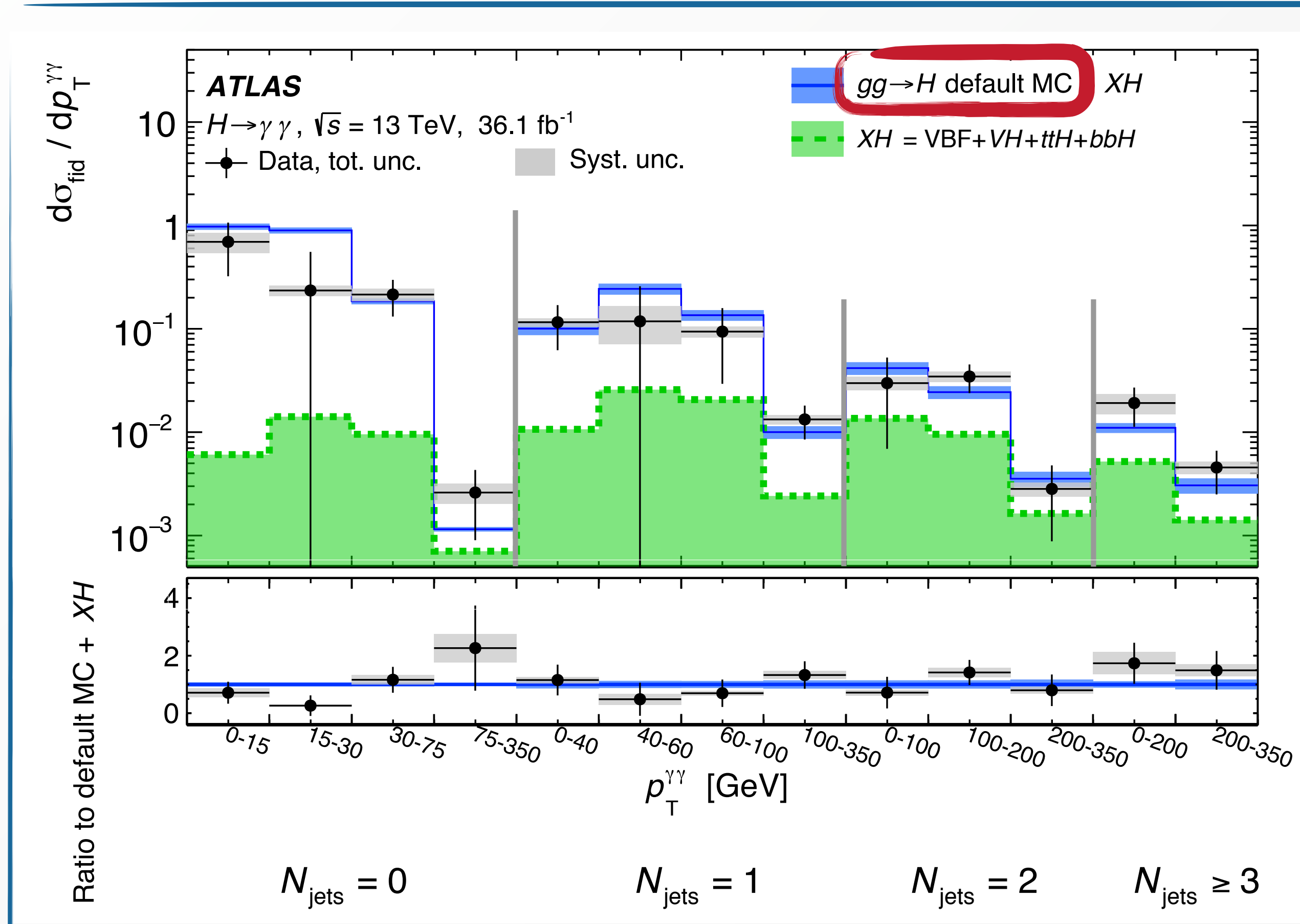
# The Higgs transverse momentum



[ATLAS 1802.04146]

- Relatively easy to measure
- Sensitivity to New Physics (e.g. **light Yukawa** couplings, **trilinear** Higgs coupling) [Bishara *et al.* '16][Soreq *et al.* '16] [Bizon *et al.* 1610.05771]
- Experimental analyses categorize events into **jet bins** according to the jet multiplicity
- Increased sensitivity to Higgs boson kinematics, spin-CP properties, BSM effects...

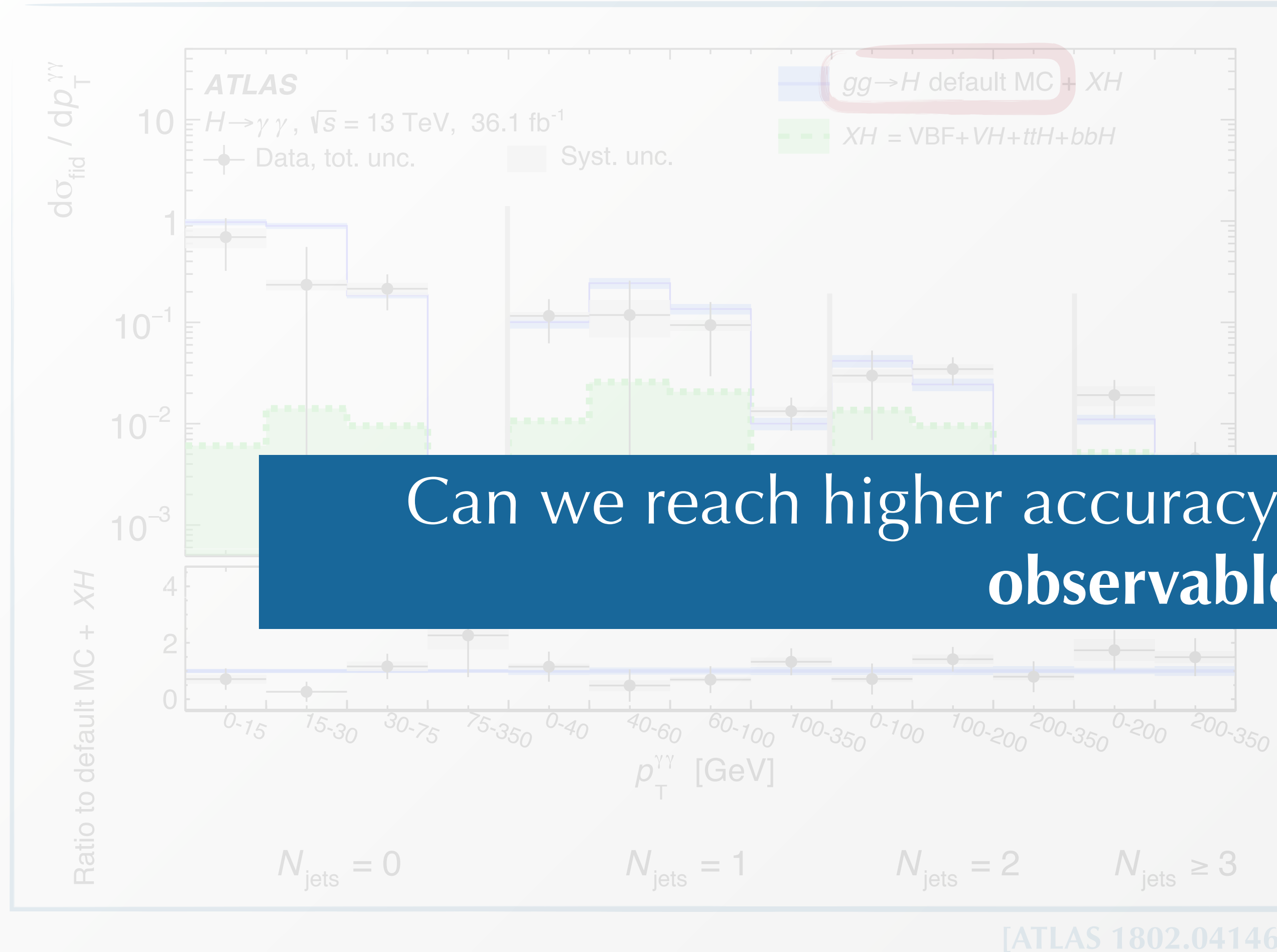
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- Current description of double-differential distributions based on predictions with **NNLO+PS accuracy** [Hamilton et al. 1309.0017]
- Similar comments apply also to other analyses (e.g. VH with boosted Higgs, W+W- production...)

# The Higgs transverse momentum

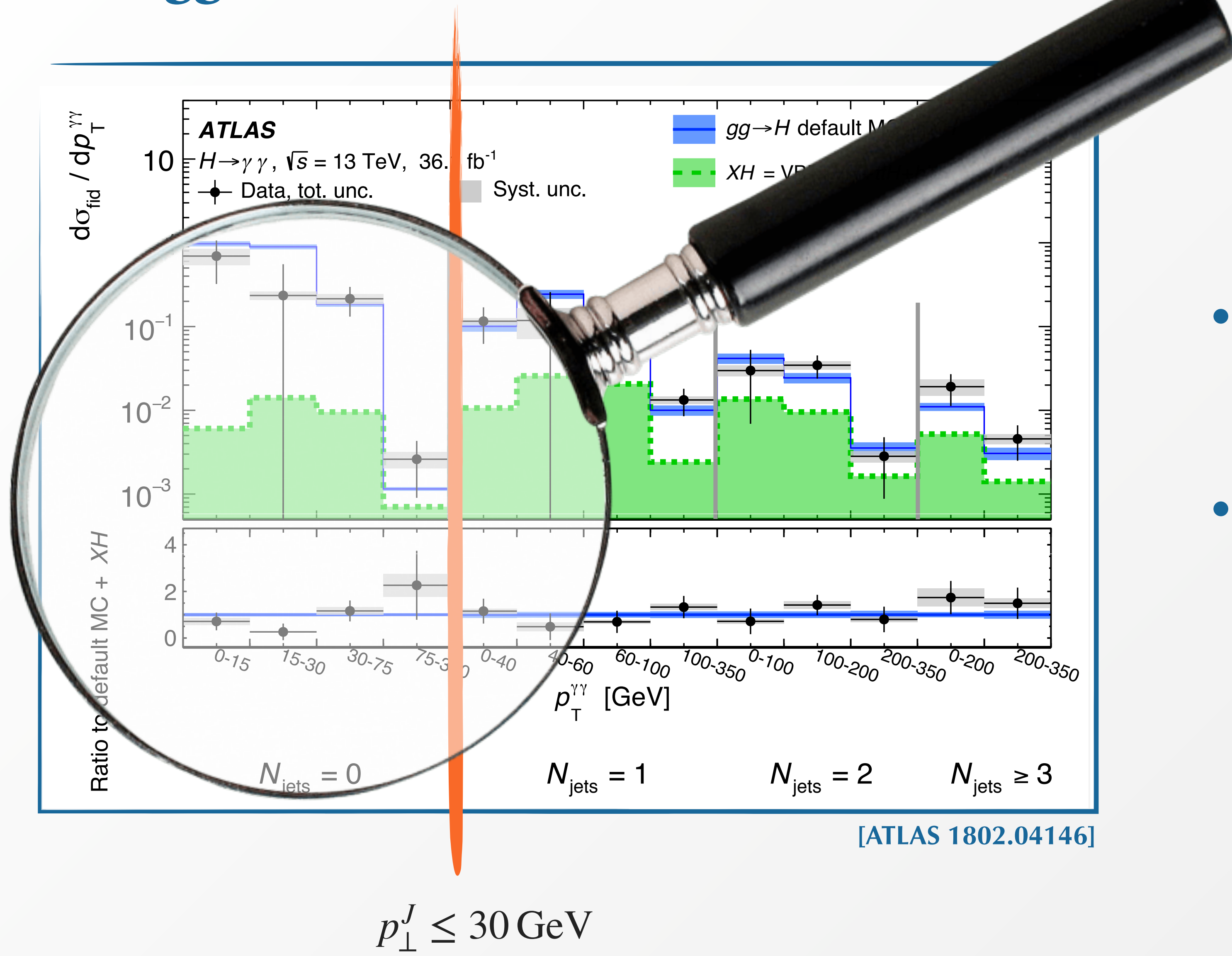


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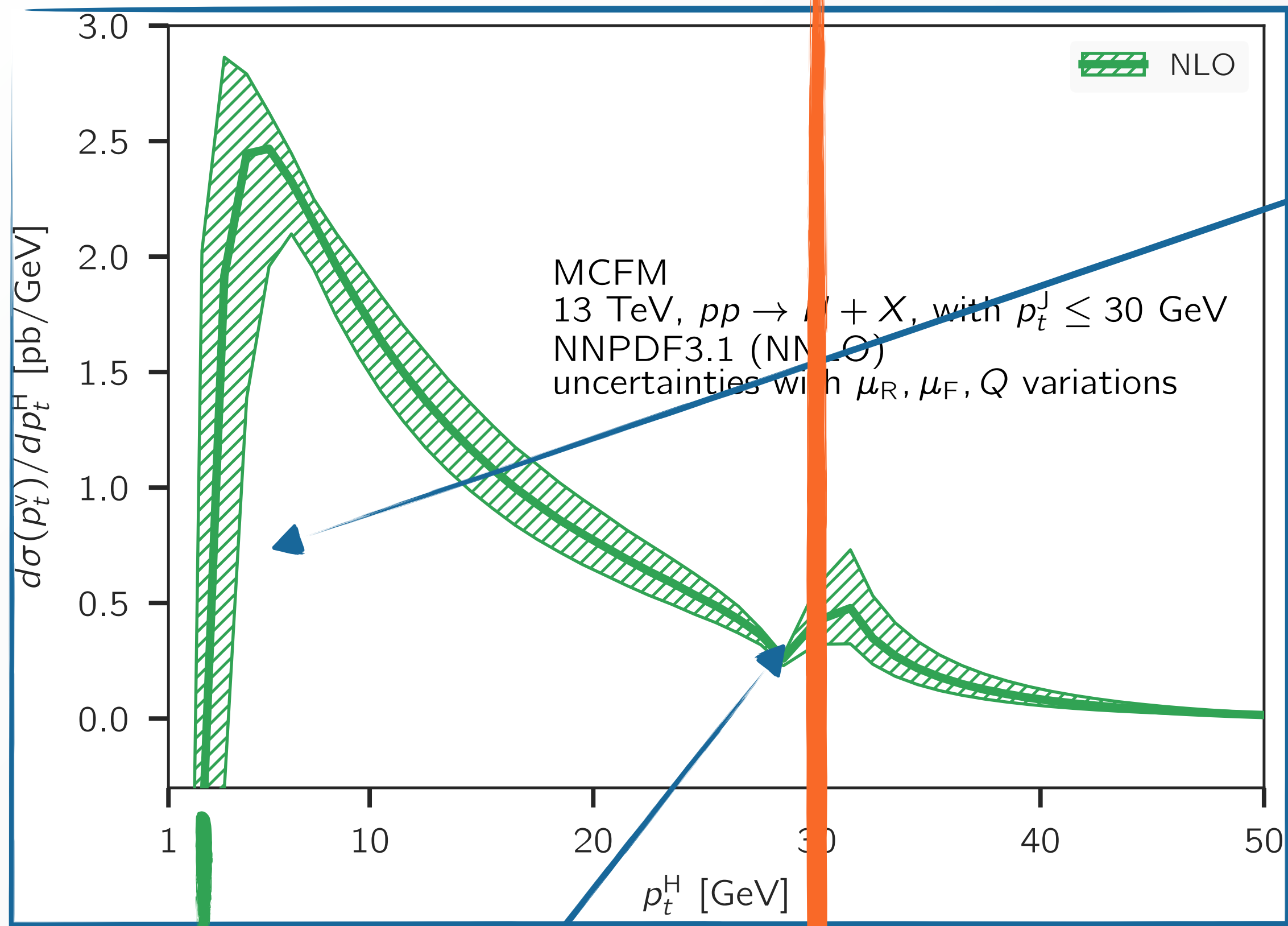
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# The Higgs transverse momentum



- Focus on the **zero-jet bin**  $p_{\perp}^J \leq p_{\perp}^{J,v}$
- Jet veto enforced to enhance the Higgs signal with respect to its backgrounds (e.g.  $W^+W^-$  event selection) or study of different production channels (e.g. STXS)

# The appearance of large logarithms

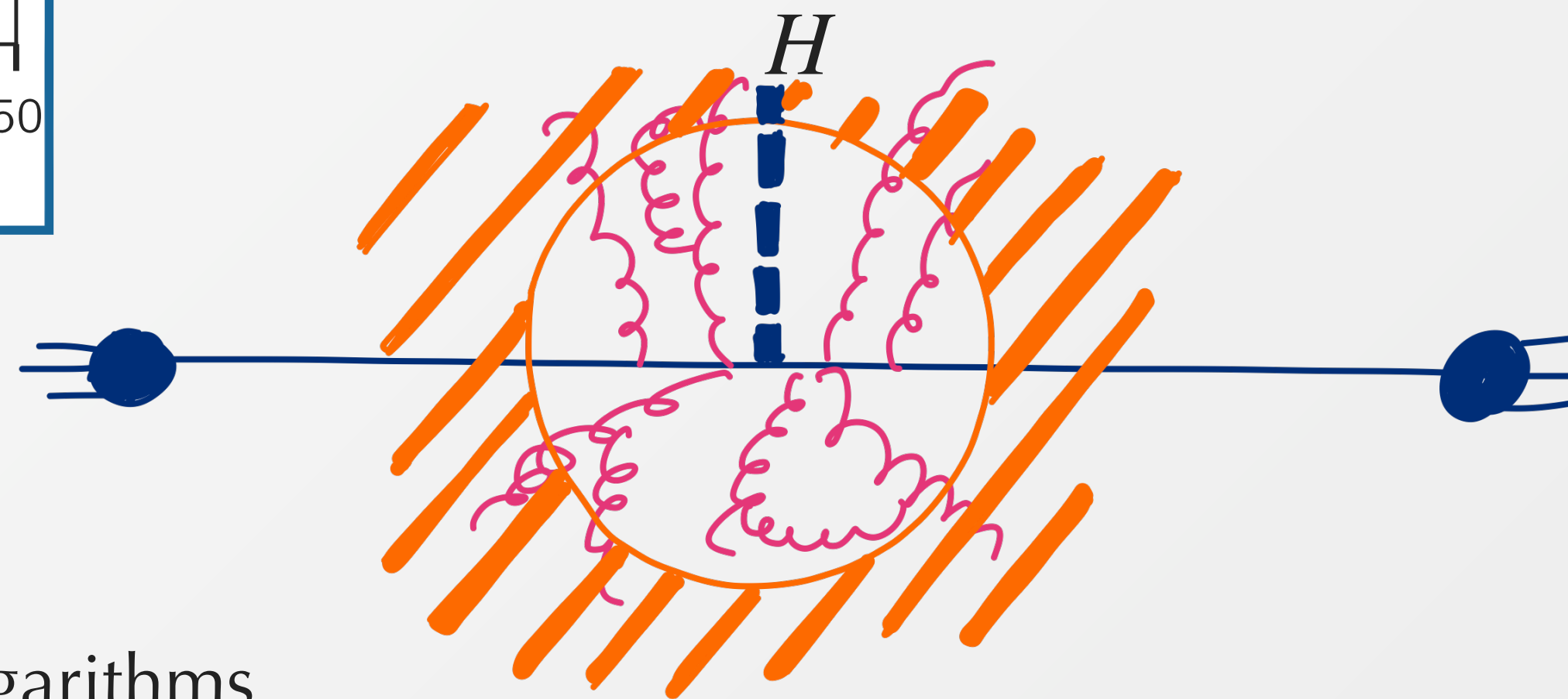


Large **transverse momentum** logarithms

$$L = \ln(p_{\perp}^H/m_H) \quad p_{\perp}^H \ll m_H$$

Large(ish) **jet veto** logarithms

$$L = \ln(p_{\perp}^{J,v}/m_H) \quad p_{\perp}^{J,v} < m_H$$



Jet veto = 30 GeV

$$L = \ln(|\vec{p}_{\perp}^H + \vec{p}_{\perp}^J|/p_{\perp}^H) \quad \text{Sudakov shoulder logarithms}$$

# The appearance of large logarithms

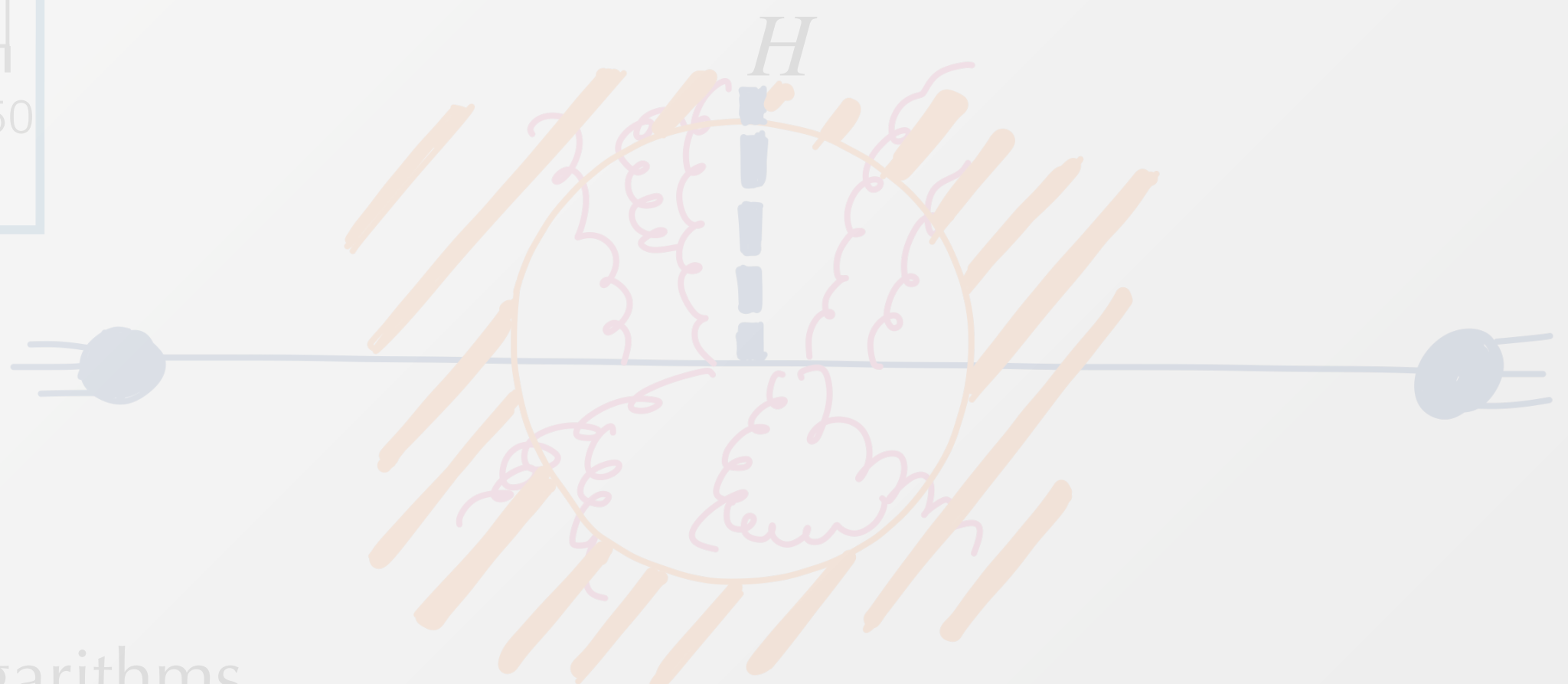


Fixed order predictions no longer reliable:  
**all order resummation** of the perturbative series mandatory

Large transverse momentum logarithms

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$$L = \ln(p_{\perp}^{J,\nu} / m_H) \quad p_{\perp}^{J,\nu} < m_H$$

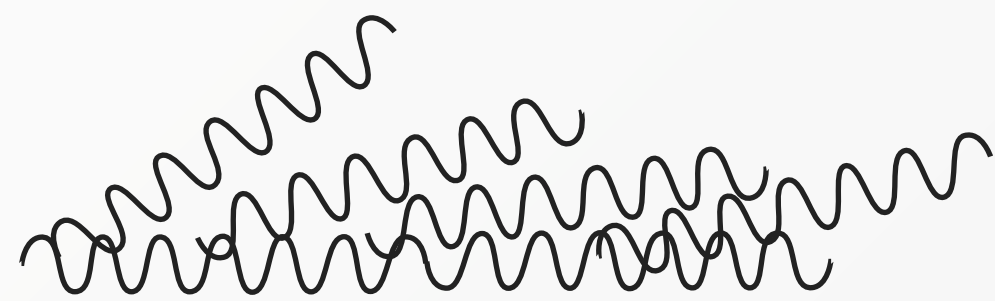


$$L = \ln(|\vec{p}_{\perp}^H + \vec{p}_{\perp}^J| / p_{\perp}^H) \quad \text{Sudakov shoulder logarithms}$$

# An example : resummation of the transverse momentum spectrum

Resummation of transverse momentum is particularly delicate because  $p_{\perp}$  is a **vectorial quantity**

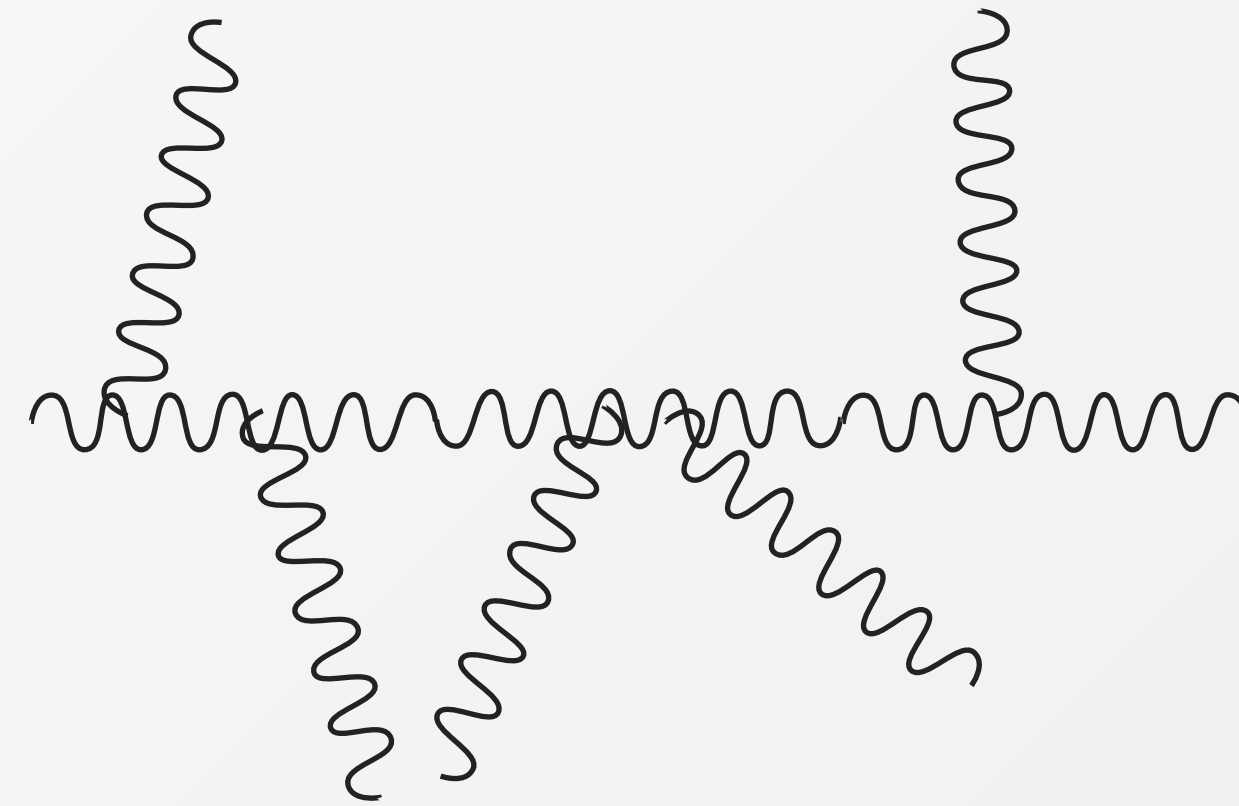
**Two concurring mechanisms** leading to a system with small  $p_{\perp}$



$$p_{\perp}^2 \sim k_{t,i}^2 \ll m_H^2$$

cross section naturally suppressed as there is no phase space left for gluon emission  
(Sudakov limit)

