
Resummation in PDF fits

Luca Rottoli

Rudolf Peierls Centre for Theoretical Physics, University of Oxford



LHC, New Physics, and the pursuit of Precision

LHC as a **discovery machine**

- ▶ Higgs Boson ✓
- ▶ BSM particles ✗ (never as of today)

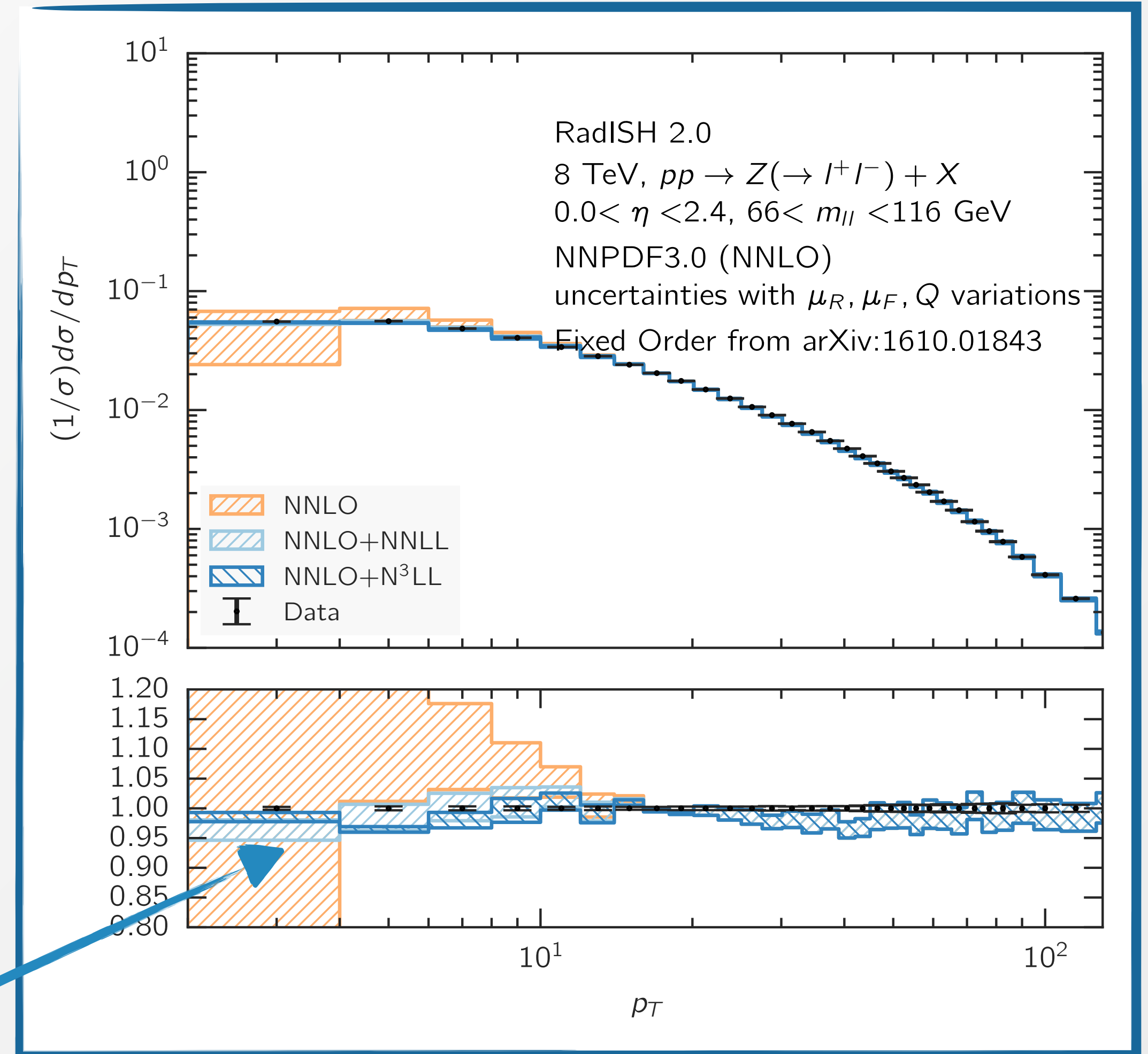
Focus in LHC run II

- ▶ Measurement of the Standard Model parameters with **very high precision**
- ▶ Signals of New Physics **beyond the Standard Model**

A theorist's Quest:

- ▶ New BSM scenarios to be tested
- ▶ New techniques to enhance signal/background ratio and isolate tiny deviations from SM predictions
- ▶ Development of **accurate** and **precise** theoretical predictions

Goal: 1% accuracy in theoretical predictions



[Bizon, Monni, Re, LR, Torielli et al, in preparation]

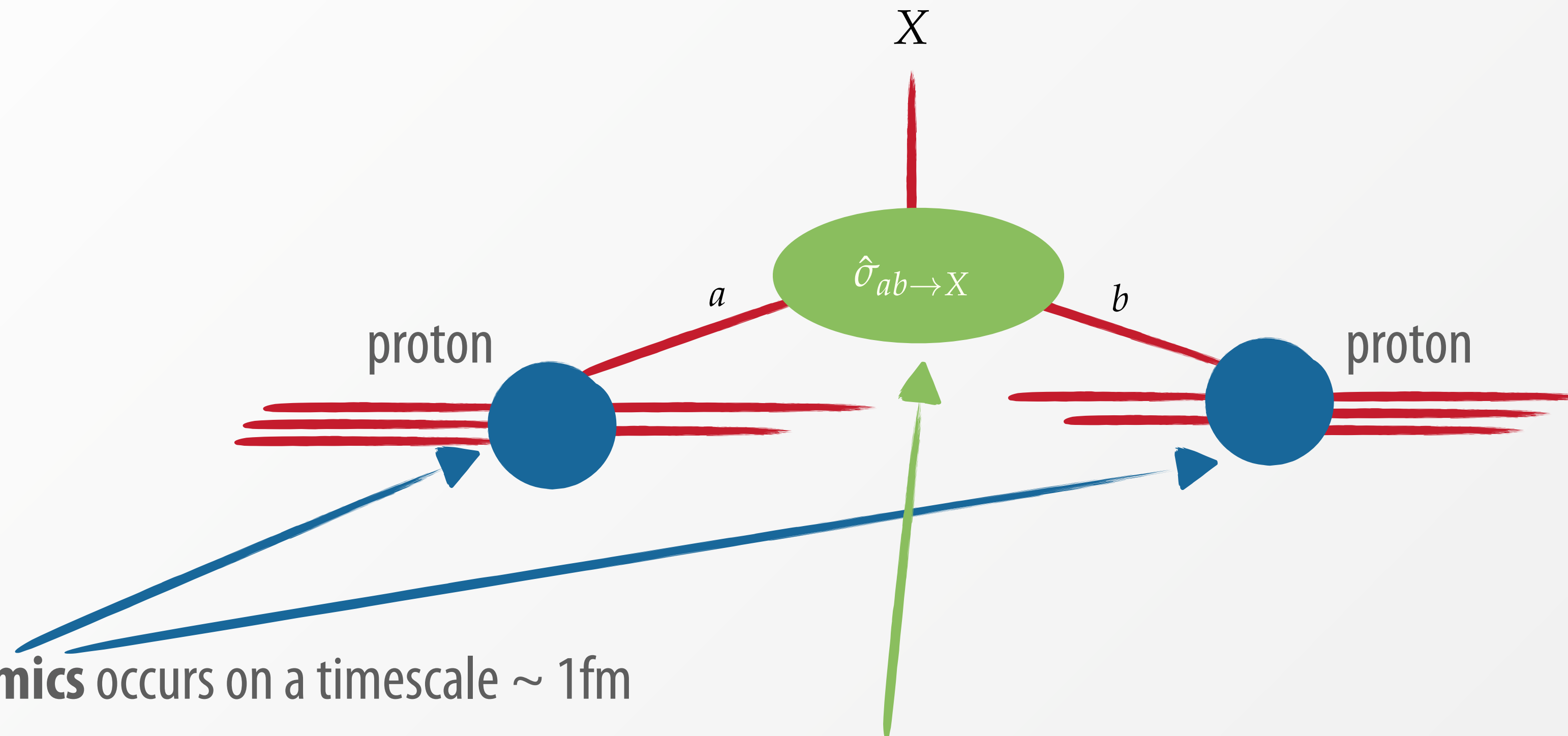
LHC, New Physics, and the pursuit of Precision

Large **HADRON** Collider



A crucial ingredient in the physics precision programme at the LHC is the **accurate understanding** of the **internal structure of the initial state hadrons**

Factorization

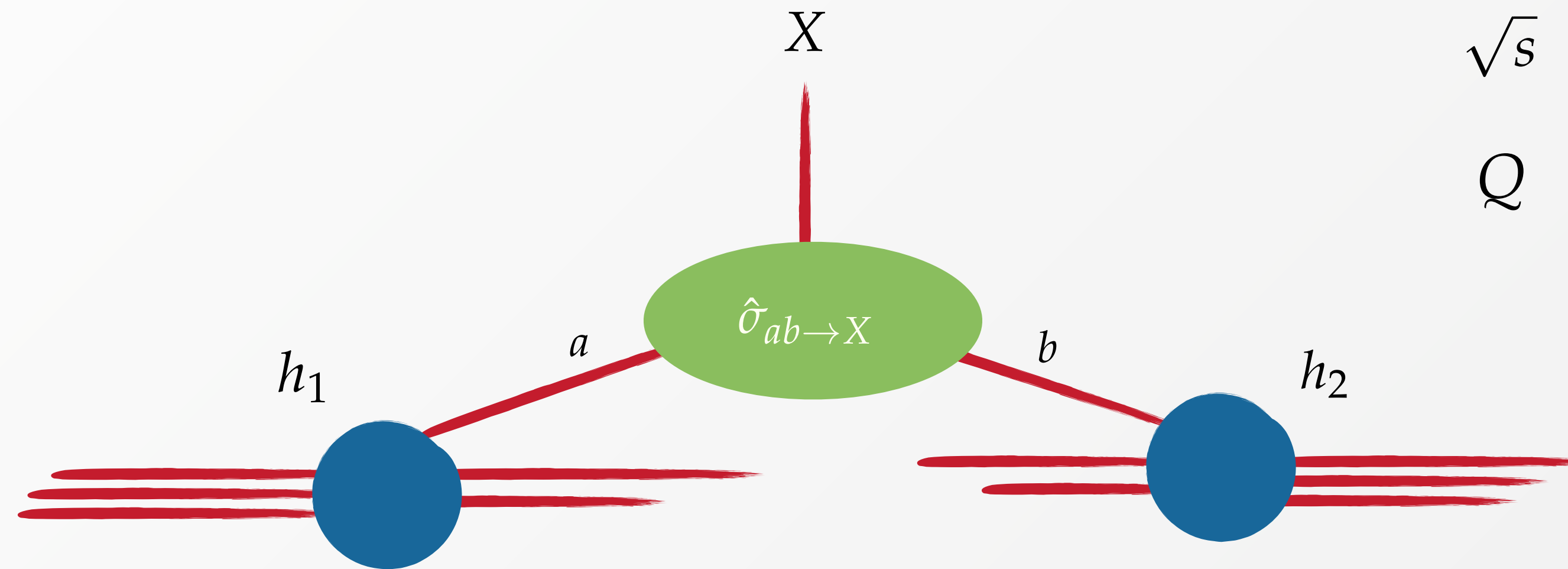


Proton's dynamics occurs on a timescale $\sim 1\text{fm}$

Production of a heavy particle e.g. Higgs Production (**hard process**) occurs on timescale $1/M_X \sim 1/100\text{ GeV} \sim 0.002\text{ fm}$

Large separation between scales allows to separate the hard process and treat it **independently** from the hadronic dynamics: **collinear factorization**

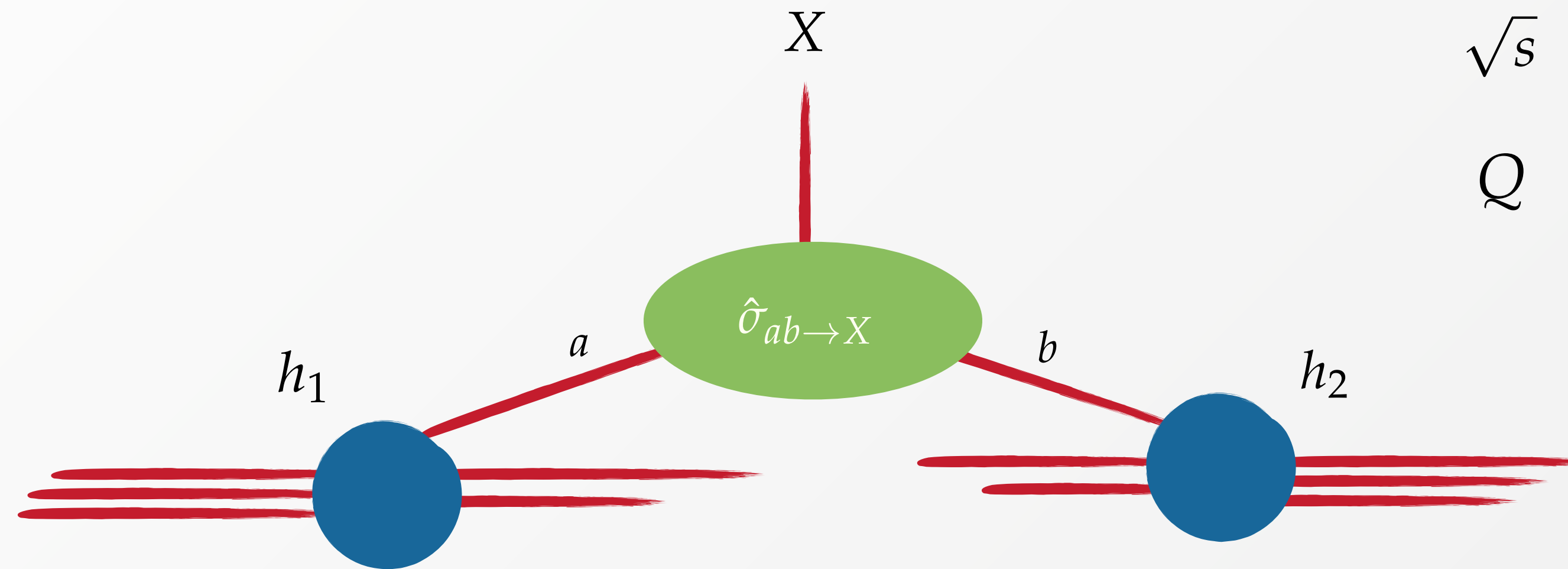
Factorization



\sqrt{s} centre-of-mass energy
 Q hard scale of the process

$$\sigma_X(s, Q^2) = \sum_{a,b} \int dx_1 dx_2 f_{a/h_1}(x_1, Q^2) f_{b/h_2}(x_2, Q^2) \hat{\sigma}_{ab \rightarrow X}(Q^2, x_1 x_2 s)$$

Factorization

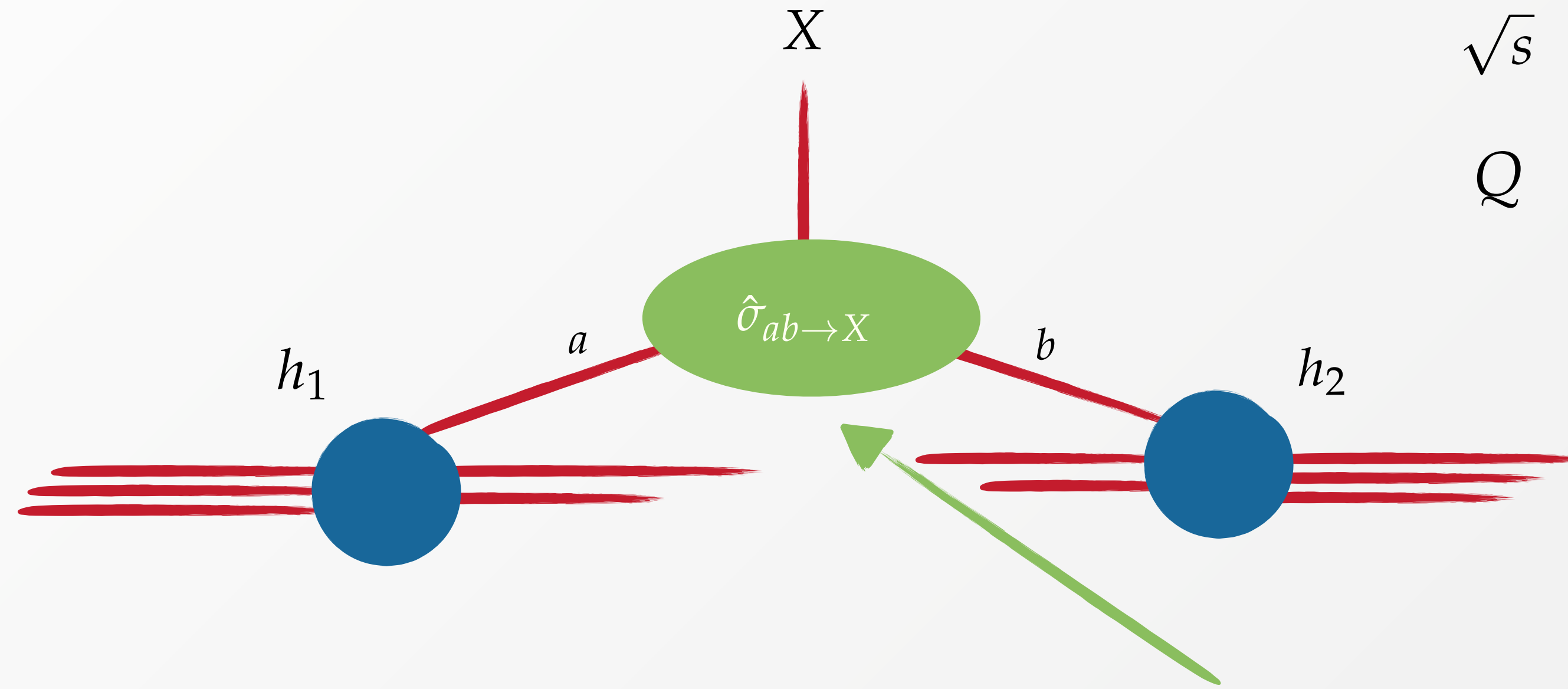


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$$\sigma_X(Q^2, s) = \sum_{a,b} f_{a/h_1}(Q^2) \otimes f_{b/h_2}(Q^2) \otimes \hat{\sigma}_{ab \rightarrow X}(Q^2, s)$$

