Theory uncertainties affecting m_W measurements

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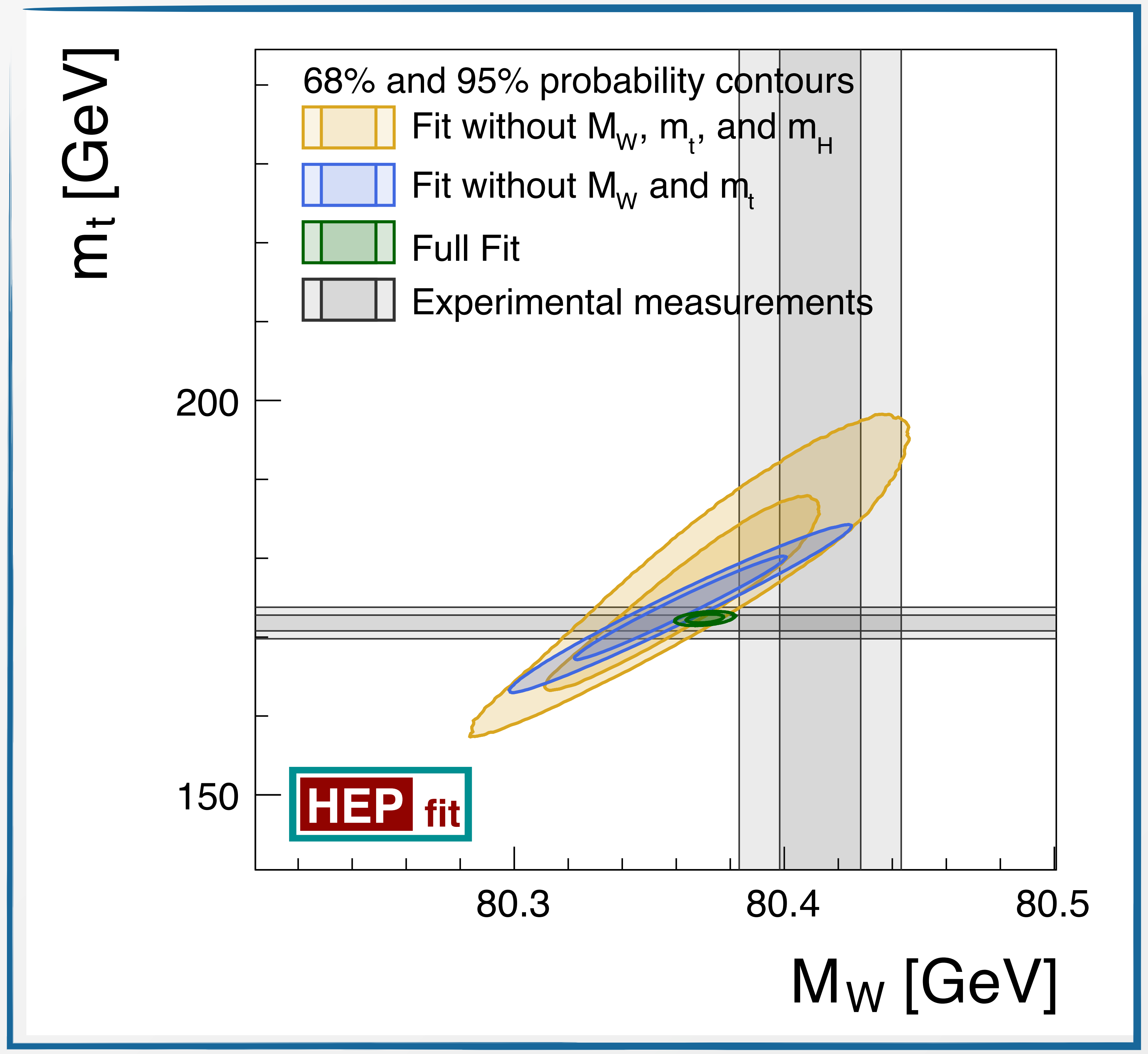
Measurements of m_W at hadron colliders

The discovery of the Higgs boson and the measurement of its mass allow for the prediction of the W mass with high precision

$$m_W = 80.350 \pm 8\,\text{GeV}$$

Which is in a 2σ agreement with the experimental average (pre-CDF II)

$$m_W = 80.385 \pm 15\,\text{GeV}$$



[de Blas, Pierini, Reina, Silvestrini '22]

Measurements of m_W at hadron colliders

Full kinematics of **charged DY production** is not accessible at hadron colliders; in particular, the invariant mass of the neutrino-lepton pair cannot be reconstructed

Reconstruction possible in the **transverse plane** (requires precise measurement of the **hadronic recoil**)

Precise determinations of the W mass exploit observables with high sensitivity to small variations $\mathcal{O}(10^{-4})$ of m_W, such as the lepton transverse momentum p_T^ℓ or the transverse mass $m_T = \sqrt{2 p_T^\ell p_T^\nu (1 - \cos\Delta\phi_{\ell\nu})}$

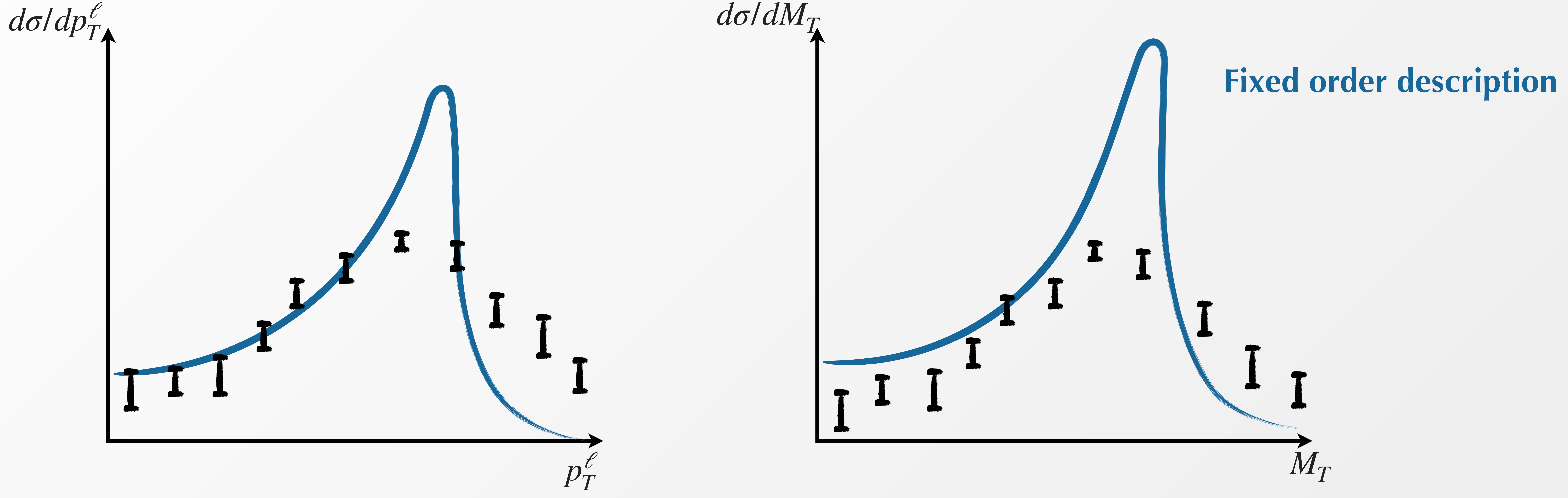
$$\frac{d\sigma}{d|p_T^\ell|^2} \sim \frac{1}{\sqrt{1 - 4\frac{|p_T^\ell|^2}{\hat{s}}}} \sim \frac{1}{\sqrt{1 - 4\frac{|p_T^\ell|^2}{m_W^2}}}$$

Jacobian peak at $p_T^\ell \sim m_W/2$

Enhanced sensitivity to m_W in both distributions at the $\mathcal{O}(10^{-3})$—$\mathcal{O}(10^{-2})$ level.

Measurements of m_W at hadron colliders

Different sensitivity to experimental uncertainties and quality of theoretical modelling



Measurements of m_W at hadron colliders

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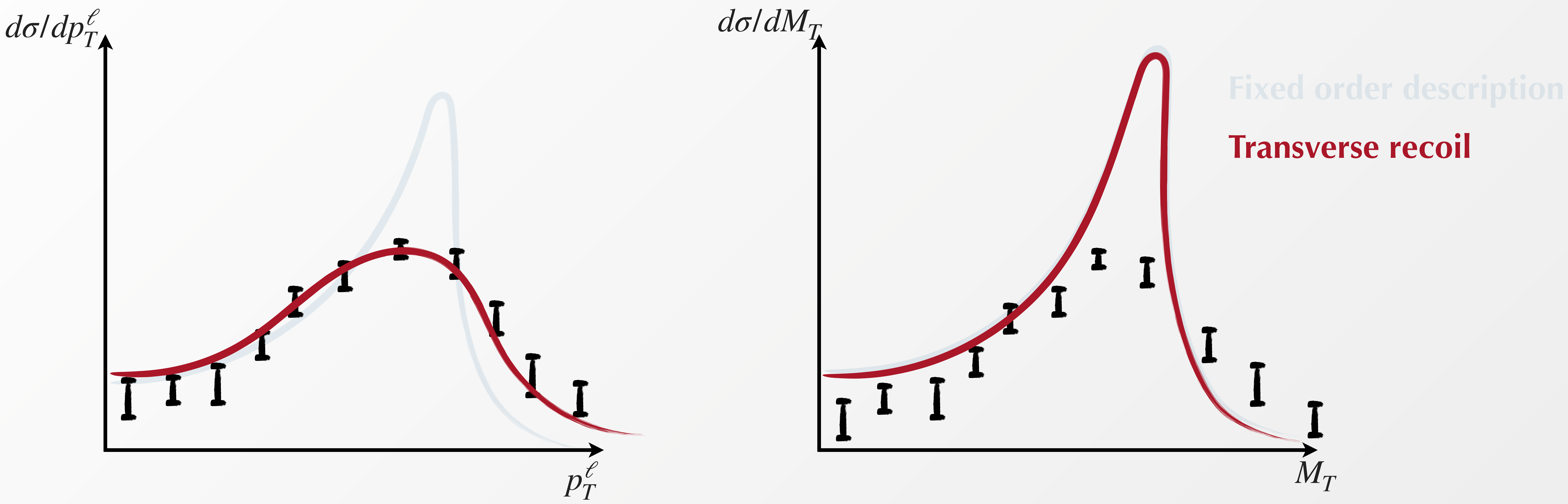
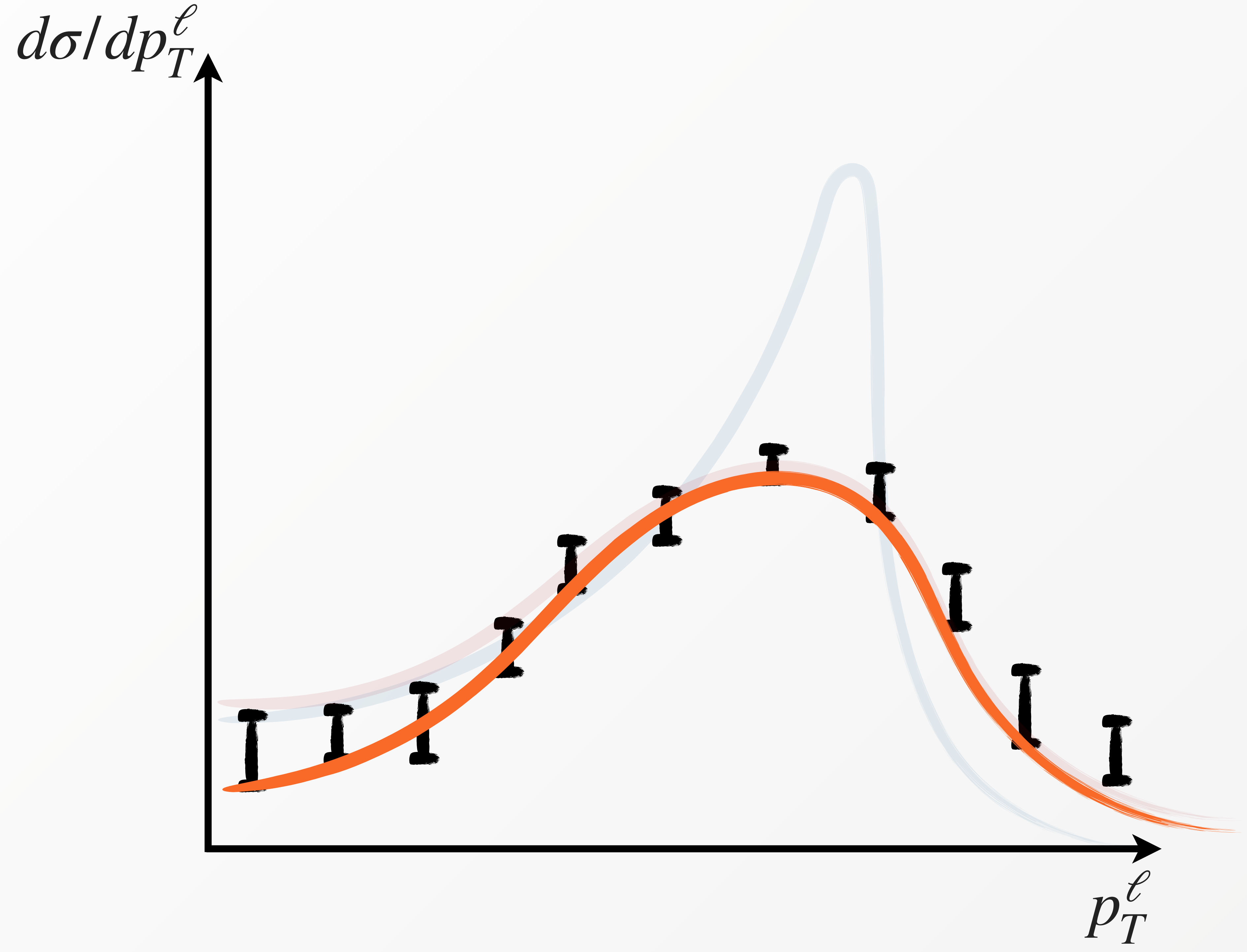


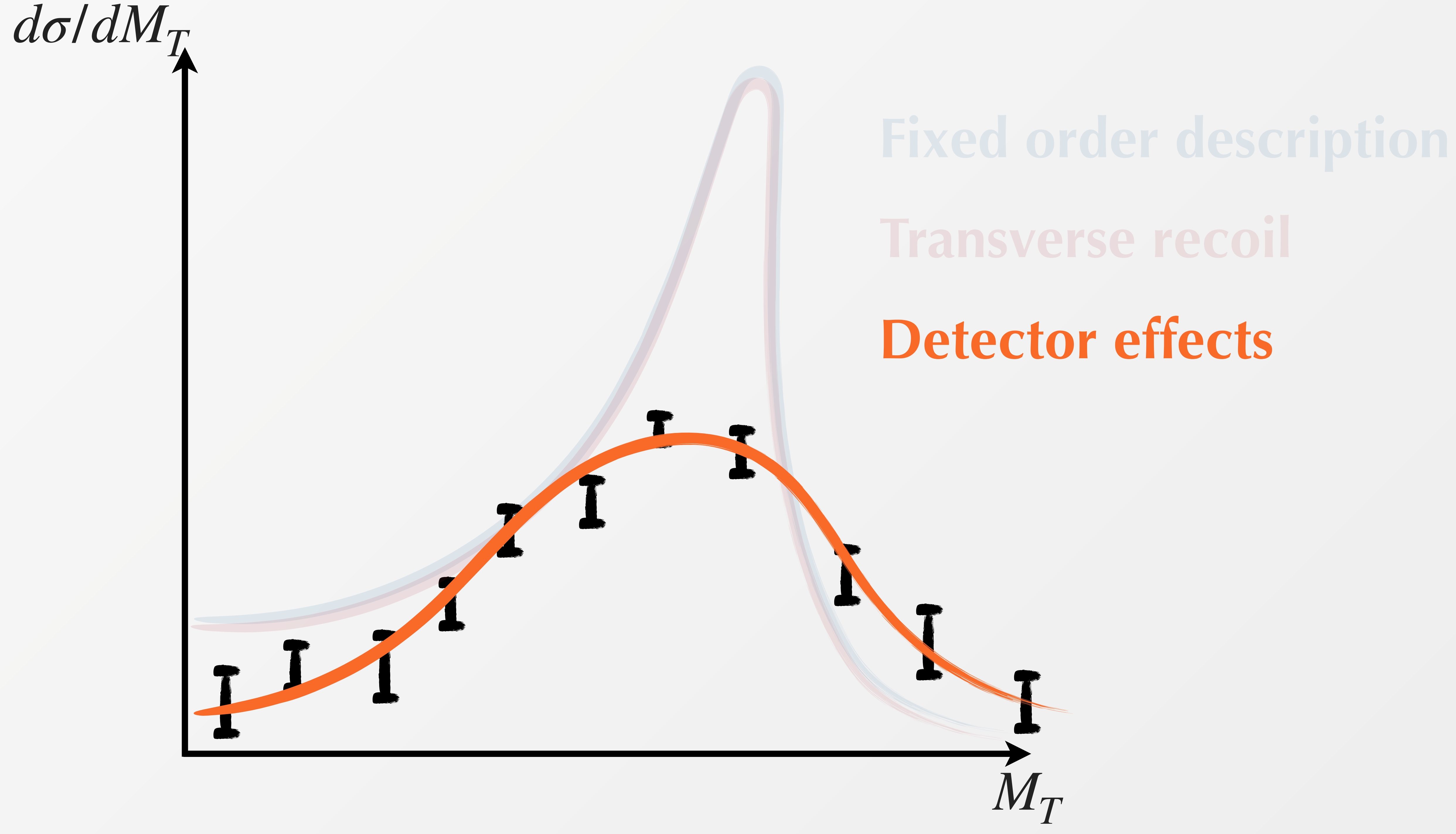
Description of the data requires:

- Modelling of **IS QCD + FS QED radiation**

Measurements of m_W at hadron colliders

Different sensitivity to experimental uncertainties and quality of theoretical modelling



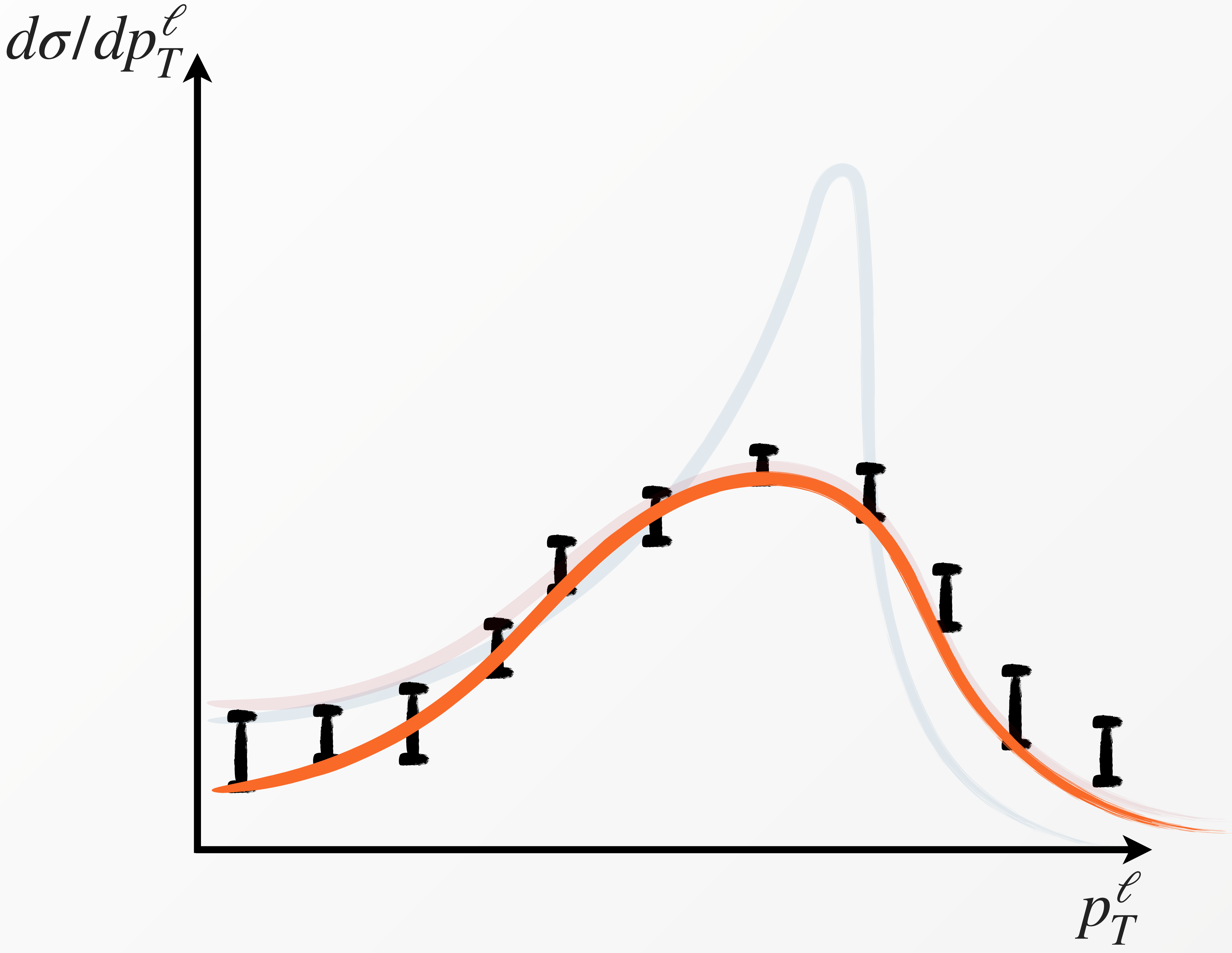


Description of the data requires:

