Transverse observables in Higgs and Drell-Yan production at N^{3}LL$'$+NNLO

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FNSNF
Swiss National Science Foundation

Based on: Re, LR & Torrielli 2104.07509

RadISH *formalism: Monni, Re, Torrielli 1604.02191, Bizon, Monni, Re, LR & Torrielli 1705.09127*

Transverse observables in colour-singlet production

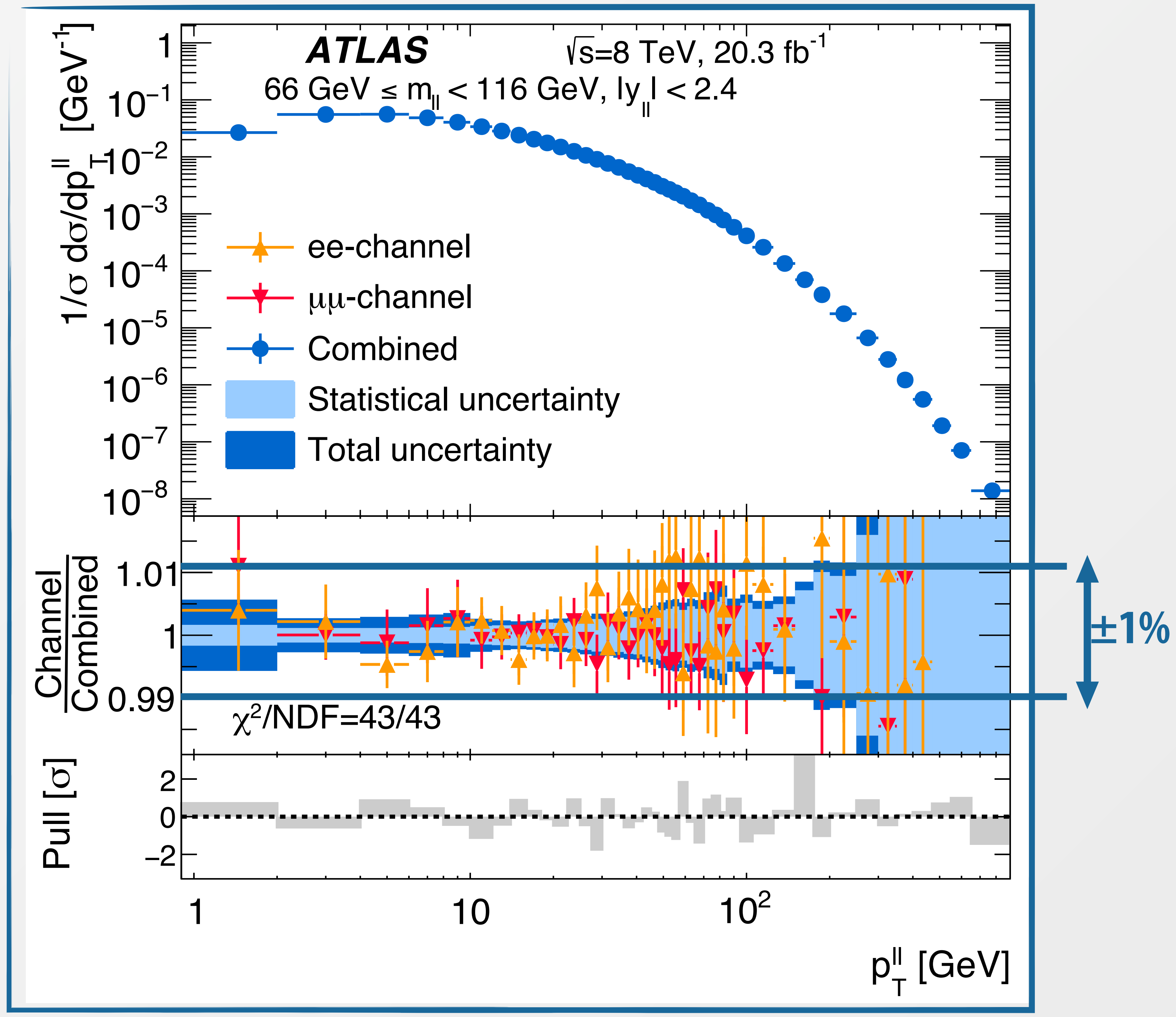
Parameterized as

$$V(k) = \left(\frac{k_t}{M}\right)^a f(\phi)$$

for a **single soft** QCD emission k **collinear** to incoming leg.
Independent of the rapidity of radiation. $V \to 0$ for soft/collinear radiation.

Inclusive observables (e.g. transverse momentum p_t) probe directly the kinematics of the colour singlet

$$V(k_1, \ldots k_n) = V(k_1 + \ldots + k_n)$$

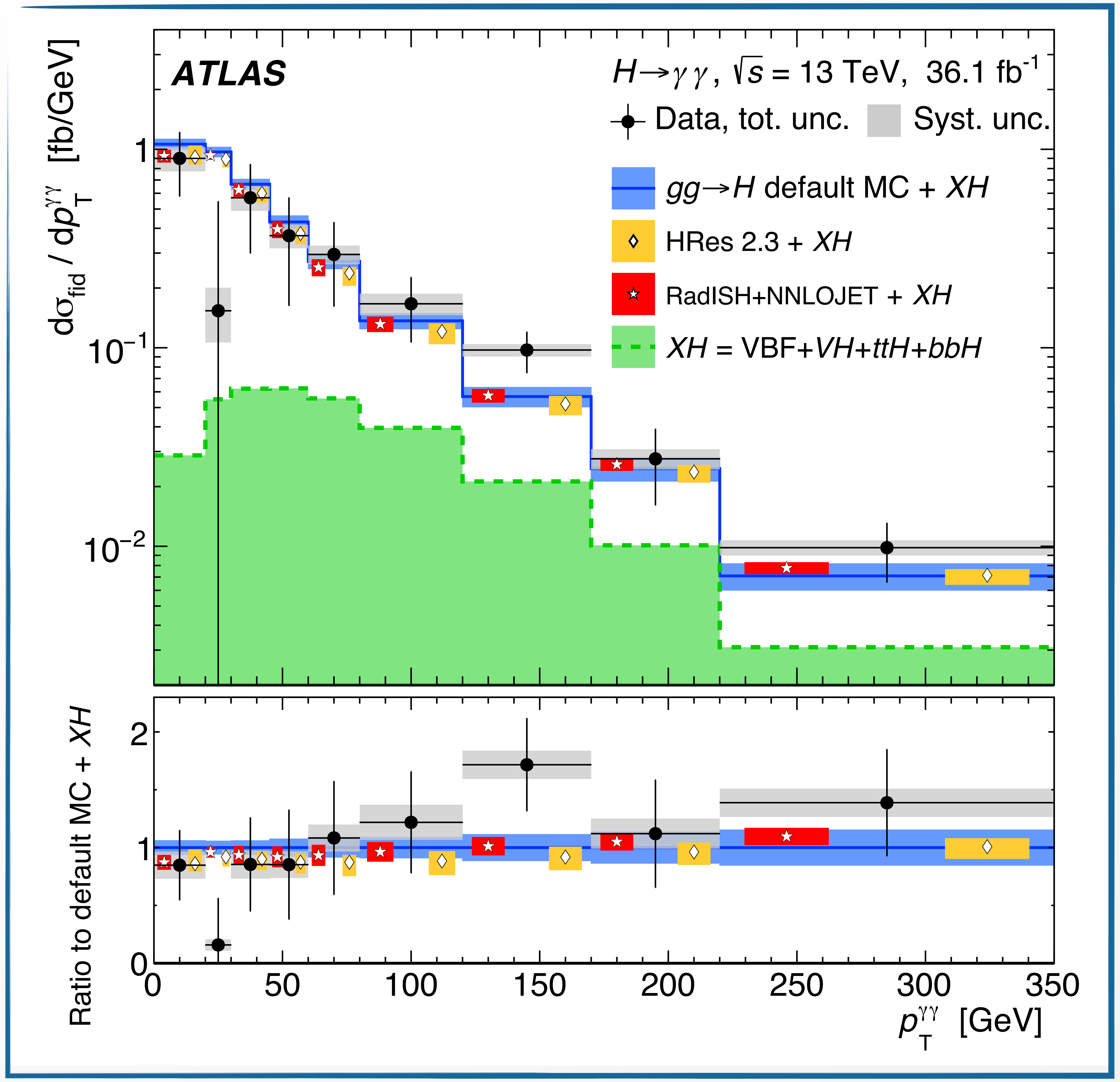


Clean experimental and theoretical environment for precision physics

- little or no sensitivity to multi-parton interactions
- reduced sensitivity to non-perturbative effects
- measured extremely precisely at experiments

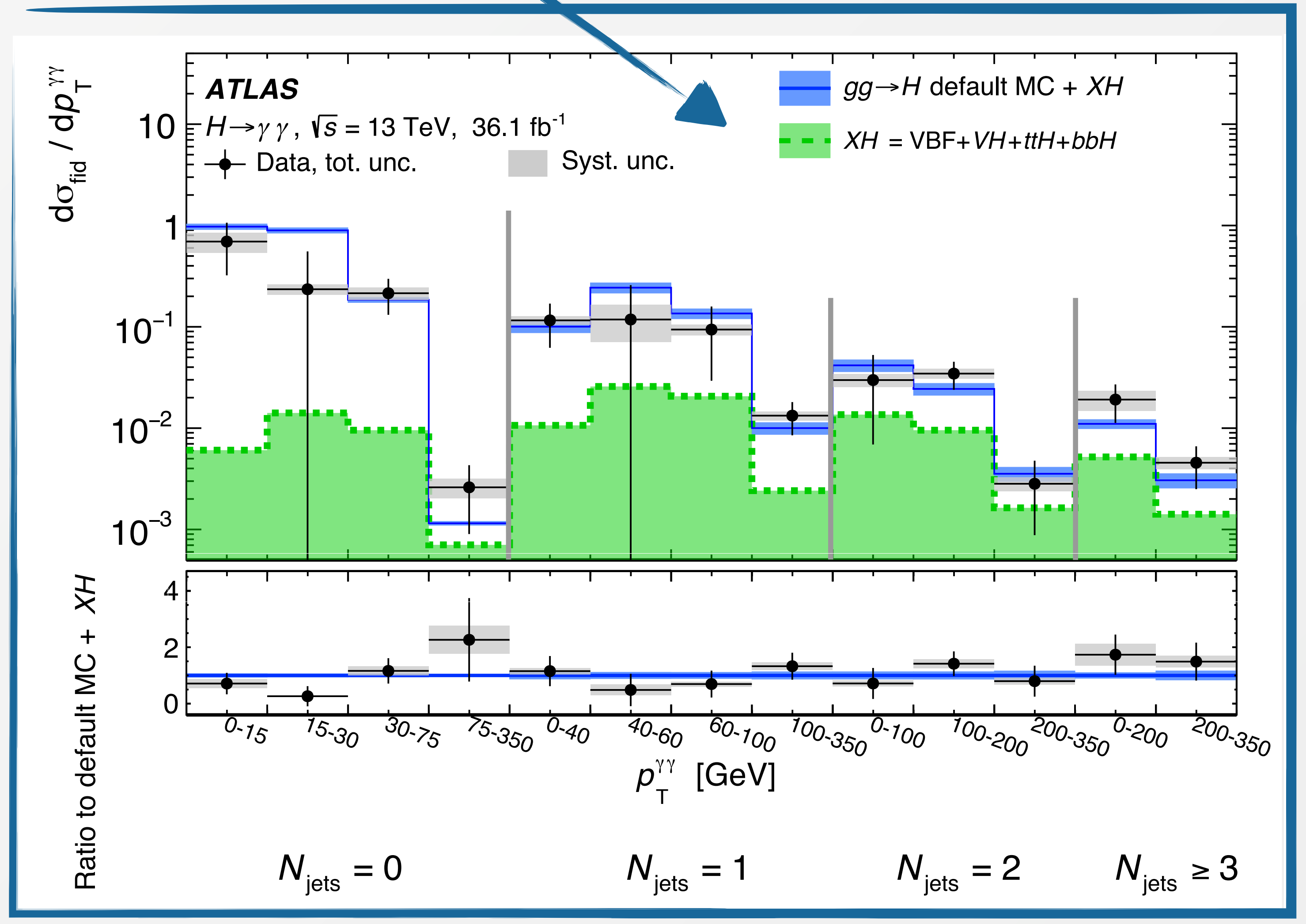
Very accurate theoretical predictions needed

Example: the Higgs transverse momentum



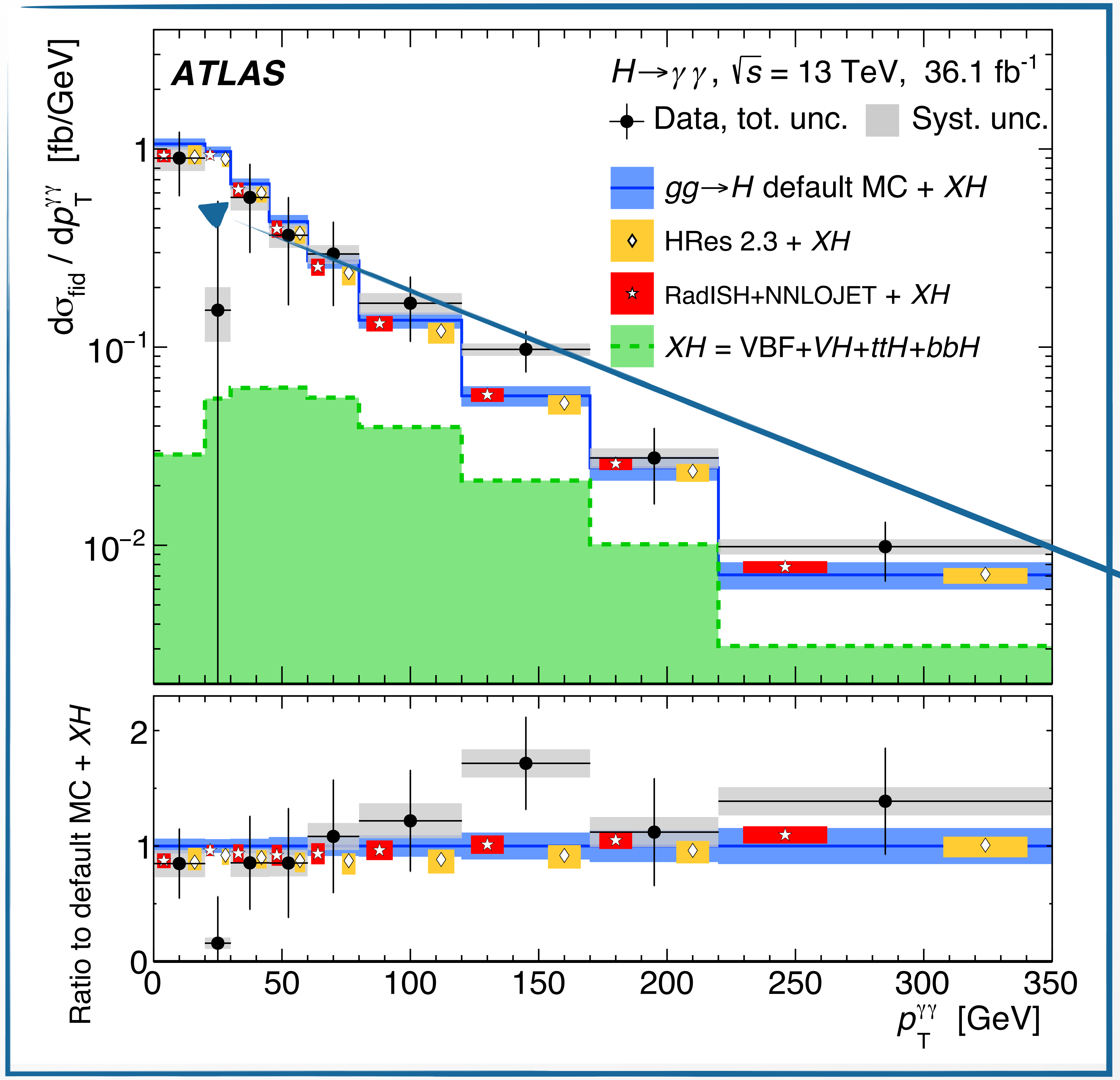
[ATLAS 1802.04146]

- Sensitivity to New Physics (e.g. **light Yukawa** couplings, **trilinear** Higgs coupling) [Bishara *et al.* '16][Soreq *et al.* '16] [Bizon *et al.* 1610.05771]
- Experimental analyses categorize events into **jet bins** according to the jet multiplicity



- Increased sensitivity to Higgs boson kinematics, spin-CP properties, BSM effects…