**Exponential suppression**



$$\sum_{i=1}^n \vec{k}_{t,i} \simeq 0$$

**Large kinematic cancellations**

$p_{\perp} \sim 0$  far from the Sudakov limit

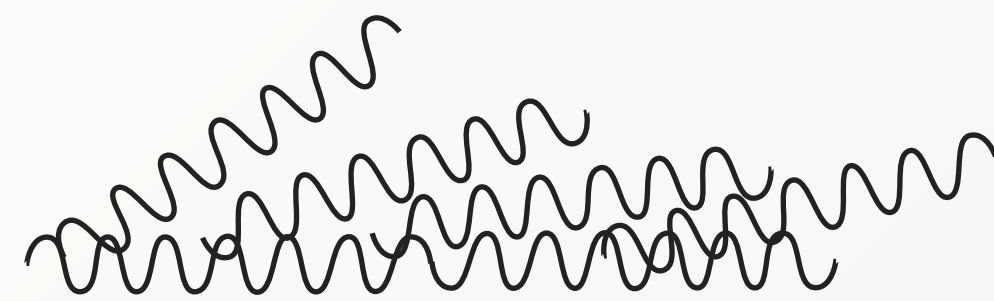
**Power suppression**



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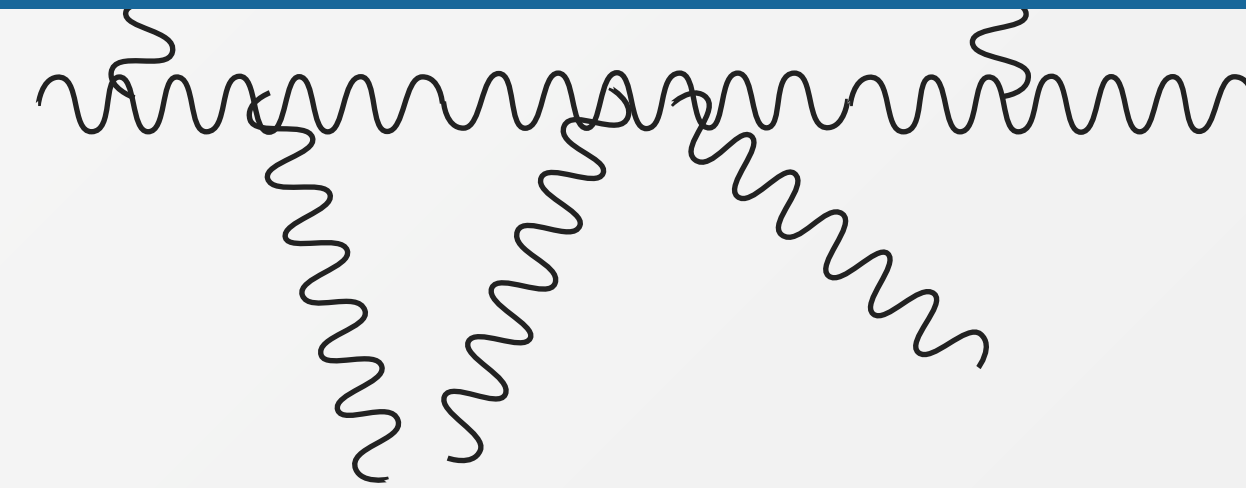


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**Dominant at small  $p_{\perp}$**



$$\sum_{i=1}^n \vec{k}_{t,i} \simeq 0$$

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**Power suppression**

# Resummation of the transverse momentum spectrum in direct space

Highly **non-trivial problem**. Solution to the problem recently formulated by extending the CAESAR/ARES approach to deal with observables with azimuthal cancellations: **RadISH** approach  
 [Monni, Re, Torrielli '16][Bizon, Monni, Re, LR, Torrielli '17]

[Banfi, Salam, Zanderighi '01, '03, '04]  
 [Banfi, McAslan, Monni, Zanderighi '15, '16]  
 [Banfi, El-Menoufi, Monni '19]

Problem recently addressed also within SCET [Ebert, Tackmann '17]

Result at NLL accuracy can be written as

$$\sigma(p_{\perp}) = \sigma_0 \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} \boxed{e^{-R(v_1)}} \text{ Simple observable} \quad v_i = k_{t,i}/m_H, \quad \zeta_i = v_i/v_1$$

$$\times \epsilon^{R'(v_1)} R'(v_1) \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'(\zeta_i v_1) \Theta \left( p_{\perp} - |\vec{k}_{t,i} + \dots + \vec{k}_{t,n+1}| \right)$$

Transfer function

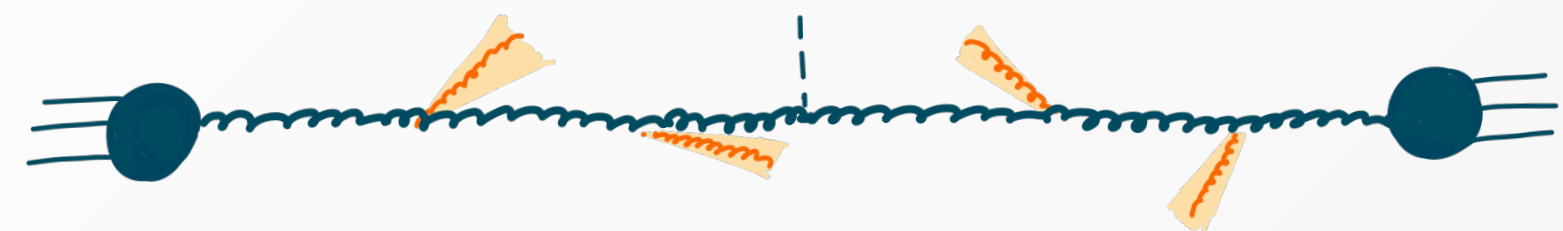
Formula can be evaluated with Monte Carlo method; dependence on  $\epsilon$  vanishes (as  $\mathcal{O}(\epsilon)$ ) and result is **finite** in four dimensions

**Subleading effects retained:** no divergence at small  $p_{\perp}$ , power-like behaviour respected

# Direct space formulation: generality

NLL result for  $p_{\perp}^J$

$$\sigma(p_{\perp}^J) = \sigma_0 e^{Lg_1(\alpha_s\beta_0L) + g_2(\alpha_s\beta_0L)}$$



[Parisi, Petronzio '79; Collins, Soper, Sterman '85]

NLL result for  $p_{\perp}^H$

$$\sigma(p_{\perp}^H) = \sigma_0 \int d^2\vec{p}_{\perp}^H \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_{\perp}^H} e^{-R_{\text{NLL}}(L)}$$



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General formula for a generic transverse observable at NLL [Bizon, Monni, Re, LR, Torrielli '17]

$$\sigma(v) = \sigma_0 \int \frac{dk_{t,1}}{k_{t,1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(k_{t,1})} R'(k_{t,1}) d\mathcal{F} \Theta(v - V(k_1, \dots, k_{n+1}))$$

$$d\mathcal{F} = e^{R'(k_{t,1})} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'(\zeta_i k_{t,1})$$

$$R_{\text{NLL}}(L) = -Lg_1(\alpha_s\beta_0L) - g_2(\alpha_s\beta_0L)$$

$$L = \ln(k_{t,1}/M)$$

$$R'_{\text{NLL}}(k_t) = 4 \left( \frac{\alpha_s^{\text{CMW}}(k_t)}{\pi} C_A \ln \frac{m_H}{k_t} - \alpha_s(k_t)\beta_0 \right)$$

**CMW scheme**

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differential control in momentum space provides guidance to **double-differential resummation**

General

$$\sigma(v) = \sigma_0 \int \frac{dk_{t,1}}{k_{t,1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(k_{t,1})} R'(k_{t,1}) d\mathcal{Z} \Theta\left(p_T^H - |\vec{k}_{t,1} + \dots + \vec{k}_{t,n+1}|\right)$$

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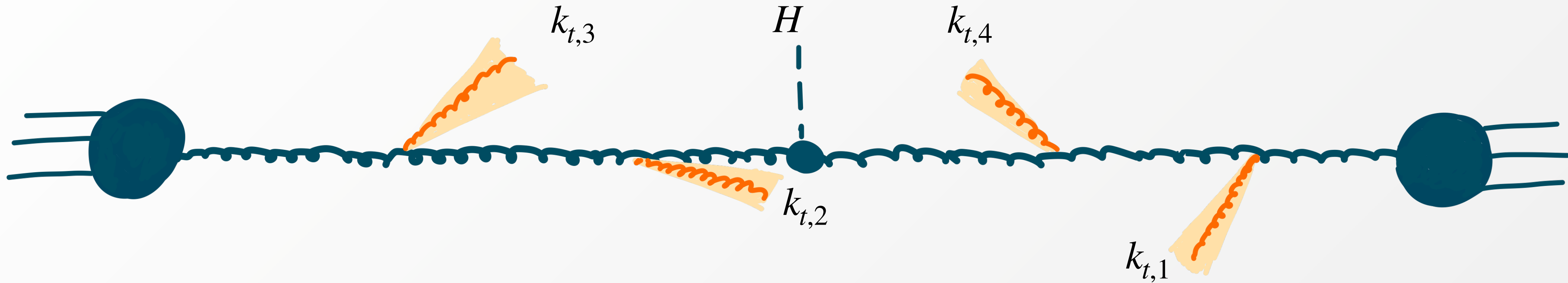
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# Double-differential resummation in direct space

At NLL, emissions are **strongly ordered** in angle.  $k_t$ -clustering algorithms associate **each emission** to a **different** jet



Double-differential resummation obtained by **combining measurement functions!**

At NLL, now including parton luminosities

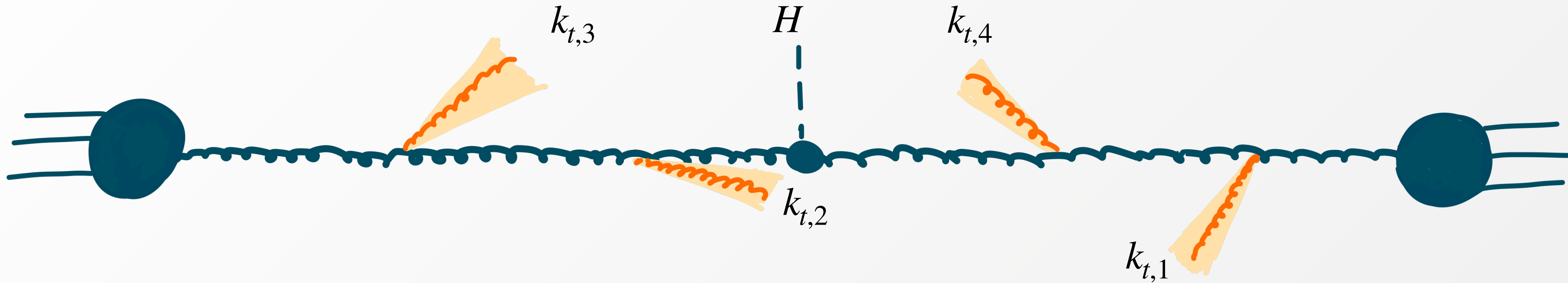
$$\sigma^{\text{incl}}(p_{\perp}^H) = \int \frac{dk_{t,1}}{k_{t,1}} \frac{d\phi_1}{2\pi} d\mathcal{L} \partial_L \left[ -e^{-R_{\text{NLL}}(k_{t,1})} \mathcal{L}_{\text{NLL}}(k_{t,1}) \right] \Theta \left( p_{\perp}^H - |\vec{k}_{t,1} + \dots + \vec{k}_{t,n+1}| \right)$$

$$\sigma^{\text{incl}}(p_{\perp}^{\text{J},\text{v}}) = \int \frac{dk_{t,1}}{k_{t,1}} \frac{d\phi_1}{2\pi} d\mathcal{L} \partial_L \left[ -e^{-R_{\text{NLL}}(k_{t,1})} \mathcal{L}_{\text{NLL}}(k_{t,1}) \right] \Theta \left( p_{\perp}^{\text{J},\text{v}} - \max \{k_{t,1}, \dots, k_{t,n+1}\} \right)$$



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$$\frac{d\sigma(p_{\perp}^{J,v}, p_{\perp}^H)}{d\Phi_B} = \int \frac{dk_{t,1}}{k_{t,1}} \frac{d\phi_1}{2\pi} d\mathcal{F} \partial_L \left[ -e^{-R_{\text{NLL}}(k_{t,1})} \mathcal{L}_{\text{NLL}}(k_{t,1}) \right] \Theta \left( p_{\perp}^{J,v} - \max \{k_{t,1}, \dots, k_{t,n+1}\} \right) \Theta \left( p_{\perp}^H - |\vec{k}_{t,1} + \dots + \vec{k}_{t,n+1}| \right)$$

$$\mathcal{L}_{\text{NLL}}(\mu) = M_{gg \rightarrow H}^2 f_g(x_1, \mu) f_g(x_2, \mu)$$

At higher orders, it contains also coefficient functions and hard-virtual coefficients

# Double-differential resummation in direct space at NNLL

Additional corrections must be included at NNLL [Banfi et al. '12][Becher et al. '13][Stewart et al. '14]

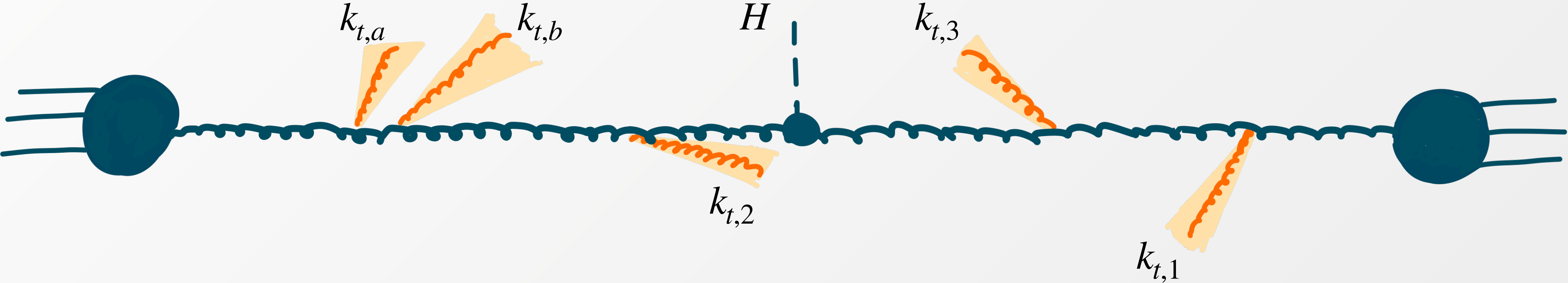
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- **clustering correction**: jet algorithm can cluster two **independent** emissions into the same jet

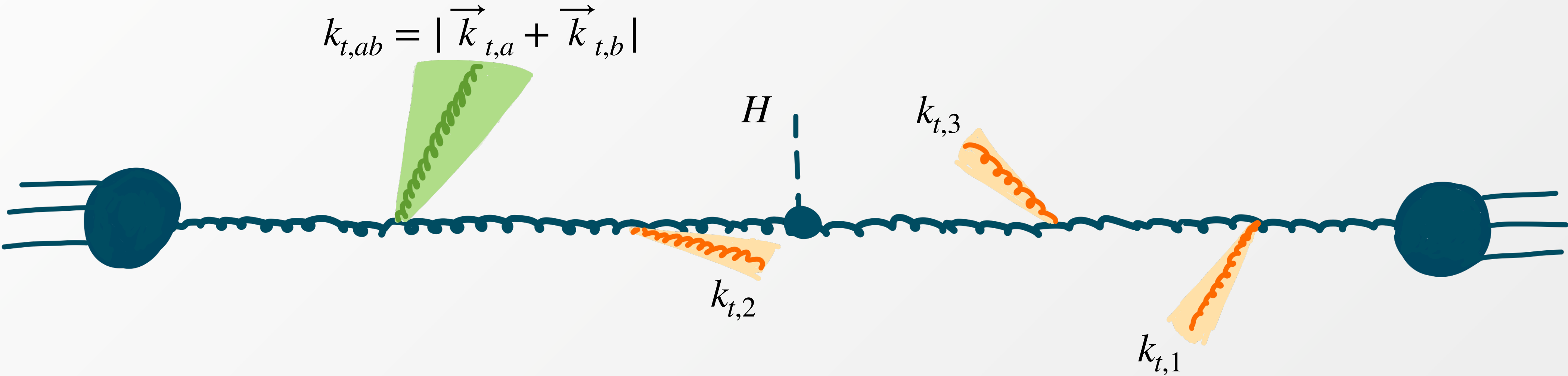


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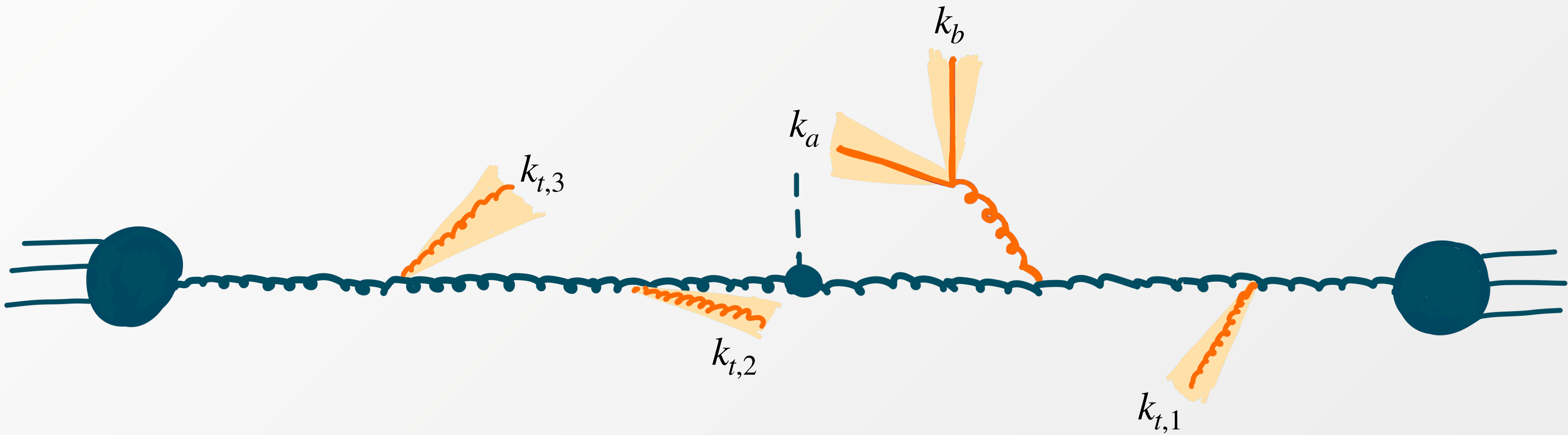


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- **correlated correction**: amends the inclusive treatment of the correlated squared amplitude for two emission accounting for configurations where the two **correlated** emissions (non abelian) are not clustered in the same jet

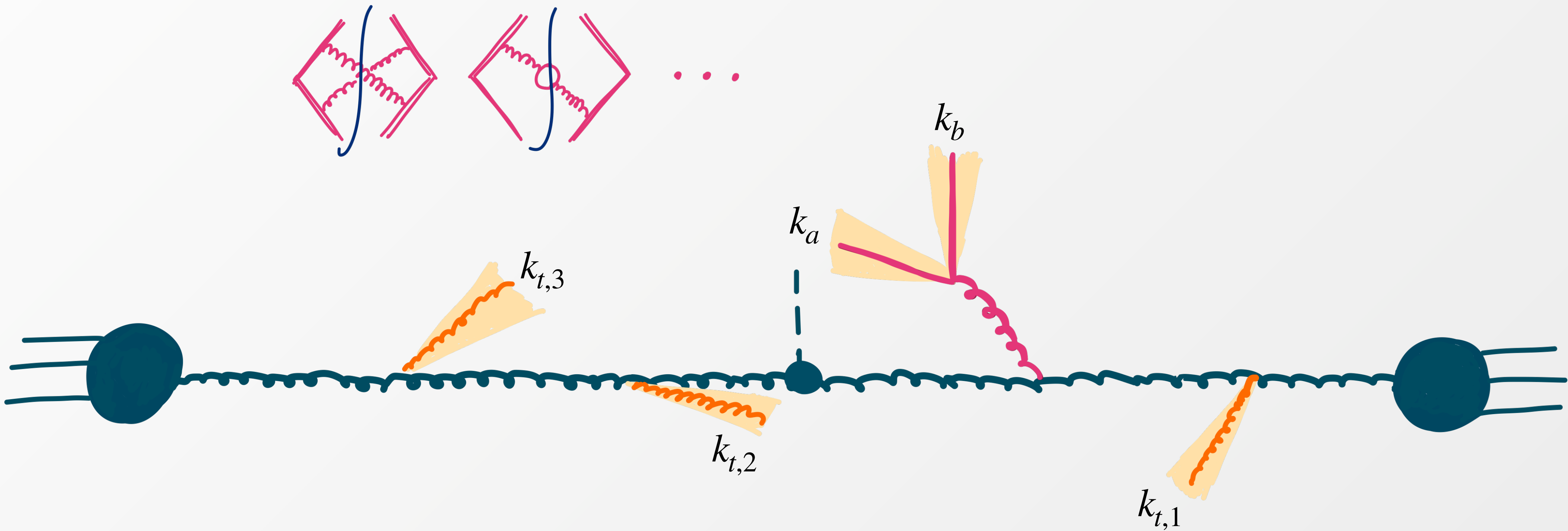


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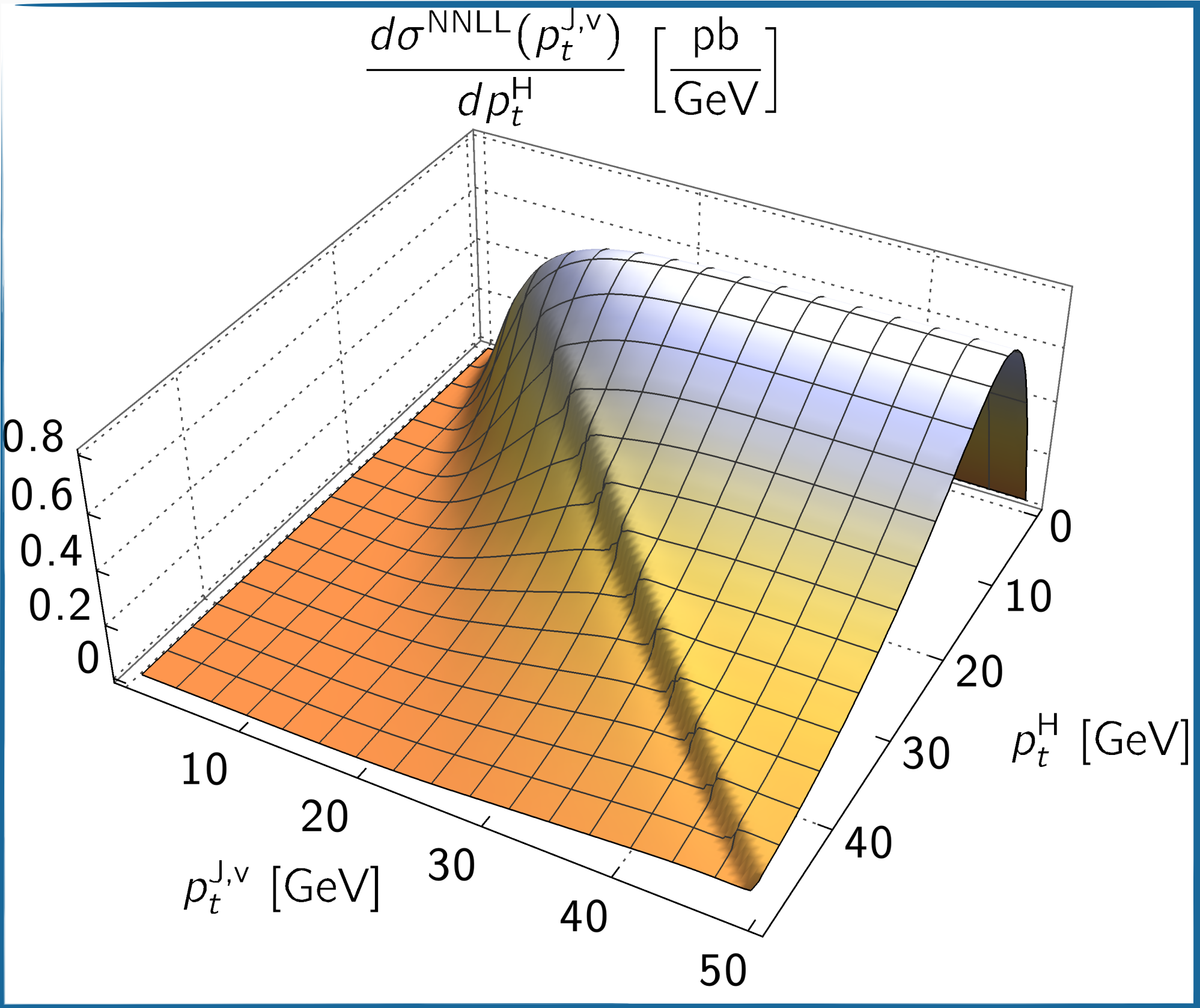
The same philosophy used at NLL can be applied at NNLL

where e.g

$$\begin{aligned}
 \sigma_{\text{clust}}^{\text{NNLL}}(p_{\perp}^{\text{J,v}}, p_{\perp}^{\text{H}}) &\simeq \int_0^{\infty} \frac{dk_{t,1}}{k_{t,1}} \frac{d\phi_1}{2\pi} \int d\mathcal{F} e^{-R(k_{t,1})} 8 C_A^2 \frac{\alpha_s^2(k_{t,1})}{\pi^2} \Theta\left(p_{\perp}^{\text{J,v}} - \max_{i>1} \{k_{t,i}\}\right) \\
 &\times \int_0^{k_{t,1}} \frac{dk_{t,s_1}}{k_{t,s_1}} \frac{d\phi_{s_1}}{2\pi} \int_{-\infty}^{\infty} d\Delta\eta_{1s_1} J_{1s_1}(R) \left[ \Theta\left(p_{\perp}^{\text{J,v}} - |\vec{k}_{t,1} + \vec{k}_{t,s_1}|\right) - \Theta\left(p_{\perp}^{\text{J,v}} - k_{t,1}\right) \right] \\
 &\times \Theta\left(p_{\perp}^{\text{H}} - |\vec{k}_{t,1} + \dots + \vec{k}_{t,n+1} + \vec{k}_{t,s_1}|\right)
 \end{aligned}$$

