Resummed predictions for the transverse momentum spectrum of EW bosons at the LHC

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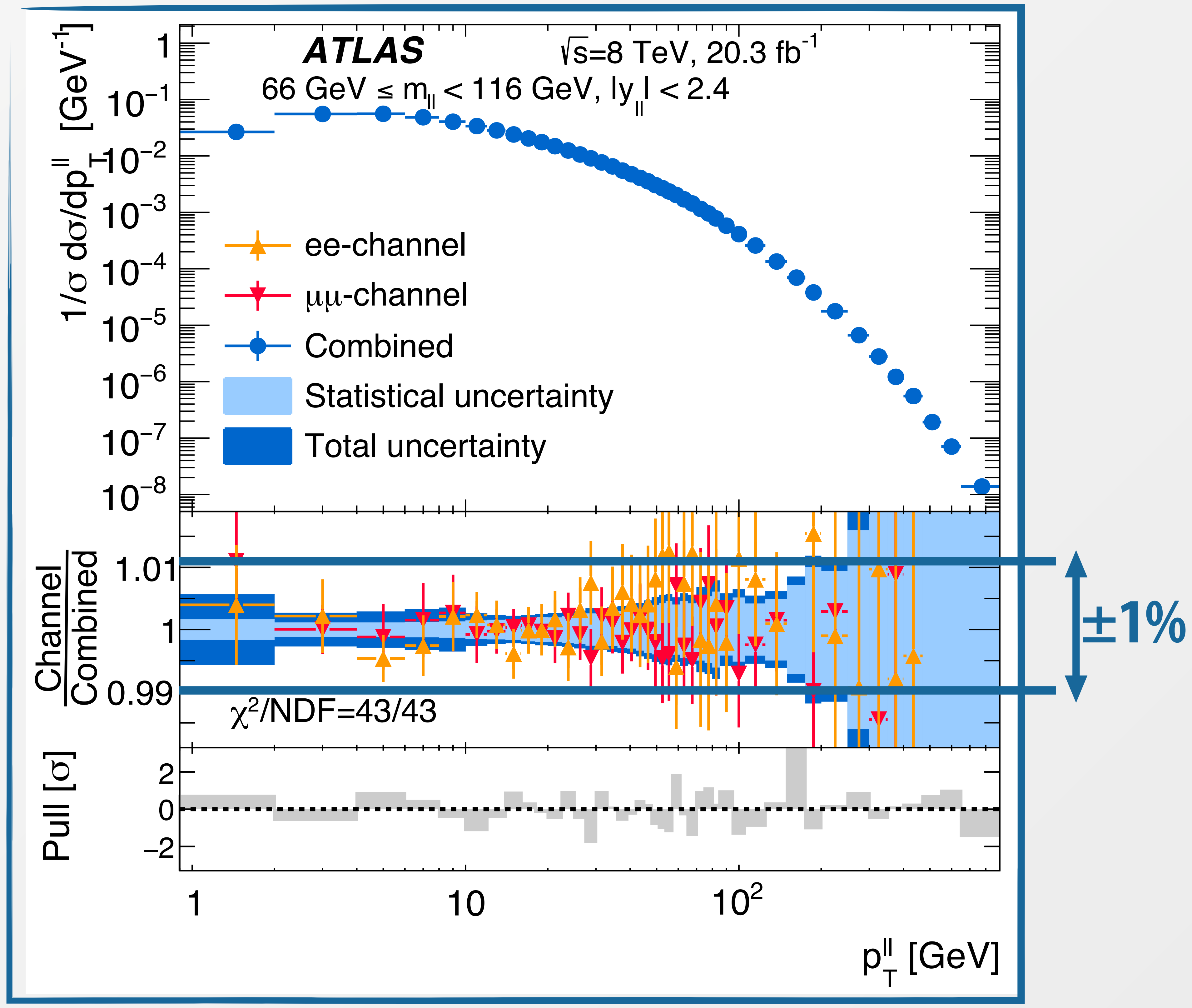
The quest for precision

Transverse observables are a **clean experimental and theoretical environment** for precision physics

Inclusive observables (e.g. transverse momentum p_t) probe directly the kinematics of the colour singlet

$$V(k_1, \ldots k_n) = V(k_1 + \ldots + k_n)$$

- negligible or no sensitivity to multi-parton interactions
- reduced sensitivity to non-perturbative effects
- measured **extremely precisely at experiments**, challenging current theoretical predictions

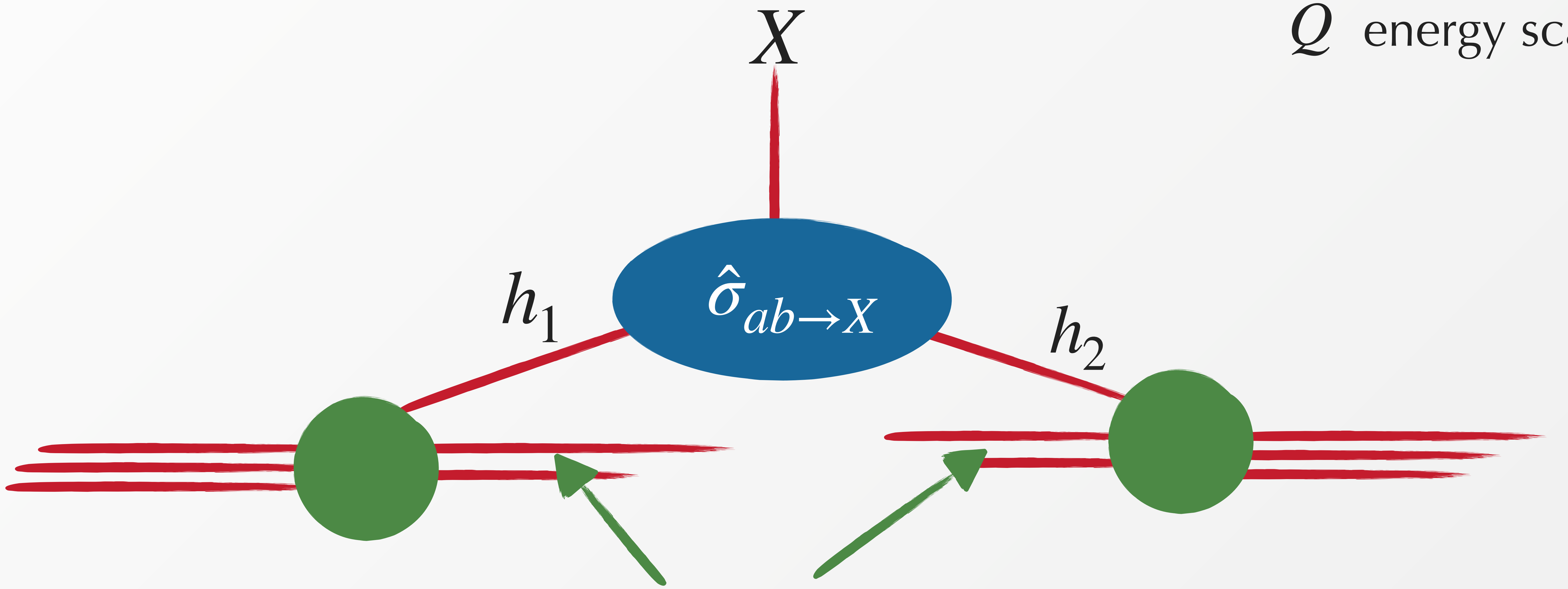


Important implications for extraction of SM parameters (strong coupling and PDF determination, ***W* mass measurements**…)

Precision physics at the LHC: theory

Key concept: **collinear factorization**

$\sqrt{s}$ centre-of-mass energy
Q energy scale of the process



$$\sigma(s,Q^2) = \sum_{a,b} \int dx_1 dx_2\, f_{a/h_1}(x_1,Q^2) f_{b/h_2}(x_2,Q^2)\, \hat{\sigma}_{ab\to X}(Q^2, x_1 x_2 s) + \mathcal{O}(\Lambda^p_{\mathrm{QCD}}/Q^p)$$

Parton Distribution Functions (PDFs)

Long-distance, non-perturbative, universal objects

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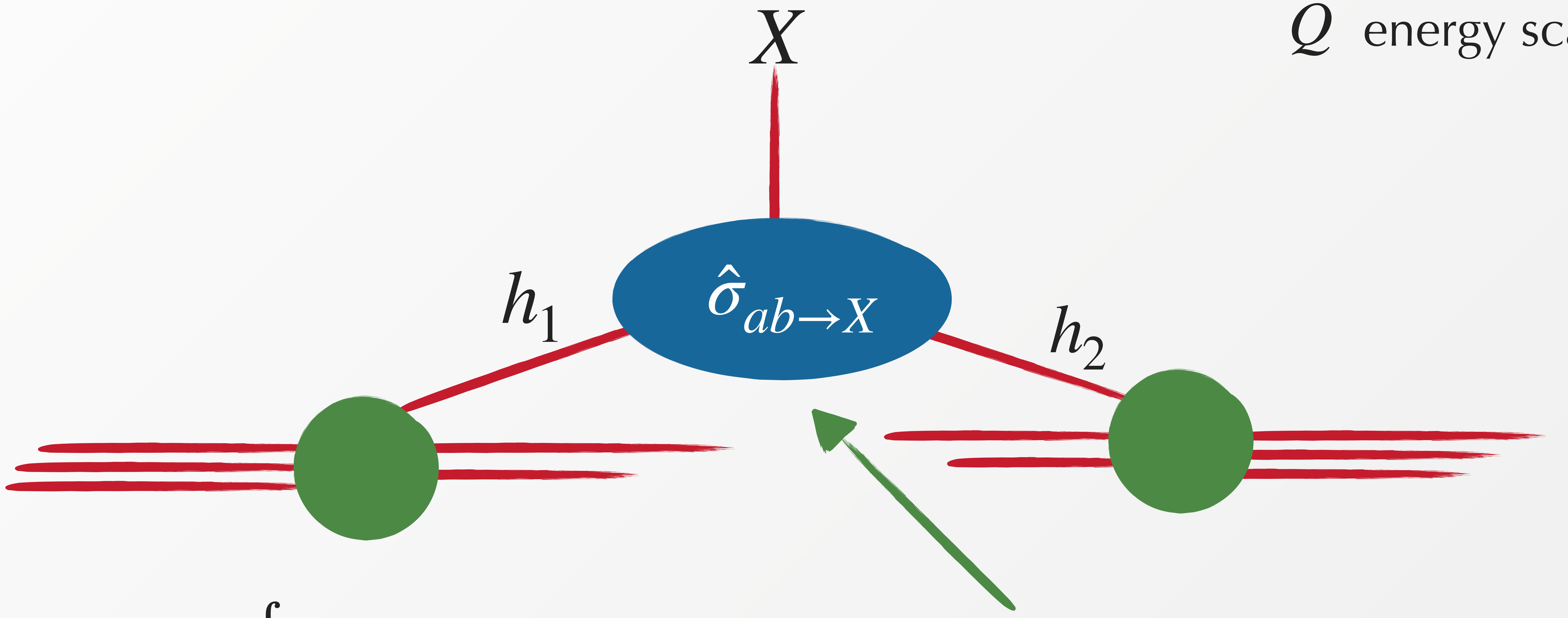

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Hard-scattering matrix element

Short-distance, perturbative, process-dependent

Precision physics at the LHC: theory

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Input parameters:

strong coupling α_s

PDFs f

few percent uncertainty; improvable

Non-perturbative effects

percent effect; not yet under control

Precision physics at the LHC: theory

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$$\tilde{\sigma} = 1 + \alpha_s \tilde{\sigma}_1 + \alpha_s^2 \tilde{\sigma}_2 + \alpha_s^3 \tilde{\sigma}_3 + \ldots$$

LO **NLO** **NNLO** **N^{3}LO**

$\alpha_s \sim 0.1$

δ~10-20% **NLO**

δ~1-5% **NNLO** (or even N^{3}LO)

NLO now standard and largely automated

NNLO available for an increasing number of processes

N^{3}LO Higgs production in gluon fusion and VBF (hadron-collider processes)

QCD beyond fixed order

Perturbative QCD at fixed order

$$\tilde{\sigma} = 1 + \alpha_s \tilde{\sigma}_1 + \alpha_s^2 \tilde{\sigma}_2 + \alpha_s^3 \tilde{\sigma}_3 + \dots$$

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LO NLO NNLO N^{3}LO

Assumption: perturbative coefficients $\tilde{\sigma}_n$ are well behaved (renormalon ambiguity)

Many observables studied at the LHC depend on more than one scale; **single** or **double** logs of the ratio of those scales at all orders in perturbation theory

$$(\alpha_s \ln R)^n \qquad (\alpha_s \ln^2 R)^n$$

If the logarithms are large the convergence of the series is spoiled

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Perturbative QCD at fixed order

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LO NLO NNLO N³LO

Assumption: p…

Fixed order predictions no longer reliable: all order resummation of the perturbative series mandatory

Many observab… of those scales at all orders in perturbation theory

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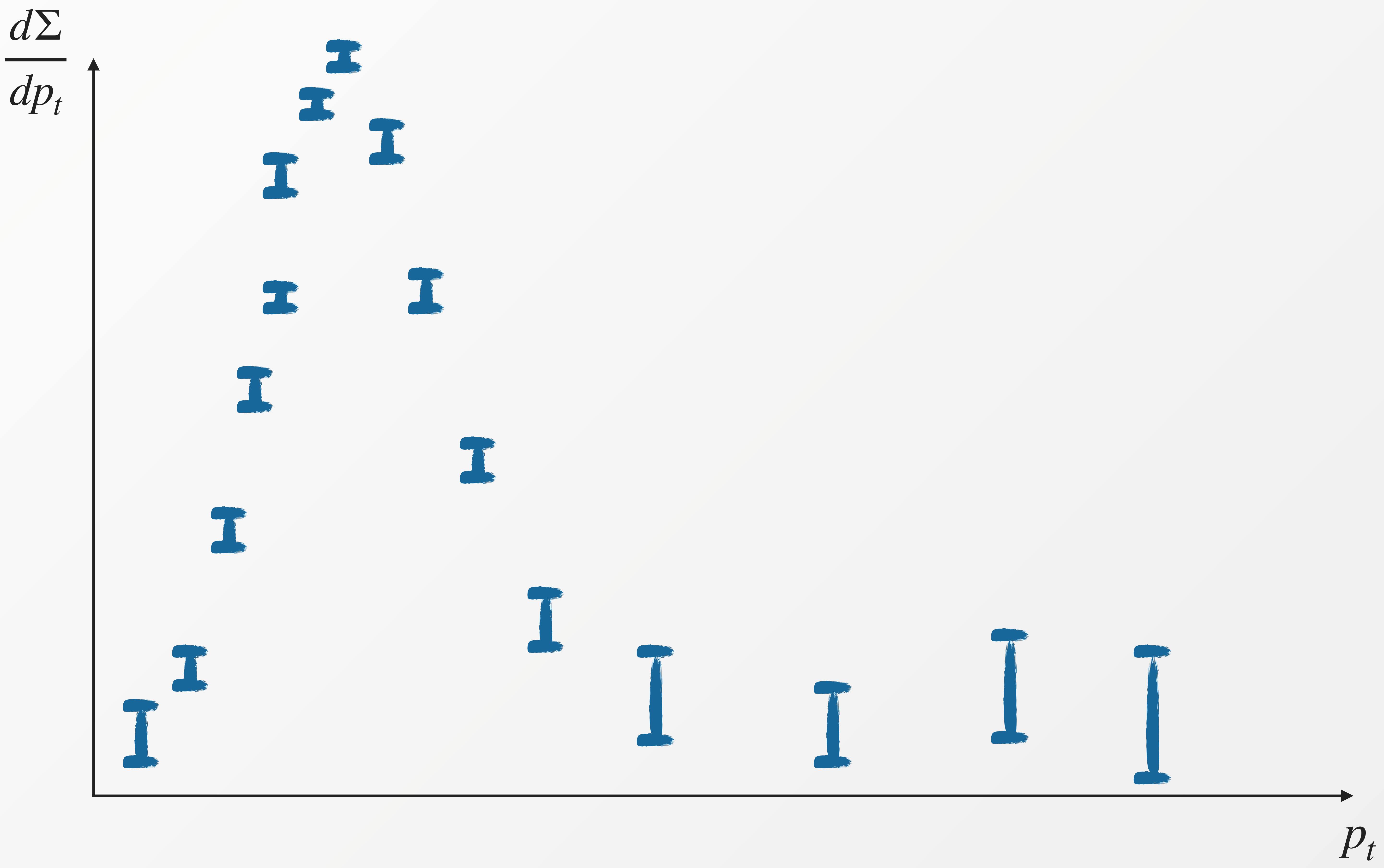
Resum what?

