Non-local subtractions for jet processes

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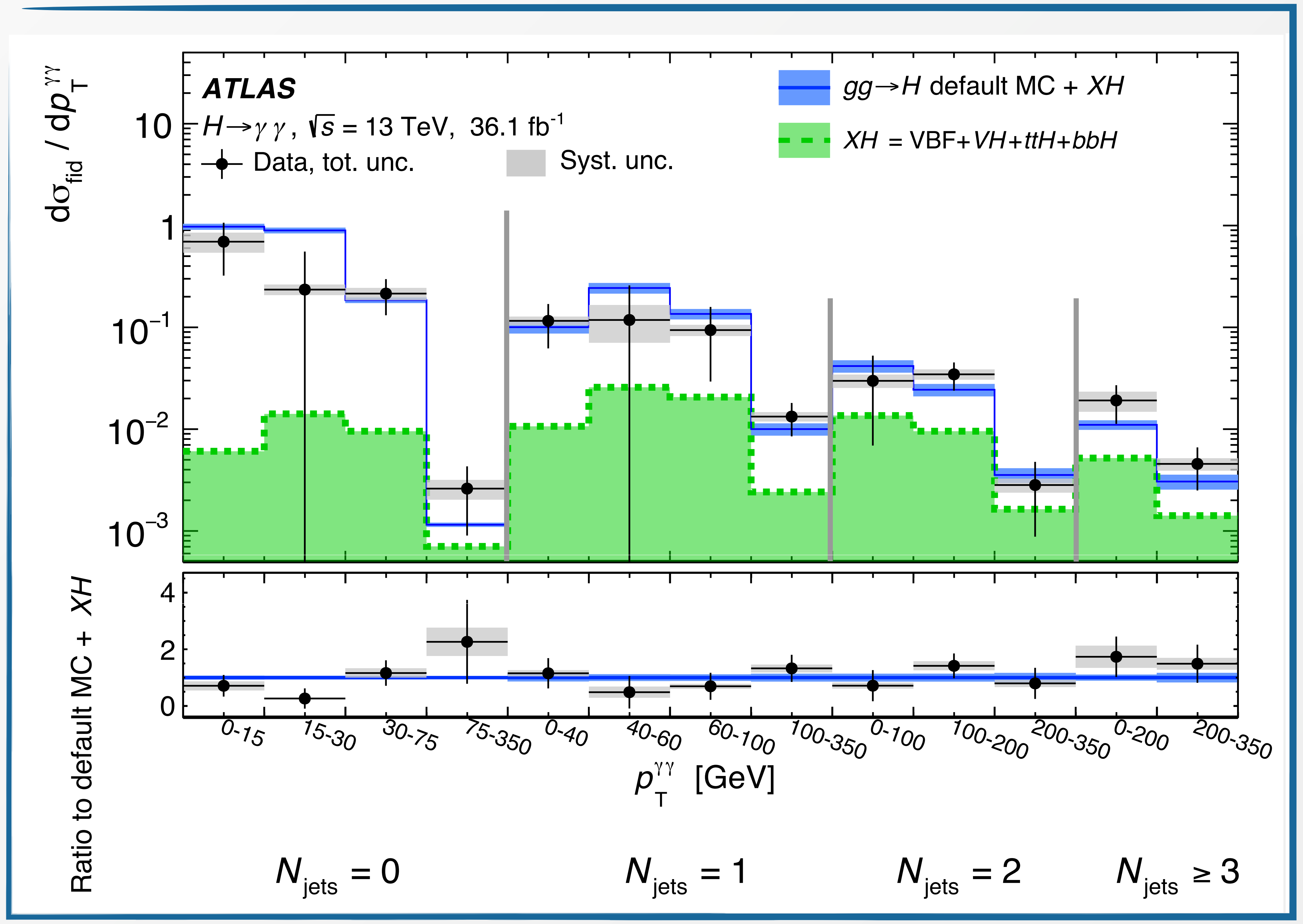
Buonocore, Grazzini, Haag, LR 2110.06913, Buonocore, Grazzini, Haag, LR, Savoini 2201.11519 + ongoing work

Jet physics at the LHC

- Jets are **ubiquitous** at the LHC
- Experimental analyses categorize events into **jet bins** according to the jet multiplicity
- E.g. $pp \to H + X$: **enhanced sensitivity** to Higgs boson kinematics, spin-CP properties, BSM effects…