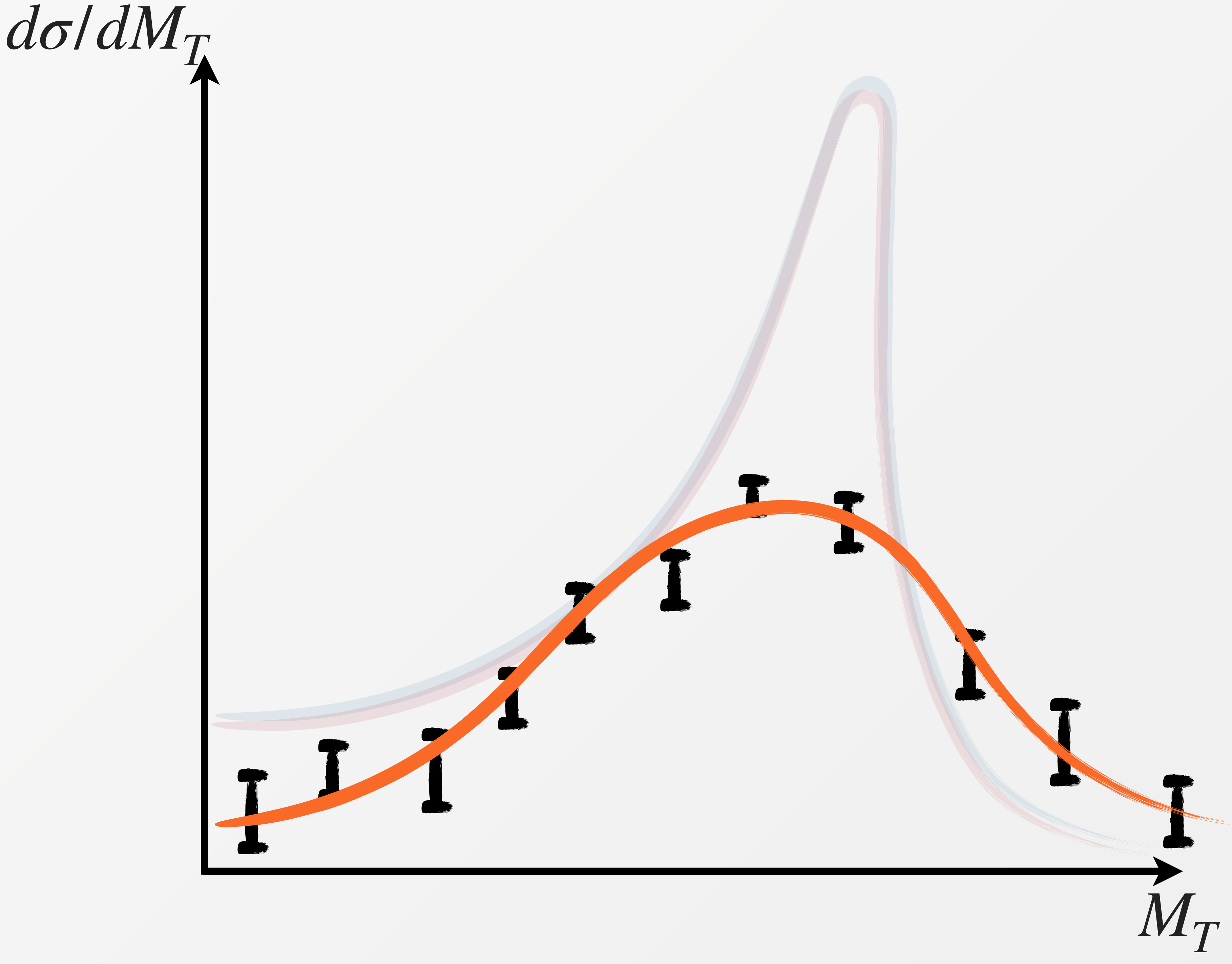
- Modelling of **IS QCD + FS QED radiation**
- Modelling of the **smearing** of the distributions due to the reconstruction of the neutrino in the transverse plane

Measurements of m_W at hadron colliders

Different sensitivity to experimental uncertainties and quality of theoretical modelling



Mostly **QCD + QED radiation**

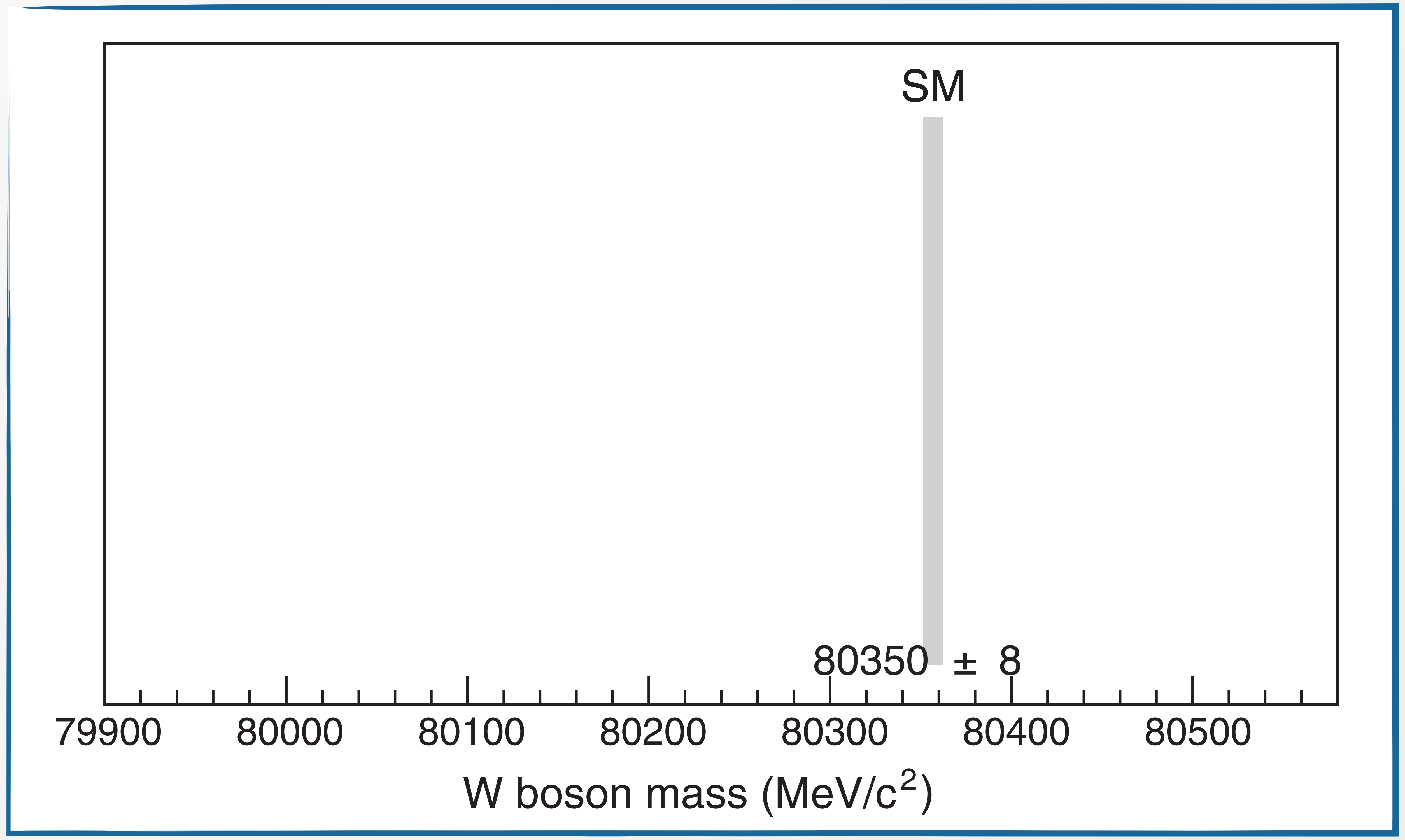


Mostly **detector effects**

$$m_T = \sqrt{2p_T^\ell p_T^\nu(1 - \cos\Delta\phi_{\ell\nu})}$$

Requires precise determination of the neutrino transverse momentum: **challenging at the LHC** due to worse control of the hadronic recoil

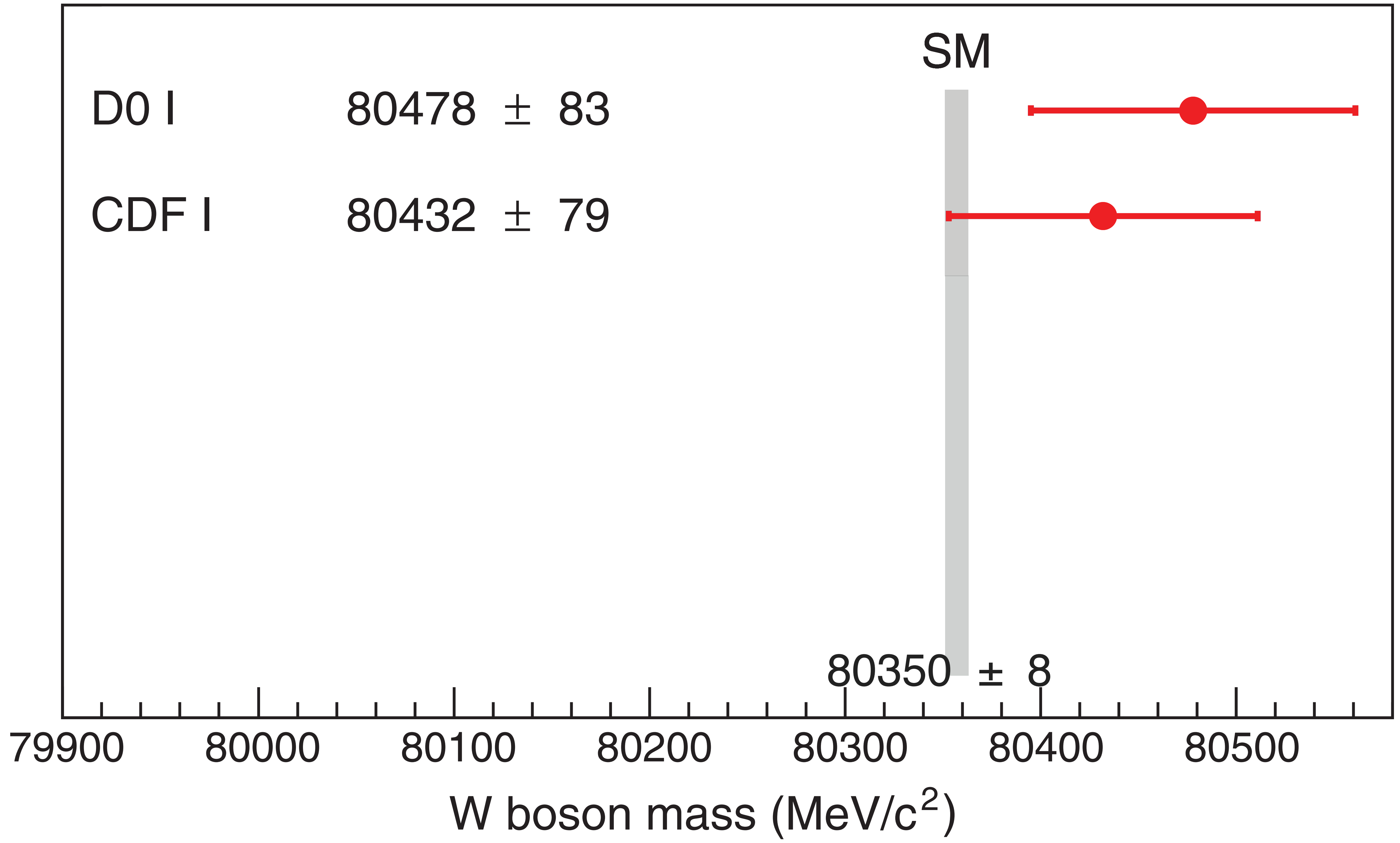
Measurements of m_W at hadron colliders



Measurements of m_W at hadron colliders

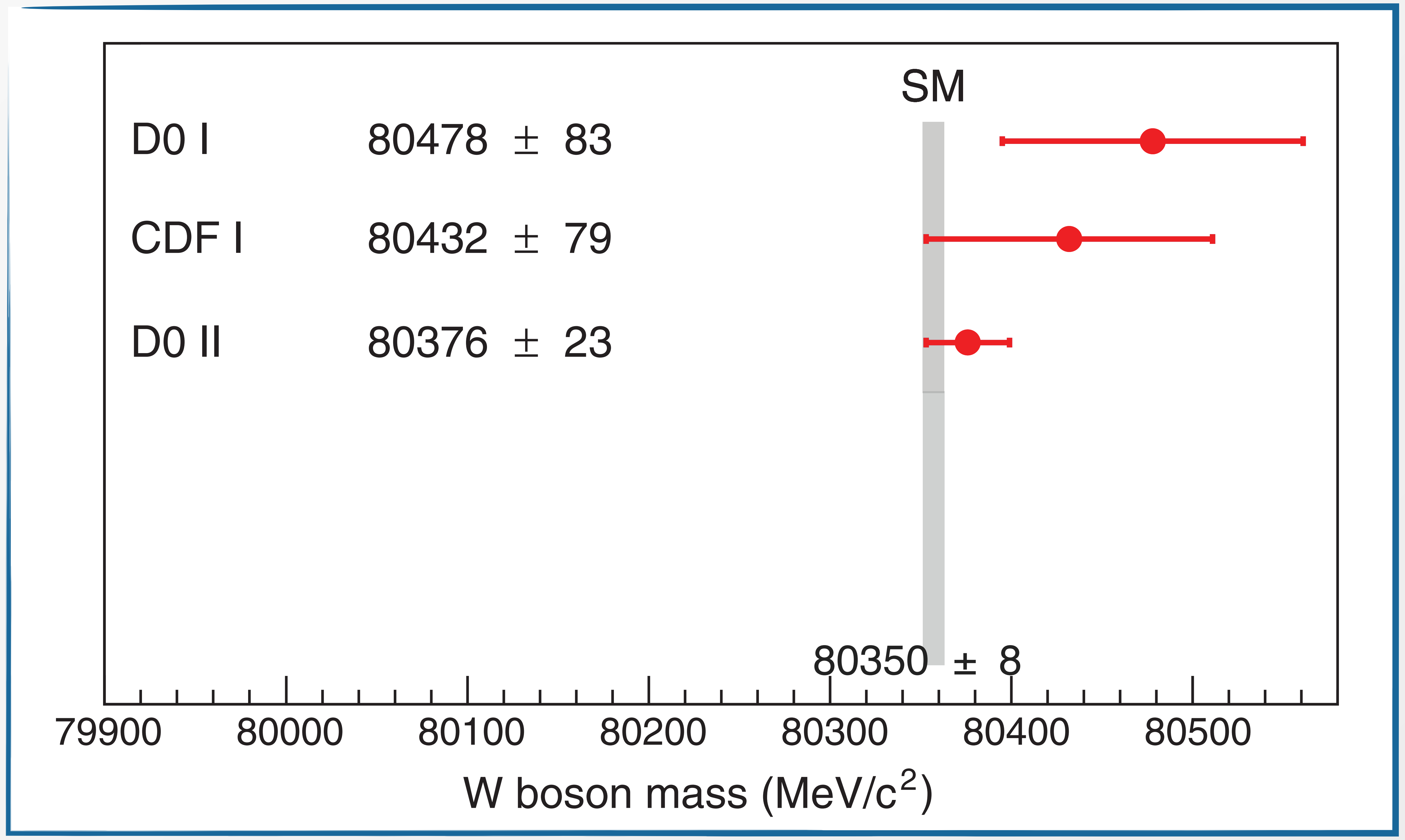
CDF I (2000)
ResBos
$(N)NLL_{QCD}+NLO_{QCD}$
MRST and CTEQ-5 PDFs
[CDF collaboration, hep-ex/0007044.pdf]

D0 I (2002)
ResBos
$(N)NLL_{QCD}+NLO_{QCD}$
MRST and CTEQ-5 PDFs
[D0 Collaboration hep-ex/0204014]



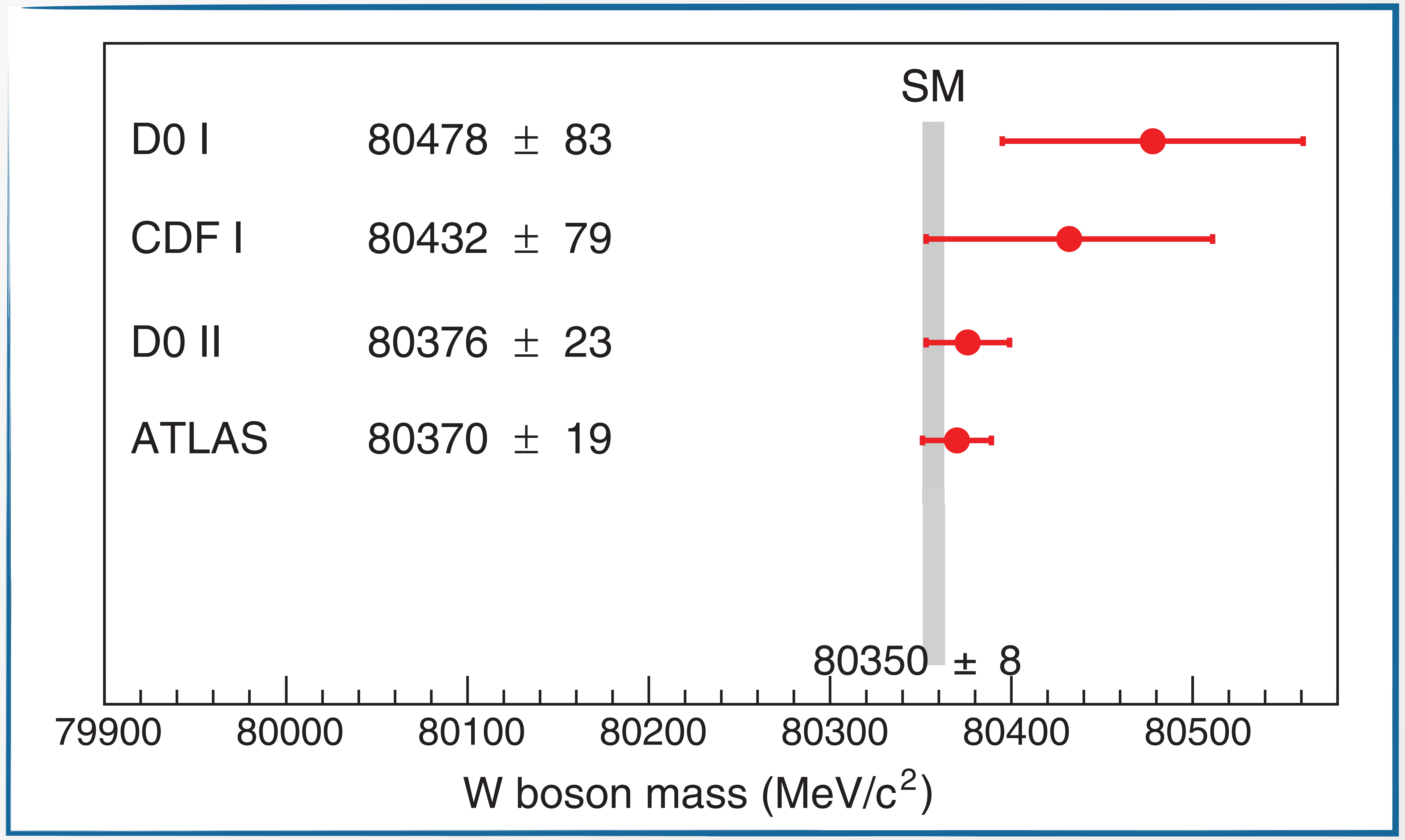
Measurements of m_W at hadron colliders

D0 II (2012)
ResBos
(N)NLL$_{QCD}$+NLO$_{QCD}$
CTEQ6.6 PDFs
QED modelling with PHOTOS
[D0 Collaboration 1203.0293]

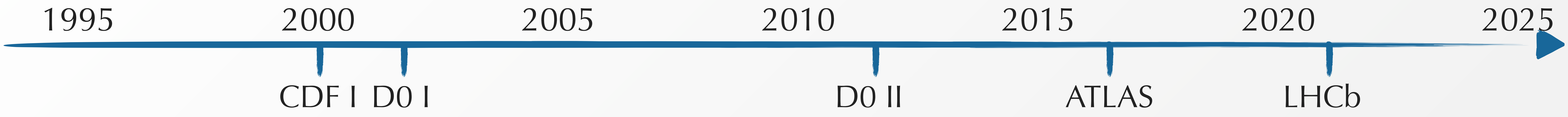


Measurements of m_W at hadron colliders

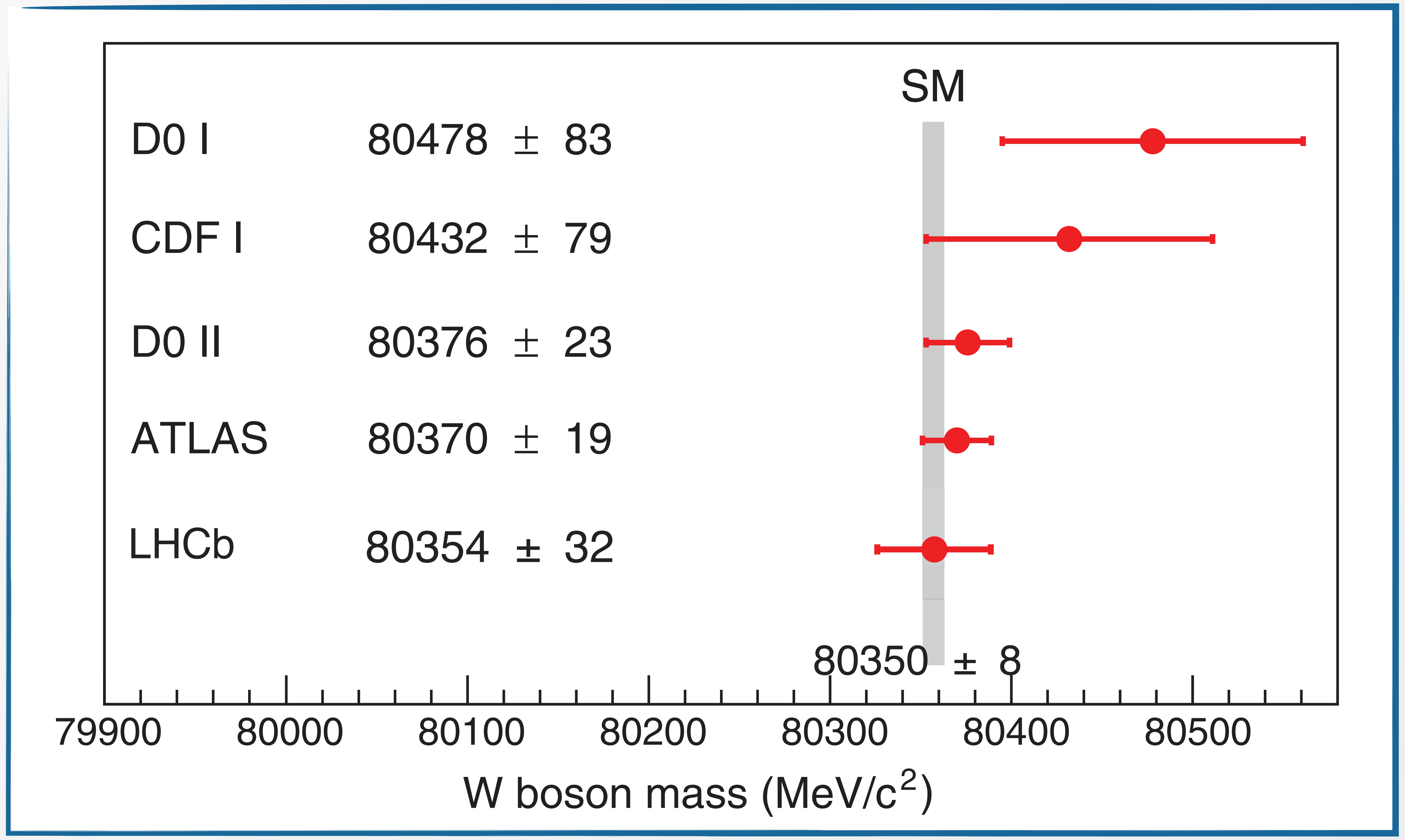
ATLAS (2017)
`POWHEG+PYTHIA 8` (+ NNLO reweighting)
(N)LL$_{QCD}$+(N)NLO$_{QCD}$
CT10 PDFs
QED modelling with `PHOTOS`
[ATLAS Collaboration 1701.07240]