Example: the Higgs transverse momentum

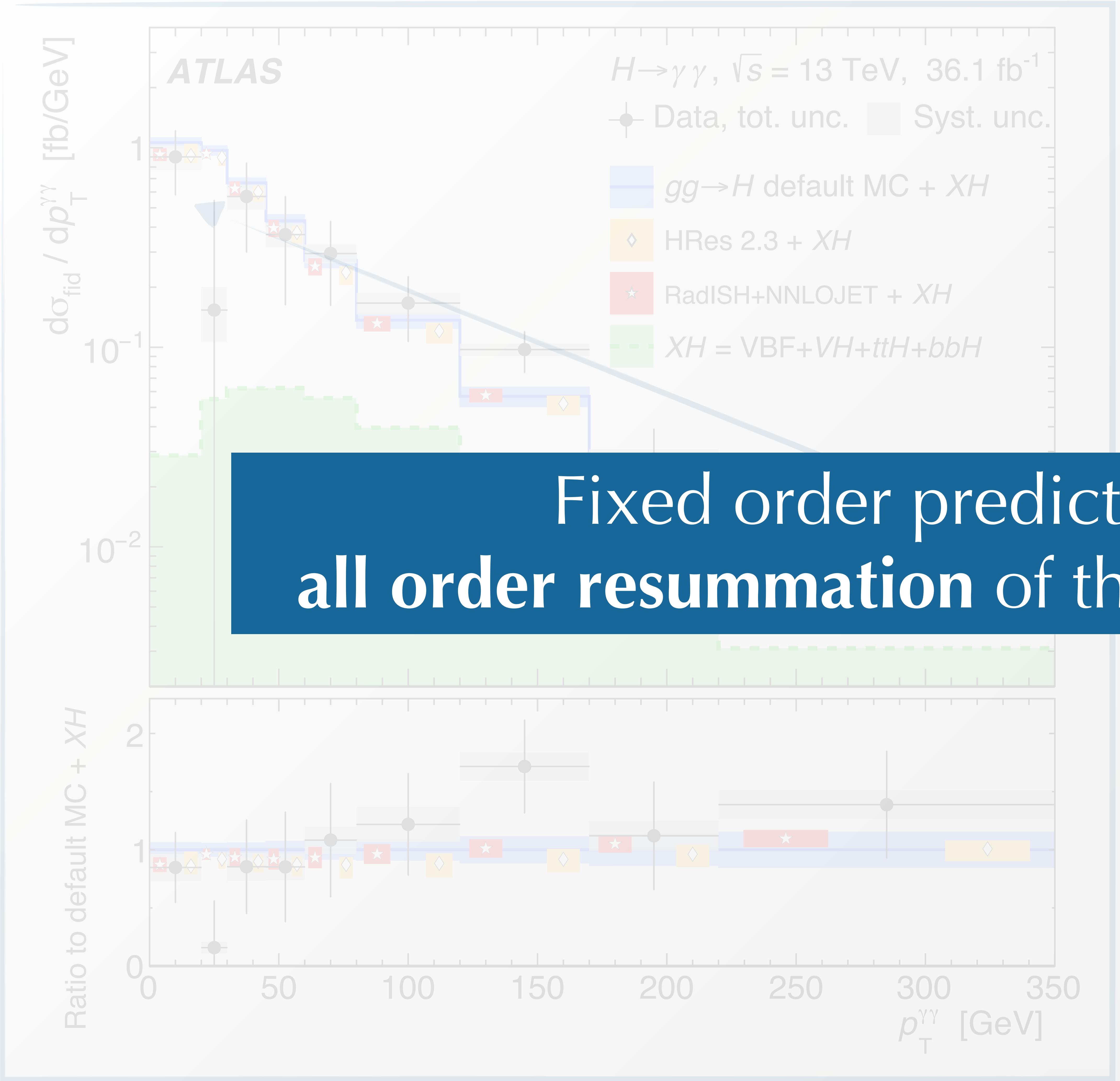


[ATLAS 1802.04146]

Large **transverse momentum** logarithms

$$L = \ln(p_t^H / m_H) \qquad p_t^H \ll m_H$$

Example: the Higgs transverse momentum



[ATLAS 1802.04146]

Fixed order predictions no longer reliable:
all order resummation of the perturbative series mandatory

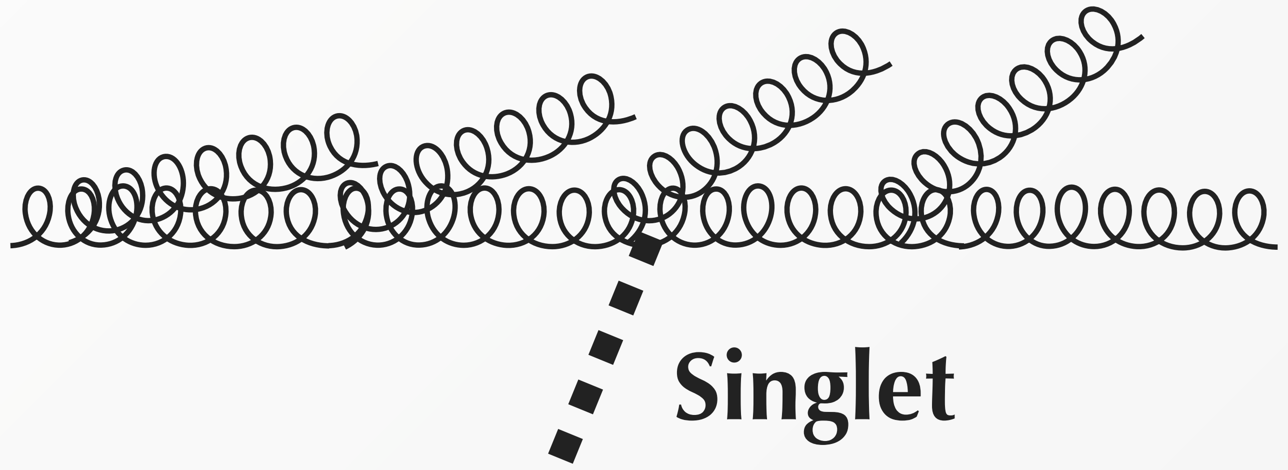
Large **transverse momentum** logarithms

$$L = \ln(p_t^H/m_H) \qquad p_t^H \ll m_H$$

Resummation of the transverse momentum spectrum

Resummation of transverse momentum is delicate because p_t is a **vectorial quantity**

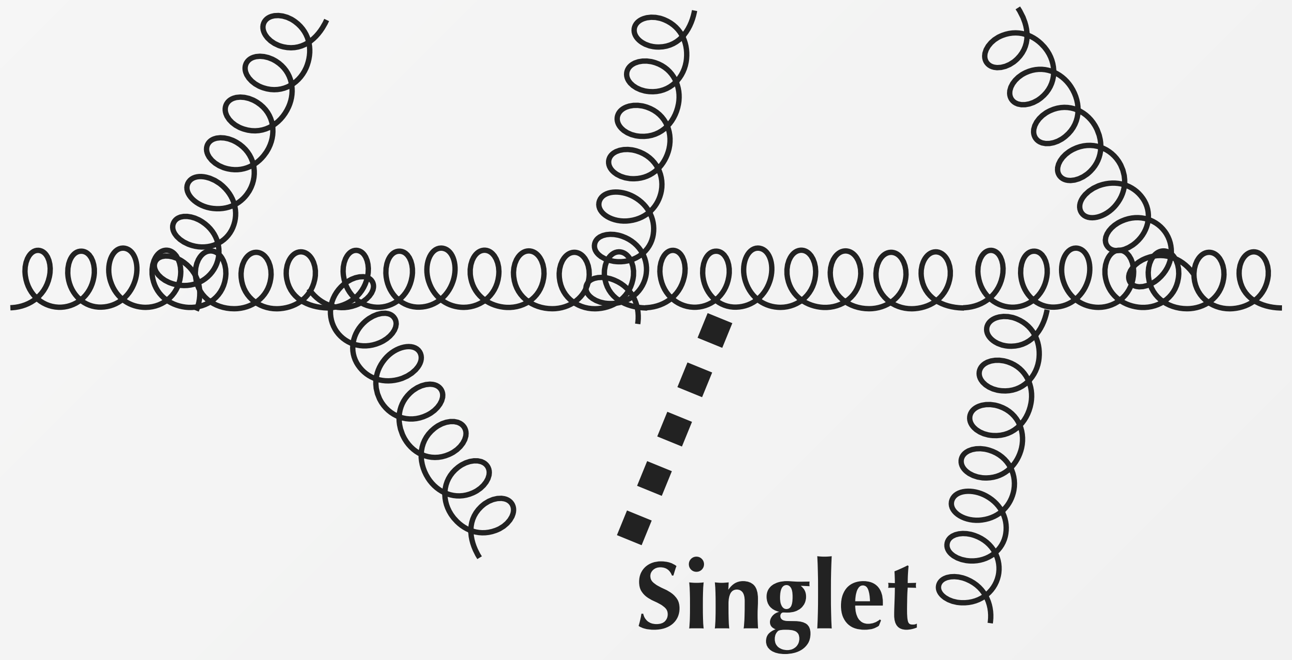
Two concurring mechanisms leading to a system with small p_t



$$p_t^2 \sim k_{t,i}^2 \ll m_H^2$$

cross section naturally suppressed as there is no phase space left for gluon emission (Sudakov limit)

Exponential suppression



$$\sum_{i=1}^{n} \vec{k}_{t,i} \simeq 0$$

Large kinematic cancellations

p_t ~0 far from the Sudakov limit

Power suppression

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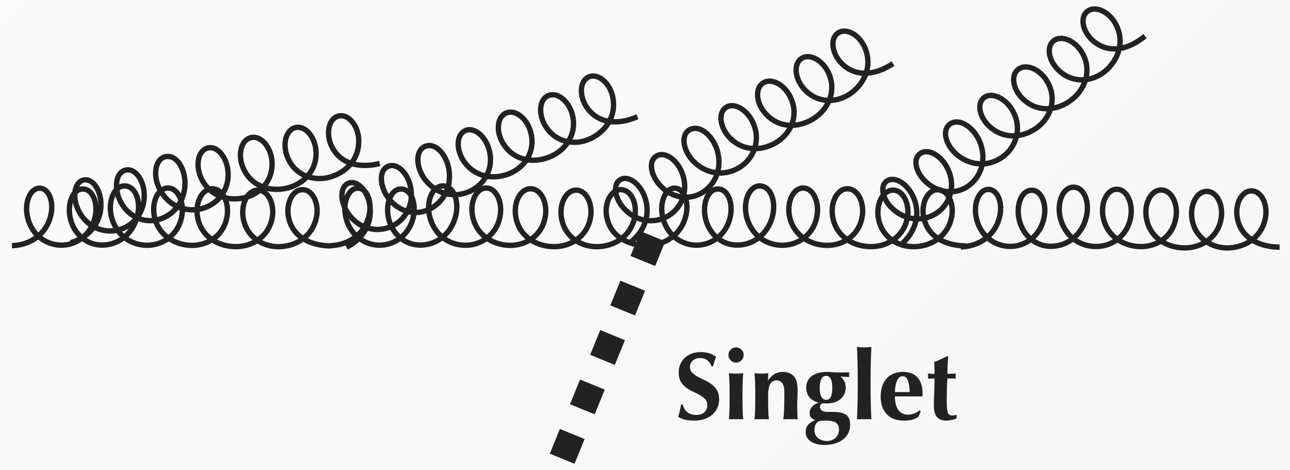

$$p_t^2 \sim k_{t,i}^2 \ll m_H^2$$

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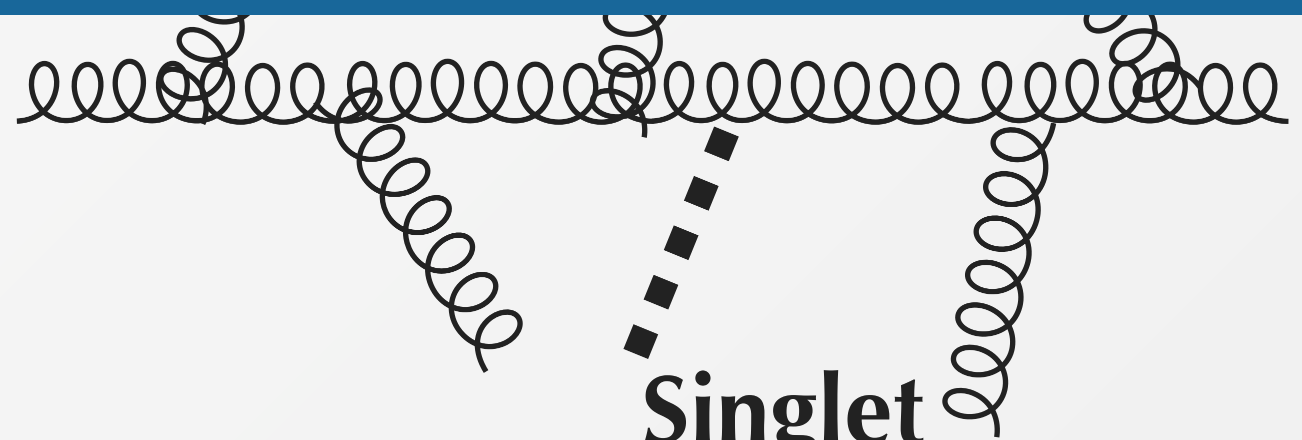
Exponential suppression

Dominant at small p_t

[Parisi, Petronzio, 1979]



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Large kinematic cancellations

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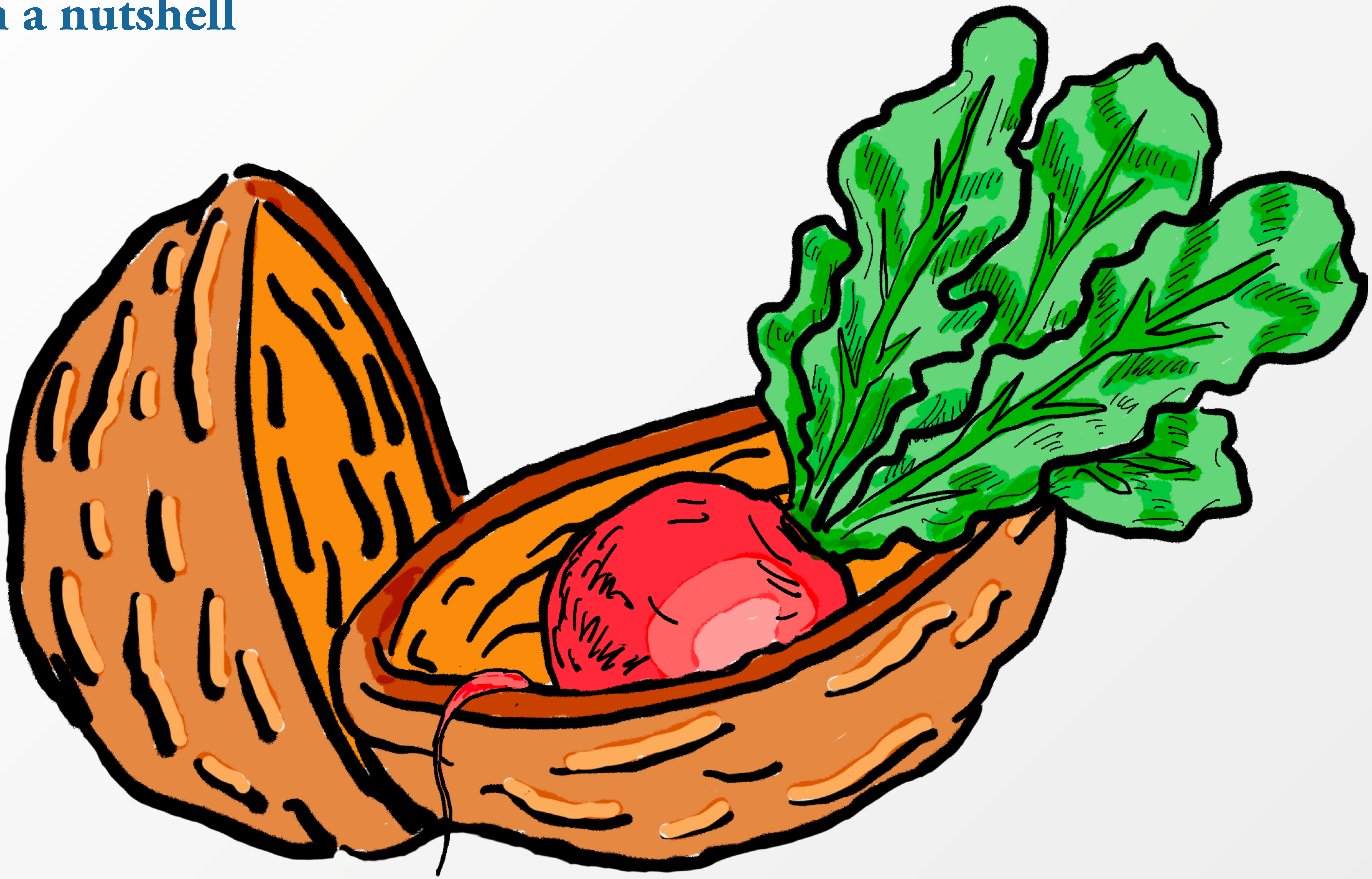
Power suppression

RadISH in a nutshell



RadISH in a nutshell

Resummation of the p_t spectrum in direct space

Result at NLL accuracy (with fixed PDFs) can be written as

$$\sigma(p_t) = \sigma_0 \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(v_1)}$$

Unresolved

$$v_i = k_{t,i}/m_H, \quad \zeta_i = v_i/v_1$$

$$\times\, \epsilon^{R'(v_1)} R'\left(v_1\right) \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^{1} \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'\left(\zeta_i v_1\right)\, \Theta\left(p_t - |\,\vec{k}_{t,i} + \cdots \vec{k}_{t,n+1}|\,\right)$$

Resolved

RadISH in a nutshell

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Unresolved

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Resolved

Formula can be evaluated with Monte Carlo method; dependence on ϵ vanishes (as $\mathcal{O}(\epsilon)$) and result is **finite** in four dimensions

Sudakov and **azimuthal mechanisms** accounted for, **no assumption** on $k_{t,i}$ vs p_t hierarchy.