Factorization



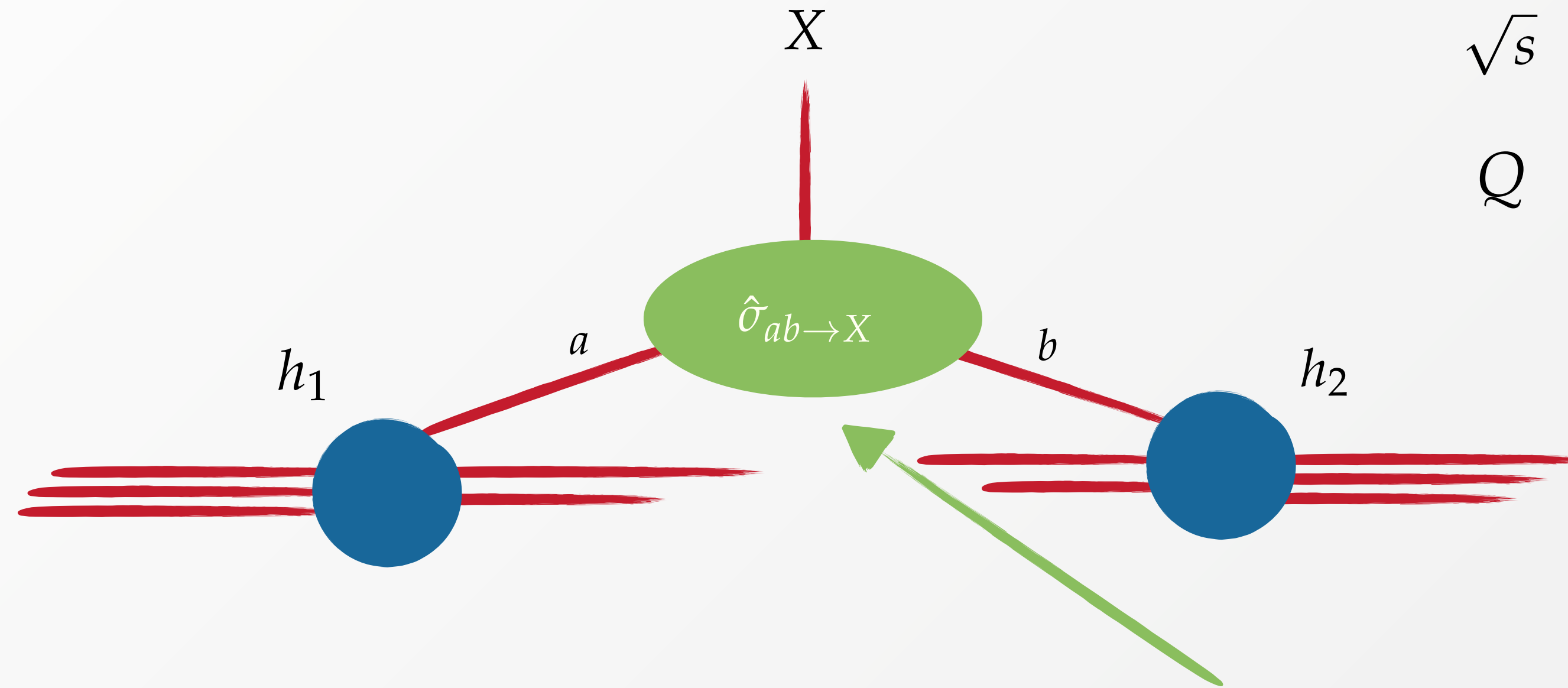
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partonic cross-section

short-distance: **perturbative**

Factorization



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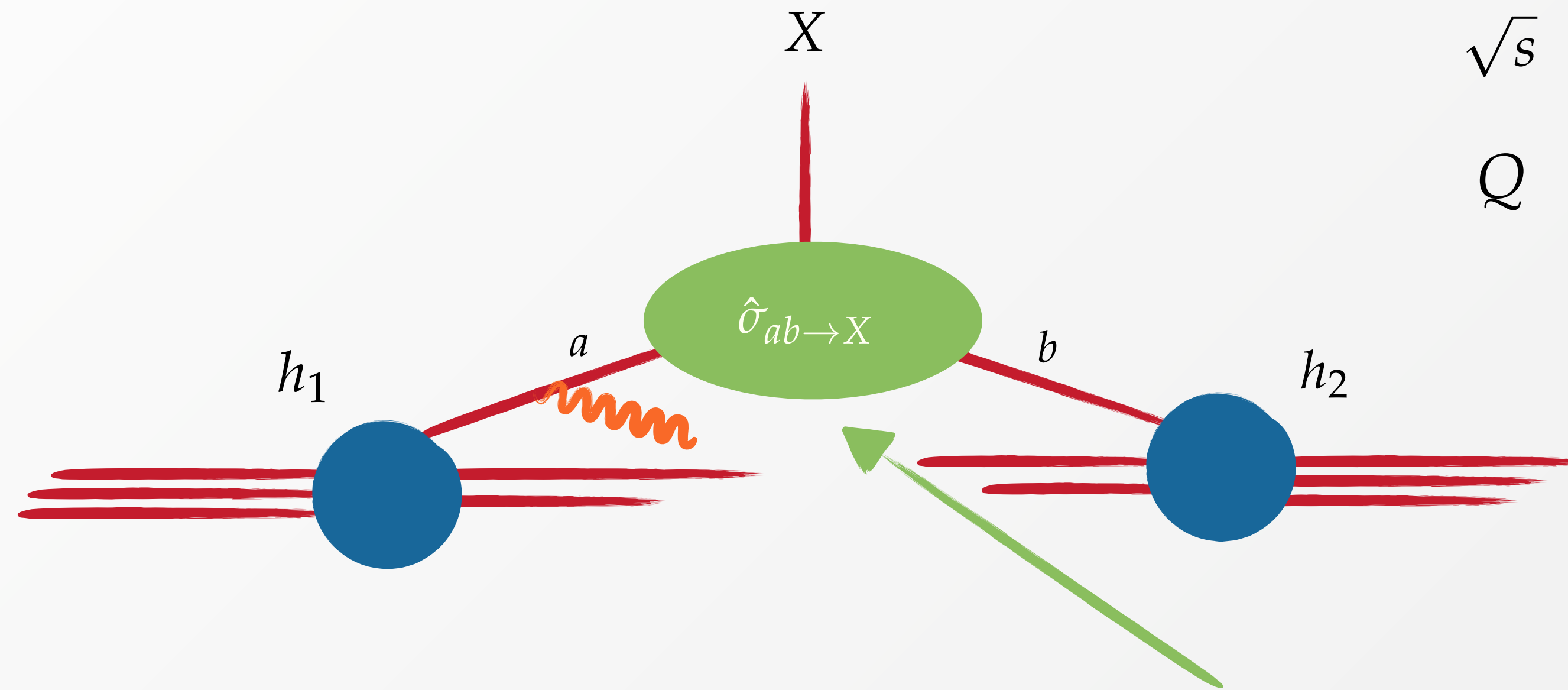
short-distance: **perturbative**

QCD at short distance is perturbative (**asymptotic freedom**)

$$\hat{\sigma} = \hat{\sigma}_0(1 + \dots)$$

L0

Factorization



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 Q hard scale of the process

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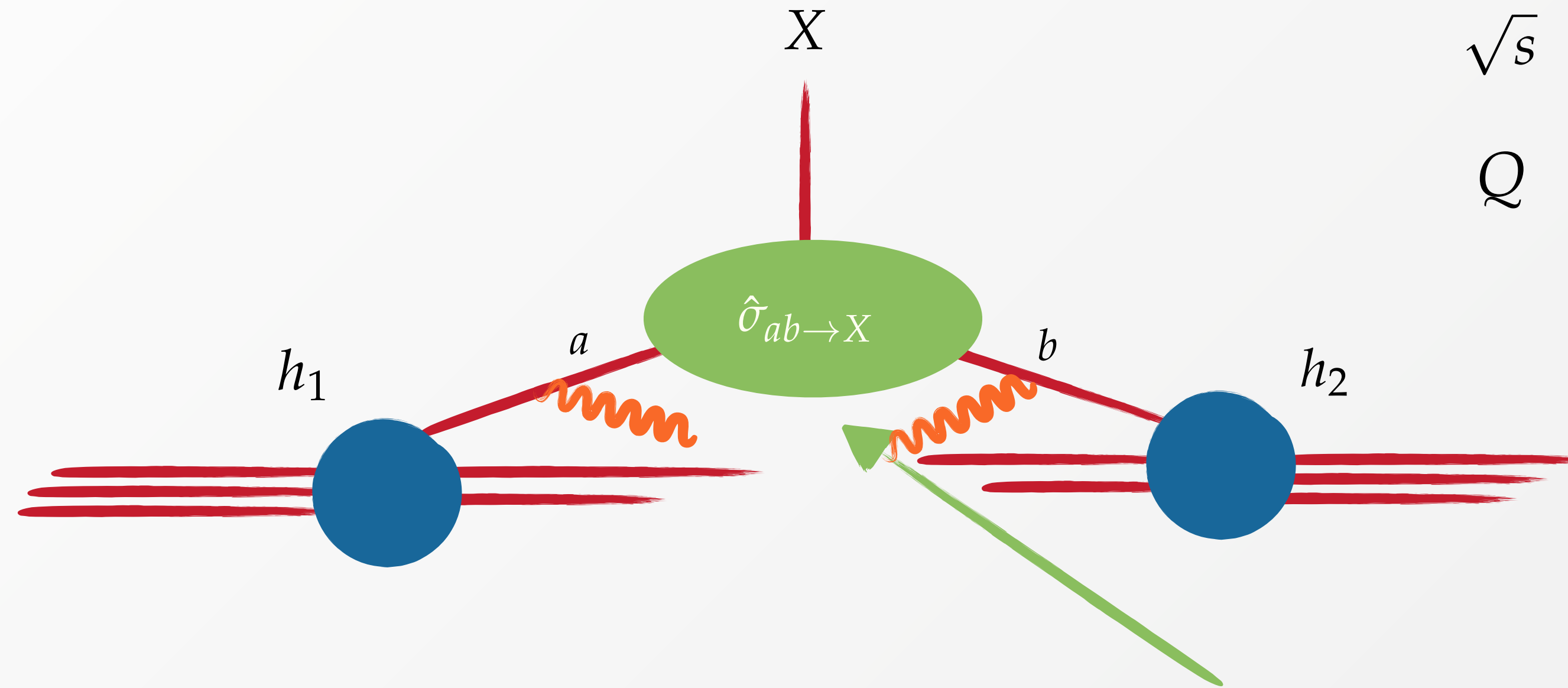
short-distance: **perturbative**

QCD at short distance is perturbative (**asymptotic freedom**)

$$\hat{\sigma} = \hat{\sigma}_0 (1 + \alpha_s c_1 + \alpha_s^2 c_2 + \dots)$$

NLO

Factorization



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$$\sigma_X(s, Q^2) = \sum_{a,b} \int dx_1 dx_2 f_{a/h_1}(x_1, Q^2) f_{b/h_2}(x_2, Q^2) \hat{\sigma}_{ab \rightarrow X}(Q^2, x_1 x_2 s)$$

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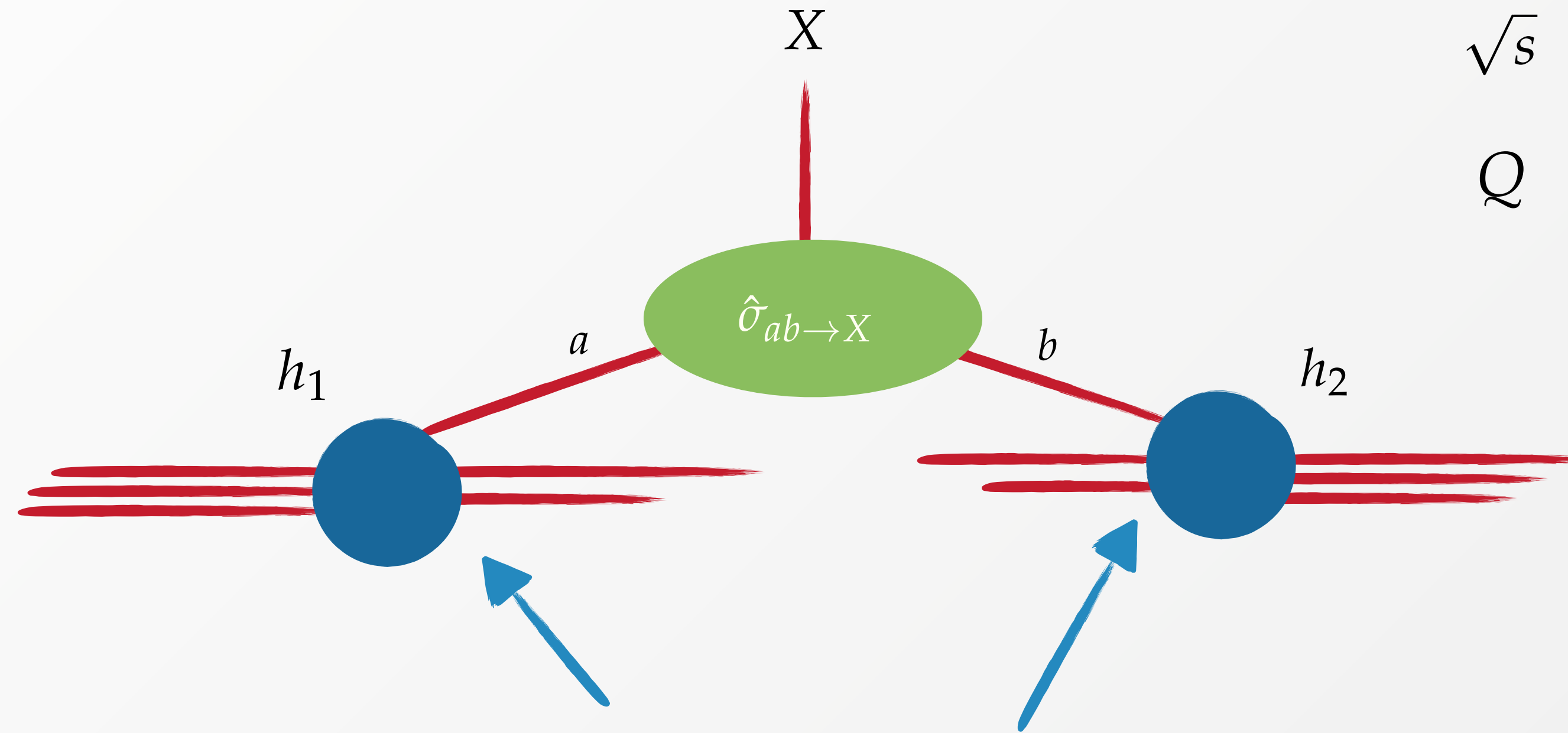
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Parton Distribution Functions (PDFs)

long-distance: **non-perturbative**

Parton distribution functions (PDFs) are **universal** objects which encode information on the substructure of the proton and which describe the dynamics of **quarks** and **gluons (partons)**

PDFs are currently **extracted from experiments**

Parton Distribution Functions

PDFs depend on two kinematic variables $f(x, Q^2)$

fraction of the momentum of the proton

Parton Distribution Functions

PDFs depend on two kinematic variables $f(x, Q^2)$

Scale of the process

Parton Distribution Functions

PDFs depend on two kinematic variables $f(x, Q^2)$ and are parametrized at an initial scale Q_0

Parton Distribution Functions

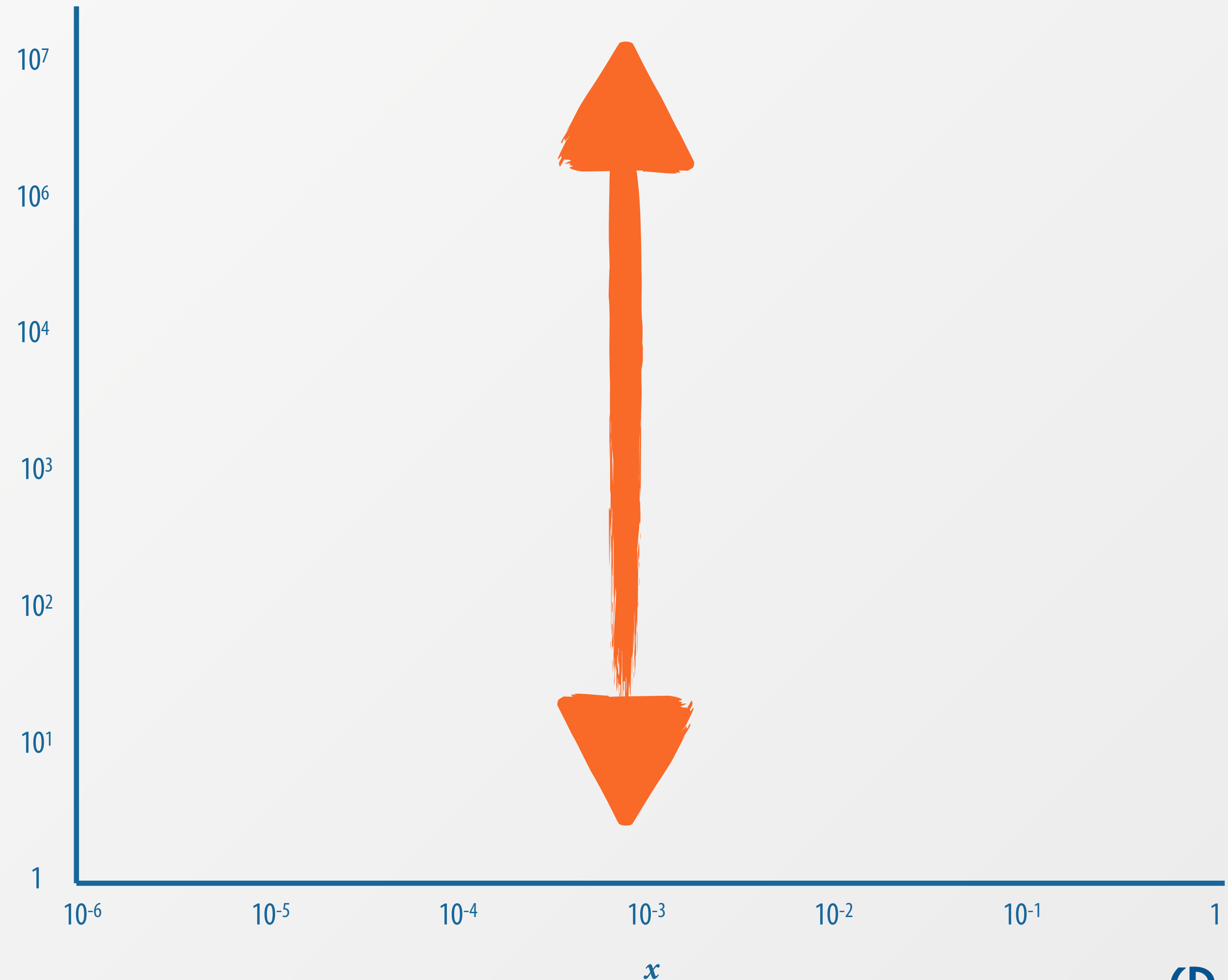
PDFs depend on two kinematic variables $f(x, Q^2)$

and are parametrized at an initial scale Q_0

Evolution in Q^2 is encoded in **DGLAP equation**

$$Q^2 \frac{\partial}{\partial Q^2} f_i(x, Q^2) = \int_x^1 \frac{dz}{z} P_{ij} \left(\frac{x}{z}, \alpha_s(Q^2) \right) f_j(z, Q^2)$$

Q^2 [GeV]



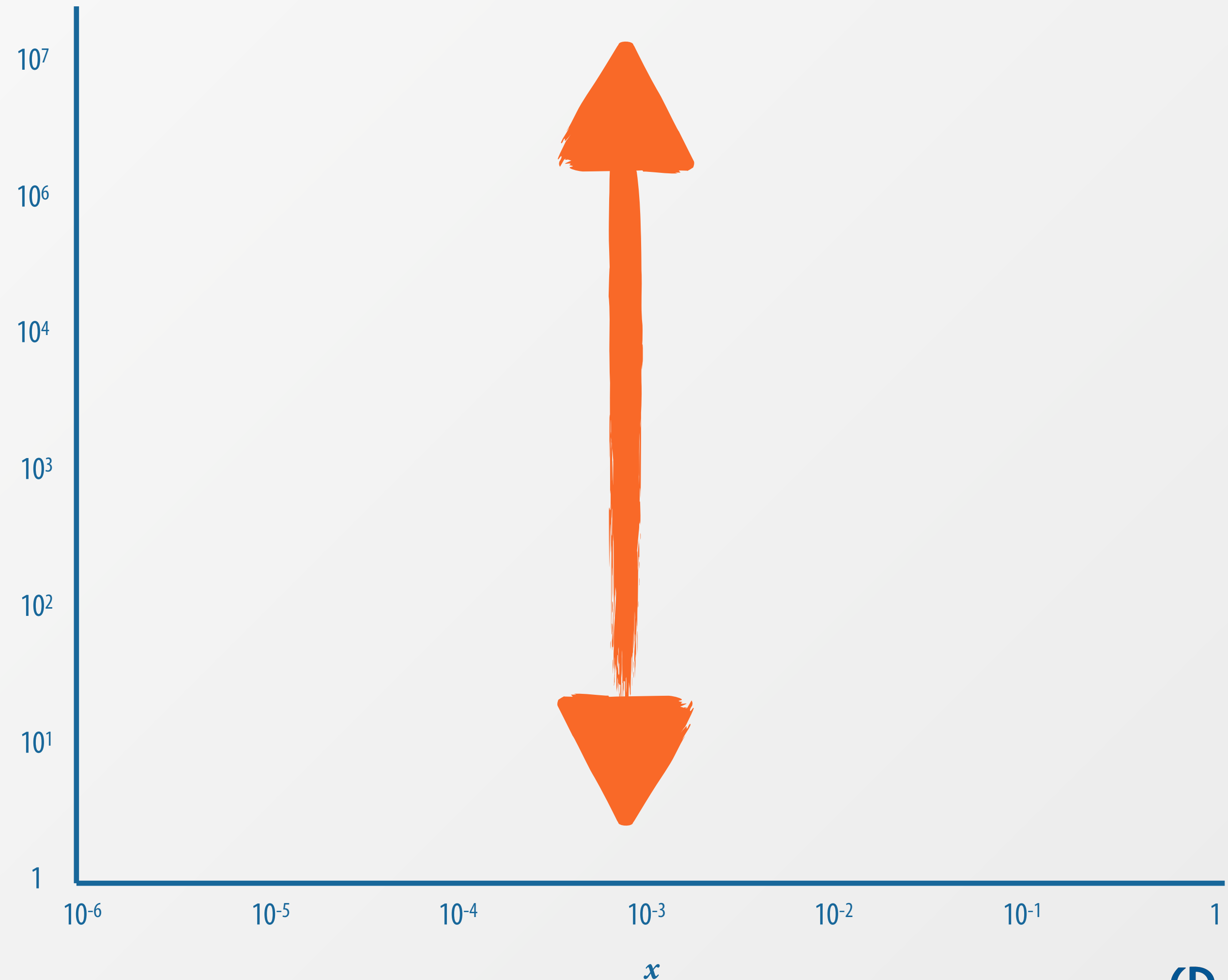
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splitting functions Q^2 [GeV]