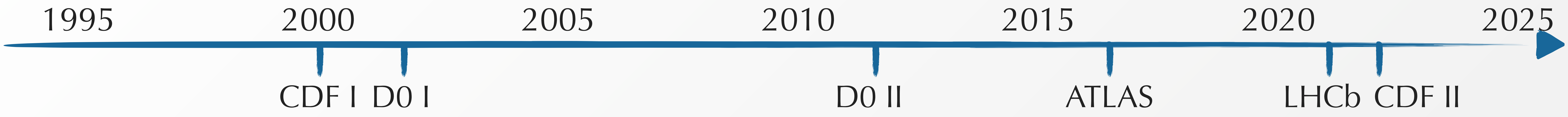
Measurements of m_W at hadron colliders



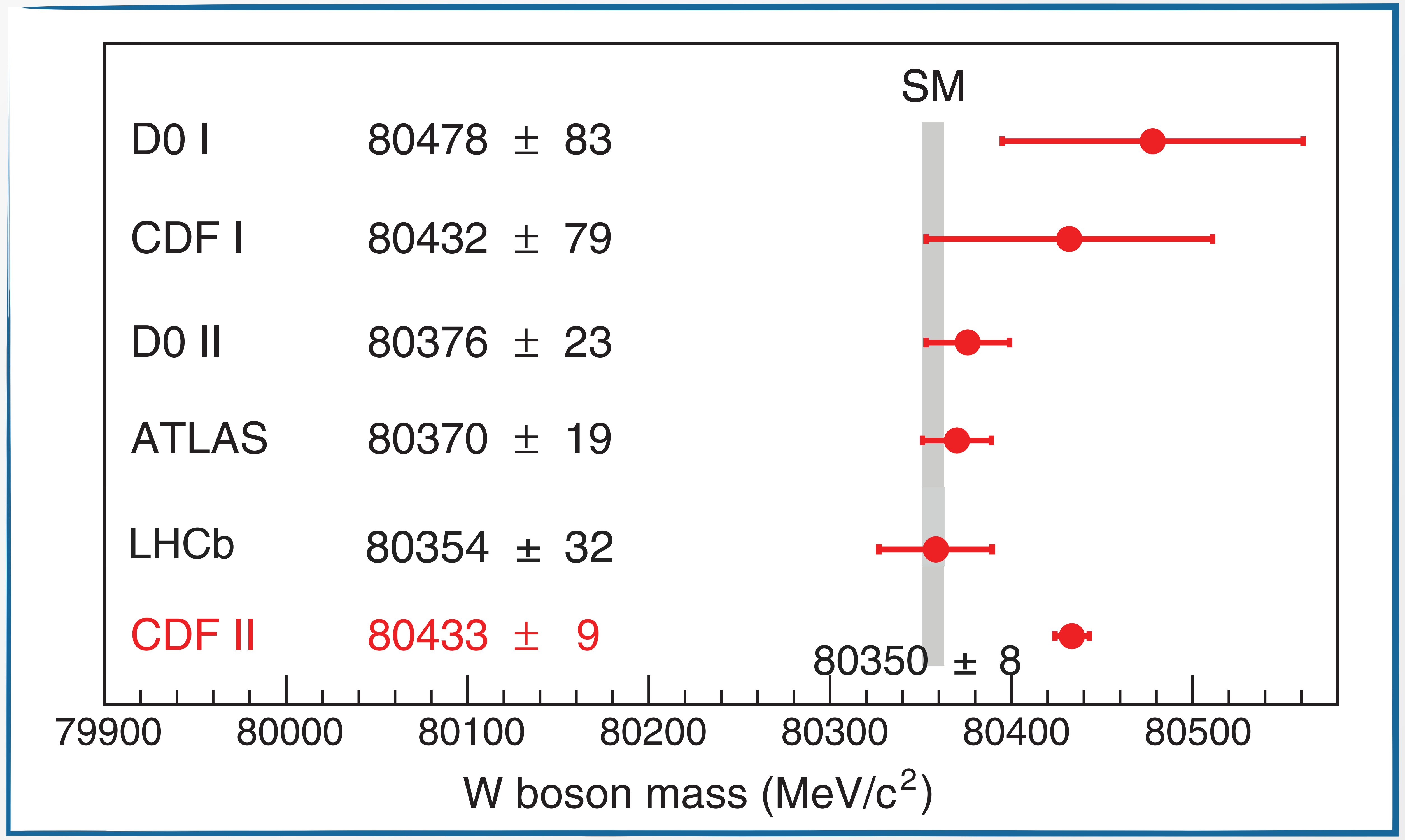
LHCb (2021)
POWHEG+PYTHIA 8 (+NNLO reweighting)
(N)LL$_{QCD}$+(N)NLO$_{QCD}$
CT10 PDFs
QED modelling with PHOTOS, PYTHIA, HERWIG
[LHCb Collaboration 2109.01113]



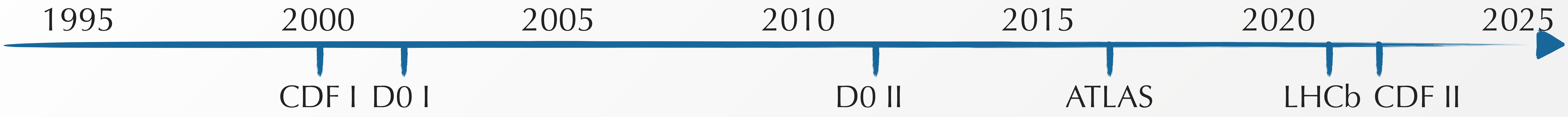
Measurements of m_W at hadron colliders



CDF II (2022)
`ResBos`
(N)NLL$_{QCD}$+NLO$_{QCD}$
NNPDF3.1 PDFs
QED modelling with `PHOTOS, HORACE`
[CDF Collaboration, Science 376, 170–176 (2022)]



Measurements of m_W at hadron colliders



CDF II measurement features **extremely aggressive** estimates for theory uncertainties, especially when compared to CDF I results

Source of uncertainty	$W \to e\nu$	$W \to \mu\nu$	Common
Lepton scale	75	85	
Lepton resolution	25	20	
PDFs	15	15	15
P_T^W	15	20	3
Recoil	37	35	
Higher order QED	20	10	5
Trigger and lepton ID bias		$15 \oplus 10$	
Backgrounds	5	25	
Total	92	103	16

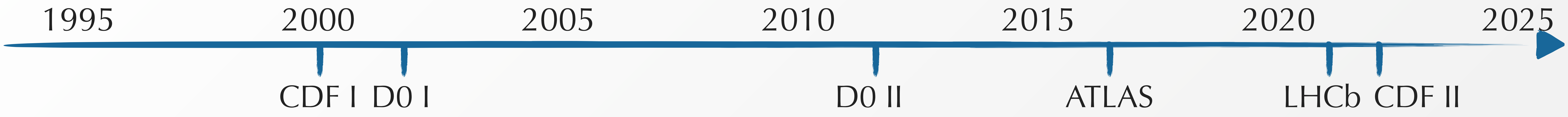
[CDF collaboration, hep-ex/0007044.pdf]

Table 2. Uncertainties on the combined M_W result.

Source	Uncertainty (MeV)
Lepton energy scale	3.0
Lepton energy resolution	1.2
Recoil energy scale	1.2
Recoil energy resolution	1.8
Lepton efficiency	0.4
Lepton removal	1.2
Backgrounds	3.3
p_T^Z model	1.8
p_T^W/p_T^Z model	1.3
Parton distributions	3.9
QED radiation	2.7
W boson statistics	6.4
Total	9.4

[CDF Collaboration, Science 376, 170–176 (2022)]

Measurements of m_W at hadron colliders



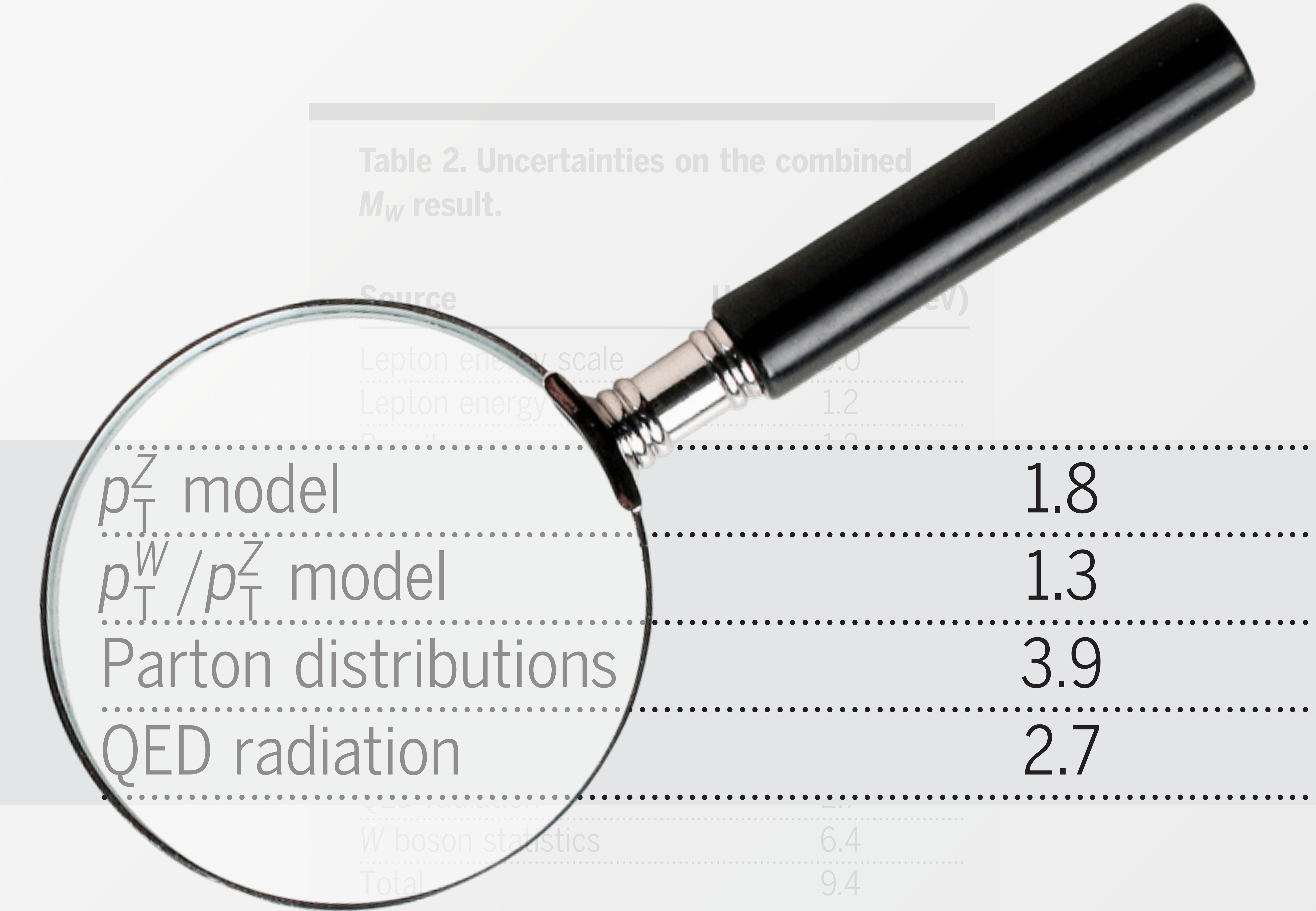
CDF II measurement features **extremely aggressive** estimates for theory uncertainties, especially when compared to CDF I results

But also when compared to a more recent analysis with lower luminosity, as all the errors are reduced by a factor 2-3

$p_T(W)$ model	5
Parton distributions	10
QED radiation	4

[CDF collaboration, 1203.0275]

CDF II, 2.2 fb^{-1} (2012) $m_W = 80.387 \pm 19$



CDF II, 8.8 fb^{-1} (2022) $m_W = 80.433 \pm 9$

Measurements of m_W at hadron colliders

1995 — 2000 — 2005 — 2010 — 2015 — 2020 — 2025

CDF I, D0 I, D0 II, ATLAS, LHCb, CDF II

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$p_T(W)$ model
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[CDF

p_T^Z model

	1.8
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Source	Uncertainty (MeV)
Lepton energy scale	3.0
Lepton energy resolution	1.2
Recoil energy scale	1.2
	1.8
	0.4
	1.2
	3.3
	1.8
	1.3
	3.9
	2.7
	6.4
	9.4

Do these error reflect an **improved theoretical understanding** of Z/W production at hadron colliders?

CDF II, 2.2 fb^{-1} (2012) $m_W = 80.387 \pm 19$

CDF II, 8.8 fb^{-1} (2022) $m_W = 80.433 \pm 9$

Measurements of m_W at hadron colliders

CDF II, 2.2 fb^{-1}
(2012)

CDF II, 8.8 fb^{-1}
(2022)

Not really: despite being published 10 years apart, the two analyses share most of the same underlying **theoretical model**

ResBos
(N)NLL$_{QCD}$+NLO$_{QCD}$
CTEQ6.6 NLO PDFs
QED modelling with PHOTOS+HORACE

ResBos
(N)NLL$_{QCD}$+NLO$_{QCD}$
NNPDF3.1 NNLO PDFs
QED modelling with PHOTOS+HORACE

Reduction of the theoretical error obtained via additional **data constraint** and use of **more modern PDF sets**

Theory uncertainties and m_W measurements

ResBos

(N)NLL$_{QCD}$+NLO$_{QCD}$

NNPDF3.1 NNLO PDFs

QED modelling with PHOTOS+HORACE

Theory uncertainties and m_W measurements

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QED modelling with PHOTOS+HORACE

"Estimating theory uncertainties is more of an art than a science"

Stefano Forte

PDFs and their uncertainties

ResBos

(N)NLL$_{QCD}$+NLO$_{QCD}$

NNPDF3.1 NNLO PDFs

QED modelling with PHOTOS+HORACE

PDFs and their uncertainties

Uncertainties related to PDFs can have different origin:

- Uncertainty propagated from the statistical and systematic errors on the measurements used in their determination (canonical "**PDF uncertainty**")
- Theoretical uncertainties of the predictions used in PDF fits, such as missing higher order uncertainty: these are starting to be addressed only recently, and are typically not included in the nominal PDF uncertainty [Abdul Khalek, Ball, LR, et al, (NNPDF Coll.), 1905.04311] [J. McGowan, T. Cridge, L. Harland-Lang, R. Thorne (MSHT Coll.) 2207.04739]

Comparisons between different groups used to assess sources of **methodological uncertainty** in the PDF extraction

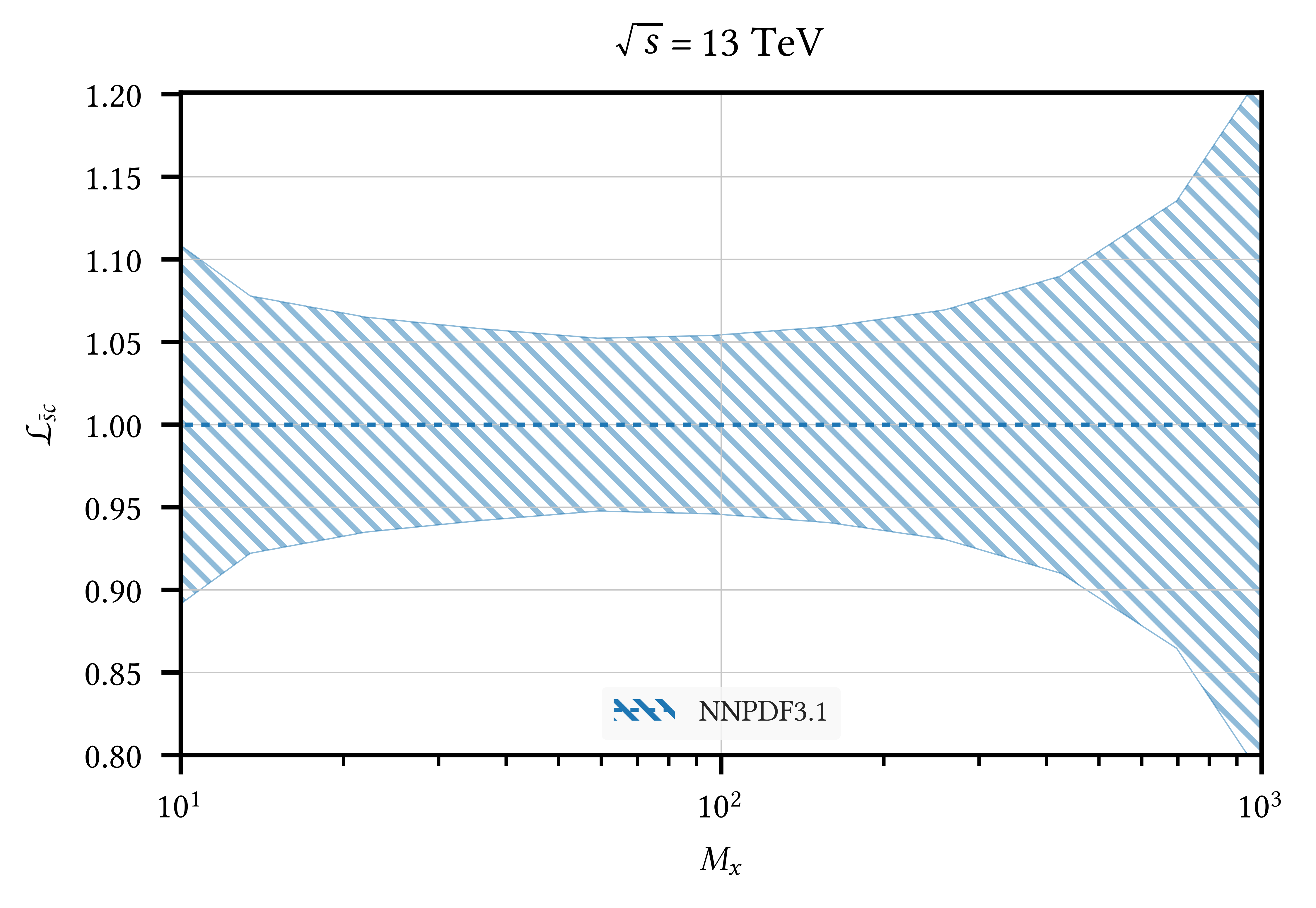
m_W measurements typically include the nominal PDF uncertainty and, more conservatively, they also assess that it encompasses the envelope of various PDF sets

PDFs and their uncertainties

Numerous studies on the impact of PDF uncertainties have been performed at both colliders
[Tevatron 0707.0085,0708.3642,0908.0766,1203.0275,1203.0293,1307.7627] [Bozzi, Citelli, Rojo, Vesterinen, Vicini 1104.2056, 1501.05587, 1508.06954]
[ATLAS 1701.07240] [Kotwal PRD 98, 033008] [Manca, Cerri, Foppiani, Rolandi 1707.09344] [Bianchini, Rolandi 1902.03028] [Farry, Lupton, Pili, Vesterinen, 1902.04323]
[Bagnaschi, Vicini 1910.04726] [Hussein, Isaacson, Huston 1905.00110] [Gao, Liu, Xie 2205.03942]

The relative size of PDFs uncertainties at the Tevatron and at the LHC is affected by the different **centre-of-mass** energy of the collision and the **different initial states**

PDFs uncertainties **not an obstacle at Tevatron**; they have long been considered a **limiting factor at the LHC** due to the smaller values of the partonic x probed (higher collider energy) and the larger contribution from the second quark generation



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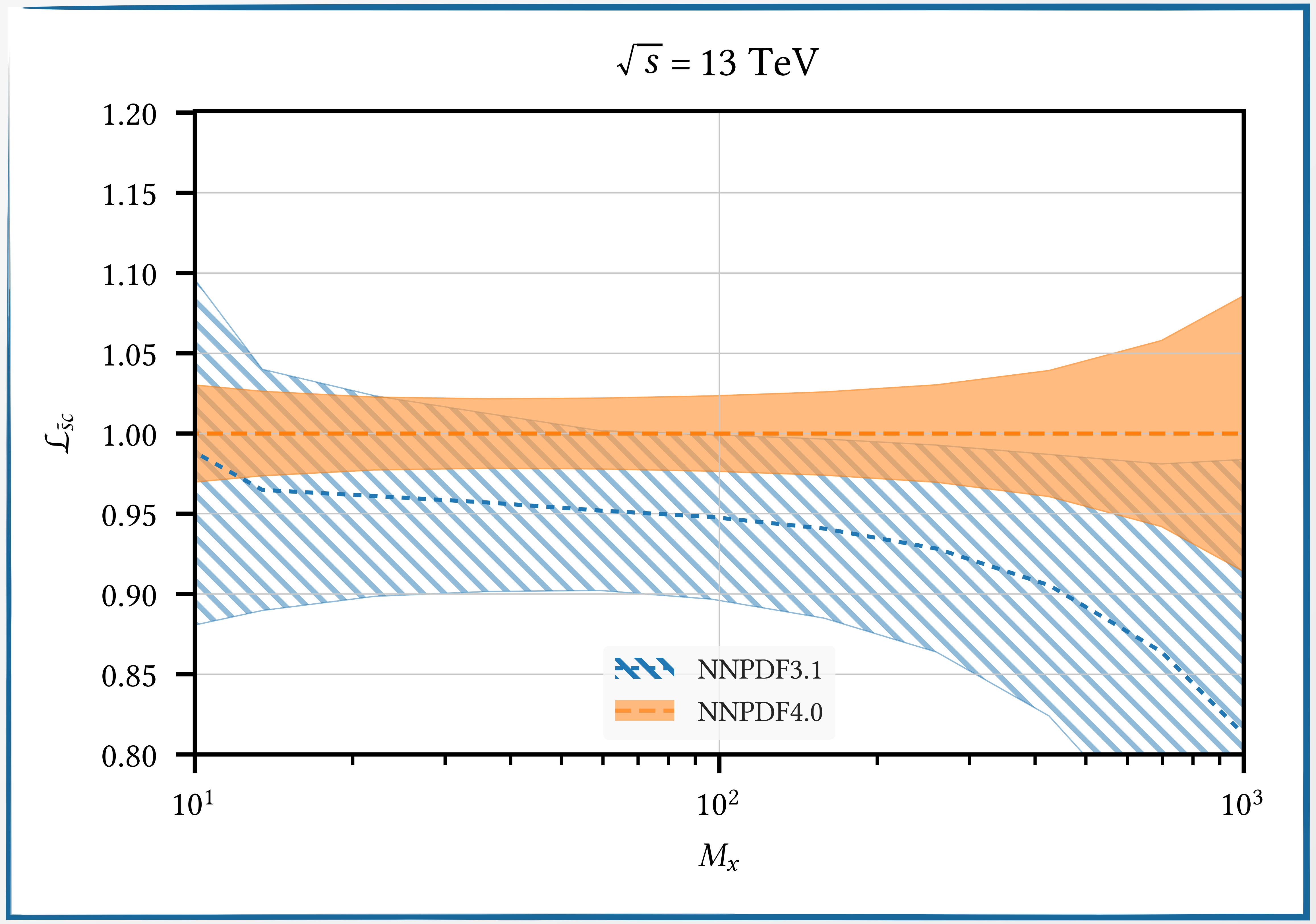
Latest generation of NNPDF parton densities (large number of LHC datasets included, new machine-learning based methodology) achieves **substantial reduction** of PDF uncertainty