Logarithmic accuracy defined in terms of $\ln(m_H/k_{t1})$

Result formally equivalent to the b-space formulation [Bizon, Monni, Re, LR, Torrielli '17]

All-order formula in Mellin space [Bizon, Monni, Re, LR, Torrielli '17]

Now include effect of **collinear radiation** and terms beyond NLL accuracy

$$\frac{d\Sigma(v)}{d\Phi_B} = \int_{\mathscr{C}_1} \frac{dN_1}{2\pi i} \int_{\mathscr{C}_2} \frac{dN_2}{2\pi i} x_1^{-N_1} x_2^{-N_2} \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \mathbf{f}^T_{N_1}(\mu_0) \hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) \mathbf{f}_{N_2}(\mu_0),$$

Unresolved

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-\mathbf{R}(\epsilon k_{t1})}$$

$$\times \exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$

Resolved

$$\times \sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi} \mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right),$$

$$\times \sum_{\ell_i=1}^{2}\left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi} \mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right)$$

$$v = p_t/M$$

All-order formula in Mellin space [Bizon, Monni, Re, LR, Torrielli '17]

Now include effect of **collinear radiation** and terms beyond NLL accuracy

$$\frac{d\Sigma(v)}{d\Phi_B} = \int_{\mathscr{C}_1} \frac{dN_1}{2\pi i} \int_{\mathscr{C}_2} \frac{dN_2}{2\pi i} x_1^{-N_1} x_2^{-N_2} \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \mathbf{f}^T_{N_1}(\mu_0) \hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) \mathbf{f}_{N_2}(\mu_0),$$

Unresolved

Sudakov radiator

$v = p_t/M$

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-\mathbf{R}(\epsilon k_{t1})}$$

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Resolved

$$\times \sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi} \mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$

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All-order formula in Mellin space [Bizon, Monni, Re, LR, Torrielli '17]

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Unresolved

Hard-virtual coefficient

hard-momentum region
of the virtual corrections
to the Born process

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-\mathbf{R}(\epsilon k_{t1})}$$

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$v = p_t/M$

Resolved

$$\times \sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi}\mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$

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Collinear coefficient functions and their RGE

Final state parton momenta ~ collinear to the momentum of the initial-state partons

Unresolved

Resolved

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-\mathbf{R}(\epsilon k_{t1})}$$

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All-order formula in Mellin space [Bizon, Monni, Re, LR, Torrielli '17]

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Unresolved

DGLAP evolution

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0))H(\mu_R)\mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right]\int_0^M\frac{dk_{t1}}{k_{t1}}\int_0^{2\pi}\frac{d\phi_1}{2\pi}e^{-\mathbf{R}(\epsilon k_{t1})}$$

$v = p_t/M$

$$\times\exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\frac{\alpha_s(k_t)}{\pi}\mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$

Resolved

$$\times\sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi}\mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$

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Inclusion of N³LL′ effects in RadISH [Re, LR, Torrielli '21]

Capture **all constant terms** of relative order $\mathcal{O}(\alpha_s^3)$

- α_s^3 is N⁴LL (since $\alpha_s^n L^{n-3}$) but sufficient to get all $\alpha_s^n L^{2n-6}$ in the cumulant
- Allows for the computation of **N³LO cross section** for H, DY production based on p_t-slicing methods

[Billis et al. '21][Cieri et al. '21] [Chen et al. '21]

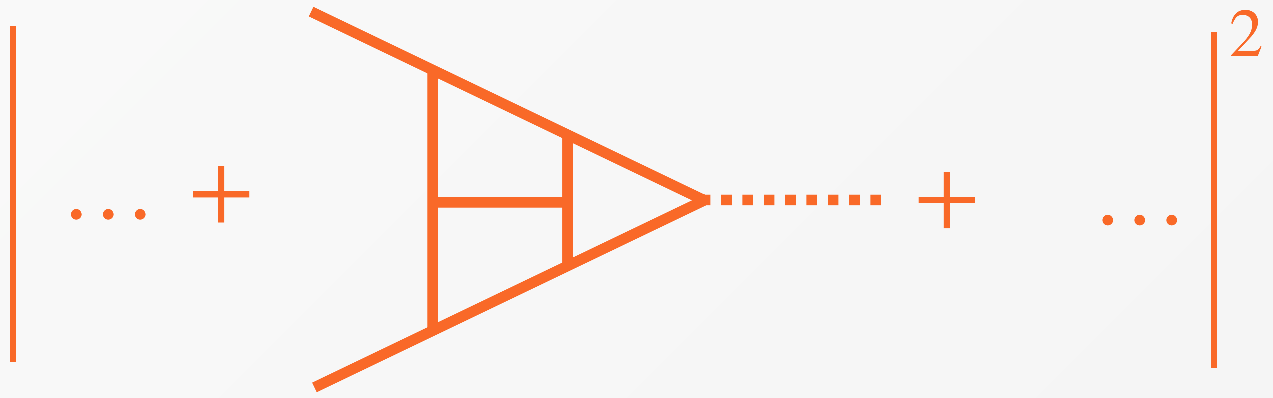
→ *Tongzhi's talk*

Inclusion of N³LL′ effects in RadISH [Re, LR, Torrielli '21]

Sources of N³LL′ correction, neglected in previous RadISH implementation

Three-loop hard-virtual coefficient

$$H(\alpha_s) = 1 + \left(\frac{\alpha_s}{2\pi}\right) H_1 + \left(\frac{\alpha_s}{2\pi}\right)^2 H_2 + \left(\frac{\alpha_s}{2\pi}\right)^3 H_3$$



[Gehrmann et al. '10]

$$\begin{aligned}
\hat{\Sigma}^{c_1,c_2}_{N_1,N_2}(v) = {}& \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-\mathbf{R}(\epsilon k_{t1})} \\
& \times \exp\left\{ - \sum_{\ell=1}^{2} \left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t)) \right) \right\} \\
& \times \sum_{\ell_1=1}^{2} \left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi} \mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1})) \right) \\
& \times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^{1} \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right), \\
& \times \sum_{\ell_i=1}^{2} \left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi} \mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti})) \right)
\end{aligned}$$

Three-loop Wilson coefficient for Higgs EFT

[Schroder, Steinhauser '05]

Inclusion of N³LL′ effects in RadISH [Re, LR, Torrielli '21]

Sources of N³LL′ correction, neglected in previous RadISH implementation

Three-loop coefficient functions

$$C(\alpha_s, z) = \delta(1-z) + \left(\frac{\alpha_s}{2\pi}\right) C_1(z) + \left(\frac{\alpha_s}{2\pi}\right)^2 C_2(z) + \left(\frac{\alpha_s}{2\pi}\right)^3 C_3(z)$$

[Li, Zhu '16][Vladimirov '16][Luo et al. '19][Ebert et al. '20]

$$\hat{\Sigma}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-\mathbf{R}(\epsilon k_{t1})}$$

$$\times \exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$

$$\times \sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi}\mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right),$$

$$\times \sum_{\ell_i=1}^{2}\left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi}\mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right)$$

For Higgs production: **two-loop G coefficient functions**

[Catani, Grazzini '11]

$$G(\alpha_s, z) = \left(\frac{\alpha_s}{2\pi}\right) G_1(z) + \left(\frac{\alpha_s}{2\pi}\right)^2 G_2(z)$$

[Luo et al. '19]

Inclusion of N³LL′ effects in RadISH [Re, LR, Torrielli '21]

Sources of N³LL′ correction, neglected in previous RadISH implementation

Constants terms coming from the Sudakov

$$R(k_{t1}) = -\log\frac{M}{k_{t1}}g_1 - g_2 - \left(\frac{\alpha_s}{\pi}\right)g_3 - \left(\frac{\alpha_s}{\pi}\right)^2 g_4 - \left(\frac{\alpha_s}{\pi}\right)^3 g_5$$

Resummation scale $Q \sim M$

$$\ln\frac{M}{k_{t1}} \to \ln\frac{Q}{k_{t1}} + \ln\frac{M}{Q}$$

Constant terms expanded in α_s and included in H

$$\hat{\Sigma}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0))H(\mu_R)\mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right]\int_0^M\frac{dk_{t1}}{k_{t1}}\int_0^{2\pi}\frac{d\phi_1}{2\pi}e^{-\mathbf{R}(\epsilon k_{t1})}$$

$$\times\exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\frac{\alpha_s(k_t)}{\pi}\mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$

$$\times\sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi}\mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$

$$\times\sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=2}^{n+1}\int_\epsilon^1\frac{d\zeta_i}{\zeta_i}\int_0^{2\pi}\frac{d\phi_i}{2\pi}\Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1})\right),$$

$$\times\sum_{\ell_i=1}^{2}\left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi}\mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right)$$

Inclusion of N³LL′ effects in RadISH [Re, LR, Torrielli '21]

Sources of N³LL′ correction, neglected in previous RadISH implementation

Constants terms coming from resolved contributions

$$\Gamma(\alpha_s) = \Gamma^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)\Gamma^{(1)}$$

$$\Gamma^{(C)}(\alpha_s) = \left(\frac{\alpha_s}{2\pi}\right)^2 \Gamma^{(C,1)}$$

$$\begin{aligned}
\hat{\Sigma}^{c_1,c_2}_{N_1,N_2}(v) &= \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0))H(\mu_R)\mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right]\int_0^M \frac{dk_{t1}}{k_{t1}}\int_0^{2\pi}\frac{d\phi_1}{2\pi}e^{-\mathbf{R}(\epsilon k_{t1})} \\
&\times \exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\frac{\alpha_s(k_t)}{\pi}\boldsymbol{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\boldsymbol{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\} \\
&\times \sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi}\boldsymbol{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \boldsymbol{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right) \\
&\times \sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=2}^{n+1}\int_\epsilon^1\frac{d\zeta_i}{\zeta_i}\int_0^{2\pi}\frac{d\phi_i}{2\pi}\Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1})\right), \\
&\times \sum_{\ell_i=1}^{2}\left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi}\boldsymbol{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \boldsymbol{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right)
\end{aligned}$$

Momentum-space formula at N³LL

$$
\begin{aligned}
\frac{d\Sigma(v)}{d\Phi_B} = & \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \partial_L \left(-e^{-R(k_{t1})} \mathscr{L}_{\mathrm{N^3LL}}(k_{t1})\right) \int d\mathscr{Z} \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right) \\
& + \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(k_{t1})} \int d\mathscr{Z} \int_0^1 \frac{d\zeta_s}{\zeta_s} \frac{d\phi_s}{2\pi} \left\{ \left(R'(k_{t1}) \mathscr{L}_{\mathrm{NNLL}}(k_{t1}) - \partial_L \mathscr{L}_{\mathrm{NNLL}}(k_{t1}) \right) \right. \\
& \times \left(R''(k_{t1}) \ln \frac{1}{\zeta_s} + \frac{1}{2} R'''(k_{t1}) \ln^2 \frac{1}{\zeta_s} \right) - R'(k_{t1}) \left(\partial_L \mathscr{L}_{\mathrm{NNLL}}(k_{t1}) - 2 \frac{\beta_0}{\pi} \alpha_s^2(k_{t1}) \hat{P}^{(0)} \otimes \mathscr{L}_{\mathrm{NLL}}(k_{t1}) \ln \frac{1}{\zeta_s} \right) \\
& \left. + \frac{\alpha_s^2(k_{t1})}{\pi^2} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathscr{L}_{\mathrm{NLL}}(k_{t1}) \right\} \\
& \times \left\{ \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_s)\right) - \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right) \right\} \\
& + \frac{1}{2} \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(k_{t1})} \int d\mathscr{Z} \int_0^1 \frac{d\zeta_{s1}}{\zeta_{s1}} \frac{d\phi_{s1}}{2\pi} \int_0^1 \frac{d\zeta_{s2}}{\zeta_{s2}} \frac{d\phi_{s2}}{2\pi} R'(k_{t1}) \left\{ \mathscr{L}_{\mathrm{NLL}}(k_{t1}) \left(R''(k_{t1})\right)^2 \ln \frac{1}{\zeta_{s1}} \ln \frac{1}{\zeta_{s2}} - \partial_L \mathscr{L}_{\mathrm{NLL}}(k_{t1}) R''(k_{t1}) \left(\ln \frac{1}{\zeta_{s1}} + \ln \frac{1}{\zeta_{s2}} \right) \right. \\
& \left. + \frac{\alpha_s^2(k_{t1})}{\pi^2} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathscr{L}_{\mathrm{NLL}}(k_{t1}) \right\} \\
& \times \left\{ \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_{s1}, k_{s2})\right) - \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_{s1})\right) - \right. \\
& \left. \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_{s2})\right) + \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right) \right\} + \mathcal{O}\left(\alpha_s^n \ln^{2n-6} \frac{1}{v}\right).
\end{aligned}
$$

Momentum-space formula at N^{3}LL$'$

$$
\begin{aligned}
\frac{d\Sigma(v)}{d\Phi_B} = & \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \partial_L \left(-e^{-R(k_{t1})} \mathscr{L}_{\mathrm{N^3LL'}}(k_{t1})\right) \int d\mathcal{Z} \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right) \\
& + \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(k_{t1})} \int d\mathcal{Z} \int_0^1 \frac{d\zeta_s}{\zeta_s} \frac{d\phi_s}{2\pi} \Bigg\{ \left(R'(k_{t1}) \mathscr{L}_{\mathrm{NNLL}}(k_{t1}) - \partial_L \mathscr{L}_{\mathrm{NNLL}}(k_{t1}) \right) \\
& \times \left(R''(k_{t1}) \ln\frac{1}{\zeta_s} + \frac{1}{2} R'''(k_{t1}) \ln^2 \frac{1}{\zeta_s} \right) - R'(k_{t1}) \left(\partial_L \mathscr{L}_{\mathrm{NNLL}}(k_{t1}) - 2\frac{\beta_0}{\pi} \alpha_s^2(k_{t1}) \hat{P}^{(0)} \otimes \mathscr{L}_{\mathrm{NLL}}(k_{t1}) \ln\frac{1}{\zeta_s} \right) \\
& + \frac{\alpha_s^2(k_{t1})}{\pi^2} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathscr{L}_{\mathrm{NLL'}}(k_{t1}) - \beta_0 \frac{\alpha_s^3(k_{t1})}{\pi^2} \left(\hat{P}^{(0)} \otimes \hat{C}^{(1)} + \hat{C}^{(1)} \otimes \hat{P}^{(0)} \right) \otimes \mathscr{L}_{\mathrm{NLL}}(k_{(t1)}) + \frac{\alpha_s^3(k_{t1})}{\pi^2} 2\beta_0 \ln\frac{1}{\zeta_s} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathscr{L}_{\mathrm{NLL}}(k_{(t1)}) \\
& + \frac{\alpha_s^3(k_{t1})}{2\pi^2} \left(\hat{P}^{(0)} \otimes \hat{P}^{(1)} + \hat{P}^{(1)} \otimes \hat{P}^{(0)} \right) \otimes \mathscr{L}_{\mathrm{NLL}}(k_{(t1)}) \Bigg\} \times \left\{ \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_s)\right) - \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right) \right\} \\
& + \frac{1}{2} \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(k_{t1})} \int d\mathcal{Z} \int_0^1 \frac{d\zeta_{s1}}{\zeta_{s1}} \frac{d\phi_{s1}}{2\pi} \int_0^1 \frac{d\zeta_{s2}}{\zeta_{s2}} \frac{d\phi_{s2}}{2\pi} R'(k_{t1}) \Bigg\{ \mathscr{L}_{\mathrm{NLL}}(k_{t1}) \left(R''(k_{t1})\right)^2 \ln\frac{1}{\zeta_{s1}} \ln\frac{1}{\zeta_{s2}} - \partial_L \mathscr{L}_{\mathrm{NLL}}(k_{t1}) R''(k_{t1}) \left(\ln\frac{1}{\zeta_{s1}} + \ln\frac{1}{\zeta_{s2}} \right) \\
& + \frac{\alpha_s^2(k_{t1})}{\pi^2} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathscr{L}_{\mathrm{NLL}}(k_{t1}) + \frac{\alpha_s^2(k_{t1})}{\pi^2} \left(\ln\frac{1}{\zeta_{s1}} + \ln\frac{1}{\zeta_{s2}} \right) R''(k_{t1}) \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathscr{L}_{\mathrm{NLL}}(k_{(t1)}) - \ln\frac{1}{\zeta_{s1}} \ln\frac{1}{\zeta_{s2}} (R''(k_{t1})^2 \partial_L \mathscr{L}_{\mathrm{NLL}}(k_{(t1)}) \\
& + \frac{\alpha_s^2(k_{t1})}{\pi^3} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathscr{L}_{\mathrm{NLL}}(k_{t1}) \Bigg\} \times \Big\{ \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_{s1}, k_{s2})\right) - \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_{s1})\right) - \\
& \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_{s2})\right) + \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right) \Big\} + \mathcal{O}\left(\alpha_s^n \ln^{2n-7} \frac{1}{v} \right)
\end{aligned}
$$