And analogously for other contributions

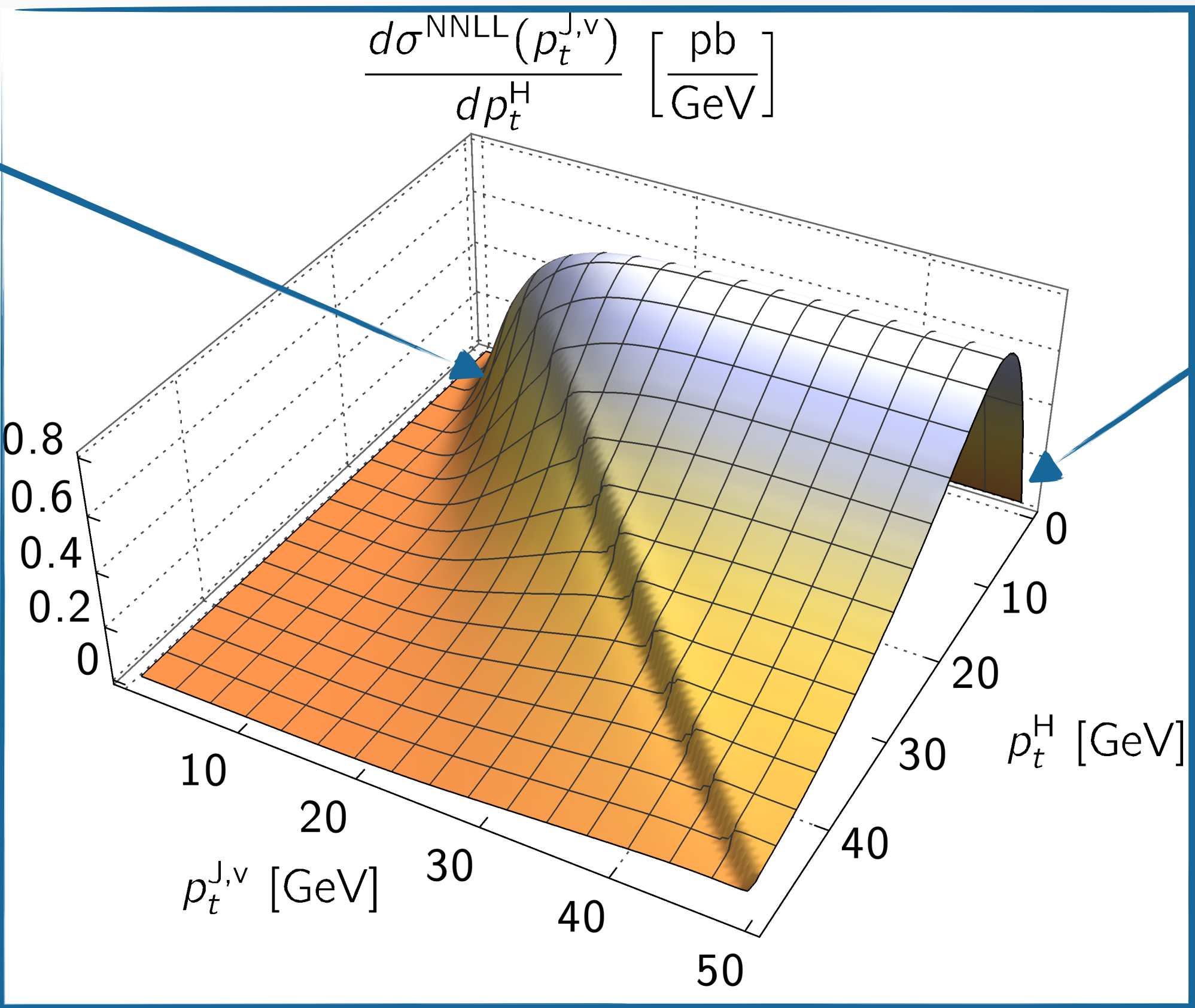
# NNLL cross section differential in $p_{\perp}^H$ , cumulative in $p_{\perp}^J \leq p_{\perp}^{J,v}$





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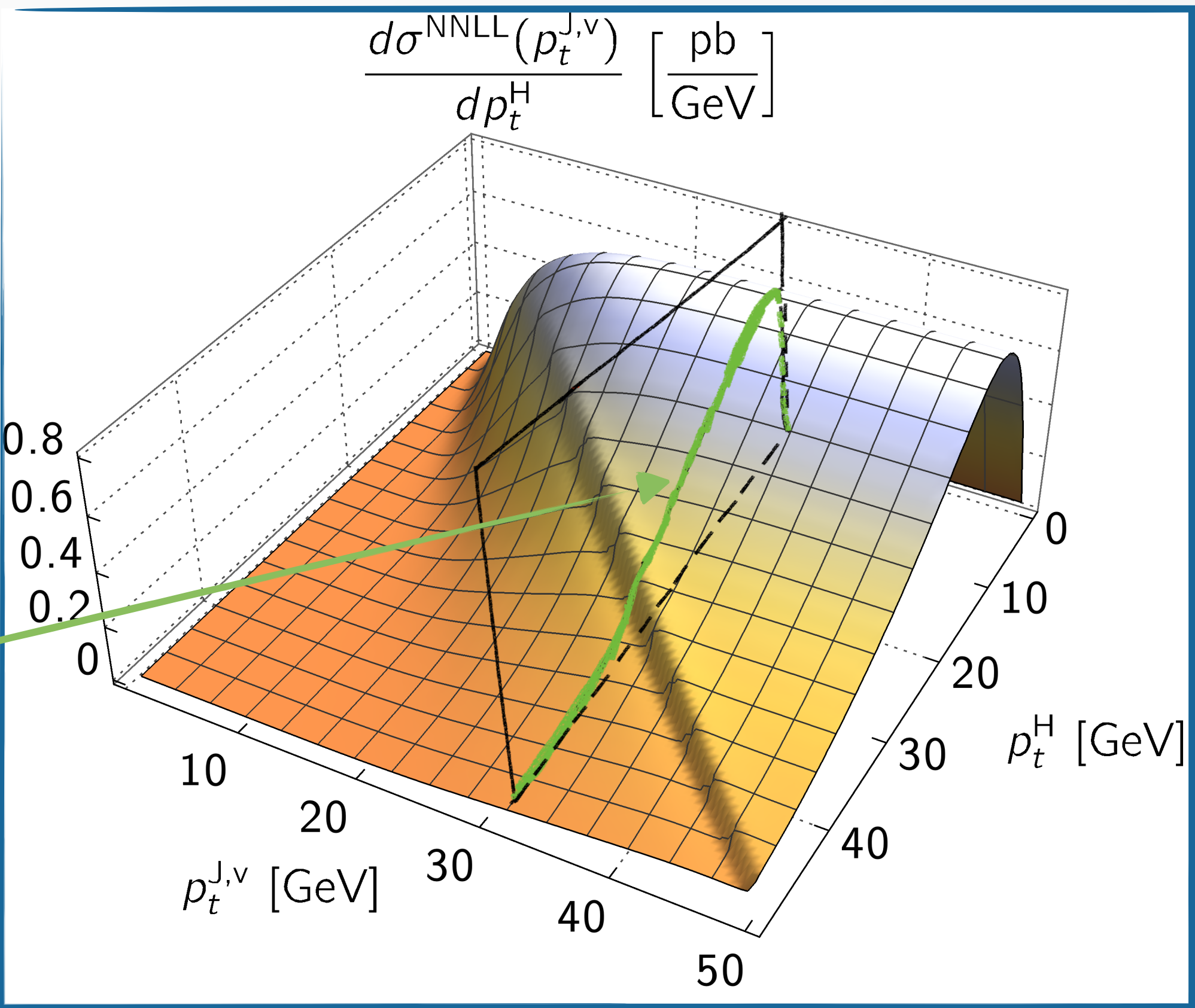
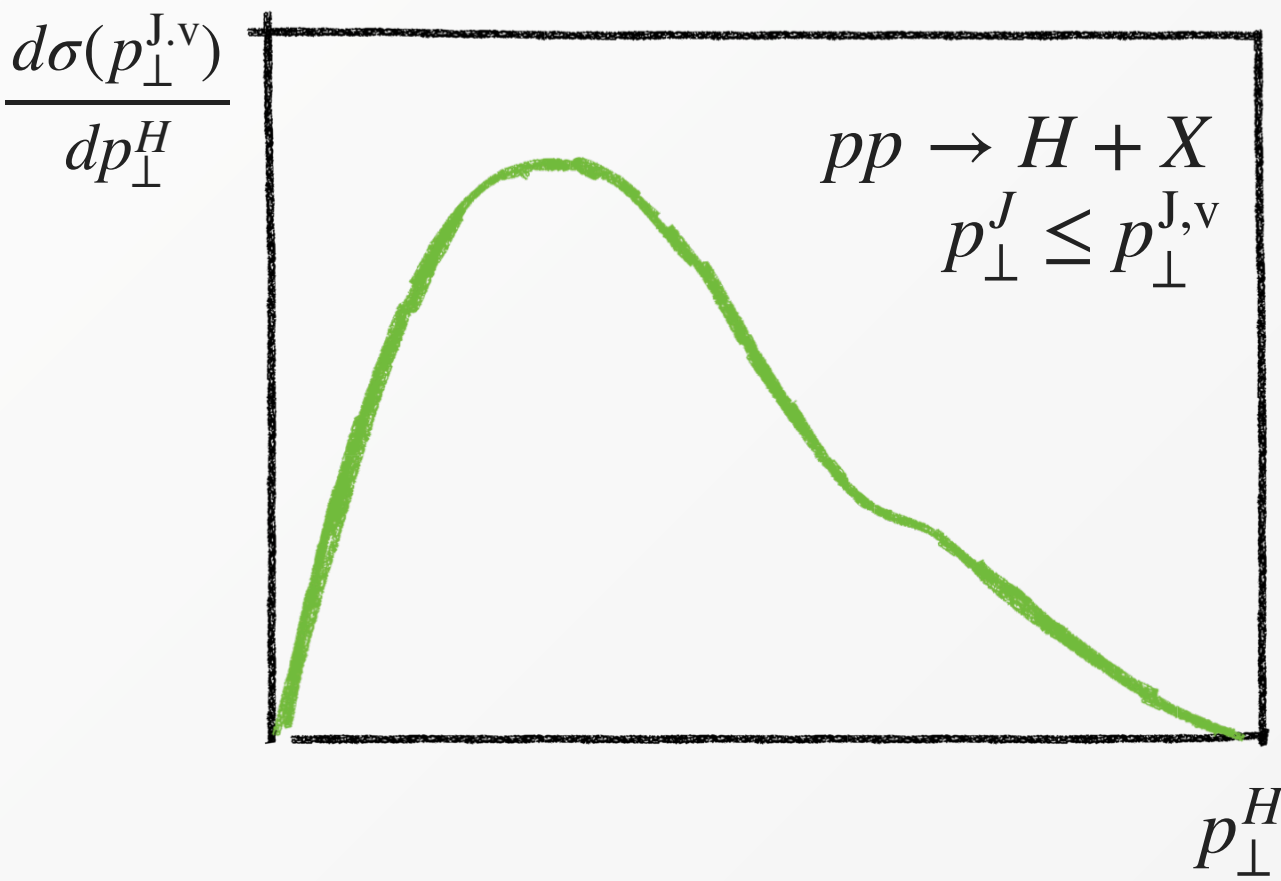
Sudakov suppression at small  $p_{\perp}^J$



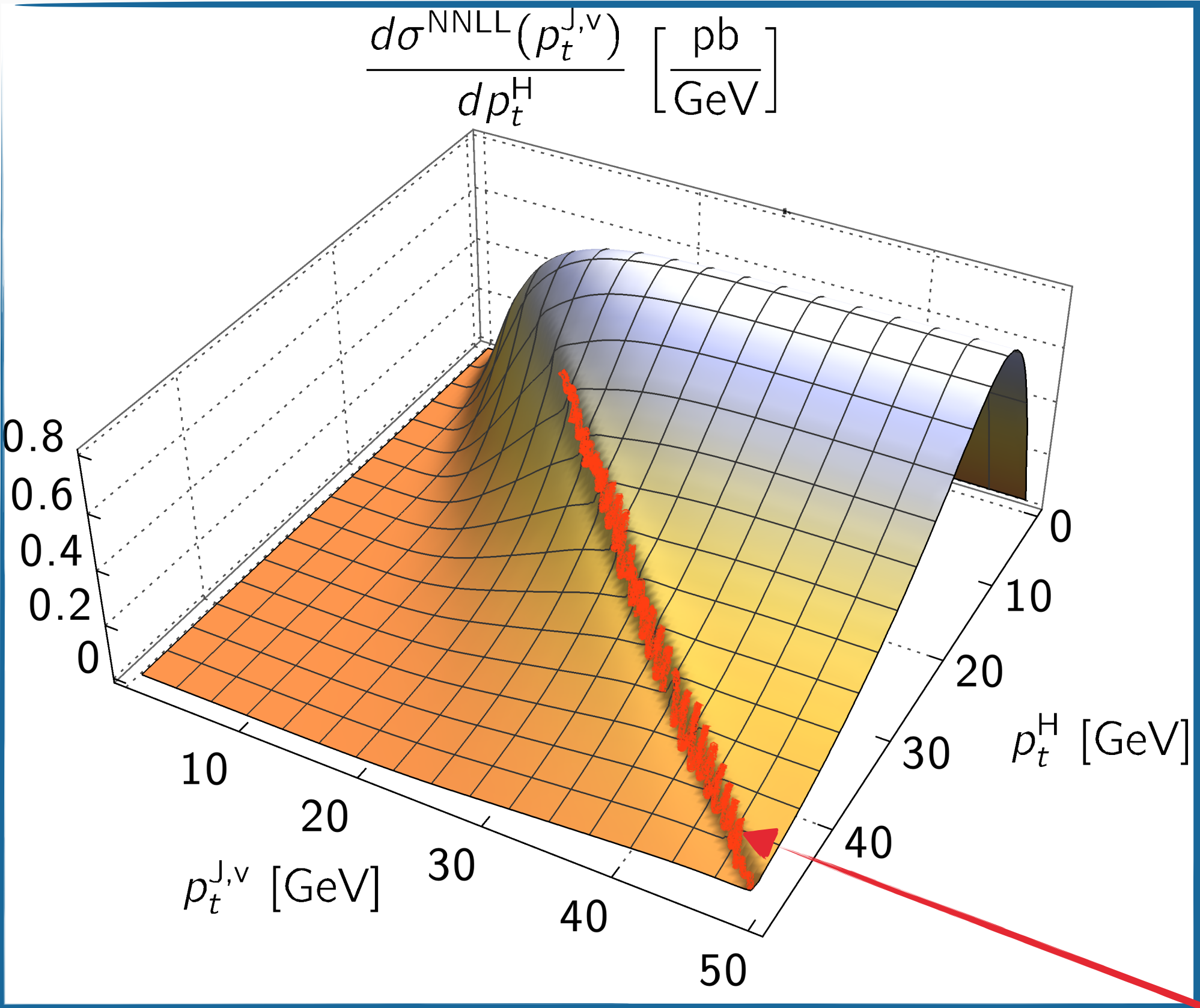
Peaked structure (Sudakov) + power-like suppression at very small  $p_{\perp}^H$

# NNLL cross section differential in $p_{\perp}^H$ , cumulative in $p_{\perp}^J \leq p_{\perp}^{J,v}$

At a given value of  $p_{\perp}^{J,v}$  it corresponds to the  $p_{\perp}^H$  cross section in the 0-jet bin



# NNLL cross section differential in $p_{\perp}^H$ , cumulative in $p_{\perp}^J \leq p_{\perp}^{J,v}$



[Catani, Webber '97]

**Sudakov shoulder:** integrable singularity beyond LO at  $p_{\perp}^H \simeq p_{\perp}^{J,v}$

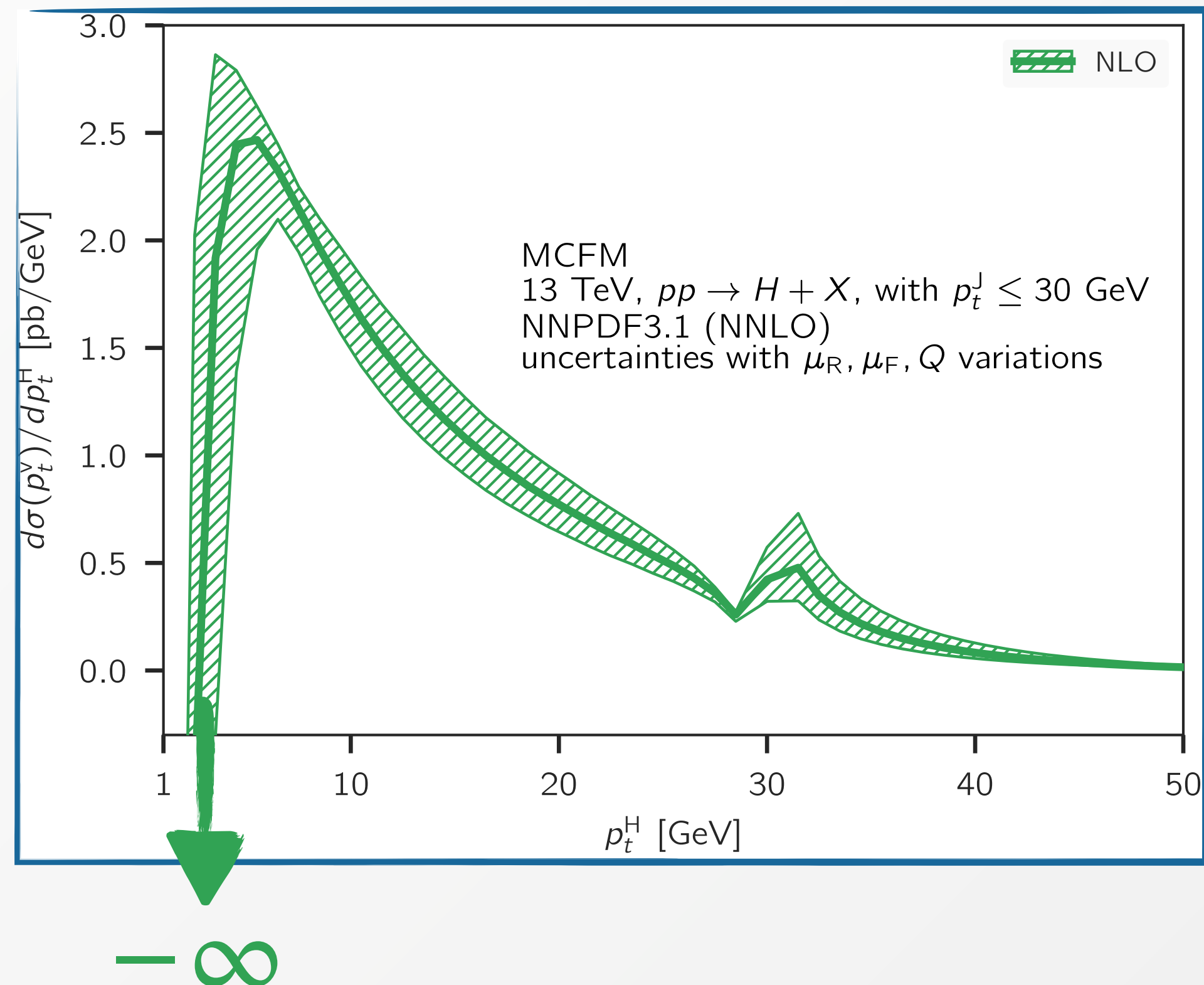
Logarithms associated to the Shoulder are resummed in the limit  $p_{\perp}^H \sim p_{\perp}^{J,v} \ll m_H$

# LHC results: Higgs transverse momentum with a jet veto

Multiplicative matching to fixed order (NLO H+j from MCFM, NNLO H from ggHiggs)

[Campbell, Ellis, Giele, '15]

[Bonvini et al '13]



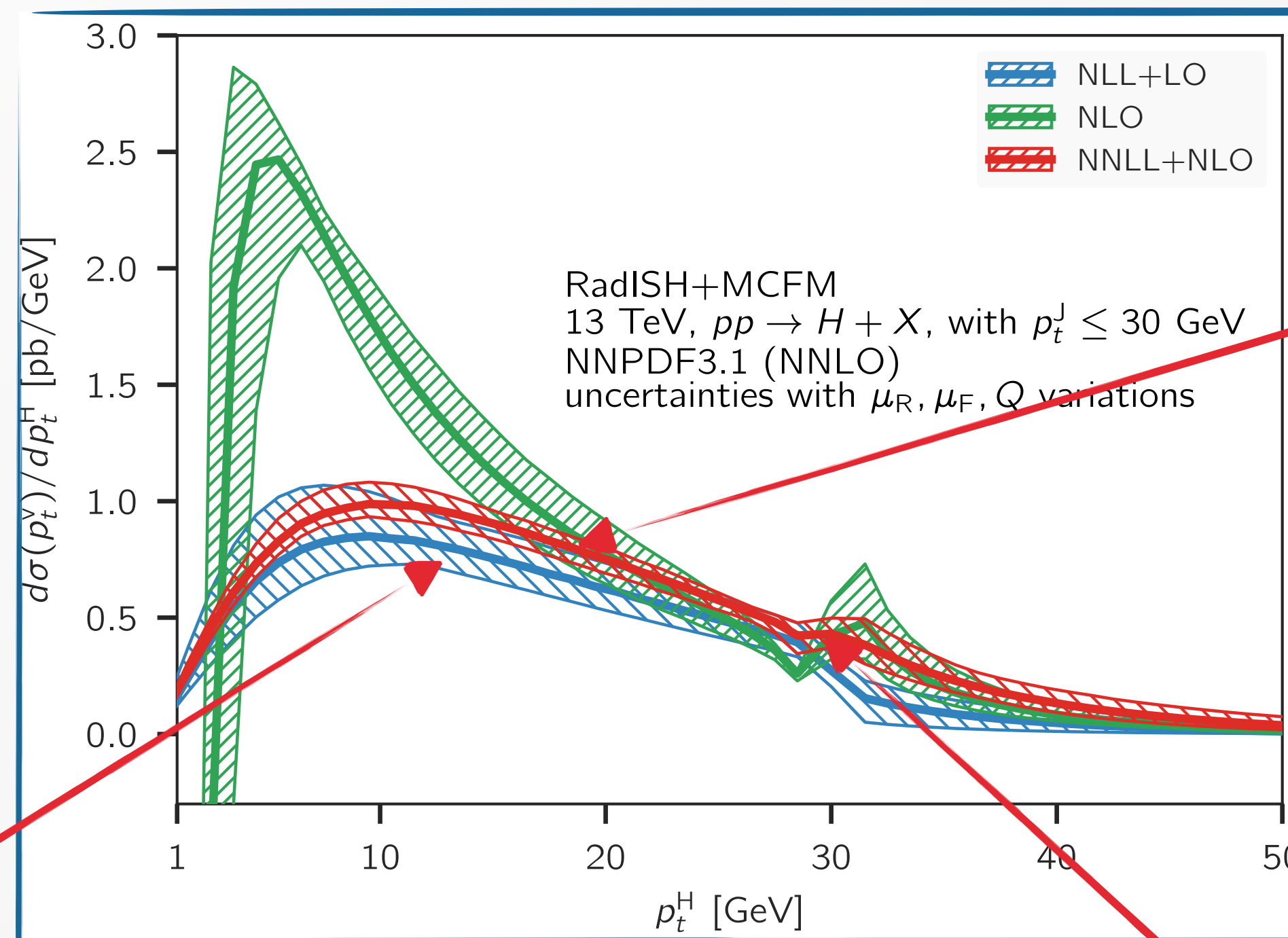
# LHC results: Higgs transverse momentum with a jet veto

Multiplicative matching to fixed order (NLO H+j from MCFM, NNLO H from ggHiggs)

[Campbell, Ellis, Giele, '15]

[Bonvini et al '13]

residual uncertainties at  
**NNLL+NLO** at the 10% level



large K-factor becomes relevant  
at larger  $p_{\perp}^H$

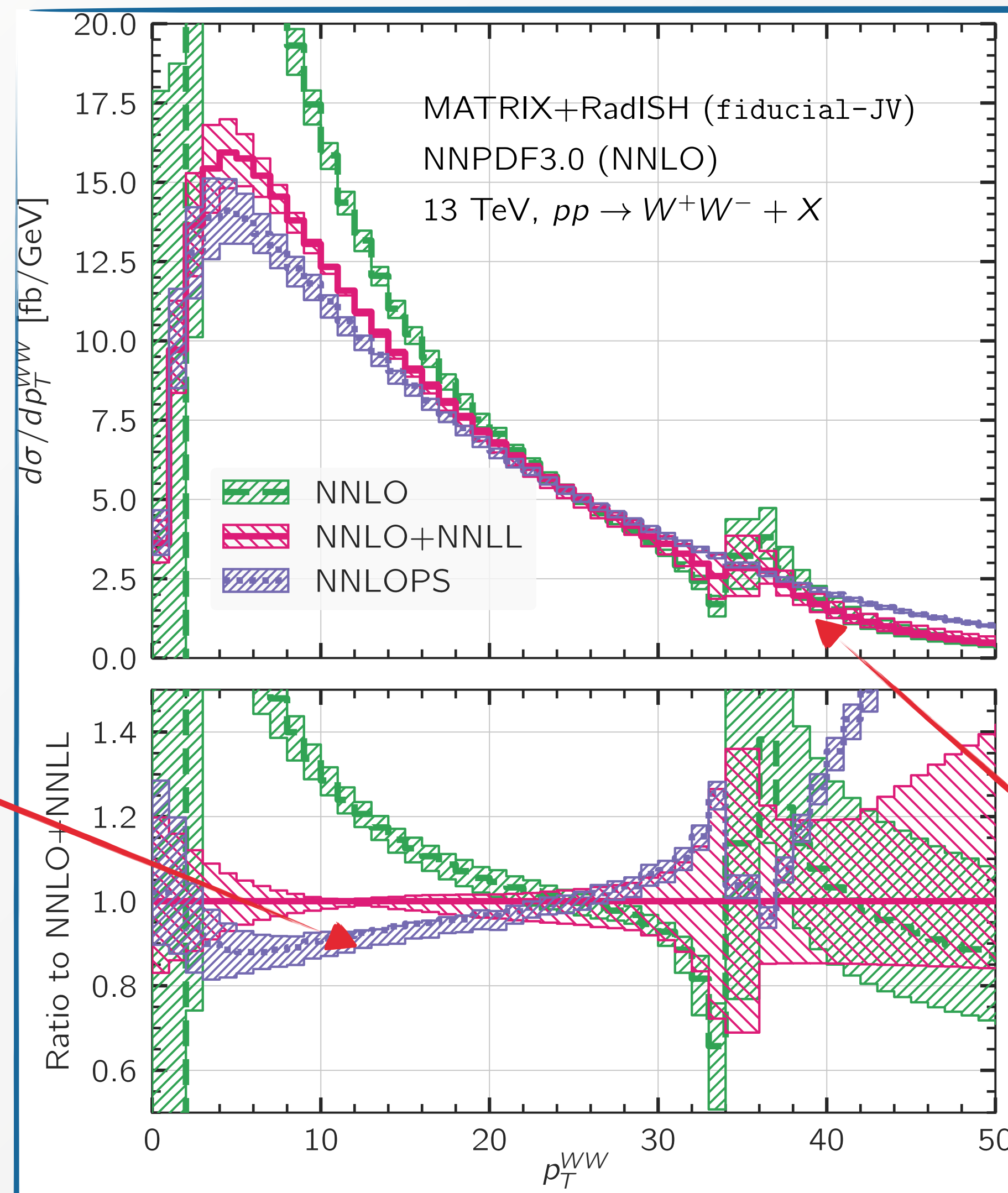
good perturbative convergence to the left of the shoulder  
above, multi-particle configurations play a substantial role

much reduced sensitivity  
to the Sudakov shoulder  
with respect to NLO  
spectrum