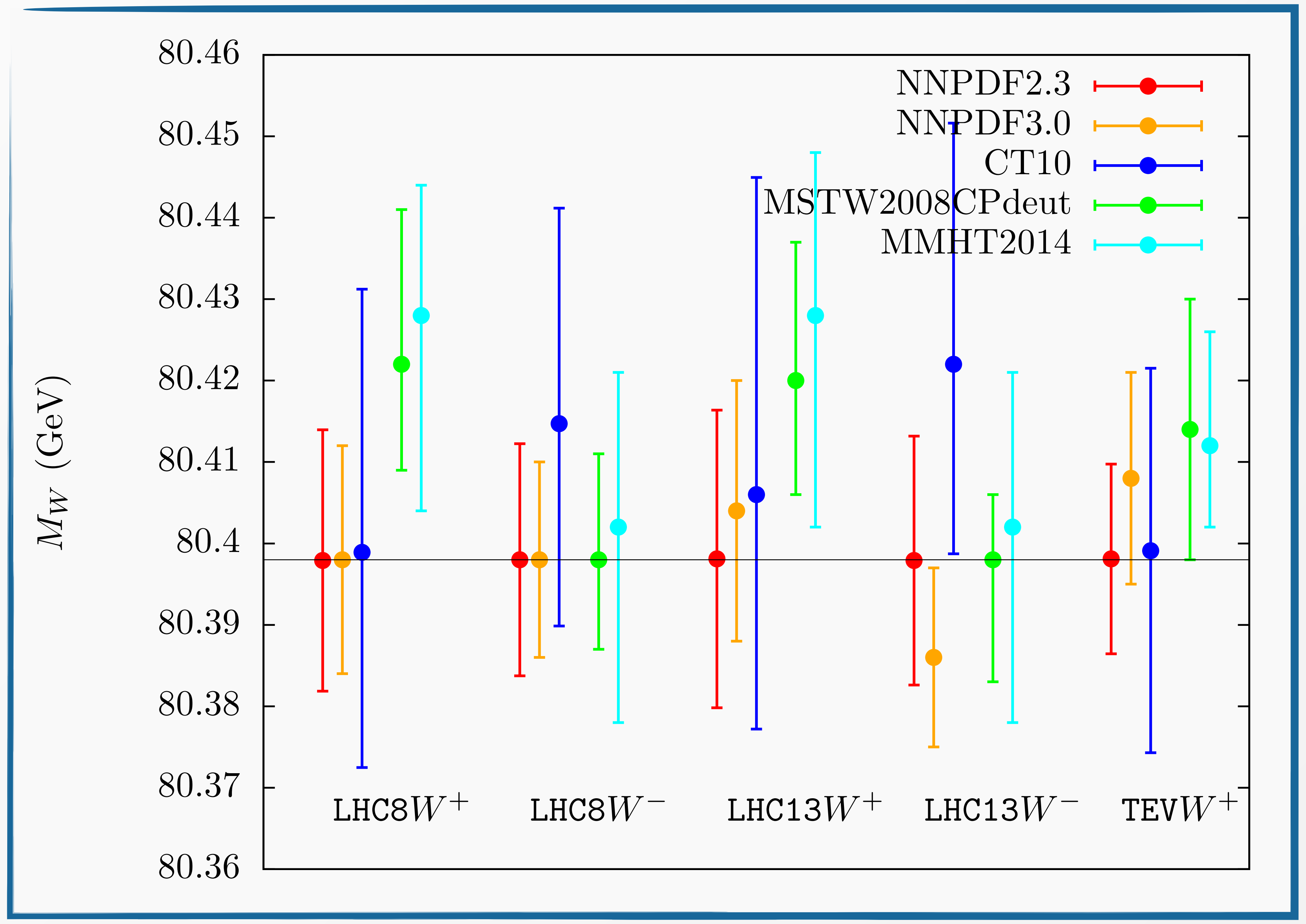


PDFs and their uncertainties: template fits

PDF-induced uncertainty typically computed by generating templates with a given PDF member *i* for various values of m_W, and subsequently fitting all other members *j* defining a proper figure of merit

$$\chi^2_{i,j} = \sum_{k \in \text{bins}} \frac{(T^j_k - D^i_k)^2}{\sigma^2_k}$$

Once the preferred value for m_W for each member has been determined by minimising the figure of merit, compute PDF-induced uncertainty



[Bozzi, Citelli, Vicini 1501.05587]

PDF uncertainties with this strategy are **relatively large at the LHC**, with a resulting uncertainty larger than 10 MeV and considerably large spreads between different PDF sets

Cfr. ~ 4 MeV quoted by CDF II with NNLO PDFs

4 MeV also claimed by CDF II to be the shift between NNPDF3.1 NNLO and ~**15 years old** NLO CTEQ6.6 PDFs

PDFs and their uncertainties: bin-by-bin correlations

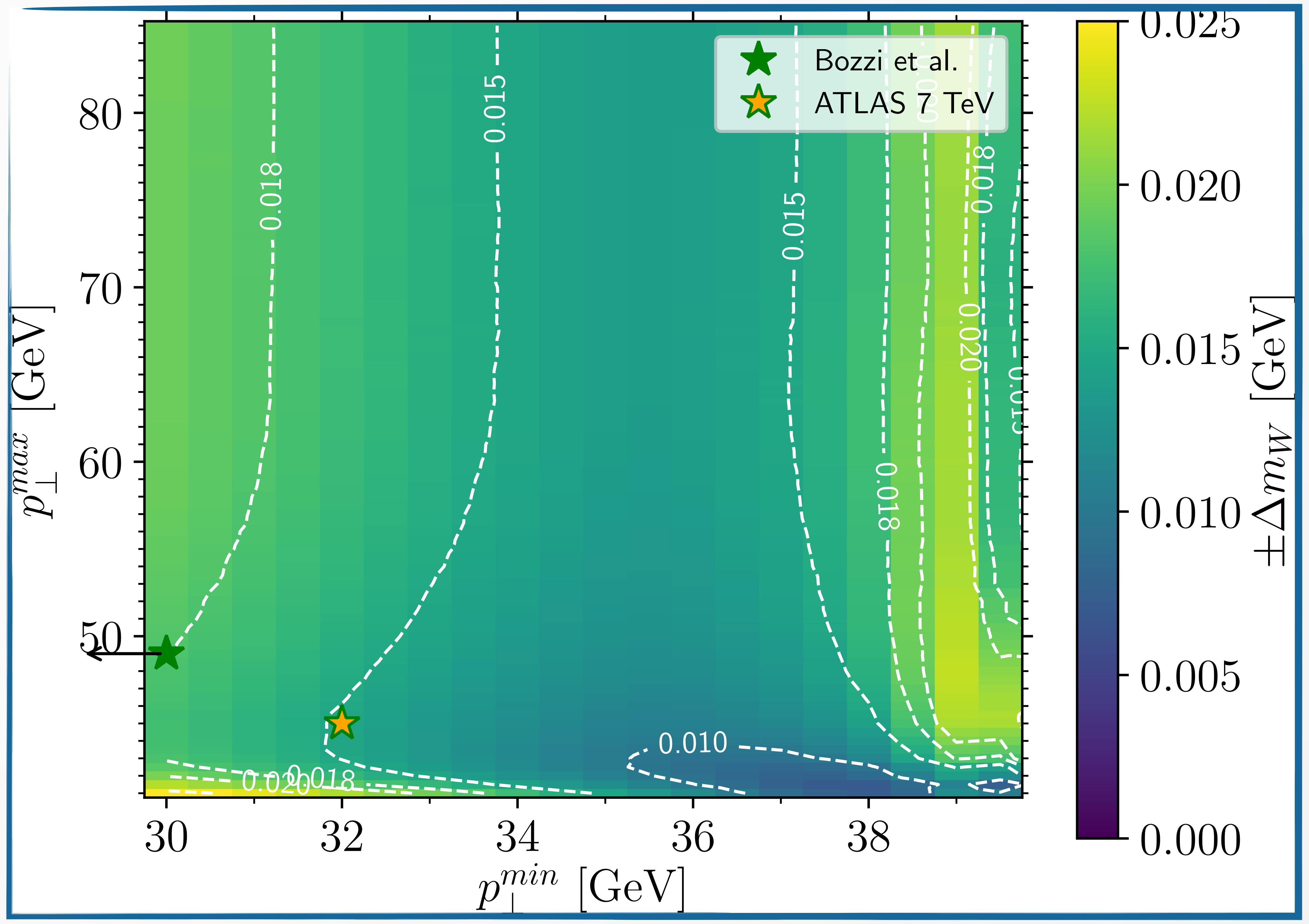
[Bagnaschi, Vicini 1910.04726]

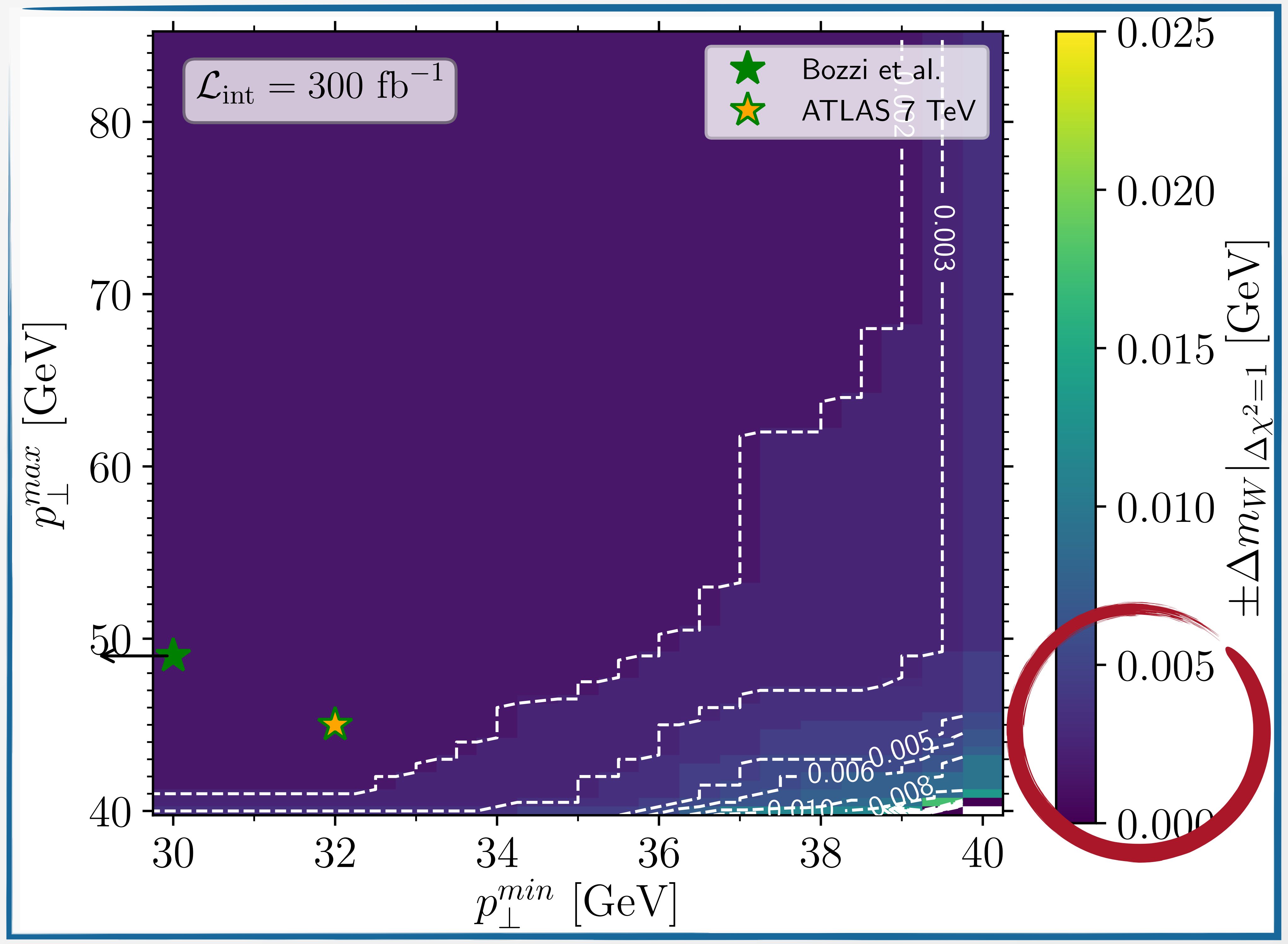
This estimate does not take into account **bin-by-bin correlations** between PDF replicas

$$(\Sigma_{\rm PDF})_{ij} = \langle (\mathcal{T} - \langle \mathcal{T} \rangle_{\rm PDF})_i (\mathcal{T} - \langle \mathcal{T} \rangle_{\rm PDF})_j \rangle_{\rm PDFs}$$

Compute χ^2 taking into account bin-by-bin correlations introducing a covariance matrix in the definition

$$\chi^2_{i,\min} = \sum_{k,l \in \text{bins}} (T_{0,i} - D)_k (C^{-1})_k (T_{0,i} - D)_l \qquad \forall m_{W,i} \qquad C = \Sigma_{\rm PDF}$$





Reduced sensitivity to the PDF uncertainty, if other source of errors are under control

QED modelling and its uncertainty

ResBos

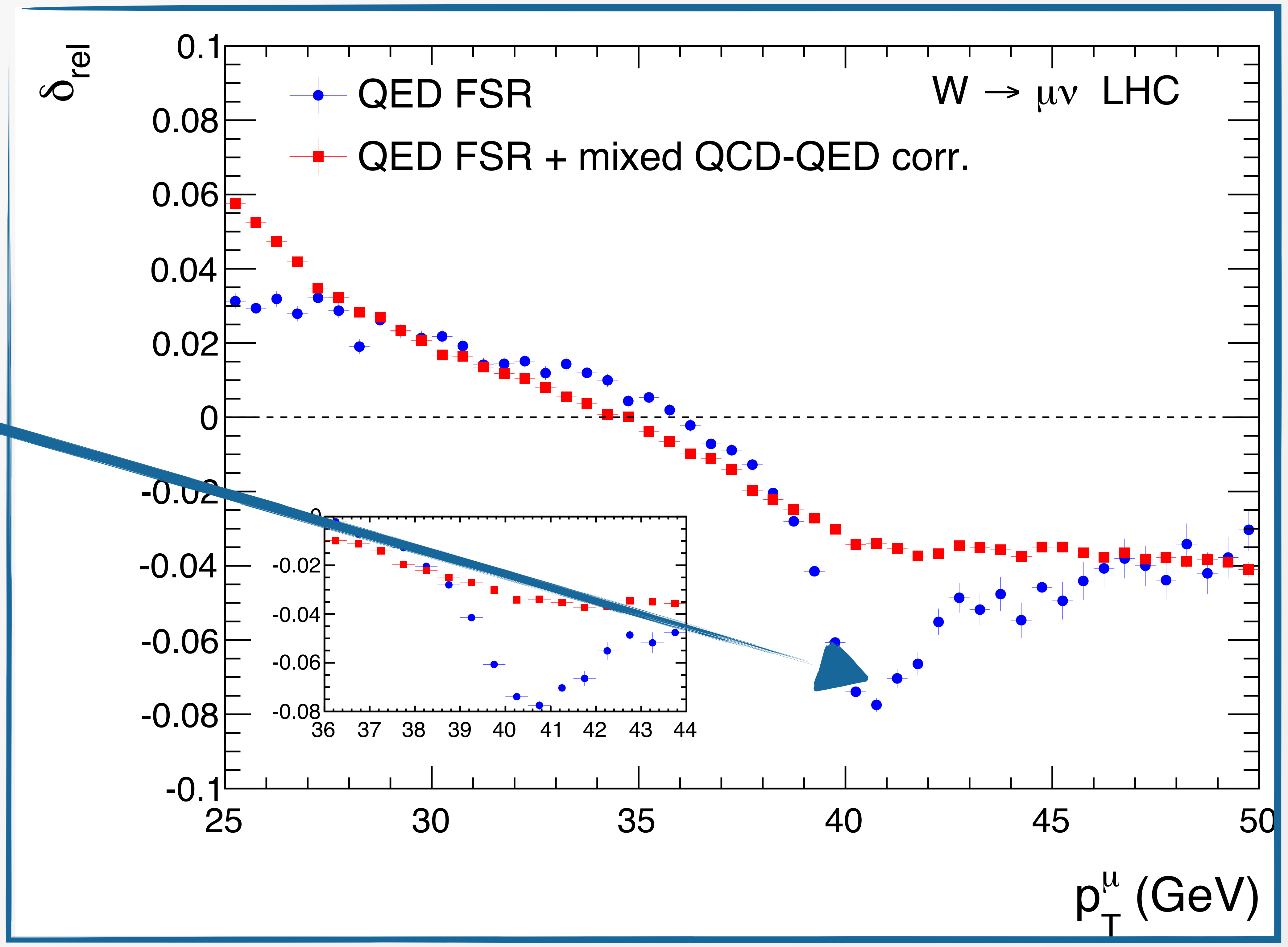
(N)NLL$_{QCD}$+NLO$_{QCD}$

NNPDF3.1 NNLO PDFs

QED modelling with PHOTOS+HORACE

Impact of EW and mixed QCD×EW corrections

Both p_T^ℓ and m_T features large radiative corrections due to **QED final state radiation** at the **jacobian peak**



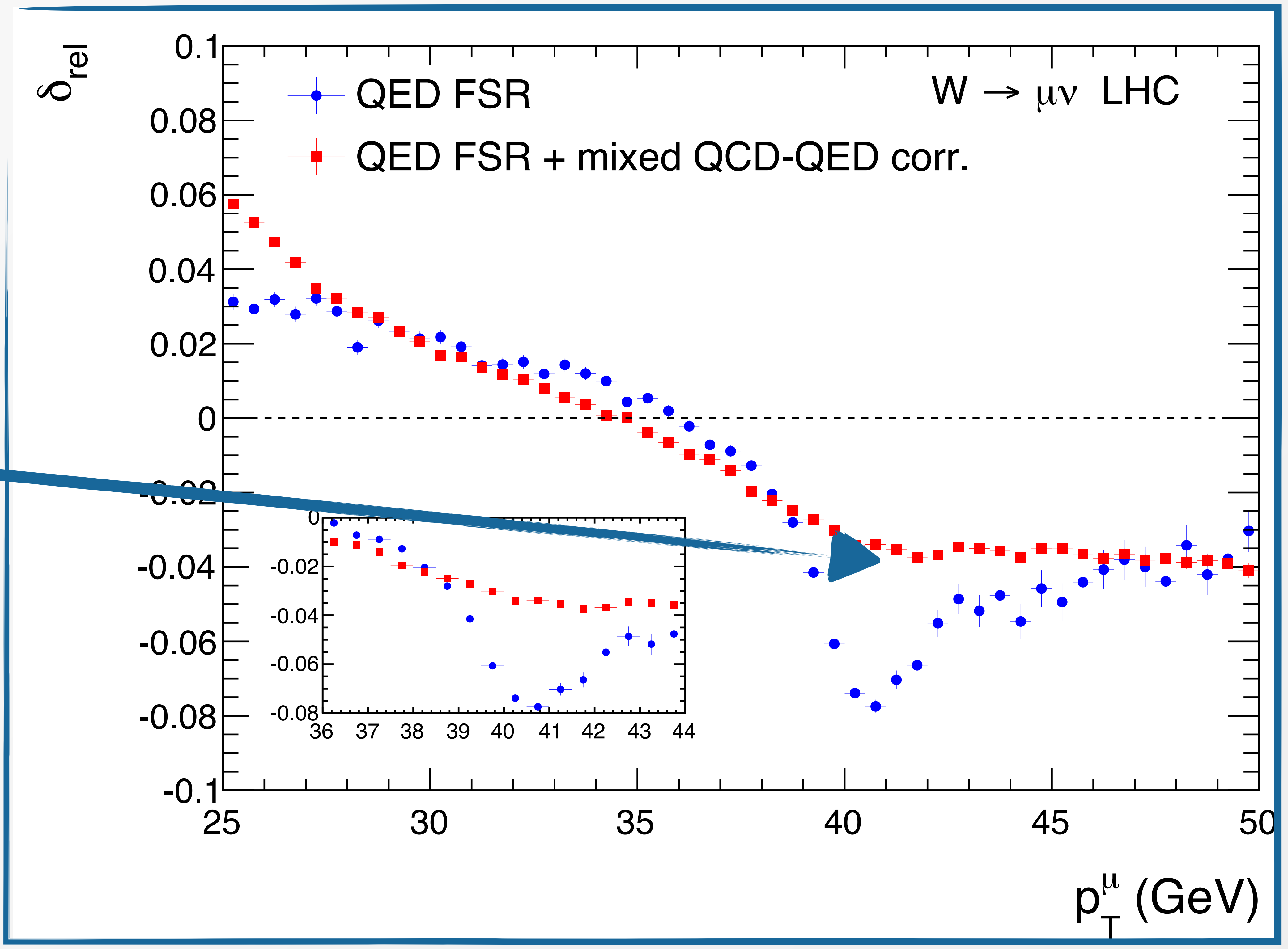
[Carloni Calame, Chiesa, Martinez, Montagna, Nicrosini, Piccinini, Vicini 1612.02841]

Impact of EW and mixed QCD×EW corrections

Both p_T^ℓ and m_T features large radiative corrections due to **QED final state radiation** at the **jacobian peak**

The precise shape of p_T^ℓ at the Jacobian peak is determined by the **interplay of QCD and QED corrections**

Impact of mixed corrections minor for m_T



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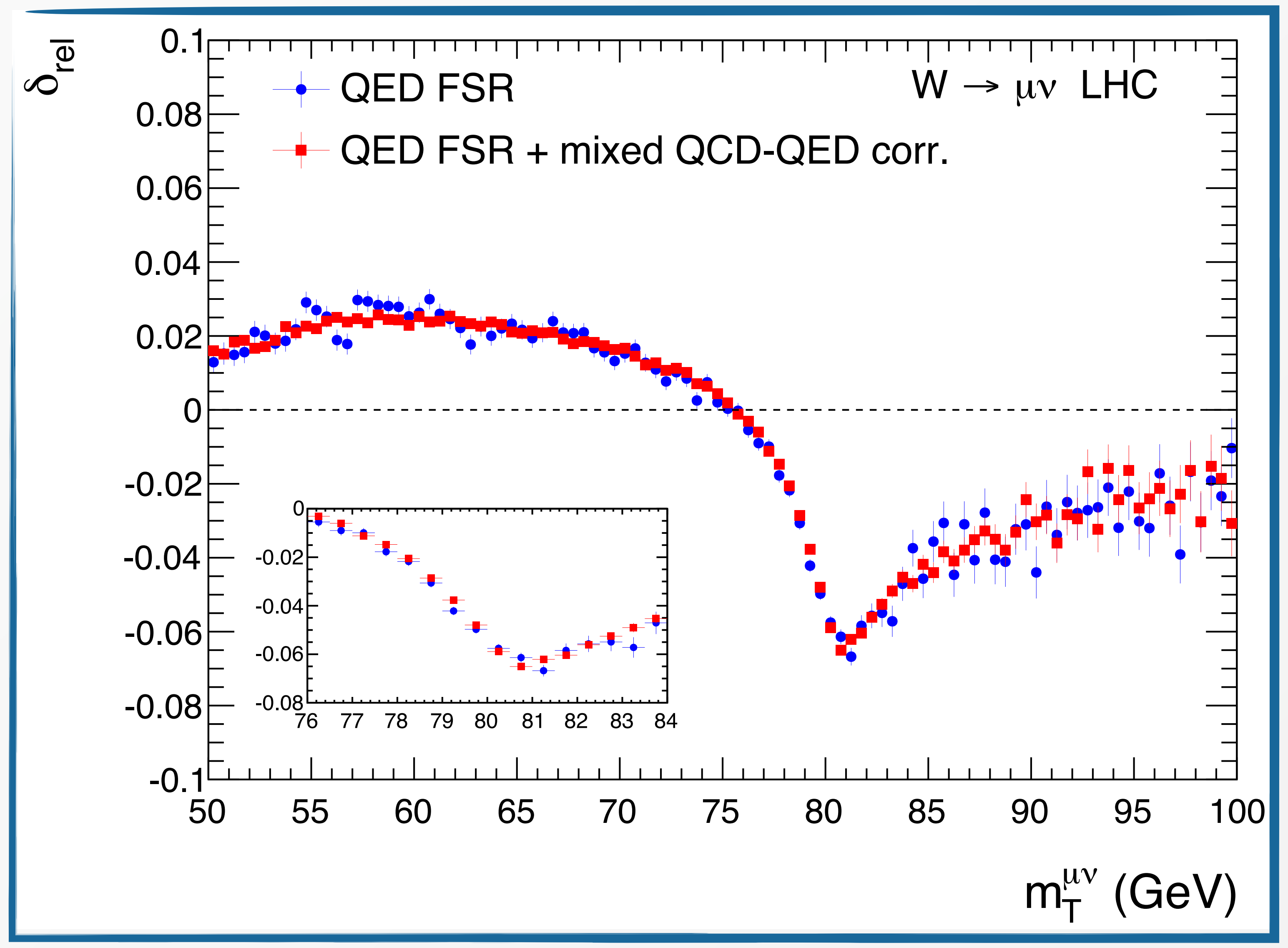