Luminosity factors now contains H_3 and C_3

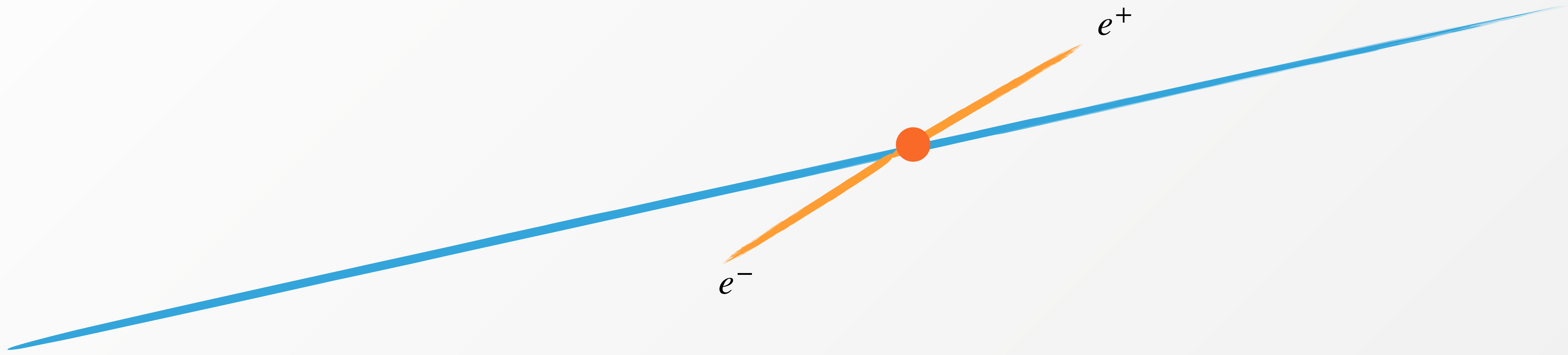
New structures appearing at α_s^3

Convolution structure obtained after Mellin inversion

Extra column of logs predicted

Inclusion of transverse recoil effects

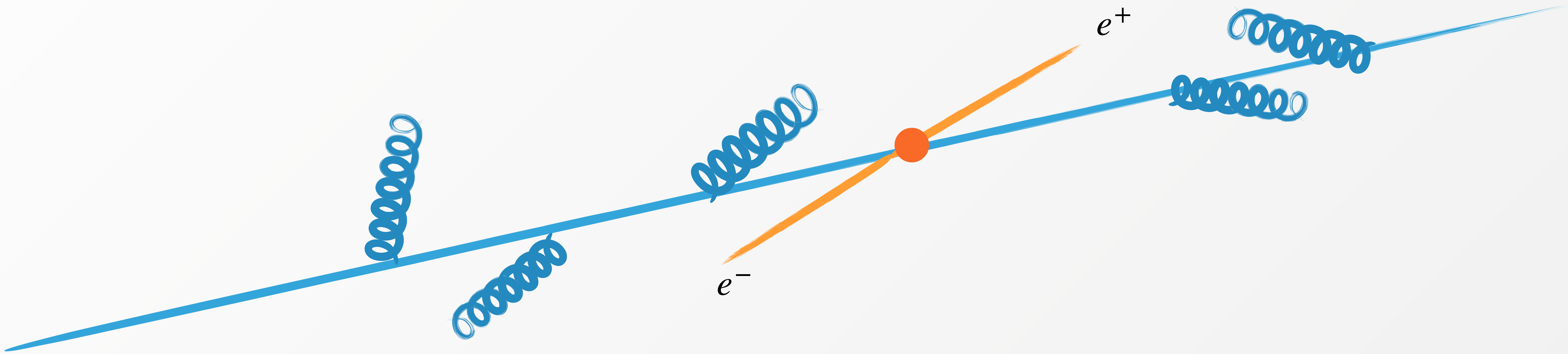
[Catani et al. '15]



Born matrix element
evaluated at $p_t = 0$

Inclusion of transverse recoil effects

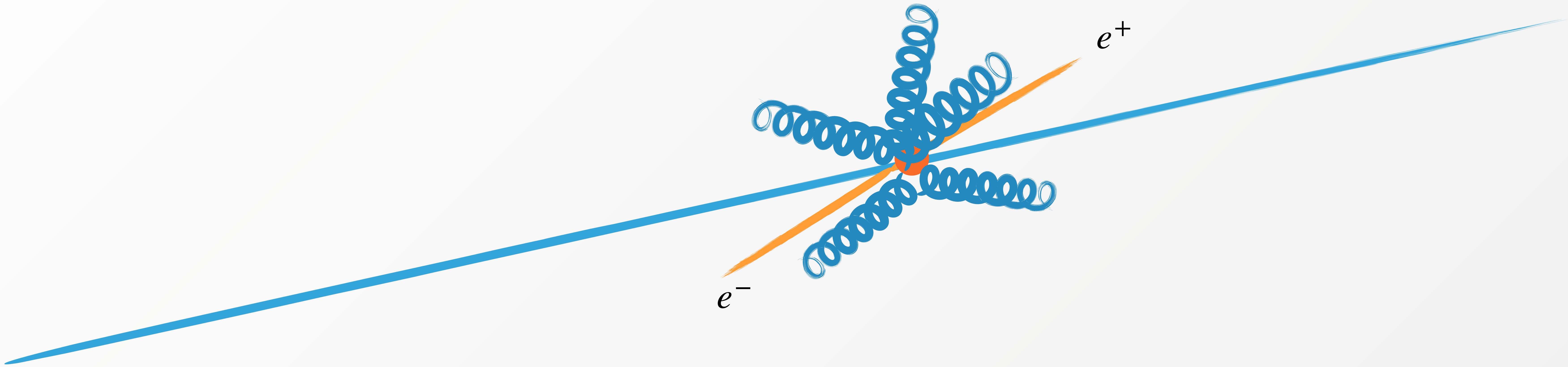
[Catani et al. '15]



Generate singlet p_t by QCD radiation

Inclusion of transverse recoil effects

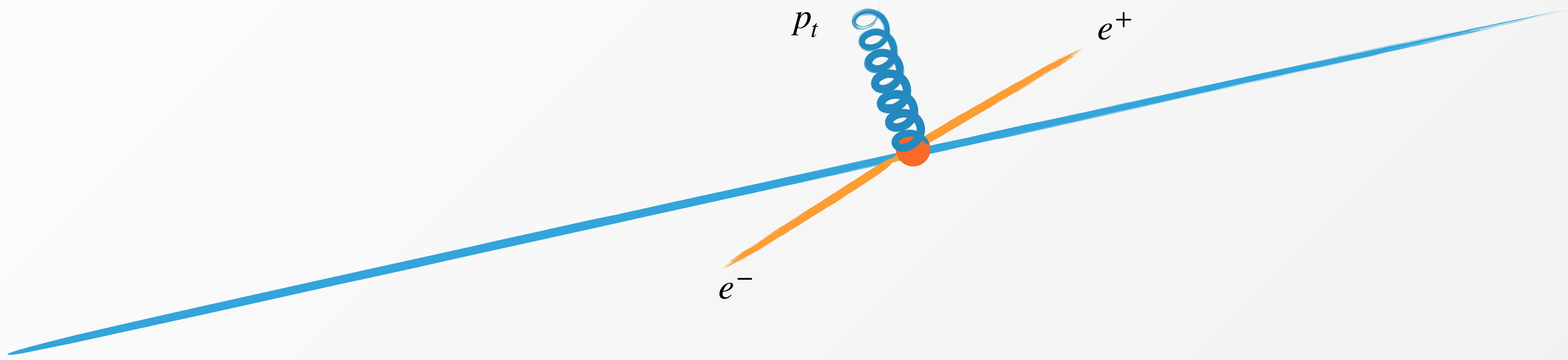
[Catani et al. '15]



Generate singlet p_t by QCD radiation

Inclusion of transverse recoil effects

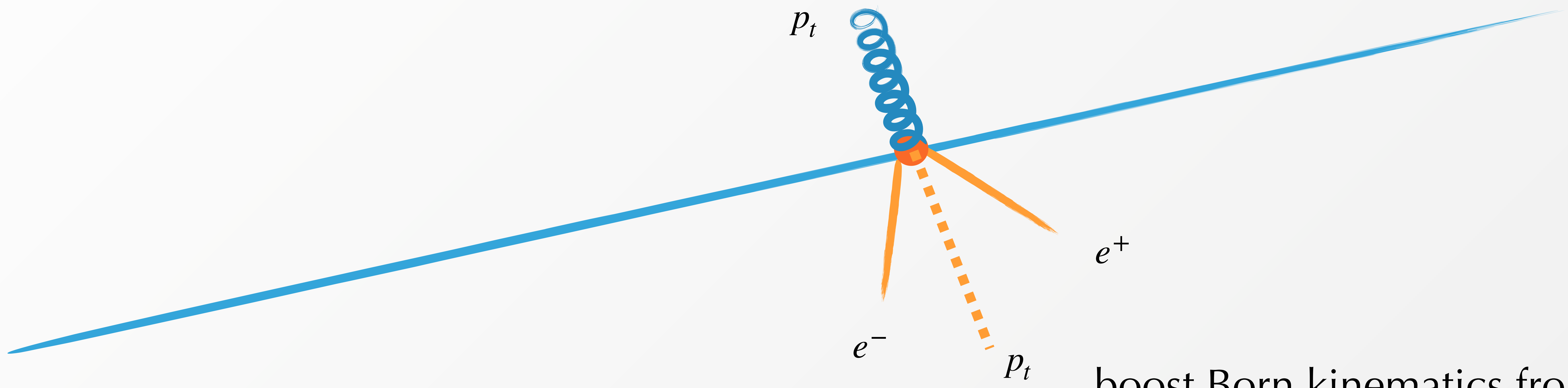
[Catani et al. '15]



Generate singlet p_t by QCD radiation

Inclusion of transverse recoil effects

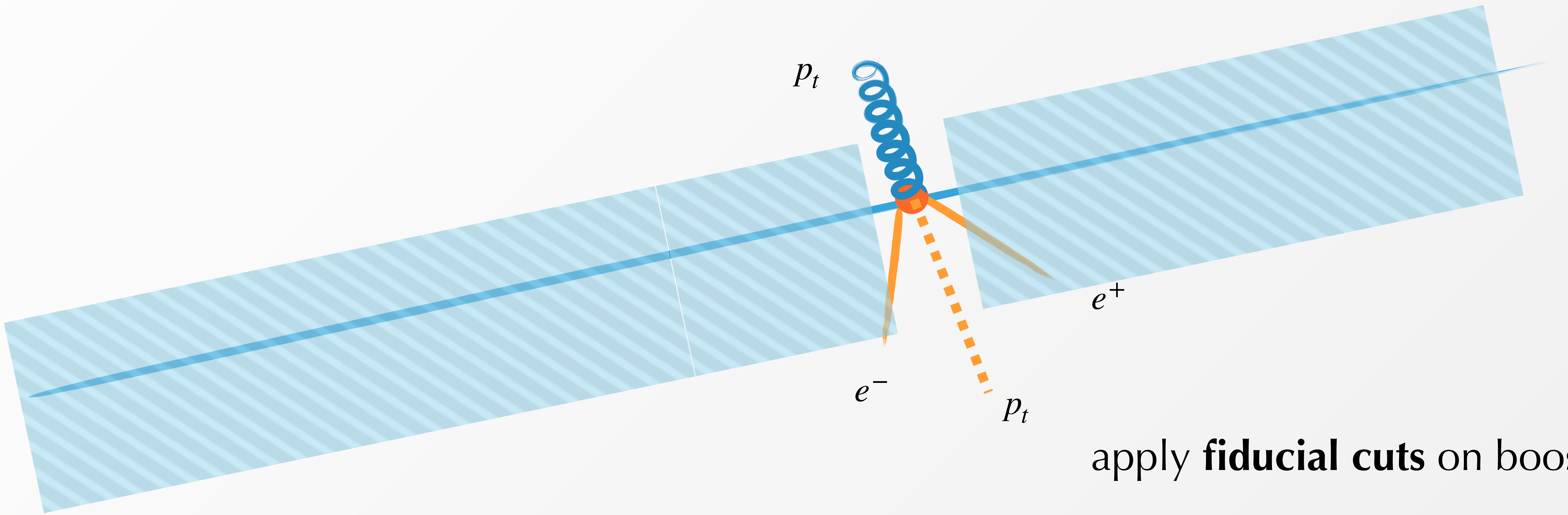
[Catani et al. '15]



boost Born kinematics from boson rest frame (e.g. CS) to lab frame with that p_t

Inclusion of transverse recoil effects

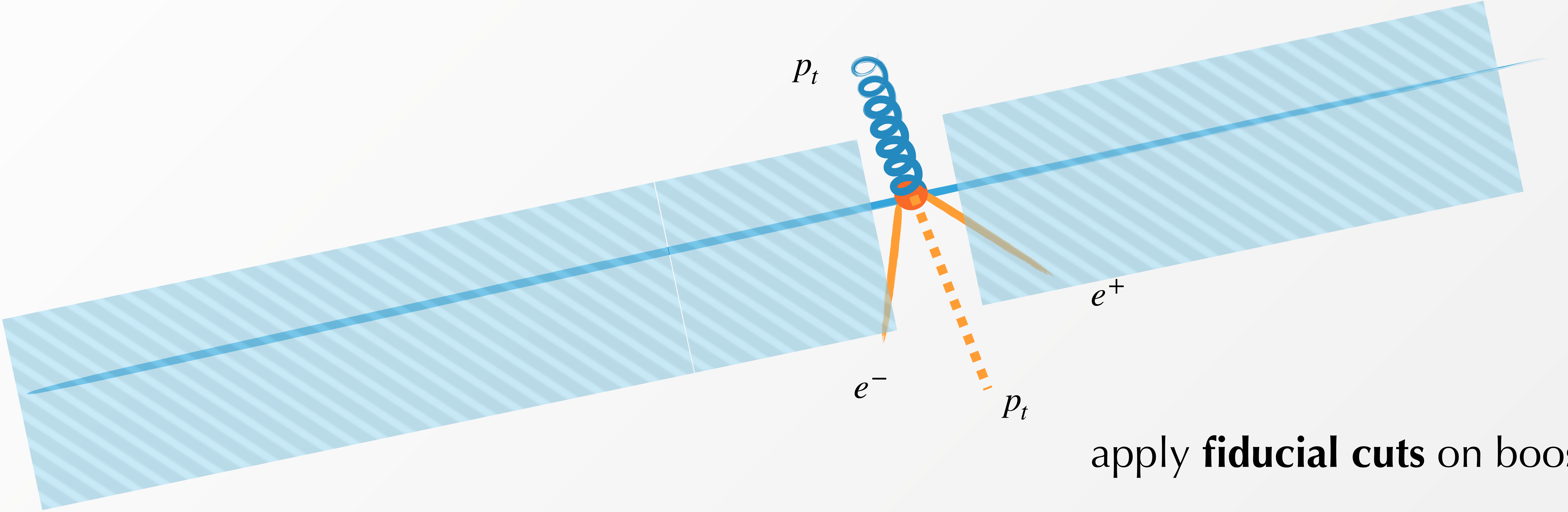
[Catani et al. '15]



apply **fiducial cuts** on boosted Born kinematics

Inclusion of transverse recoil effects

[Catani et al. '15]