Parton Distribution Functions

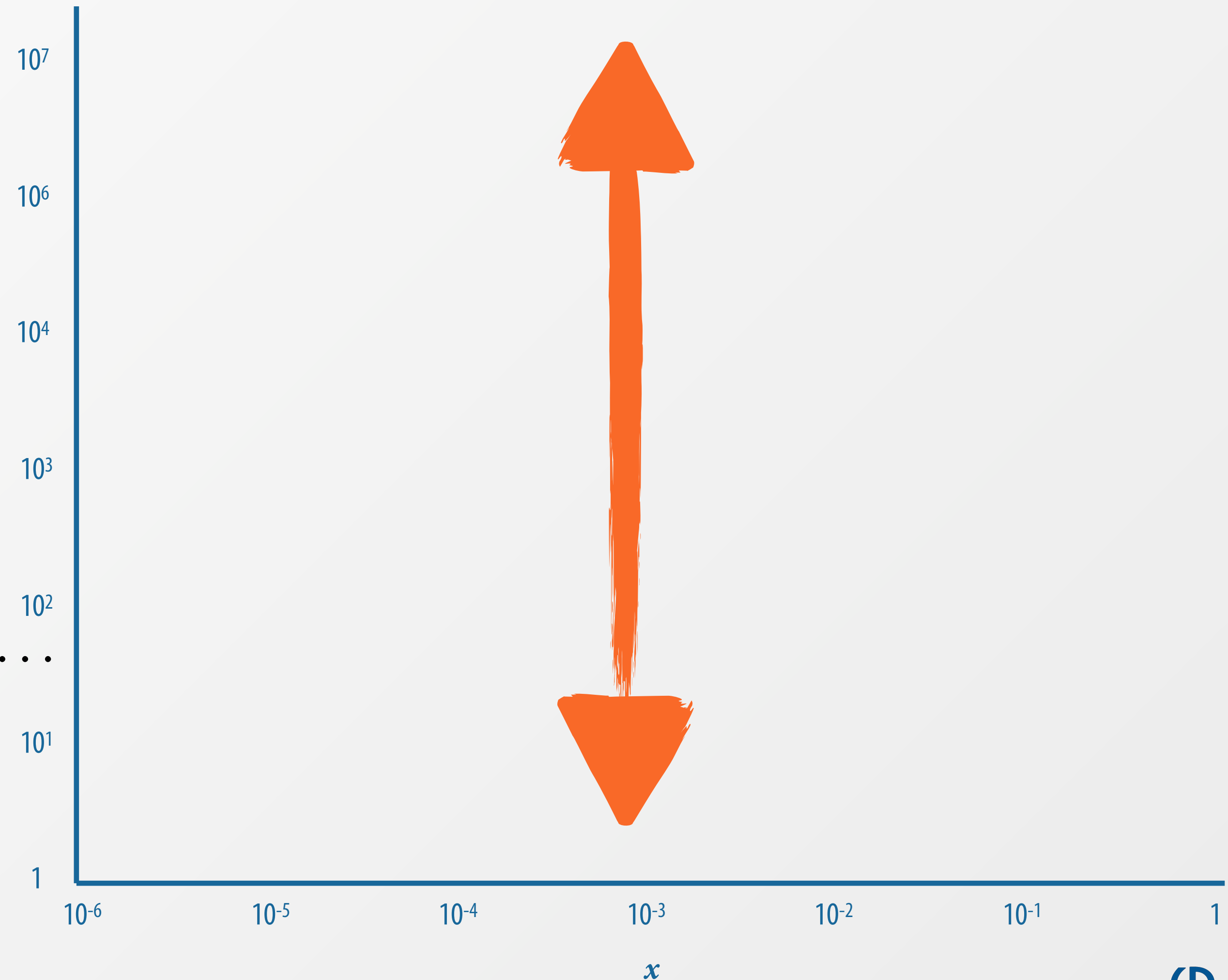
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splitting functions Q^2 [GeV]

$$P_{ij} \left(x, \alpha_s(Q^2) \right) = P_{ij}^{(0)}(x) + \alpha_s P_{ij}^{(1)}(x) + \alpha_s^2 P_{ij}^{(2)}(x) + \dots$$



DGLAP equation

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$2n_f + 1$ **coupled** differential equation
number of (active) flavours

However, strong interactions do not tell apart quarks and antiquarks (**charge conjugation** and $SU(n_f)$ **flavour symmetry**)

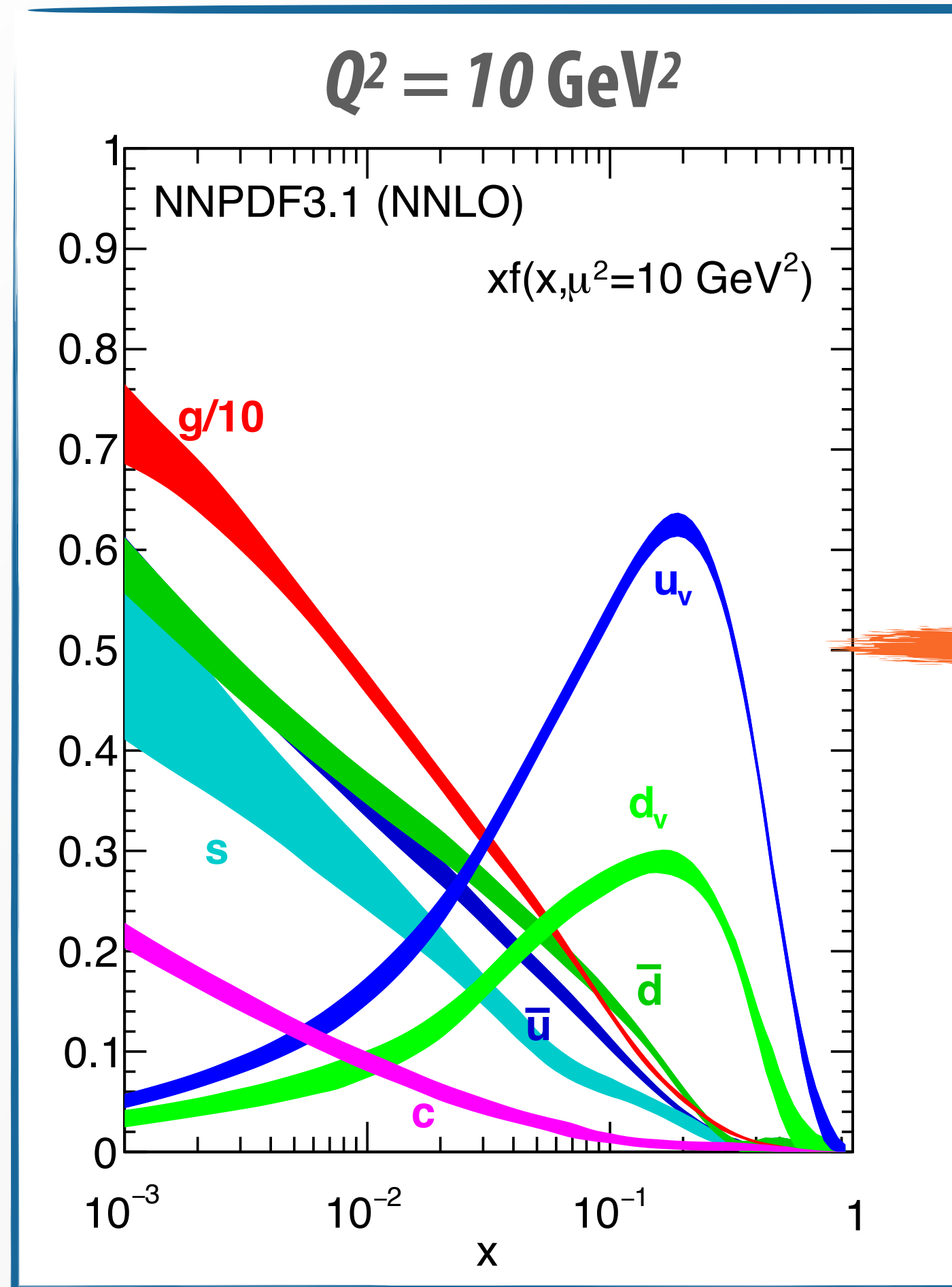
$$P_{q_i q_j} = P_{\bar{q}_i \bar{q}_j}, \quad P_{q_i \bar{q}_j} = P_{\bar{q}_i q_j}, \quad P_{q_i g} = P_{\bar{q}_i g} \equiv P_{qg}, \quad P_{g q_i} = P_{g \bar{q}_i} \equiv P_{gq}$$

Only **singlet combination** couples to **gluon**

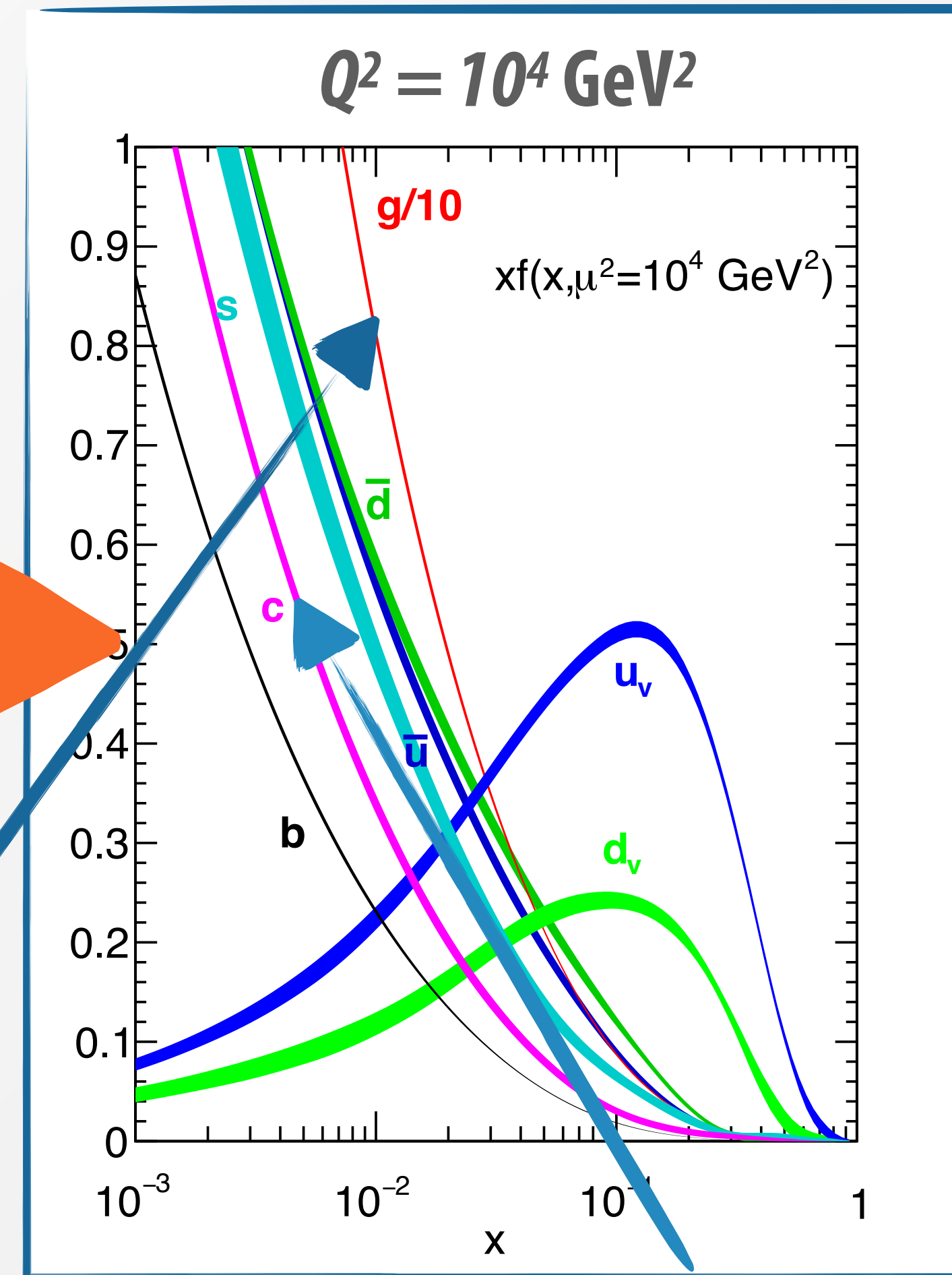
$$\Sigma(x, Q^2) = \sum_i [q_i(x, t) + \bar{q}_i(x, t)]$$

$$Q^2 \frac{\partial}{\partial \ln Q^2} \begin{pmatrix} \Sigma \\ g \end{pmatrix} = \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} \Sigma \\ g \end{pmatrix}$$

DGLAP equation



Q^2 evolution



growth of small-x gluon

Parton lose momentum and shifts at smaller values of x

Parton Distribution Function Fits

$$\sigma_X(Q^2, s) = \sum_{a,b} f_{a/h_1}(Q^2) \otimes f_{b/h_2}(Q^2) \otimes \hat{\sigma}_{ab \rightarrow X}(Q^2, s)$$

$$Q^2 \frac{d}{dQ^2} f_i(Q^2) = P_{ij}(\alpha_s(Q^2)) \otimes f_j(Q^2)$$

theoretical prediction (to be compared with data)

theoretical input

Parton Distribution Function Fits

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theoretical prediction (to be compared with data)

$$Q^2 \frac{d}{dQ^2} f_i(Q^2) = P_{ij}(\alpha_s(Q^2)) \otimes f_j(Q^2)$$

theoretical input

PDF fits are typically based on **fixed-order** theory...

$$\hat{\sigma} = \hat{\sigma}_0(1 + \alpha_s c_1 + \alpha_s^2 c_2 + \dots)$$

$$P_{ij}(x, \alpha_s(Q^2)) = P_{ij}^{(0)}(x) + \alpha_s P_{ij}^{(1)}(x) + \alpha_s^2 P_{ij}^{(2)}(x) + \dots$$

...but is fixed-order theory always good enough?

Large logarithms

Single (double) logarithmic enhancement

$$\alpha_s^k \ln^j \quad 0 \leq j \leq (2)k$$

Perturbative convergence is spoiled when

$$\alpha_s \ln^{(2)} \sim 1$$

e.g. small- x behaviour of splitting functions

$$xP(x, \alpha_s) = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{2\pi}\right)^n \left[\sum_{m=1}^n A_{m-1}^{(n)} \ln^{m-1} \frac{1}{x} + x\bar{P}^{(n)}(x) \right]$$

Instability at small- x

Finite in the limit $x \rightarrow 0$



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Instability at small- x

All-order resummation of the logarithmically enhanced terms

($n \geq 0, m=n$) leading-logarithm (LL x), ($n \geq 0, m=n, n-1$) next-to-leading-logarithm (NLL x), etc.

Resummation in PDF fits

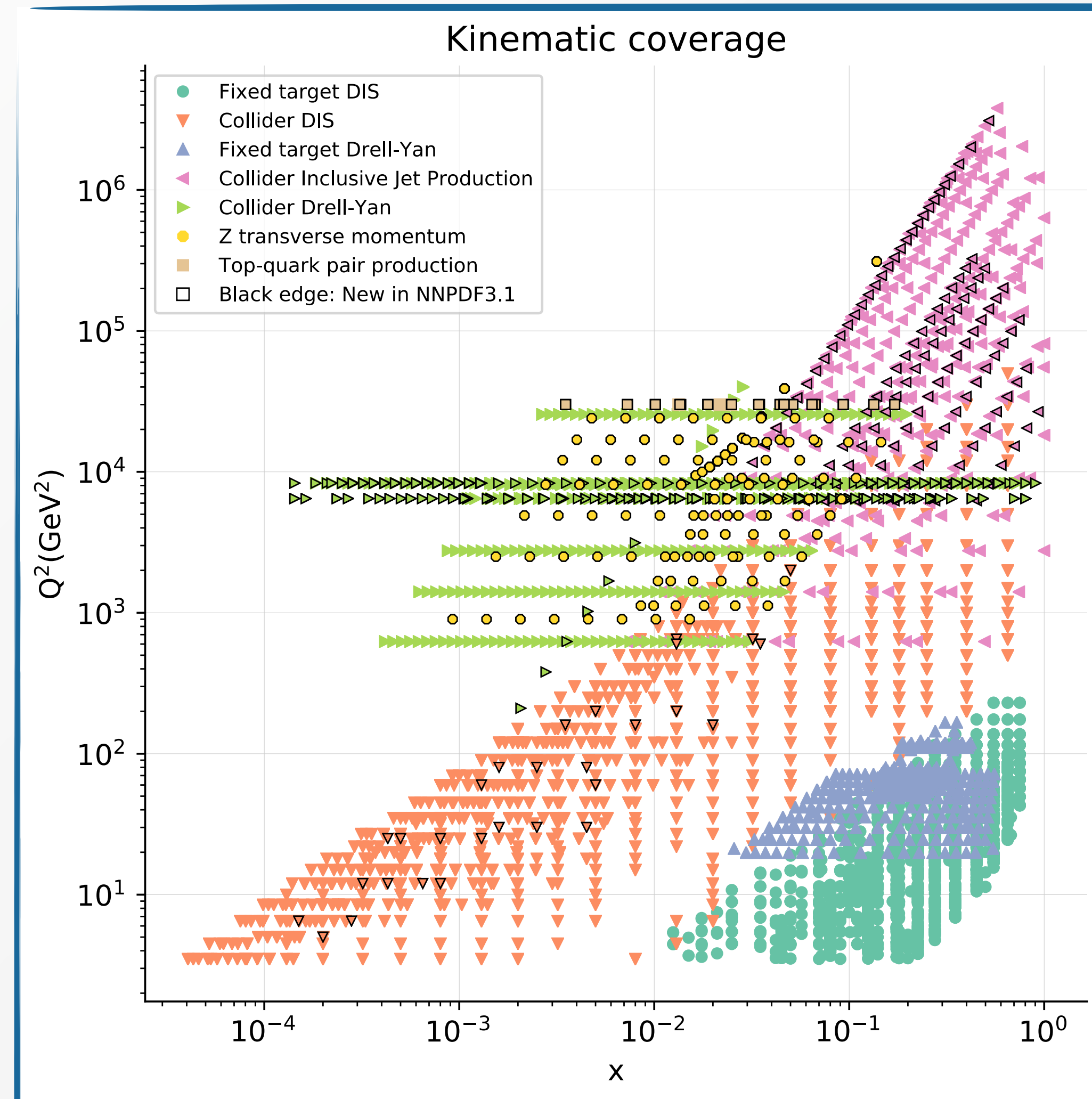
Including resummation in PDF fits:

- ▶ provides **consistent predictions** when resummed computations are used
- ▶ improves the **quality** of the PDF fits
- ▶ helps in investigating the impact of **missing higher orders**