Impact of EW and mixed QCD×EW corrections

Largest shifts induced by **QED FSR**

Subleading EW effects induce **few MeV** shifts

	$pp \to W^+$, $\sqrt{s} = 14$ TeV Templates accuracy: LO Pseudo–data accuracy	M_W shifts (MeV) $W^+ \to \mu^+\nu$ M_T	 p_T^ℓ	 $W^+ \to e^+\nu$ M_T	 p_T^ℓ
1	HORACE only FSR-LL at $\mathcal{O}(\alpha)$	-94±1	-104±1	-204±1	-230±2
2	HORACE FSR-LL	-89±1	-97±1	-179±1	-195±1
3	HORACE NLO-EW with QED shower	-90±1	-94±1	-177±1	-190±2
4	HORACE FSR-LL + Pairs	-94±1	-102±1	-182±2	-199±1
5	PHOTOS FSR-LL	-92±1	-100±2	-182±1	-199±2

	$p\bar{p} \to W^+$, $\sqrt{s} = 1.96$ TeV Templates accuracy: NLO-QCD+QCD$_{PS}$ Pseudodata accuracy	 QED FSR	M_W shifts (MeV) $W^+ \to \mu^+\nu$ M_T	 p_T^ℓ	 $W^+ \to e^+\nu$(dres) M_T	 p_T^ℓ
1	NLO-QCD+(QCD+QED)$_{PS}$	PYTHIA	-91±1	-308±4	-37±1	-116±4
2	NLO-QCD+(QCD+QED)$_{PS}$	PHOTOS	-83±1	-282±4	-36±1	-114±3
3	NLO-(QCD+EW)-two-rad+(QCD+QED)$_{PS}$	PYTHIA	-86±1	-291±3	-38±1	-115±3
4	NLO-(QCD+EW)-two-rad+(QCD+QED)$_{PS}$	PHOTOS	-85±1	-290±4	-37±2	-113±3

[Carloni Calame, Chiesa, Martinez, Montagna, Nicrosini, Piccinini, Vicini 1612.02841]

Analyses do include the bulk of the QCD×QED corrections

The impact on the m_W shifts of the mixed QCD×QED corrections strongly depends on the underlying QCD model

Note: in this approach non-factorizable contributions are neglected

Impact of EW and mixed QCD×EW corrections

Largest shifts induced by **QED FSR**

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$pp \to W^+$, $\sqrt{s} = 14$ TeV Templates accuracy: LO Pseudo–data accuracy		M_W shifts (MeV)			
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$p\bar{p} \to W^+$, $\sqrt{s} =$ Templates accuracy: NL Pseudodata accuracy
1 NLO-QCD+(QCD+QED)$_{PS}$
2 NLO-QCD+(QCD+QED)$_{PS}$
3 NLO-(QCD+EW)-two-rad+(Q
4 NLO-(QCD+EW)-two-rad+(Q

[Carloni Calame, Chiesa, Martinez, Montagna, Nicrosini, Piccinini, Vicini 1612.02841]

Analyses do include the bulk of the ... ections

... e m_W shifts of the ... D corrections strongly ... nderlying QCD model

Note: in this approach non-factorizable contributions are neglected

Do we have a precise understanding of **mixed QCD×EW** corrections?

Progress in mixed QCD×EW corrections

Complete set of corrections to neutral and charged current Drell-Yan production recently obtained by two groups

NNLO QCD-EW corrections to charged-current DY (2-loop contributions in pole approximation).
[Buonocore, Grazzini, Kallweit, Savoini, Tramontano 2102.12539]

exact NNLO QCD-EW corrections to neutral-current DY
[Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini, 2106.11953]
[Armadillo, Bonciani, Devoto, Rana, Vicini 2201.01754]
[Buccioni, Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, Röntsch, Signorile-Signorile 2203.11237]

Impact of mixed $\mathcal{O}(\alpha_s\alpha)$ corrections estimated to be potentially relevant for $\mathcal{O}(10\,\mathrm{MeV})$ extraction at the LHC
[Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch 2103.02671]

Matching of such corrections to **QCD and QED all-order resummation** of high relevance for accurate and precise analysis of the p_T^ℓ distribution

Combination of QCD+QED resummation so far available only for on-shell Z production
[Cieri, Ferrera, Sborlini '18]

QCD modelling and its uncertainty

ResBos

$(N)NLL_{QCD}$+NLO_{QCD}

NNPDF3.1 NNLO PDFs

QED modelling with PHOTOS+HORACE

QCD modelling and its uncertainty

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QED modelling with PHOTOS+HORACE

“RESBOS offers one of the most accurate theoretical calculations available for these processes”

CDF Collaboration et al., Science 376, 170–176 (**2022**)

Progress in resummed theoretical calculations

$(N)NLL_{QCD}+NLO_{QCD}$
[Balasz, Yuan '97]

`ResBos`
[Balasz, Yuan '97]

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Progress in resummed theoretical calculations

1995 2000 2005 2010 2015 2020 2025

CDF I D0 I D0 II ATLAS LHCb CDF II

$(\mathrm{N})\mathrm{NLL}_{\mathrm{QCD}}+\mathrm{NLO}_{\mathrm{QCD}}$
[Balasz, Yuan '97]

$\mathrm{NNLL}'_{\mathrm{QCD}}+\mathrm{NNLO}_{\mathrm{QCD}}$
[Bozzi, Catani, Ferrera, De Florian, Grazzini '10]

$\mathrm{N}^3\mathrm{LL}_{\mathrm{QCD}}+\mathrm{N}^3\mathrm{LO}_{\mathrm{QCD}}$
[Bizon, Monni, Re, LR, Torrielli '17]

$\mathrm{N}^3\mathrm{LL}'_{\mathrm{QCD}}+\mathrm{N}^3\mathrm{LO}_{\mathrm{QCD}}$
[Camarda, Cieri, Ferrera '21]

[Chen, Gehrmann, Glover, Huss, Monni, Re, LR, Torrielli '22]

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CDF I (≈2001), D0 I (≈2002), D0 II (≈2012), ATLAS (≈2017), LHCb (≈2021), CDF II (≈2022)

$(N)NLL_{QCD}+NLO_{QCD}$
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DYQT
[Bozzi, Catani, Ferrera, De Florian, Grazzini '10]

DYRES
[Catani, Ferrera, De Florian, Grazzini '15]

CUTE
[Becher, Neubert , Wilhelm '11]

$N^3LL_{QCD}+N^3LO_{QCD}$
[Bizon, Monni, Re, LR, Torrielli '17]

ResBos2
[Isaacson '17]

RadISH
[Bizon, Monni, Re, LR, Torrielli '17]

$N^3LL'_{QCD}+N^3LO_{QCD}$
[Camarda, Cieri, Ferrera '21]

[Chen, Gehrmann, Glover, Huss, Monni, Re, LR, Torrielli '22]

[Neumann, Campbell '22]

DYTURBO
[Camarda et al,'19]

MATRIX+ RadISH
[Kallweit, Wiesemann, Re, LR, '20]

MCFM+CUTE
[Becher, Neumann '20]

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[Camarda et al,'19]

`MATRIX+RadISH`
[Kallweit, Wiesemann, Re, LR, '20]

`RadISH`
[Bizon, Monni, Re, LR, Torrielli '17]

Many codes long available, with higher formal accuracy than `ResBos`

`CUTE`
[Becher, Neubert , Wilhelm '11]

`MCFM+CUTE`
[Becher, Neumann '20]

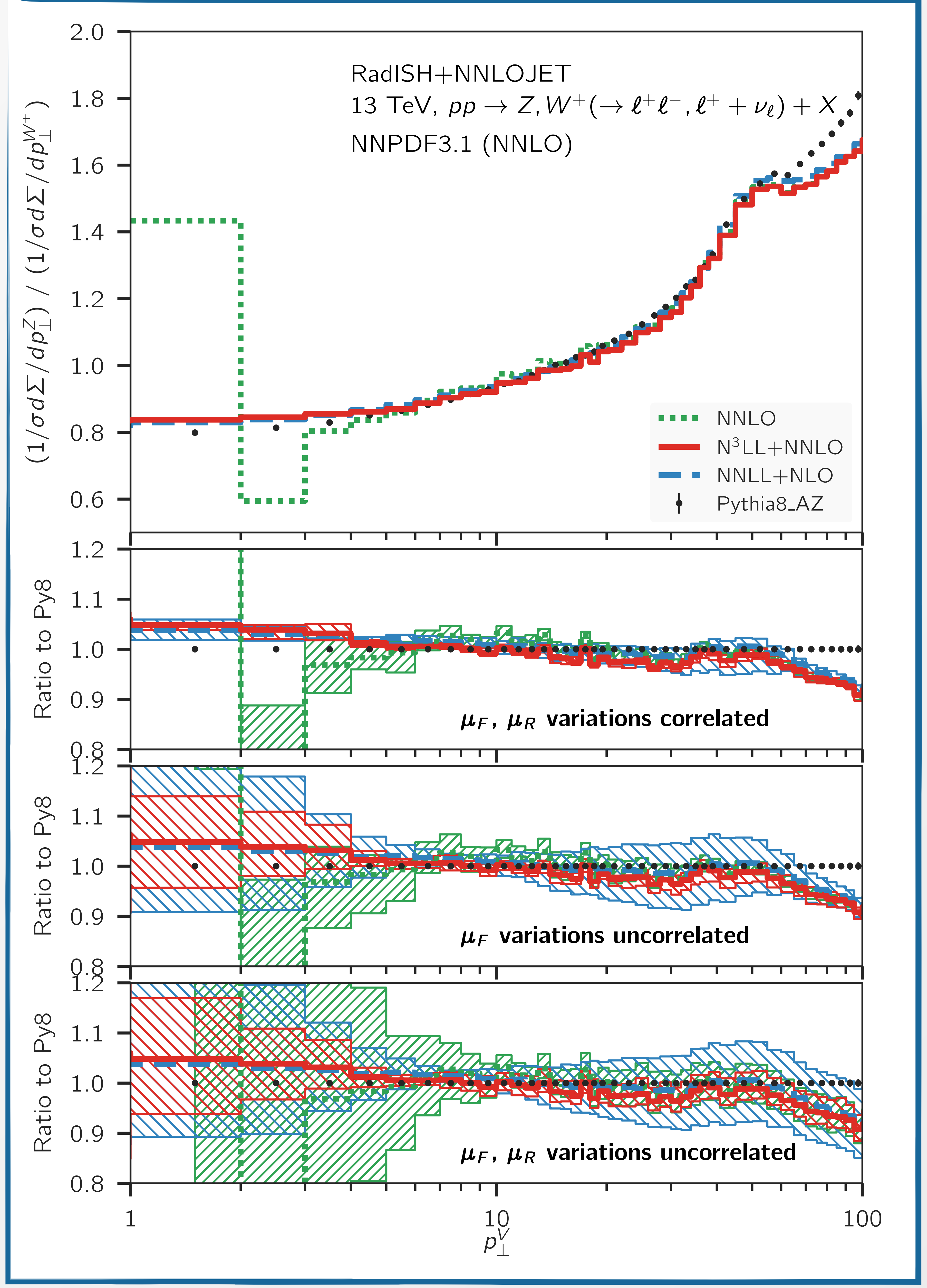
Progress in higher-order theoretical calculations

Possible to assess behaviour of the perturbative series at higher orders for crucial observables such as p_T^Z/p_T^W ratio

$$\frac{1}{\sigma^W}\frac{d\sigma^W}{p_\perp^W} \sim \frac{1}{\sigma^Z_{\text{data}}}\frac{d\sigma^Z_{\text{data}}}{p_\perp^Z}\frac{\frac{1}{\sigma^W_{\text{theory}}}\frac{d\sigma^W_{\text{theory}}}{p_\perp^W}}{\frac{1}{\sigma^Z_{\text{theory}}}\frac{d\sigma^Z_{\text{theory}}}{p_\perp^Z}}$$

Stability of the ratio indicates **high level of correlation** between the two spectra

Comparison with tuned event generator such as `PYTHIA*` however indicates that full correlation might be too strong an assumption



[Bizon, de Ridder, Gehrmann, Glover, Huss, Chen, Monni, Re, LR, Walker, 1905.05171]

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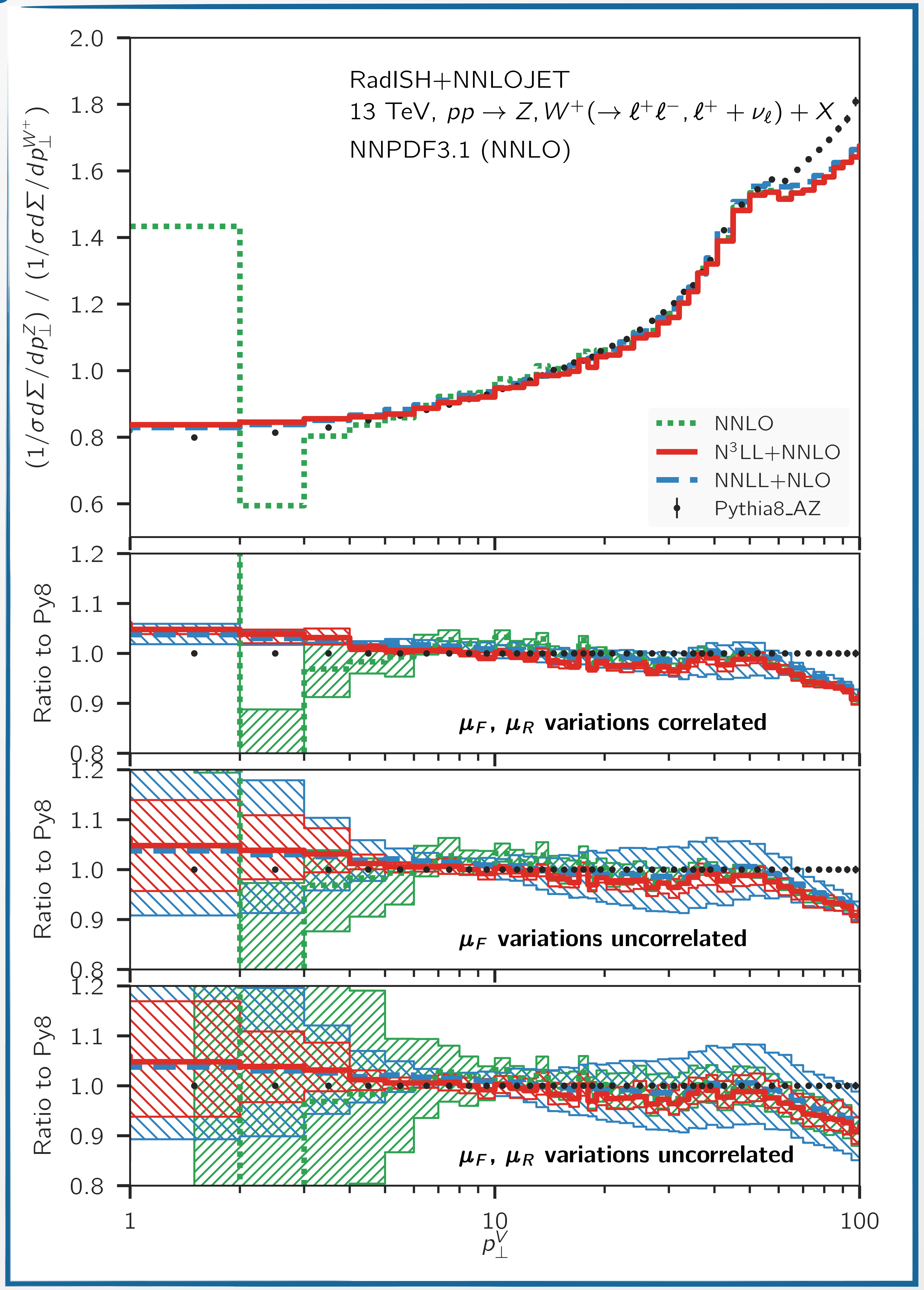
* *"PYTHIA is not QCD"*

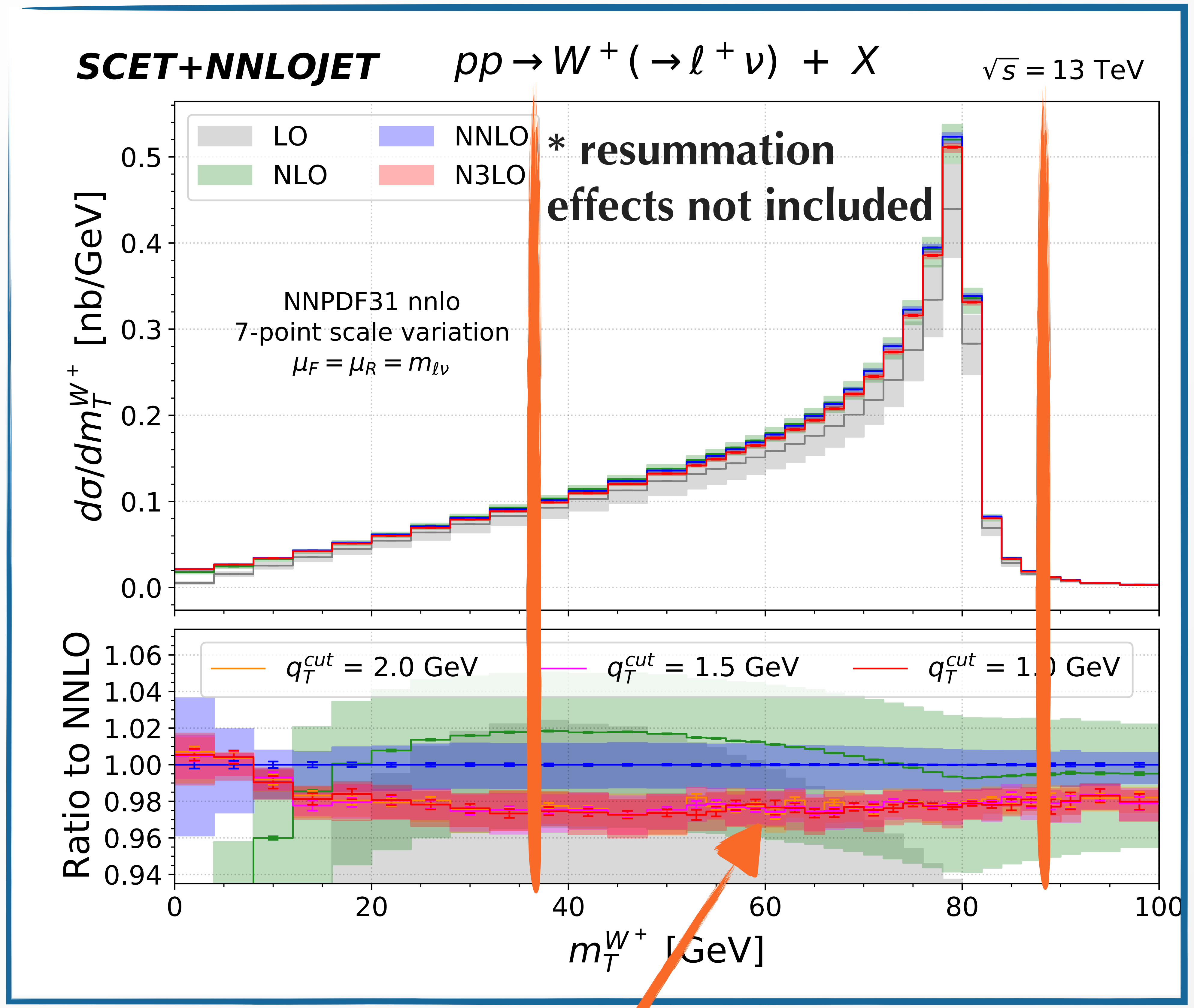
[Kirill Melnikov, QCD@LHC 2016]



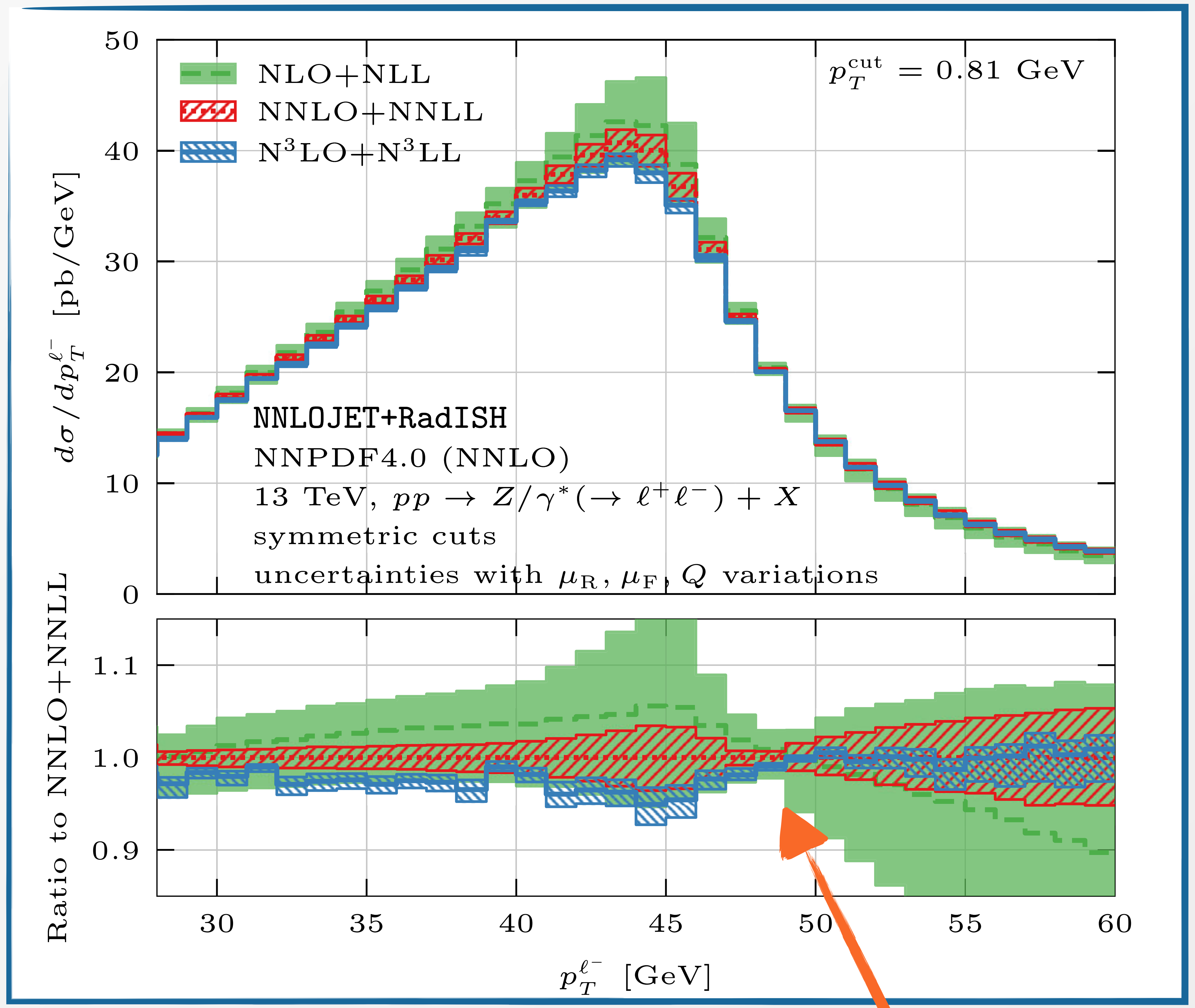
[Bizon, de Ridder, Gehrmann, Glover, Huss, Chen, Monni, Re, LR, Walker, 1905.05171]