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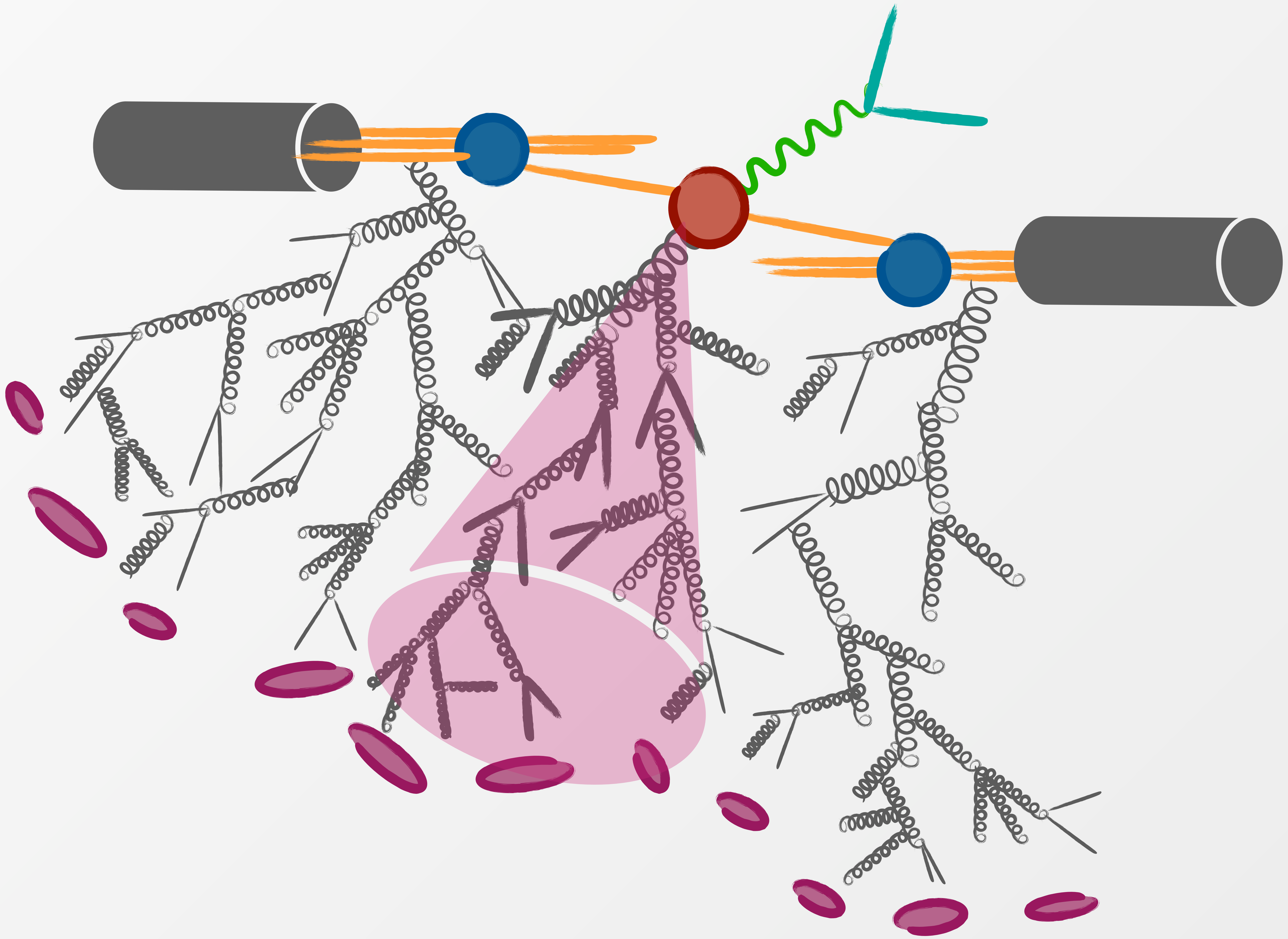
- Experimental analyses categorize events into **jet bins** according to the jet multiplicity
- E.g. $pp \to H + X$: **enhanced sensitivity** to Higgs boson kinematics, spin-CP properties, BSM effects…
- Description of jet processes requires an understanding of QCD across a wide range of energy scales
- Additional theoretical challenges in processes with one or more jets