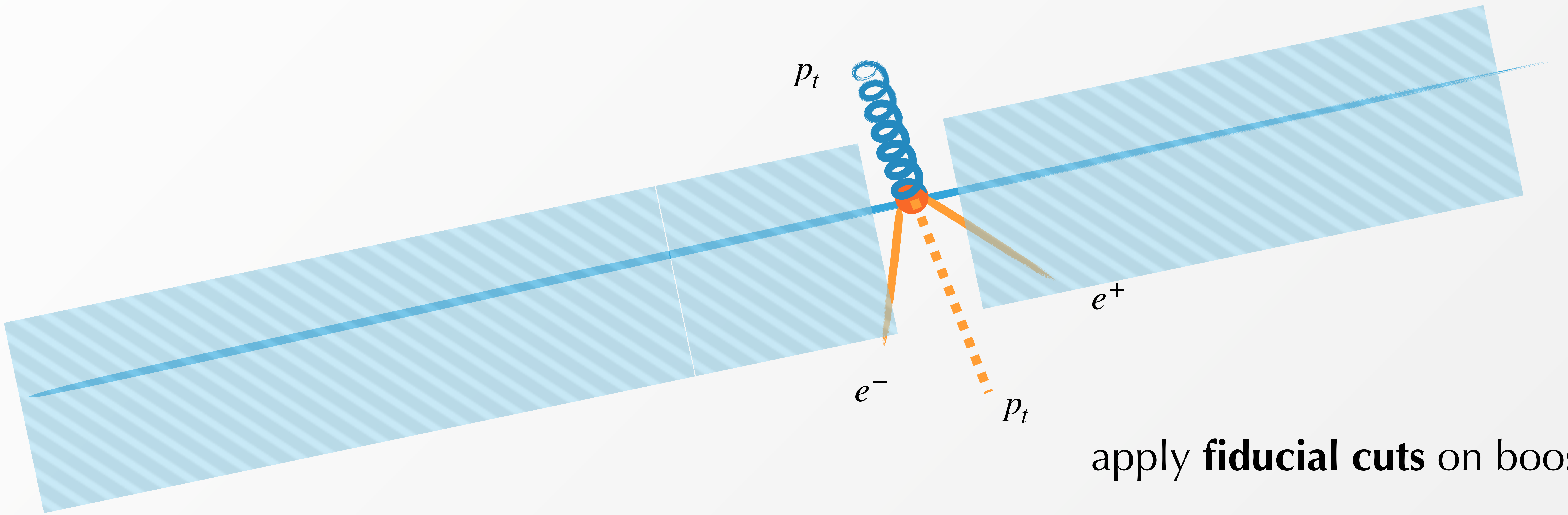


apply **fiducial cuts** on boosted Born kinematics

Sufficient to capture the **full linear fiducial power correction** for p_t [Ebert et al. '20]

Inclusion of transverse recoil effects

[Catani et al. '15]



apply **fiducial cuts** on boosted Born kinematics

Implementation in RadISH:

- Each contribution in the resummation formula boosted in the corresponding frame
- Derivative of the expansion computed on-the-fly, boost computed according to the value of p_t

Results

Matching to fixed order

Two different families of **matching schemes**, defined at the **differential** level (due to the inclusion of **recoil effects**)

Additive matching

$$\frac{d\Sigma_{\rm add}^{{\rm N}^k{\rm LL}^{(\prime)}}(v)}{dv} = \left(\frac{d\Sigma^{{\rm N}^k{\rm LL}^{(\prime)}}(v)}{dv} - \frac{d\Sigma_{\rm exp}^{{\rm N}^k{\rm LL}^{(\prime)}}(v)}{dv}\right) Z(v) + \frac{d\Sigma^{{\rm N}^{k-1}{\rm LO}}(v)}{dv}$$

Multiplicative matching

$$\frac{d\Sigma_{\rm mult}^{{\rm N}^k{\rm LL}^{(\prime)}}(v)}{dv} = \left(\frac{d\Sigma^{{\rm N}^k{\rm LL}^{(\prime)}}(v)/dv}{d\Sigma_{\rm exp}^{{\rm N}^k{\rm LL}^{(\prime)}}(v)/dv}\right)^{Z(v)} \frac{d\Sigma^{{\rm N}^{k-1}{\rm LO}}(v)}{dv}$$

At NNLO+N^{3}LL$'$ the two matching schemes are on equal footing, differences starts at α_s^4

Damping function (does not act on linear power corrections)

$$Z(v) = \left[1 - (v/v_0)^2\right]^3 \Theta(v_0 - v)$$

v_0 varied in the interval [2/3, 3/2] around central value to **estimate matching uncertainty**

Central value $v_0 = 1$ for $p_\perp$ and $v_0 = 1/2$ for ϕ_η^*

Drell-Yan production: setup

Drell-Yan fiducial region defined as [ATLAS 2019]

$$p_t^{\ell^\pm} > 27\,\mathrm{GeV}\,, \qquad |\eta^{\ell^\pm}| < 2.5\,, \qquad 66\,\mathrm{GeV} < m_{\ell\ell} < 116\,\mathrm{GeV}$$

Central scales chosen as

$$\mu_R = \kappa_R M_t \qquad \mu_F = \kappa_F M_t, \qquad Q = \kappa_Q M_{\ell\ell} \qquad M_t = \sqrt{M_{\ell\ell}^2 + p_t^{\ell\ell 2}}$$

In resummed predictions $M_t \to M_{\ell\ell} = M_t + \mathcal{O}\left(\frac{p_t^{\ell\ell}}{M_{\ell\ell}}\right)^2$

Scale uncertainty:

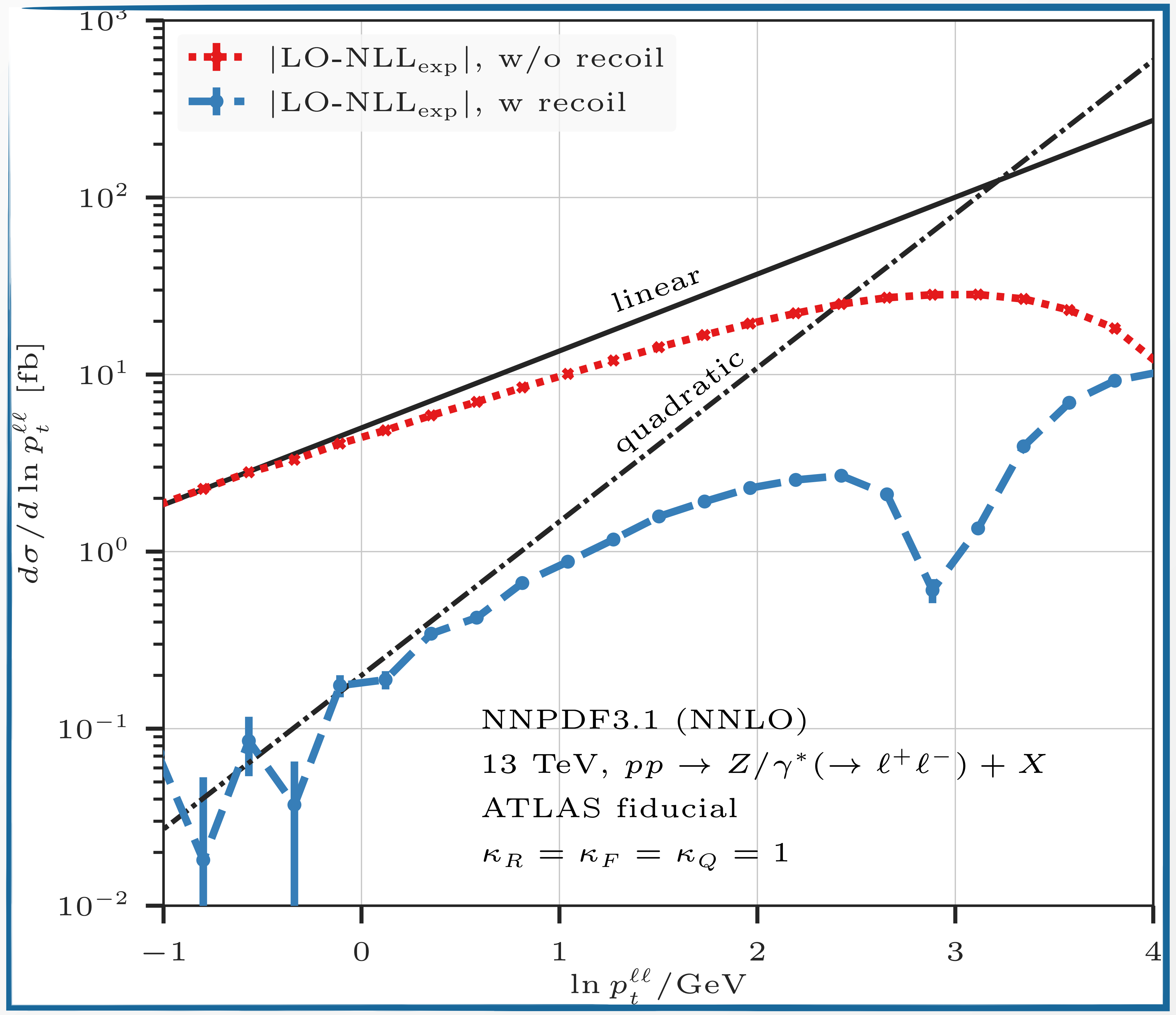
[canonical 7 scale variation + variation of κ_Q by a factor of 2 for central μ_R, μ_F] $\times$ 3 values of v_0 $\rightarrow$ **27 variations**

NNPDF31 NNLO parton densities with $\alpha_s = 0.118$. NNLO predictions from NNLOJET

Transverse recoil effects in fiducial DY setup

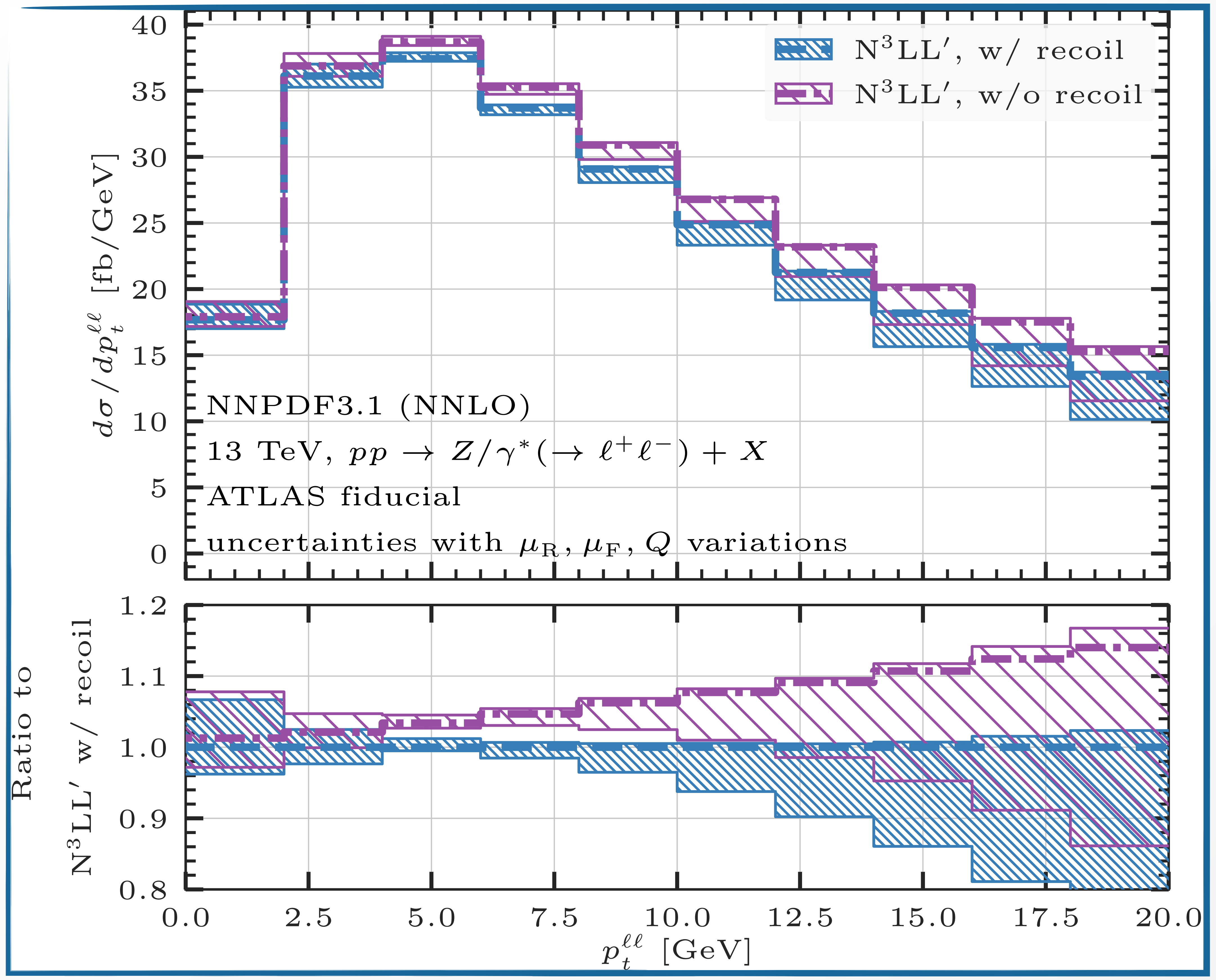
Symmetric cuts on the dileptons induce linear power corrections in the fiducial spectrum

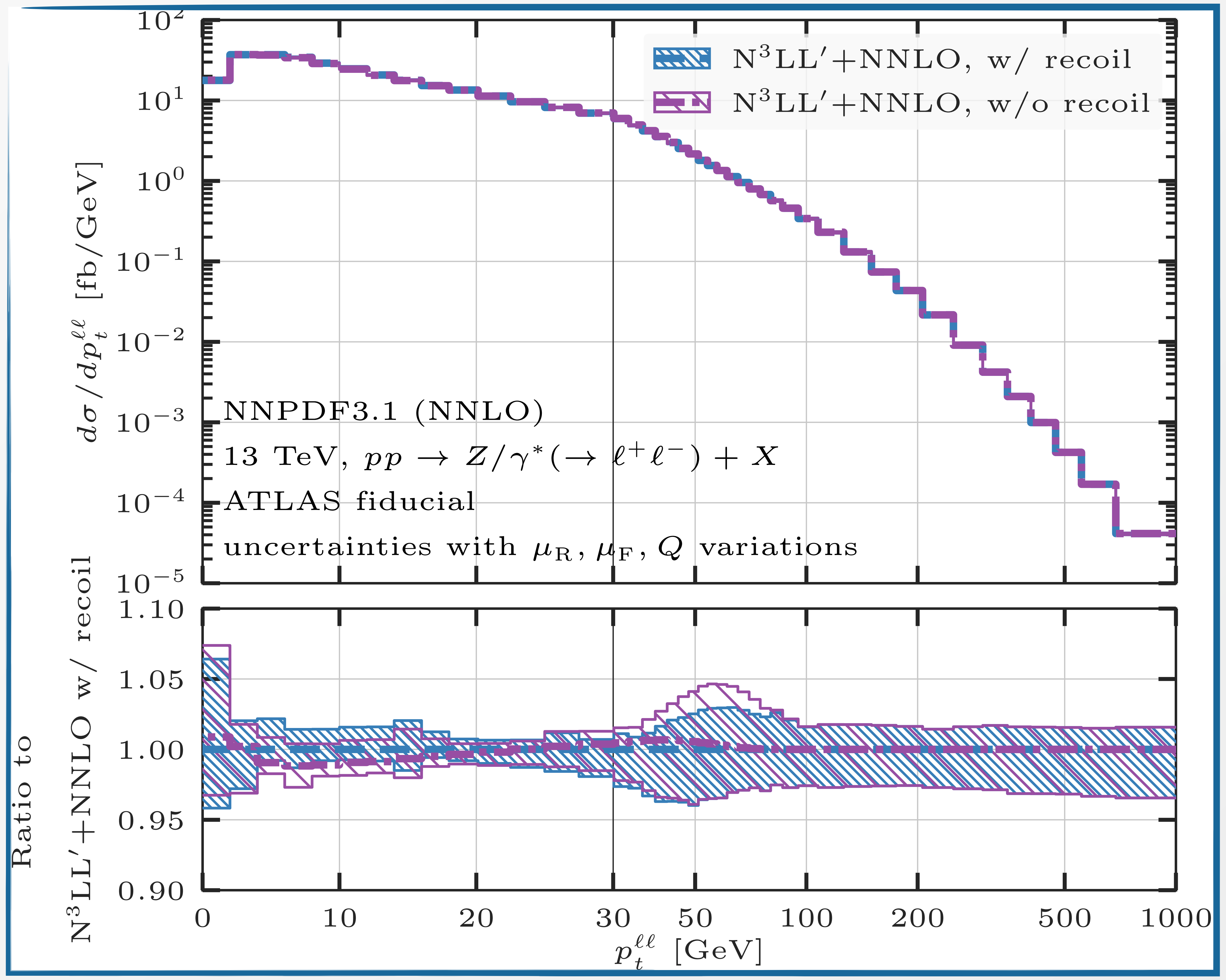
Can be avoided by suitable choice of cuts [Salam, Slade '21]



Recoil effectively captures the **full linear fiducial power correction** for p_t

Transverse recoil effects in fiducial DY setup





At the pure resummed level recoil prescription captures whole linear power corrections from fiducial cuts