Progress in higher-order theoretical calculations

Shape of differential spectra is affected by higher order predictions



[Gehrmann, Glover, Huss, Chen, Yang, Zhu 2205.11426]



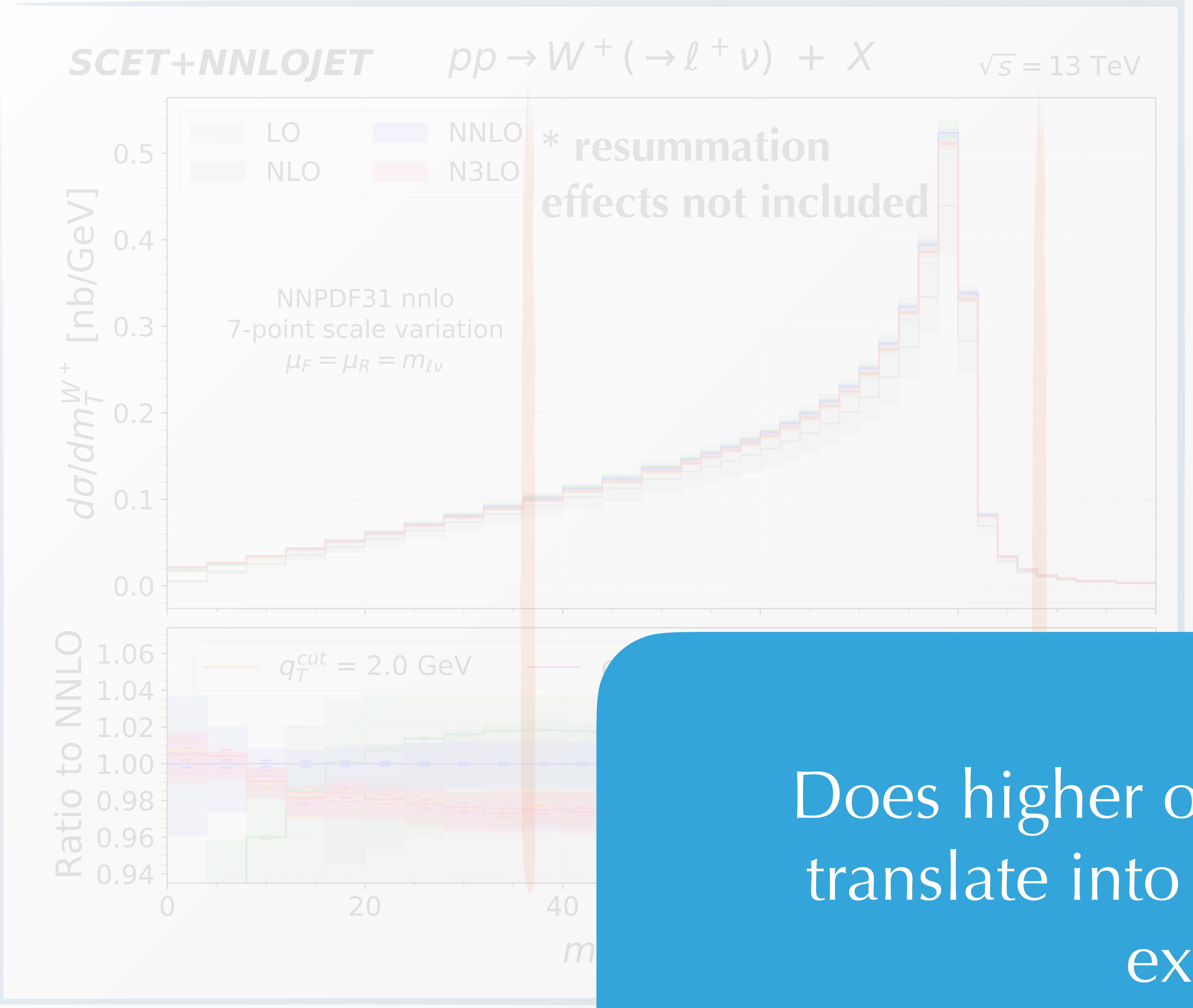
[Gehrmann, Glover, Huss, Chen, Monni, Re, LR, Torrielli, 2203.01565]

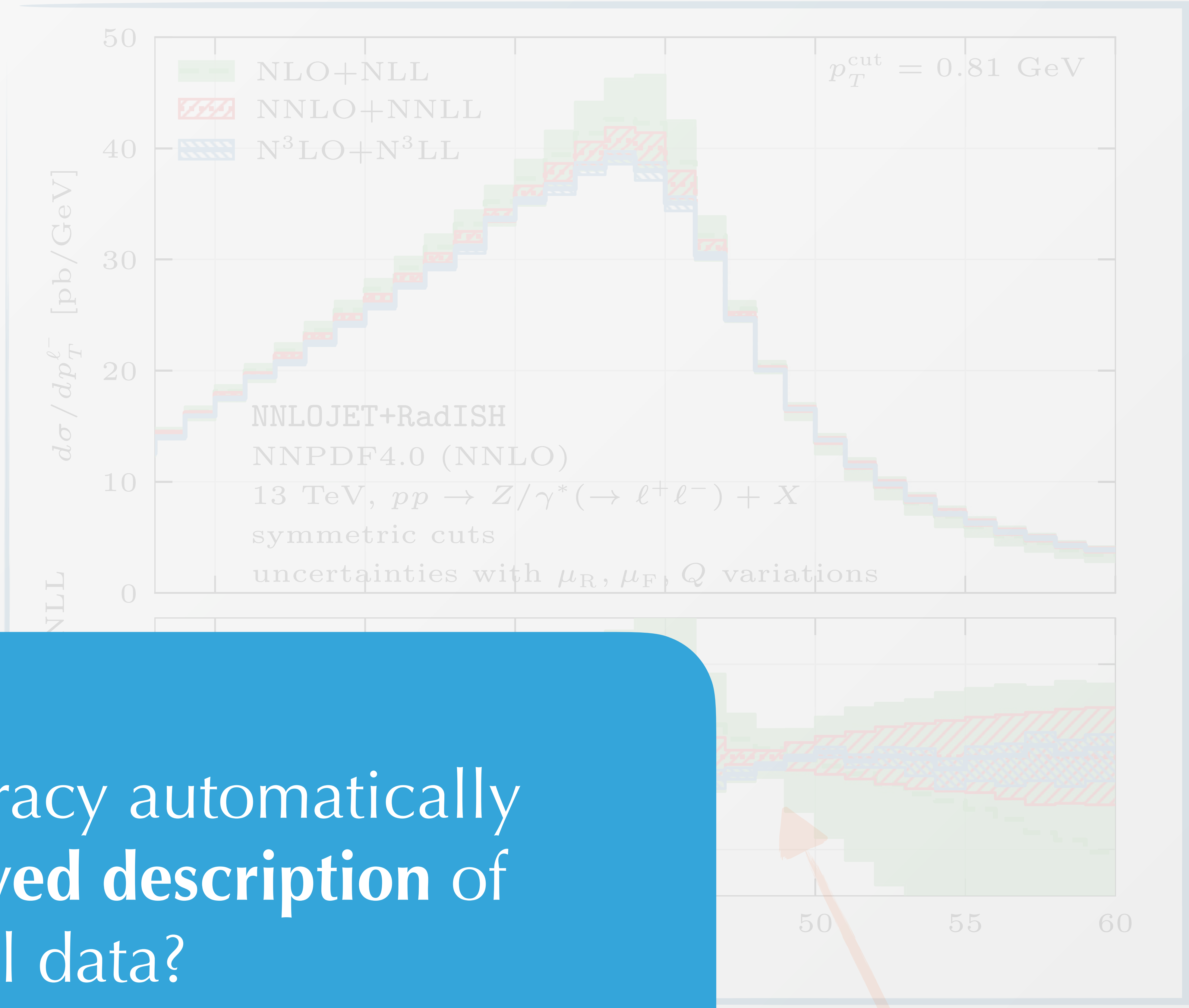
Impact of **N^{3}LO$_{QCD}$** corrections relatively flat in the fit window for m_T

N^{3}LL$'_{QCD}$+N^{3}LO$_{QCD}$ modifies the shape after the Jacobian peak for p_T^ℓ

Progress in higher-order theoretical calculations

Shape of differential spectra is affected by higher order predictions





[Gehrmann, Glover, H

, LR, Torrielli, 2203.01565]

Does higher order accuracy automatically translate into an **improved description** of experimental data?

Impact of N^3LO_{QCD} corrections relatively flat in the fit window for m_T

$N^3LL'_{QCD}$+N^3LO_{QCD} modifies the shape after the Jacobian peak for p_T^ℓ

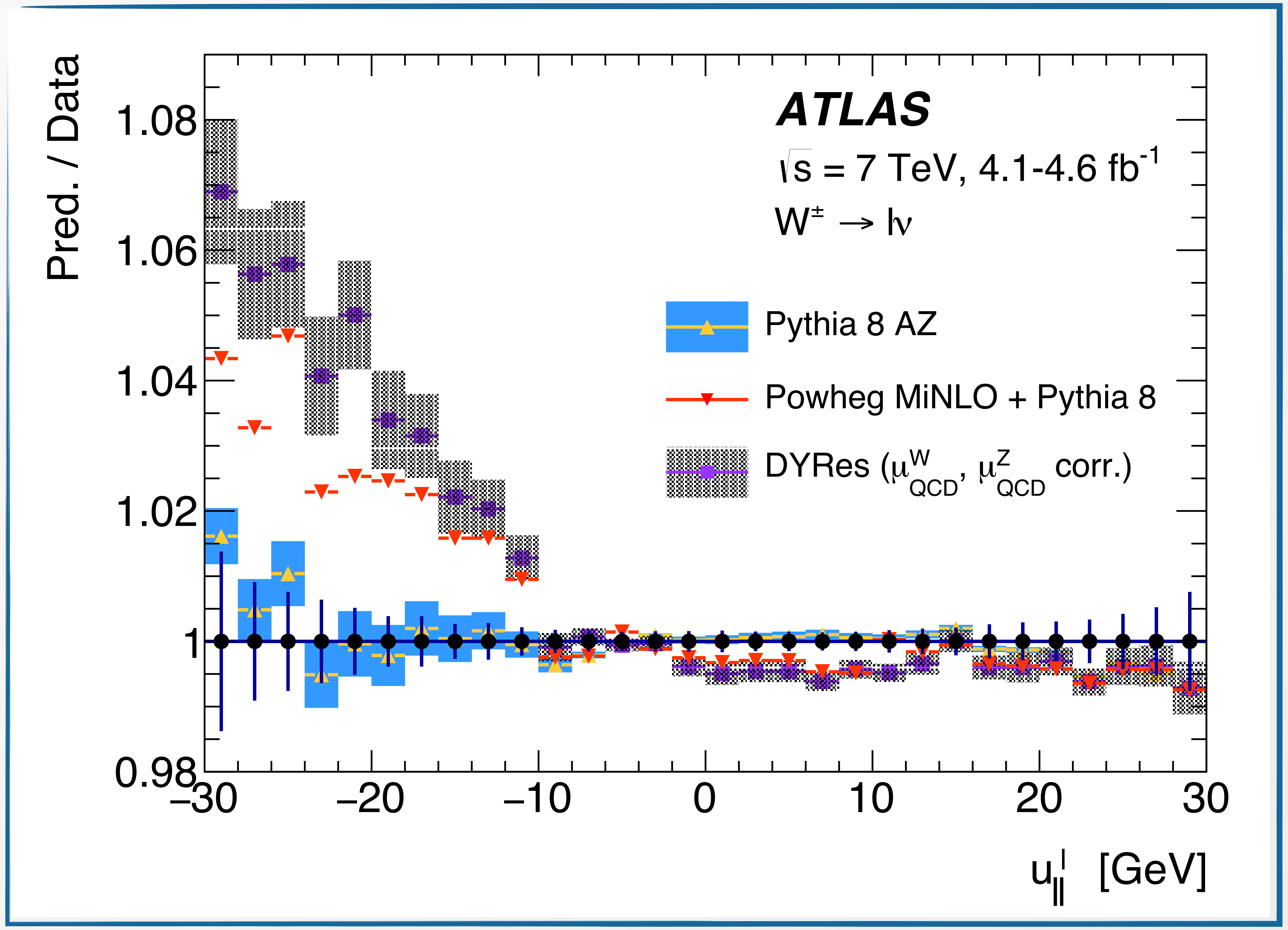
Description of experimental data

The **description of experimental data** plays a crucial role

Predictions with formal higher accuracy could still be outperformed by **tuned** parton shower simulation

In the ATLAS analysis `ResBos` code was found **unable** to satisfactorily describe hadronic recoil

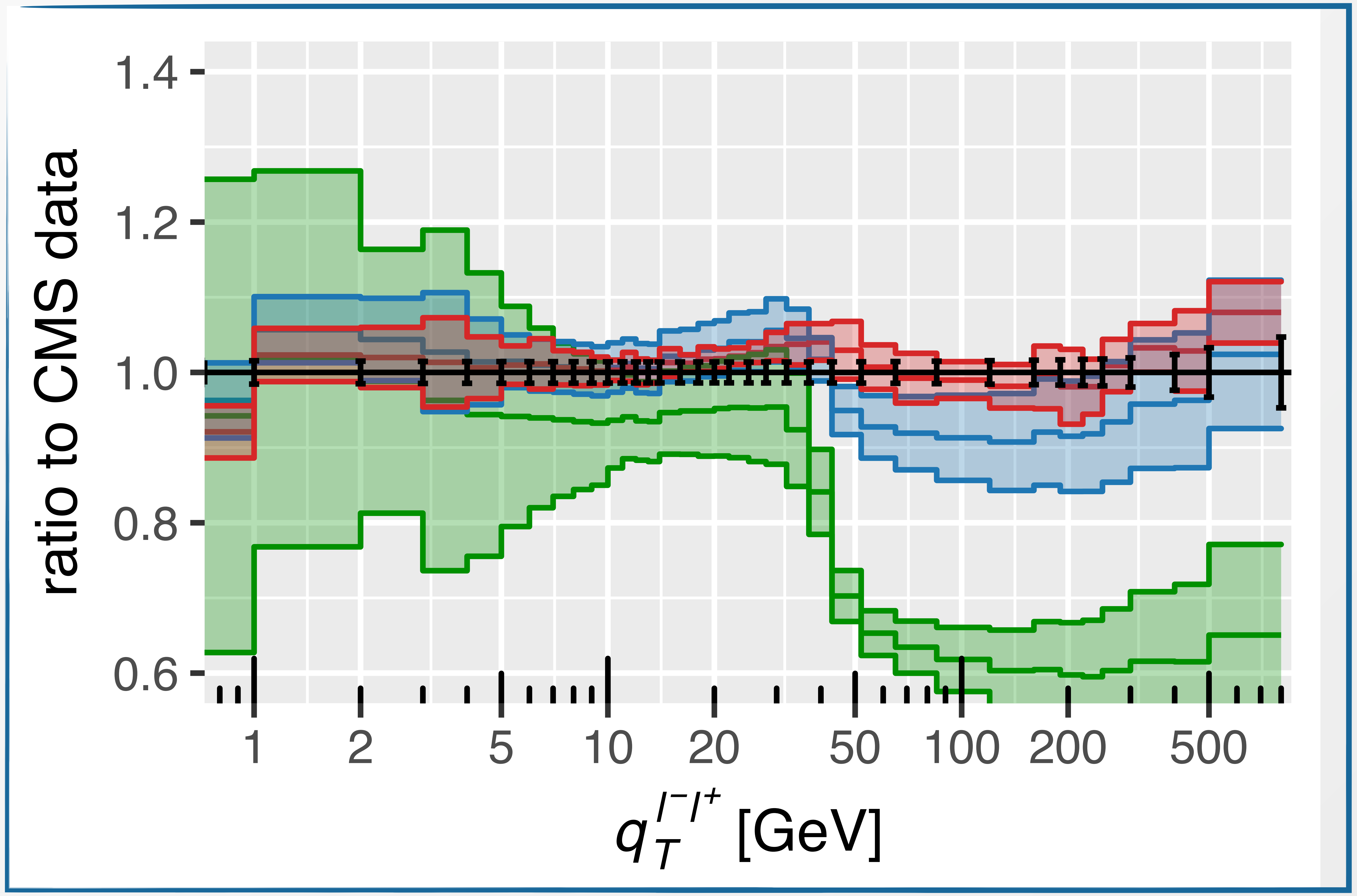
$u_{||}^{\ell}$: projection of hadronic recoil on the axis parallel to reconstructed charged-lepton transverse momentum



[ATLAS '17]

Description of experimental data

The theoretical progress made in the the past 5 years has **significantly improved** the description of the experimental data, pinning down the theoretical uncertainties to the **few percent level** in the description of differential spectra



[Neumann, Campbell '22]

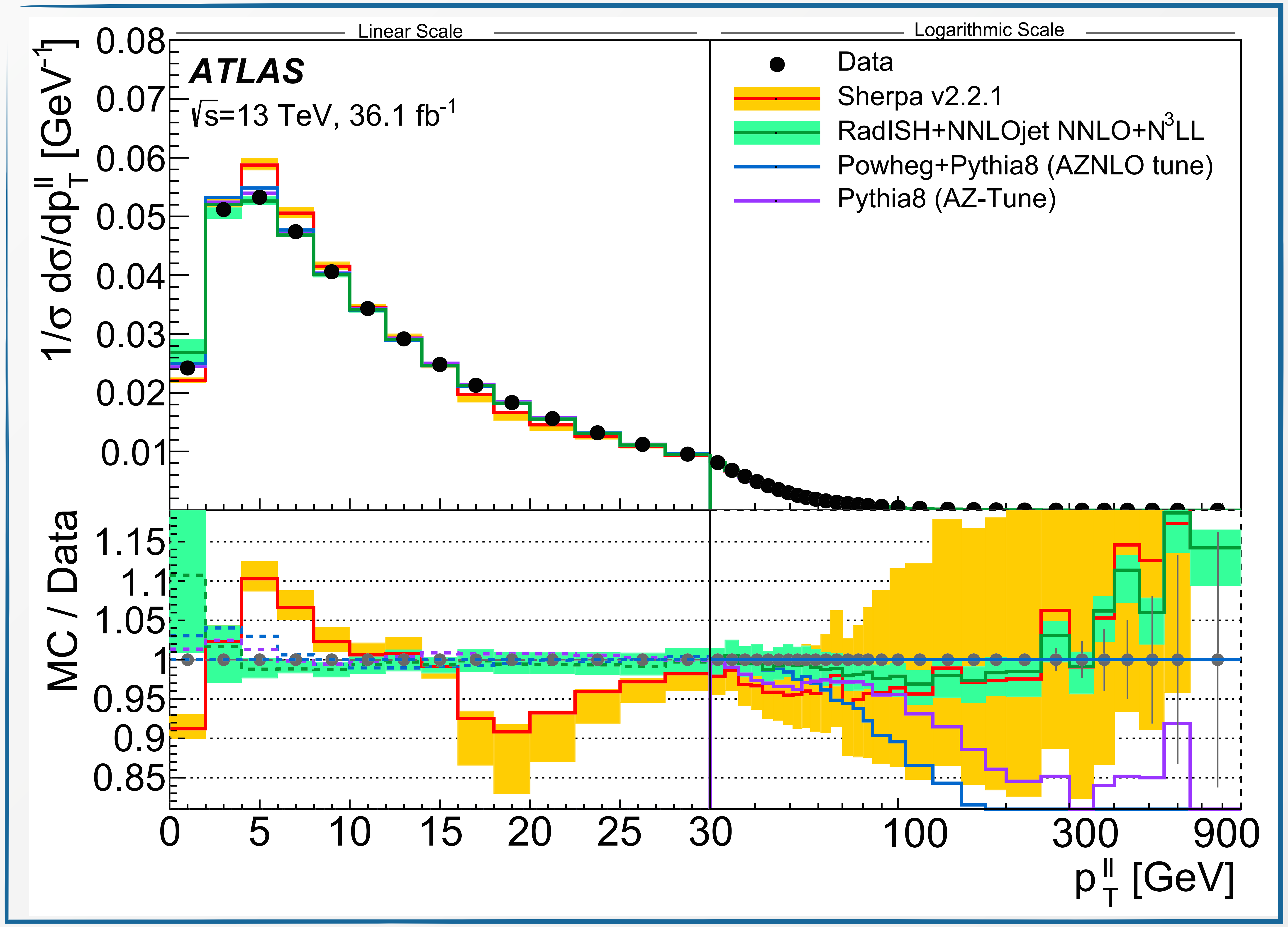
blue curve: **$NNLL'_{QCD}$+$NNLO_{QCD}$**

red curve: **$N^3LL'_{QCD}$+N^3LO_{QCD}**

Description of experimental data

Theoretical predictions now are capable of describing the data **precisely** across a wide range of scales

green curve: N^3LL_{QCD}+N^3LO_{QCD}



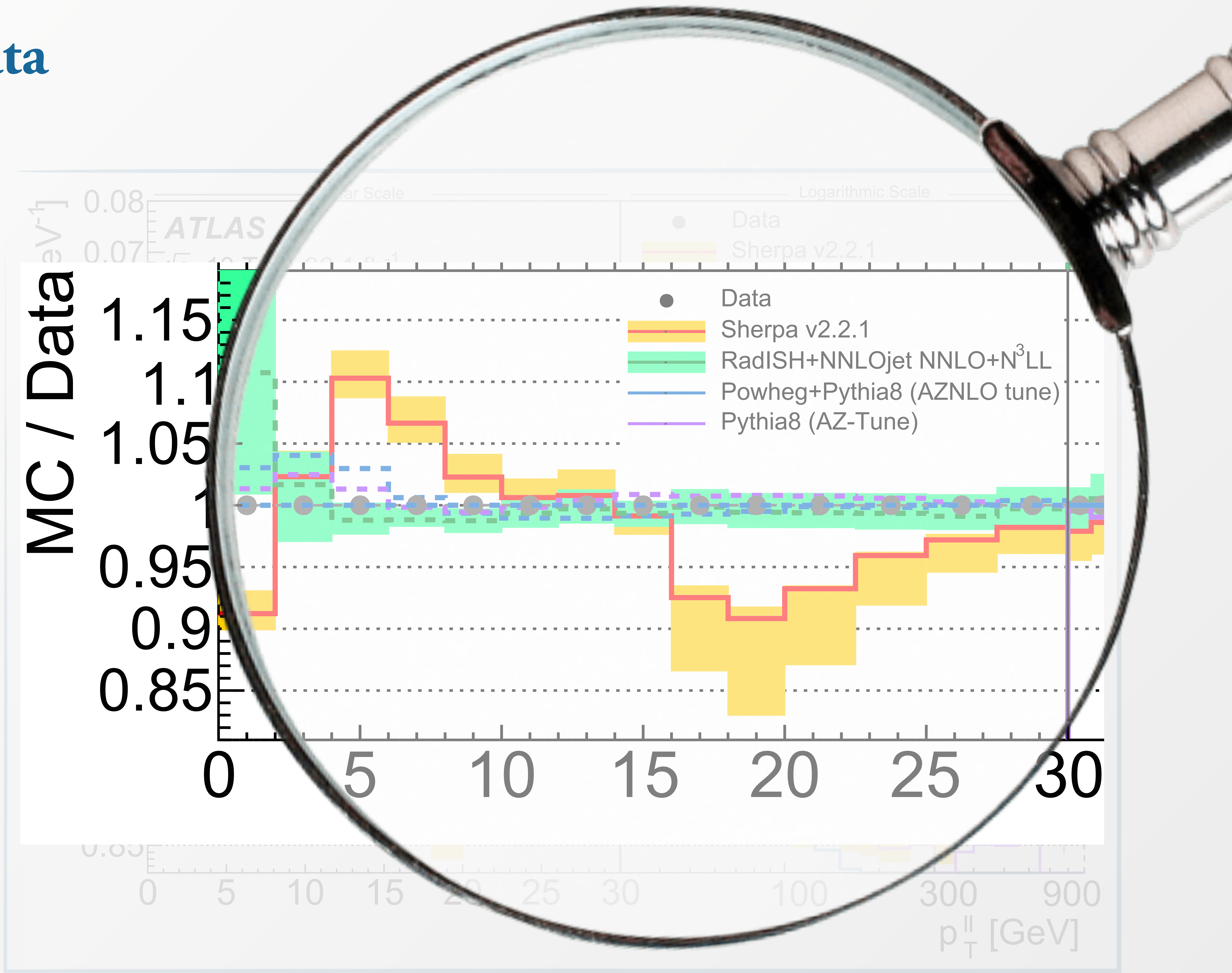
[ATLAS '20]