Example: **transverse momentum distribution** in Higgs production



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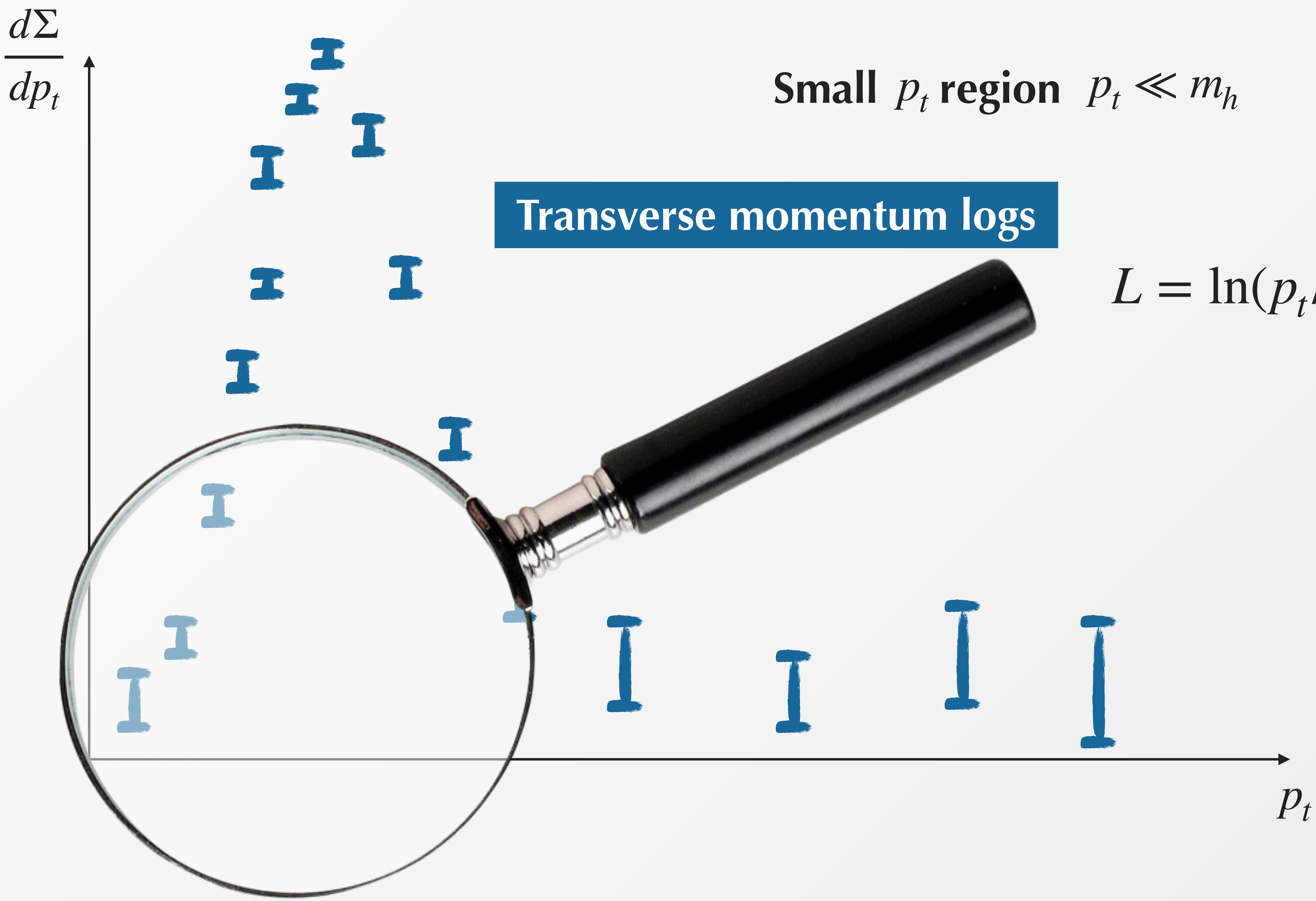

Small p_t region $p_t \ll m_h$

Transverse momentum logs

$$L = \ln(p_t/m_h)$$

Resum what?

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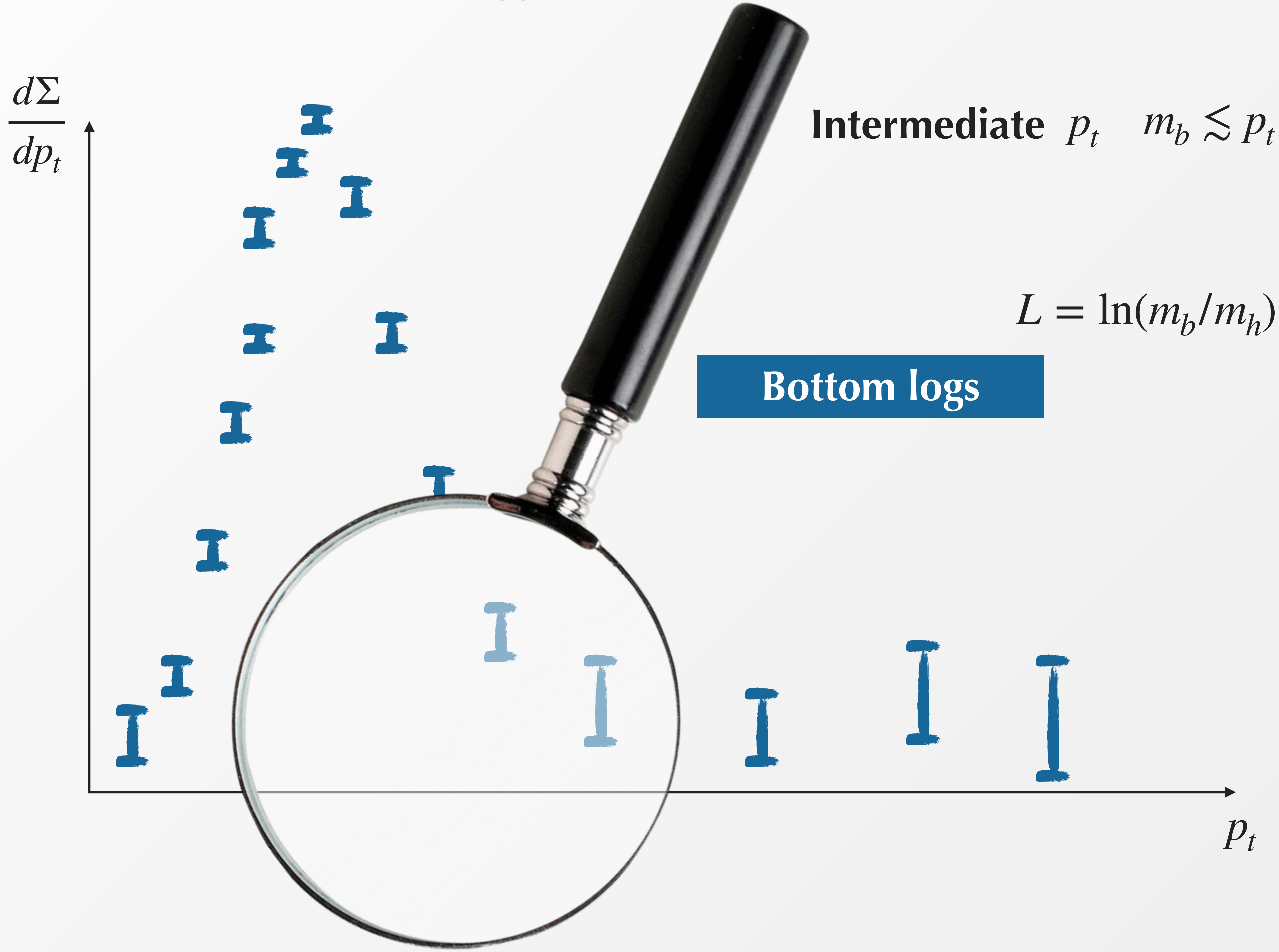

Intermediate p_t $m_b \lesssim p_t \lesssim m_t$

$L = \ln(m_b/m_h)$

Bottom logs

Resum what?

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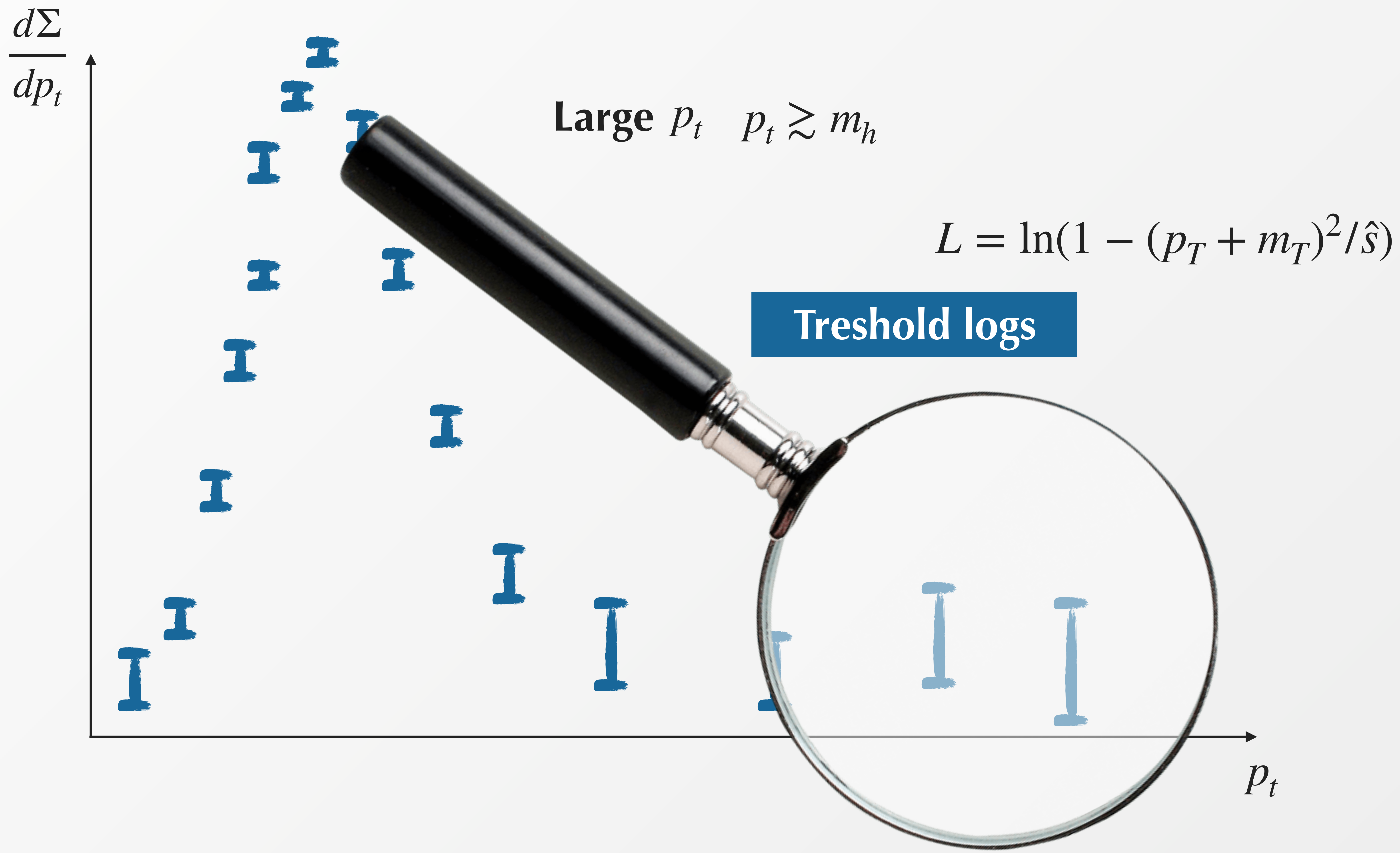

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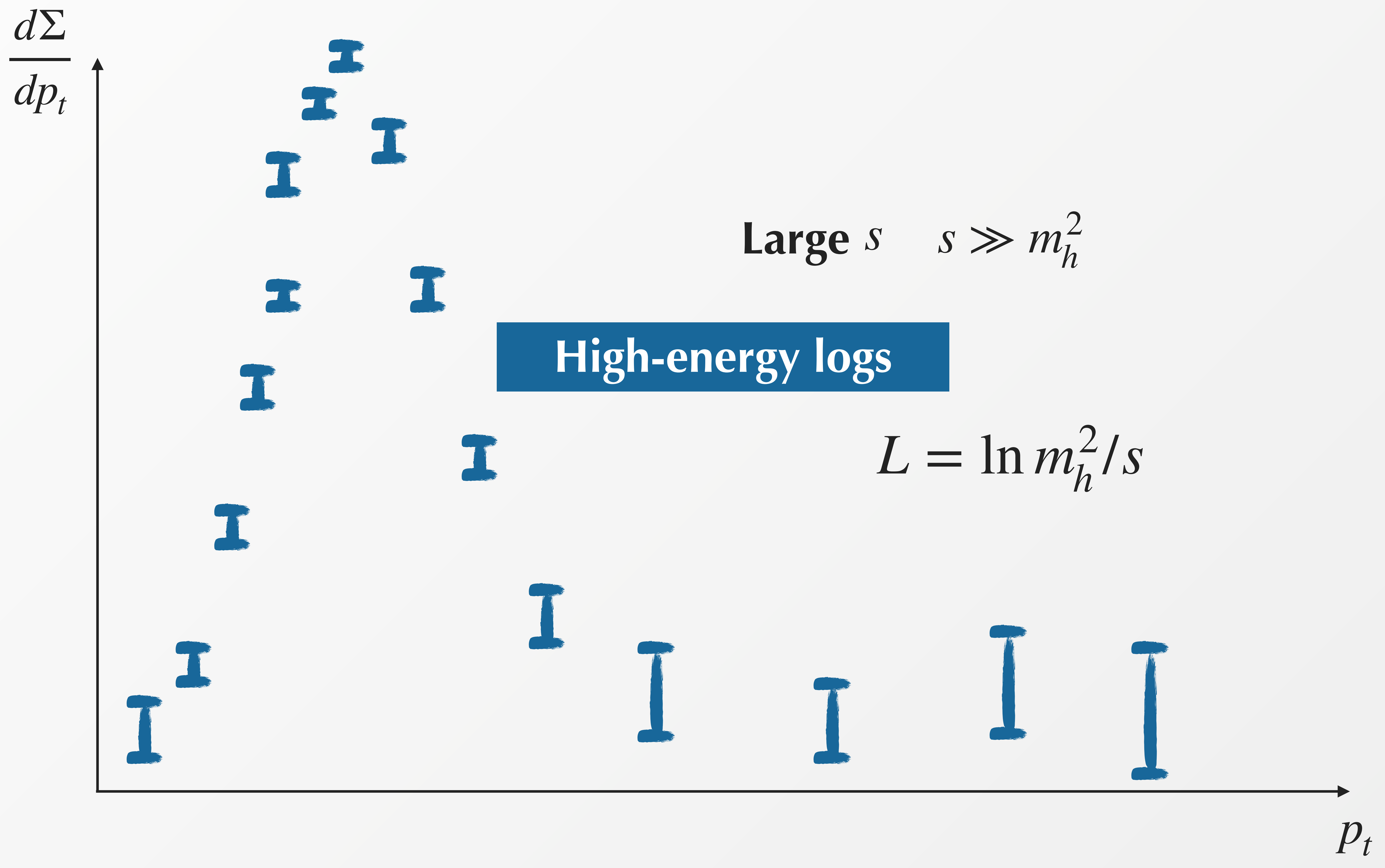