... it brings us closer to 'all-order' PDFs

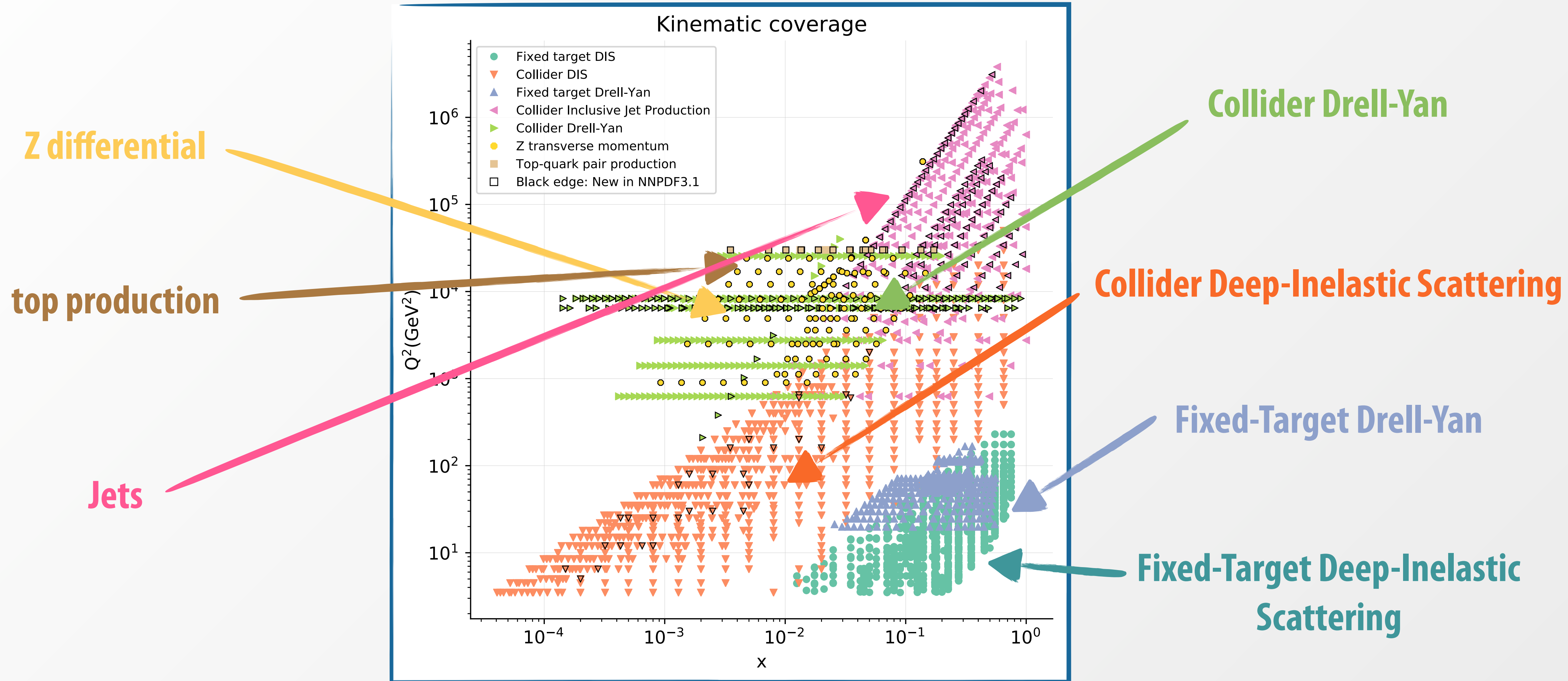
Global PDF fits

Processes used in global PDF fits

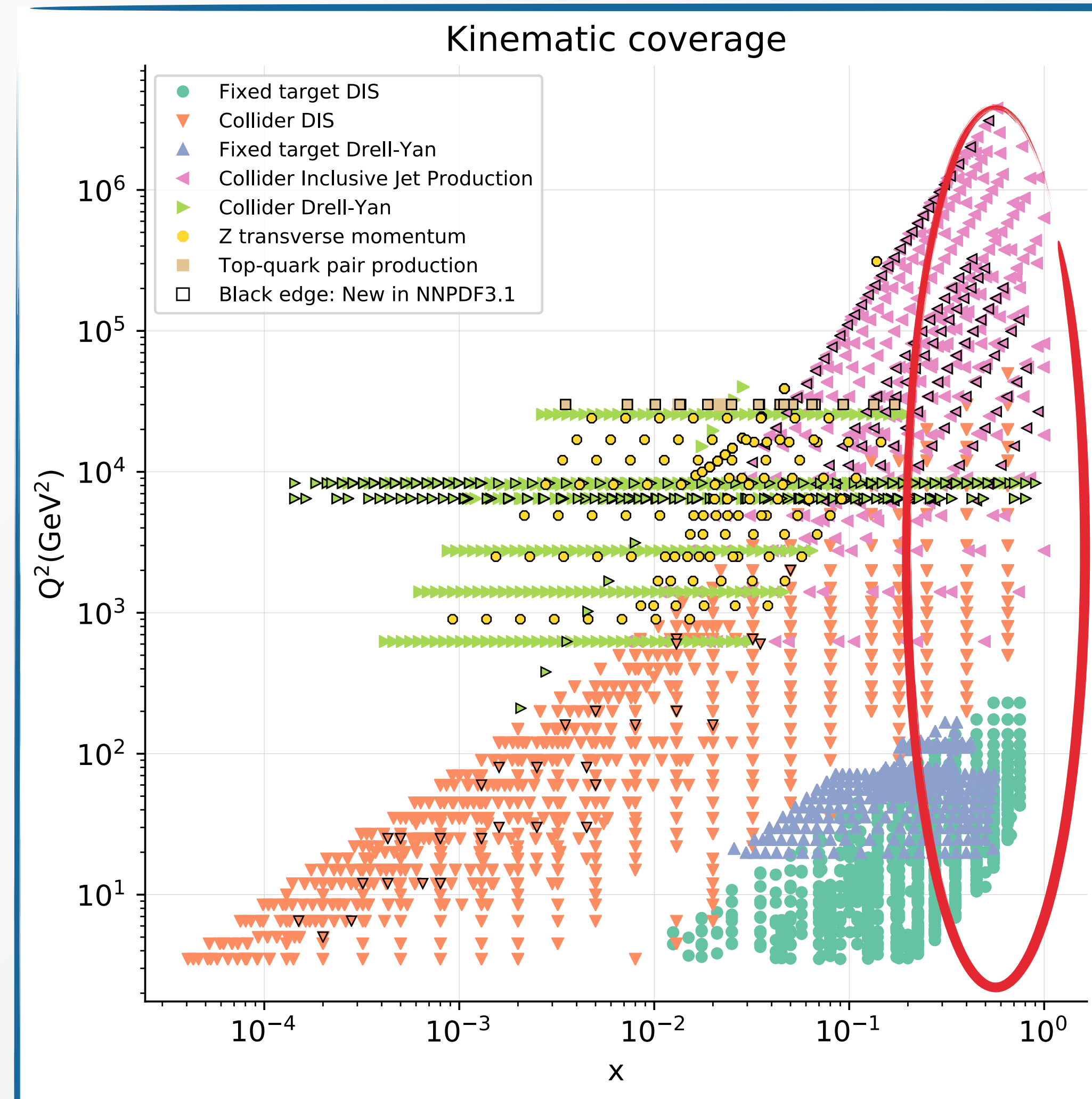


Global PDF fits

Processes used in global PDF fits



Resummation in global PDF fits



Large x : **threshold** resummation

double logs due to **soft** gluon emission

$$\left(\frac{\ln^k(1-x)}{(1-x)} \right)_+$$

[Bonvini, Marzani, Rojo, LR, Ubiali, Ball, Bertone, Carrazza, Hartland 1507.01006]

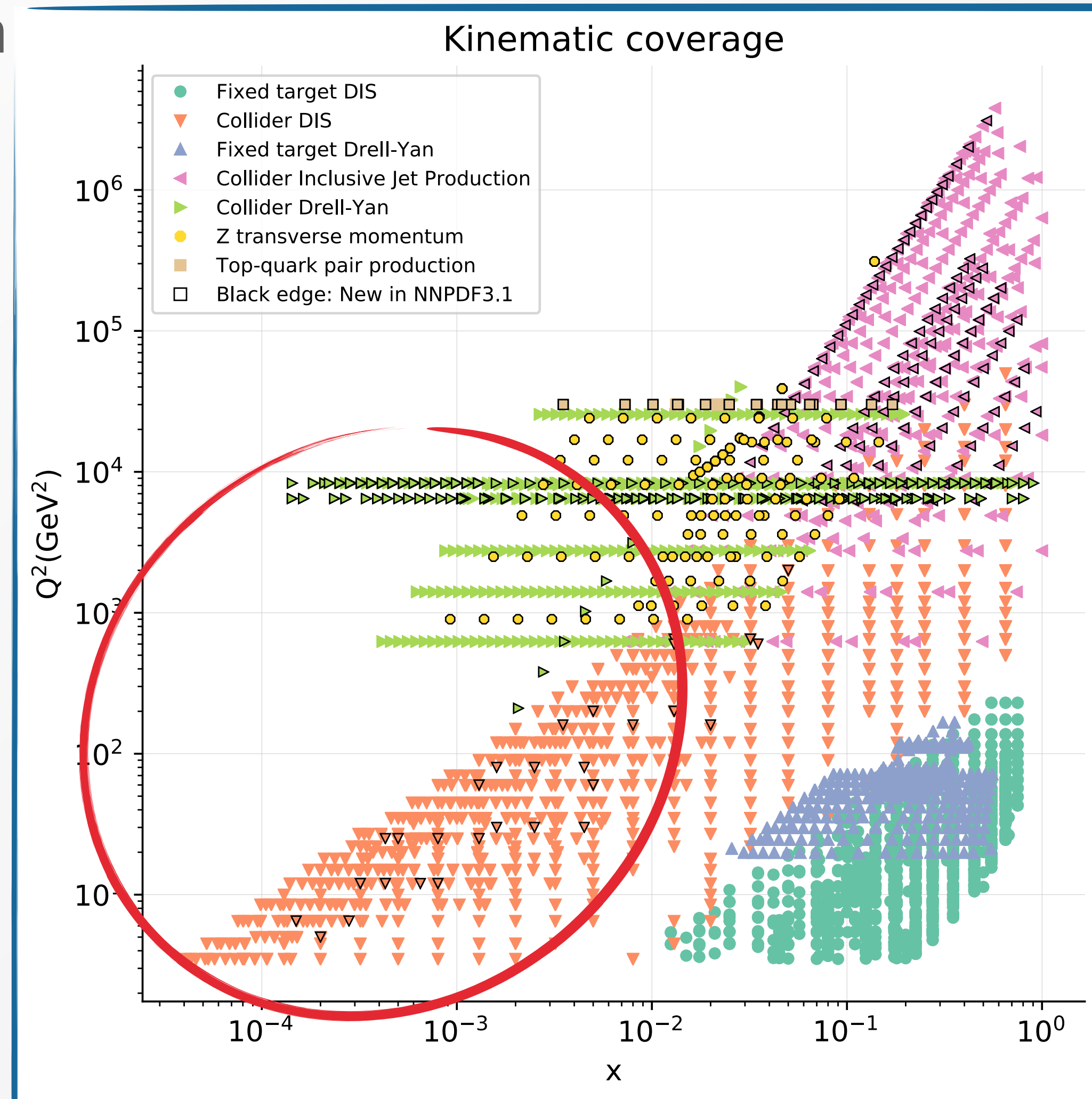
Resummation in global PDF fits

Small x : **high-energy** resummation

single logs due to high-energy gluon emission

$$\frac{1}{x} \ln^k x$$

[Ball,Bertone,Bonvini,Marzani,Rojo,LR
1710.05935]



Resummation in global PDF fits

Resummation affects:

Observable (coefficient functions)

$$\sigma = \sigma_0 \mathbf{C}(\alpha_s(\mu)) \otimes f(\mu) [\otimes f(\mu)]$$

Evolution (splitting functions)

$$\mu^2 \frac{d}{d\mu^2} f(\mu) = \mathbf{P}(\alpha_s(\mu)) \otimes f(\mu)$$

	observable (coefficient function)	evolution (splitting function)
small x	NLLx*	NLLx
large x	(N)NNLL	—

*starts at NLLx

PDFs with large- x resummation

[Bonvini,Marzani,Rojo,LR,Ubiali,Ball,Bertone, Carrazza,Hartland 1507.01006]

PDFs with large- x resummation: NNPDF3.0res

[Bonvini,Marzani,Rojo,LR,Ubiali,Ball,Bertone,
Carrazza,Hartland 1507.01006]

Datasets considered in NNPDF3.0res

process	observable	included?
DIS	$d\sigma/(dx dQ^2)$ (NC, CC, F2c...)	✓
DY Z/ γ	$d\sigma/(dy dM^2)$	✓
DY W	differential in lepton kinematics	✗ no public code available yet
$t\bar{t}$	total σ	✓
jets	inclusive $d\sigma/(dy dp_T)$	✗ NLL known to be poor

Accuracy is **competitive** with global fit, except for large- x gluon (jets not included)

Resummation is included supplementing fixed-order computations with **K-factors**

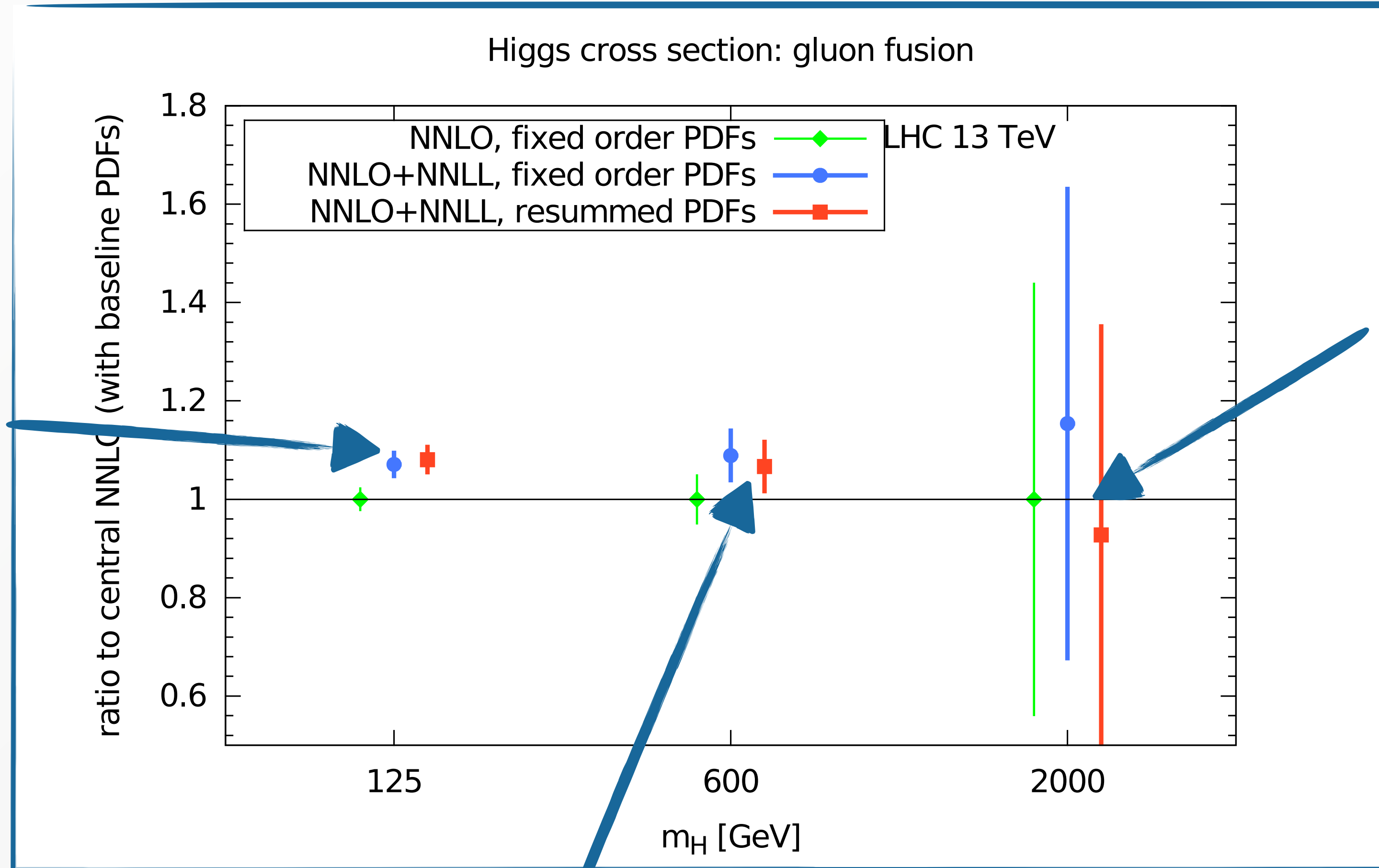
$$K^{N^k LO + N^k LL} = \frac{\sigma^{N^k LO + N^k LL}}{\sigma^{N^k LO}}$$

public code **TROLL**

www.ge.infn.it/~bonvini/troll

NNPDF3.0res: Impact on phenomenology

Higgs Production



SM Higgs is not affected by resummation of PDFs

$m_H \sim 2$ TeV NNLO+NNLL with resummed PDFs is similar to FO PDFs (larger uncertainty)

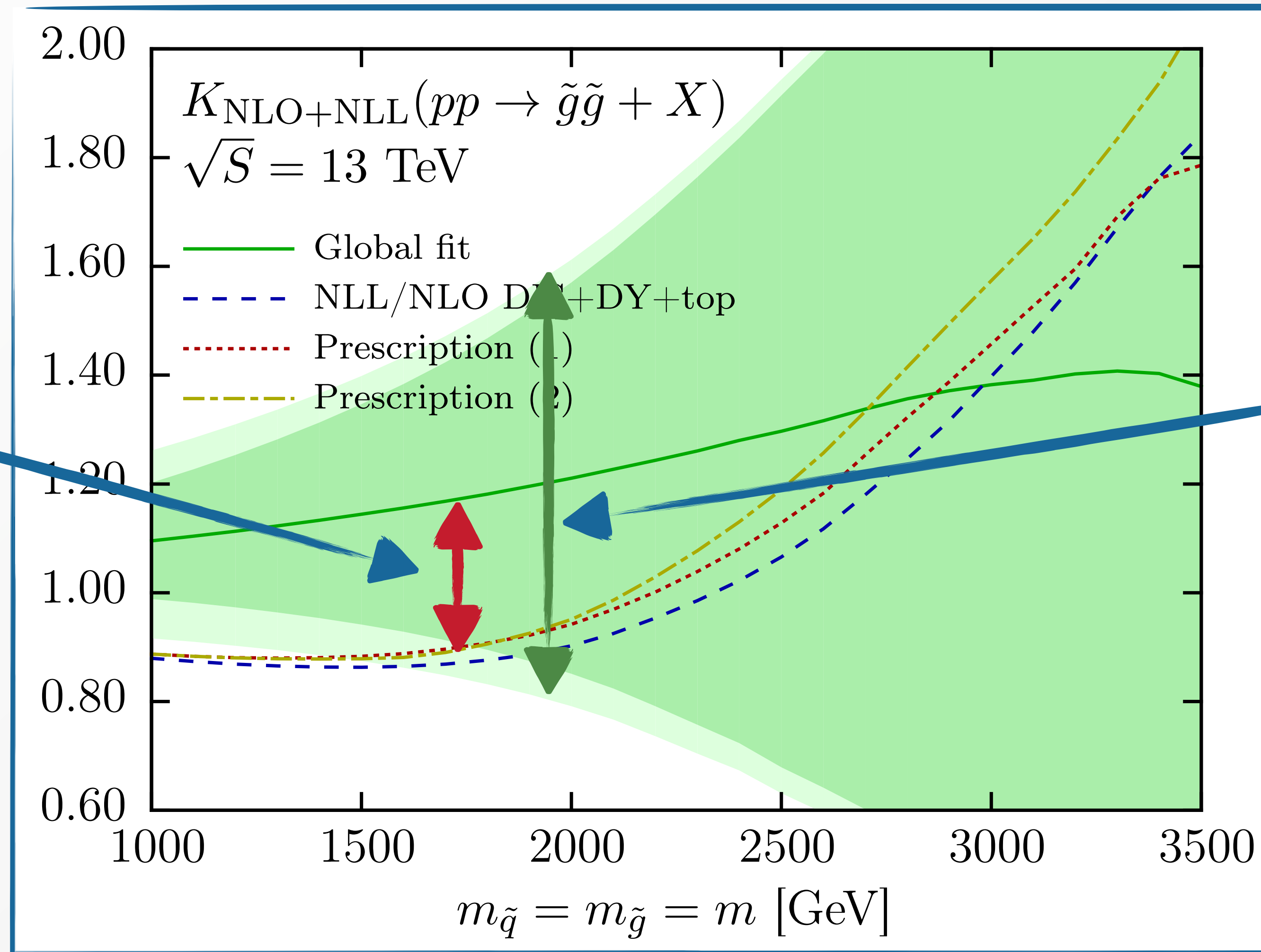
$m_H \sim 600$ GeV cancellation of 1/2 of the enhancement

NNPDF3.0res: Impact on phenomenology

[Beenakker,Borschensky,Krämer,Kulesza,Laenen,Marzani,Rojo 1510.00375]

Susy particles

Predictions for MSSM particles are modified when using resummed PDFs



However, PDF errors are very large

Outlook

- ▶ First ever **global** fit of PDFs with **threshold resummation**
- ▶ PDFs are **suppressed in the large- x region**; at intermediate values of x quark PDFs are slightly enhanced (sum rule); negligible effects at $x < 0.01$
- ▶ Inclusion of resummation **compensates the enhancement** from resummation in partonic cross sections
- ▶ Consistent resummed calculations might be closer to fixed order results

Limitations: larger uncertainties due to **reduced dataset**.

Methodology enables to have truly global resummed PDFs when calculations for missing processes will be available.

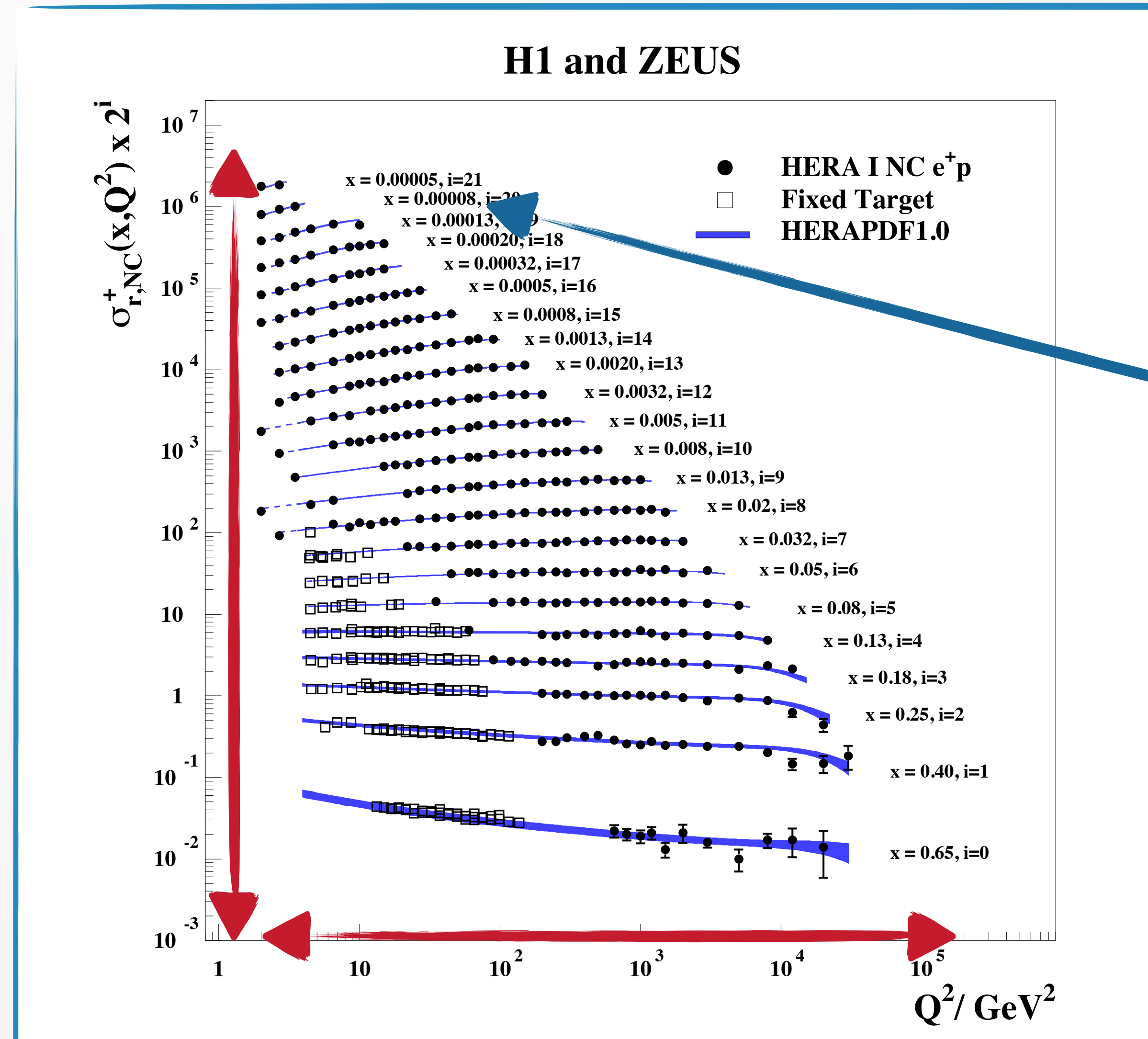
New processes to be included: DY Z/ γ (Zp_T), $t\bar{t}$ (differential)...