# LHC applications: $W^+W^-$ production

**Fiducial region** defined by a rather stringent jet veto

NNLL+NLO spectrum obtained by interfacing RadISH with MATRIX [Grazzini, Kallweit, Rathlev, Wiesemann '15, '17]



**RadISH+MATRIX** interface  
for generic  $2 \rightarrow 1$  and  $2 \rightarrow 2$   
colour singlet processes

[Kallweit, Wiesemann, Re, LR 2004.07720]

[Wiesemann, Re, Zanderighi '18]

Comparison with NNLOPS  
result (much lower log  
accuracy) shows differences  
at the  $\mathcal{O}(10\%)$  level

reduced sensitivity to the  
Sudakov shoulder with  
respect to NLO spectrum

# Summary

- Precision of the data demands an increasing theoretical accuracy at the **multi-differential level** to fully exploit LHC potential
- First **joint resummation** for a **double-differential** kinematic observable involving a **jet algorithm** in hadronic collisions
- Formalism has been extended to **more complex final states**;  $2 \rightarrow 1$  and  $2 \rightarrow 2$  colour singlet processes available via upcoming MATRIX+RadISH interface

# Backup



# All-order structure of the matrix element

$$v = p_t/M$$

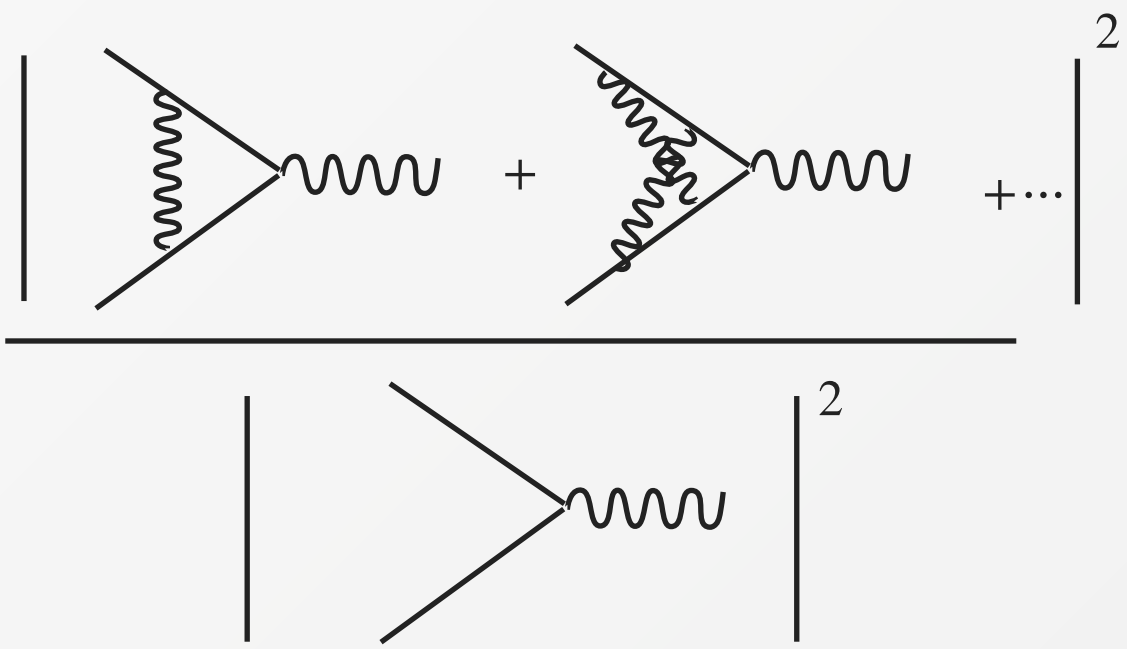
single-particle phase space

matrix element for  $n$  real emissions

$$\Sigma(v) = \int d\Phi_B \tilde{\mathcal{V}}(\Phi_B) \sum_{n=0}^{\infty} \int \prod_{i=1}^n [dk_i] |\mathcal{M}(\Phi_B, k_1, \dots, k_n)|^2 \Theta(v - V(\{\Phi_B\}, k_1, \dots, k_n))$$

all-order form factor  
(virtuals)

e.g. [Dixon, Magnea, Sterman '08]



# Transverse observable resummation with RadISH

1. Establish a **logarithmic counting** for the squared matrix element  $|\mathcal{M}(\Phi_B, k_1, \dots, k_n)|^2$

Decompose the squared amplitude in terms of  **$n$ -particle correlated blocks**, denoted by  $|\tilde{\mathcal{M}}(k_1, \dots, k_n)|^2$   
 ( $|\tilde{\mathcal{M}}(k_1)|^2 = |\mathcal{M}(k_1)|^2$ )

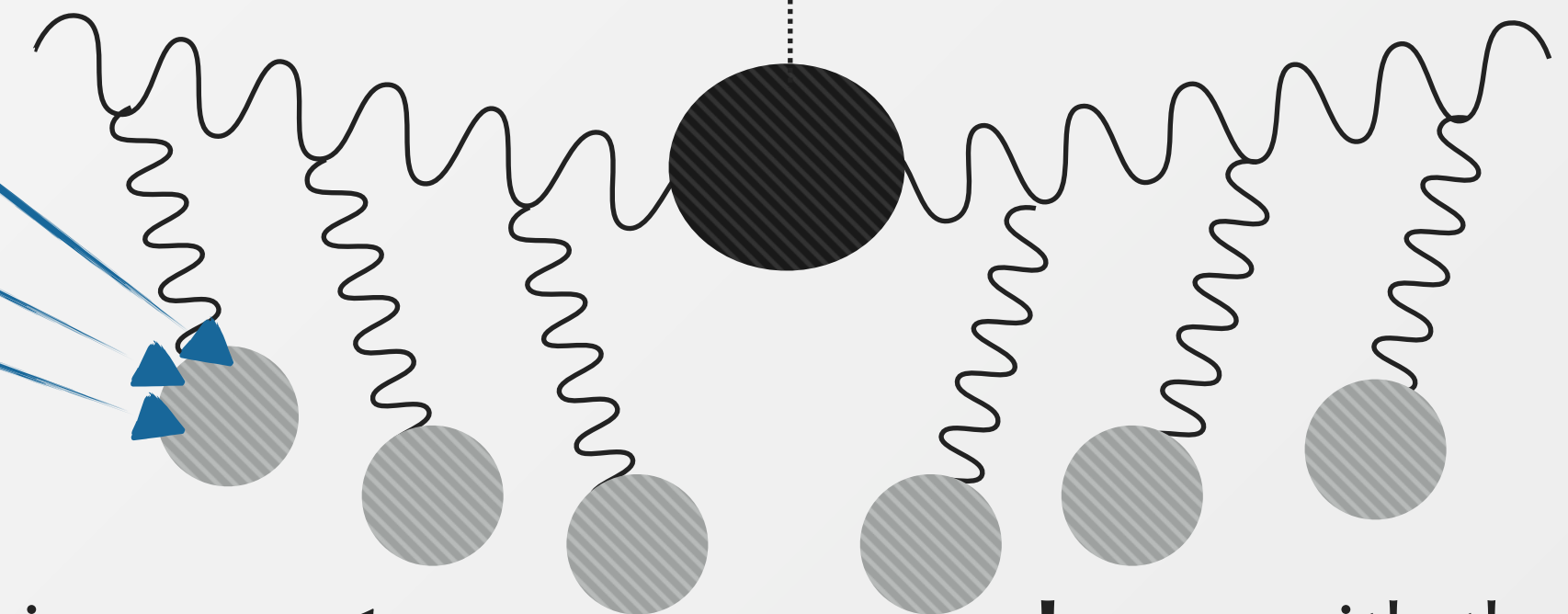
$$\sum_{n=0}^{\infty} |\mathcal{M}(\Phi_B, k_1, \dots, k_n)|^2 = |\mathcal{M}_B(\Phi_B)|^2$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \left\{ \prod_{i=1}^n \left( |\mathcal{M}(k_i)|^2 + \int [dk_a][dk_b] |\tilde{\mathcal{M}}(k_a, k_b)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} - \vec{k}_{ti}) \delta(Y_{ab} - Y_i) \right. \right. \\ \left. \left. + \int [dk_a][dk_b][dk_c] |\tilde{\mathcal{M}}(k_a, k_b, k_c)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} + \vec{k}_{tc} - \vec{k}_{ti}) \delta(Y_{abc} - Y_i) + \dots \right) \right\} \equiv |\mathcal{M}_B(\Phi_B)|^2 \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^n |\mathcal{M}(k_i)|_{\text{inc}}^2$$

\*expression valid for inclusive observables

$$|\tilde{\mathcal{M}}(k_1)|^2 = \frac{|\mathcal{M}(k_1)|^2}{|\mathcal{M}_B|^2} = |\mathcal{M}(k_1)|^2$$

$$|\tilde{\mathcal{M}}(k_1, k_2)|^2 = \frac{|\mathcal{M}(k_1, k_2)|^2}{|\mathcal{M}_B|^2} - \frac{1}{2!} |\mathcal{M}(k_1)|^2 |\mathcal{M}(k_2)|^2$$



Upon integration over the phase space, the expansion can be put in a **one to one correspondence** with the logarithmic structure

**Systematic recipe to include terms up to the desired logarithmic accuracy**

# Resummation in direct space: the $p_t$ case

2. Exploit rIRC safety to single out the IRC singularities of the real matrix element and achieve the **cancellation of the exponentiated divergences** of virtual origin

Introduce a slicing parameter  $\epsilon \ll 1$  such that all inclusive blocks with  $k_{t,i} < \epsilon k_{t,1}$ , with  $k_{t,1}$  hardest emission, can be neglected in the computation of the observable

$$\Sigma(v) = \int d\Phi_B |\mathcal{M}_B(\Phi_B)|^2 \mathcal{V}(\Phi_B) \quad \text{unresolved emissions}$$

$$\times \int [dk_1] |\mathcal{M}(k_1)|_{\text{inc}}^2 \left( \sum_{l=0}^{\infty} \frac{1}{l!} \int \prod_{i=2}^{l+1} [dk_j] |\mathcal{M}(k_j)|_{\text{inc}}^2 \Theta(\epsilon V(k_1) - V(k_j)) \right)$$

$$\times \left( \sum_{m=0}^{\infty} \frac{1}{m!} \int \prod_{i=2}^{m+1} [dk_i] |\mathcal{M}(k_i)|_{\text{inc}}^2 \Theta(V(k_i) - \epsilon V(k_1)) \Theta(v - V(\Phi_B, k_1, \dots, k_{m+1})) \right) \quad \text{resolved emissions}$$

**Unresolved emission** doesn't contribute to the evaluation of the observable: it can be exponentiated directly and employed to cancel the virtual divergences, giving rise to a Sudakov radiator

$$\mathcal{V}(\Phi_B) \exp \left\{ \int [dk] |\mathcal{M}(k)|_{\text{inc}}^2 \Theta(\epsilon V(k_1) - V(k)) \right\} \simeq e^{-R(\epsilon V(k_1))}$$

# Resummation in direct space: the $p_t$ case

Result at NLL accuracy can be written as

$$\Sigma(v) = \sigma^{(0)} \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(\epsilon v_1)} R'(v_1) \quad v_i = V(k_i), \quad \zeta_i = v_i/v_1$$
$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'(\zeta_i v_1) \Theta(v - V(\Phi_B, k_1, \dots, k_{n+1}))$$

Formula can be evaluated with Monte Carlo method; dependence on  $\epsilon$  vanishes exactly and result is finite in four dimensions

It contains **subleading effect** which in the original CAESAR approach are disposed of by expanding  $R$  and  $R'$  around  $v$

~~$$R(\epsilon v_1) = R(v) + \frac{dR(v)}{d \ln(1/v)} \ln \frac{v}{\epsilon v_1} + \mathcal{O}\left(\ln^2 \frac{v}{\epsilon v_1}\right)$$
$$R'(v_i) = R'(v) + \mathcal{O}\left(\ln \frac{v}{v_i}\right)$$~~

**Not possible!** valid only if the ratio  $v_i/v$  remains of order one in the whole emission phase space, but for observables which feature kinematic cancellations there are configurations with  $v_i \gg v$ . **Subleading effects necessary**

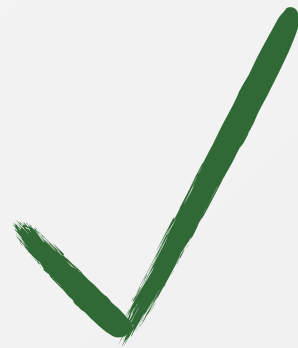
# Resummation in direct space: the $p_t$ case

Result at NLL accuracy can be written as

$$\Sigma(v) = \sigma^{(0)} \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(\epsilon v_1)} R'(v_1) \quad v_i = V(k_i), \quad \zeta_i = v_i/v_1$$
$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'(\zeta_i v_1) \Theta(v - V(\Phi_B, k_1, \dots, k_{n+1}))$$

Formula can be evaluated with Monte Carlo method; dependence on  $\epsilon$  vanishes exactly and result is finite in four dimensions

Convenient to perform an expansion around  $k_{t1}$  (more efficient and simpler implementation)

$$R(\epsilon k_{t1}) = R(k_{t1}) + \frac{dR(k_{t1})}{d \ln(1/k_{t1})} \ln \frac{1}{\epsilon} + \mathcal{O}\left(\ln^2 \frac{1}{\epsilon}\right)$$
$$R'(k_{ti}) = R'(k_{t1}) + \mathcal{O}\left(\ln \frac{k_{t1}}{k_{ti}}\right)$$


**Subleading effects retained:** no divergence at small  $v$ , power-like behaviour respected

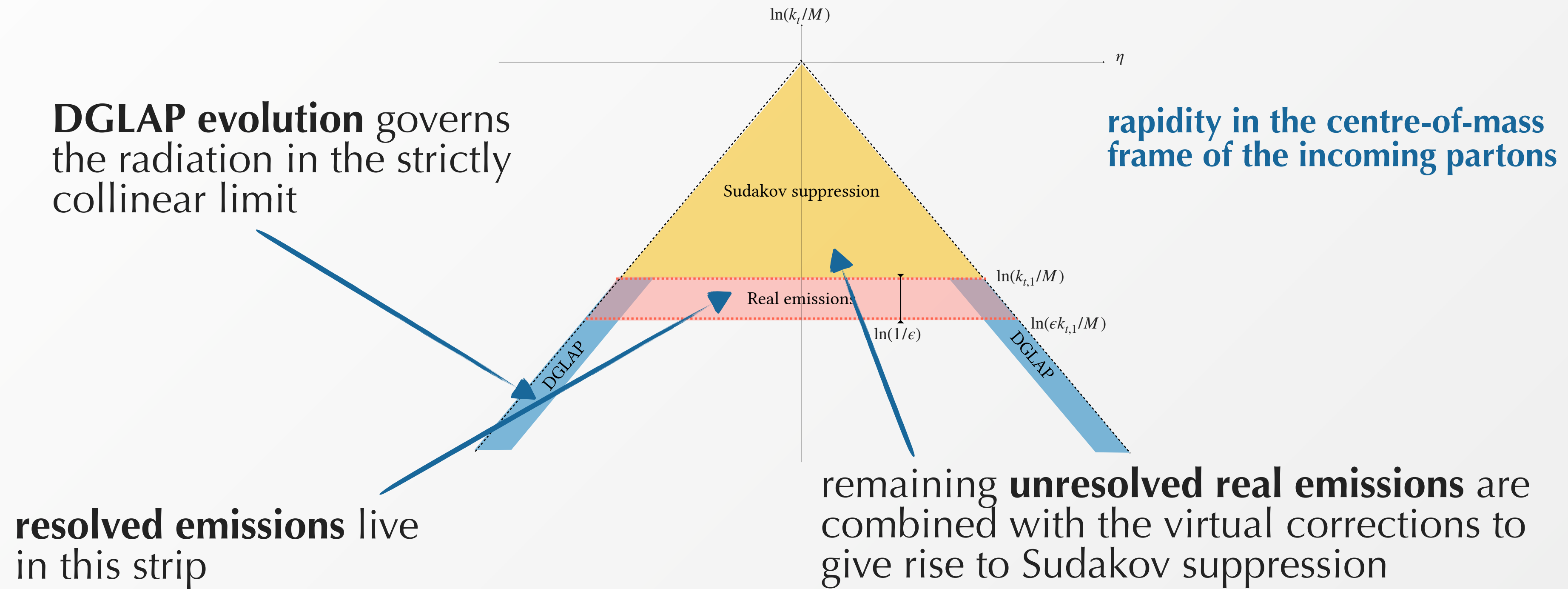
**Logarithmic accuracy** defined in terms of  $\ln(M/k_{t1})$

Result formally equivalent to the  $b$ -space formulation

# Parton luminosities

Consider configurations in which emissions are ordered in  $k_{t,i}$ ,  $k_{t,1}$  hardest emission

Phase space for each secondary emission can be depicted in the Lund diagram



- DGLAP evolution can be performed **inclusively** up to  $\epsilon k_{t,1}$  thanks to rIRC safety
- In the **overlapping region** hard-collinear emissions modify the observable's value: the evolution should be performed exclusively (unintegrated in  $k_t$ )
- At NLL the real radiation can be approximated with its soft limit: DGLAP can be performed inclusively up to  $k_{t,1}$  (i.e. one can evaluate  $\mu_F = k_{t,1}$ )

# Beyond NLL

Extension to NNLL and beyond requires the systematic inclusion of the correlated blocks necessary to achieve the desired logarithmic accuracy

Moreover, one needs to **relax a series of assumptions** which give rise to subleading corrections neglected at NLL (for instance, exact rapidity bounds). These corrections can be included systematically by including additional terms in the expansion

$$R(\epsilon v_1) = R(v_1) + \frac{dR(v_1)}{d \ln(1/v_1)} \ln \frac{1}{\epsilon} + \mathcal{O} \left( \ln^2 \frac{1}{\epsilon} \right)$$

Finally, one needs to specify a complete treatment for **hard-collinear radiation**. Starting at NNLL one or more real emissions can be hard and collinear to the emitting leg, and the available phase space for subsequent real emissions changes

Two classes of contributions:

- one soft by construction and which is analogous to the  $R'$  contribution

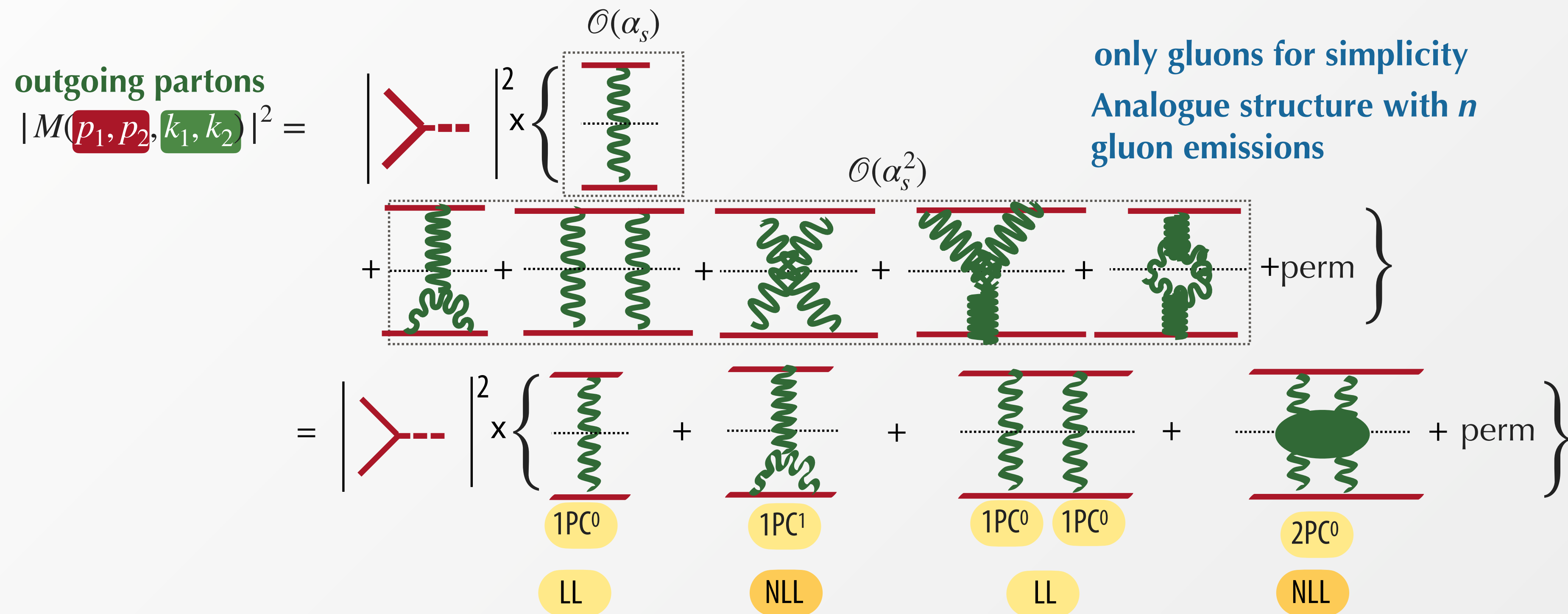
$$R'(v_i) = R'(v_1) + \mathcal{O} \left( \ln \frac{v_1}{v_i} \right)$$

- another hard and collinear (exclusive DGLAP step): last step of DGLAP evolution must be performed unintegrated in  $k_t$

# Logarithmic counting

Necessary to establish a **well defined logarithmic counting**: possible to do that by decomposing the squared amplitude in terms of n-particle correlated blocks (**nPC**)

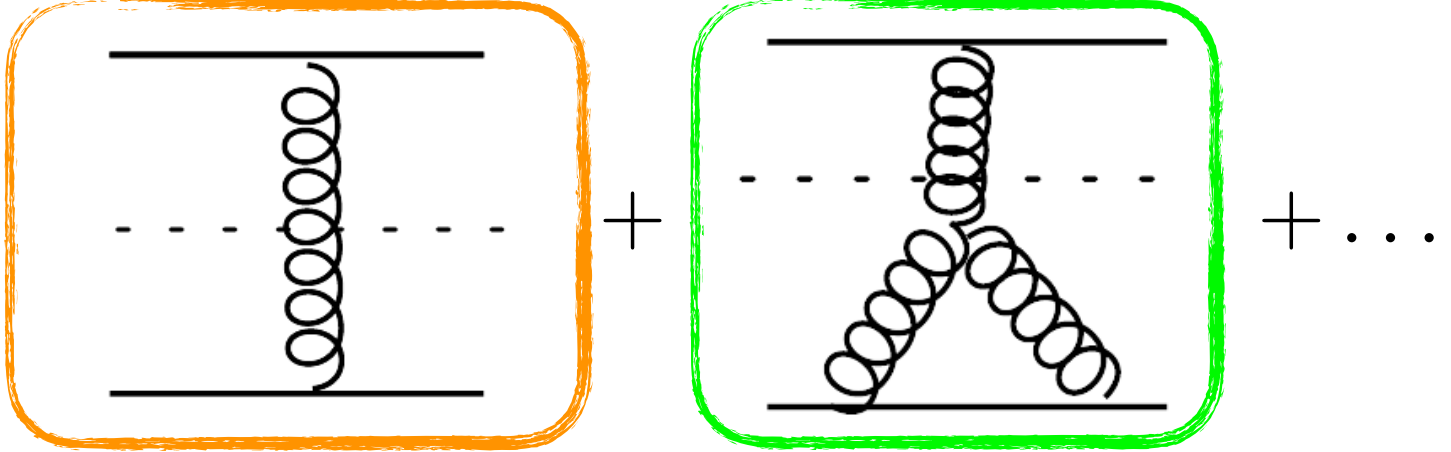
e.g.  $pp \rightarrow H +$  emission of up to 2 (soft) gluons  $O(\alpha_s^2)$



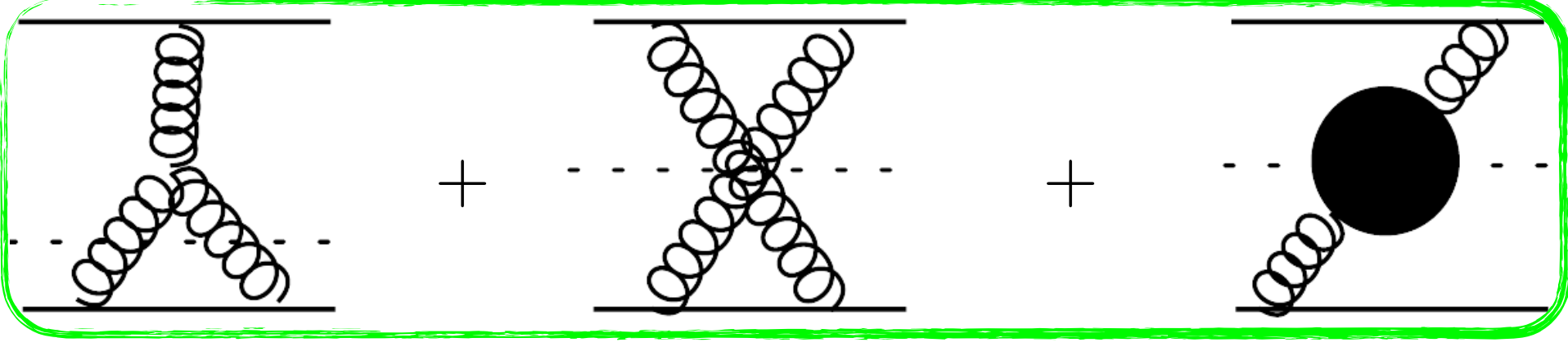
Logarithmic counting defined in terms of **nPC blocks** (owing to rIRC safety of the observable)



# Logarithmic counting: correlated blocks

$$|\tilde{M}(k_a)|^2 = \frac{|M(\tilde{p}_1, \tilde{p}_2, k_a)|^2}{|M_B(\tilde{p}_1, \tilde{p}_2)|^2} = |M(k_a)|^2 \longrightarrow$$


$$\underbrace{\hspace{10em}}_{\alpha_s L^2} + \dots$$

$$|\tilde{M}(k_a, k_b)|^2 = \frac{|M(\tilde{p}_1, \tilde{p}_2, k_a, k_b)|^2}{|M_B(\tilde{p}_1, \tilde{p}_2)|^2} - \frac{1}{2!} \underbrace{|M(k_a)|^2 |M(k_b)|^2}_{\alpha_s^2 L^4} \longrightarrow$$


$$\underbrace{\hspace{10em}}_{\alpha_s^2 L^3 + \alpha_s^2 L^2} + \dots$$

15 this LL is absorbed in the resummation of  $|M(k)|^2$

Thanks to P. Monni

# Resummation at NLL accuracy

Final result at NLL

$$\begin{aligned} \frac{d\Sigma(v)}{d\Phi_B} &= \int \frac{dk_{t,1}}{k_{t,1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(k_{t,1})} \epsilon^{R'(k_{t,1})} \mathcal{L}_{\text{NLL}}(k_{t,1}) R'(k_{t,1}) \\ &\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'(\zeta_i k_{t,1}) \Theta(v - V(\Phi_B, k_1, \dots, k_{n+1})) \end{aligned}$$

This formula can be evaluated by means of fast Monte Carlo methods **RadISH** (**R**adiation off **I**nitial **S**tate **H**adrons)