Large s $s \gg m_h^2$

High-energy logs

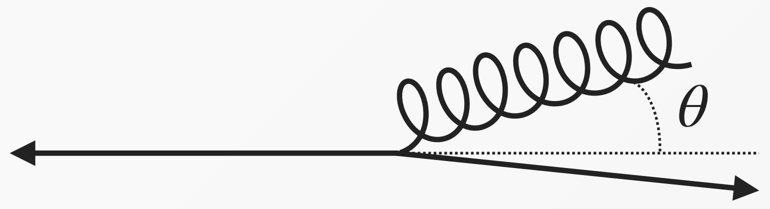
$$L = \ln m_h^2/s$$

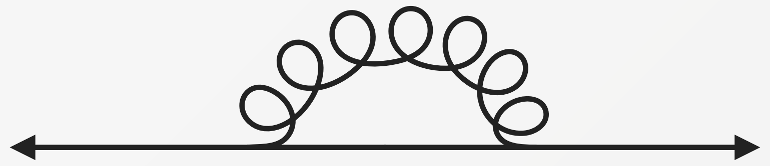
It's not a bug, it's a feature

Real emission diagrams singular for **soft/collinear emission**. Singularities are cancelled by virtual counterparts for IRC safe observables

Consider processes where real radiation is **constrained** in a corner of the phase space, (exclusive boundary of the phase space, **restrictive cuts**)

$$\tilde{\sigma}_1(v) \sim \underbrace{\int \frac{d\theta}{\theta}\frac{dE}{E}\Theta\left(v - E\theta/Q\right)} - \underbrace{\int \frac{d\theta}{\theta}\frac{dE}{E}}$$





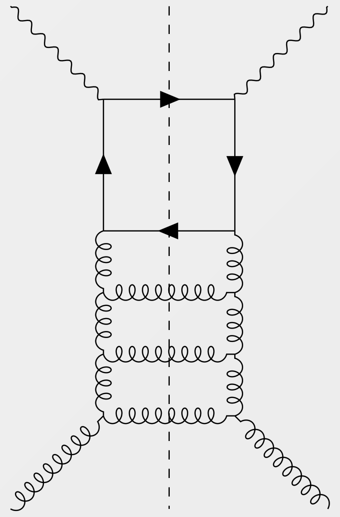
$v \to 0$ observable can become negative even in the perturbative regime

$$\sim \; -\int \frac{dE}{E}\frac{d\theta}{\theta}\Theta(E\theta/Q - v) \sim -\frac{1}{2}\ln^2 v$$

Sudakov logarithms

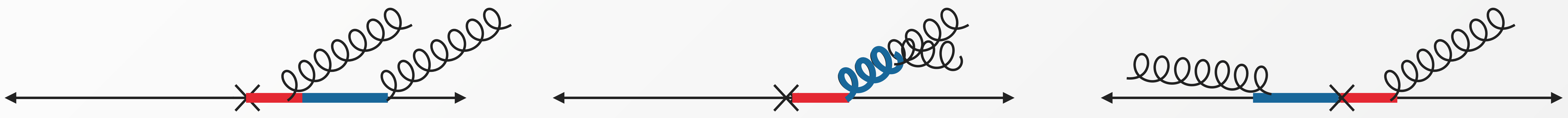
Double logarithms **leftovers** of the real-virtual cancellation of IRC divergences

Single logarithms appear also when **exchanged gluon** is soft (**no collinear contribution**). **High-energy resummation** of $\alpha_s \ln m^2/s$



Making pQCD great again: all-order resummation

Soft-collinear emission of two gluons



Two propagators nearly on shell, 4 divergences. Diagrams can potentially give $\alpha_s^2 \ln^4 v$

All order structure

$$\tilde{\sigma}(v) = \sum_{n=0}^{\infty} \alpha_s^n \sum_{m=1}^{2n} c_{nm} L^m + \ldots \qquad L = \ln(v)$$

Origin of the logs is simple. Resum them to all orders by **reorganizing** the series

$$\tilde{\sigma}(v) = \boxed{f_1(\alpha_s L^2)} + \frac{1}{L} f_2(\alpha_s L^2) + \ldots$$

Leading logarithmic (LL) resummation of the perturbative series

Accurate for $L \sim 1/\sqrt{\alpha_s}$

All-order resummation

$$\tilde{\sigma}(v) = f_1(\alpha_s L^2) + \frac{1}{L} f_2(\alpha_s L^2) + \ldots$$

*È la somma che fa il totale

All-order resummation: exponentiation

Independent emissions $k_1, \ldots k_n$ (plus corresponding virtual contributions) in the soft and collinear limit (**eikonal approximation**)

$$d\Phi_n |\mathcal{M}(k_1, \ldots k_n)|^2 \to \frac{1}{n!}\alpha_s^n \prod_{i=1}^{n} \frac{dE_i}{E_i}\frac{d\theta_i}{\theta_i}$$

Calculate observable with arbitrary number of emissions: **exponentiation**

$$\tilde{\sigma} \simeq \sum_{n=0}^{\infty} \frac{1}{n!}\alpha_s^n \prod_{i=1}^{n} \int \frac{dE_i}{E_i}\frac{d\theta_i}{\theta_i}\Theta(E_i\theta_i/Q - v) \simeq e^{-\alpha_s L^2}$$

[Sudakov '54]

Sudakov suppression

Price for constraining real radiation

Exponentiated form allows for a **more powerful reorganization**

$$\tilde{\sigma}(v) = \exp\left[\sum_n \left(\underbrace{\mathcal{O}(\alpha_s^n L^{n+1})}_{\textbf{LL}} + \underbrace{\mathcal{O}(\alpha_s^n L^n)}_{\textbf{NLL}} + \underbrace{\mathcal{O}(\alpha_s^n L^{n-1})}_{\textbf{NNLL}} + \ldots\right)\right]$$

Region of applicability now valid up to $L \sim 1/\alpha_s$, successive terms suppressed by α_s

Exponentiation not always possible, e.g. Jade Jet Resolution [Brown, Stirling '90] or jet mass pruning (convolution of two exponentials) [Dasgupta, Marzani, Salam '13]

All-order resummation: (re)-factorization

Phase-space constraints do not usually factorize in **direct space**

$$\tilde{\sigma}(v) \sim \int \prod_i^n [dk_i] \mathcal{M}(k_1, \ldots, k_n) \Theta_{\rm PS}(v - V(k_1, \ldots k_n))$$

Solution: move to **conjugate space** where phase space factorization is manifest

e.g. p_t resummation
[Parisi, Petronzio '79; Collins, Soper, Sterman '85]

$$\delta^{(2)}\left(\vec{p}_t - \sum_{i=1}^n \vec{k}_{t,i}\right) = \int d^2b \frac{1}{4\pi^2} e^{i\vec{b}\cdot\vec{p}_t} \prod_{i=1}^n e^{-i\vec{b}\cdot\vec{k}_{t,i}}$$

two-dimensional momentum conservation

Exponentiation in conjugate space; **inverse transform** to move back to direct space

Extremely successful approach

direct QCD

- Catani, Trentadue, Mangano, Marchesini, Webber, Nason, Dokshitzer…
- Collins, Soper, Sterman, Laenen, Magnea…

SCET

- Manohar, Bauer, Stewart, Becher, Neubert….

+ many others!

Emphasis on properties of QCD matrix elements and QCD radiation

Factorization properties in the singular region and associated RGE (factorization → evolution → resummation)

SCET vs. dQCD **not an issue** [Sterman *et al.* '13, '14][Bonvini, Forte, Ghezzi, Ridolfi, LR '12, '13, '14][Becher, Neubert *et al.* '08, '11, 14]

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Limitation: it is process-dependent, and must be performed manually and analytically for each observable (error prone)

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Is it possible to achieve resummation without the need to establish factorization properties on a case-by-case basis?

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Is it possible to achieve resummation without the need to establish factorization properties on a case-by-case basis?

Yes

CAESAR/ARES approach: towards automated resummation

Translate the resummability of the observable into properties of the observable in the presence of multiple radiation: **recursive infrared and collinear (rIRC) safety** [Banfi, Salam, Zanderighi '01, '03, '04]

a) in the presence of multiple soft and/or collinear emissions the observable has the **same scaling properties** as with just one of them
b) there exists a **resolution scale** q_0, **independent of the observable**, such that emissions below q_0 do not contribute significantly to the observable's value.

$$\tilde{\sigma}(v) \sim \int d[k_1] e^{-R(q_0 V(k_1))}$$

$$\times \left(\sum_{m=0}^{\infty} \frac{1}{m!} \int \prod_{i=2}^{m+1} [dk_i] |\mathcal{M}(k_i)|^2 \Theta(V(k_i) - q_0 V(k_1)) \Theta\left(v - V(k_1, \ldots, k_{m+1})\right) \right)$$

Unresolved emission can be treated as **totally uncorrelated** ⟶ **exponentiation**

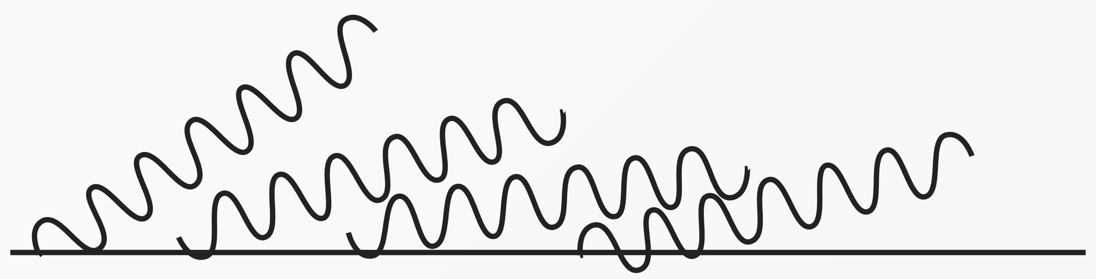
Resolved emission treated exclusively with **Monte Carlo methods**

Method entirely formulated in direct space

The curious case of the transverse momentum

Resummation of transverse momentum is particularly delicate because p_t is a **vectorial quantity**

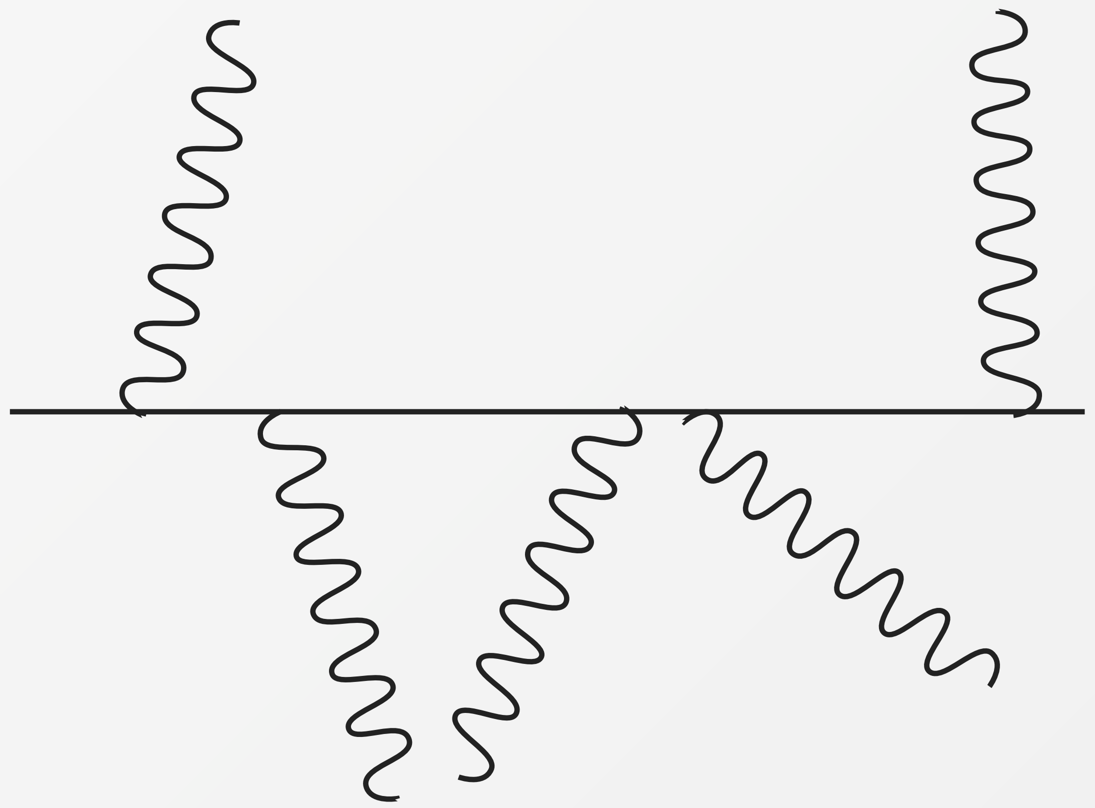
Two concurring mechanisms leading to a system with small p_t



$$p_t^2 \sim k_{t,i}^2 \ll M^2$$

cross section naturally suppressed as there is no phase space left for gluon emission (Sudakov limit)

Exponential suppression



$$\sum_{i=1}^{n} \vec{k}_{t,i} \simeq 0$$

Large kinematic cancellations

p_t ~0 far from the Sudakov limit

Power suppression

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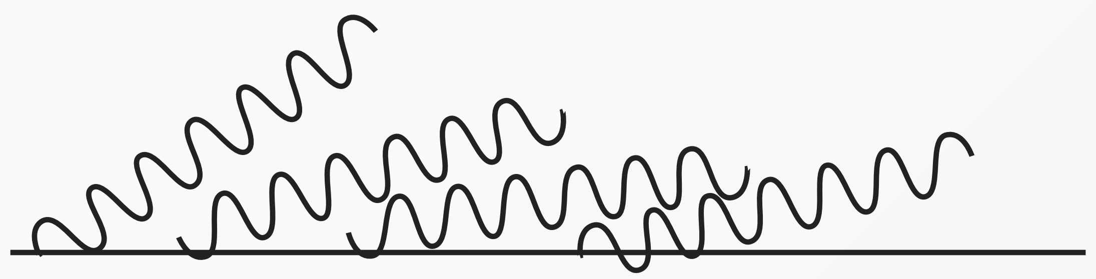

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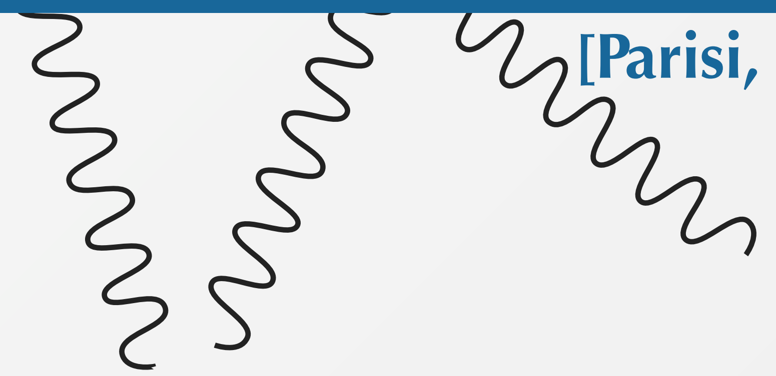
Exponential suppression

Dominant at small p_t



[Parisi, Petronzio '78]

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Power suppression

Resummation in direct space: the p_t case

Non-trivial problem: not possible to find a closed analytic expression in direct space which is both

a) free of logarithmically subleading corrections

b) free of singularities at finite p_t values

[Frixione, Nason, Ridolfi '98]

A naive logarithmic counting at small p_t is not sensible, as one loses the **correct power-suppressed scaling** if only logarithms are retained

It is not possible to reproduce a power-like behaviour with logs of p_t/M

Can we apply the CAESAR method to transverse-momentum resummation?