Fixed-order calculations

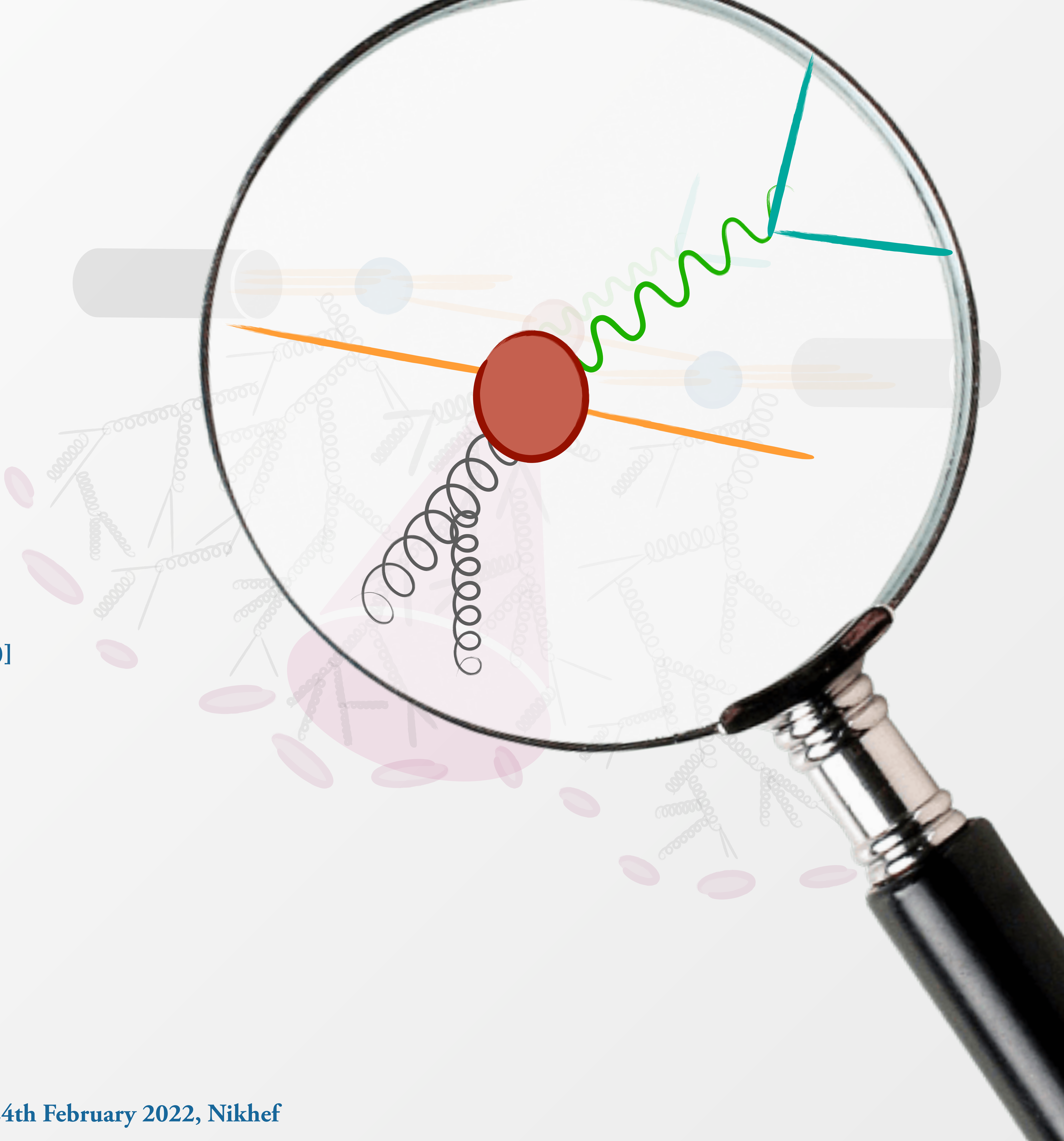
- **Complex singularity structure** for processes with one or more jets

- Fixed order calculations at NNLO accuracy require efficient subtraction methods to extract and cancel virtual and real singularities

- $V + j$ NNLO calculations available with local and non-local subtraction methods
  [Caola, Melnikov, Schulze]
  [Chen, Gehrmann, Gehrmann-De Ridder, Glover, Huss + others (NNLOJET)]
  [Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello, Williams]