Parton luminosity at NLL reads

$$\mathcal{L}_{\text{NLL}}(k_{t,1}) = \sum_c \frac{d|M_B|_{c\bar{c}}^2}{d\Phi_B} f_c(x_1, k_{t,1}^2) f_{\bar{c}}(x_2, k_{t,1}^2)$$

At higher logarithmic accuracy, it includes **coefficient functions** and **hard-virtual** corrections

# Result at N<sup>3</sup>LL accuracy

$$\begin{aligned}
\frac{d\Sigma(v)}{d\Phi_B} &= \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \partial_L \left( -e^{-R(k_{t1})} \mathcal{L}_{N^3LL}(k_{t1}) \right) \int d\mathcal{Z}[\{R', k_i\}] \Theta(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1})) \\
&+ \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(k_{t1})} \int d\mathcal{Z}[\{R', k_i\}] \int_0^1 \frac{d\zeta_s}{\zeta_s} \frac{d\phi_s}{2\pi} \left\{ \left( R'(k_{t1}) \mathcal{L}_{NNLL}(k_{t1}) - \partial_L \mathcal{L}_{NNLL}(k_{t1}) \right) \right. \\
&\times \left( R''(k_{t1}) \ln \frac{1}{\zeta_s} + \frac{1}{2} R'''(k_{t1}) \ln^2 \frac{1}{\zeta_s} \right) - R'(k_{t1}) \left( \partial_L \mathcal{L}_{NNLL}(k_{t1}) - 2 \frac{\beta_0}{\pi} \alpha_s^2(k_{t1}) \hat{P}^{(0)} \otimes \mathcal{L}_{NLL}(k_{t1}) \ln \frac{1}{\zeta_s} \right) \\
&\left. + \frac{\alpha_s^2(k_{t1})}{\pi^2} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathcal{L}_{NLL}(k_{t1}) \right\} \left\{ \Theta(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1}, k_s)) - \Theta(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1})) \right\} \\
&+ \frac{1}{2} \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(k_{t1})} \int d\mathcal{Z}[\{R', k_i\}] \int_0^1 \frac{d\zeta_{s1}}{\zeta_{s1}} \frac{d\phi_{s1}}{2\pi} \int_0^1 \frac{d\zeta_{s2}}{\zeta_{s2}} \frac{d\phi_{s2}}{2\pi} R'(k_{t1}) \\
&\times \left\{ \mathcal{L}_{NLL}(k_{t1}) (R''(k_{t1}))^2 \ln \frac{1}{\zeta_{s1}} \ln \frac{1}{\zeta_{s2}} - \partial_L \mathcal{L}_{NLL}(k_{t1}) R''(k_{t1}) \left( \ln \frac{1}{\zeta_{s1}} + \ln \frac{1}{\zeta_{s2}} \right) \right. \\
&\left. + \frac{\alpha_s^2(k_{t1})}{\pi^2} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathcal{L}_{NLL}(k_{t1}) \right\} \\
&\times \left\{ \Theta(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1}, k_{s1}, k_{s2})) - \Theta(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1}, k_{s1})) - \right. \\
&\left. \Theta(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1}, k_{s2})) + \Theta(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1})) \right\} + \mathcal{O} \left( \alpha_s^n \ln^{2n-6} \frac{1}{v} \right), \quad (3.18)
\end{aligned}$$

[Bizon, Monni, Re, LR, Torrielli '17]

All ingredients to perform resummation at **N<sup>3</sup>LL accuracy** are now available

[Catani *et al.* '11, '12][Gehrmann *et al.* '14][Li, Zhu '16, Vladimirov '16][Moch *et al.* '18, Lee *et al.* '19]

Fixed-order predictions now available at **NNLO**

[A. Gehrmann-De Ridder *et al.* '15, 16, '17][Boughezal *et al.* '15, 16]

Online LHCP 2020, Paris, 29 May 2020

# Matching with fixed order

Multiplicative matching performed at the **double-cumulant level**

fixed-order double-cumulative result at NNLO

double-cumulative result at NNLL

$$\sigma_{\text{NNLO}}(p_{\perp}^H < p_{\perp}^{H,v}, p_{\perp}^J < p_{\perp}^{J,v}) = \sigma_{\text{NNLO}} - \int \Theta(p_{\perp}^H > p_{\perp}^{H,v}) \vee \Theta(p_{\perp}^J > p_{\perp}^{J,v}) d\sigma_{H+J,\text{NLO}}$$

$$\sigma_{\text{match}}(p_{\perp}^H < p_{\perp}^{H,v}, p_{\perp}^J < p_{\perp}^{J,v}) = \frac{\sigma_{\text{NNLL}}(p_{\perp}^H < p_{\perp}^{H,v}, p_{\perp}^J < p_{\perp}^{J,v})}{\sigma_{\text{NNLL}}(\{p_{\perp}^{J,v}, p_{\perp}^{H,v}\} \rightarrow \infty)} \left[ \sigma_{\text{NNLL}}(\{p_{\perp}^{J,v}, p_{\perp}^{H,v}\} \rightarrow \infty) \frac{\sigma_{\text{NNLO}}(p_{\perp}^H < p_{\perp}^{H,v}, p_{\perp}^J < p_{\perp}^{J,v})}{\sigma_{\text{NNLL,exp}}(p_{\perp}^H < p_{\perp}^{H,v}, p_{\perp}^J < p_{\perp}^{J,v})} \right]_{\mathcal{O}(\alpha_s^2)}$$

asymptotic limit of the NNLL result

expansion of the double-cumulative result at NNLL

- NNLL+NNLO result for  $p_{\perp}^{J,v}$  recovered for  $p_{\perp}^{H,v} \rightarrow \infty$
- **NNLO constant** included through multiplicative matching (NNLL' accuracy)

# Matching to fixed order: multiplicative matching

Cumulative cross section should reduce to the fixed order at large  $v$

$$\Sigma_{\text{matched}}^{\text{mult}}(v) \sim \Sigma_{\text{res}}(v) \left[ \frac{\Sigma_{\text{f.o.}}(v)}{\Sigma_{\text{res}}(v)} \right]_{\text{expanded}}$$

$$\Sigma_{\text{f.o.}}(v) = \sigma_{\text{f.o.}} - \int_v^\infty \frac{d\sigma}{dv} dv$$

- allows to include constant terms from NNLO (if N<sup>3</sup>LO total xs available)
- physical suppression at small  $v$  cures potential instabilities

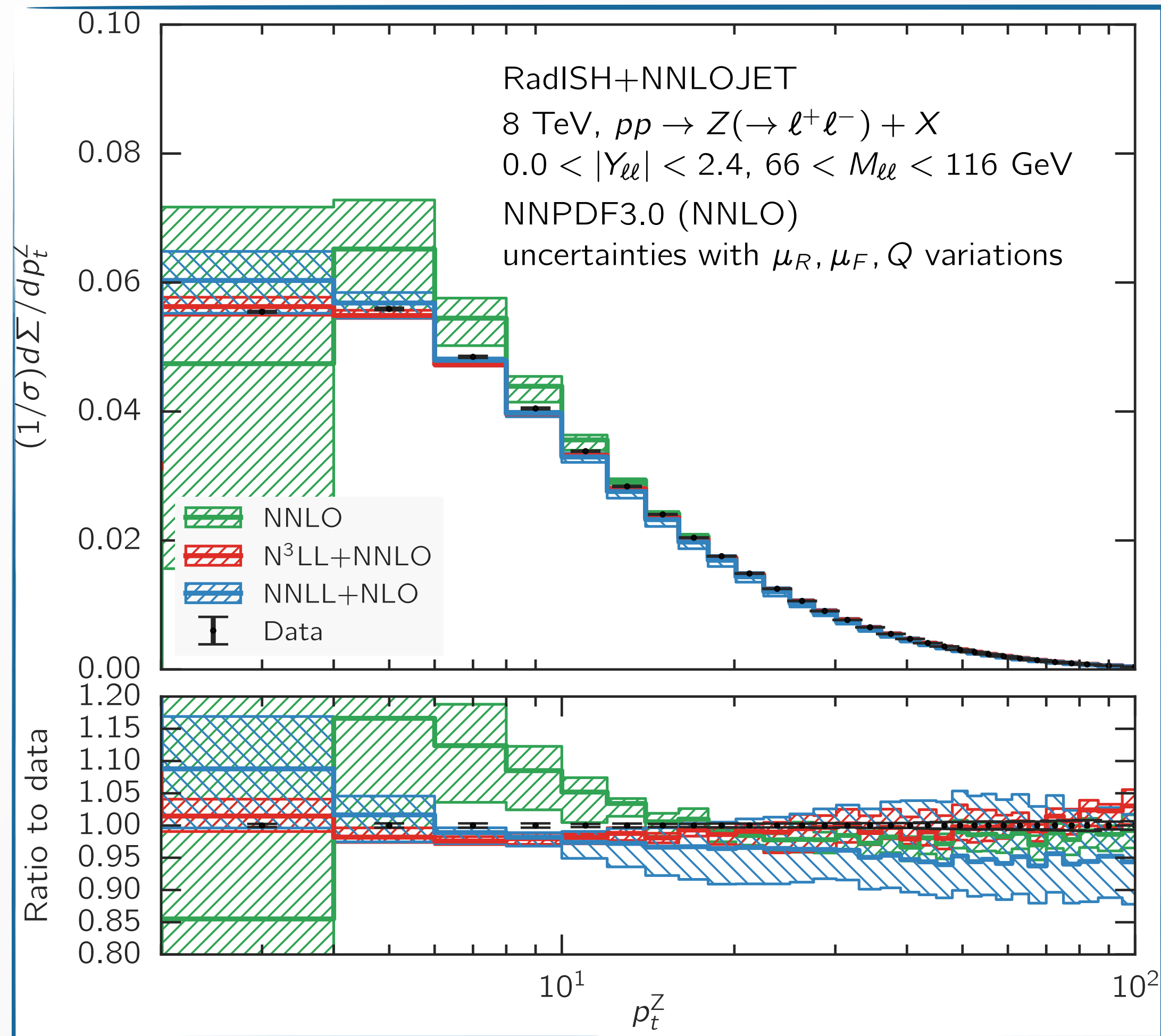
To ensure that resummation does not affect the hard region of the spectrum when the matching is performed we introduce **modified logarithms**

This corresponds to restrict the rapidity phase space at large  $k_t$   $\int_{-\ln Q/k_{t,i}}^{\ln Q/k_{t,i}} d\eta \rightarrow \int_{-\ln Q/k_{t,1}}^{\ln Q/k_{t,1}} d\eta \rightarrow \int_{-\epsilon}^{\epsilon} d\eta \rightarrow 0$

$$\ln(Q/k_{t1}) \rightarrow \frac{1}{p} \ln \left( 1 + \left( \frac{Q}{k_{t1}} \right)^p \right)$$

$Q$  : **perturbative resummation scale**  
used to probe the size of subleading logarithmic corrections  
 $p$  : arbitrary matching parameter

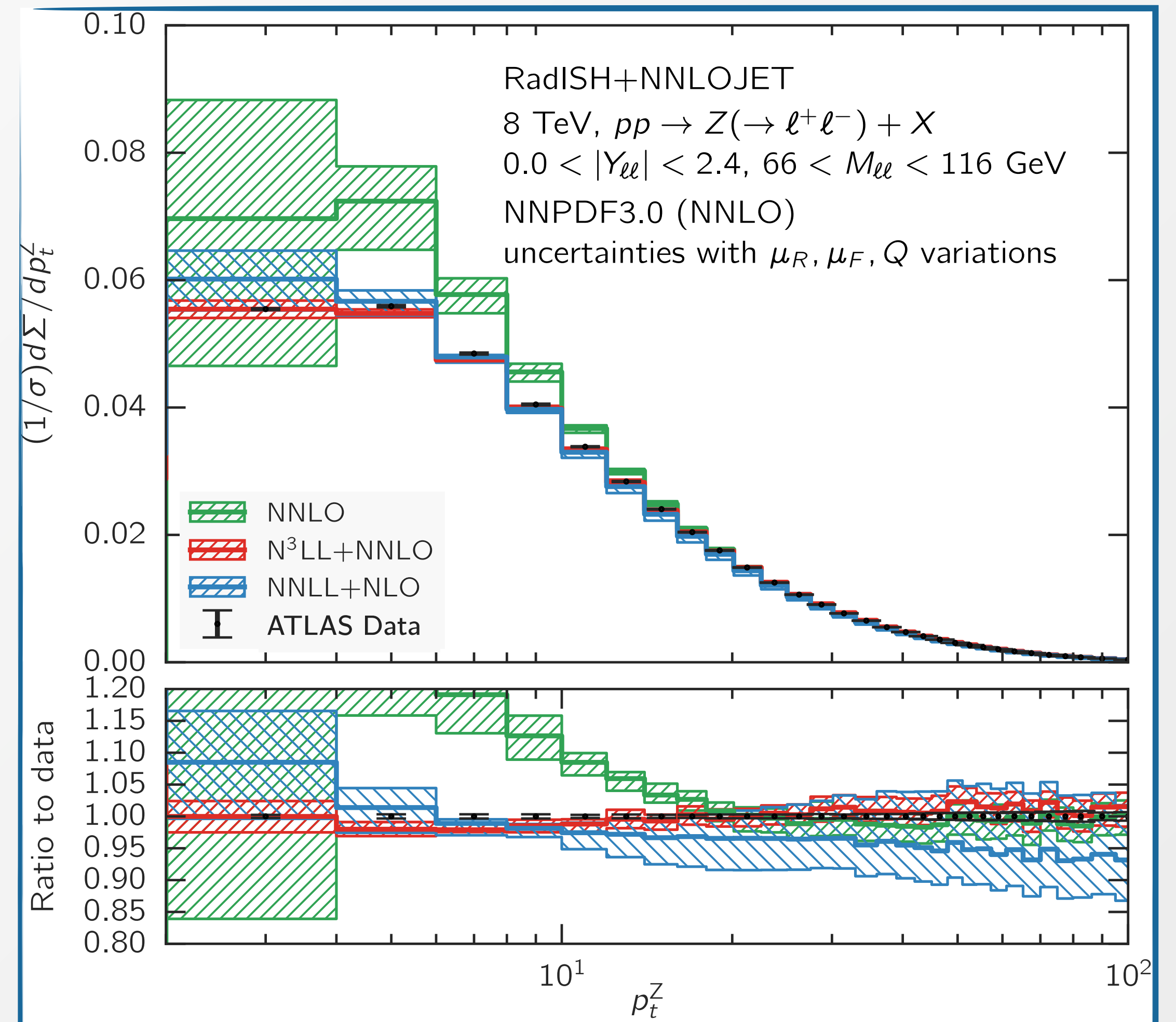
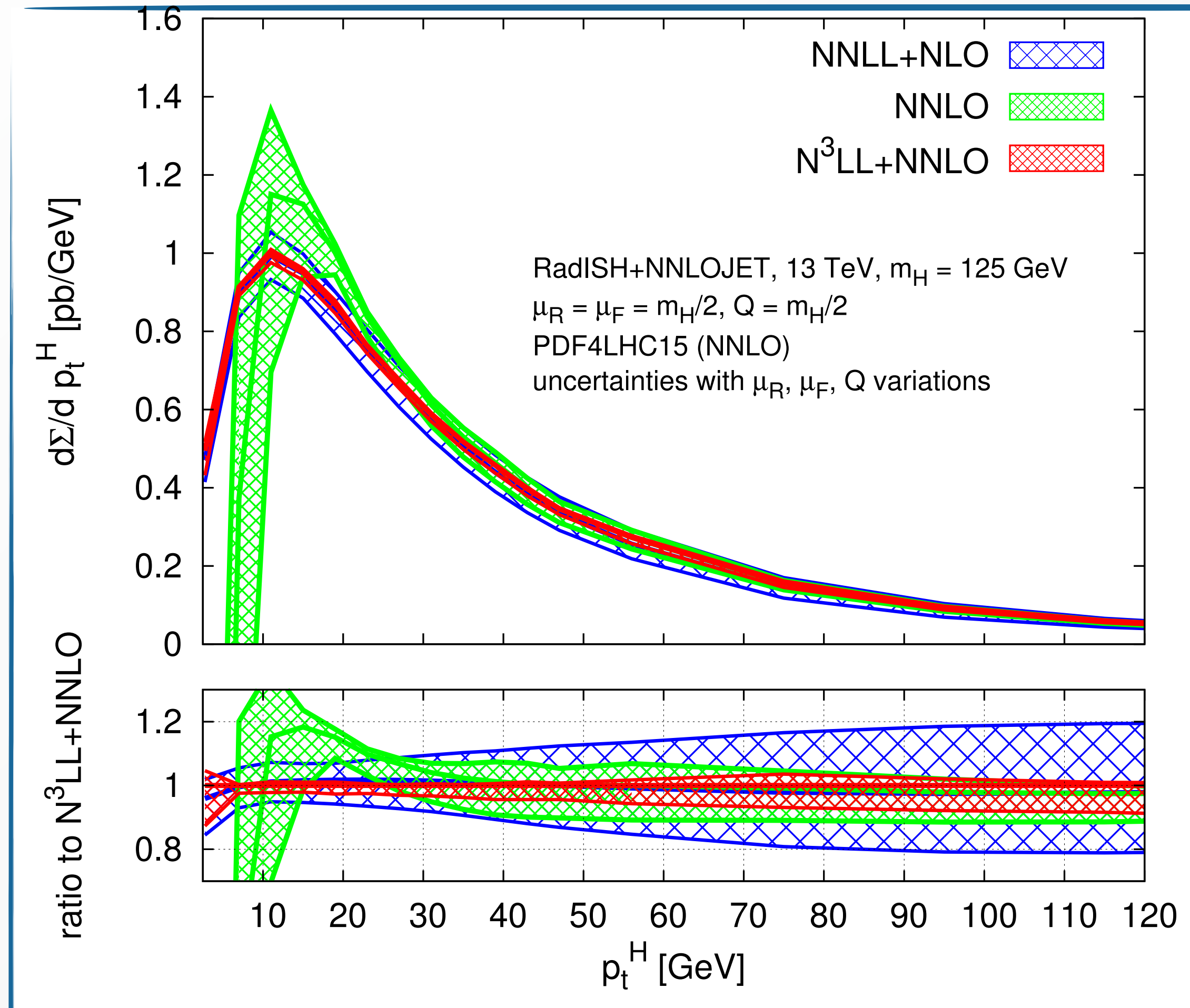
# Predictions for the $Z$ spectrum at 8 TeV



- Good description of the data in all fiducial regions
- Perturbative uncertainty at the few percent level, still does not match the precision of the ATLAS data

# Resummation of the transverse momentum spectrum at $N^3LL+NNLO$

$N^3LL$  result matched to NNLO  $H+j$ ,  $Z+j$ ,  $W^\pm+j$  [Bizon, LR *et al.* '18, '19]



# Theoretical predictions for $Z$ and $W$ observables at 13 TeV

Bizon, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Monni, Re, LR, Walker, 190x.xxxx

Results obtained using the following fiducial cuts (agreed with ATLAS)

$$p_t^{\ell^\pm} > 25 \text{ GeV}, \quad |\eta^{\ell^\pm}| < 2.5, \quad 66 \text{ GeV} < M_{\ell\ell} < 116 \text{ GeV}$$
$$p_t^\ell > 25 \text{ GeV}, \quad |\eta^\ell| < 2.5, \quad \cancel{E}_T^{\nu_e} > 25 \text{ GeV}, \quad m_T > 50 \text{ GeV}$$

using NNPDF3.1 with  $\alpha_s(M_Z)=0.118$  and setting the central scales to

$$\mu_R = \mu_F = M_T = \sqrt{M_{\ell\ell'}^2 + p_T^2}, \quad Q = \frac{M_{\ell\ell'}}{2}$$

5 flavour (massless) scheme: no HQ effects, LHAPDF PDF thresholds

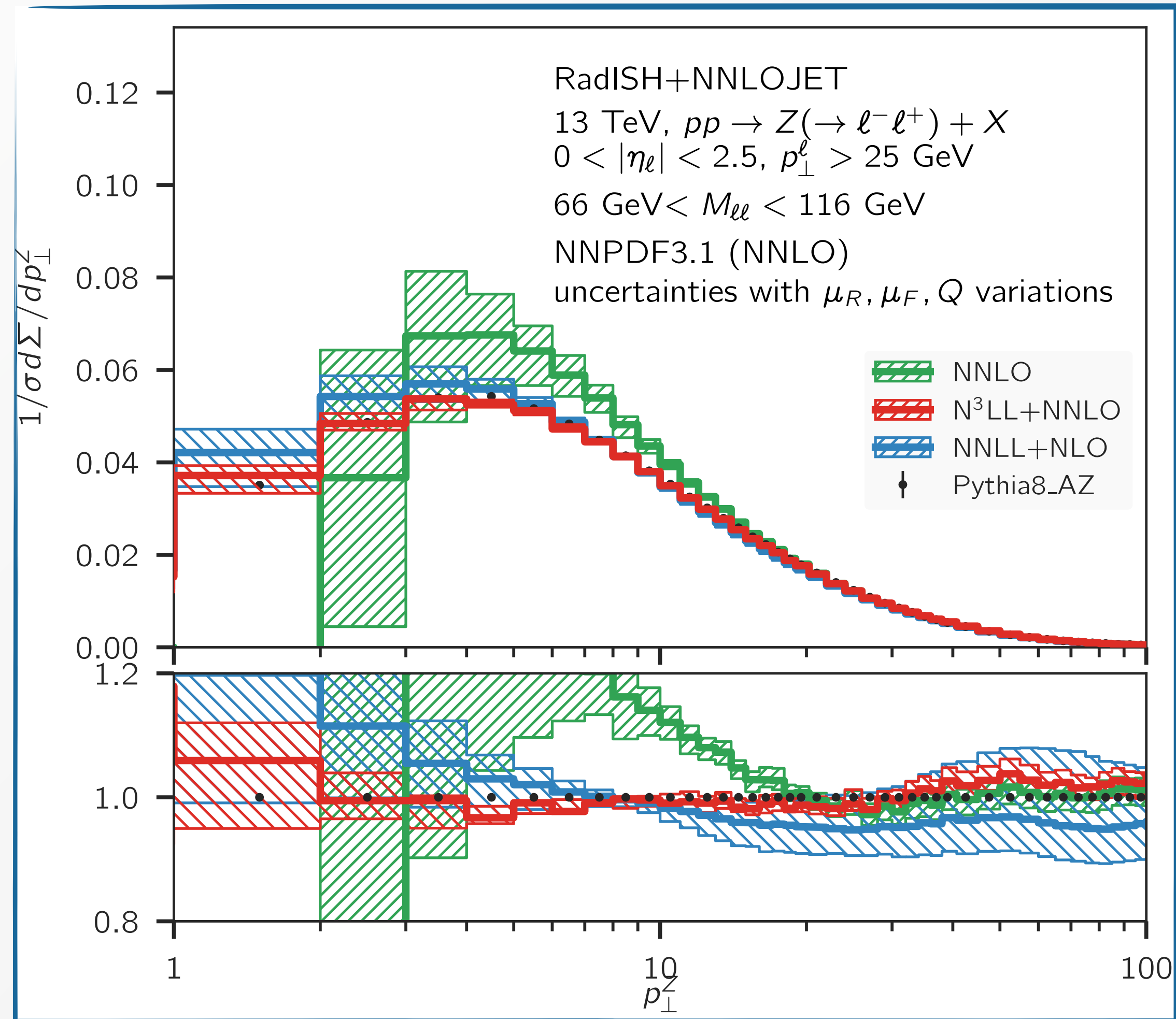
Scale uncertainties estimated by varying **renormalization** and **factorization** scale by a factor of two around their central value (**7 point variation**) and varying the **resummation** scale by a factor of 2 around its central value for factorization and renormalization scales set to their central value: **9 point envelope**