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Yes!

[Monni, Re, Torrielli '16]
[Bizon, Monni, Re, LR, Torrielli '17]

All-order structure of the matrix element

$v = p_t/M$

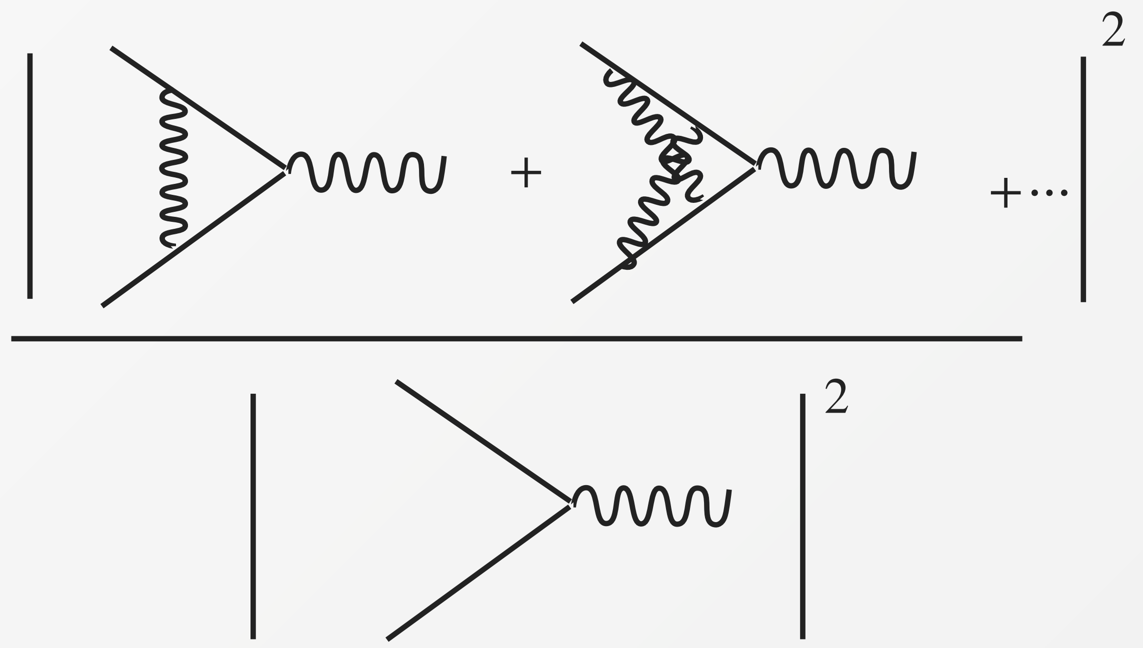
single-particle phase space

matrix element for *n* real emissions

$$\Sigma(v) = \int d\Phi_B \mathcal{V}(\Phi_B) \sum_{n=0}^{\infty} \int \prod_{i=1}^{n} [dk_i] |\mathcal{M}(\Phi_B, k_1, \ldots k_n)|^2 \Theta(v - V(\{\Phi_B\}, k_1, \ldots k_n))$$

all-order form factor (virtuals)

e.g. [Dixon, Magnea, Sterman '08]



Transverse observable resummation with RadISH

1. Establish a **logarithmic counting** for the squared matrix element $|\mathscr{M}(\Phi_B, k_1, \ldots k_n)|^2$

Transverse observable resummation with RadISH

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Decompose the squared amplitude in terms of ***n*-particle correlated blocks,** denoted by $|\tilde{\mathcal{M}}(k_1, \ldots, k_n)|^2$
$(|\tilde{\mathcal{M}}(k_1)|^2 = |\mathcal{M}(k_1)|^2)$

$$\sum_{n=0}^{\infty} |\mathcal{M}(\Phi_B, k_1, \ldots, k_n)|^2 = |\mathcal{M}_B(\Phi_B|^2$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \left\{ \prod_{i=1}^{n} \left(|\mathcal{M}(k_i)|^2 + \int [dk_a][dk_b] |\tilde{\mathcal{M}}(k_a, k_b)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} - \vec{k}_{ti}) \delta(Y_{ab} - Y_i) \right.\right.$$

$$\left.\left. + \int [dk_a][dk_b][dk_c] |\tilde{\mathcal{M}}(k_a, k_b, k_c)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} + \vec{k}_{tc} - \vec{k}_{ti}) \delta(Y_{abc} - Y_i) + \ldots \right) \right\} \equiv |\mathcal{M}_B(\Phi_B)|^2 \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} |\mathcal{M}(k_i)|^2_{\text{inc}}$$

LL

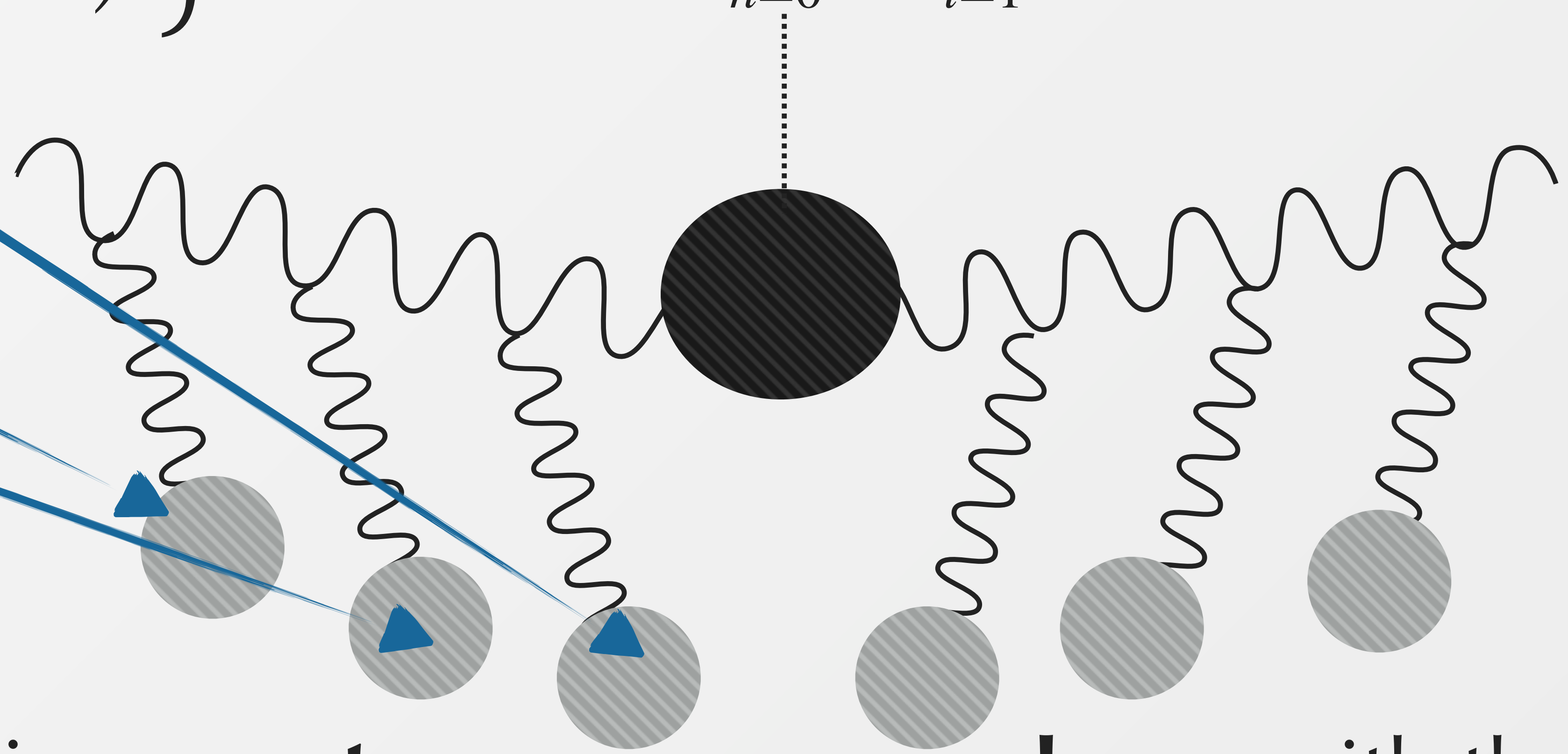
NLL

NNLL

*expression valid for inclusive observables

$$|\tilde{M}(k_1)|^2 = \frac{|M(k_1)|^2}{|M_B|^2} = |M(k_1)|^2$$

$$|\tilde{M}(k_1, k_2)|^2 = \frac{|M(k_1, k_2)|^2}{|M_B|^2} - \frac{1}{2!} |M(k_1)|^2 M|(k_2)|^2$$



Upon integration over the phase space, the expansion can be put in a **one to one correspondence** with the logarithmic structure

Systematic recipe to include terms up to the desired logarithmic accuracy

Transverse observable resummation with RadISH

2. Exploit rIRC safety to single out the IRC singularities of the real matrix element and achieve the **cancellation of the exponentiated divergences** of virtual origin

Resummation in direct space: the p_t case

2. Exploit rIRC safety to single out the IRC singularities of the real matrix element and achieve the **cancellation of the exponentiated divergences** of virtual origin

Introduce a slicing parameter $\epsilon \ll 1$ such that all inclusive blocks with $k_{t,i} < \epsilon k_{t,1}$, with $k_{t,1}$ hardest emission, can be neglected in the computation of the observable

$$\Sigma(v) = \int d\Phi_B |\mathcal{M}_B(\Phi_B)|^2 \mathcal{V}(\Phi_B)$$

unresolved emissions

$$\times \int [dk_1] |\mathcal{M}(k_1)|^2_{\rm inc} \left(\sum_{l=0}^{\infty} \frac{1}{l!} \int \prod_{j=2}^{l+1} [dk_j] |\mathcal{M}(k_j)|^2_{\rm inc} \Theta(\epsilon V(k_1) - V(k_j)) \right)$$

$$\times \left(\sum_{m=0}^{\infty} \frac{1}{m!} \int \prod_{i=2}^{m+1} [dk_i] |\mathcal{M}(k_i)|^2_{\rm inc} \Theta(V(k_i) - \epsilon V(k_1)) \Theta\left(v - V(\Phi_B, k_1, \ldots, k_{m+1})\right) \right)$$

resolved emissions

Unresolved emission doesn't contribute to the evaluation of the observable: it can be exponentiated directly and employed to cancel the virtual divergences, giving rise to a Sudakov radiator

$$\mathcal{V}(\Phi_B) \exp\left\{ \int [dk] |\mathcal{M}(k)|^2_{\rm inc} \Theta(\epsilon V(k_1) - V(k)) \right\} \simeq e^{-R(\epsilon V(k_1))}$$

Resummation in direct space: the p_t case

Result at NLL accuracy can be written as

$$\Sigma(v) = \sigma^{(0)} \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(\epsilon v_1)} R'\left(v_1\right) \qquad v_i = V(k_i), \quad \zeta_i = v_i/v_1$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^{1} \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'\left(\zeta_i v_1\right) \, \Theta\left(v - V(\Phi_B, k_1, \ldots, k_{n+1})\right)$$

Formula can be evaluated with Monte Carlo method; dependence on ϵ vanishes exactly and result is finite in four dimensions

It contains **subleading effect** which in the original CAESAR approach are disposed of by expanding R and R' around v

$$R(\epsilon v_1) = R(v) + \frac{dR(v)}{d\ln(1/v)} \ln \frac{v}{\epsilon v_1} + \mathcal{O}\left(\ln^2 \frac{v}{\epsilon v_1}\right)$$

$$R'\left(v_i\right) = R'(v) + \mathcal{O}\left(\ln \frac{v}{v_i}\right)$$

Not possible! valid only if the ratio v_i/v remains of order one in the whole emission phase space, but for observables which feature kinematic cancellations there are configurations with $v_i \gg v$. **Subleading effects necessary**

Resummation in direct space: the p_t case

Result at NLL accuracy can be written as

$$\Sigma(v) = \sigma^{(0)} \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(\epsilon v_1)} R'\left(v_1\right) \qquad v_i = V(k_i), \quad \zeta_i = v_i/v_1$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'\left(\zeta_i v_1\right) \, \Theta\left(v - V(\Phi_B, k_1, \ldots, k_{n+1})\right)$$

Formula can be evaluated with Monte Carlo method; dependence on ϵ vanishes exactly and result is finite in four dimensions

Convenient to perform an expansion around k_{t1} (more efficient and simpler implementation)

$$R(\epsilon k_{t1}) = R(k_{t1}) + \frac{dR(k_{t1})}{d\ln(1/k_{t1})} \ln\frac{1}{\epsilon} + \mathcal{O}\left(\ln^2\frac{1}{\epsilon}\right)$$

$$R'\left(k_{ti}\right) = R'(k_{t1}) + \mathcal{O}\left(\ln\frac{k_{t1}}{k_{ti}}\right)$$

Subleading effects retained: no divergence at small v, power-like behaviour respected

Logarithmic accuracy defined in terms of $\ln(M/k_{t1})$

Result formally equivalent to the b-space formulation

Resummation at NLL accuracy

Final result at NLL

$$\frac{d\Sigma(v)}{d\Phi_B} = \int \frac{dk_{t,1}}{k_{t,1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(k_{t,1})} \epsilon^{R'(k_{t,1})} \mathscr{L}_{\mathrm{NLL}}(k_{t,1}) R'\left(k_{t,1}\right)$$
$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^{1} \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'\left(\zeta_i k_{t,1}\right) \Theta\left(v - V(\Phi_B, k_1, \ldots, k_{n+1})\right)$$

This formula can be evaluated by means of fast Monte Carlo methods `RadISH (Radiation off Initial State Hadrons)`

Parton luminosity at NLL reads

$$\mathscr{L}_{\mathrm{NLL}}(k_{t,1}) = \sum_c \frac{d|M_B|^2_{c\bar{c}}}{d\Phi_B} f_c\left(x_1, k_{t,1}^2\right) f_{\bar{c}}\left(x_2, k_{t,1}^2\right)$$

At higher logarithmic accuracy, it includes **coefficient functions** and **hard-virtual** corrections