Description of experimental data

Theoretical predictions now are capable of describing the data **precisely** across a wide range of scales

green curve: N³LL$_{QCD}$+N³LO$_{QCD}$

And are on par, if not better, than parton showers predictions that have been **tuned to experimental data**



[ATLAS '20]

N.B. : `RadISH+NNLOJET` predictions **do not** include any non-perturbative modelling at low q_T

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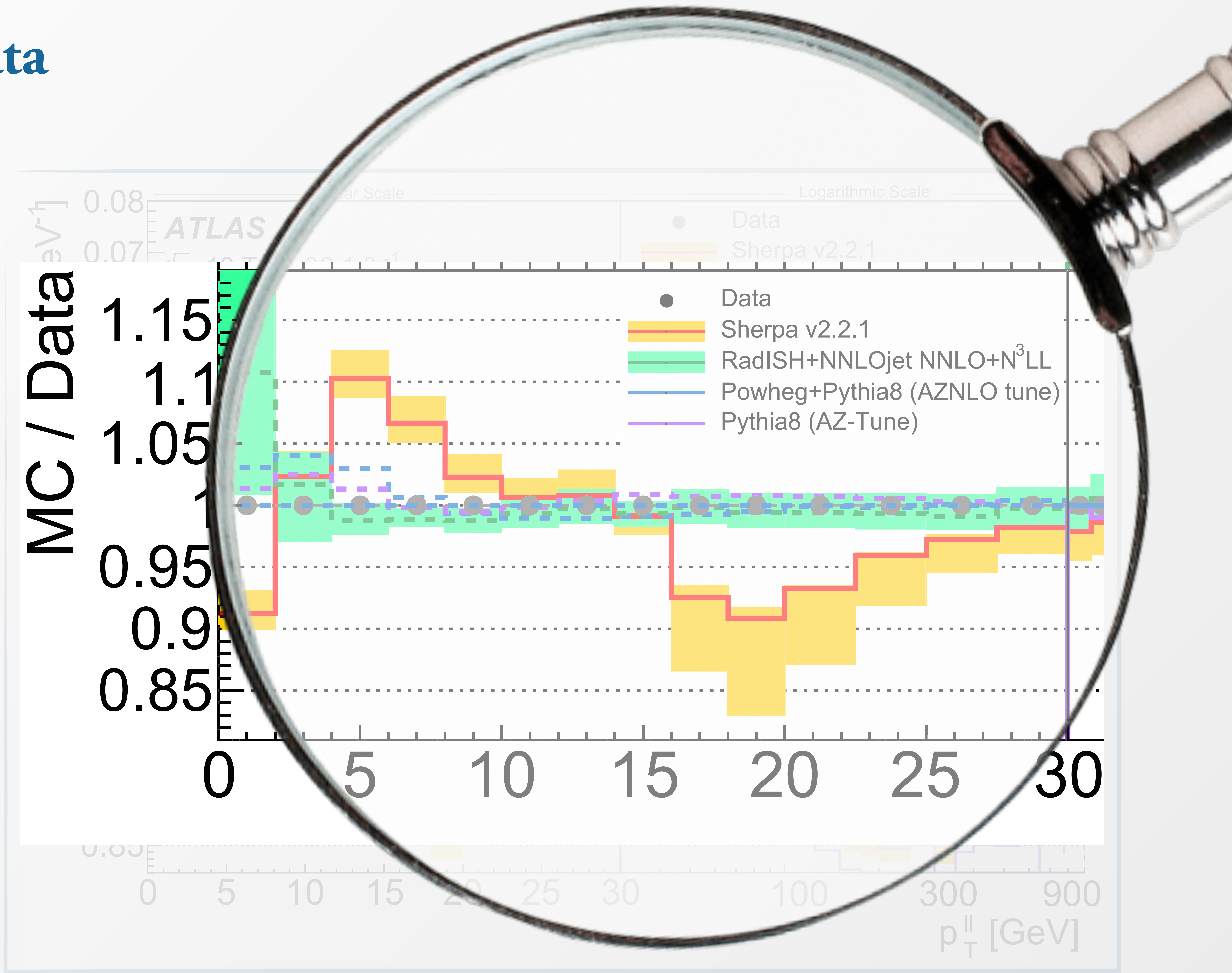

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[ATLAS '20]

N.B. : RadISH+NNLOJET predictions **do not** include any non-perturbative modelling at low q_T

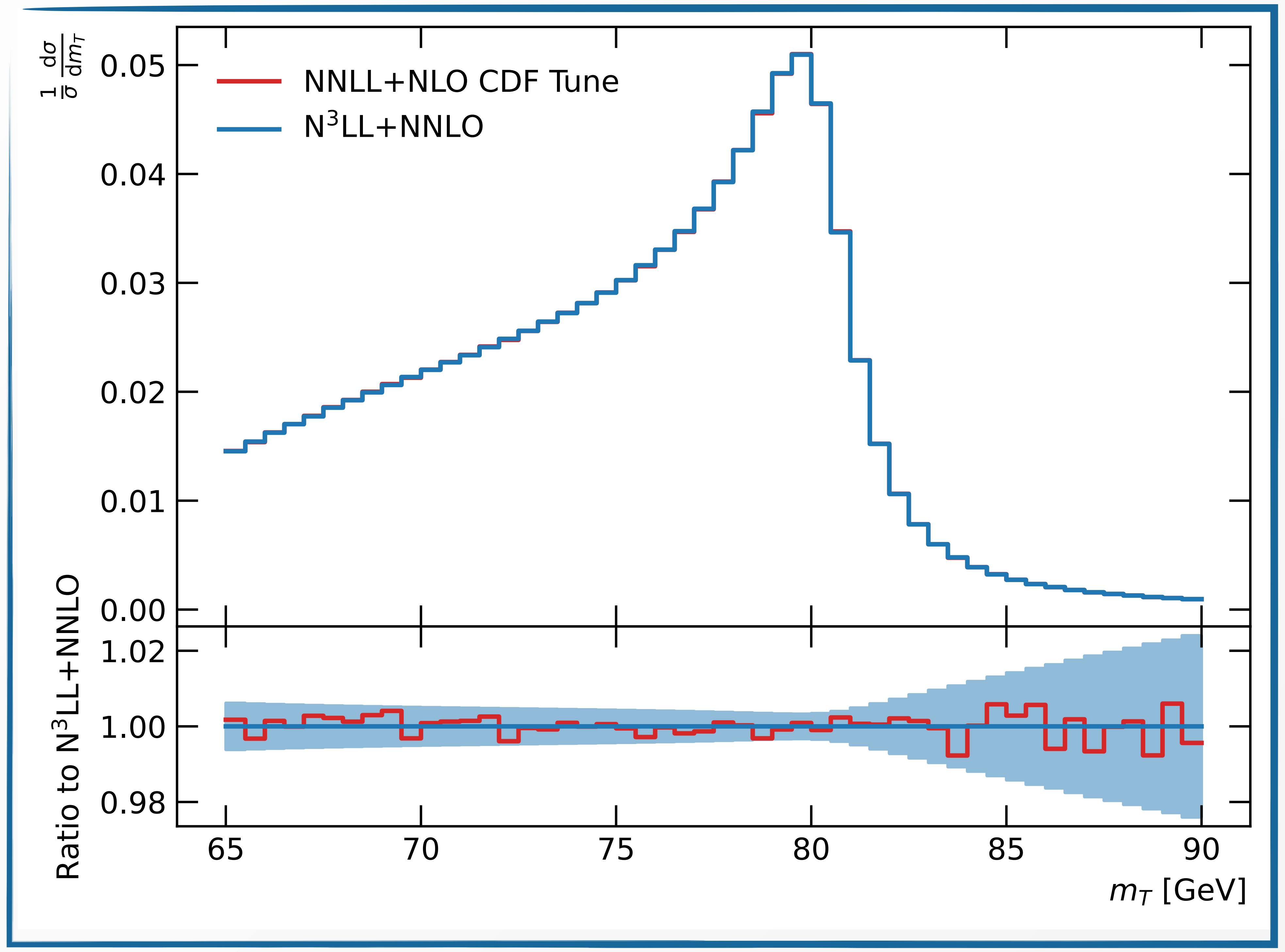
I told you so

What is the impact of **higher-order predictions** on the **extraction** of m_W?

Higher order effects and tuning

Impact of MHOU estimated comparing ResBos2 (**N^3LL_{QCD}+(N)NLO$_{QCD}$**) to ResBos (**(N)NLL$_{QCD}$+(N)LO$_{QCD}$**)

[Isaacson, Fu, Yuan 2205.02788]
[CERN-LPCC-2022-06]



1. ResBos first tuned to reproduce ResBos2 for Z p_T
2. Predictions computed for W differential distribution and compared to ResBos2 results

Shift estimated to be $\mathcal{O}(10\,\mathrm{MeV})$ towards the SM results

Is 10 MeV to be interpreted just as a shift or as **estimate** for the **theory uncertainty**?

Cfr. $\mathcal{O}(1-2\,\mathrm{MeV})$ uncertainty quoted in CDF II

Is it legitimate to allow for variations of the parameters of the perturbative component (such as α_s) to describe data?

Is this estimate stable upon variations of the **underlying resummation formalism**? **Independent analysis** with another tool necessary to confirm these estimates

Although it is not realistic to assume that each analysis could be performed multiple times with different tools, the impact of varying the underlying QCD model should be assessed, see e.g. recent LHCb analysis

Higher order effects and tuning

Impact of MHOU estimated comparing ResBos2 (N^3LL_{QCD}+(N)NLO_{QCD}) to ResBos ((N)NLL_{QCD}+(N)LO_{QCD})

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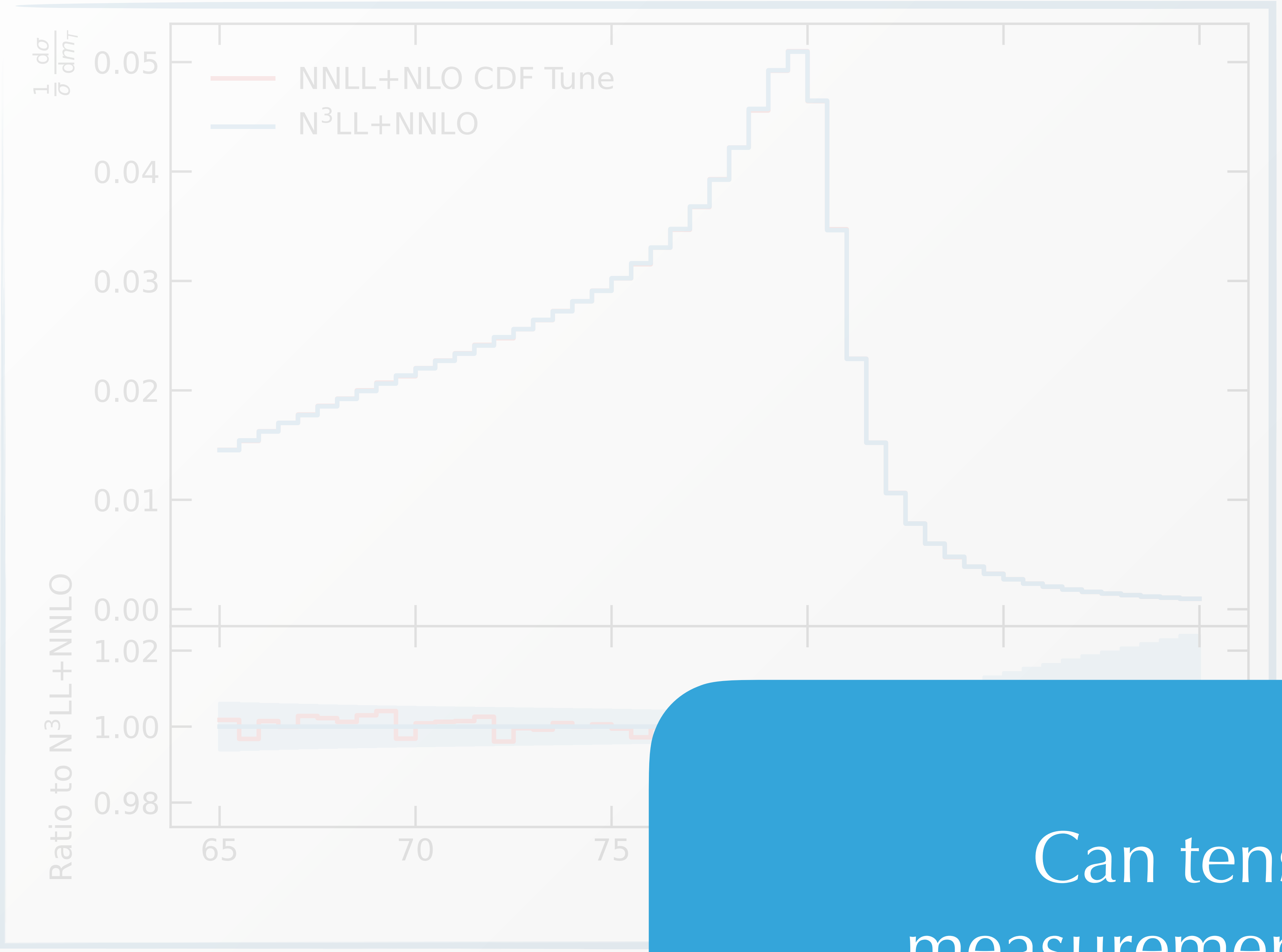
[CERN-LPCC-2022-06]



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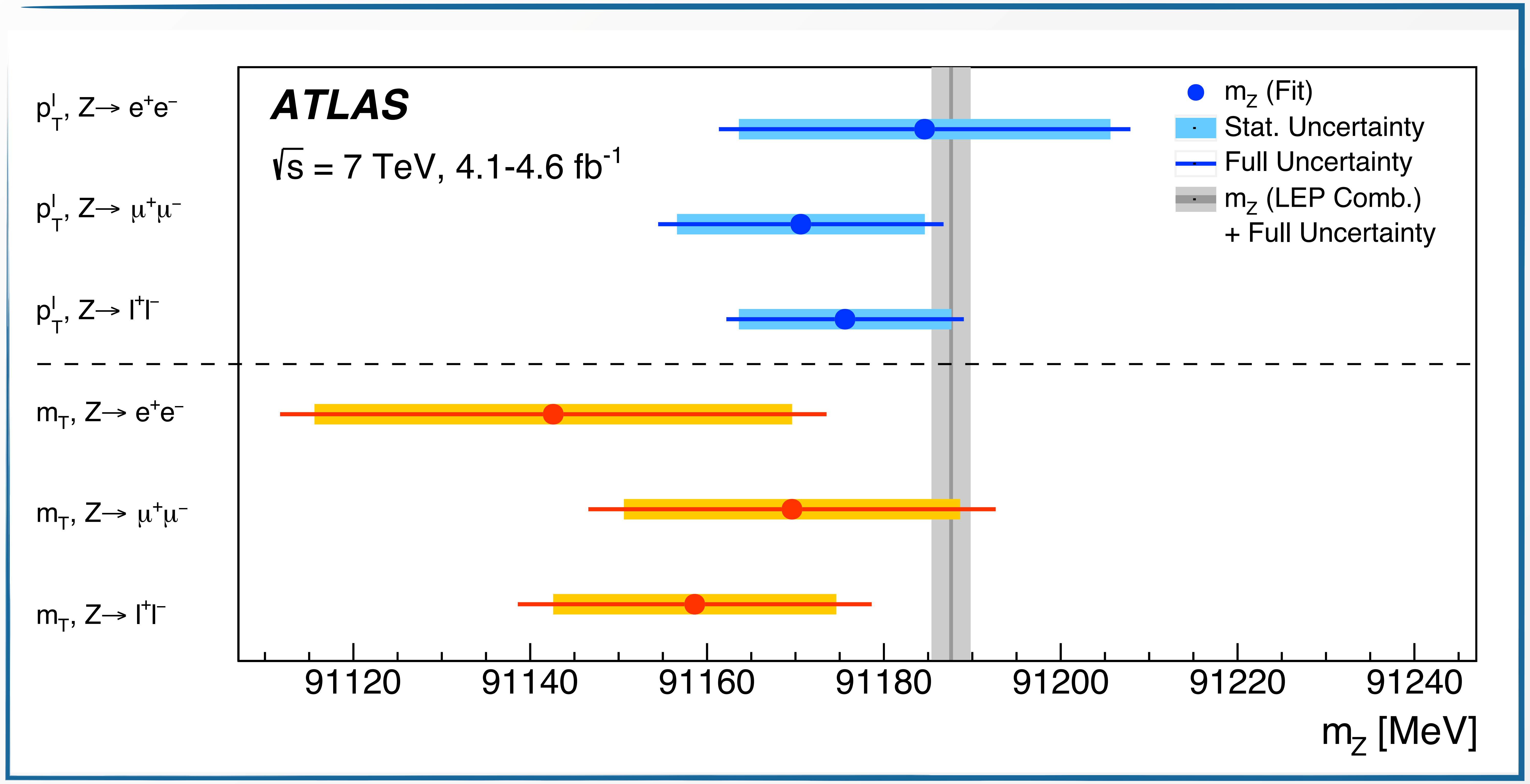
… of the parameters of … α_s) to describe data?

Is this estimate stable up… …ndent analysis with another tool necessary t…

Can tension with SM and other measurements be explained entirely by missing higher order effects?

Although it is not realistic to assume that each analysis could be performed multiple times with different tools, the impact of varying the underlying QCD model should be assessed, see e.g. recent LHCb analysis

Controlling systematics



[ATLAS '17]

Robust check of many underlying systematics (although not sensible to modelling of p_T^Z/p_T^W ratio) can be performed by extracting the Z mass using template fit technique

"Quia vidisti me, credidisti; beati, qui non viderunt et crediderunt"

Johannes 20, 29

[because thou hast seen me, thou hast believed: blessed *are* they that have not seen, and *yet* have believed]

THE STANDARD MODEL AND MISSING E_T

OR

THE MANY ROADS TO PARADISE

Stephen D. Ellis[‡]

CERN - Geneva

cuts. We are thus able to sum the contributions of MANY SMALL sources which were absent in previous studies and which can, in the sum, yield a sizeable result. These are the "many roads" of the title. [This concern, that many small numbers can yield a large sum, was colourfully voiced at the meeting by G. Altarelli who described his vision of a mixture of small effects -- the Altarelli cocktail -- leading to the observed signal.] Before proceeding to the

Template fits on shapes and their limitations

Template fit strategy relies on **tuning** of the model parameters, both perturbative and non-perturbative (strong coupling, non-perturbative component, intrinsic k_T) to fit p_T^Z

Program	χ^2/ndf	α_s	
DYTurbo	208.1/13	0.1180	$g = 0.523 \pm 0.047\,\mathrm{GeV}^2$
POWHEGPythia	30.3/12	0.1248 ± 0.0004	$k_\mathrm{T}^\mathrm{intr} = 1.470 \pm 0.130\,\mathrm{GeV}$
POWHEGHerwig	55.6/12	0.1361 ± 0.0001	$k_\mathrm{T}^\mathrm{intr} = 0.802 \pm 0.053\,\mathrm{GeV}$
Herwig	41.8/12	0.1352 ± 0.0002	$k_\mathrm{T}^\mathrm{intr} = 0.753 \pm 0.052\,\mathrm{GeV}$
Pythia, CT09MCS	69.0/12	0.1287 ± 0.0004	$k_\mathrm{T}^\mathrm{intr} = 2.113 \pm 0.032\,\mathrm{GeV}$
Pythia, NNPDF31	62.1/12	0.1289 ± 0.0004	$k_\mathrm{T}^\mathrm{intr} = 2.109 \pm 0.032\,\mathrm{GeV}$

[LHCb '21]

Value of χ^2 lowered considerably after the fit, where good description of the data can be achieved

Result of the fit (default model of LHCb requires α_s~0.130) begs the question whether the QCD modelling has anything to do with perturbative QCD

Theory errors cannot be accommodated easily in the fitting procedure unless one uses a probabilistic definition of the theory uncertainty (which requires to go beyond canonical scale variation)

[Cacciari, Houdeau 1105.5152] [Bonvini 2006.16293]

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21]

21]

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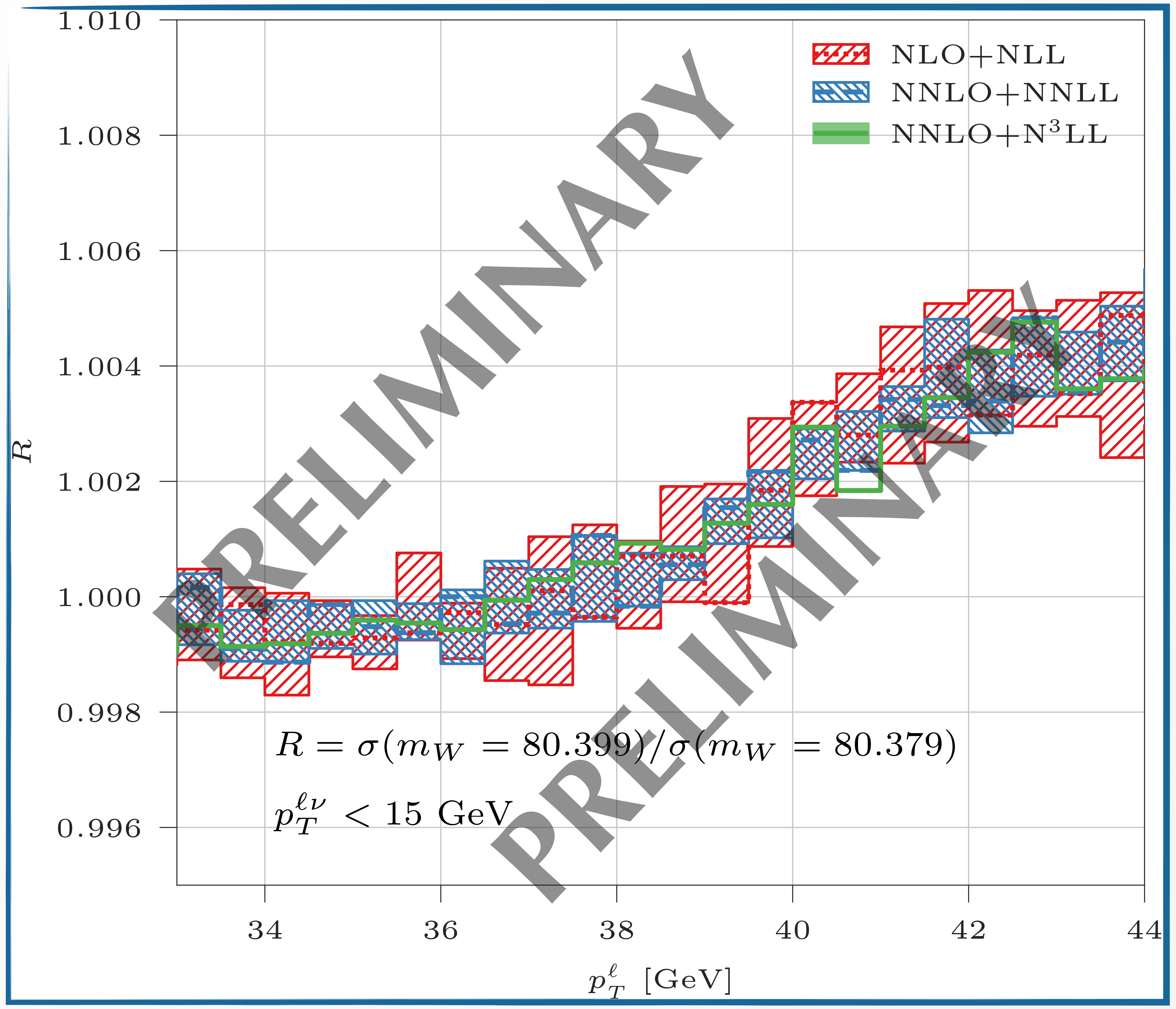
Is there any other approach capable of extracting m_W from data?