Matching parameter  $p$  set to 4 as a default

**No non perturbative parameters included** in the following

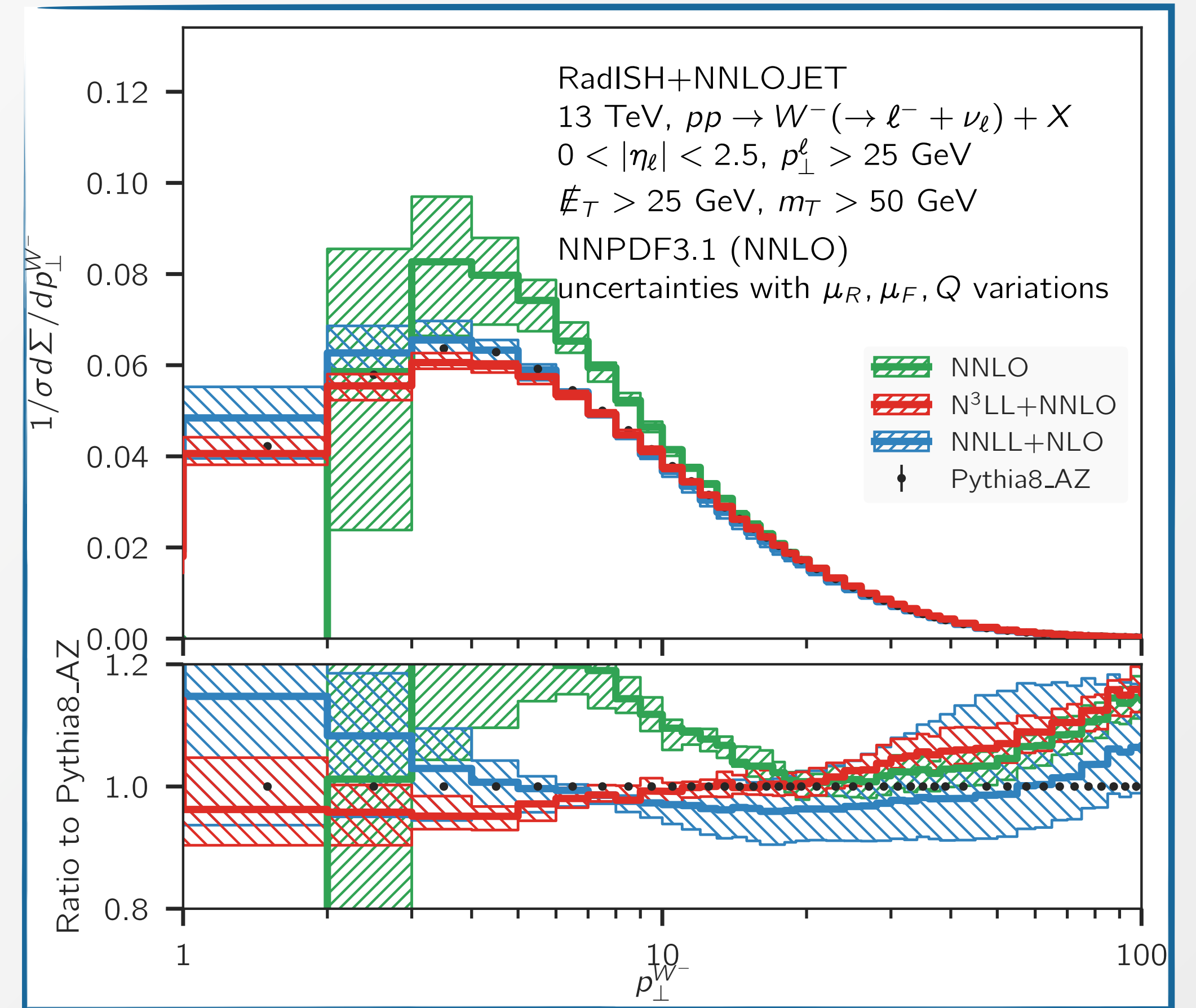
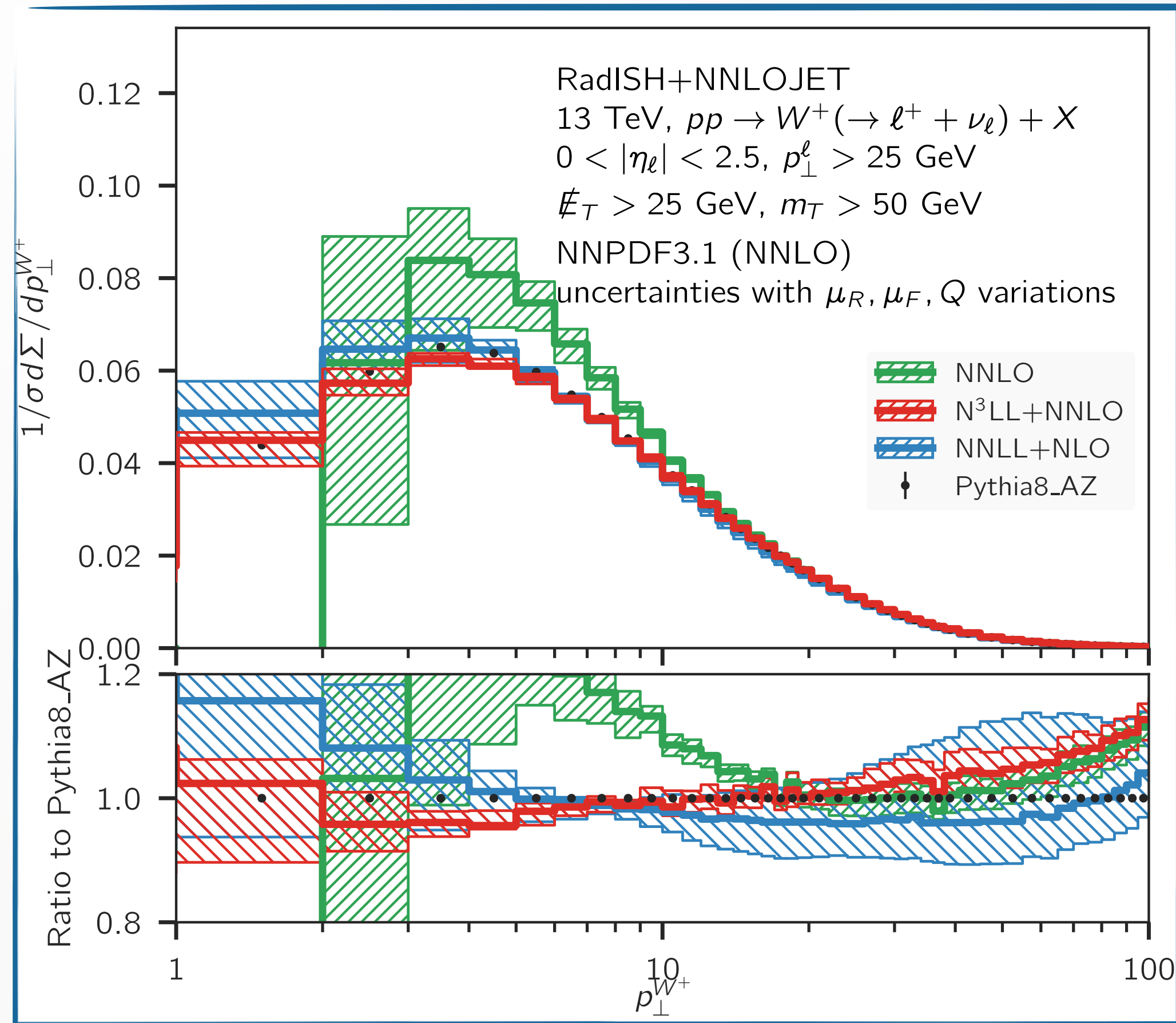


# Predictions for the $Z$ spectrum



*Thanks to Jan Kretzschmar for providing the  
PYTHIA8 AZ tune results*

# Predictions for the $W^+$ and $W^-$ spectra



# Ratio of differential distributions

Z and W production share a similar pattern of QCD radiative corrections

Crucial to understand correlation between Z and W spectra to exploit data-driven predictions

$$\frac{1}{\sigma^W} \frac{d\sigma^W}{p_{\perp}^W} \sim \frac{1}{\sigma_{\text{data}}^Z} \frac{d\sigma_{\text{data}}^Z}{p_{\perp}^Z} \frac{\frac{1}{\sigma_{\text{theory}}^W} \frac{d\sigma_{\text{theory}}^W}{p_{\perp}^W}}{\frac{1}{\sigma_{\text{theory}}^Z} \frac{d\sigma_{\text{theory}}^Z}{p_{\perp}^Z}}$$

Several choices are possible:

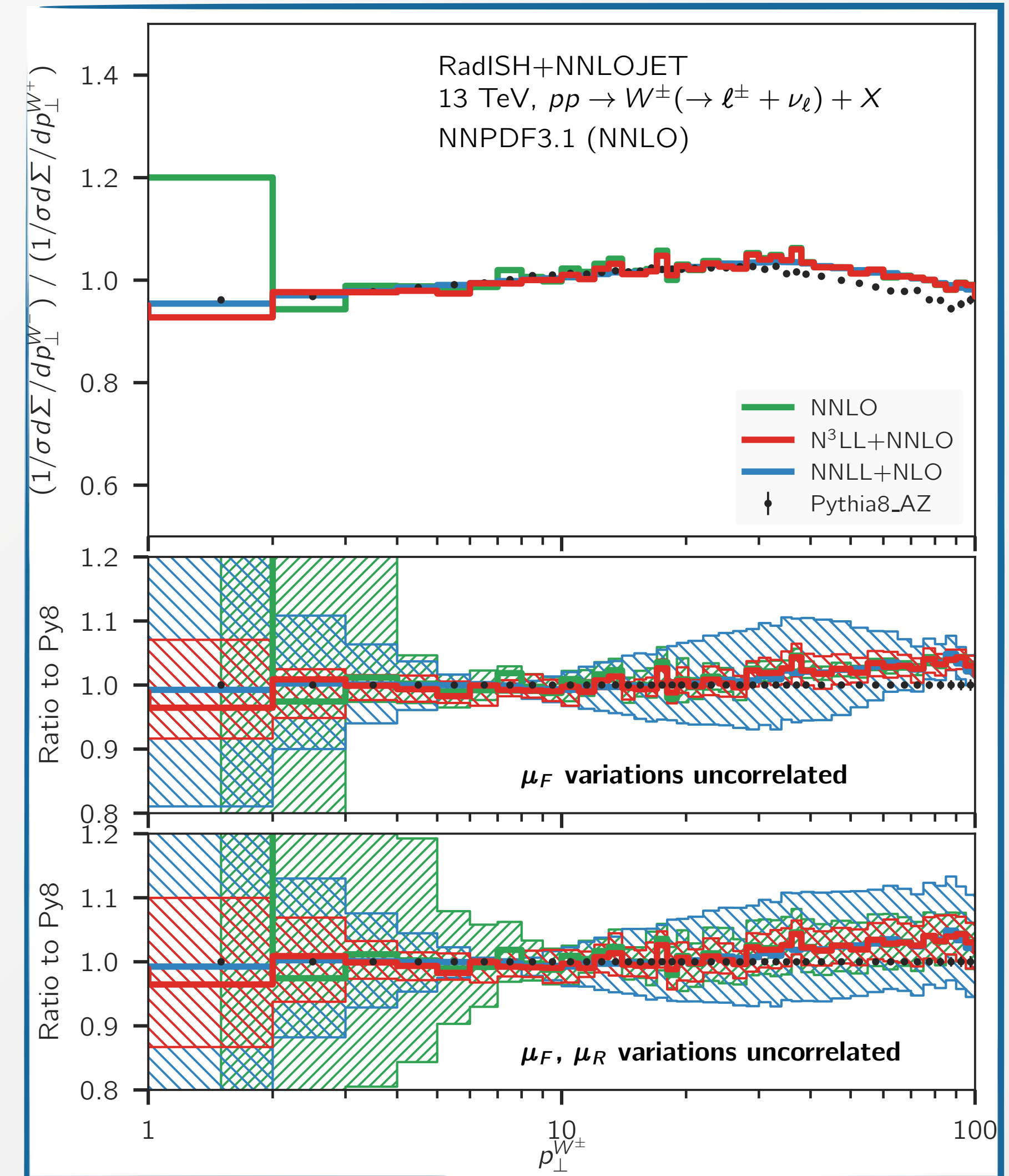
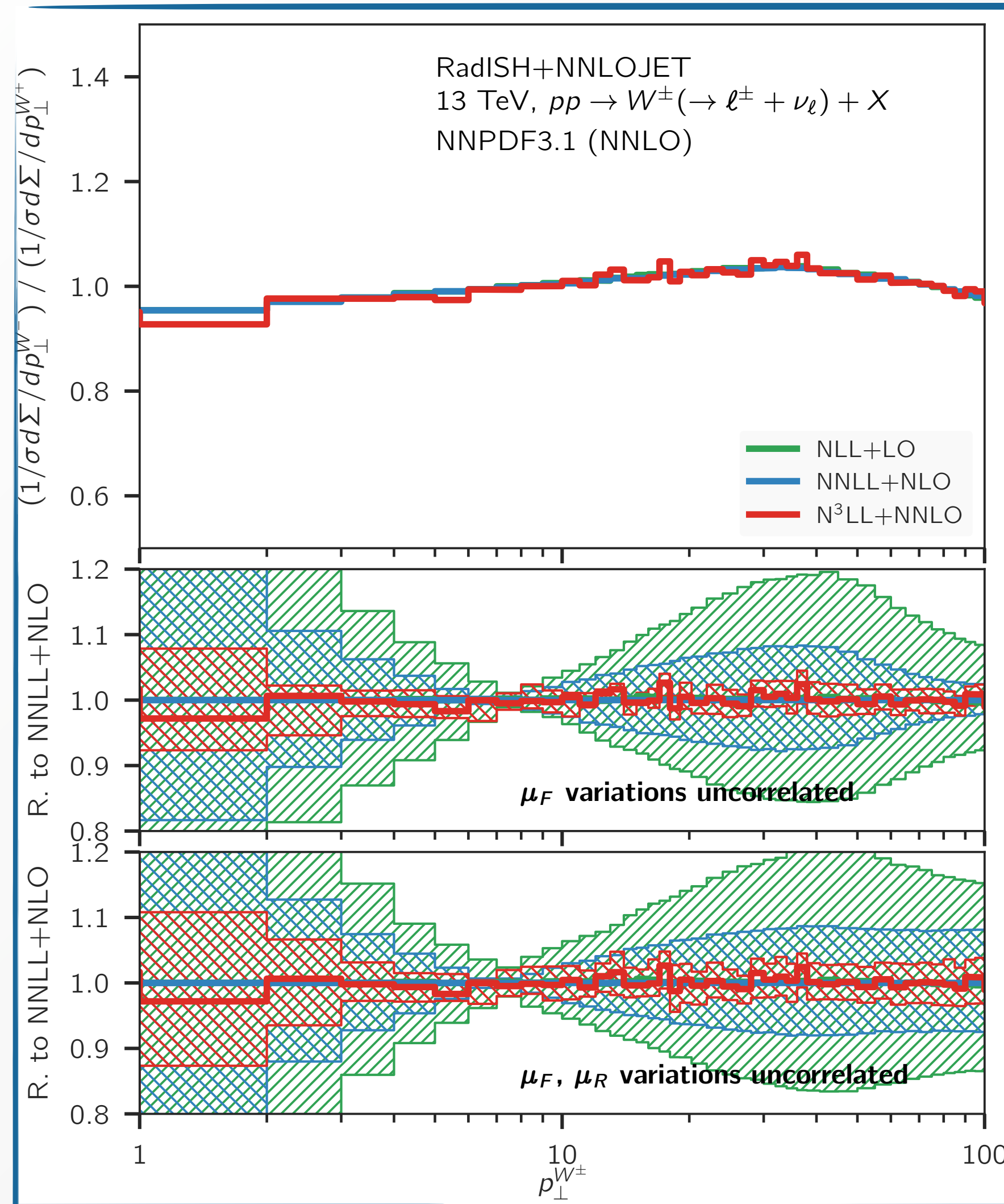
- **Correlate resummation and renormalisation** scale variations, keep **factorisation scale uncorrelated**, while keeping

$$\frac{1}{2} \leq \frac{\mu_{\text{F}}^{\text{num}}}{\mu_{\text{F}}^{\text{den}}} \leq 2$$

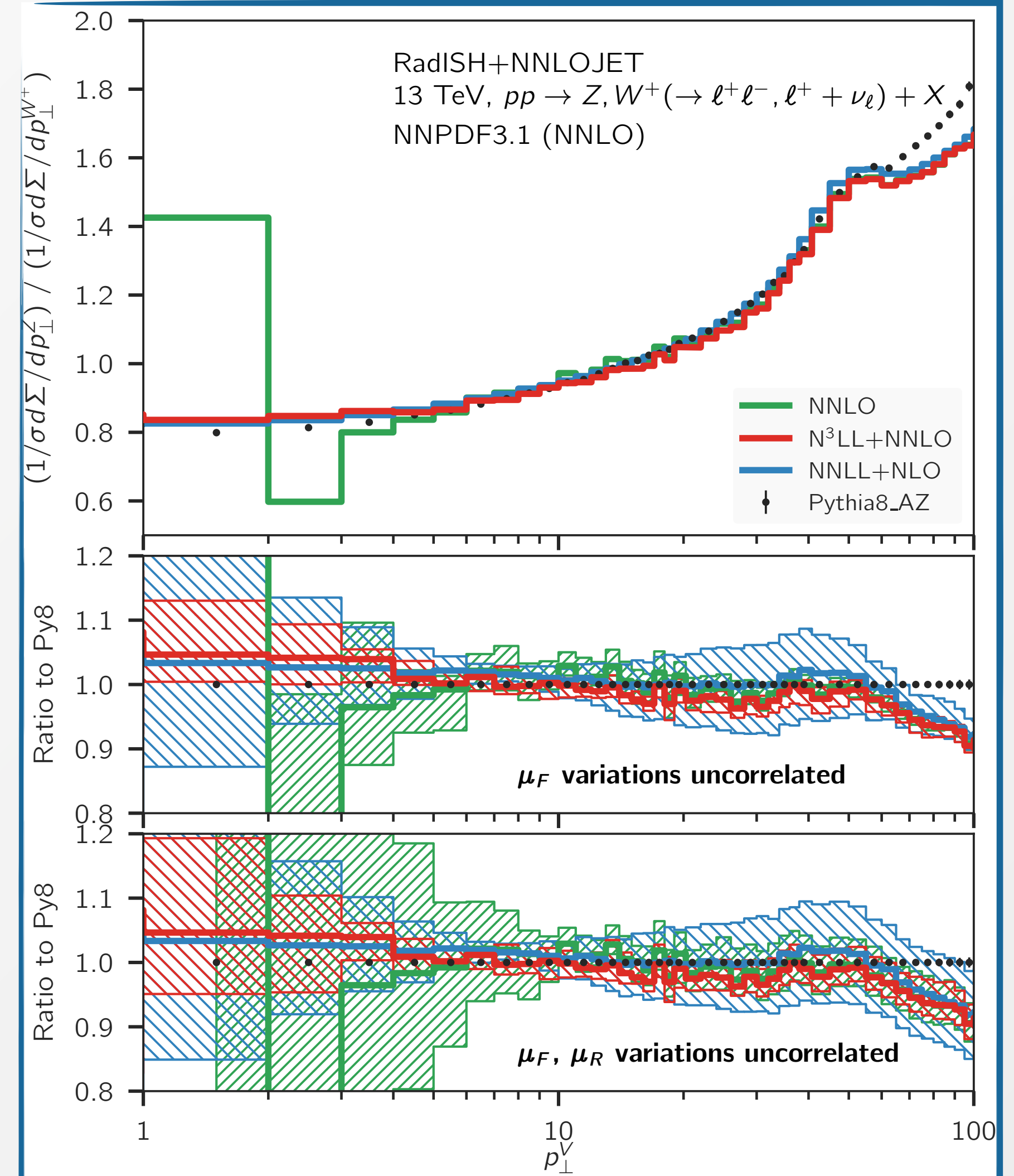
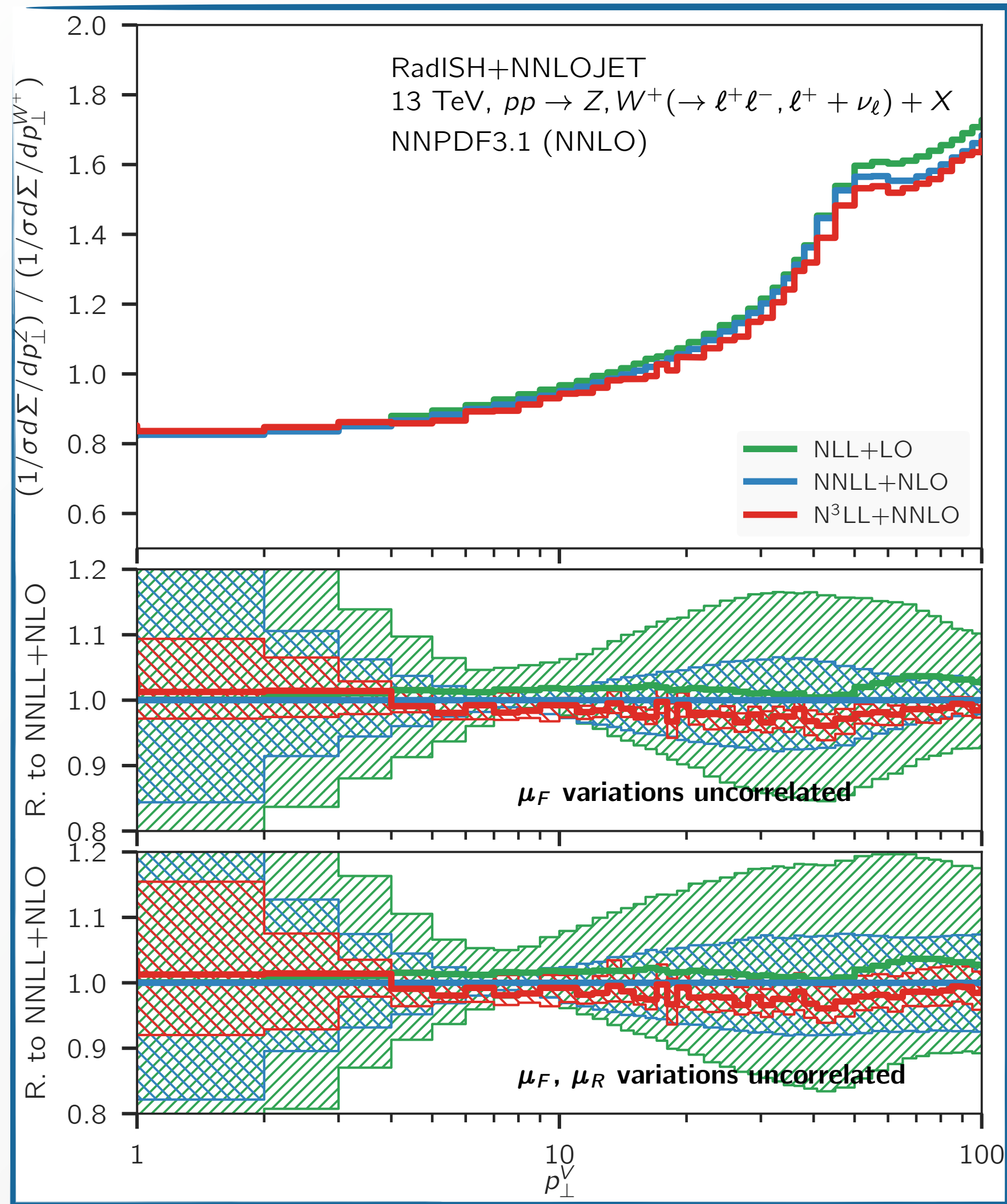
- More **conservative** estimate: vary both **renormalisation and factorisation scales in an uncorrelated** way with

$$\frac{1}{2} \leq \frac{\mu^{\text{num}}}{\mu^{\text{den}}} \leq 2$$

# Results for $W^-/W^+$ ratio



# Results for $Z/W^+$ ratio



# Equivalence with $b$ -space formulation

$$\frac{d\Sigma(v)}{d\Phi_B} = \int_{\mathcal{E}_1} \frac{dN_1}{2\pi i} \int_{\mathcal{E}_2} \frac{dN_2}{2\pi i} x_1^{-N_1} x_2^{-N_2} \sum_{c_1, c_2} \frac{d|M_B|_{c_1 c_2}^2}{d\Phi_B} \mathbf{f}_{N_1}^T(\mu_0) \hat{\Sigma}_{N_1, N_2}^{c_1, c_2}(v) \mathbf{f}_{N_2}(\mu_0)$$

unresolved  
emission + virtual  
corrections

$$\hat{\Sigma}_{N_1, N_2}^{c_1, c_2}(v) = \left[ \mathbf{C}_{N_1}^{c_1; T}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}_{N_2}^{c_2}(\alpha_s(\mu_0)) \right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} \\ \times e^{-\mathbf{R}(ek_{t1})} \exp \left\{ - \sum_{\ell=1}^2 \left( \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \Gamma_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \Gamma_{N_\ell}^{(C)}(\alpha_s(k_t)) \right) \right\}$$

Result valid for  
all inclusive  
observables (e.g.  
 $p_t, \varphi^*$ )

resolved  
emission

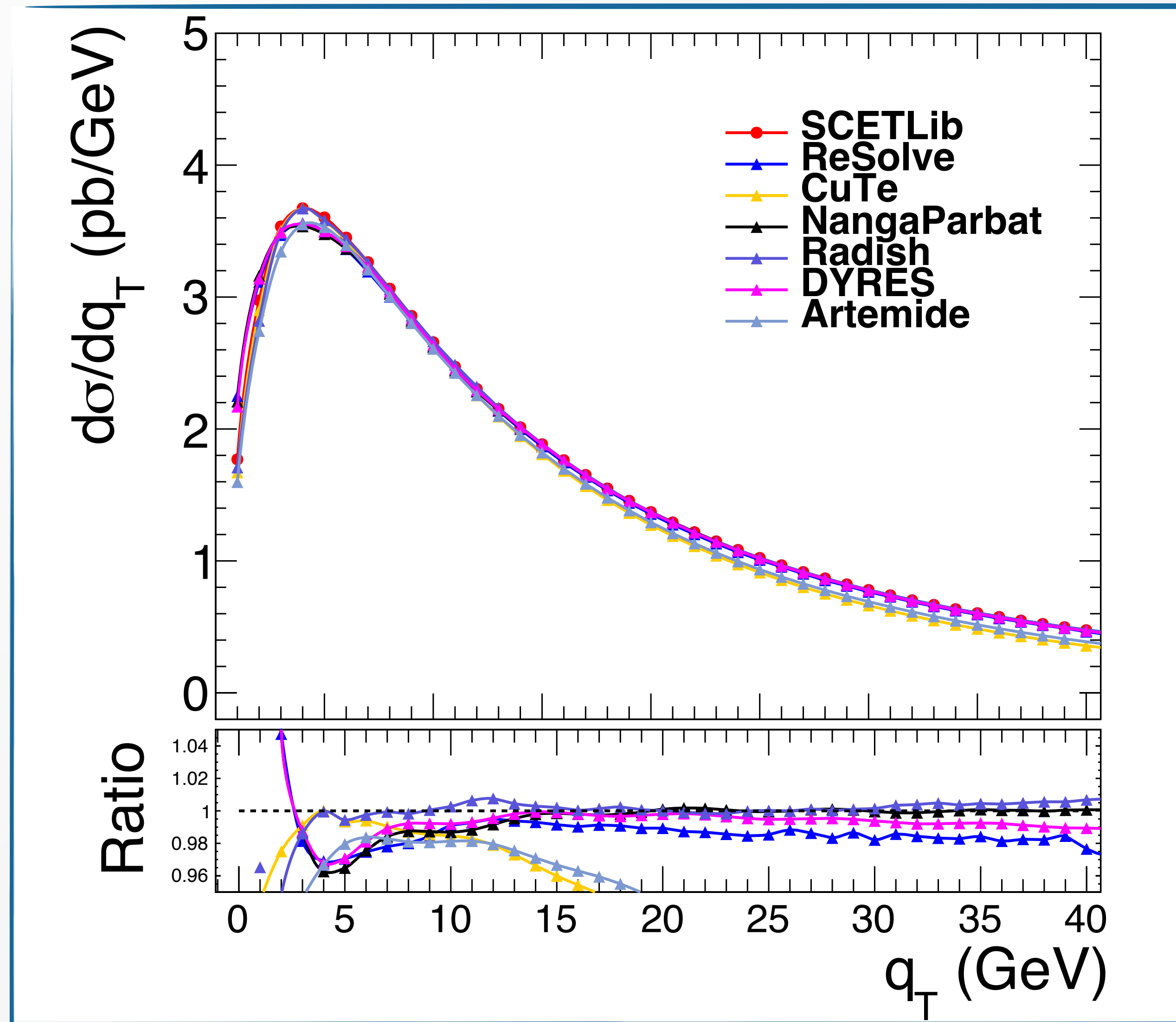
$$\sum_{\ell_1=1}^2 \left( \mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi} \Gamma_{N_{\ell_1}}(\alpha_s(k_{t1})) + \Gamma_{N_{\ell_1}}^{(C)}(\alpha_s(k_{t1})) \right) \\ \times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \sum_{\ell_i=1}^2 \left( \mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi} \Gamma_{N_{\ell_i}}(\alpha_s(k_{ti})) + \Gamma_{N_{\ell_i}}^{(C)}(\alpha_s(k_{ti})) \right) \\ \times \Theta(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1}))$$

Formulation **equivalent to  $b$ -space** result (up to a **scheme change** in the anomalous dimensions)

$$\frac{d^2\Sigma(v)}{d\Phi_B dp_t} = \sum_{c_1, c_2} \frac{d|M_B|_{c_1 c_2}^2}{d\Phi_B} \int b db p_t J_0(p_t b) \mathbf{f}^T(b_0/b) \mathbf{C}_{N_1}^{c_1; T}(\alpha_s(b_0/b)) H(M) \mathbf{C}_{N_2}^{c_2}(\alpha_s(b_0/b)) \mathbf{f}(b_0/b) \\ \times \exp \left\{ - \sum_{\ell=1}^2 \int_0^M \frac{dk_t}{k_t} \mathbf{R}'_{\ell}(k_t) (1 - J_0(bk_t)) \right\} \quad (1 - J_0(bk_t)) \simeq \Theta(k_t - \frac{b_0}{b}) + \frac{\zeta_3}{12} \frac{\partial^3}{\partial \ln(Mb/b_0)^3} \Theta(k_t - \frac{b_0}{b})$$

**N<sup>3</sup>LL effect: absorbed in the definition  
of  $H_2, B_3, A_4$  coefficients wrt to CSS**

# Equivalence with $b$ -space formulation

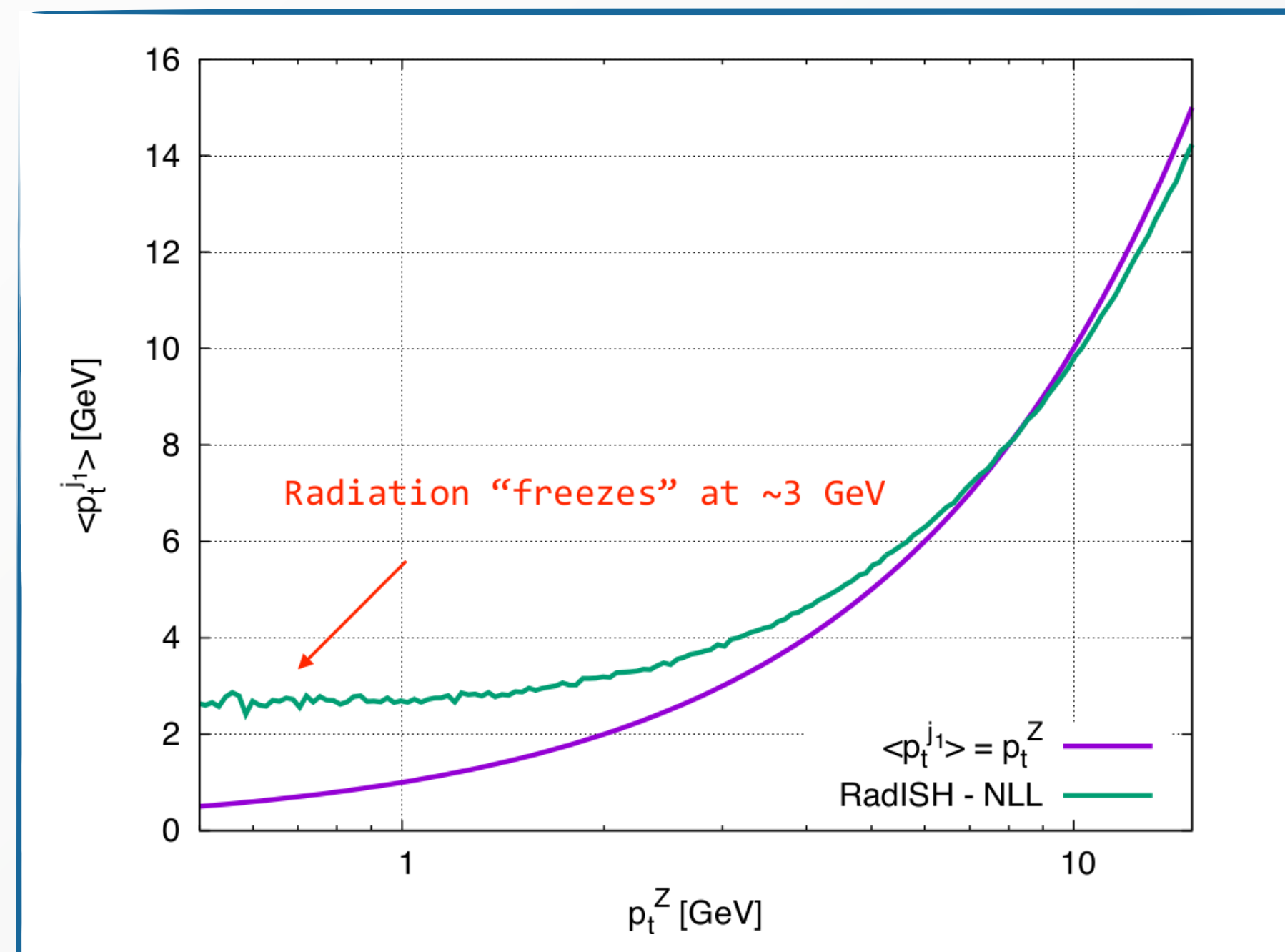


# The Landau pole and the small $p_T$ limit

Running coupling  $\alpha_s(k_{t1}^2)$  and Sudakov radiator hit Landau pole at

$$\alpha_s(\mu_R^2)\beta_0 \ln Q/k_{t1} = \frac{1}{2} \quad k_{t1} \sim 0.01 \text{ GeV}, \quad \mu_R = Q = m_Z$$

Only real cutoff in the calculation: emission probability is set to zero below this scale and parton densities are frozen.