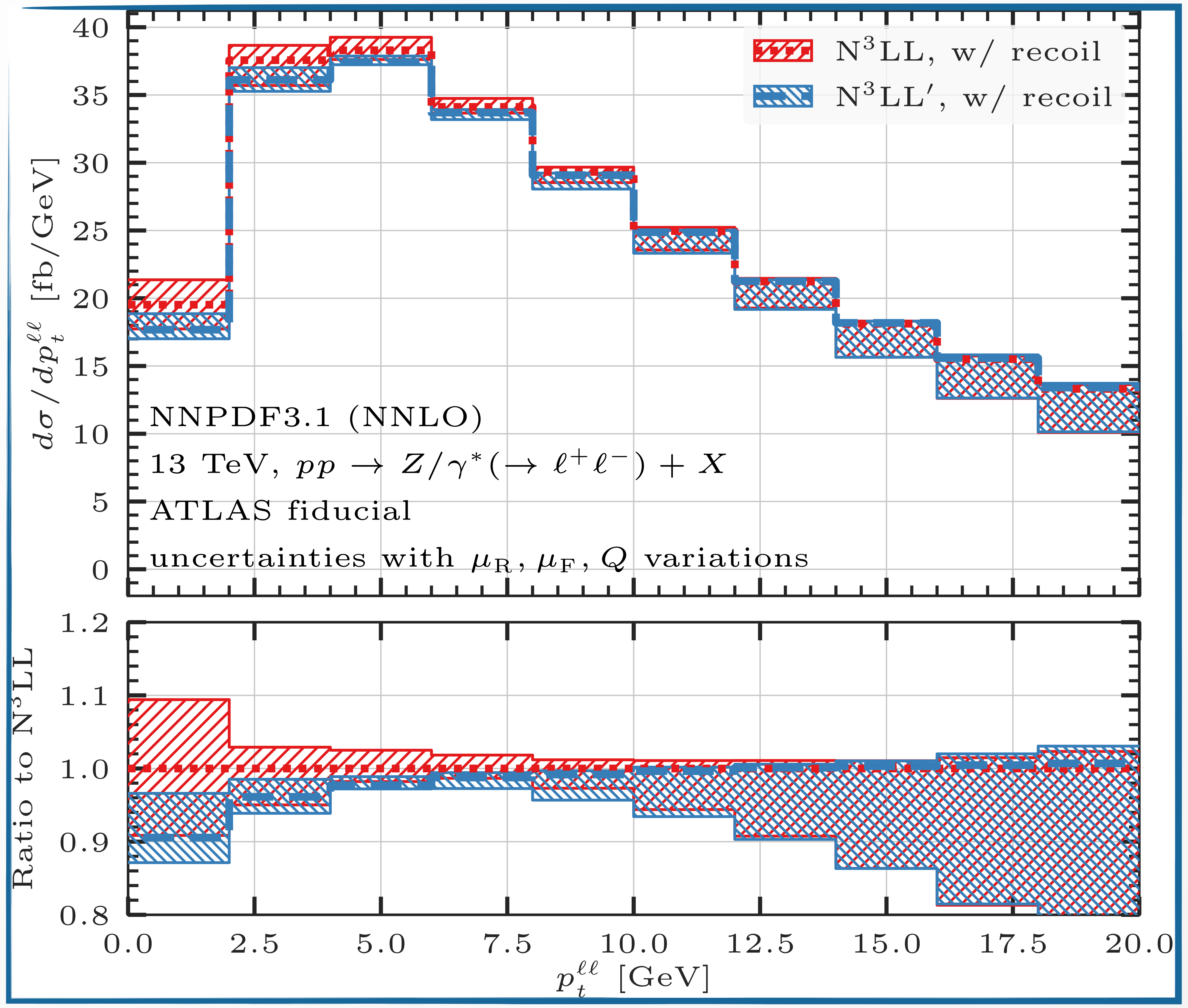
Effect reduce at 1-2% level after matching to fixed order (effect becomes $\mathcal{O}(\alpha_s^4)$)

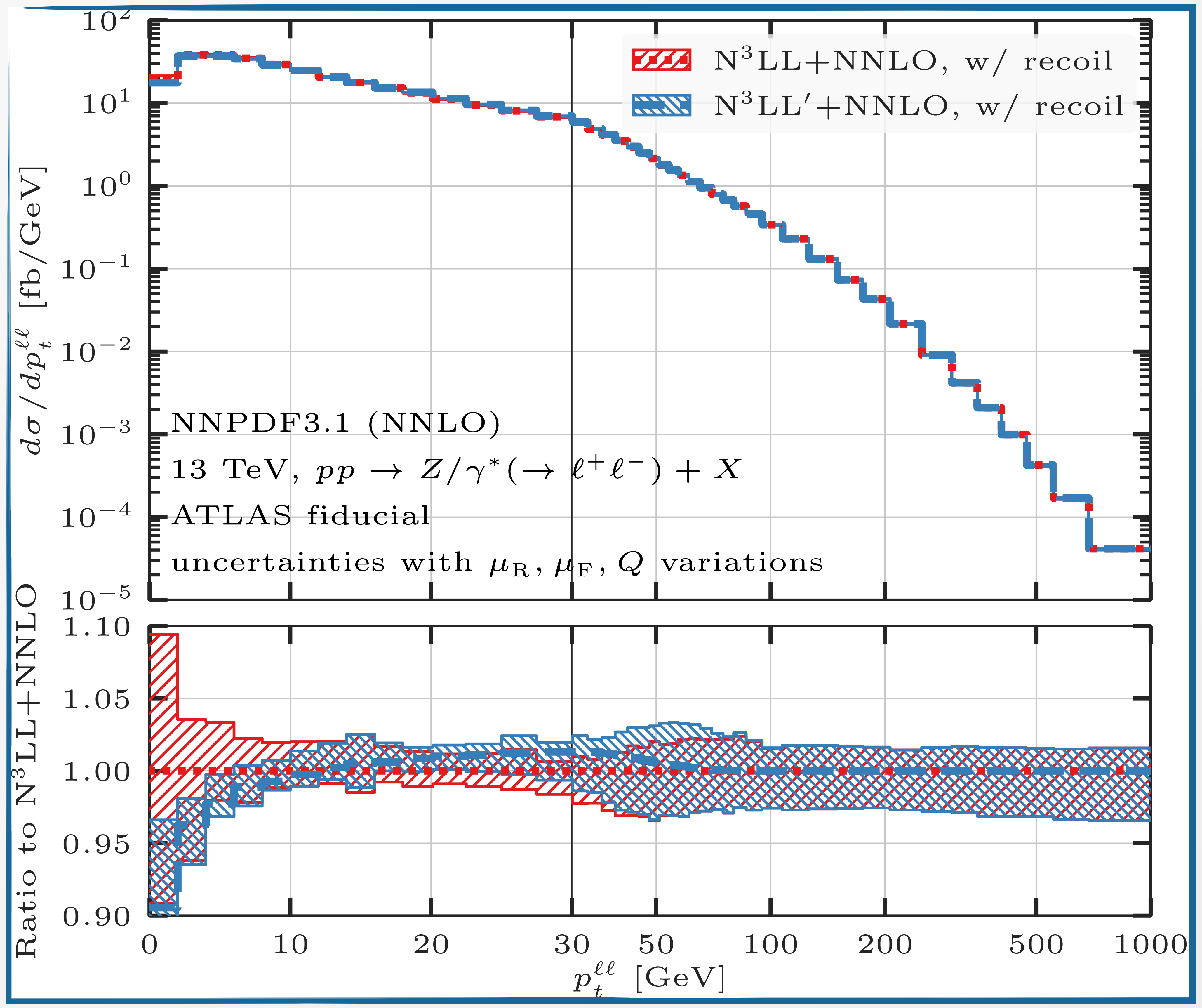
$$\ln(Q/k_{t1}) \to 1/p \ln(1 + (Q/k_{t1})^p)$$

Pure resummed: band widening due to power corrections due to modified logs

$$\int_0^M \frac{dk_{t1}}{k_{t1}} \to \int_0^\infty \frac{dk_{t1}}{k_{t1}} \frac{(Q/k_{t1})^p}{1 + (Q/k_{t1})^p}$$

Drell-Yan production: N^3LL' effects





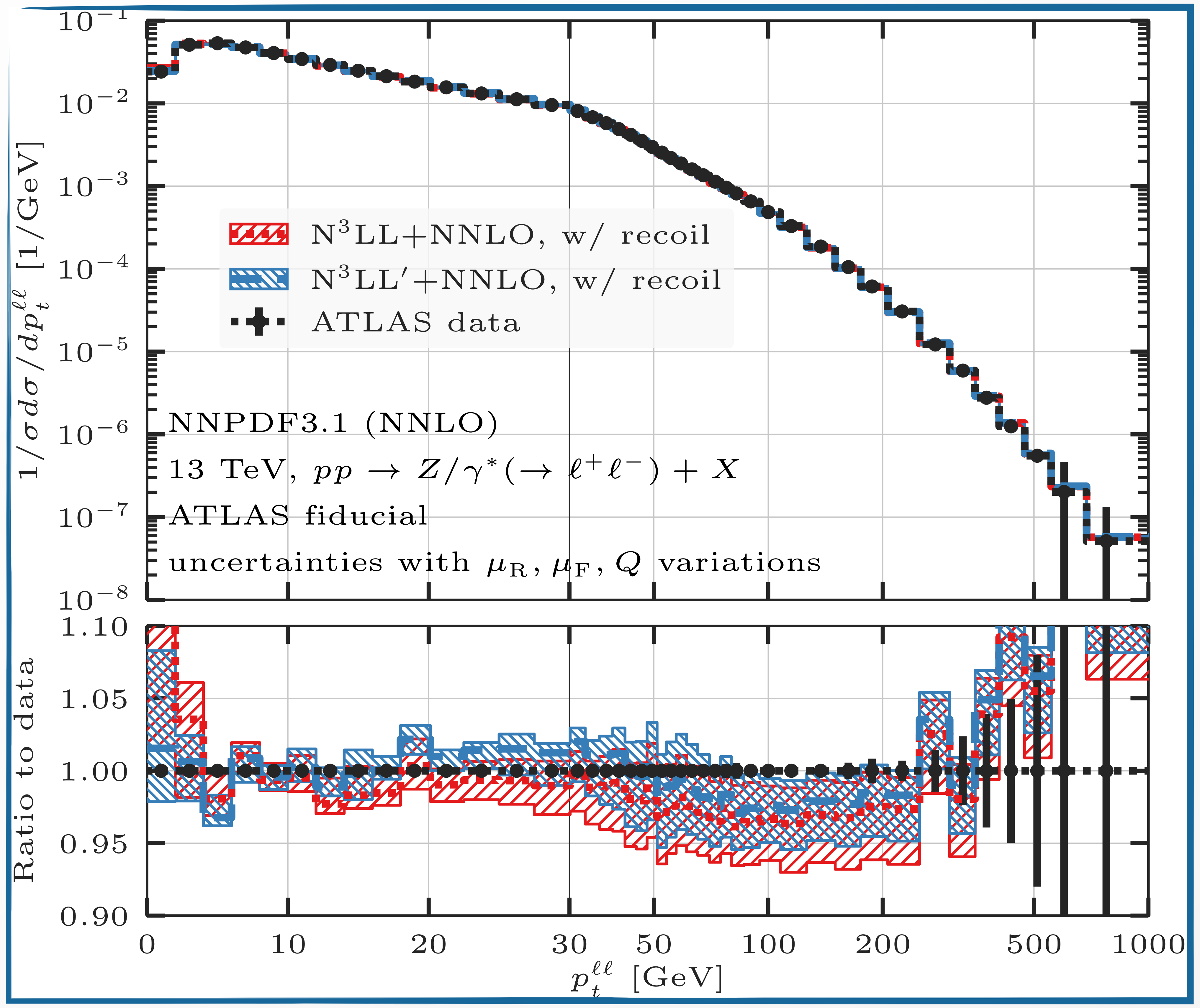
Reduction in theoretical uncertainty below 10 GeV

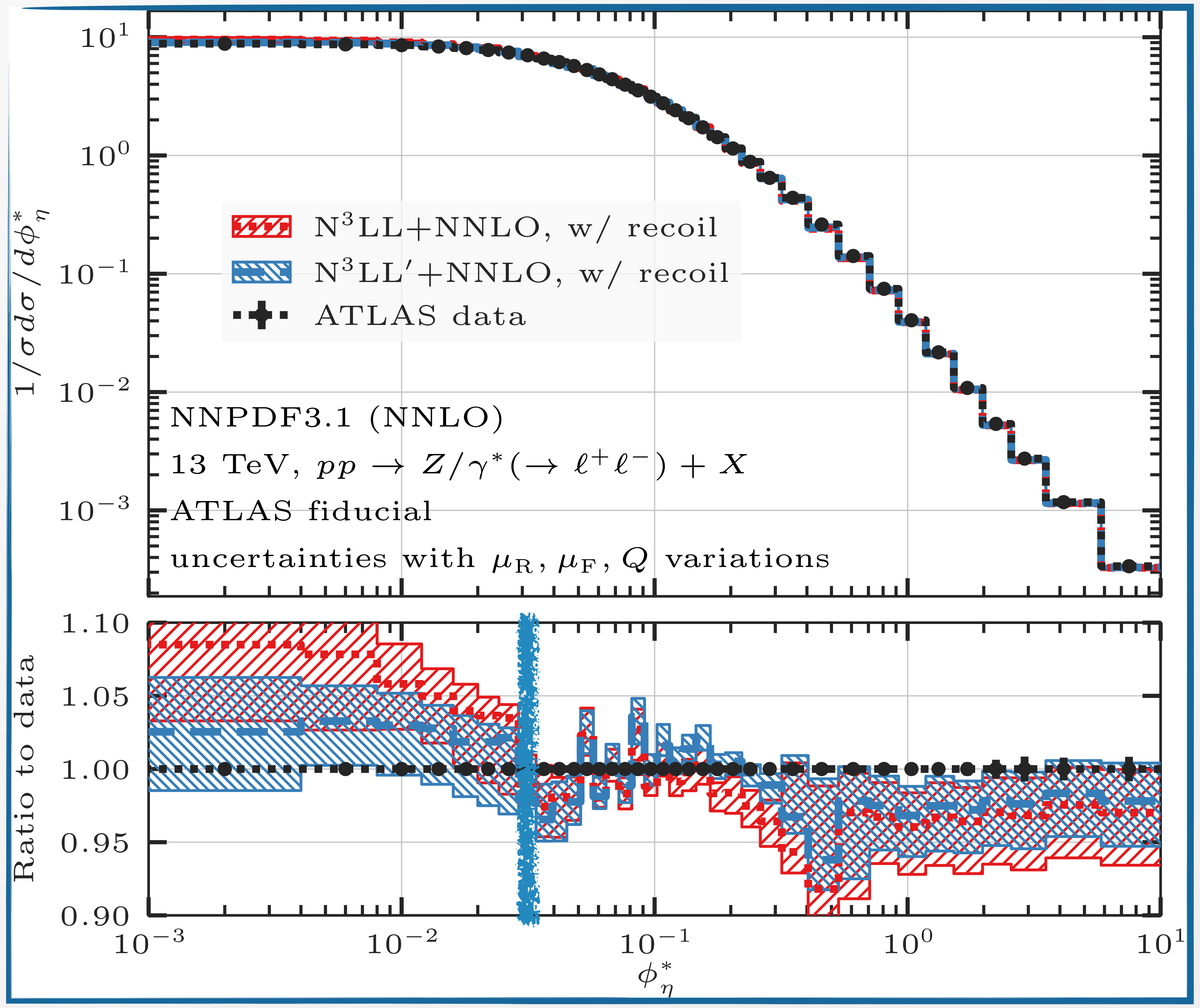
$\kappa_R = \kappa_F = 1, \qquad \kappa_Q = 1/2$

Modification at the **5-10% level** below 10 GeV (similar effect, but larger, present at NNLL vs NNLL′)

Minor differences with respect to N^3LL for value of p_t larger than 5 GeV

Drell-Yan production: comparison with ATLAS data





N³LL′+NNLO **improves the description of data** w.r.t. N³LL+NNLO

Theoretical uncertainties at the **few percent level** across the whole range

High statistic runs needed for the description of ϕ^*_η in the singular region (fixed-order component set to 0)

Marginal effect of recoil after matching (1-2% effect)

Higgs production: setup

Higgs fiducial region defined as [ATLAS 2018]

$$\min(p_t^{\gamma_1}, p_t^{\gamma_2}) > 31.25\ \text{GeV}\,, \qquad \max(p_t^{\gamma_1}, p_t^{\gamma_2}) > 43.75\ \text{GeV}$$

$$0 < |\eta^{\gamma_{1,2}}| < 1.37 \quad \text{or} \quad 1.52 < |\eta^{\gamma_{1,2}}| < 2.37\,, \qquad |Y_{\gamma\gamma}| < 2.37$$