Result at N^{3}LL accuracy

$$
\begin{aligned}
\frac{d\Sigma(v)}{d\Phi_B} &= \int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}\partial_L\left(-e^{-R(k_{t1})}\mathcal{L}_{\mathrm{N^3LL}}(k_{t1})\right)\int d\mathcal{Z}[\{R',k_i\}]\Theta\left(v - V(\{\tilde p\},k_1,\dots,k_{n+1})\right)\\
&+ \int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}e^{-R(k_{t1})}\int d\mathcal{Z}[\{R',k_i\}]\int_0^1\frac{d\zeta_s}{\zeta_s}\frac{d\phi_s}{2\pi}\Bigg\{\left(R'(k_{t1})\mathcal{L}_{\mathrm{NNLL}}(k_{t1}) - \partial_L\mathcal{L}_{\mathrm{NNLL}}(k_{t1})\right)\\
&\times\left(R''(k_{t1})\ln\frac{1}{\zeta_s}+\frac{1}{2}R'''(k_{t1})\ln^2\frac{1}{\zeta_s}\right) - R'(k_{t1})\left(\partial_L\mathcal{L}_{\mathrm{NNLL}}(k_{t1}) - 2\frac{\beta_0}{\pi}\alpha_s^2(k_{t1})\hat P^{(0)}\otimes\mathcal{L}_{\mathrm{NLL}}(k_{t1})\ln\frac{1}{\zeta_s}\right)\\
&+\frac{\alpha_s^2(k_{t1})}{\pi^2}\hat P^{(0)}\otimes\hat P^{(0)}\otimes\mathcal{L}_{\mathrm{NLL}}(k_{t1})\Bigg\}\Big\{\Theta\left(v - V(\{\tilde p\},k_1,\dots,k_{n+1},k_s)\right) - \Theta\left(v - V(\{\tilde p\},k_1,\dots,k_{n+1})\right)\Big\}\\
&+\frac{1}{2}\int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}e^{-R(k_{t1})}\int d\mathcal{Z}[\{R',k_i\}]\int_0^1\frac{d\zeta_{s1}}{\zeta_{s1}}\frac{d\phi_{s1}}{2\pi}\int_0^1\frac{d\zeta_{s2}}{\zeta_{s2}}\frac{d\phi_{s2}}{2\pi}R'(k_{t1})\\
&\times\Bigg\{\mathcal{L}_{\mathrm{NLL}}(k_{t1})\left(R''(k_{t1})\right)^2\ln\frac{1}{\zeta_{s1}}\ln\frac{1}{\zeta_{s2}} - \partial_L\mathcal{L}_{\mathrm{NLL}}(k_{t1})R''(k_{t1})\left(\ln\frac{1}{\zeta_{s1}}+\ln\frac{1}{\zeta_{s2}}\right)\\
&+\frac{\alpha_s^2(k_{t1})}{\pi^2}\hat P^{(0)}\otimes\hat P^{(0)}\otimes\mathcal{L}_{\mathrm{NLL}}(k_{t1})\Bigg\}\\
&\times\Big\{\Theta\left(v - V(\{\tilde p\},k_1,\dots,k_{n+1},k_{s1},k_{s2})\right) - \Theta\left(v - V(\{\tilde p\},k_1,\dots,k_{n+1},k_{s1})\right) -\\
&\Theta\left(v - V(\{\tilde p\},k_1,\dots,k_{n+1},k_{s2})\right) + \Theta\left(v - V(\{\tilde p\},k_1,\dots,k_{n+1})\right)\Big\} + \mathcal{O}\left(\alpha_s^n\ln^{2n-6}\frac{1}{v}\right), \qquad (3.18)
\end{aligned}
$$

[Bizon, Monni, Re, LR, Torrielli '17]

All ingredients to perform resummation at **N^{3}LL accuracy** are now available

[Catani *et al.* '11, '12][Gehrmann *et al.* '14][Li, Zhu '16, Vladimirov '16][Moch *et al.* '18, Lee *et al.* '19]

Fixed-order predictions now available at **NNLO**

[A. Gehrmann-De Ridder *et al.* '15, 16, '17][Boughezal *et al.* '15, 16]

Matching to fixed order: multiplicative matching

Cumulative cross section should reduce to the fixed order at large v

$$\Sigma^{\text{mult}}_{\text{matched}}(v) \sim \Sigma_{\text{res}}(v) \left[\frac{\Sigma_{\text{f.o.}}(v)}{\Sigma_{\text{res}}(v)} \right]_{\text{expanded}}$$

$$\Sigma_{\text{f.o}}(v) = \sigma_{f.o.} - \int_v^{\infty} \frac{d\sigma}{dv} dv$$

- allows to include constant terms from NNLO (if N[3]LO total xs available)
- physical suppression at small v cures potential instabilities

To ensure that resummation does not affect the hard region of the spectrum when the matching is performed we introduce **modified logarithms**

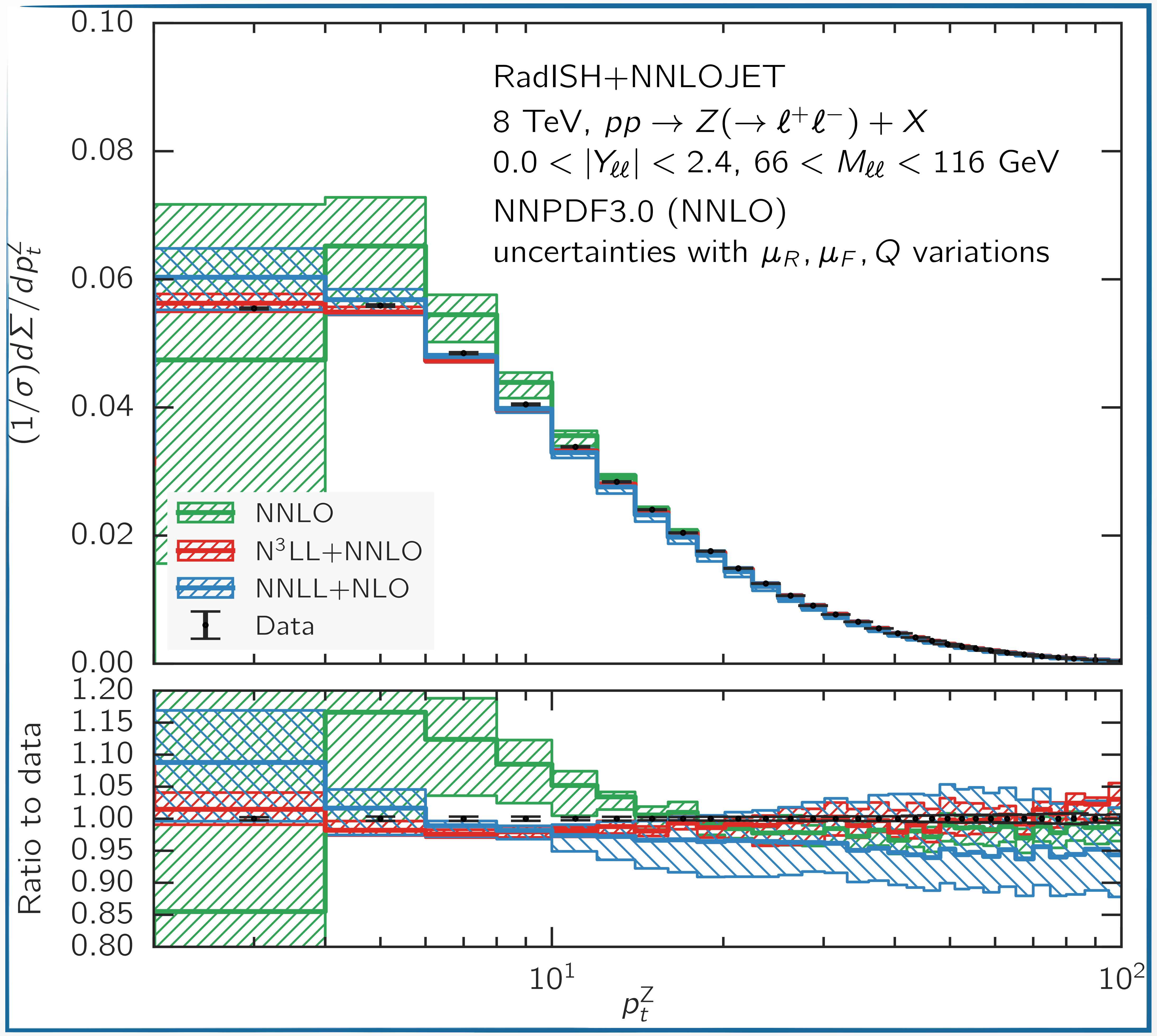
This corresponds to restrict the rapidity phase space at large k_t

$$\ln(Q/k_{t1}) \to \frac{1}{p} \ln \left(1 + \left(\frac{Q}{k_{t1}} \right)^p \right)$$

Q : **perturbative resummation scale** used to probe the size of subleading logarithmic corrections

p : arbitrary matching parameter

Predictions for the Z spectrum at 8 TeV



- Good description of the data in all fiducial regions
- Perturbative uncertainty at the few percent level, still does not match the precision of the ATLAS data

Theoretical predictions for Z and W observables at 13 TeV

Bizon, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Monni, Re, LR, Walker, 190x.xxxx

Results obtained using the following fiducial cuts (agreed with ATLAS)

$$p_t^{\ell^\pm} > 25\,\text{GeV}, \quad |\eta^{\ell^\pm}| < 2.5, \quad 66\,\text{GeV} < M_{\ell\ell} < 116\,\text{GeV}$$

$$p_t^{\ell} > 25\,\text{GeV}, \quad |\eta^{\ell}| < 2.5, \quad \not{E}_T^{\nu_\ell} > 25\,\text{GeV}, \quad m_T > 50\,\text{GeV}$$

using NNPDF3.1 with $\alpha_s(M_Z)$=0.118 and setting the central scales to

$$\mu_R = \mu_F = M_T = \sqrt{M_{\ell\ell'}^2 + p_T^2}, \quad Q = \frac{M_{\ell\ell'}}{2}$$

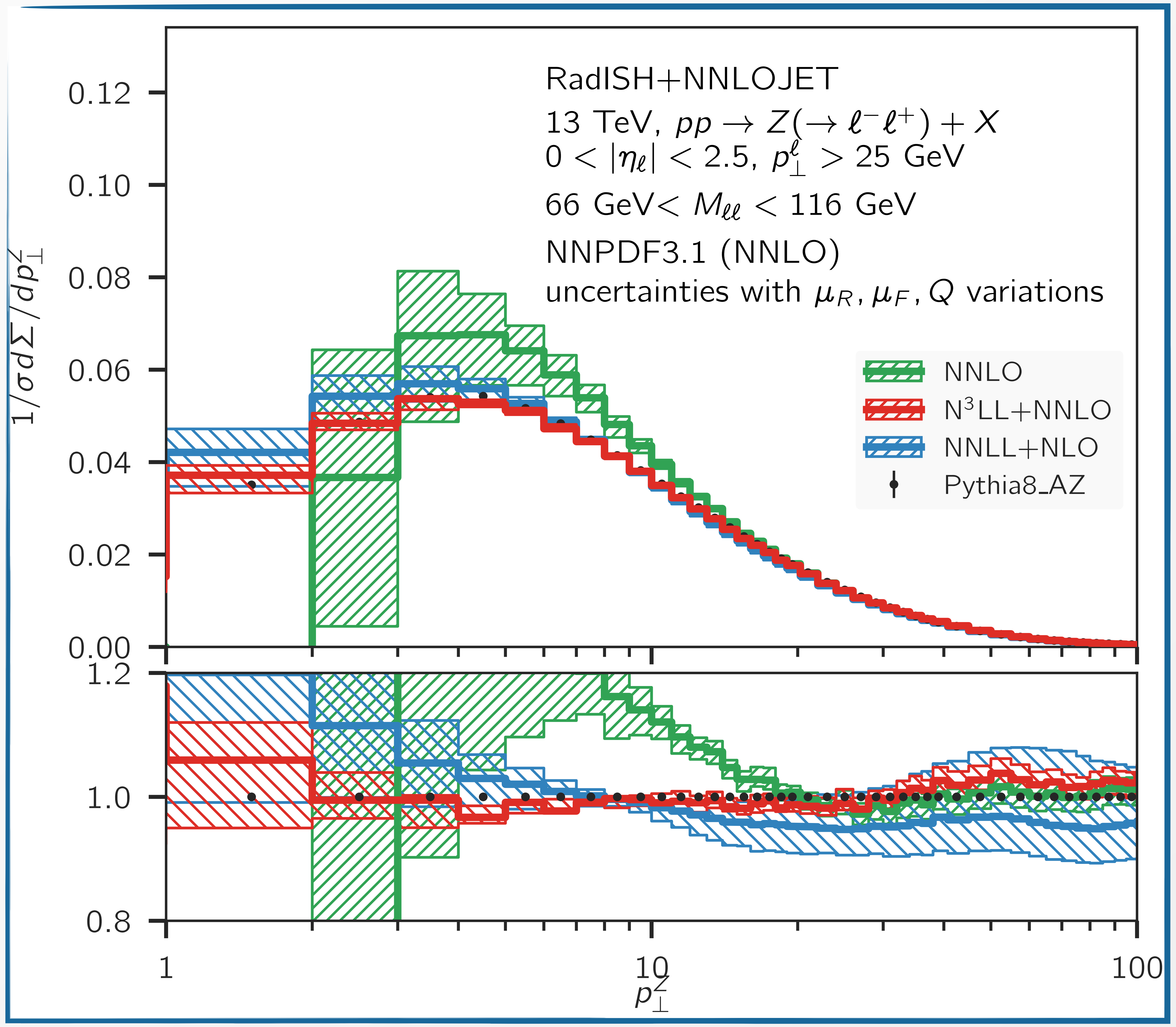
5 flavour (massless) scheme: no HQ effects, LHAPDF PDF thresholds

Scale uncertainties estimated by varying **renormalization** and **factorization** scale by a factor of two around their central value (**7 point variation**) and varying the **resummation** scale by a factor of 2 around its central value for factorization and renormalization scales set to their central value: **9 point envelope**

Matching parameter p set to 4 as a default

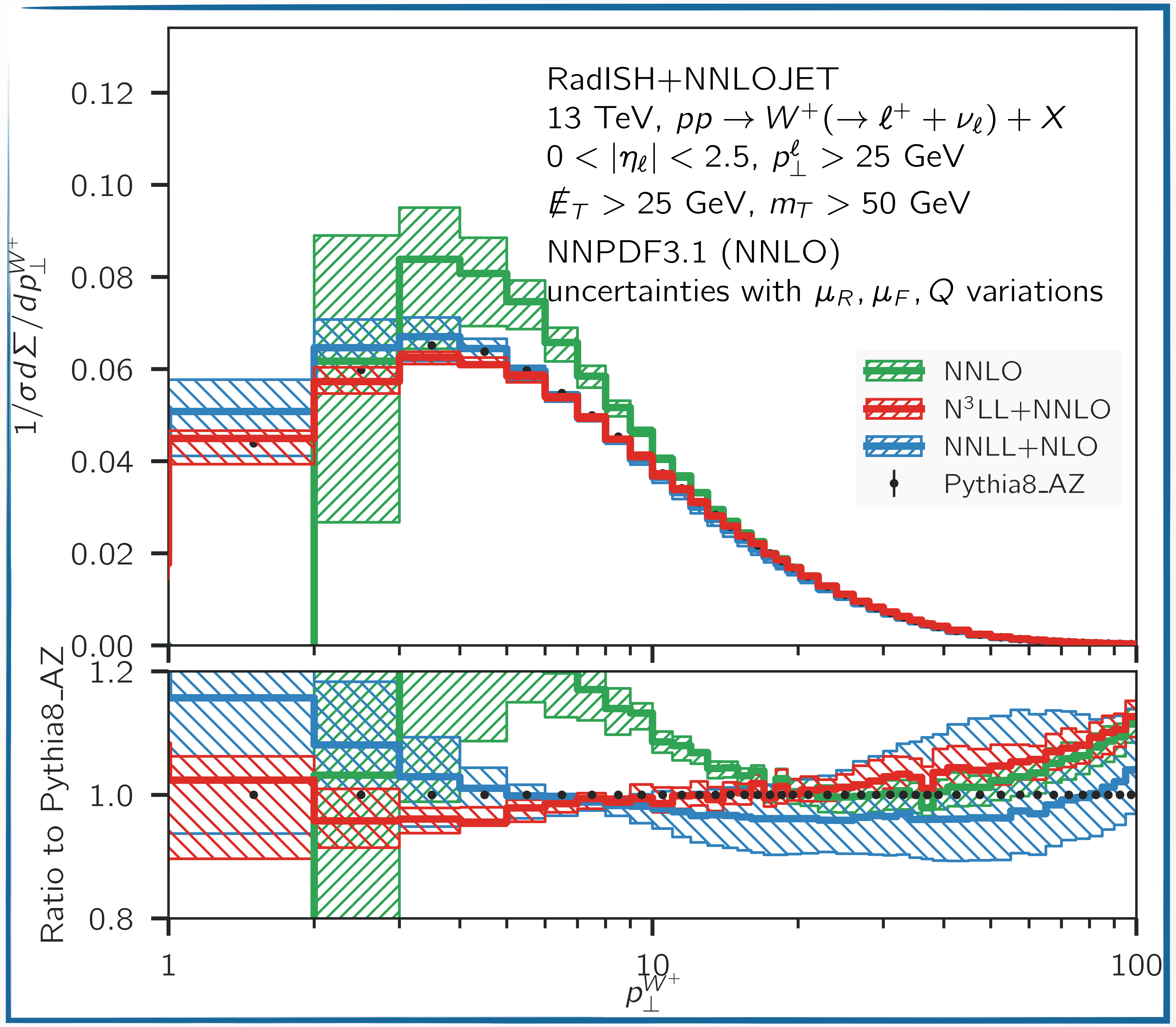
No non perturbative parameters included in the following