At small  $p_t$  the large azimuthal cancellations dominate over the Sudakov suppression: the cutoff is never an issue in practice

$$\frac{d^2\Sigma(v)}{dp_t d\Phi_B} \simeq 2\sigma^{(0)}(\Phi_B)p_t \left( \frac{\Lambda_{\text{QCD}}^2}{M^2} \right)^{\frac{16}{25} \ln \frac{41}{16}}$$

Thanks to P. Monni



# Behaviour at small $p_t$

Explicit evaluation shows that the Parisi-Petronzio perturbative scaling at small  $p_t$  is reproduced. At NLL, Drell-Yan pair production,  $n_f=4$

$$\frac{d^2\Sigma(v)}{dp_t d\Phi_B} = 4 \sigma^{(0)}(\Phi_B) p_t \int_{\Lambda_{\text{QCD}}}^M \frac{dk_{t1}}{k_{t1}^3} e^{-R(k_{t1})} \simeq 2\sigma^{(0)}(\Phi_B) p_t \left( \frac{\Lambda_{\text{QCD}}^2}{M^2} \right)^{\frac{16}{25} \ln \frac{41}{16}}$$

As now higher logarithmic terms (up to N<sup>3</sup>LL) are under control, the coefficient of this scaling can be systematically improved in *perturbation* theory (non-perturbative effects – of the same order – not considered)

N<sup>3</sup>LL calculation allows one to have control over the terms of relative order  $O(\alpha_s^2)$ . Scaling  $L \sim 1/\alpha_s$  valid in the deep infrared regime.

# Numerical implementation

$$\frac{d\Sigma(p_t)}{d\Phi_B} = \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} \partial_L \left( -e^{-R'(k_{t1})} \mathcal{L}_{\text{NLL}}(k_{t1}) \right) \times$$

$$\times \underbrace{\epsilon^{R'(k_{t1})} \sum_{n=0}^{\infty} \frac{1}{n!} \left( \prod_{i=2}^{n+1} \int_{\epsilon k_{t1}}^{k_{t1}} \frac{dk_{ti}}{k_{ti}} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'(k_{t1}) \right)}_{\equiv \int d\mathcal{Z}[\{R', k_i\}] \Theta(p_t - |\vec{k}_{t1} + \dots + \vec{k}_{t(n+1)}|)} \Theta(p_t - |\vec{k}_{t1} + \dots + \vec{k}_{t(n+1)}|).$$

►  $L = \ln(M/k_{t1})$ ; luminosity  $\mathcal{L}_{\text{NLL}}(k_{t1}) = \sum_{c_1, c_2} \frac{d|M_B|^2_{c_1 c_2}}{d\Phi_B} f_{c_1}(x_1, k_{t1}) f_{c_2}(x_2, k_{t1})$ .

►  $\int d\mathcal{Z}[\{R', k_i\}] \Theta$  finite as  $\epsilon \rightarrow 0$ :

$$\epsilon^{R'(k_{t1})} = 1 - R'(k_{t1}) \ln(1/\epsilon) + \dots = 1 - \int_{\epsilon k_{t1}}^{k_{t1}} R'(k_{t1}) + \dots,$$

$$\int d\mathcal{Z}[\{R', k_i\}] \Theta = \left[ 1 - \int_{\epsilon k_{t1}}^{k_{t1}} R'(k_{t1}) + \dots \right] \left[ \Theta(p_t - |\vec{k}_{t1}|) + \int_{\epsilon k_{t1}}^{k_{t1}} R'(k_{t1}) \Theta(p_t - |\vec{k}_{t1} + \vec{k}_{t2}|) + \dots \right]$$

$$= \underbrace{\Theta(p_t - |\vec{k}_{t1}|)}_{\epsilon \rightarrow 0} + \int_0^{k_{t1}} R'(k_{t1}) \underbrace{\left[ \Theta(p_t - |\vec{k}_{t1} + \vec{k}_{t2}|) - \Theta(p_t - |\vec{k}_{t1}|) \right]}_{\text{finite: real-virtual cancellation}} + \dots$$

► Evaluated with Monte Carlo techniques:  $\int d\mathcal{Z}[\{R', k_i\}]$  is generated as a parton shower over secondary emissions.

# Numerical implementation

- ▶ Secondary radiation:

$$\begin{aligned}d\mathcal{Z}[\{R', k_i\}] &= \sum_{n=0}^{\infty} \frac{1}{n!} \left( \prod_{i=2}^{n+1} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \int_{\epsilon k_{t1}}^{k_{t1}} \frac{dk_{ti}}{k_{ti}} R'(k_{t1}) \right) \epsilon^{R'(k_{t1})} \\ &= \sum_{n=0}^{\infty} \left( \prod_{i=2}^{n+1} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \int_{\epsilon k_{t1}}^{k_{t(i-1)}} \frac{dk_{ti}}{k_{ti}} R'(k_{t1}) \right) \epsilon^{R'(k_{t1})}, \\ \epsilon^{R'(k_{t1})} &= e^{-R'(k_{t1}) \ln 1/\epsilon} = \prod_{i=2}^{n+2} e^{-R'(k_{t1}) \ln k_{t(i-1)}/k_{ti}},\end{aligned}$$

with  $k_{t(n+2)} = \epsilon k_{t1}$ .

- ▶ Each secondary emissions has differential probability

$$dw_i = \frac{d\phi_i}{2\pi} \frac{dk_{ti}}{k_{ti}} R'(k_{t1}) e^{-R'(k_{t1}) \ln k_{t(i-1)}/k_{ti}} = \frac{d\phi_i}{2\pi} d \left( e^{-R'(k_{t1}) \ln k_{t(i-1)}/k_{ti}} \right).$$

- ▶  $k_{t(i-1)} \geq k_{ti}$ . Scale  $k_{ti}$  extracted by solving  $e^{-R'(k_{t1}) \ln k_{t(i-1)}/k_{ti}} = r$ , with  $r$  random number extracted uniformly in  $[0, 1]$ . Shower ordered in  $k_{ti}$ .
- ▶ Extract  $\phi_i$  randomly in  $[0, 2\pi]$ .

# Joint resummation in direct space

$$\begin{aligned}
\sigma_{\text{incl}}^{\text{NNLL}}(p_t^{\text{J,v}}, p_t^{\text{H,v}}) &= \int_0^{p_t^{\text{J,v}}} \frac{dk_{t,1}}{k_{t,1}} \frac{d\phi_1}{2\pi} \int d\mathcal{Z} \left\{ \frac{d}{dL_{t,1}} \left[ -e^{-R_{\text{NNLL}}(L_{t,1})} \mathcal{L}_{\text{NNLL}}(\mu_{\text{F}} e^{-L_{t,1}}) \right] \Theta\left(p_t^{\text{H,v}} - |\vec{k}_{t,1} + \dots + \vec{k}_{t,n+1}|\right) \right. \\
&+ e^{-R_{\text{NLL}}(L_{t,1})} \hat{R}'(k_{t,1}) \int_0^{k_{t,1}} \frac{dk_{t,s_1}}{k_{t,s_1}} \frac{d\phi_{s_1}}{2\pi} \left[ \left( \delta \hat{R}'(k_{t,1}) + \hat{R}''(k_{t,1}) \ln \frac{k_{t,1}}{k_{t,s_1}} \right) \mathcal{L}_{\text{NLL}}(\mu_{\text{F}} e^{-L_{t,1}}) - \frac{d}{dL_{t,1}} \mathcal{L}_{\text{NLL}}(\mu_{\text{F}} e^{-L_{t,1}}) \right] \\
&\times \left. \left[ \Theta\left(p_t^{\text{H,v}} - |\vec{k}_{t,1} + \dots + \vec{k}_{t,n+1} + \vec{k}_{t,s_1}|\right) - \Theta\left(p_t^{\text{H,v}} - |\vec{k}_{t,1} + \dots + \vec{k}_{t,n+1}|\right) \right] \right\}, \quad (38)
\end{aligned}$$

$$\begin{aligned}
\sigma_{\text{clust}}^{\text{NNLL}}(p_t^{\text{J,v}}, p_t^{\text{H,v}}) &= \int_0^\infty \frac{dk_{t,1}}{k_{t,1}} \frac{d\phi_1}{2\pi} \int d\mathcal{Z} e^{-R_{\text{NLL}}(L_{t,1})} \mathcal{L}_{\text{NLL}}(\mu_{\text{F}} e^{-L_{t,1}}) 8 C_A^2 \frac{\alpha_s^2}{\pi^2} \frac{L_{t,1}}{(1 - 2\beta_0 \alpha_s L_{t,1})^2} \Theta\left(p_t^{\text{J,v}} - \max_{i>1} \{k_{t,i}\}\right) \\
&\times \left\{ \int_0^{k_{t,1}} \frac{dk_{t,s_1}}{k_{t,s_1}} \frac{d\phi_{s_1}}{2\pi} \int_{-\infty}^\infty d\Delta\eta_{1s_1} J_{1s_1}(R) \left[ \Theta\left(p_t^{\text{J,v}} - |\vec{k}_{t,1} + \vec{k}_{t,s_1}|\right) - \Theta\left(p_t^{\text{J,v}} - k_{t,1}\right) \right] \Theta\left(p_t^{\text{H,v}} - |\vec{k}_{t,1} + \dots + \vec{k}_{t,n+1} + \vec{k}_{t,s_1}|\right) \right. \\
&+ \frac{1}{2!} \hat{R}'(k_{t,1}) \int_0^{k_{t,1}} \frac{dk_{t,s_1}}{k_{t,s_1}} \frac{dk_{t,s_2}}{k_{t,s_2}} \frac{d\phi_{s_1}}{2\pi} \frac{d\phi_{s_2}}{2\pi} \int_{-\infty}^\infty d\Delta\eta_{s_1s_2} J_{s_1s_2}(R) \left[ \Theta\left(p_t^{\text{J,v}} - |\vec{k}_{t,s_1} + \vec{k}_{t,s_2}|\right) - \Theta\left(p_t^{\text{J,v}} - \max\{k_{t,s_1}, k_{t,s_2}\}\right) \right] \\
&\times \left. \Theta\left(p_t^{\text{H,v}} - |\vec{k}_{t,1} + \dots + \vec{k}_{t,n+1} + \vec{k}_{t,s_1} + \vec{k}_{t,s_2}|\right) \Theta\left(p_t^{\text{J,v}} - k_{t,1}\right) \right\}, \quad (42)
\end{aligned}$$

$$\begin{aligned}
\sigma_{\text{correl}}^{\text{NNLL}}(p_t^{\text{J,v}}, p_t^{\text{H,v}}) &= \int_0^\infty \frac{dk_{t,1}}{k_{t,1}} \frac{d\phi_1}{2\pi} \int d\mathcal{Z} e^{-R_{\text{NLL}}(L_{t,1})} \mathcal{L}_{\text{NLL}}(\mu_{\text{F}} e^{-L_{t,1}}) 8 C_A^2 \frac{\alpha_s^2}{\pi^2} \frac{L_{t,1}}{(1 - 2\beta_0 \alpha_s L_{t,1})^2} \Theta\left(p_t^{\text{J,v}} - \max_{i>1} \{k_{t,i}\}\right) \\
&\times \left\{ \int_0^{k_{t,1}} \frac{dk_{t,s_1}}{k_{t,s_1}} \frac{d\phi_{s_1}}{2\pi} \int_{-\infty}^\infty d\Delta\eta_{1s_1} \mathcal{C}\left(\Delta\eta_{1s_1}, \Delta\phi_{1s_1}, \frac{k_{t,1}}{k_{t,s_1}}\right) (1 - J_{1s_1}(R)) \right. \\
&\times \left[ \Theta\left(p_t^{\text{J,v}} - k_{t,1}\right) - \Theta\left(p_t^{\text{J,v}} - |\vec{k}_{t,1} + \vec{k}_{t,s_1}|\right) \right] \Theta\left(p_t^{\text{H,v}} - |\vec{k}_{t,1} + \dots + \vec{k}_{t,n+1} + \vec{k}_{t,s_1}|\right) \\
&+ \frac{1}{2!} \hat{R}'(k_{t,1}) \int_0^{k_{t,1}} \frac{dk_{t,s_1}}{k_{t,s_1}} \frac{dk_{t,s_2}}{k_{t,s_2}} \frac{d\phi_{s_1}}{2\pi} \frac{d\phi_{s_2}}{2\pi} \int_{-\infty}^\infty d\Delta\eta_{s_1s_2} \mathcal{C}\left(\Delta\eta_{s_1s_2}, \Delta\phi_{s_1s_2}, \frac{k_{t,s_2}}{k_{t,s_1}}\right) (1 - J_{s_1s_2}(R)) \Theta\left(p_t^{\text{J,v}} - k_{t,1}\right) \\
&\times \left. \left[ \Theta\left(p_t^{\text{J,v}} - \max\{k_{t,s_1}, k_{t,s_2}\}\right) - \Theta\left(p_t^{\text{J,v}} - |\vec{k}_{t,s_1} + \vec{k}_{t,s_2}|\right) \right] \Theta\left(p_t^{\text{H,v}} - |\vec{k}_{t,1} + \dots + \vec{k}_{t,n+1} + \vec{k}_{t,s_1} + \vec{k}_{t,s_2}|\right) \right\}. \quad (43)
\end{aligned}$$

# All-order resummation: CAESAR/ARES approach

## Solution 2:

Translate the resummability into properties of the observable in the presence of multiple radiation:  
**recursive infrared and collinear (rIRC) safety**

[Banfi, Salam, Zanderighi '01, '03, '04]

[Banfi, McAslan, Monni, Zanderighi, El-Menoufi '14, '18]

**Simple observable** easy to calculate

$$\tilde{\sigma} \sim \int \frac{dv_1}{v_1} \Sigma_s(v_1) \mathcal{F}(v, v_1)$$

**Transfer function** relates the resummation of the full observable to the one of the simple observable.

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i.e. conditional probability

Separation obtained by introducing a **resolution scale**  $q_0 = \epsilon k_{t,1}$

$$\tilde{\sigma} \sim \int [dk_1] e^{-R(q_0)} \rightarrow \text{exponentiation}$$

**Unresolved emission** can be treated as **totally unconstrained**

$$\times |\mathcal{M}(k_1)|^2 \left( \sum_{m=0}^{\infty} \frac{1}{m!} \int \prod_{i=2}^{m+1} [dk_i] |\mathcal{M}(k_i)|^2 \Theta(V(k_i) - q_0) \Theta(v - V(k_1, \dots, k_{m+1})) \right)$$

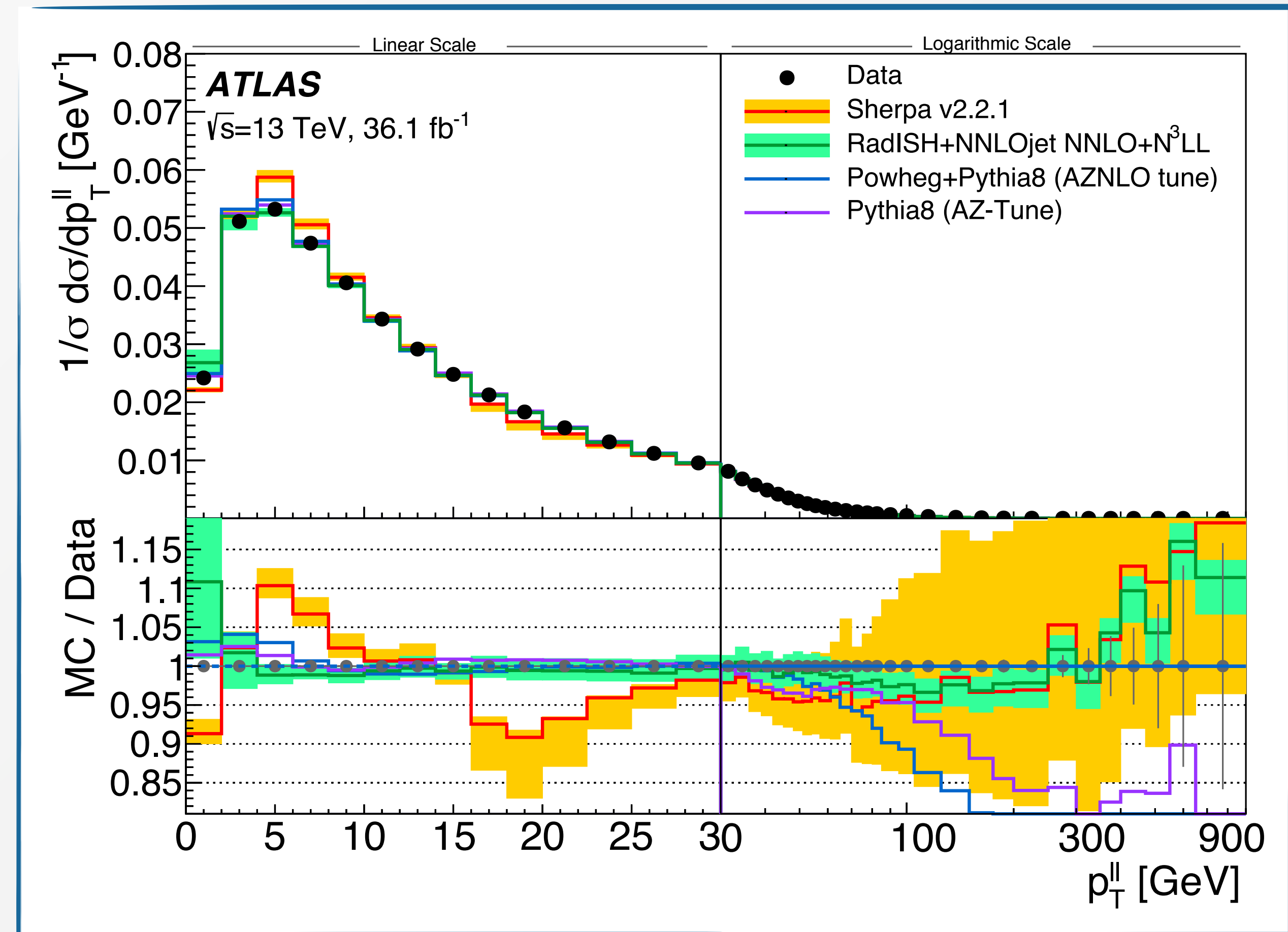
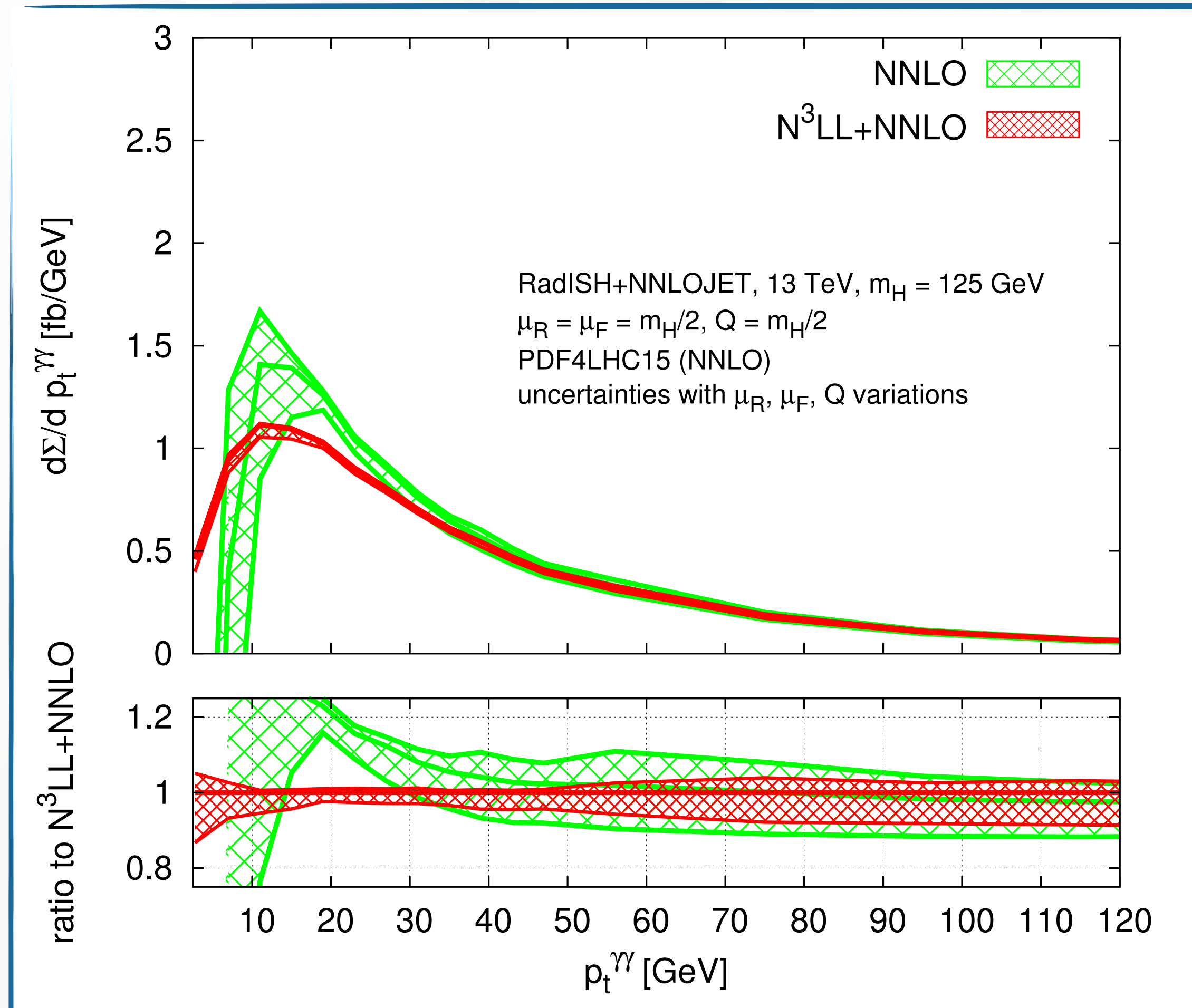
**Resolved emission** treated exclusively with **Monte Carlo methods**. Integral is finite, can be integrated in d=4 with a computer

Approach recently formulated within SCET language [Bauer, Monni '18, '19 + ongoing work]

Method entirely formulated in **direct space**

# Resummation of the transverse momentum spectrum at $N^3LL+NNLO$

$N^3LL$  result matched to NNLO  $H+j$ ,  $Z+j$ ,  $W^\pm+j$  [Bizon, LR *et al.* '17, '18, '19]



[ATLAS 1912.02844]

$H+j$  at same accuracy also in SCET [Chen *et al.* '18]

# Resummation of the transverse momentum spectrum in $b$ space

## Solution 1:

move to **conjugate space** where phase space factorization is manifest

$p_{\perp}$  resummation  
[Parisi, Petronzio '79; Collins, Soper, Sterman '85]

$$\delta^{(2)}\left(\vec{p}_t - \sum_{i=1}^n \vec{k}_{t,i}\right) = \int d^2b \frac{1}{4\pi^2} e^{i\vec{b}\cdot\vec{p}_t} \prod_{i=1}^n e^{-i\vec{b}\cdot\vec{k}_{t,i}}$$

two-dimensional momentum conservation

Exponentiation in conjugate space

virtual corrections

$$\sigma = \sigma_0 \int d^2\vec{p}_{\perp}^H \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_{\perp}^H} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^n \int [dk_i] |M(k_i)|^2 \left( e^{i\vec{b}\cdot\vec{k}_{t,i}} - 1 \right) = \sigma_0 \int d^2\vec{p}_{\perp}^H \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_{\perp}^H} e^{-R_{\text{NLL}}(L)}$$