PDFs with small- x resummation

[Ball,Bertone,Bonvini,Marzani,Rojo,LR 1710.05935]

Need for small- x resummation?

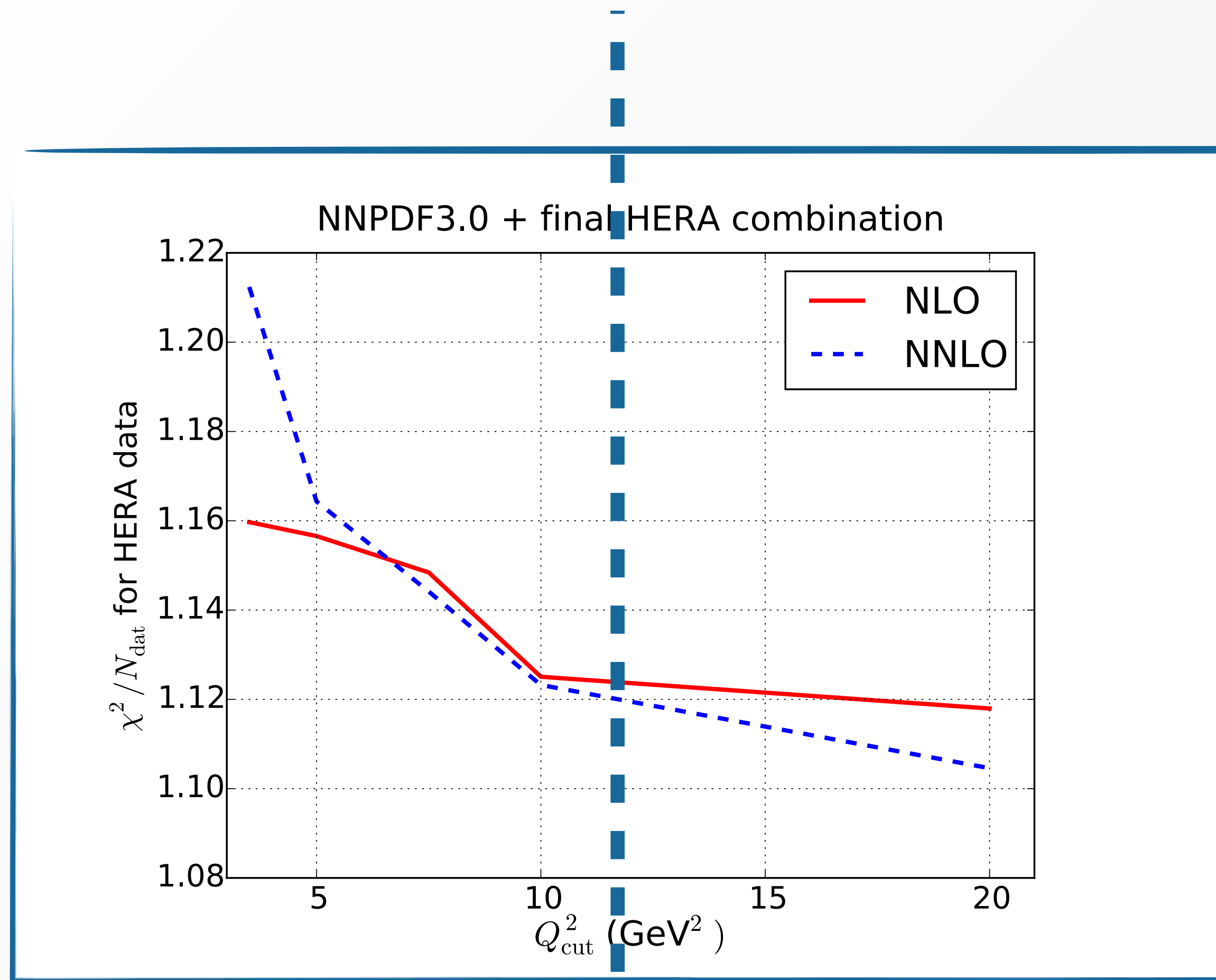
Deep Inelastic Scattering HERA dataset



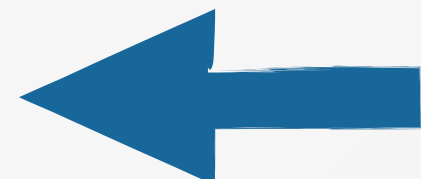
data collected down to very small x

Very good agreement over vast range of x and Q^2

Need for small- x resummation?



more points at
small x included



Courtesy of Juan Rojo

Description of HERA data poorer when data points at smaller values of x are included and fixed-order theory is used



Fixed order theory could be not sufficient to describe data points at small x and/or small Q^2

Effect is more pronounced if NNLO theory is used

This may indicate the need for
small- x resummation

(very brief) Overview of small- x resummation

Small- x resummation based on kt -factorization and BFKL. Developed mostly in the 90s-00s

[Catani,Ciafaloni,Colferai,Hautmann,Salam,Stasto][Altarelli,Ball,Forte] [Thorne,White]

Affects both **evolution** (LL_x , NLL_x) and **coefficient functions** (NLL_x , lowest logarithmic order) in the singlet sector

Splitting functions are resummed using **ABF** (Altarelli,Ball,Forte) procedure

New formalism for **coefficient function** [Bonvini,Marzani,Peraro 1607.02153] and further improvements on the ABF formalism [Bonvini,Marzani,Muselli,Peraro 1708.07510]

Resummed splitting functions and coefficient functions available through public code **HELL**

www.ge.infn.it/~bonvini/hell

Use in PDF fits possible thanks to the interface with **APFEL**

apfel.hepforge.org

Towards a global small- x resummed fit

All ingredients for a PDF fit to **DIS data** are now available

In principle, one should add additional processes:

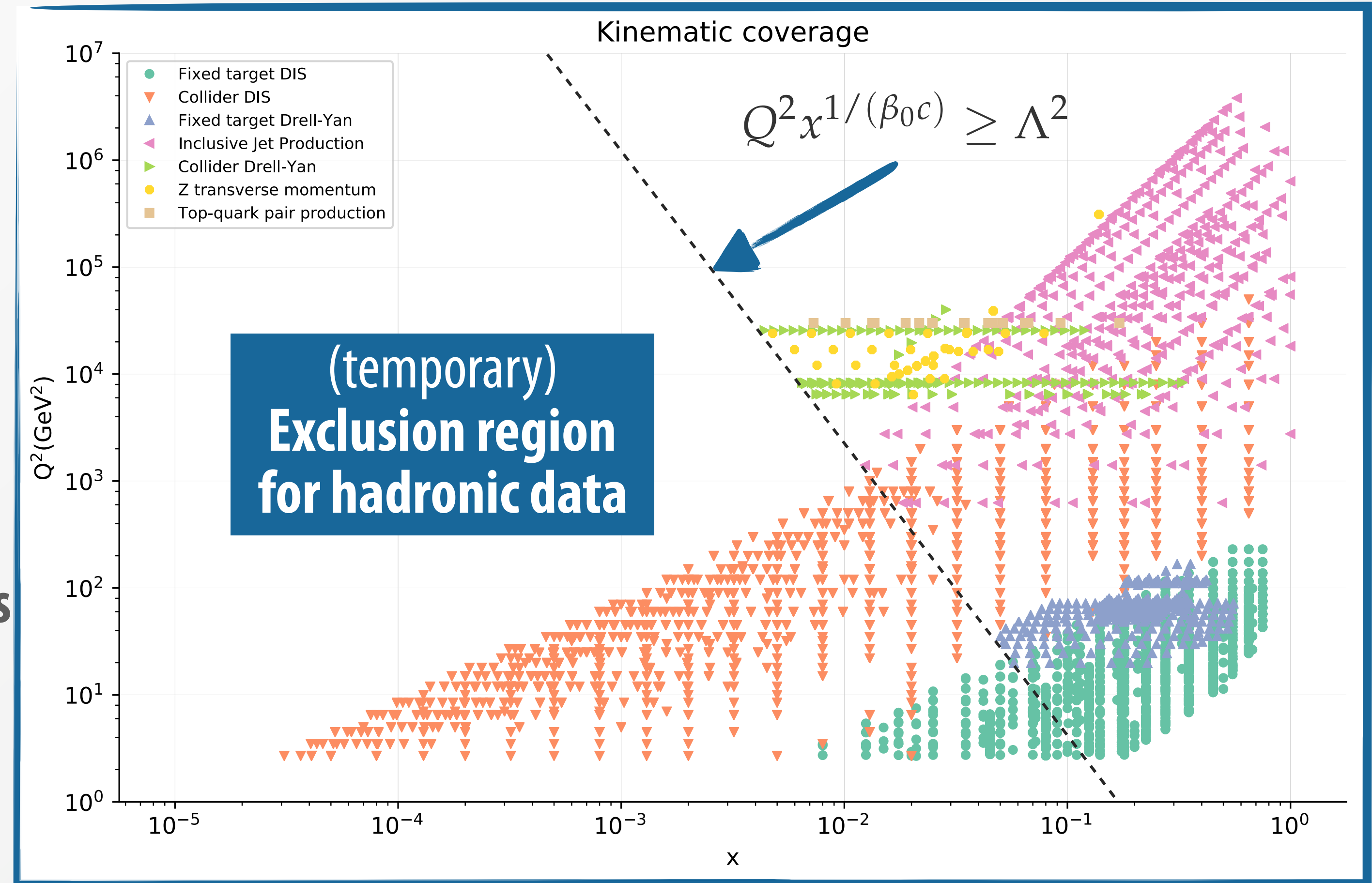
- ▶ DY
- ▶ Jets
- ▶ top
- ▶ ...

Ongoing work in this direction

However, a global fit is possible if **conservative cuts** on hadronic data are applied and points which may feature small- x enhancement are excluded

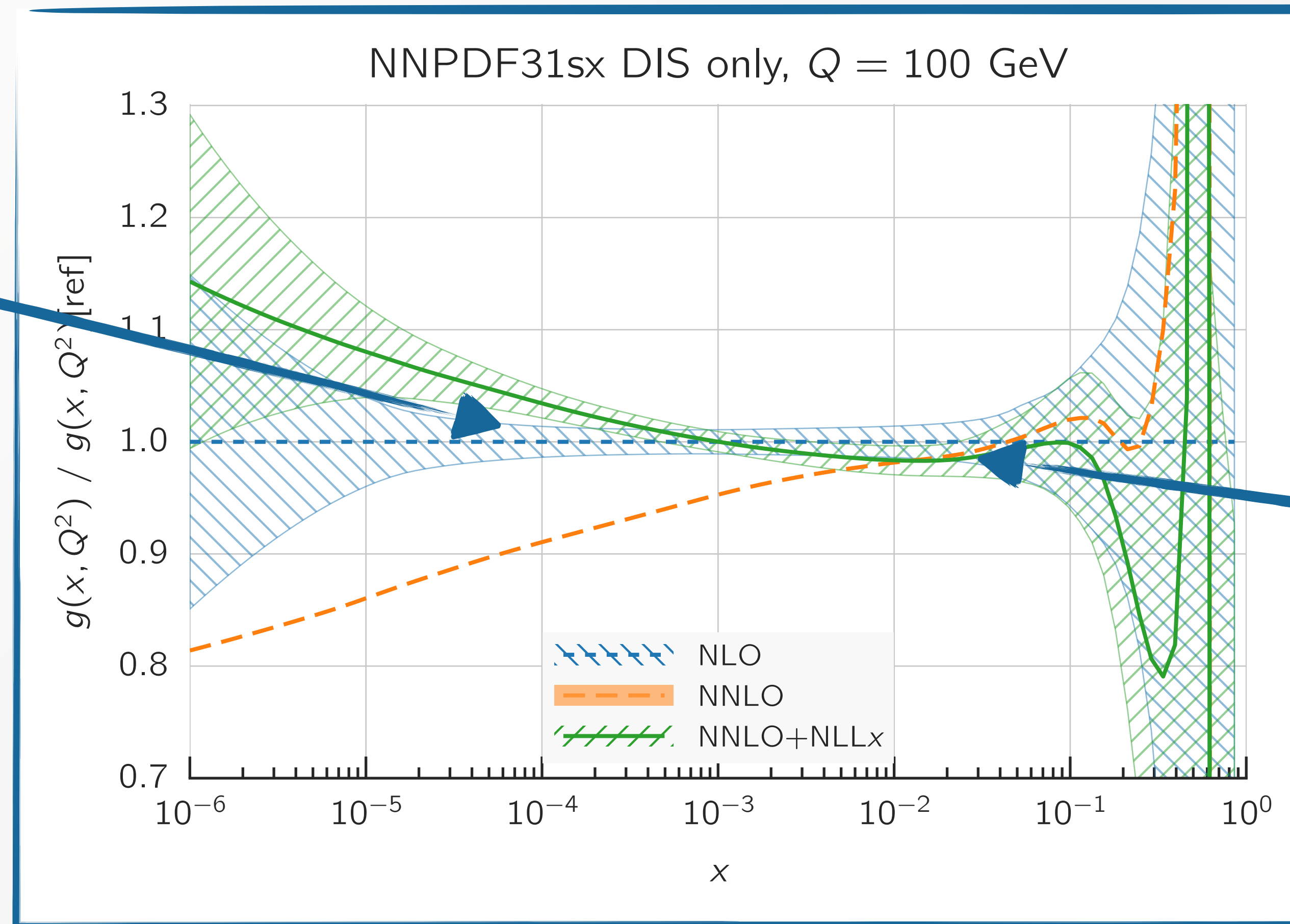
$$\alpha_s(Q^2) \log \frac{1}{x} \geq c \sim 1$$

Value of c (slope of the line) selects the exclusion region



NNPDF31sx: impact on PDFs

stabilization of the gluon with respect to the perturbative order



PDFs compatible within error at medium and large x

NNPDF31sx: fit quality

	χ^2/N_{dat}			$\Delta\chi^2$	χ^2/N_{dat}			$\Delta\chi^2$
	NLO	NLO+NLL x			NNLO	NNLO+NLL x		
NMC	1.35	1.35	+1	1.30	1.33	+9		
SLAC	1.16	1.14	-1	0.92	0.95	+2		
BCDMS	1.13	1.15	+12	1.18	1.18	+3		
CHORUS	1.07	1.10	+20	1.07	1.07	-2		
NuTeV dimuon	0.90	0.84	-5	0.97	0.88	-7		
HERA I+II incl. NC	1.12	1.12	-2	1.17	1.11	-62		
HERA I+II incl. CC	1.24	1.24	-	1.25	1.24	-1		
HERA σ_c^{NC}	1.21	1.19	-1	2.33	1.14	-56		
HERA F_2^b	1.07	1.16	+3	1.11	1.17	+2		
DY E866 $\sigma_{\text{DY}}^d/\sigma_{\text{DY}}^p$	0.37	0.37	-	0.32	0.30	-		
DY E886 σ^p	1.06	1.10	+3	1.31	1.32	-		
DY E605 σ^p	0.89	0.92	+3	1.10	1.10	-		
CDF Z rap	1.28	1.30	-	1.24	1.23	-		
CDF Run II k_t jets	0.89	0.87	-2	0.85	0.80	-4		
D0 Z rap	0.54	0.53	-	0.54	0.53	-		
D0 $W \rightarrow e\nu$ asy	1.45	1.47	-	3.00	3.10	+1		
D0 $W \rightarrow \mu\nu$ asy	1.46	1.42	-	1.59	1.56	-		
ATLAS total	1.18	1.16	-7	0.99	0.98	-2		
ATLAS W, Z 7 TeV 2010	1.52	1.47	-	1.36	1.21	-1		
ATLAS HM DY 7 TeV	2.02	1.99	-	1.70	1.70	-		
ATLAS W, Z 7 TeV 2011	3.80	3.73	-1	1.43	1.29	-1		
ATLAS jets 2010 7 TeV	0.92	0.87	-4	0.86	0.83	-2		
ATLAS jets 2.76 TeV	1.07	0.96	-6	0.96	0.96	-		
ATLAS jets 2011 7 TeV	1.17	1.18	-	1.10	1.09	-1		
ATLAS Z p_T 8 TeV (p_T^l, M_{ll})	1.21	1.24	+2	0.94	0.98	+2		
ATLAS Z p_T 8 TeV (p_T^l, y_l)	3.89	4.26	+2	0.79	1.07	+2		
ATLAS σ_{tt}^{tot}	2.11	2.79	+2	0.85	1.15	+1		
ATLAS $t\bar{t}$ rap	1.48	1.49	-	1.61	1.64	-		
CMS total	0.97	0.92	-13	0.86	0.85	-3		
CMS Drell-Yan 2D 2011	0.77	0.77	-	0.58	0.57	-		
CMS jets 7 TeV 2011	0.88	0.82	-9	0.84	0.81	-3		
CMS jets 2.76 TeV	1.07	0.98	-7	1.00	1.00	-		
CMS Z p_T 8 TeV (p_T^l, y_l)	1.49	1.57	+1	0.73	0.77	-		
CMS σ_{tt}^{tot}	0.74	1.28	+2	0.23	0.24	-		
CMS $t\bar{t}$ rap	1.16	1.19	-	1.08	1.10	-		
Total	1.117	1.120	+11	1.130	1.100	-121		

$\chi^2_{\text{NNLO+NLLx}}$ smallest

1.10 vs ~1.12 (NLO & NLO+NLLx), 1.13 (NNLO)

NNPDF31sx: fit quality

	χ^2/N_{dat}			$\Delta\chi^2$	χ^2/N_{dat}			$\Delta\chi^2$
	NLO	NLO+NLLx			NNLO	NNLO+NLLx		
NMC	1.35	1.35	+1	1.30	1.33	+9		
SLAC	1.16	1.14	-1	0.92	0.95	+2		
BCDMS	1.13	1.15	+12	1.18	1.18	+3		
CHORUS	1.07	1.10	+20	1.07	1.07	-2		
NuTeV dimuon	0.90	0.84	-5	0.97	0.88	-7		
HERA I+II incl. NC	1.12	1.12	-2	1.17	1.11	-62		
HERA I+II incl. CC	1.24	1.24	-	1.25	1.24	-1		
HERA σ_c^{NC}	1.21	1.19	-1	2.33	1.14	-56		
HERA F_2^b	1.07	1.16	+3	1.11	1.17	+2		
DY E866 $\sigma_{\text{DY}}^d/\sigma_{\text{DY}}^p$	0.37	0.37	-	0.32	0.30	-		
DY E886 σ^p	1.06	1.10	+3	1.31	1.32	-		
DY E605 σ^p	0.89	0.92	+3	1.10	1.10	-		
CDF Z rap	1.28	1.30	-	1.24	1.23	-		
CDF Run II k_t jets	0.89	0.87	-2	0.85	0.80	-4		
D0 Z rap	0.54	0.53	-	0.54	0.53	-		
D0 $W \rightarrow e\nu$ asy	1.45	1.47	-	3.00	3.10	+1		
D0 $W \rightarrow \mu\nu$ asy	1.46	1.42	-	1.59	1.56	-		
ATLAS total	1.18	1.16	-7	0.99	0.98	-2		
ATLAS W, Z 7 TeV 2010	1.52	1.47	-	1.36	1.21	-1		
ATLAS HM DY 7 TeV	2.02	1.99	-	1.70	1.70	-		
ATLAS W, Z 7 TeV 2011	3.80	3.73	-1	1.43	1.29	-1		
ATLAS jets 2010 7 TeV	0.92	0.87	-4	0.86	0.83	-2		
ATLAS jets 2.76 TeV	1.07	0.96	-6	0.96	0.96	-		
ATLAS jets 2011 7 TeV	1.17	1.18	-	1.10	1.09	-1		
ATLAS Z p_T 8 TeV (p_T^l, M_{ll})	1.21	1.24	+2	0.94	0.98	+2		
ATLAS Z p_T 8 TeV (p_T^l, y_l)	3.89	4.26	+2	0.79	1.07	+2		
ATLAS σ_{tt}^{tot}	2.11	2.79	+2	0.85	1.15	+1		
ATLAS $t\bar{t}$ rap	1.48	1.49	-	1.61	1.64	-		
CMS total	0.97	0.92	-13	0.86	0.85	-3		
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sensible improvement in the $\chi^2 \dots$

$$(\chi^2_{\text{NNLO}} - \chi^2_{\text{NNLO+NLLx}}) = -121$$

NNPDF31sx: fit quality

	χ^2/N_{dat}			$\Delta\chi^2$	χ^2/N_{dat}			$\Delta\chi^2$
	NLO	NLO+NLL x			NNLO	NNLO+NLL x		
NMC	1.35	1.35	+1	1.30	1.33	+9		
SLAC	1.16	1.14	-1	0.92	0.95	+2		
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HERA I+II incl. CC	1.24	1.24	-	1.25	1.24	-1		
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Total	1.117	1.120	+11	1.130	1.100	-121		