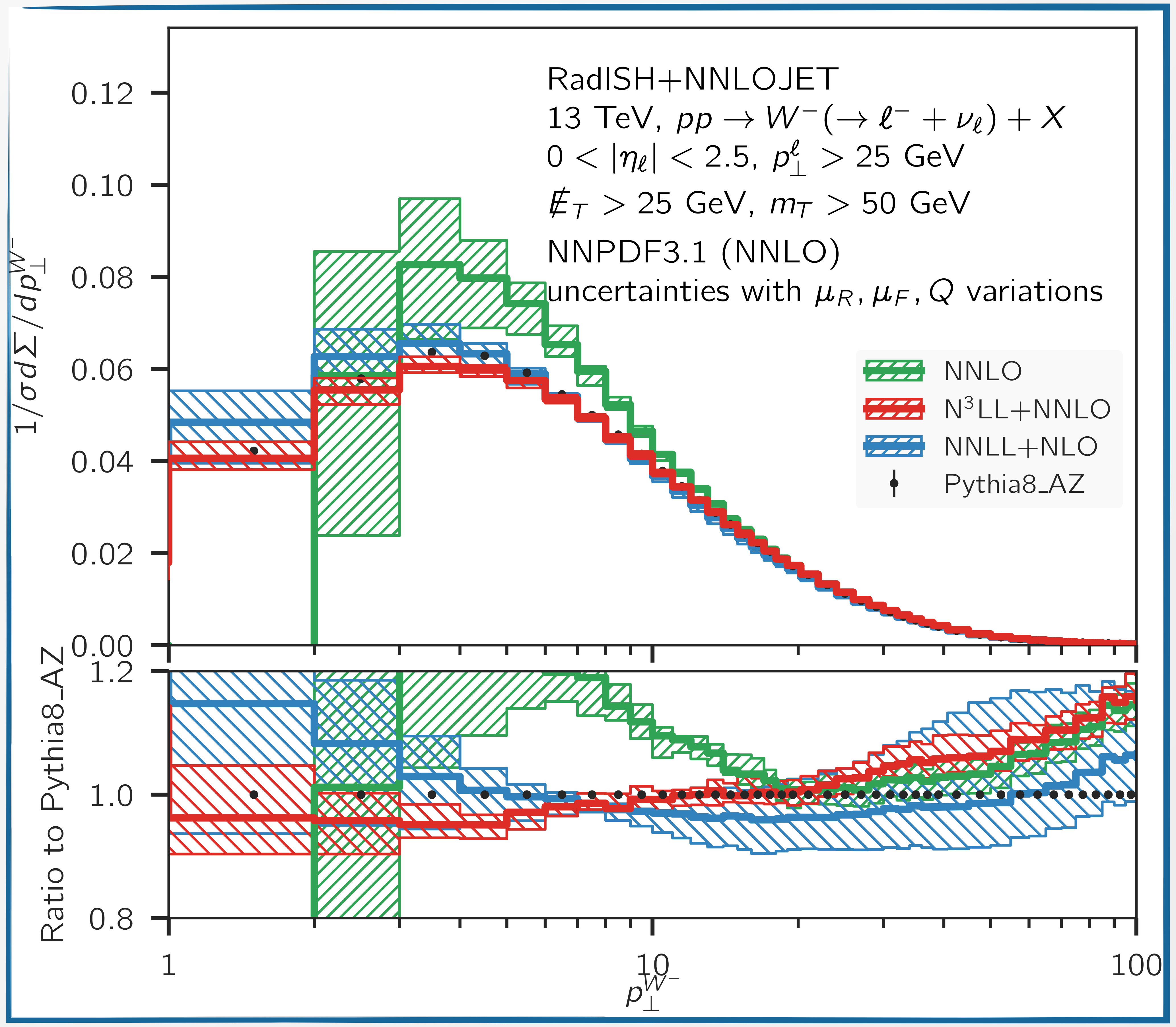
Predictions for the Z spectrum



Thanks to Jan Kretzschmar for providing the PYTHIA8 AZ tune results

Predictions for the W^+ and W^- spectra





Ratio of differential distributions

Z and W production share a similar pattern of QCD radiative corrections

Crucial to understand correlation between Z and W spectra to exploit data-driven predictions

$$\frac{1}{\sigma^W}\frac{d\sigma^W}{p_\perp^W} \sim \frac{1}{\sigma^Z_{\mathrm{data}}}\frac{d\sigma^Z_{\mathrm{data}}}{p_\perp^Z}\,\frac{\frac{1}{\sigma^W_{\mathrm{theory}}}\frac{d\sigma^W_{\mathrm{theory}}}{p_\perp^W}}{\frac{1}{\sigma^Z_{\mathrm{theory}}}\frac{d\sigma^Z_{\mathrm{theory}}}{p_\perp^Z}}$$

Several choices are possible:

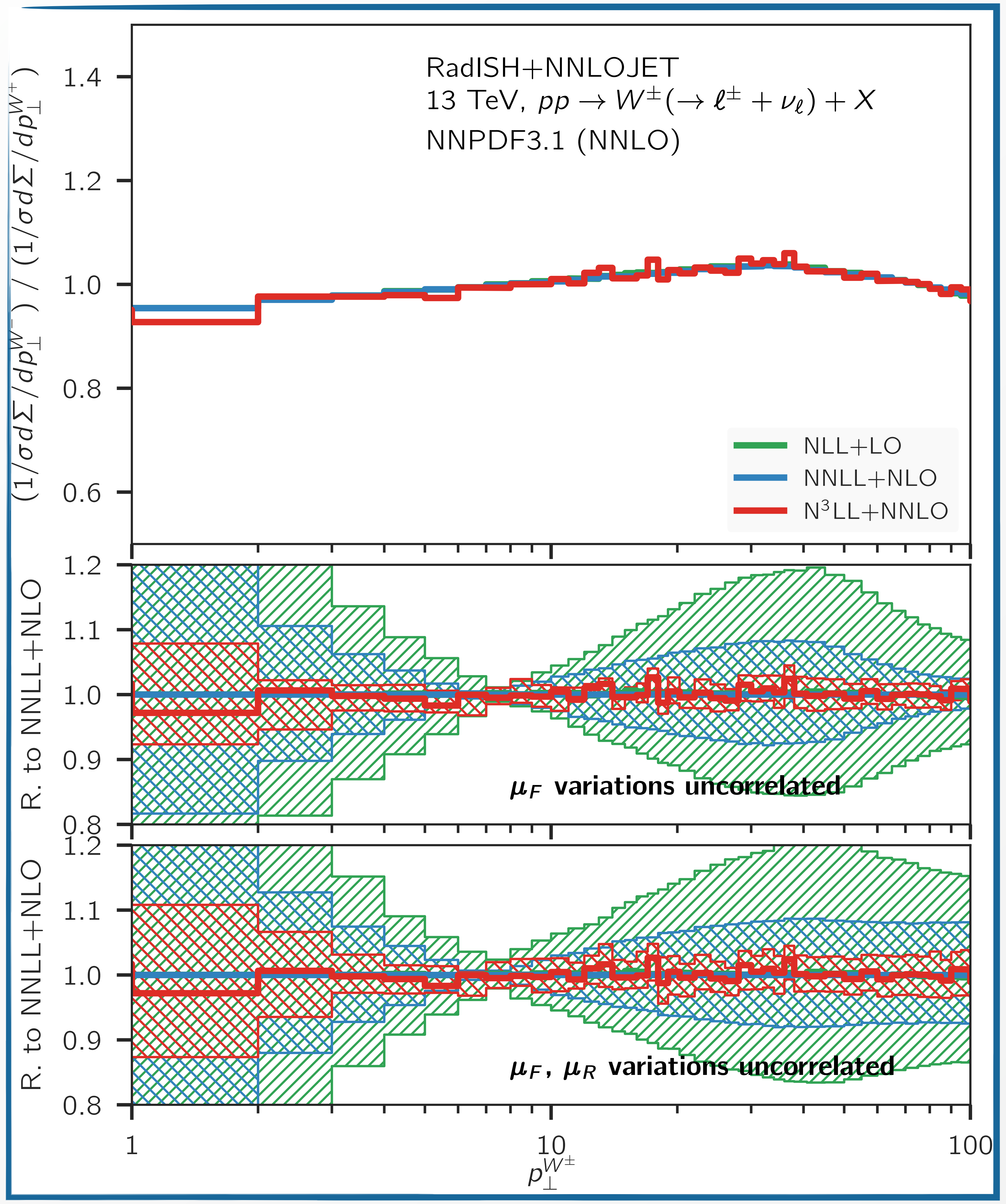
- **Correlate resummation and renormalisation** scale variations, keep **factorisation scale uncorrelated**, while keeping

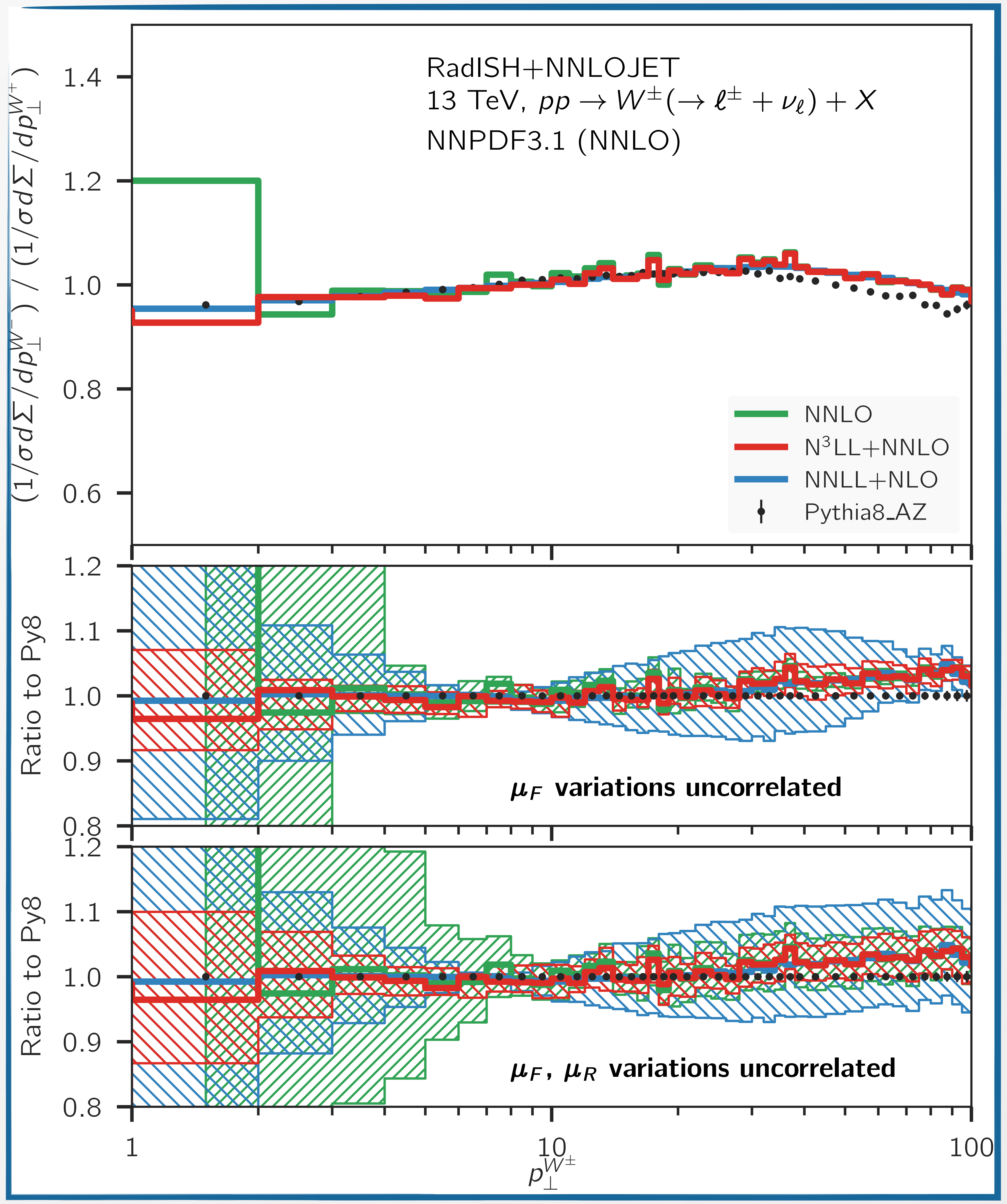
$$\frac{1}{2} \leq \frac{\mu_{\mathrm{F}}^{\mathrm{num}}}{\mu_{\mathrm{F}}^{\mathrm{den}}} \leq 2$$

- More **conservative** estimate: vary both **renormalisation and factorisation scales in an uncorrelated** way with

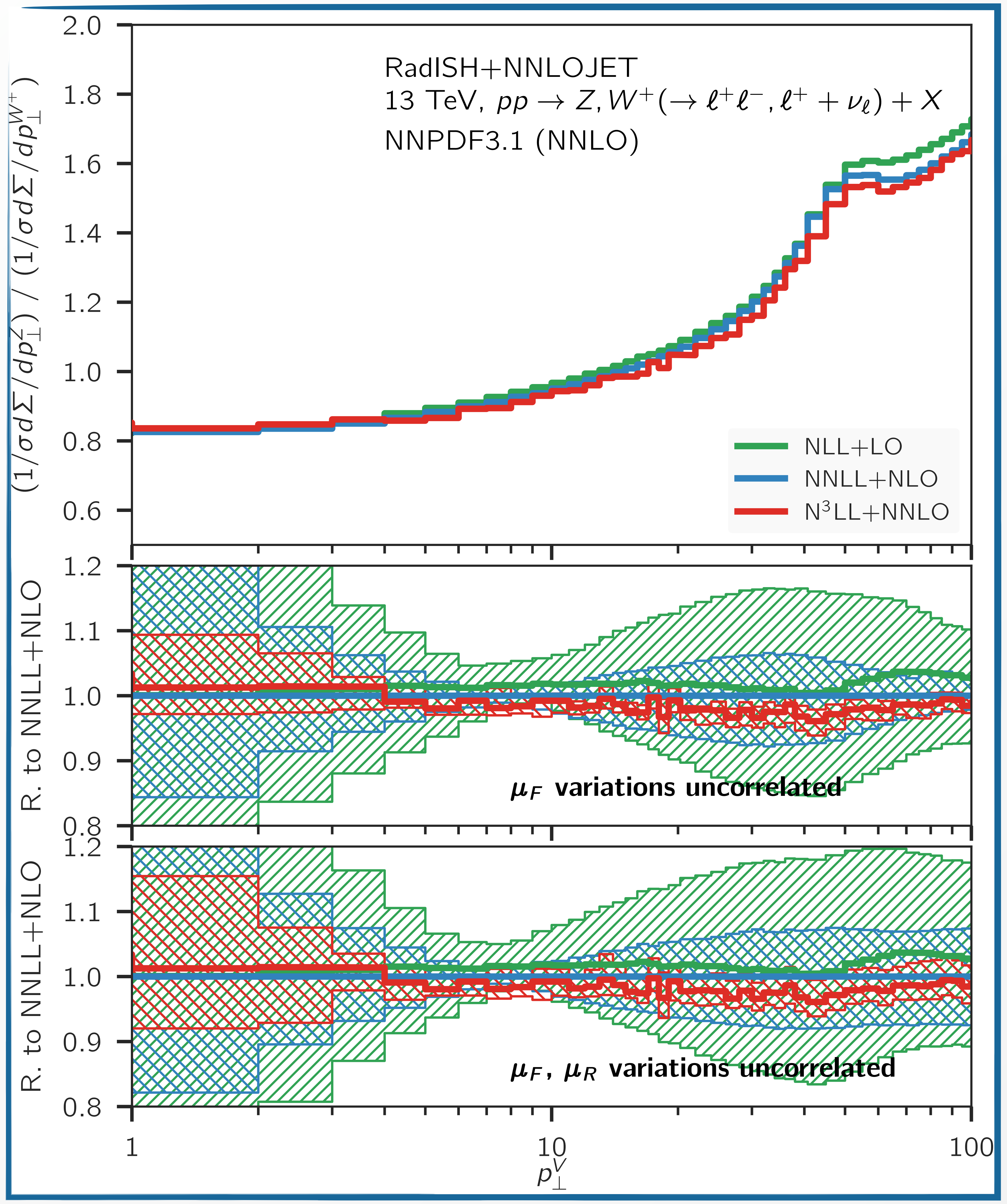
$$\frac{1}{2} \leq \frac{\mu^{\mathrm{num}}}{\mu^{\mathrm{den}}} \leq 2$$