- $pp \to 2j$ and even $pp \to 3j$ recently computed
  [H.Chawdhry, M.Czakon, A.Mitov, R.Poncelet] ($pp \to 2j$ and $pp \to 3j$)
  [NNLOJET] ($pp \to 2j$)

- **Computationally expensive** (100k-1M CPU hours); **no public code** available

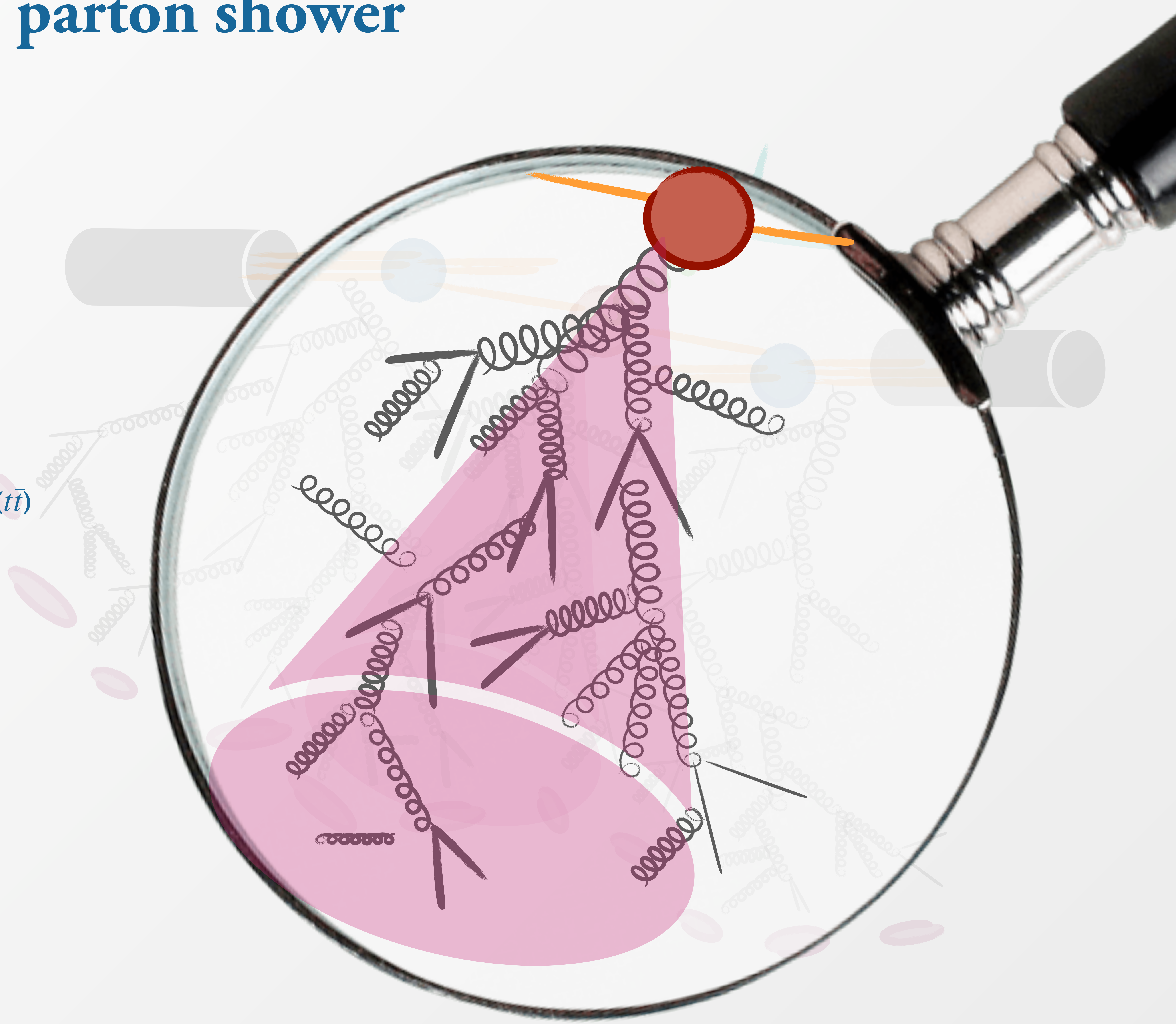


All-order calculations and matching to parton shower

- Resummation structure for jet observables complicated by the presence of **multiple emitters**