sensible improvement in the $\chi^2 \dots$
 mostly driven by **HERA DIS data**

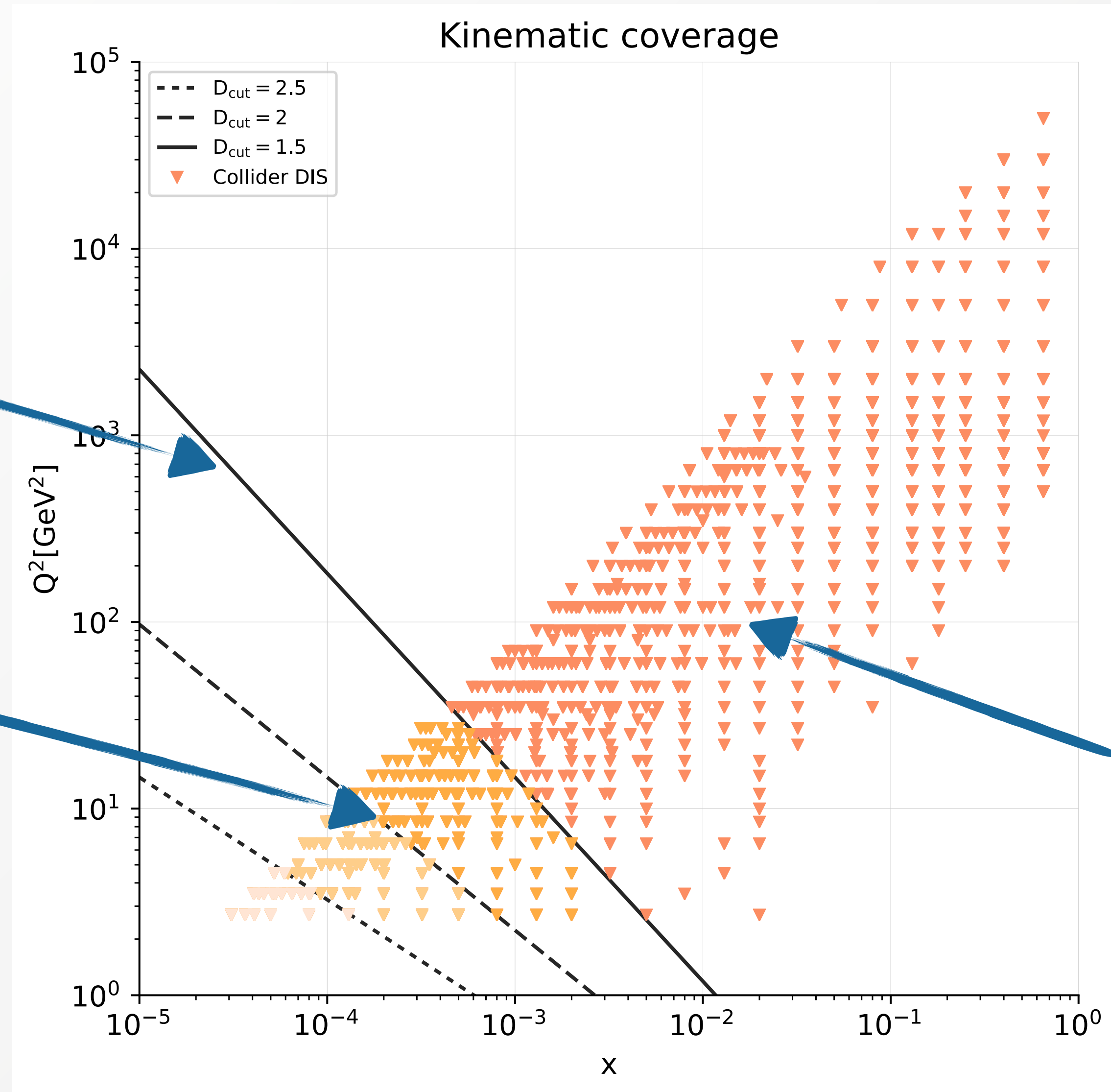
Small- x resummation and HERA data

Compute the χ^2 removing data points in the region where resummation effects are expected

cuts on DIS data

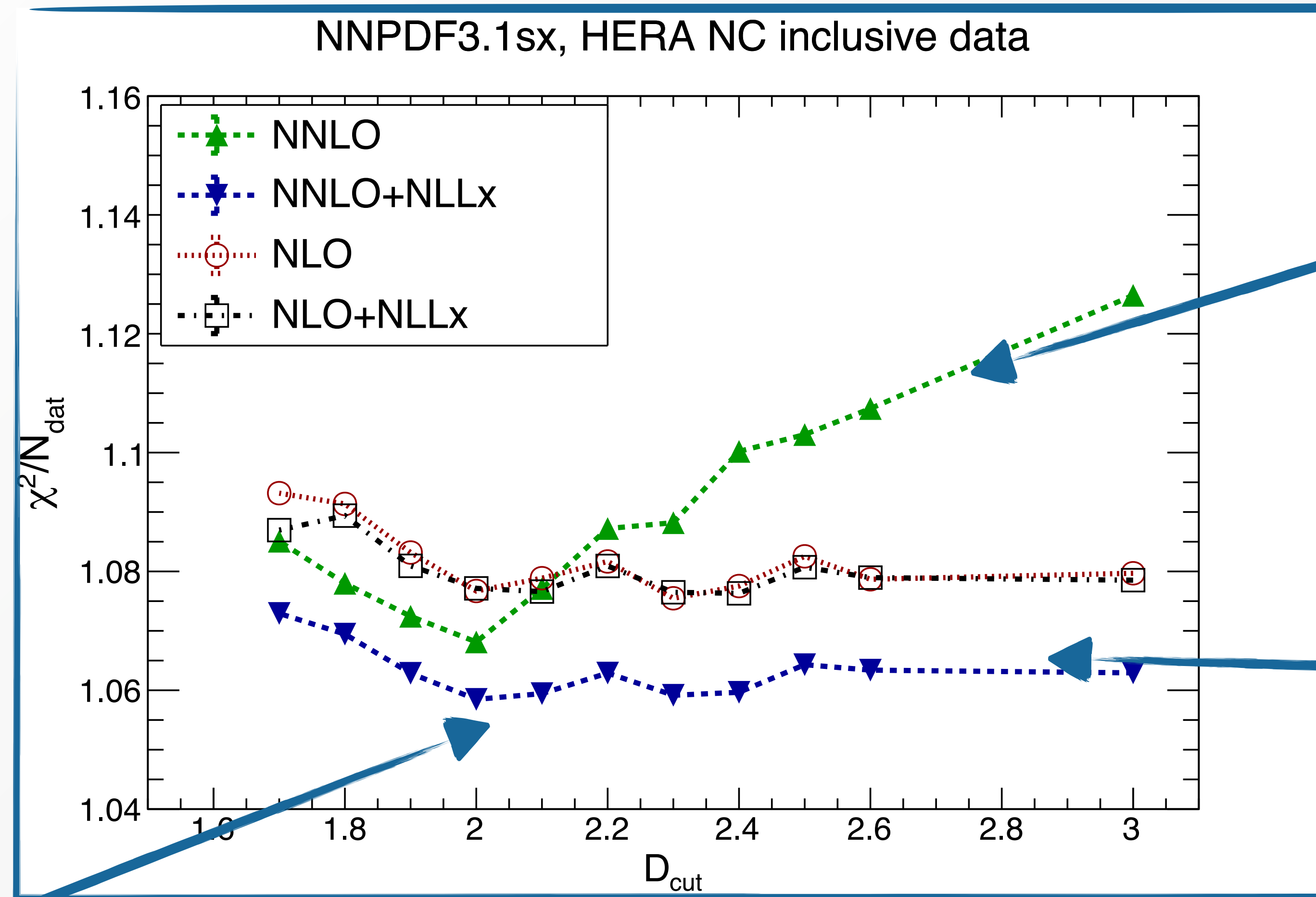
$$\alpha_s(Q^2) \ln\left(\frac{1}{x}\right) \geq D_{\text{cut}}$$

resummation effects
might be important here



fixed-order description
should be good here

Small- x resummation and HERA data

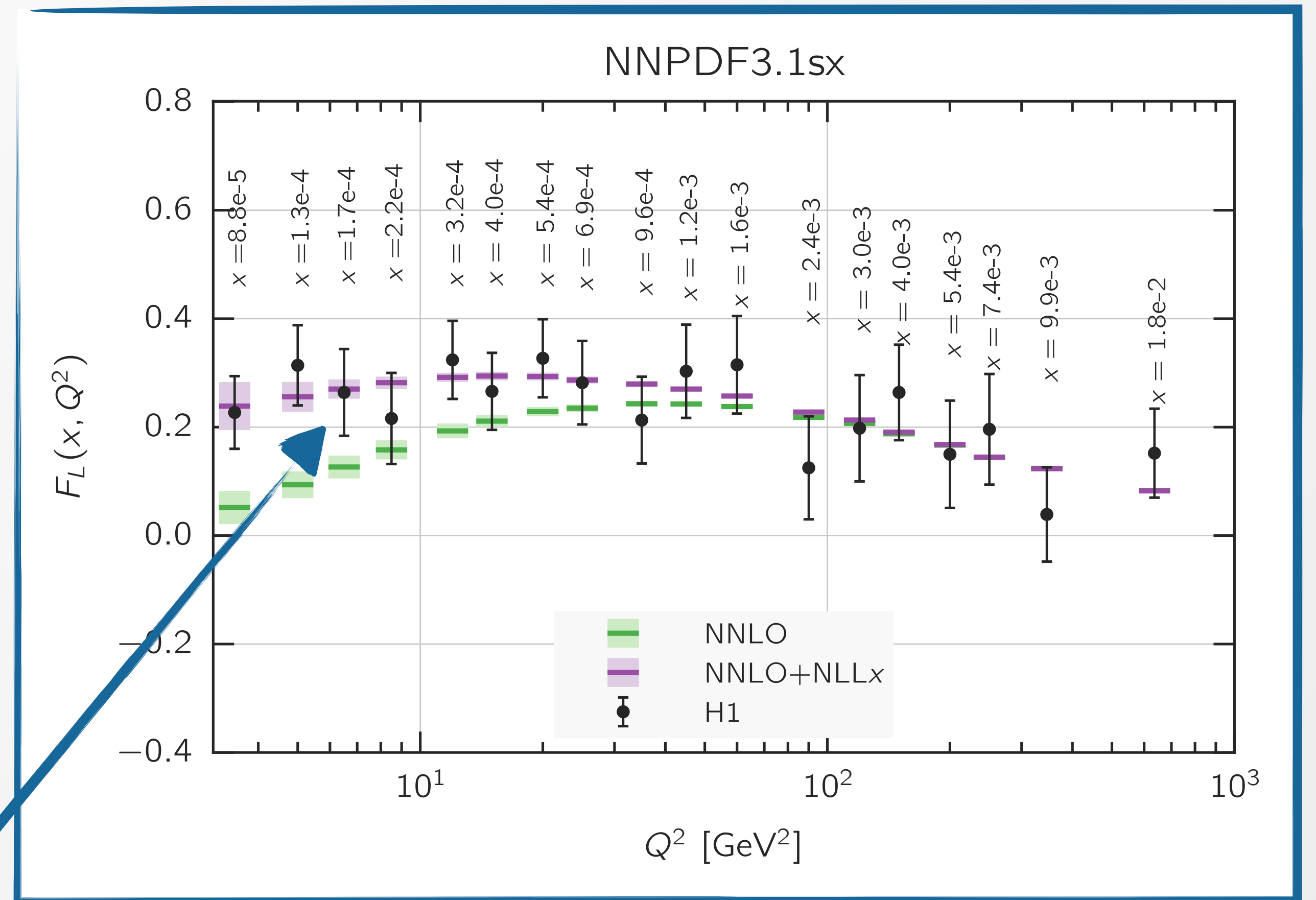
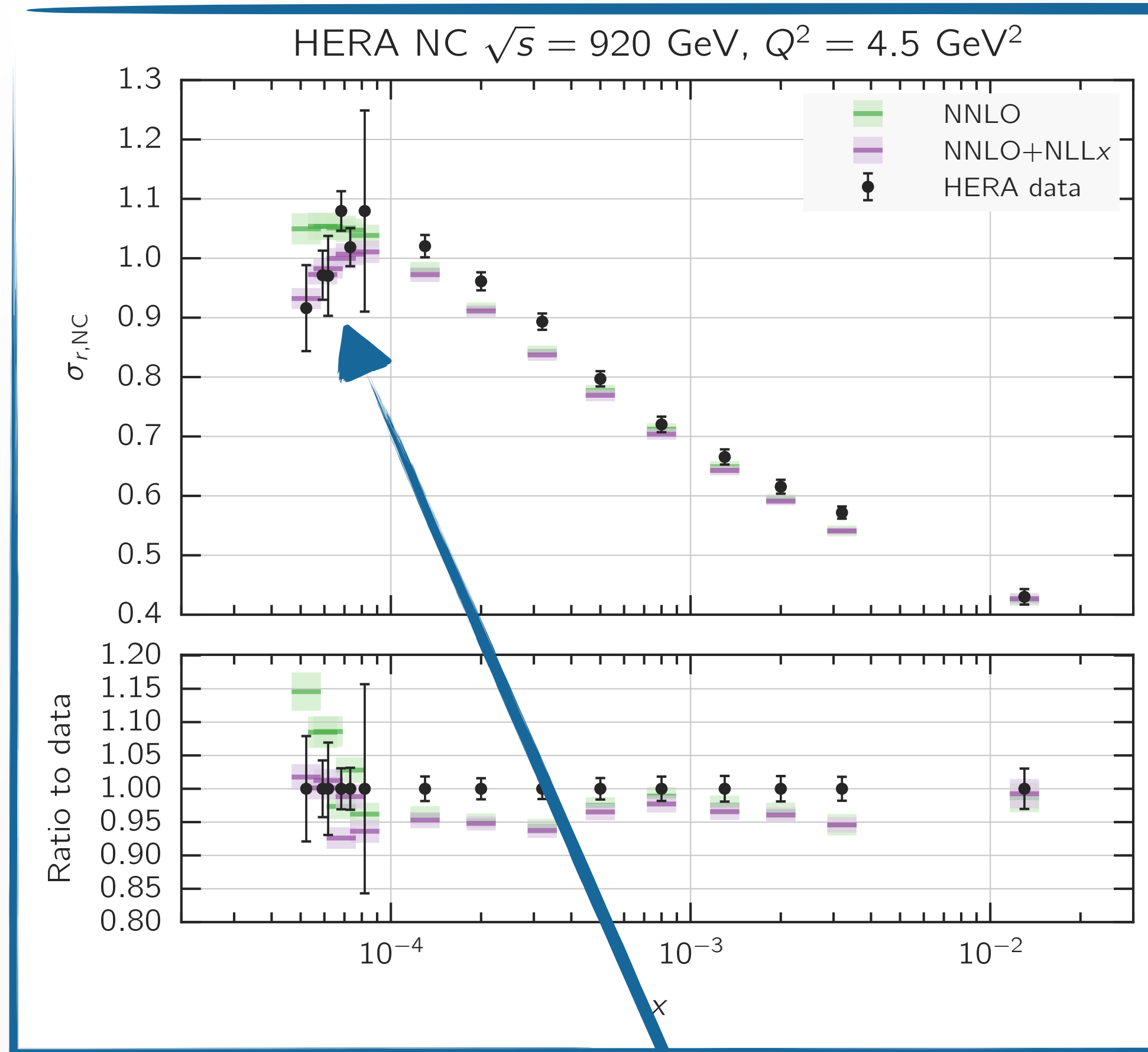


NNLO worsens if small- x data are included

NNLO+NLLx χ^2 flattens at larger values of D_{cut}

NNLO+NLLx offers the best description

Small- x resummation and HERA data



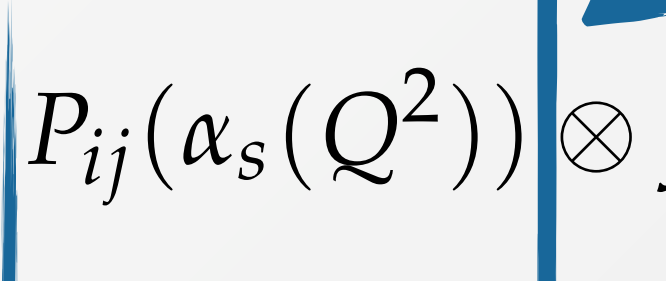
improved
description of data
at small- x and
their slope

Small- x resummation and collider processes

Need to include small- x resummation in hadronic cross section (especially DY)

First estimate of impact of resummation can be obtained by computing approximate results with resummation **only in PDF evolution**

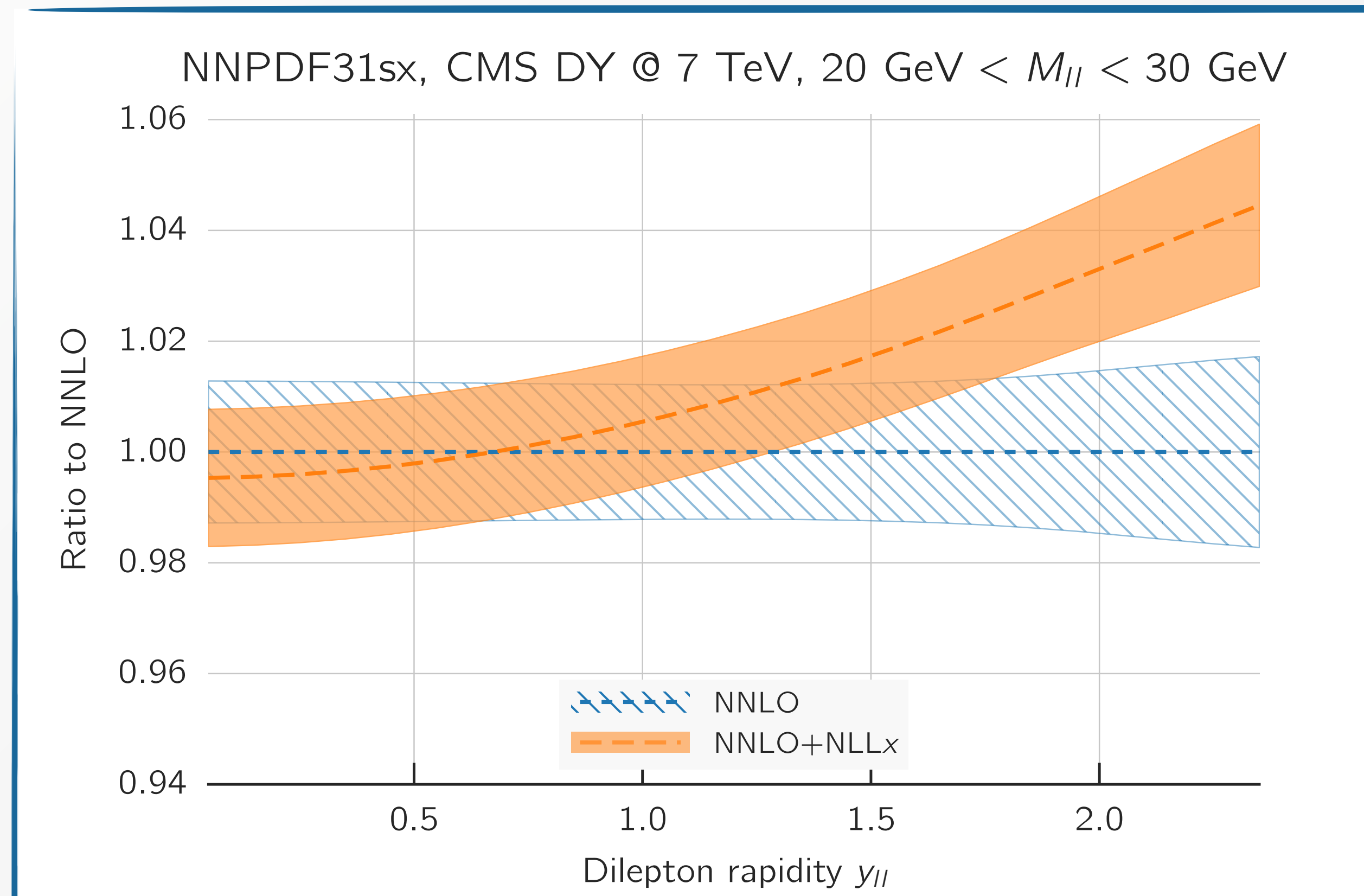
$$\sigma_X(Q^2, s) = \sum_{a,b} f_{a/h_1}(Q^2) \otimes f_{b/h_2}(Q^2) \otimes \hat{\sigma}_{ab \rightarrow X}(Q^2, s)$$

$$Q^2 \frac{d}{dQ^2} f_i(Q^2) = P_{ij}(\alpha_s(Q^2)) \otimes f_j(Q^2)$$


Small- x resummation and collider processes

Need to include small- x resummation in hadronic cross section (especially DY)

First estimate of impact of resummation can be obtained by computing approximate results with resummation **only in PDF evolution**



precision LHC phenomenology in extreme kinematic regions would require small- x resummed PDFs

Conclusions & outlook

- ▶ **First global fit** with small- x resummation in the NNPDF framework
 - ▶ Evidence that **NNLO+NLL x improves** with respect to NNLO
 - ▶ Description of the data at small x /small Q^2 significantly improves when resummation effects are included
 - ▶ Potential for reducing uncertainties for processes not necessarily related to small- x physics
-
- ▶ Computation of small- x resummation for other processes needed
 - ▶ Motivation to explore further probes of small- x dynamics at the LHC, such as low-mass DY at LHCb
 - ▶ PDF sets with **joint** (large- x + small- x) resummation?