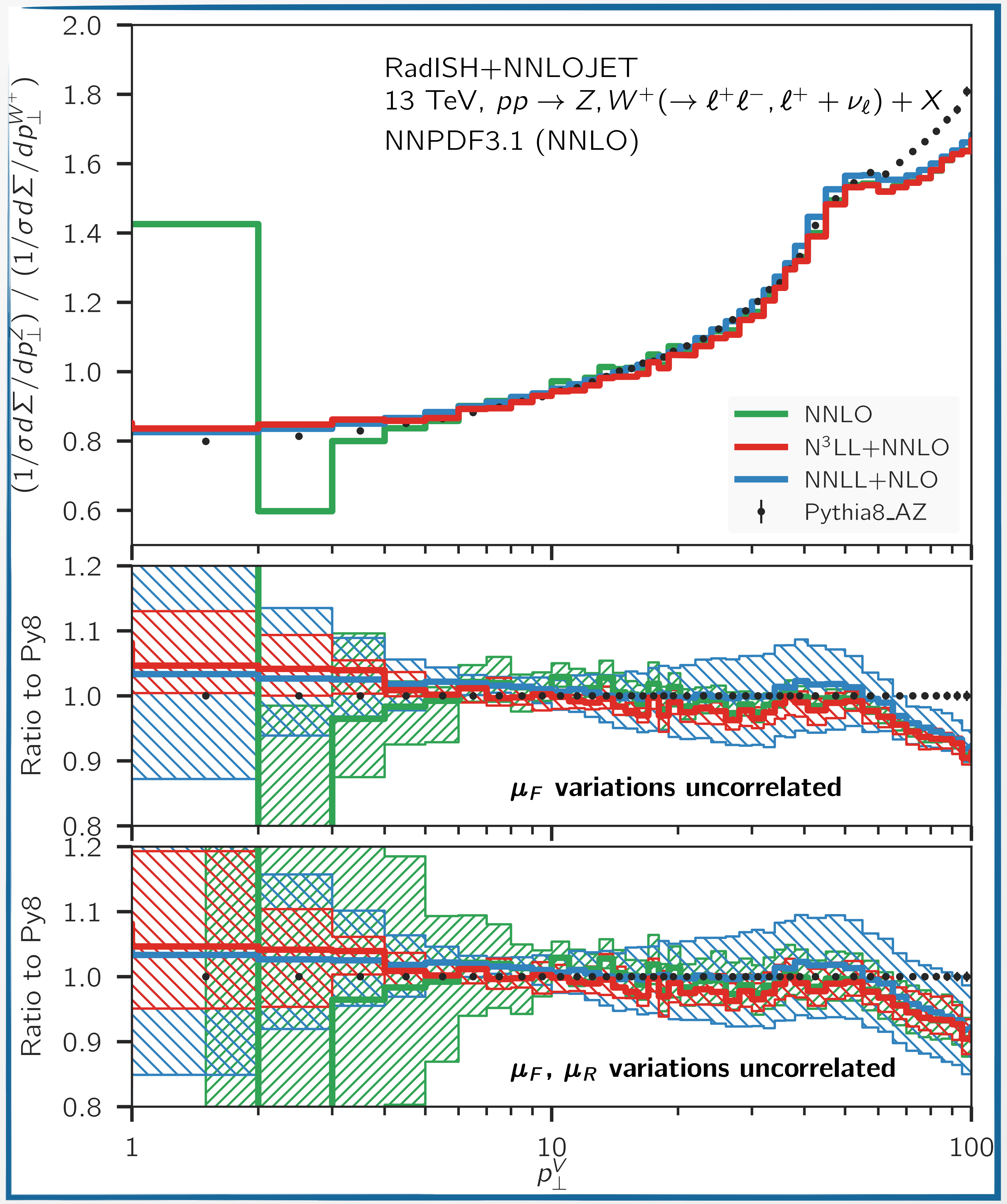
Results for W^-/W^+ ratio





Results for Z/W^+ ratio





Recapitulation

- Perturbation theory must be pushed to its limit to reduce the theory uncertainty to match the precision of the data.

- New formalism formulated in **direct space** for all-order resummation up to **N^3LL accuracy** for inclusive, transverse observables.

- Preliminary results at NNLO+N^3LL for *W* and *Z* differential distributions with **uncertainties at the few percent level**. Some discrepancies with the Pythia8 AZ tune results to be understood. Monte Carlo tunes for sub-percent precision must be handled with care.

- Preliminary results on the W^+/W^- ratios and *Z*/*W* ratios. Large correlations are observed between $\boldsymbol{W^+}$**,** $\boldsymbol{W^-}$**, and** $\boldsymbol{Z}$ production in massless QCD

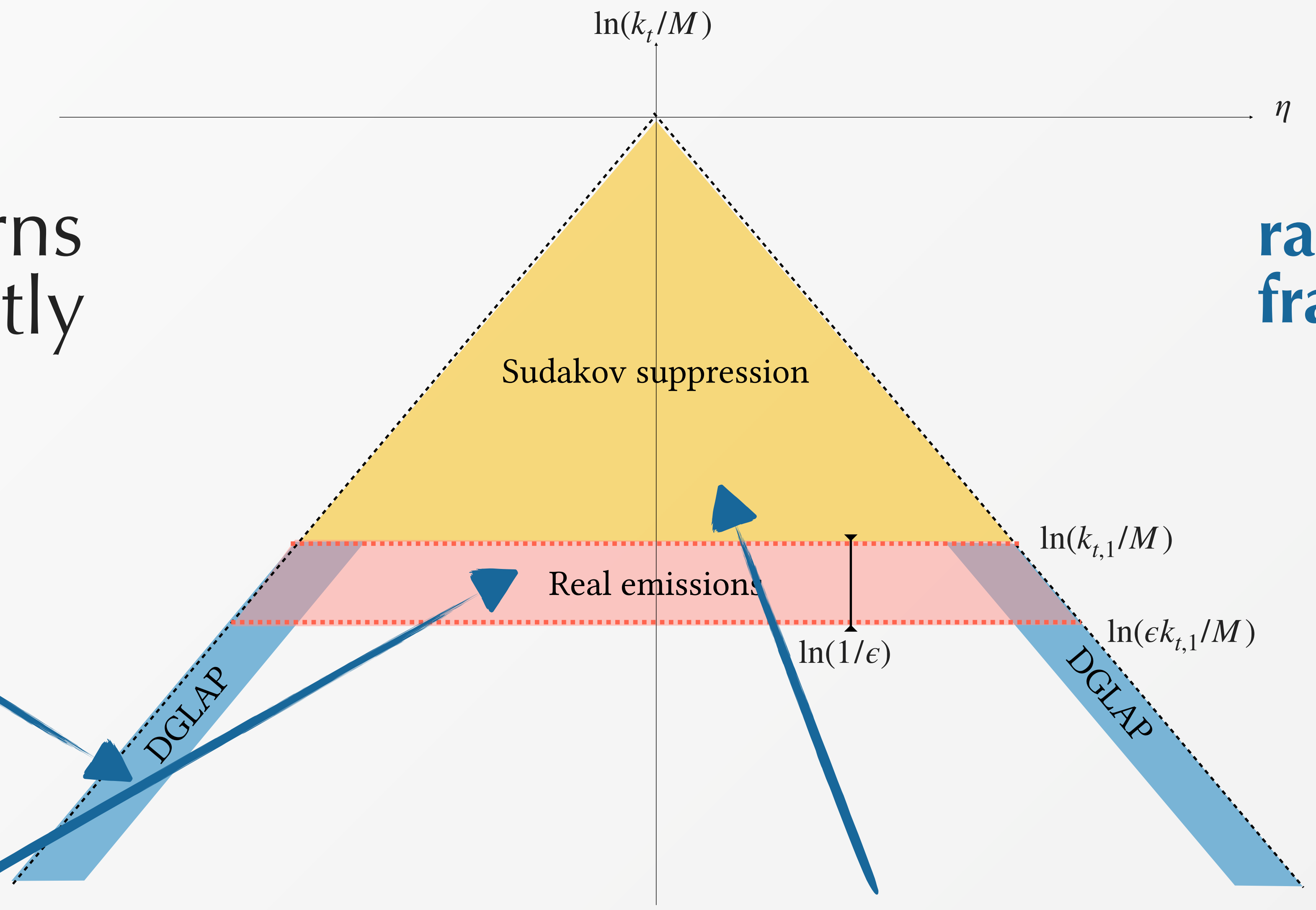
Backup

Parton luminosities

Consider configurations in which emissions are ordered in $k_{t,i}$, $k_{t,1}$ hardest emission

Phase space for each secondary emission can be depicted in the Lund diagram

DGLAP evolution governs the radiation in the strictly collinear limit



rapidity in the centre-of-mass frame of the incoming partons

resolved emissions live in this strip

remaining **unresolved real emissions** are combined with the virtual corrections to give rise to Sudakov suppression

- DGLAP evolution can be performed **inclusively** up to $\epsilon k_{t,1}$ thanks to rIRC safety
- In the **overlapping region** hard-collinear emissions modify the observable's value: the evolution should be performed exclusively (unintegrated in k_t)
- At NLL the real radiation can be approximated with its soft limit: DGLAP can be performed inclusively up to $k_{t,1}$ (i.e. one can evaluate $\mu_F = k_{t,1}$)

Beyond NLL

Extension to NNLL and beyond requires the systematic inclusion of the correlated blocks necessary to achieve the desired logarithmic accuracy

Moreover, one needs to **relax a series of assumptions** which give rise to subleading corrections neglected at NLL (for instance, exact rapidity bounds). These corrections can be included systematically by including additional terms in the expansion

$$R(\epsilon v_1) = R(v_1) + \frac{dR(v_1)}{d\ln(1/v_1)} \ln\frac{1}{\epsilon} + \mathcal{O}\left(\ln^2\frac{1}{\epsilon}\right)$$

Finally, one needs to specify a complete treatment for **hard-collinear radiation.** Starting at NNLL one or more real emissions can be hard and collinear to the emitting leg, and the available phase space for subsequent real emissions changes

Two classes of contributions:

- one soft by construction and which is analogous to the R′ contribution

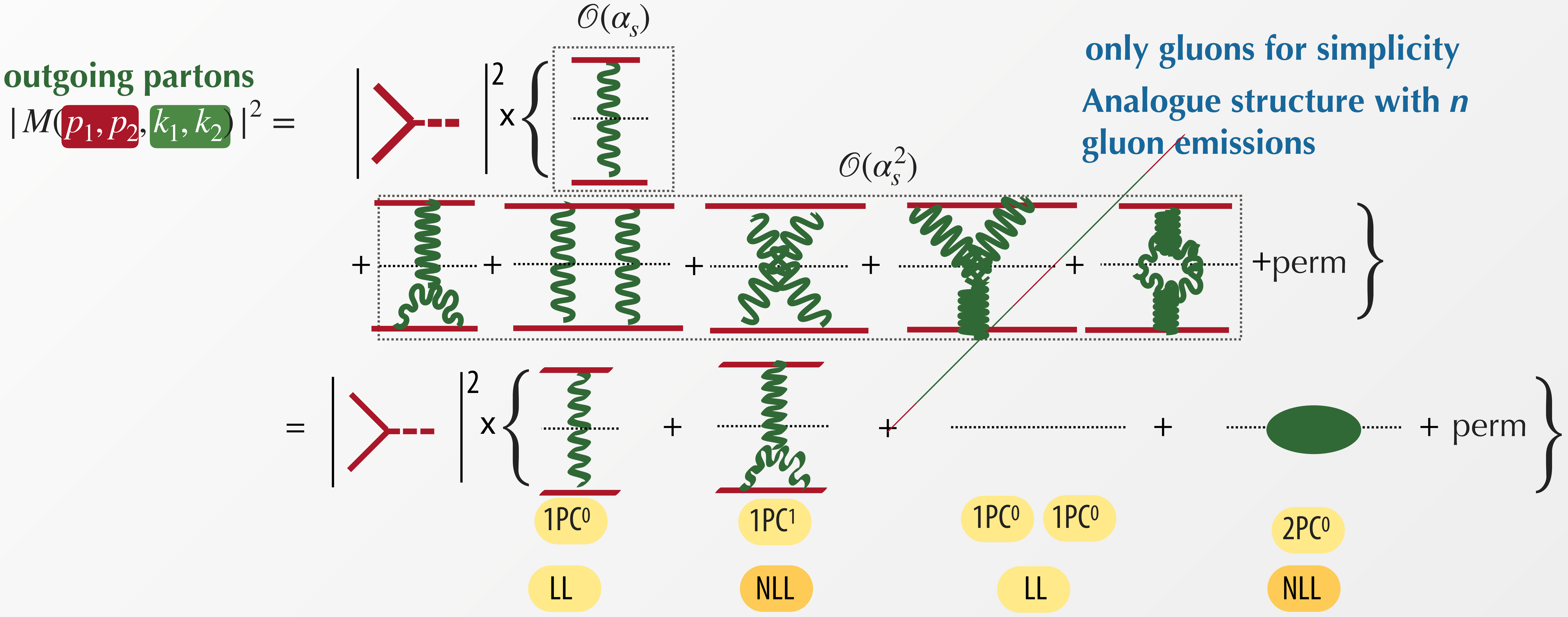
$$R'\left(v_i\right) = R'(v_1) + \mathcal{O}\left(\ln\frac{v_1}{v_i}\right)$$

- another hard and collinear (exclusive DGLAP step): last step of DGLAP evolution must be performed unintegrated in k_t

Logarithmic counting

Necessary to establish a **well defined logarithmic counting**: possibile to do that by decomposing the squared amplitude in terms of n-particle correlated blocks (**nPC**)