Central scales chosen as

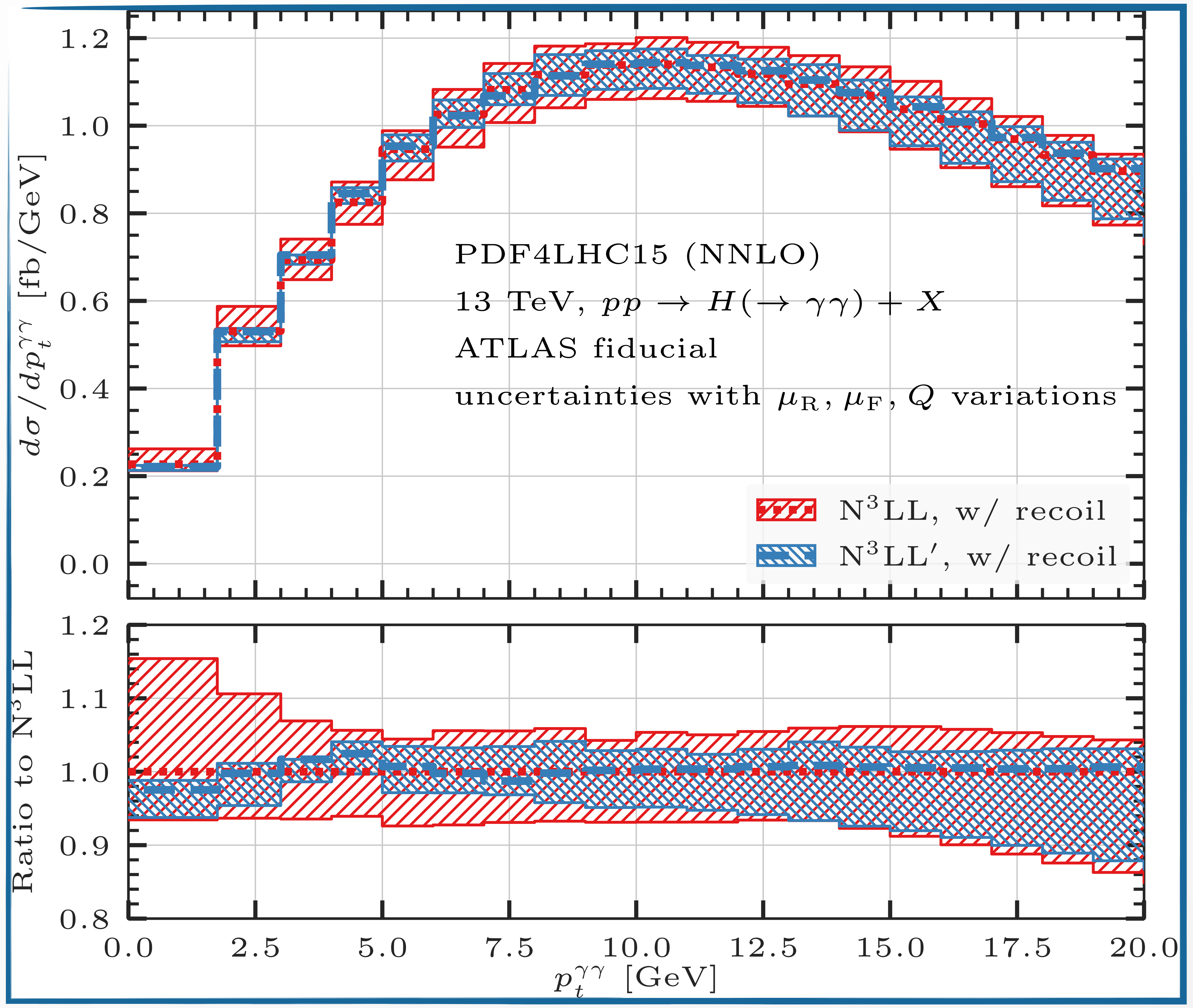
$$\mu_R = \kappa_R\, M_H \qquad \mu_F = \kappa_F\, M_H, \qquad Q = \kappa_Q\, M_H$$

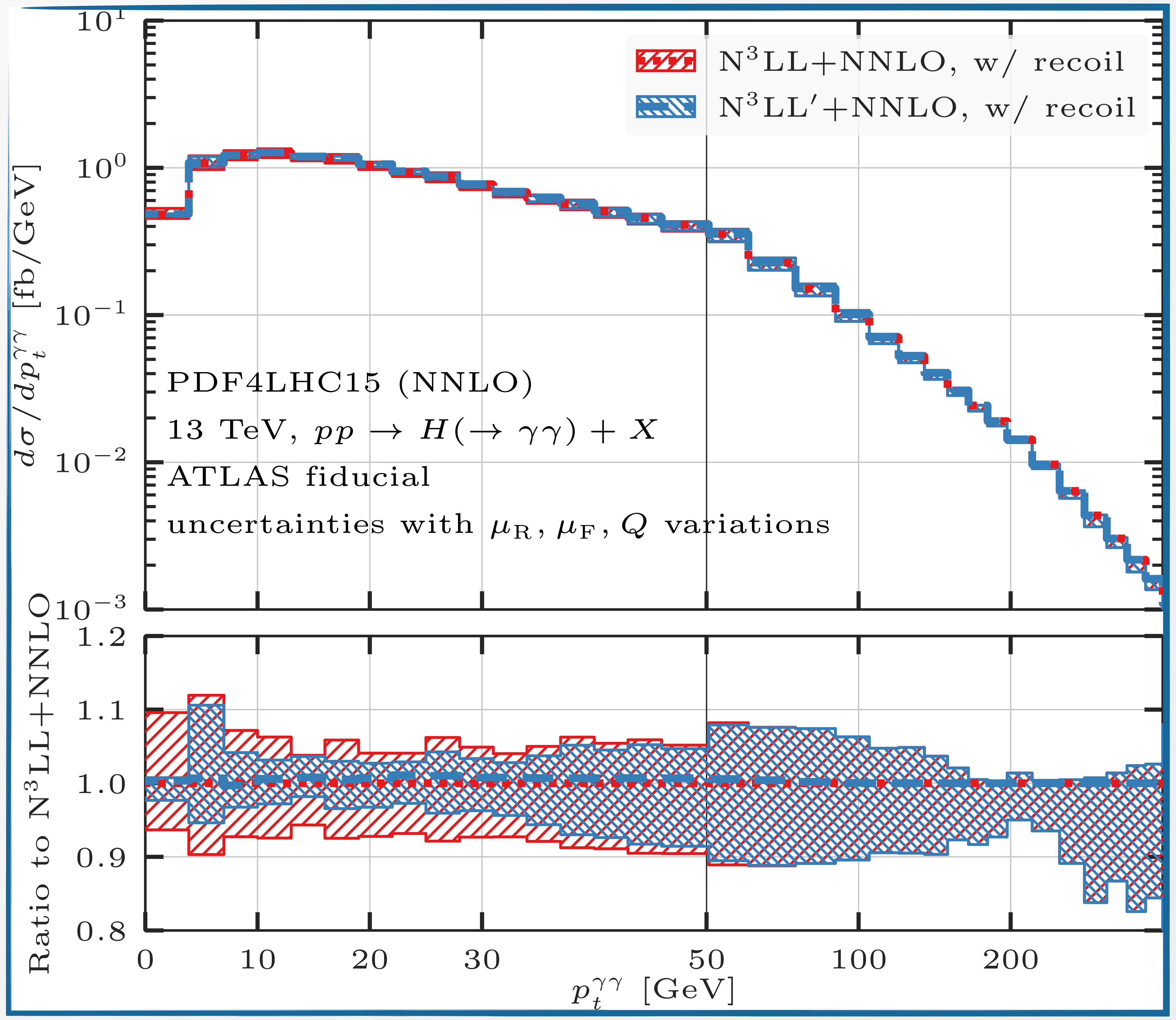
Scale uncertainty:

[canonical 7 scale variation + variation of κ_Q by a factor of 2 for central μ_R, μ_F] $\times$ 3 values of v_0 $\rightarrow$ **27 variations**

PDF4LHC15 NNLO parton densities. NNLO predictions from NNLOJET

Higgs production: N³LL′ effects



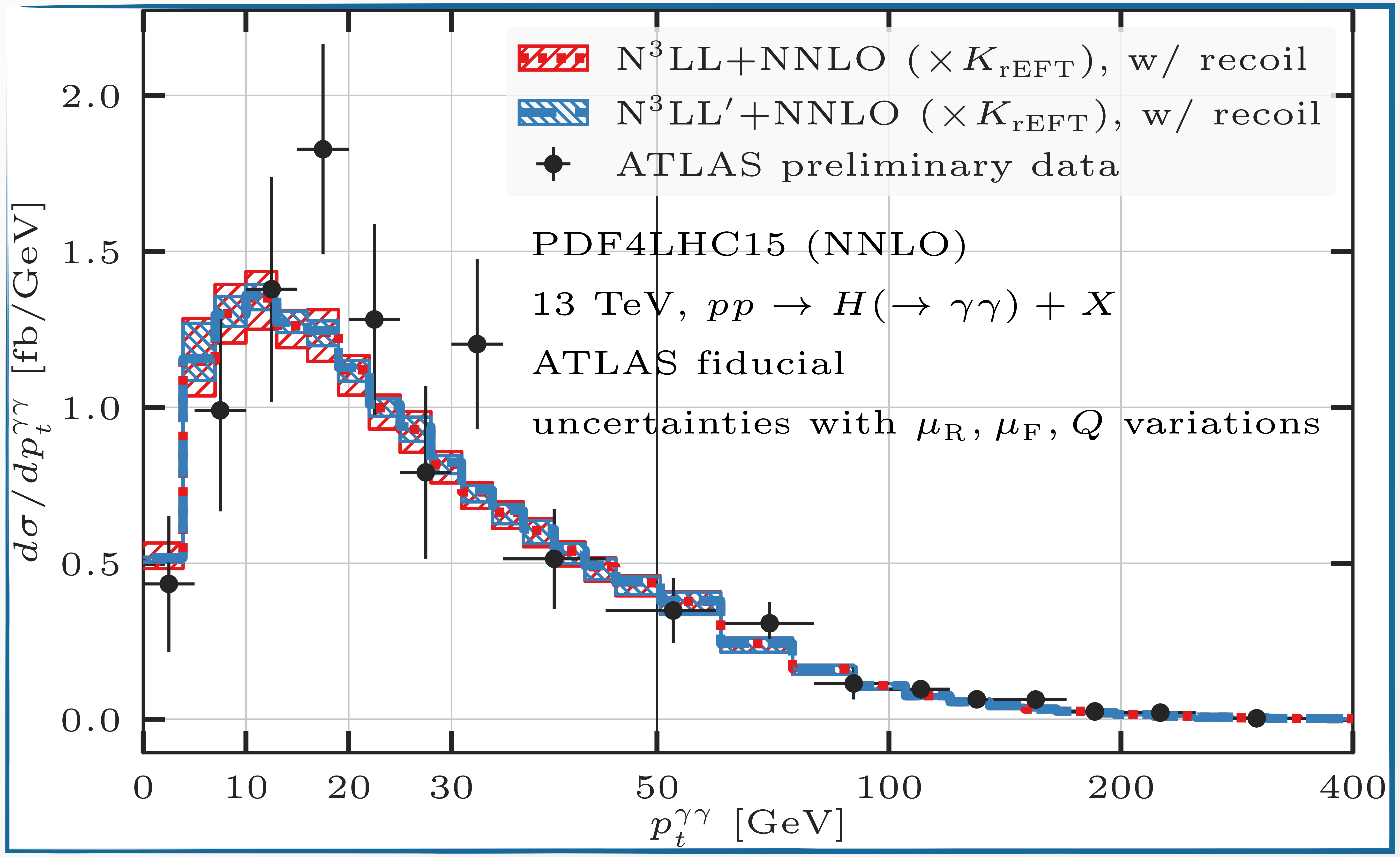


Significant reduction in theoretical uncertainty below 15 GeV, especially **below 5 GeV** $\kappa_R = \kappa_F = \kappa_Q = 1/2$

Central value almost unchanged between N³LL and N³LL′

Reduction in scale uncertainty limited at matched level (**statistical fluctuations** of the fixed order at small p_t)

Higgs production: comparison with ATLAS data



ATLAS preliminary data from https://cds.cern.ch/record/2682800

Theoretical predictions rescaled by $K_{\mathrm{rEFT}} = 1.06584$ to account for exact LO top-mass dependence

Recapitulation and outlook

- Results for singlet p_t (H/DY production) and ϕ_η^* (DY) at N^{3}LL′+NNLO accuracy by including all constant terms of relative order α_s^3 in the RadISH formalism
- Precise theoretical prediction in the fiducial region for $Z/\gamma^* \to \ell^+\ell^-$ and $H \to \gamma\gamma$
- Reduction of theoretical uncertainty at N^{3}LL′. Improved description of DY data
- Resummation uncertainty at the few percent level (DY), 5-10% level (Higgs)
- RadISH now includes recoil effects which improve the description of decay kinematics in the fiducial region. Marginal effect of recoil in matched results

Backup

Ambiguity in the definition of primed accuracy

$$\mathscr{L}_{\mathrm{NNLL}}(k_{t1}) = \sum_{c,c'} \frac{d|\mathscr{M}_B|^2_{cc'}}{d\Phi_B} \sum_{i,j} \int_{x_1}^{1} \frac{dz_1}{z_1} \int_{x_2}^{1} \frac{dz_2}{z_2} f_i\left(k_{t1}, \frac{x_1}{z_1}\right) f_j\left(k_{t1}, \frac{x_2}{z_2}\right)$$
$$\times \left\{ \delta_{ci}\,\delta_{c'j}\,\delta(1-z_1)\,\delta(1-z_2)\left(1 + \frac{\alpha_s(\mu_R)}{2\pi} H^{(1)}(\mu_R)\right) \right.$$
$$\left. + \frac{\alpha_s(\mu_R)/(2\pi)}{1 - 2\alpha_s(\mu_R)\beta_0 \ln(\mu_R/k_{t1})} \left(C^{(1)}_{ci}(z_1)\delta(1-z_2)\delta_{c'j} + \{z_1, c, i \leftrightarrow z_2, c', j\} \right) \right\}$$

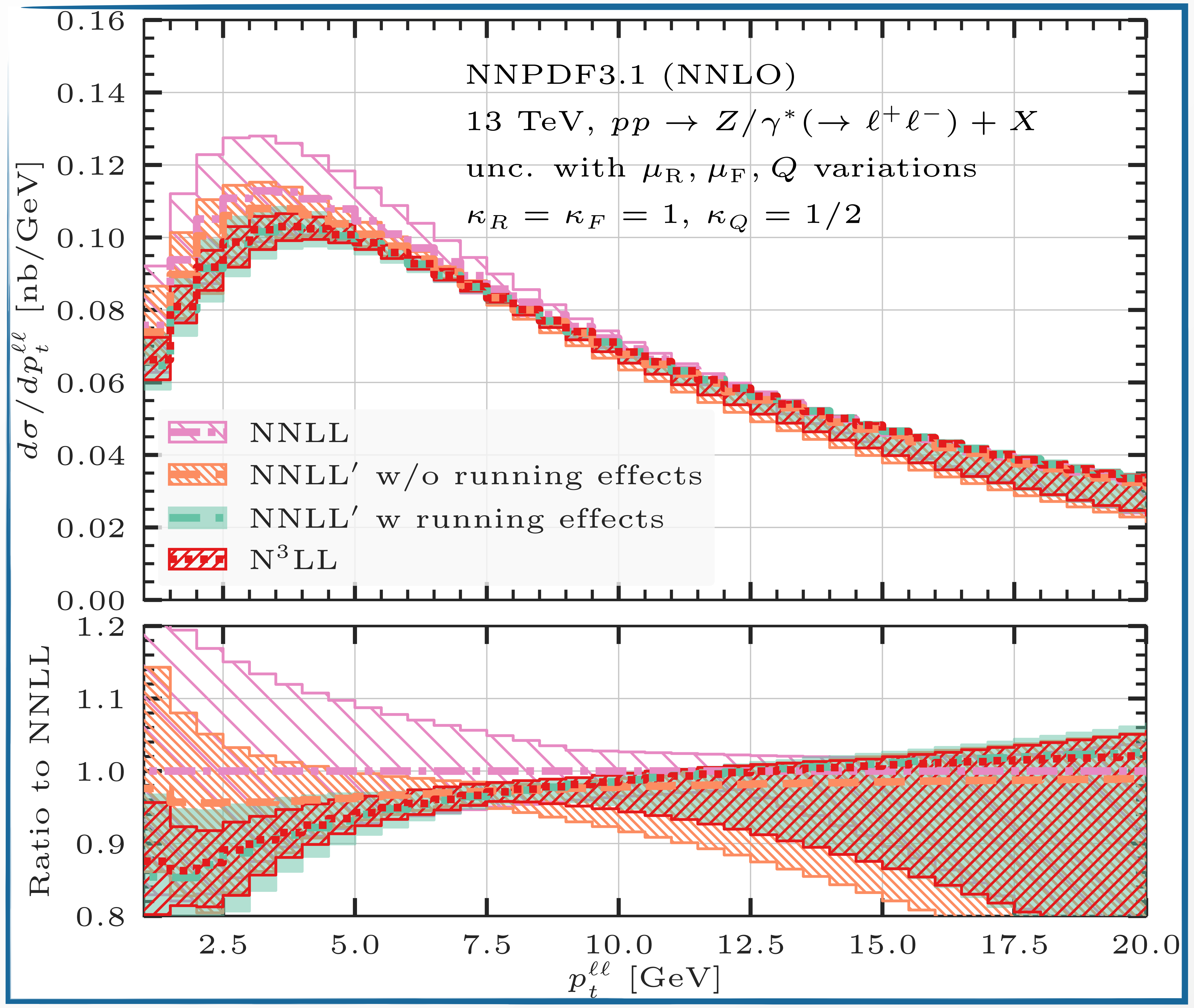
Scale at which the α_s^k term is evaluated is subleading at N^{k}LL′ accuracy