Backup

Threshold resummation in a nutshell

$$\sigma(x, Q^2) = x \int_x^1 \frac{dz}{z} \mathcal{L}\left(\frac{x}{z}, Q^2\right) \frac{\hat{\sigma}(z, Q^2)}{z}$$

Convolution integral diagonalise in **Mellin space**

$$\sigma(N, Q^2) = \mathcal{L}(N, Q^2) \sigma_0(N, Q^2) C(N)$$

Double logarithmic enhancement due to soft gluon emission

$$C(N) = 1 + \sum_{n=1}^{\infty} \alpha_s^{2n} \sum_{k=0}^{2n} c_{nk} \ln^k N + \mathcal{O}(1/N)$$

N-soft

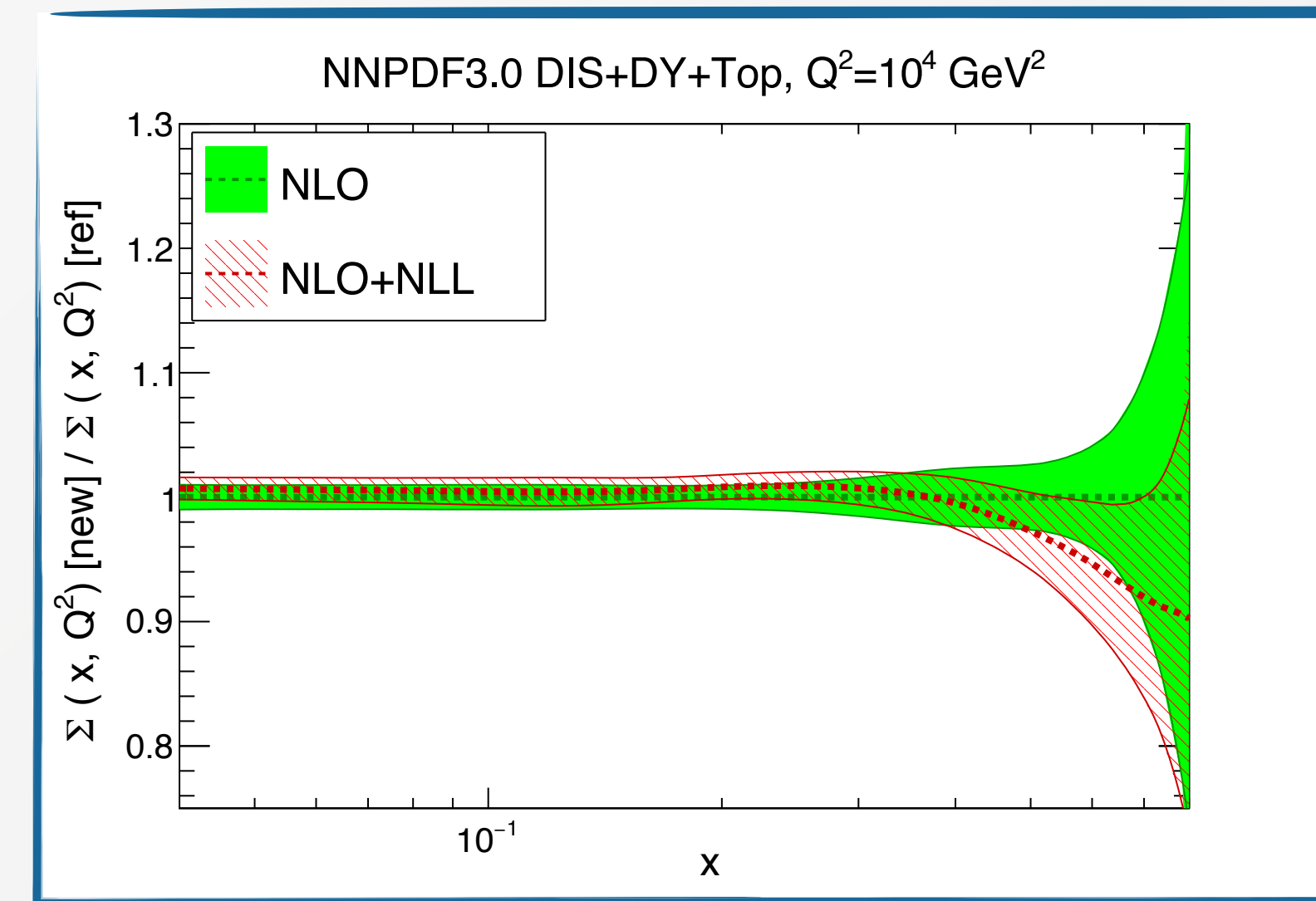
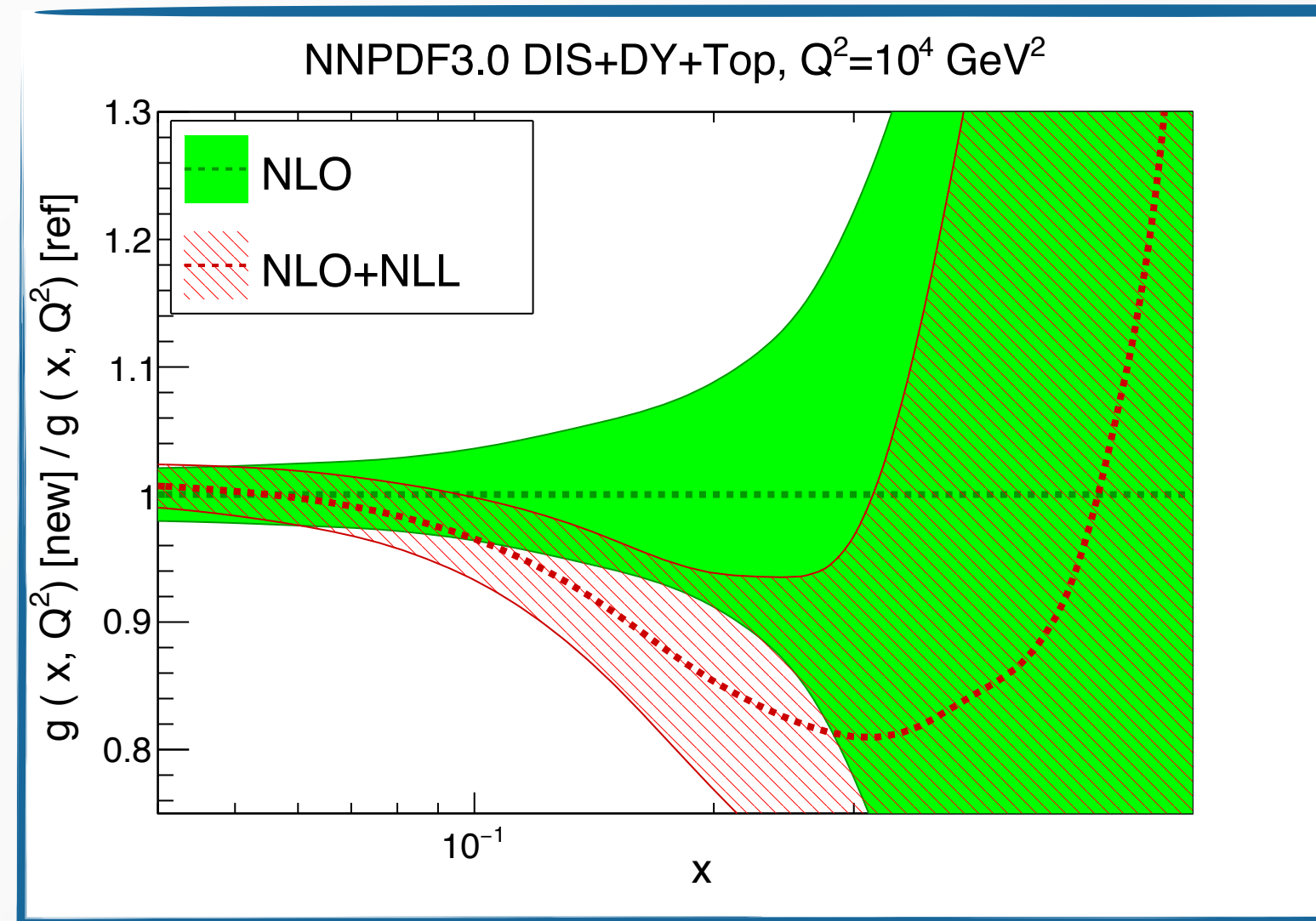
Exponentiation

$$C(N) = g_0(\alpha_s) \exp \left[\frac{1}{\alpha_s} \underbrace{g_1(\alpha_s \ln N)}_{\text{LL}} + \underbrace{g_2(\alpha_s \ln N)}_{\text{NLL}} + \alpha_s \underbrace{g_3(\alpha_s \ln N)}_{\text{NNLL}} + \dots \right]$$

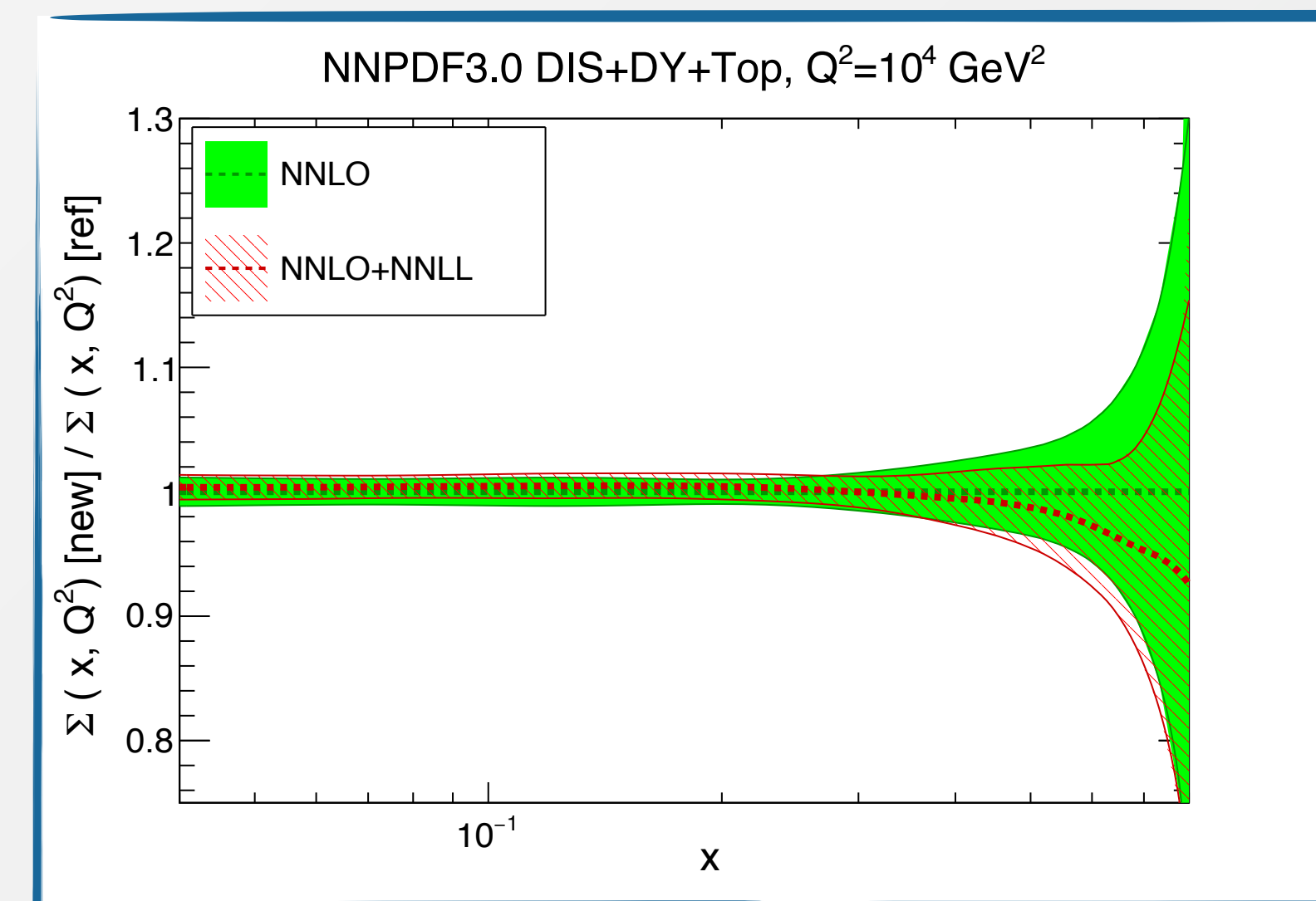
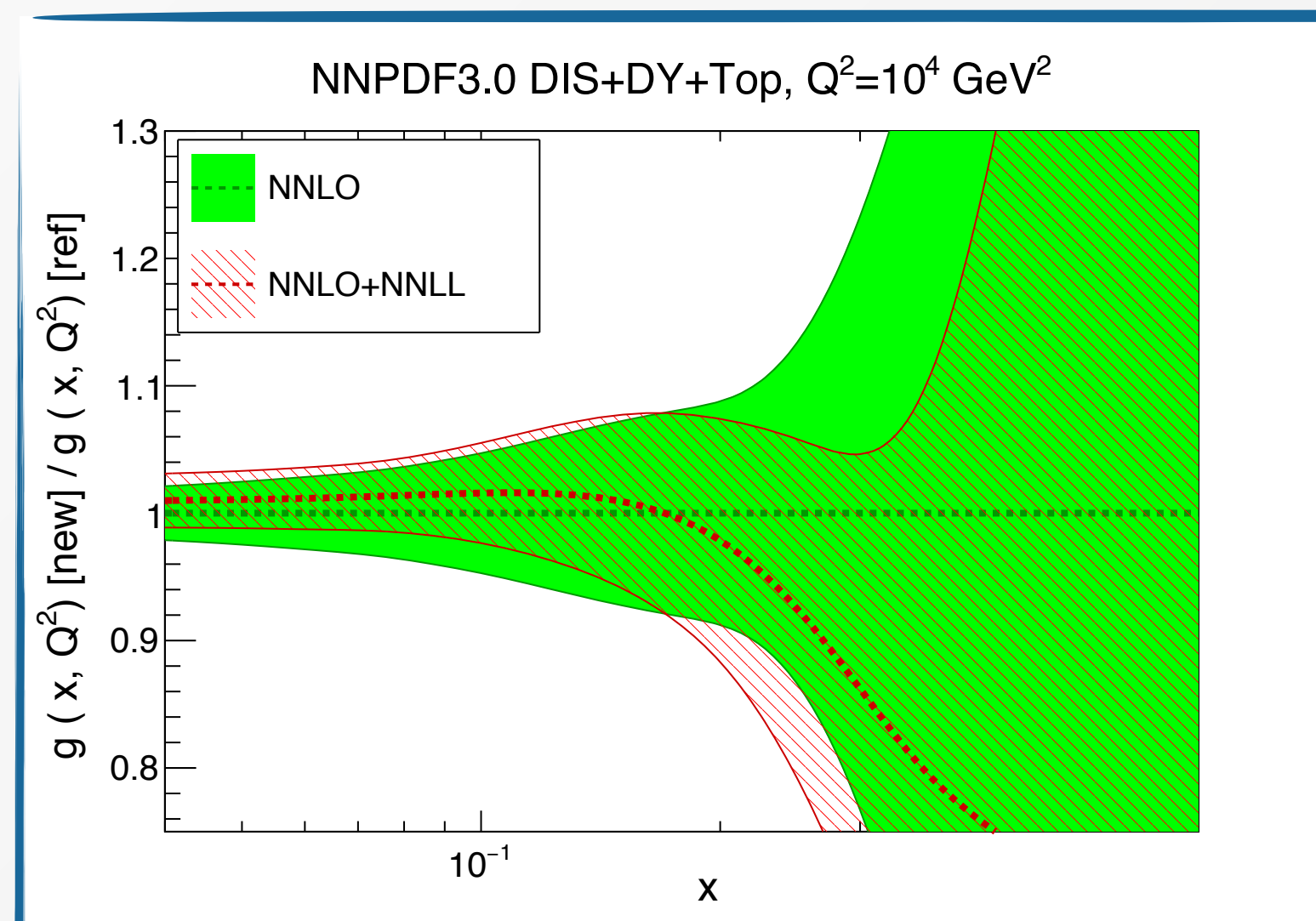
The functions g_i resum $\alpha_s^k \ln^k N$ to all orders

Impact on PDFs

NLO+NLL



NNLO+NNLL



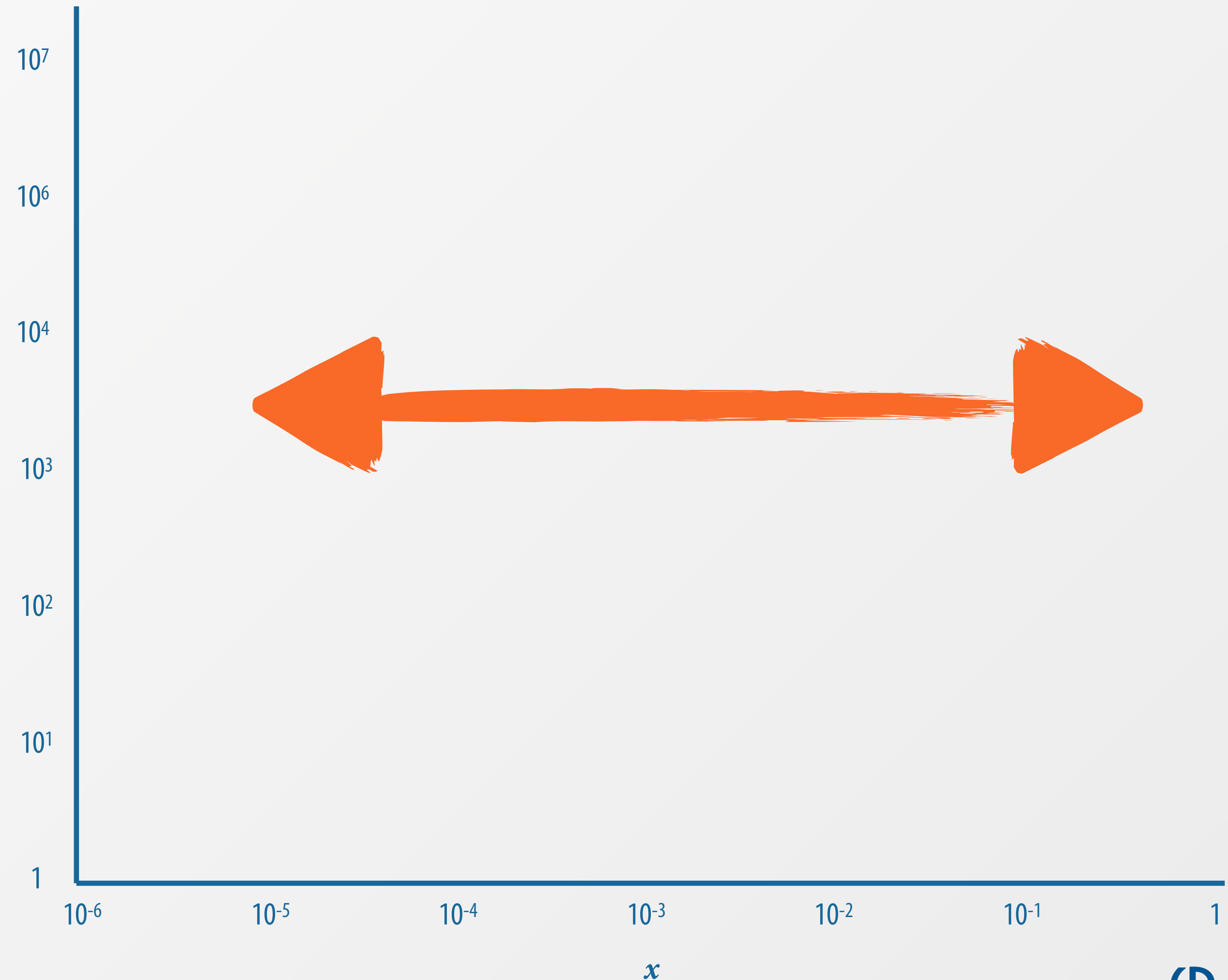
Parton Distribution Functions

PDFs depend on two kinematic variables $f(x, Q^2)$

Evolution in x^2 is encoded in **BFKL equation**

$$-x \frac{\partial}{\partial x} f_+(x, Q^2) = \int_0^\infty \frac{d\nu^2}{\nu} K\left(\frac{\mu^2}{\nu^2}, \alpha_s(Q^2)\right) f_+(x, \nu^2)$$