NLL formula with scale-independent PDFs

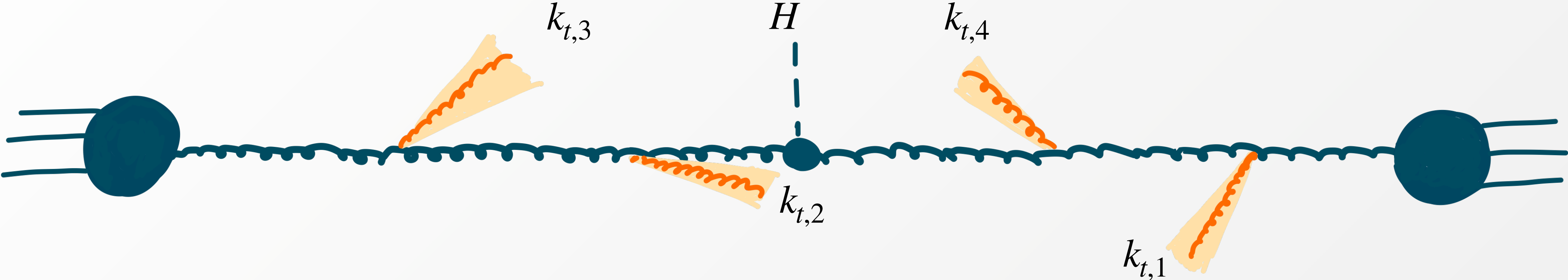
$$R_{\text{NLL}}(L) = -Lg_1(\alpha_s L) - g_2(\alpha_s L)$$

$$L = \ln(m_H b / b_0)$$



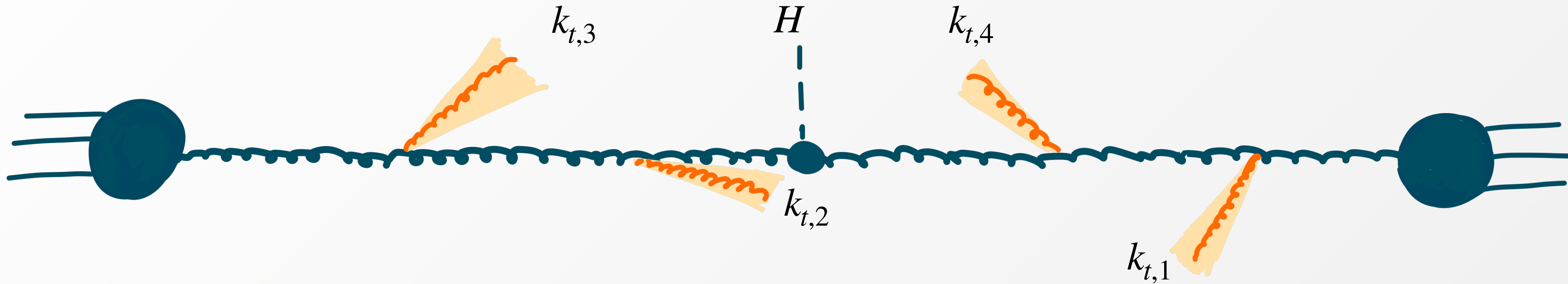
# Double-differential resummation at NLL in $b$ space

At NLL, emissions are **strongly ordered** in angle. Clustering algorithms will associate **each emission** to a **different** jet



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Additional constraint on **real radiation**

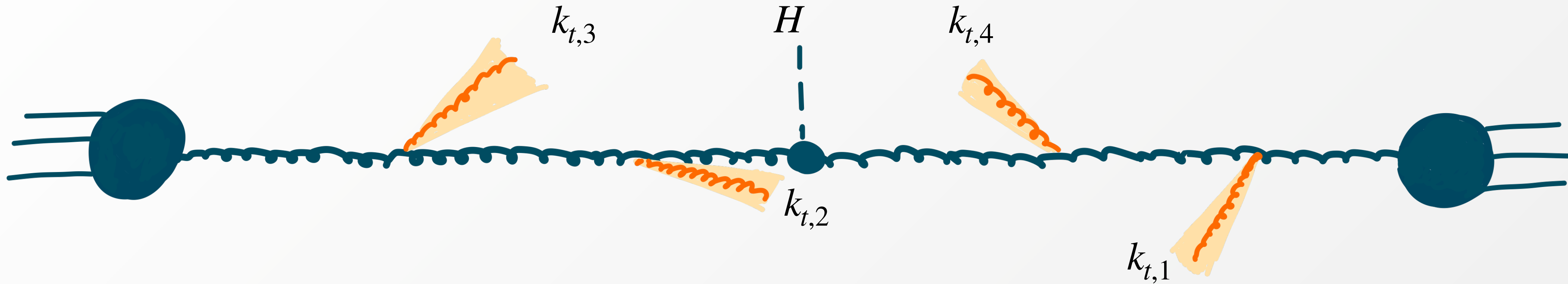
$$\Theta(p_{\perp}^{\text{J},\text{v}} - \max\{k_{t,1}, \dots, k_{t,n}\}) = \prod_{i=1}^n \Theta(p_{\perp}^{\text{J},\text{v}} - k_{t,i})$$

$p_{\perp}^H$  resummation formula

$$\frac{d\sigma}{d^2\vec{p}_{\perp}^H} = \sigma_0 \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b} \cdot \vec{p}_{\perp}^H} e^{-R_{\text{NLL}}(L)}$$

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**Joint  $p_{\perp}^H, p_{\perp}^{\text{J},\text{v}}$  resummation formula**

$$\frac{d\sigma(p_{\perp}^{\text{J},\text{v}})}{d^2\vec{p}_{\perp}^H} = \sigma_0 \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b} \cdot \vec{p}_{\perp}^H} e^{-S_{\text{NLL}}(L)}$$

**CMW scheme** [Catani, Marchesini, Webber '91]

$$S_{\text{NLL}}(L) = -Lg_1(\alpha_s L) - g_2(\alpha_s L) + \int_0^{m_H} \frac{dk_t}{k_t} R'_{\text{NLL}}(k_t) J_0(bk_t) \Theta(k_t - p_{\perp}^{\text{J},\text{v}}) \quad R'_{\text{NLL}}(k_t) = 4 \left( \frac{\alpha_s^{\text{CMW}}(k_t)}{\pi} C_A \ln \frac{m_H}{k_t} - \alpha_s(k_t) \beta_0 \right)$$

# Double-differential resummation at NNLL in $b$ space

Crucial observation: in  $b$  space the phase space constraints entirely factorize  $\longrightarrow e^{i\vec{b}\cdot\vec{k}_{t,i}}$

The jet veto constraint can be included by implementing the jet veto resummation at the  $b$ -space integrand level **directly in impact-parameter space**

**Inclusive** contribution: phase space constraint of the form

$$\Theta(p_{\perp}^{\text{J,v}} - \max\{k_{t,1}, \dots, k_{t,n}\}) = \prod_{i=1}^n \Theta(p_{\perp}^{\text{J,v}} - k_{t,i})$$

Promote radiator at NNLL

$$S_{\text{NNLL}} = -Lg_1(\alpha_s L) - g_2(\alpha_s L) - \alpha_s g_3(\alpha_s L) + \int_0^{m_H} \frac{dk_t}{k_t} R'_{\text{NNLL}}(k_t) J_0(bk_t) \Theta(k_t - p_{\perp}^{\text{J,v}})$$

# Double-differential resummation at NNLL in $b$ space

Resummation formula must be amended at NNLL [Banfi et al. '12][Becher et al. '13][Stewart et al. '14]

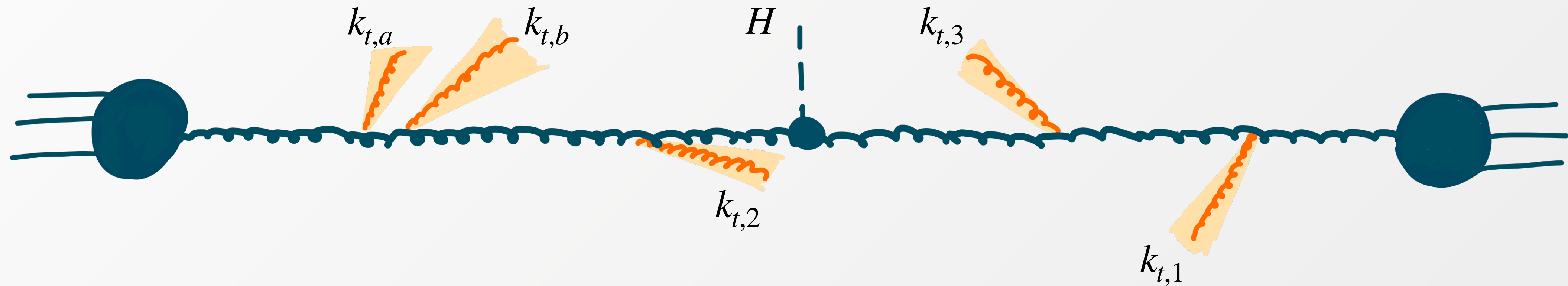
$$\frac{d\sigma(p_{\perp}^{J,V})}{d^2\vec{p}_{\perp}^H} = \sigma_0 \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_{\perp}^H} e^{-S_{\text{NNLL}}(L)} (1 + \mathcal{F}_{\text{clust}} + \mathcal{F}_{\text{correl}})$$

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- **clustering correction**: jet algorithm can cluster two **independent** emissions into the same jet

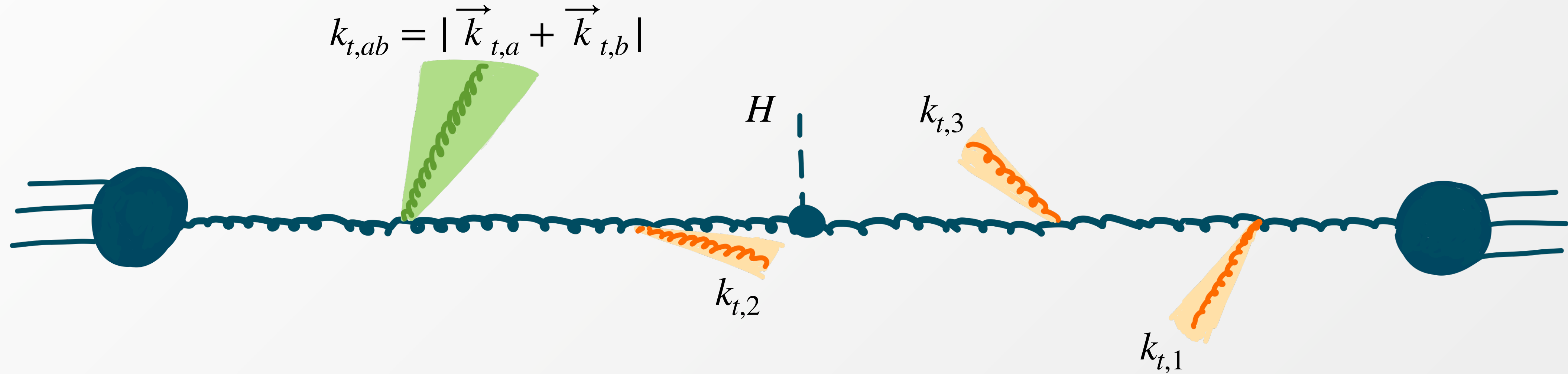


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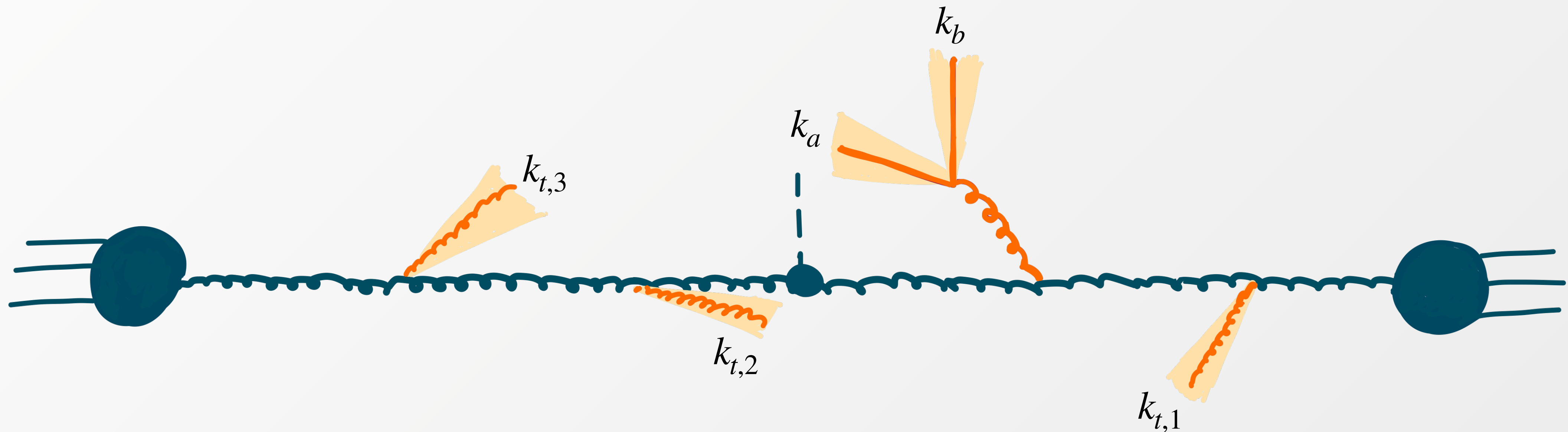


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- **correlated correction**: amends the inclusive treatment of the correlated squared amplitude for two emission accounting for configurations where the two **correlated** emissions (non abelian) are not clustered in the same jet



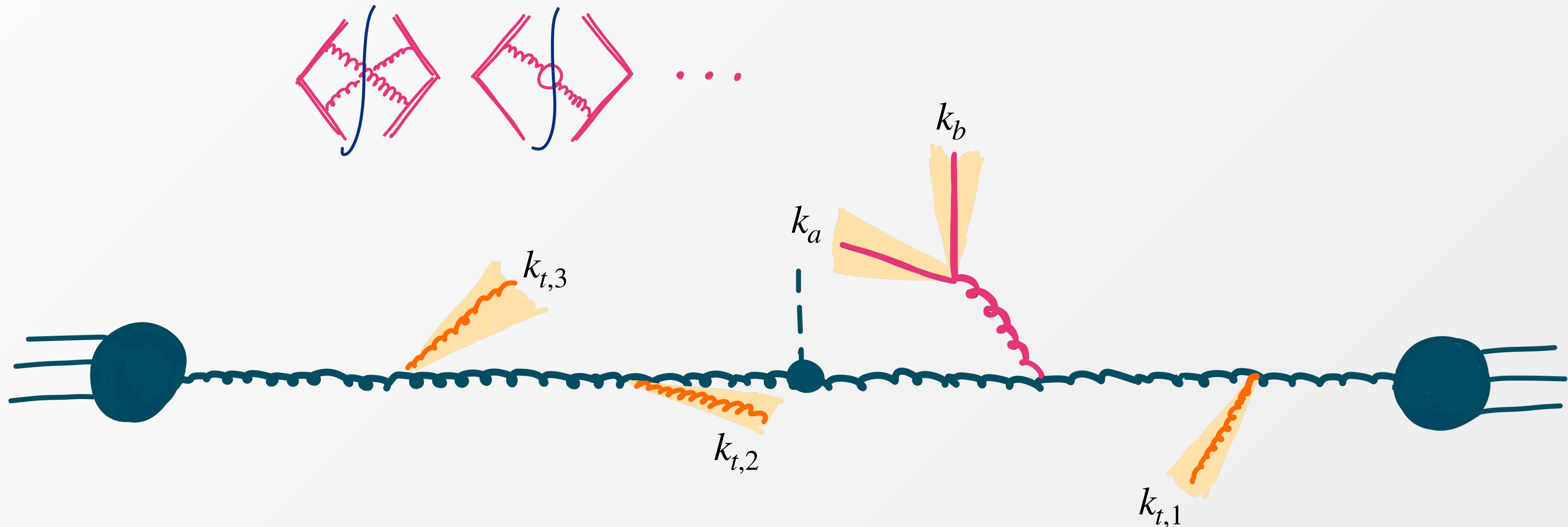


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# Double-differential resummation at NNLL in $b$ space

NNLL prediction finally requires the consistent treatment of non-soft collinear emissions off the initial state particles

Soft and non-soft emission cannot be clustered by a  $k_t$ -type jet algorithm. Non-soft collinear radiation can be handled by taking a Mellin transform of the resummed cross section, giving rise to **scale evolution of PDFs** and of the  $\mathcal{O}(\alpha_s)$  **collinear coefficient functions**

# Double-differential resummation at NNLL in $b$ space

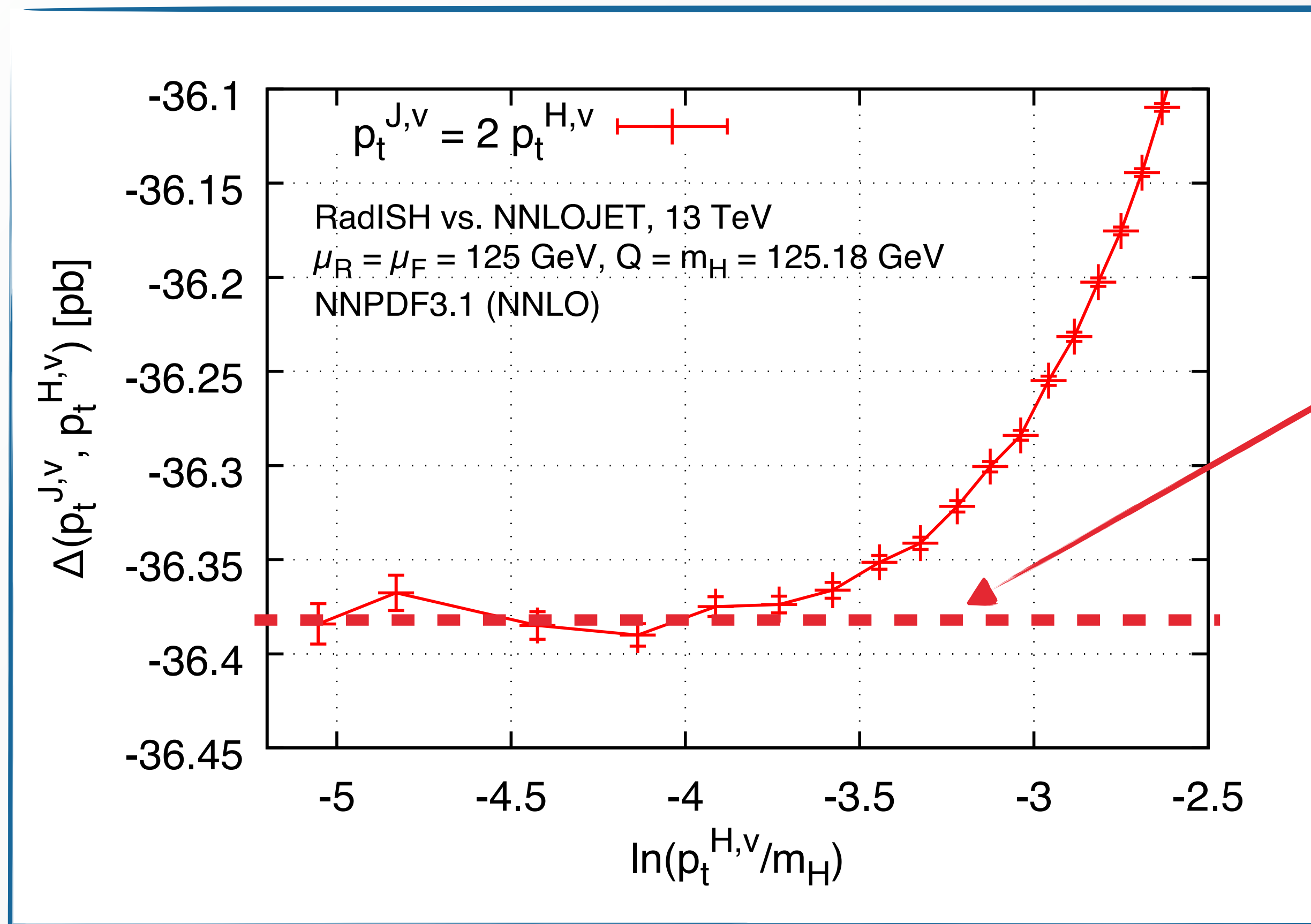
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Final result at NNLL, including **hard-virtual corrections** at  $\mathcal{O}(\alpha_s)$

$$\begin{aligned}
 \frac{d\sigma(p_{\perp}^{J,v})}{dy_H d^2\vec{p}_{\perp}^H} &= M_{\text{gg}\rightarrow\text{H}}^2 \mathcal{H}(\alpha_s(m_H)) \int_{\mathcal{C}_1} \frac{d\nu_1}{2\pi i} \int_{\mathcal{C}_2} \frac{d\nu_2}{2\pi i} x_1^{-\nu_1} x_2^{-\nu_2} \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_{\perp}^H} e^{-S_{\text{NNLL}}} (1 + \mathcal{F}_{\text{clust}} + \mathcal{F}_{\text{correl}}) \\
 &\quad \times f_{\nu_1, a_1}(b_0/b) f_{\nu_2, a_2}(b_0/b) \left[ \mathcal{P} e^{-\int_{p_{\perp}^{J,v}}^{m_H} \frac{d\mu}{\mu} \Gamma_{\nu_1}(\alpha_s(\mu)) J_0(b\mu)} \right]_{c_1 a_1} \left[ \mathcal{P} e^{-\int_{p_{\perp}^{J,v}}^{m_H} \frac{d\mu}{\mu} \Gamma_{\nu_2}(\alpha_s(\mu)) J_0(b\mu)} \right]_{c_2 a_2} \\
 &\quad \times C_{\nu_1, g b_1}(\alpha_s(b_0/b)) C_{\nu_2, g b_2}(\alpha_s(b_0/b)) \left[ \mathcal{P} e^{-\int_{p_{\perp}^{J,v}}^{m_H} \frac{d\mu}{\mu} \Gamma_{\nu_1}^{(C)}(\alpha_s(\mu)) J_0(b\mu)} \right]_{c_1 b_1} \left[ \mathcal{P} e^{-\int_{p_{\perp}^{J,v}}^{m_H} \frac{d\mu}{\mu} \Gamma_{\nu_2}^{(C)}(\alpha_s(\mu)) J_0(b\mu)} \right]_{c_2 b_2}
 \end{aligned}$$

# Accuracy check at $\mathcal{O}(\alpha_s^2)$



Comparison of the expansion of the resummed result with the fixed order at  $\mathcal{O}(\alpha_s^2)$  in the limit  $p_{\perp}^H \sim p_{\perp}^{J,v} \ll m_H$

Difference at the double-cumulative level goes to a **constant** (all logarithmic terms correctly predicted)

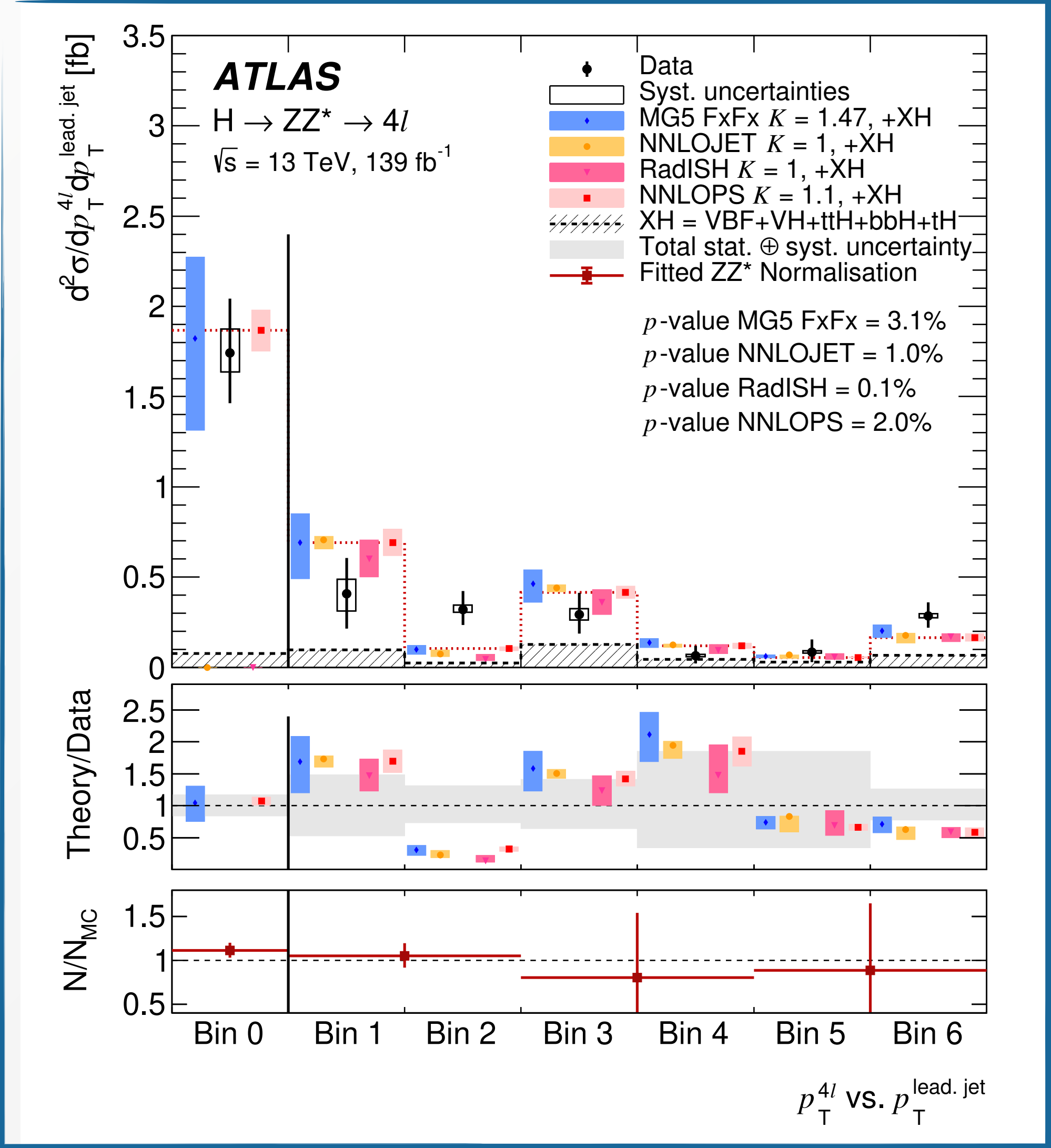
Very strong check: **NNLL resummation** of the logarithms associated to the shoulder

Analogous checks performed in the limits  $p_{\perp}^H \ll p_{\perp}^{J,v} < m_H$  and  $p_{\perp}^{J,v} \ll p_{\perp}^H < m_H$

$$\Delta(p_{\perp}^{J,v}, p_{\perp}^{H,v}) = \sigma^{\text{NNLO}}(p_{\perp}^{J,v}, p_{\perp}^{H,v}) - \sigma_{\text{exp.}}^{\text{NNLL}}(p_{\perp}^{J,v}, p_{\perp}^{H,v})$$

$$\sigma^{\text{NNLO}}(p_{\perp}^H < p_{\perp}^{H,v}, p_{\perp}^J < p_{\perp}^{J,v}) = \sigma^{\text{NNLO}} - \int \Theta(p_{\perp}^H > p_{\perp}^{H,v}) \vee \Theta(p_{\perp}^J > p_{\perp}^{J,v}) d\sigma_{H+J}^{\text{NLO}}$$

# LHC results: Higgs transverse momentum with a jet veto



[ATLAS 2004.03969]