The p_T^ℓ spectrum and its sensitivity to m_W

[LR, P. Torrielli, A. Vicini, 202SOON]



Sensitivity on m_W of the bins of the p_T^ℓ distribution can be quantified by means of the **covariance matrix** with respect to m_W variations

$$C_{ij} = \langle \sigma_i \sigma_j \rangle - \langle \sigma_i \rangle \langle \sigma_j \rangle$$

The eigenvalues of this matrix express sensitivity on m_W on **linear combinations of bins** of the distribution

Large hierarchy between the first eigenvalue and the others, suggesting that the majority of the sensitivity is captured by the largest eigenvalue

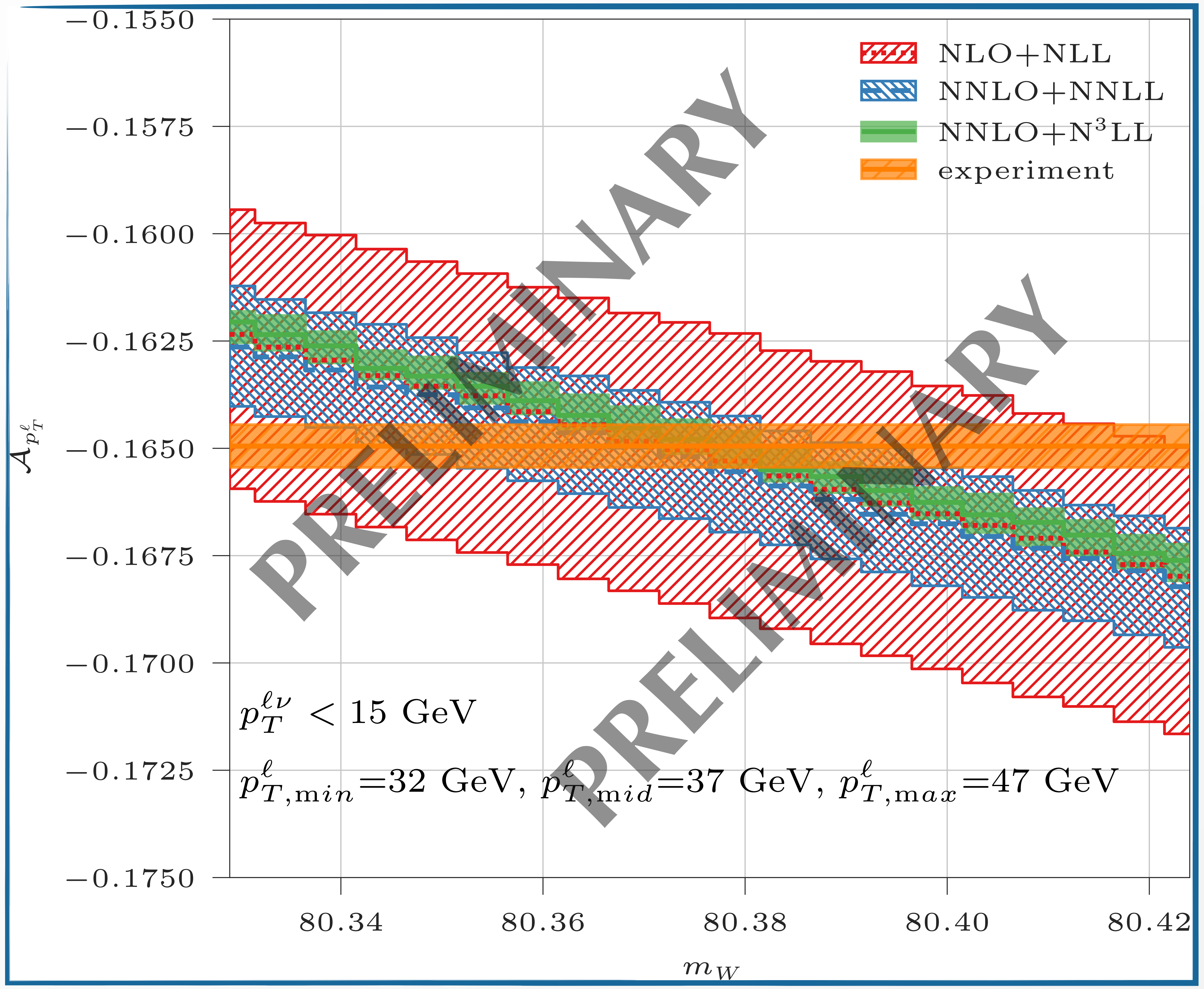
Coefficients **changes sign** around $p_T^\ell \simeq 37$ GeV

Motivates definition of a **scalar variable sensitive to m_W**

$$\mathscr{A}(p^\ell_{T,\mathrm{min}}, p^\ell_{T,\mathrm{mid}}, p^\ell_{T,\mathrm{max}}) = \frac{\sigma(p^\ell_{T,\mathrm{min}}, p^\ell_{T,\mathrm{mid}}) - \sigma(p^\ell_{T,\mathrm{mid}}, p^\ell_{T,\mathrm{max}})}{\sigma(p^\ell_{T,\mathrm{min}}, p^\ell_{T,\mathrm{mid}}) + \sigma(p^\ell_{T,\mathrm{mid}}, p^\ell_{T,\mathrm{max}})}$$

A scalar observable sensitive to m_W

[LR, P. Torrielli, A. Vicini, 202SOON]



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Sensitivity on m_W expressed by the slope of the asymmetry

Large size of the two intervals reduces the statistical error and improves the quality of the detector effects unfolding

Perturbative convergence of the observable can be studied for different intervals $[p^{\ell}_{T,\text{min}}; p^{\ell}_{T,\text{mid}}; p^{\ell}_{T,\text{max}}]$

Missing higher order uncertainty can be estimated by means of scale variations

Extraction can rely only on the information of the **charged-current Drell-Yan process**

Data-driven approach to m_W extraction

A theory-agnostic extraction of m_W

[E. Manca, PhD Thesis 2016; V. Bertacchi, Tesi di Perfezionamento 2021]

Exploit statistics collected by CMS during Run II at the LHC to extract the value of m_W simultaneously with q_T^W, y_W and polarization spectra to obtain a statistically-dominated measurement of m_W

Decoupling of the (unknown) production physics from the (known) decay physics

unpolarised cross section

W and lepton variables

$$\frac{d\sigma}{dq_{T,W}^2 dy_W d\cos\theta_\mu d\phi_\mu dm_W} = \frac{3}{16\pi} \frac{d\sigma^{U+L}}{dq_{T,W}^2 dY_W dm_W} \left[(1+\cos^2\theta_\mu) + \sum_{i=0}^{7} A_i P_i(\cos\theta_\mu, \phi_\mu) \right]$$

angular coefficients

1. Decompose inclusive $\eta^\mu \times p_T^\mu$ distribution in bins of m_W, Y_W, q_T^W for each P_i
2. Fit $\eta^\mu \times p_T^\mu$ distribution measured on data
3. Unfolding from the sole lepton kinematics to the underling boson kinematics

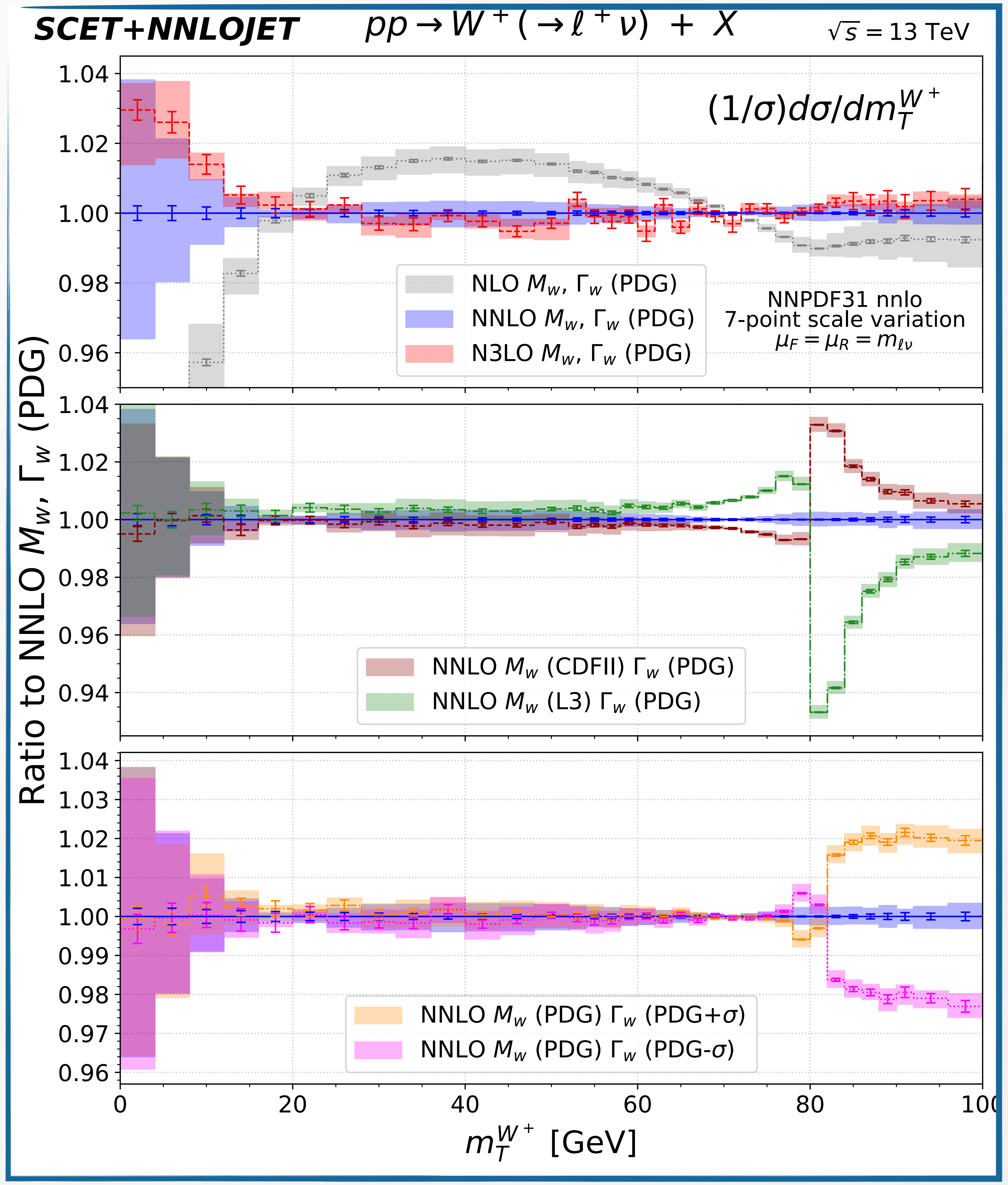
Conclusion

- Extraction of m_W mass at hadron colliders requires **accurate knowledge of QCD and QED radiation**
- Huge progress in the last few years in precise **PDF determination**, **higher-order QCD resummation** and **mixed QCD and QED corrections**
- PDF uncertainty is not a substantial limitation for a precise extraction of m_W
- Future measurement of m_W should exploit the **huge amount of theoretical work** in the last 5+ years to obtain **reliable estimates** of the theoretical uncertainties
- For such a delicate measurement, complementary approaches to extract m_W and determine its uncertainty should be used to avoid possible biases
- Shape of the p_T^ℓ distribution and presence of the Jacobian peak motivates the definition of a **scalar observable** which **maximises the sensitivity** on m_W and overcomes some limitations of templates fits performed on shapes
- Data-driven approach provides a valid alternative to extract m_W with a statistical-dominated error

"The aim of science is not to open a door to infinite wisdom, but to set a limit to infinite error"

Bertold Brecht

Backup



SCET+NNLOJET
$pp \to W^+(\to \ell^+ \nu) + X$
$\sqrt{s} = 13$ TeV
$(1/\sigma)d\sigma/dm_T^{W^+}$
NLO M_w, Γ_w (PDG)
NNLO M_w, Γ_w (PDG)
N3LO M_w, Γ_w (PDG)
NNPDF31 nnlo
7-point scale variation
$\mu_F = \mu_R = m_{\ell\nu}$
NNLO M_w (CDFII) Γ_w (PDG)
NNLO M_w (L3) Γ_w (PDG)
NNLO M_w (PDG) Γ_w (PDG+σ)
NNLO M_w (PDG) Γ_w (PDG-σ)
Ratio to NNLO M_w, Γ_w (PDG)
$m_T^{W^+}$ [GeV]

PDFs and their uncertainties: bin-by-bin correlations

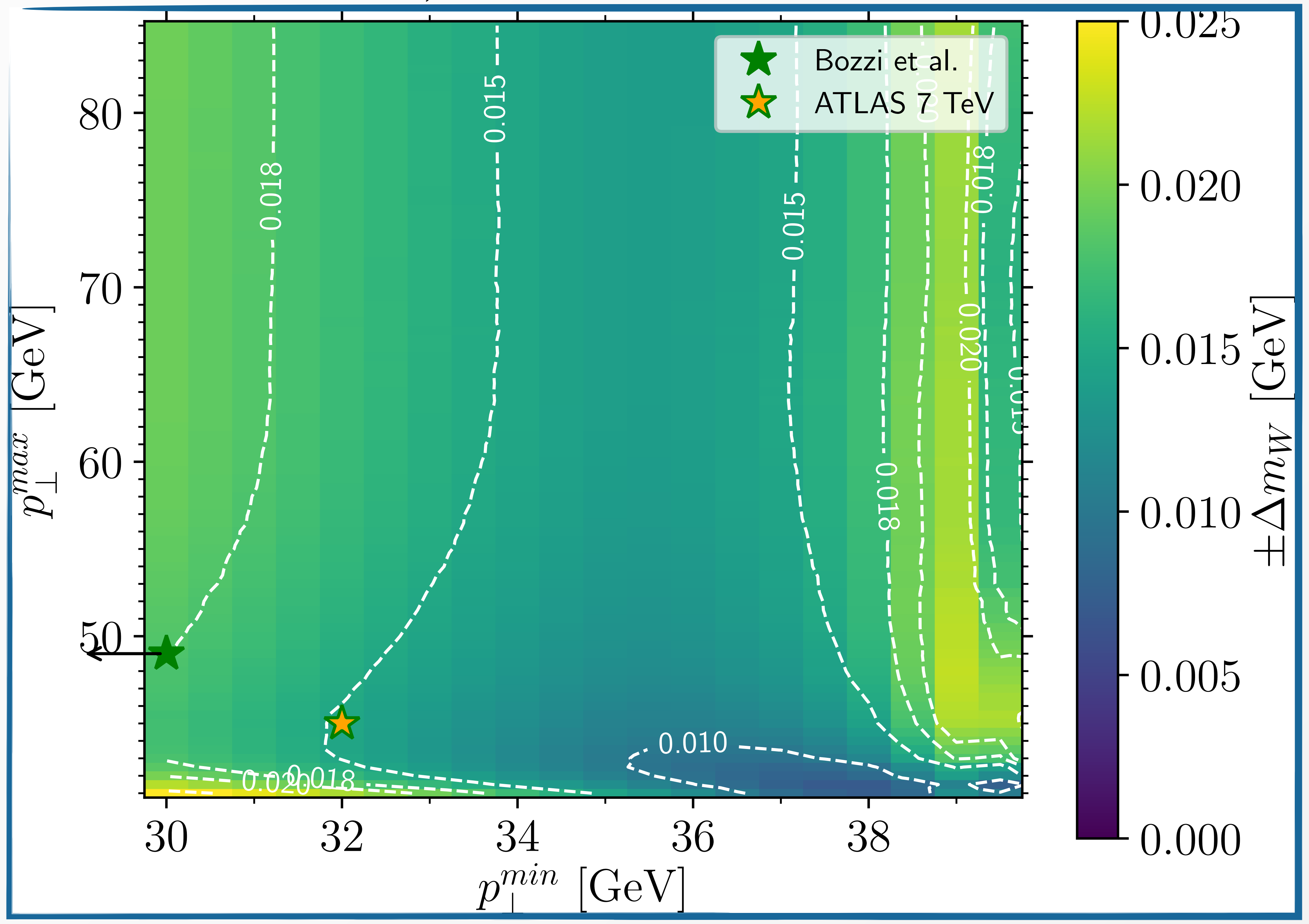
[Bagnaschi, Vicini 1910.04726]

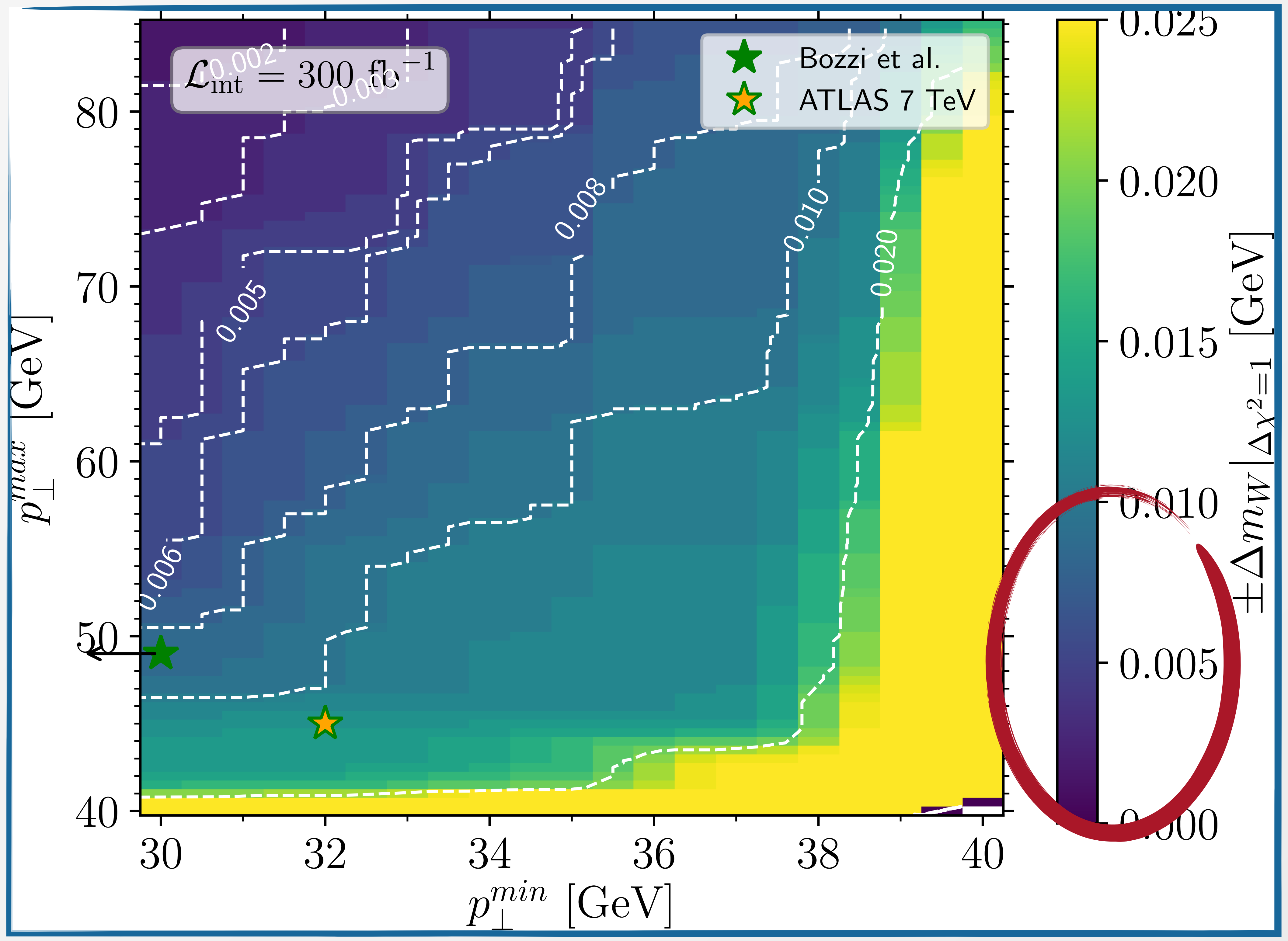
This estimate does not take into account **bin-by-bin correlations** between PDF replicas

$$(\Sigma_{\mathrm{PDF}})_{ij} = \langle (\mathcal{T} - \langle \mathcal{T} \rangle_{\mathrm{PDF}})_i (\mathcal{T} - \langle \mathcal{T} \rangle_{\mathrm{PDF}})_j \rangle_{\mathrm{PDFs}}$$

Compute χ^2 taking into account bin-by-bin correlations introducing a covariance matrix in the definition

$$\chi^2_{i,\mathrm{min}} = \sum_{k,l \in \mathrm{bins}} (T_{0,i} - D)_k (C^{-1})_k (T_{0,i} - D)_l \qquad \forall m_{W,i} \qquad C = \Sigma_{\mathrm{PDF}} + \Sigma_{\mathrm{MC}} + \Sigma_{\mathrm{stat}} + \Sigma_{\mathrm{exp,syst}}$$





Reduced sensitivity to the PDF uncertainty, if other source of errors are under control