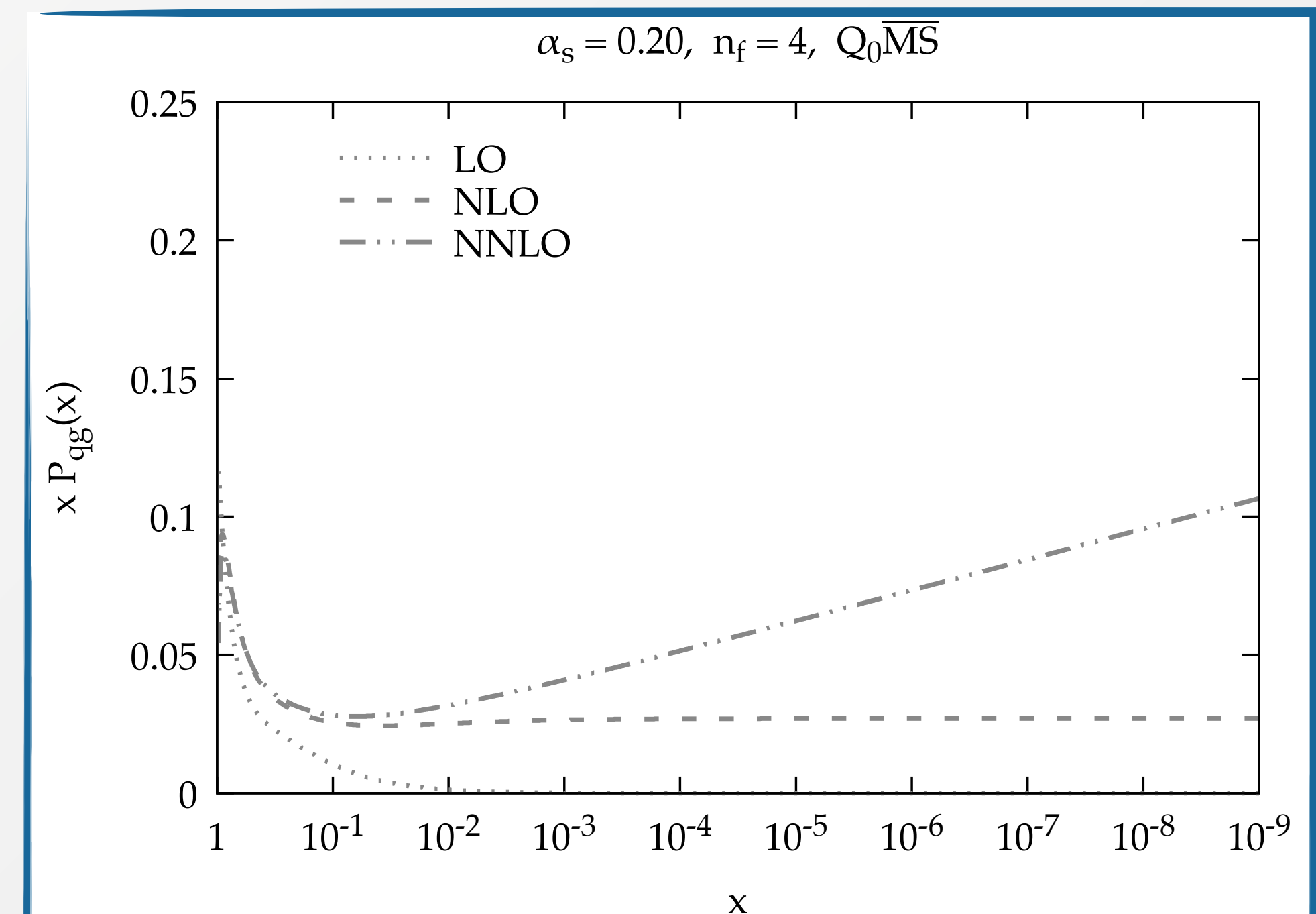
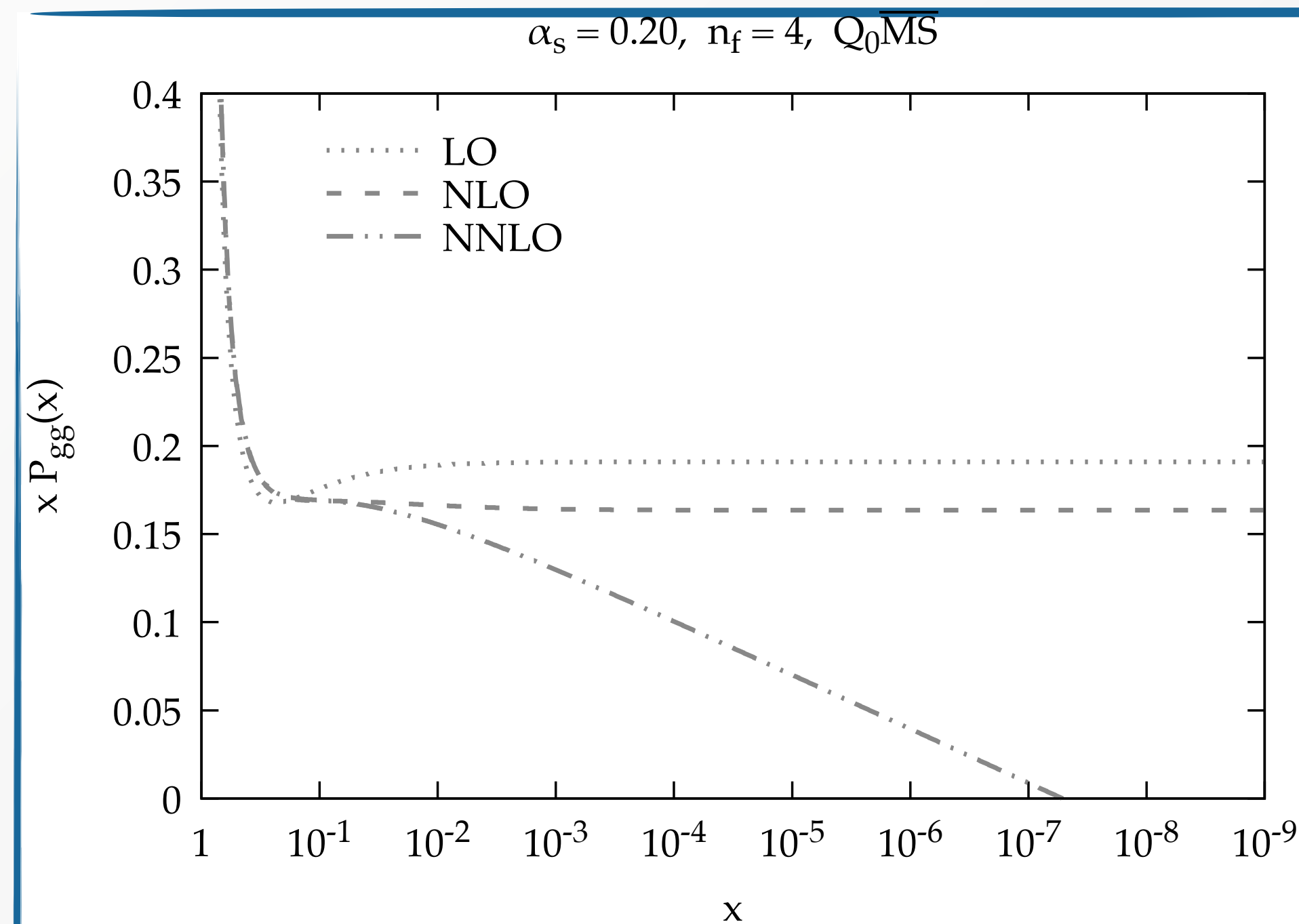
Q^2 [GeV]



Small- x resummation of DGLAP evolution

ABF procedure based on

- ▶ **duality** with BFKL evolution
- ▶ **symmetry** of the BFKL kernel
- ▶ **momentum conservation**
- ▶ resummation of (subleading, but fundamental) **running coupling effects**

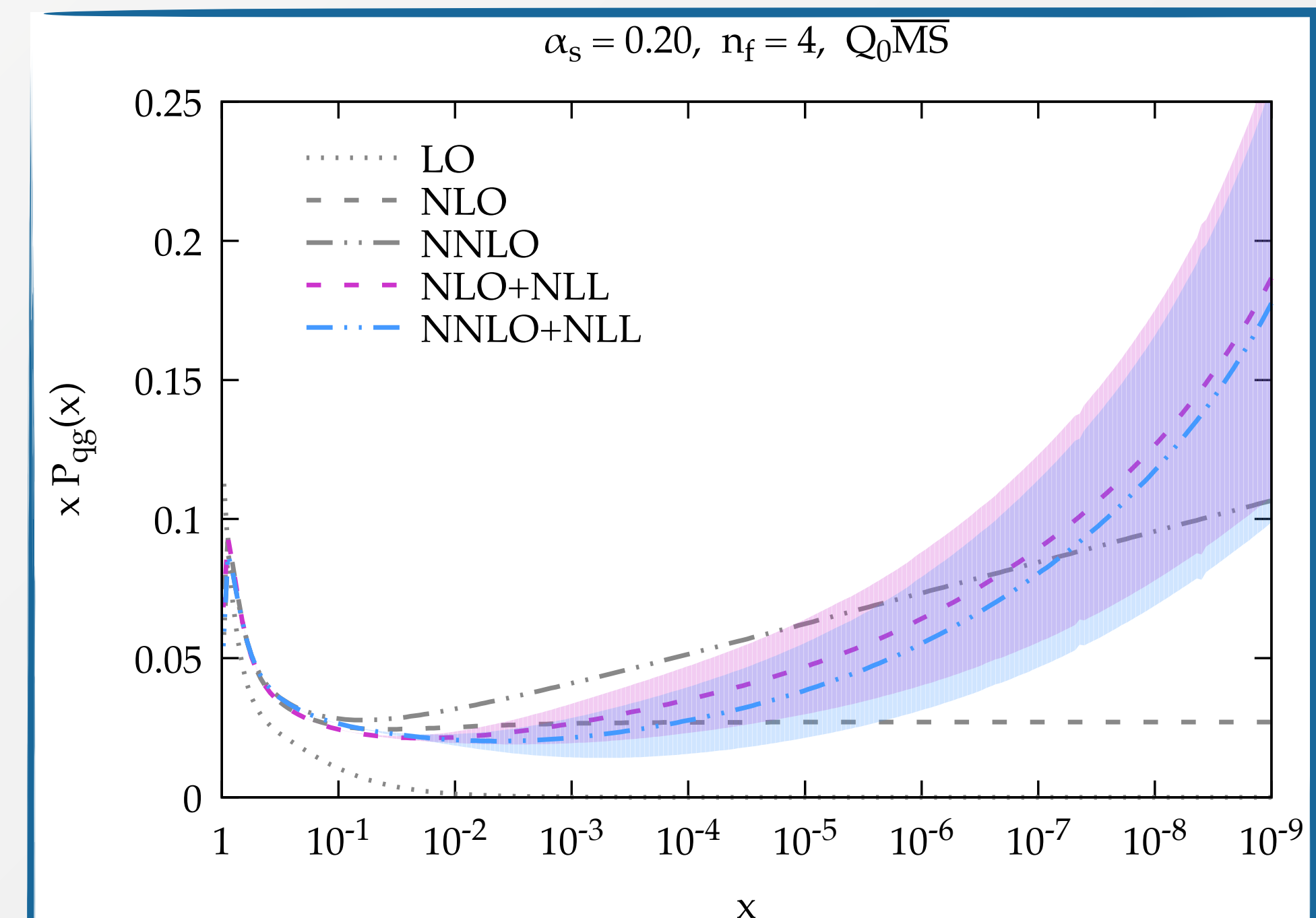
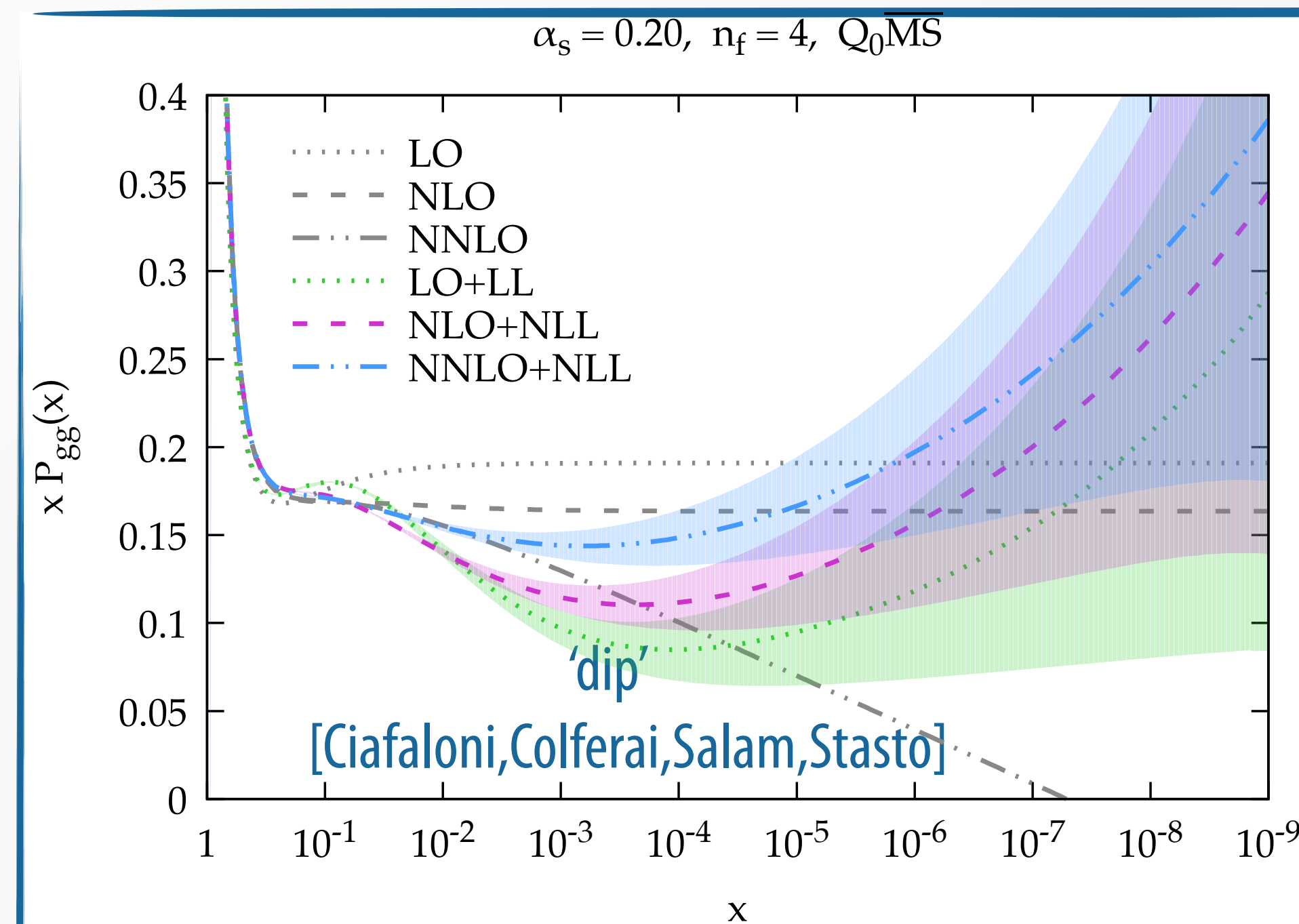


Small- x resummation of DGLAP evolution

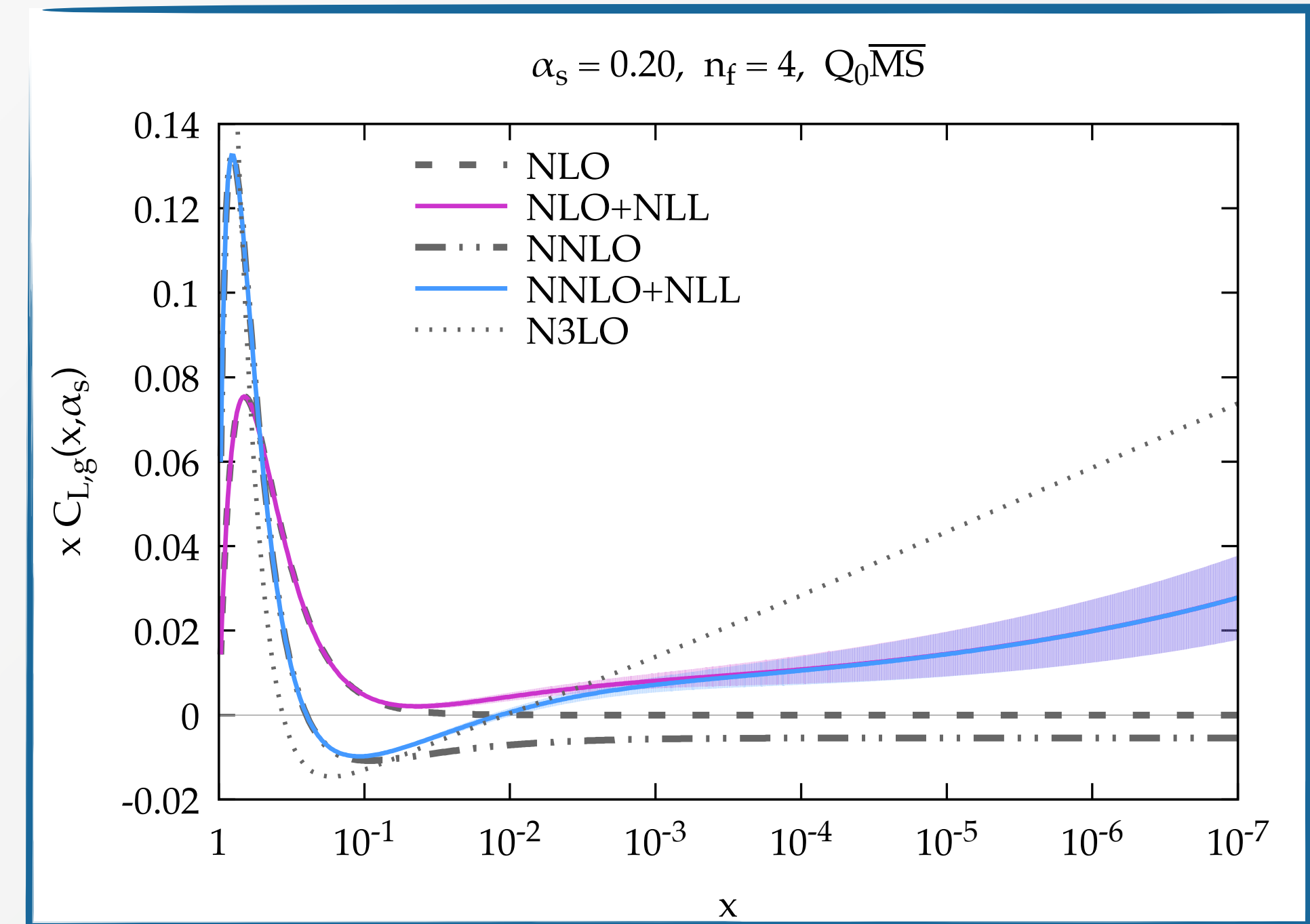
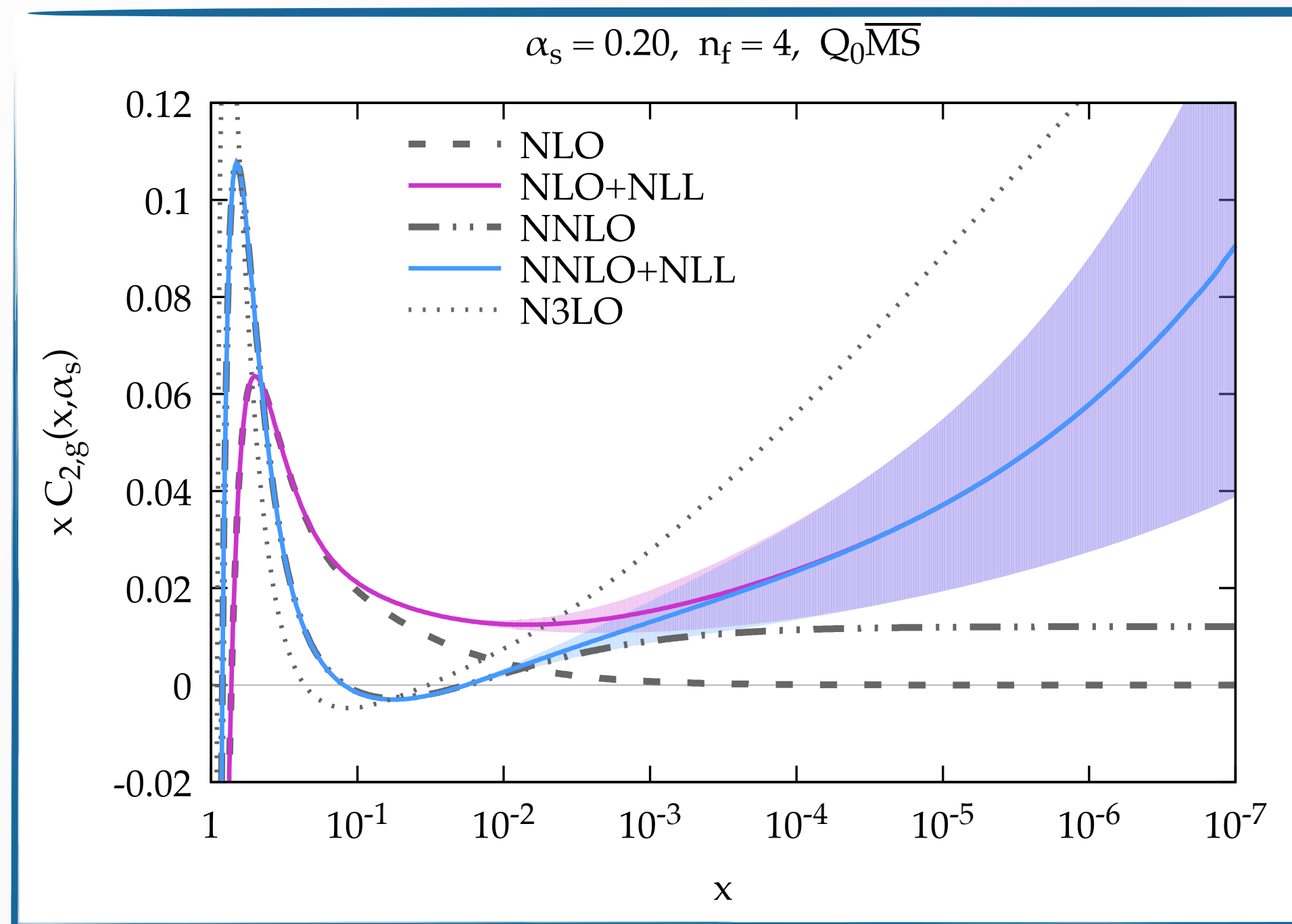
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Now matching at NNLO available!



Small- x resummation of coefficient functions



Courtesy of Marco Bonvini

► **massive** DIS coefficient functions available and implemented in **HELL**

► **VFNS** (FONLL = S-ACOT) implementation $C_{L,g}^{[n_f+1]}(m) = C_{L,g}^{[n_f]}(m), \quad C_{2,g}^{[n_f+1]}(m) = C_{2,g}^{[n_f]}(m) - K_{hg}(m)$

► resummed **matching conditions** in **HELL** $f_i^{[n_f+1]}(m) = \sum_{j=g,q,\dots,q_{n_f}} K_{ij}(m) f_j^{[n_f]}, \quad i = g, q, \dots, q_{n_f+1}$

See also [Thorne,White]