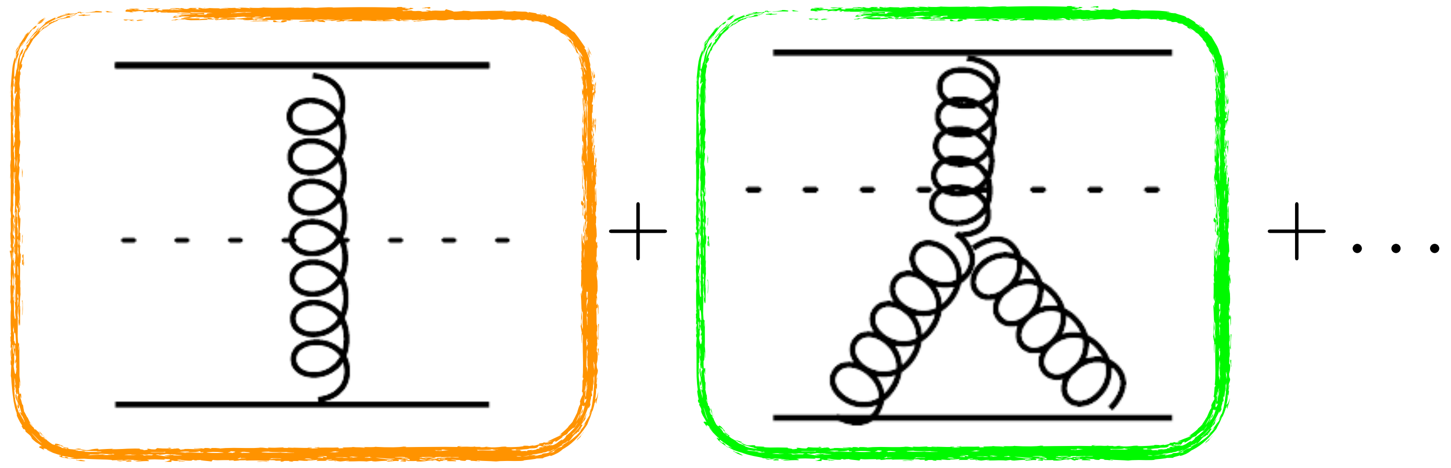
e.g. $pp \to H$ + emission of up to 2 (soft) gluons $O(\alpha_s^2)$



Logarithmic counting defined in terms of **nPC blocks** (owing to rIRC safety of the observable)

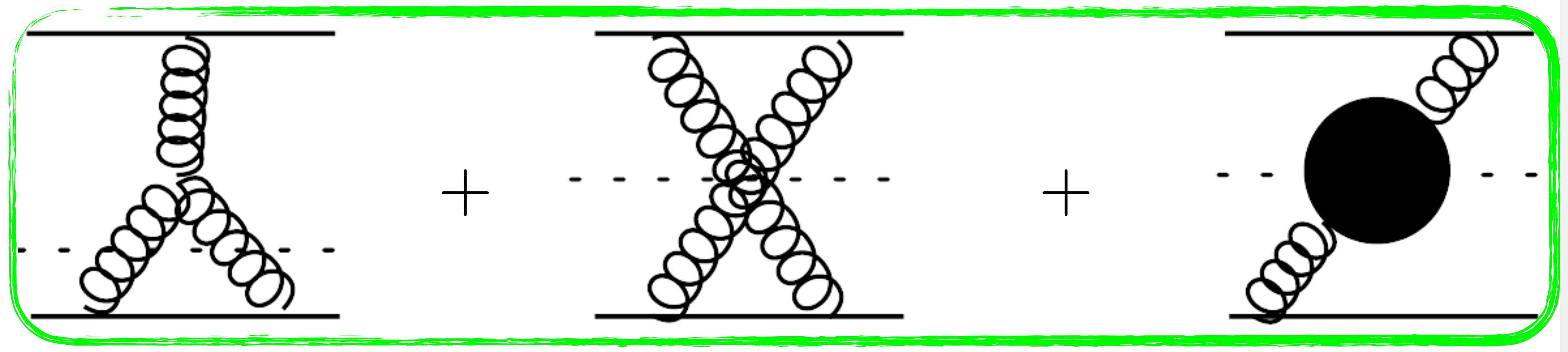
Logarithmic counting: correlated blocks

$$|\tilde{M}(k_a)|^2 = \frac{|M(\tilde{p}_1, \tilde{p}_2, k_a)|^2}{|M_B(\tilde{p}_1, \tilde{p}_2)|^2} = |M(k_a)|^2 \longrightarrow$$



$\alpha_s L^2$

$$|\tilde{M}(k_a, k_b)|^2 = \frac{|M(\tilde{p}_1, \tilde{p}_2, k_a, k_b)|^2}{|M_B(\tilde{p}_1, \tilde{p}_2)|^2} - \underbrace{\frac{1}{2!}|M(k_a)|^2 |M(k_b)|^2}_{\alpha_s^2 L^4} \longrightarrow$$



$\alpha_s^2 L^3 + \alpha_s^2 L^2$ $+ \dots$

15 this LL is absorbed in the resummation of $|M(k)|^2$

Thanks to P. Monni

Equivalence with *b*-space formulation

$$\frac{d\Sigma(v)}{d\Phi_B} = \int_{\mathscr{C}_1} \frac{dN_1}{2\pi i} \int_{\mathscr{C}_2} \frac{dN_2}{2\pi i} x_1^{-N_1} x_2^{-N_2} \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \mathbf{f}^T_{N_1}(\mu_0) \hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) \mathbf{f}_{N_2}(\mu_0)$$

unresolved emission + virtual corrections

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi}$$

$$\times e^{-\mathbf{R}(\epsilon k_{t1})} \exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$

resolved emission

$$\sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi} \mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \sum_{\ell_i=1}^{2}\left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi} \mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right)$$

$$\times \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right)$$

Result valid for all inclusive observables (e.g. p_t, φ^*)

Formulation **equivalent to *b*-space** result (up to a **scheme change** in the anomalous dimensions)

$$\frac{d^2\Sigma(v)}{d\Phi_B dp_t} = \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \int b\, db\, p_t J_0(p_t b)\, \mathbf{f}^T(b_0/b) \mathbf{C}^{c_1;T}_{N_1}(\alpha_s(b_0/b)) H(M) \mathbf{C}^{c_2}_{N_2}(\alpha_s(b_0/b)) \mathbf{f}(b_0/b)$$

$$\times \exp\left\{-\sum_{\ell=1}^{2} \int_0^M \frac{dk_t}{k_t} \mathbf{R}'_\ell(k_t)(1 - J_0(bk_t))\right\}$$

$$(1 - J_0(bk_t)) \simeq \Theta(k_t - \frac{b_0}{b}) + \frac{\zeta_3}{12} \frac{\partial^3}{\partial \ln(Mb/b_0)^3} \Theta(k_t - \frac{b_0}{b})$$

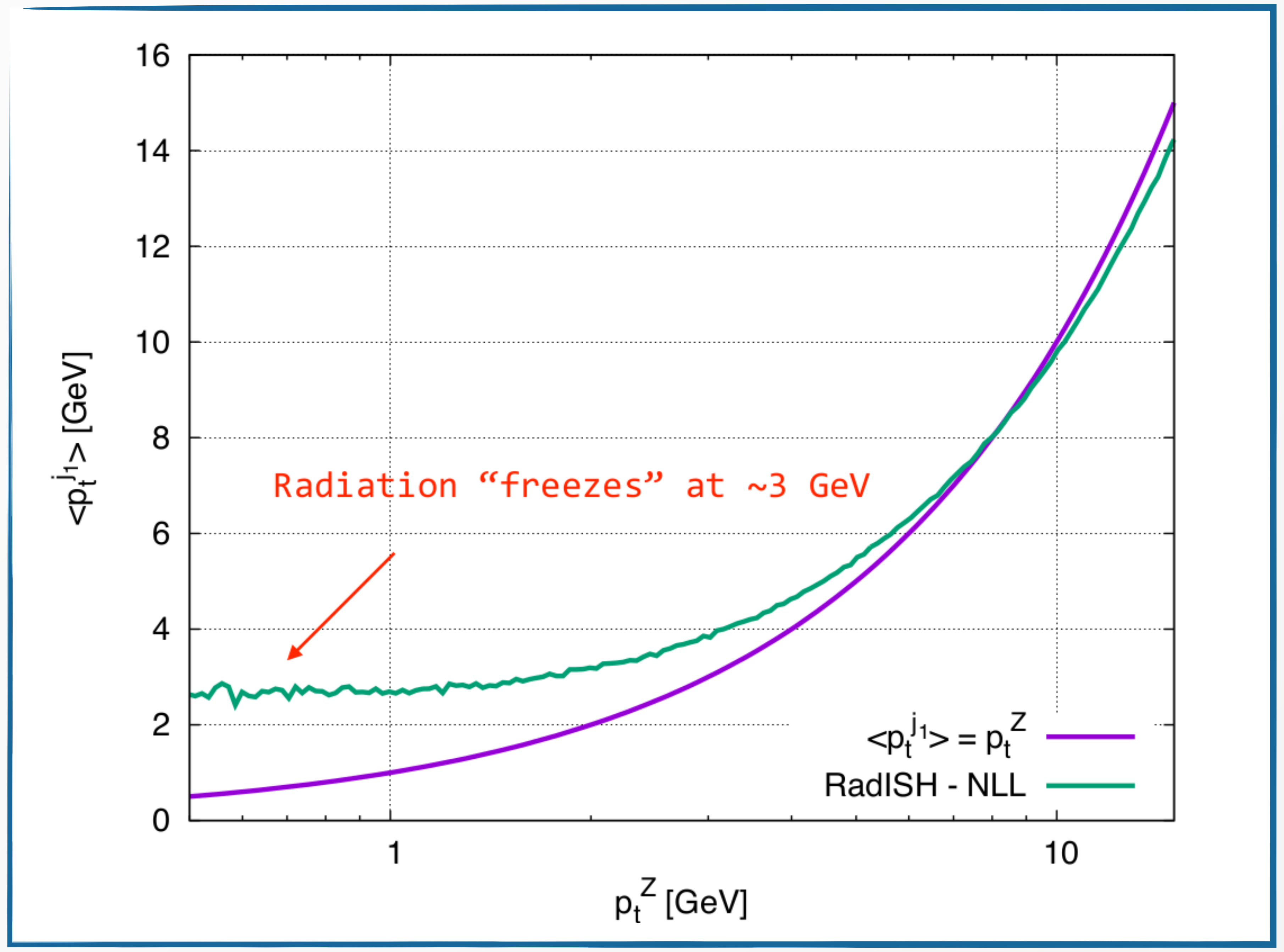
N^3LL effect: absorbed in the definition of H_2, B_3, A_4 coefficients wrt to CSS

The Landau pole and the small p_T limit

Running coupling $\alpha_s(k_{t1}{}^2)$ and Sudakov radiator hit Landau pole at

$$\alpha_s(\mu_R^2)\beta_0 \ln Q/k_{t1} = \frac{1}{2} \qquad k_{t1} \sim 0.01\,\mathrm{GeV}, \quad \mu_R = Q = m_Z$$

Only real cutoff in the calculation: emission probability is set to zero below this scale and parton densities are frozen.



At small p_t the large azimuthal cancellations dominate over the Sudakov suppression: the cutoff is never an issue in practice

$$\frac{d^2\Sigma(v)}{dp_t d\Phi_B} \simeq 2\sigma^{(0)}(\Phi_B)p_t\left(\frac{\Lambda^2_{\mathrm{QCD}}}{M^2}\right)^{\frac{16}{25}\ln\frac{41}{16}}$$

Thanks to P. Monni

Behaviour at small p_t

Explicit evaluation shows that the Parisi-Petronzio perturbative scaling at small p_t is reproduced. At NLL, Drell-Yan pair production, n_f=4

$$\frac{d^2\Sigma(v)}{dp_t d\Phi_B} = 4\,\sigma^{(0)}(\Phi_B)\,p_t \int_{\Lambda_{\rm QCD}}^{M} \frac{dk_{t1}}{k_{t1}^3} e^{-R(k_{t1})} \simeq 2\sigma^{(0)}(\Phi_B) p_t \left(\frac{\Lambda_{\rm QCD}^2}{M^2}\right)^{\frac{16}{25}\ln\frac{41}{16}}$$

As now higher logarithmic terms (up to N^3LL) are under control, the coefficient of this scaling can be systematically improved in *perturbation* theory (non-perturbative effects – of the same order – not considered)

N^3LL calculation allows one to have control over the terms of relative order $O(\alpha_s^2)$. Scaling $L \sim 1/\alpha_s$ valid in the deep infrared regime.

Numerical implementation

$$\frac{d\Sigma(p_t)}{d\Phi_B} = \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} \partial_L \left(-e^{-R'(k_{t1})} \mathcal{L}_{\rm NLL}(k_{t1})\right) \times$$
$$\times \underbrace{\epsilon^{R'(k_{t1})} \sum_{n=0}^{\infty} \frac{1}{n!} \left(\prod_{i=2}^{n+1} \int_{\epsilon k_{t1}}^{k_{t1}} \frac{dk_{ti}}{k_{ti}} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'(k_{t1})\right) \Theta(p_t - |\vec{k}_{t1} + \ldots + \vec{k}_{t(n+1)}|)}_{\equiv \int d\mathcal{Z}[\{R', k_i\}] \Theta(p_t - |\vec{k}_{t1} + \ldots + \vec{k}_{t(n+1)}|)} .$$

- $L = \ln(M/k_{t1})$; luminosity $\mathcal{L}_{\rm NLL}(k_{t1}) = \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1 c_2}}{d\Phi_B} f_{c_1}(x_1, k_{t1}) f_{c_2}(x_2, k_{t1})$.
- $\int d\mathcal{Z}[\{R', k_i\}]\Theta$ finite as $\epsilon \to 0$:

$$\epsilon^{R'(k_{t1})} = 1 - R'(k_{t1}) \ln(1/\epsilon) + \ldots = 1 - \int_{\epsilon k_{t1}}^{k_{t1}} R'(k_{t1}) + \ldots,$$

$$\int d\mathcal{Z}[\{R', k_i\}]\Theta = \left[1 - \int_{\epsilon k_{t1}}^{k_{t1}} R'(k_{t1}) + \ldots\right] \left[\Theta(p_t - |\vec{k}_{t1}|) + \int_{\epsilon k_{t1}}^{k_{t1}} R'(k_{t1}) \Theta(p_t - |\vec{k}_{t1} + \vec{k}_{t2}|) + \ldots\right]$$
$$= \Theta(p_t - |\vec{k}_{t1}|) + \underbrace{\int_0^{k_{t1}}}_{\epsilon \to 0} R'(k_{t1}) \underbrace{\left[\Theta(p_t - |\vec{k}_{t1} + \vec{k}_{t2}|) - \Theta(p_t - |\vec{k}_{t1}|)\right]}_{\text{finite: real-virtual cancellation}} + \ldots$$

- Evaluated with Monte Carlo techniques: $\int d\mathcal{Z}[\{R', k_i\}]$ is generated as a parton shower over secondary emissions.

Thanks to P. Torielli

Numerical implementation

- Secondary radiation:

$$
\begin{aligned}
d\mathcal{Z}[\{R', k_i\}] &= \sum_{n=0}^{\infty} \frac{1}{n!} \left(\prod_{i=2}^{n+1} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \int_{\epsilon k_{t1}}^{k_{t1}} \frac{dk_{ti}}{k_{ti}} R'(k_{t1}) \right) \epsilon^{R'(k_{t1})} \\
&= \sum_{n=0}^{\infty} \left(\prod_{i=2}^{n+1} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \int_{\epsilon k_{t1}}^{k_{t(i-1)}} \frac{dk_{ti}}{k_{ti}} R'(k_{t1}) \right) \epsilon^{R'(k_{t1})}, \\
\epsilon^{R'(k_{t1})} &= e^{-R'(k_{t1}) \ln 1/\epsilon} = \prod_{i=2}^{n+2} e^{-R'(k_{t1}) \ln k_{t(i-1)}/k_{ti}},
\end{aligned}
$$

with $k_{t(n+2)} = \epsilon k_{t1}$.

- Each secondary emissions has differential probability

$$
dw_i = \frac{d\phi_i}{2\pi} \frac{dk_{ti}}{k_{ti}} R'(k_{t1}) e^{-R'(k_{t1}) \ln k_{t(i-1)}/k_{ti}} = \frac{d\phi_i}{2\pi} d\left(e^{-R'(k_{t1}) \ln k_{t(i-1)}/k_{ti}} \right).
$$

- $k_{t(i-1)} \geq k_{ti}$. Scale k_{ti} extracted by solving $e^{-R'(k_{t1}) \ln k_{t(i-1)}/k_{ti}} = r$, with r random number extracted uniformly in $[0, 1]$. Shower ordered in k_{ti}.

- Extract ϕ_i randomly in $[0, 2\pi]$.

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