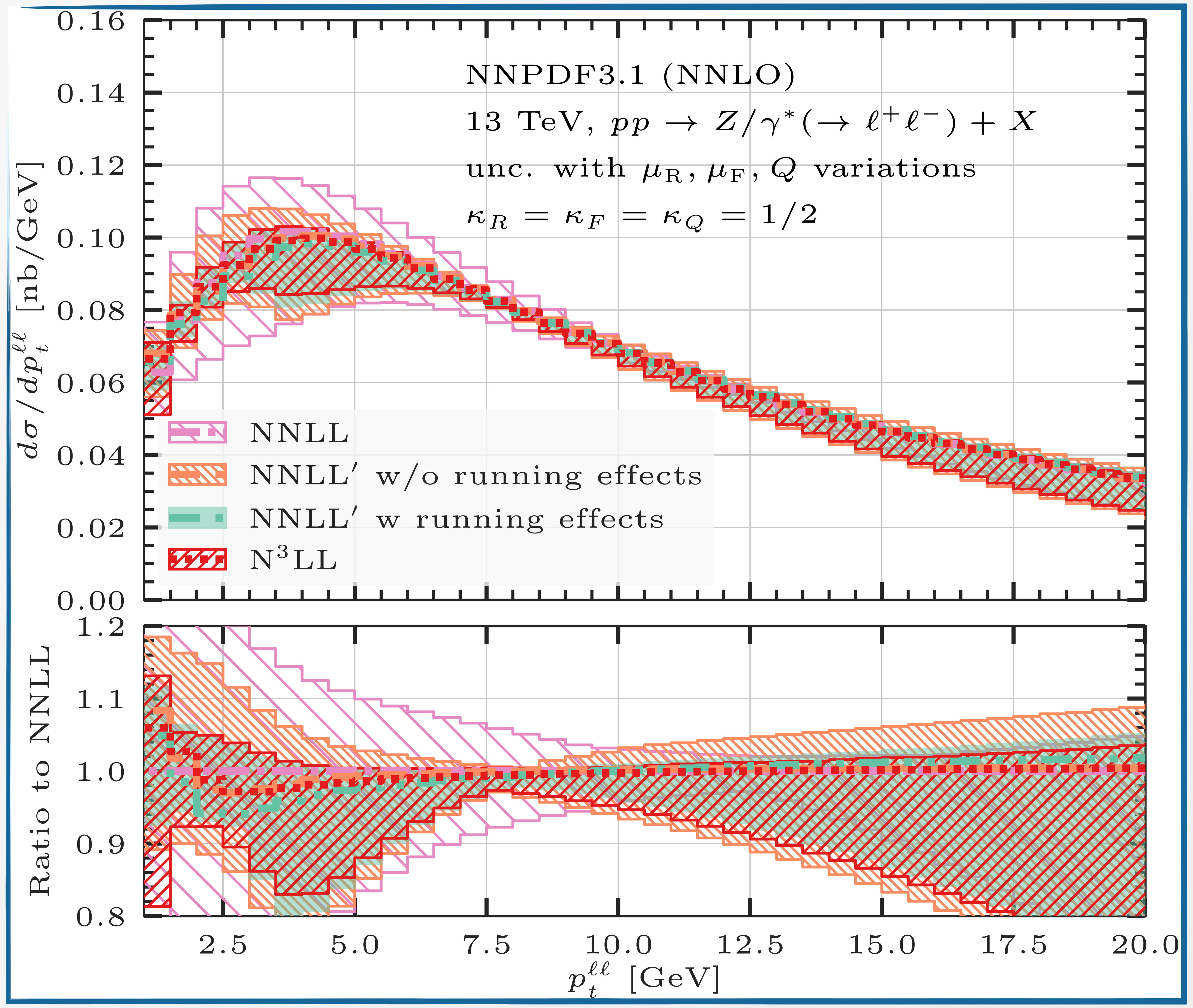
One can evaluate this contribution with $\alpha_s(\mu_R)$ → difference reflects **ambiguity** of these subleading effects

NLL′ **with running**: $\mathscr{L}_{\mathrm{NLL}'} = \mathscr{L}_{\mathrm{NNLL}}$ **Our default choice**

NLL′ **without running**: $\mathscr{L}_{\mathrm{NLL}'} = \mathscr{L}_{\mathrm{NNLL}}$ with $\alpha_s(\mu_R)$ in the C_1 component (analogously at higher orders)

Ambiguity in the definition of primed accuracy



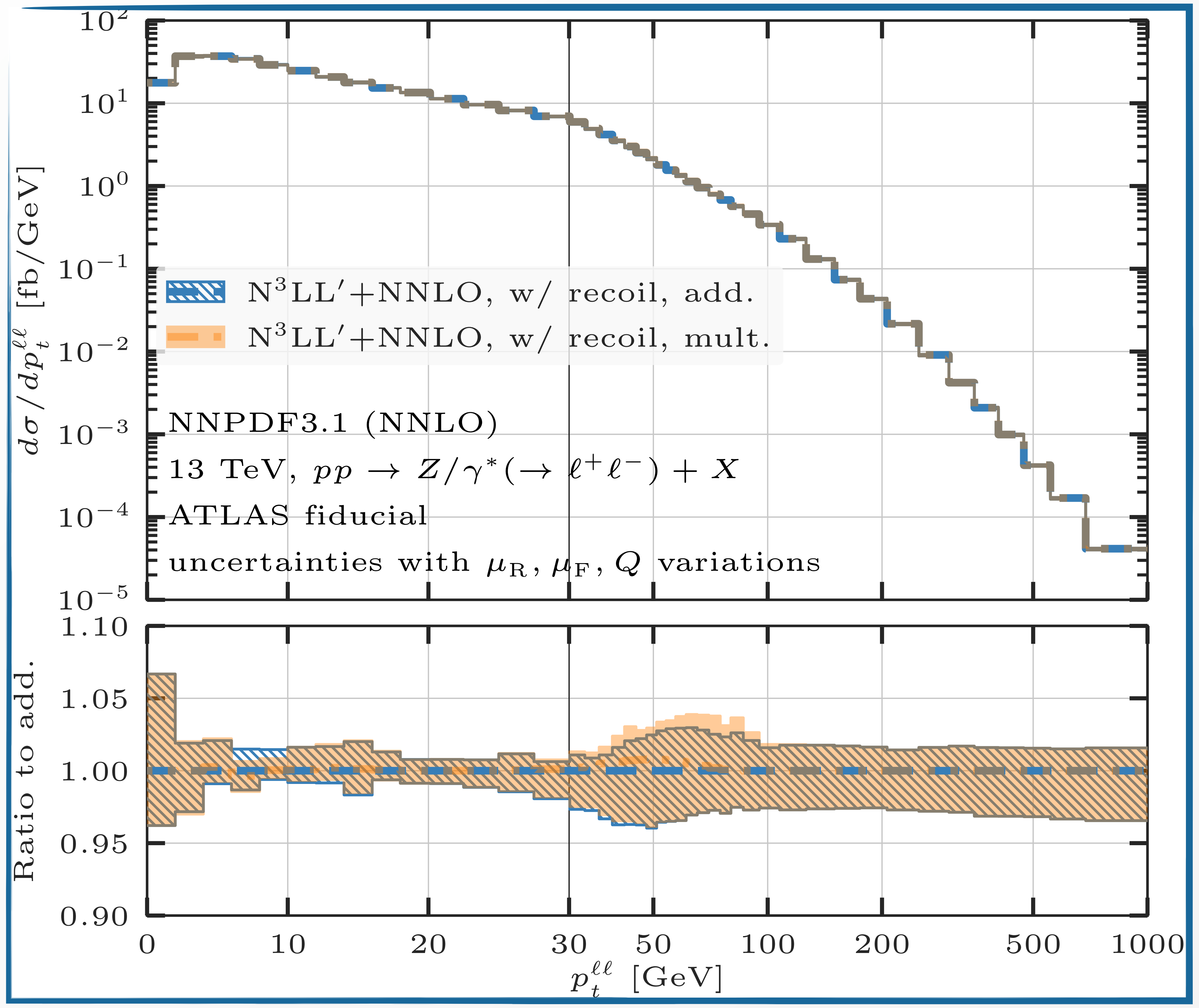


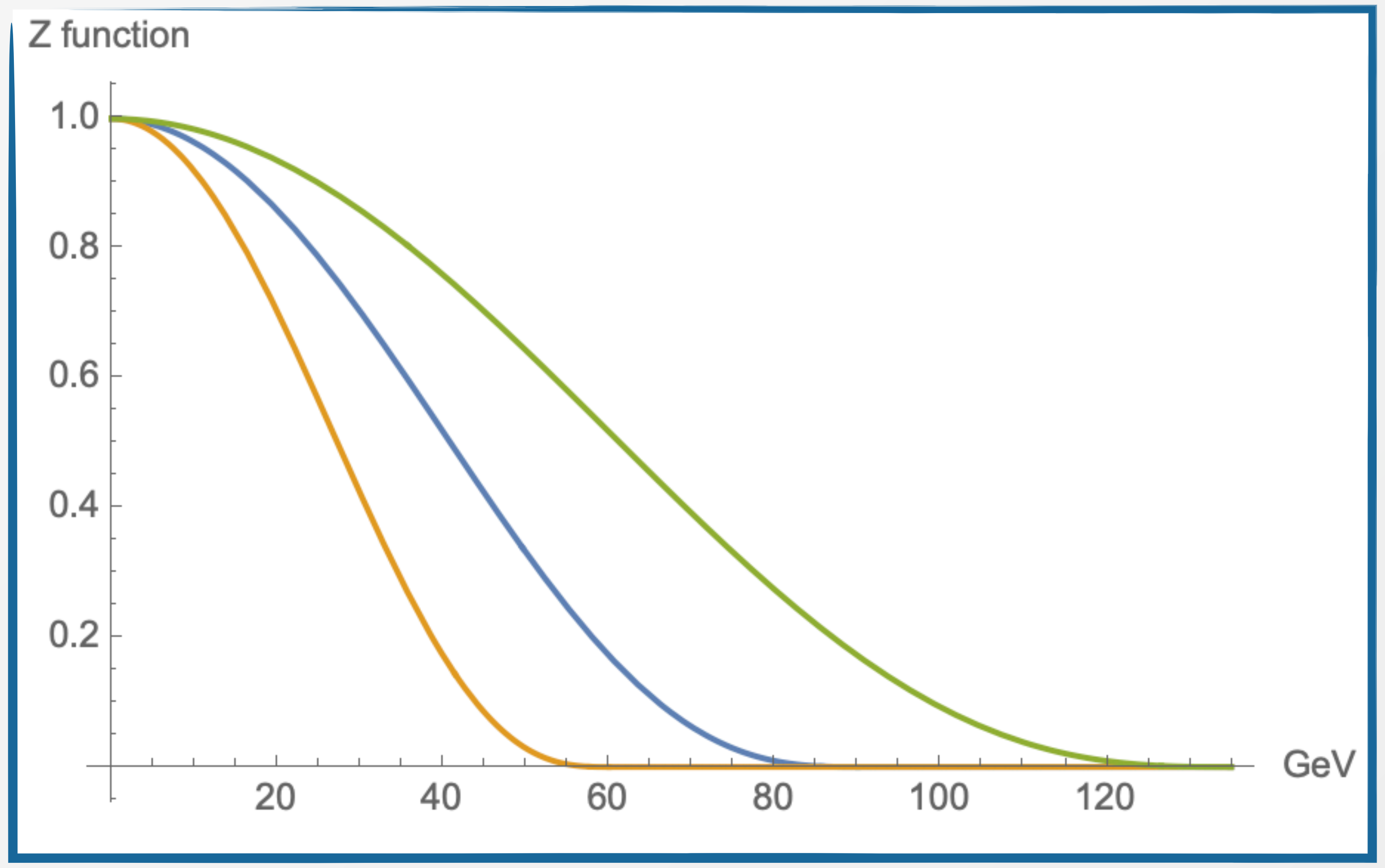
NNLL′ with and without running closer to N^{3}LL than NNLL is

NNLL′ with running band in better agreement to N^{3}LL: N^{3}LL contained within NNLL′ with running uncertainty

Band for NNLL′ with running covers difference between two NNLL′ → reliable estimate of prime ambiguity

Matching systematics





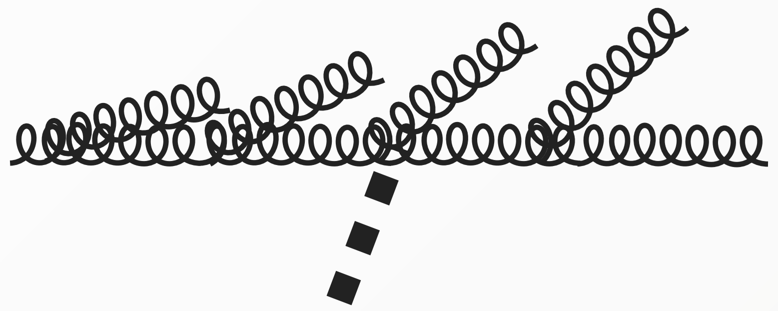
Very mild matching scheme dependence both for central results and uncertainties

Additive matching uncertainty band reliably estimate matching ambiguities

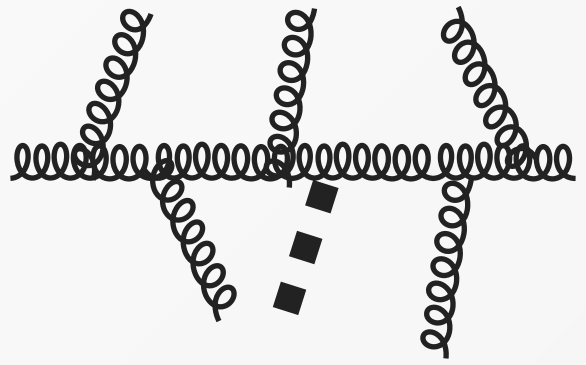
Direct space formulation: generality

NLL result for $p_\perp^H$

$$\sigma(p_\perp^H) = \sigma_0 \int d^2\overrightarrow{p}_\perp^H \int \frac{d^2\overrightarrow{b}}{4\pi^2} e^{-i\overrightarrow{b}\cdot\overrightarrow{p}_\perp^H} e^{-R_{\rm NLL}(L)}$$

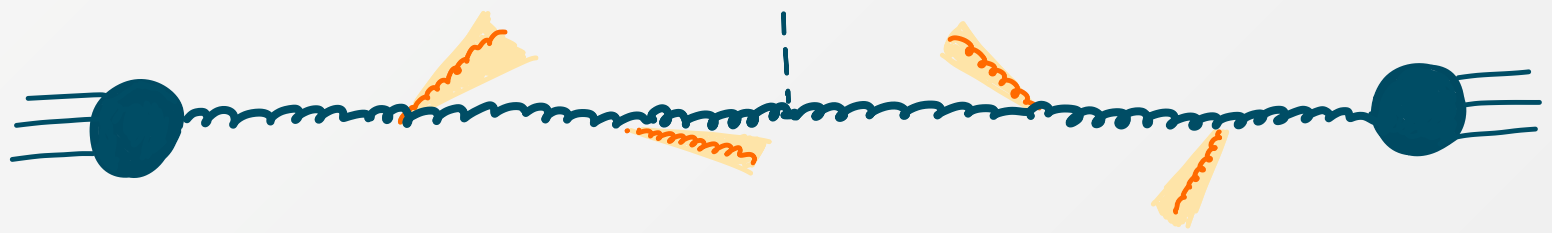


vs.



NLL result for $p_\perp^J$

$$\sigma(p_\perp^{\rm J}) = \sigma_0 e^{Lg_1(\alpha_s\beta_0 L)+g_2(\alpha_s\beta_0 L)}$$



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General formula for a generic transverse observable at NLL [Bizon, Monni, Re, LR, Torrielli '17]

$$\sigma(v) = \sigma_0 \int \frac{dk_{t,1}}{k_{t,1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(k_{t,1})} R'\left(k_{t,1}\right) d\mathcal{Z}\, \Theta\left(v - V(k_1, \ldots, k_{n+1})\right)$$

$$d\mathcal{Z} = \epsilon^{R'(k_{t,1})} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'\left(\zeta_i k_{t,1}\right)$$

$$R_{\rm NLL}(L) = -Lg_1(\alpha_s\beta_0 L) - g_2(\alpha_s\beta_0 L)$$

$$L = \ln(k_{t,1}/M)$$

$$R'_{\rm NLL}(k_t) = 4\left(\frac{\alpha_s^{\rm CMW}(k_t)}{\pi} C_A \ln\frac{m_H}{k_t} - \alpha_s(k_t)\beta_0\right)$$

CMW scheme

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differential control in momentum space provides guidance to **double-differential resummation**

[Monni, Re, LR, Torrielli '19]

$$\sigma(p_\perp^H, p_\perp^{\rm J}) = \sigma_0 \int \frac{dk_{t,1}}{k_{t,1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(k_{t,1})} R'\left(k_{t,1}\right) d\mathcal{Z}\, \Theta\left(p_T^H - |\vec{k}_{t,1} + \cdots \vec{k}_{t,n+1}|\right) \Theta\left(p_T^J - \max\{k_{t,1}, \ldots k_{t,n+1}\}\right)$$