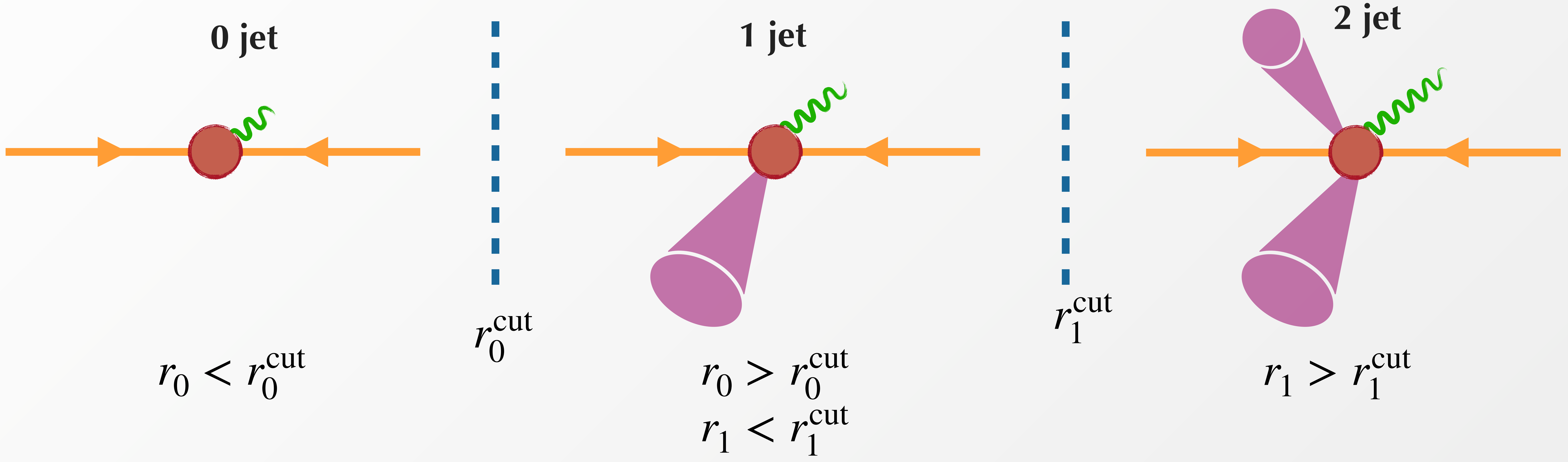
- Ingredients to reach NNLL accuracy available only for a few selected observables with three or more coloured legs
  [Bonciani, Catani, Grazzini, Sargsyan, Torre, Devoto, Mazzitelli, Kallweit]($t\bar{t}$)
  [Arpino, Banfi, El-Menoufi](three jet rate)
  [Jouttenus, Stewart, Tackmann, Waalewijn](jet mass)
  [Becher, Garcia I Tormo, Piclum](transverse thrust in pp collisions)
  [Chien, Rahn,Schrijnder van Velzen,Shao,Waalewijn.Wu]

- Matching of NNLO calculations with parton shower requires the knowledge of the same ingredients entering at NNLL′ for a **suitable resolution variable** which captures the singularities of the $N \to N+1$ (partonic) jet transition



Jet resolution variables

Resolution variables smoothly capture the transition from N to $N+1$ configurations



$0 \to 1$ jet transition: p_T^{veto}, q_T, 0-jettiness τ_0

$1 \to 2$ jet transition: two-jet resolution parameter y_{12}, 1-jettiness τ_1

Caveat: the definition of the resolution variable may or may not depend on the jet definition

The 0 jet case

- $p_T^{\rm veto}$, q_T, τ_0 are three well known variables able to discriminate the $0 \to 1$ transition and to **inclusively describe initial-state radiation**

- **Singular structure** known at (N)NNLO from the expansion of the **resummation formula** at (N)NNLL accuracy

- In the case of q_T, τ_0, the knowledge of the $\mathcal{O}(\alpha_s^2)$ terms constant terms allows for the formulation of **non-local subtraction methods** for QCD calculations at NNLO
  [Catani, Grazzini][Gaunt, Stahlhofen, Tackmann, Walsh]

- q_T and τ_0 are also used as resolution variables for **NNLO+PS event generators**

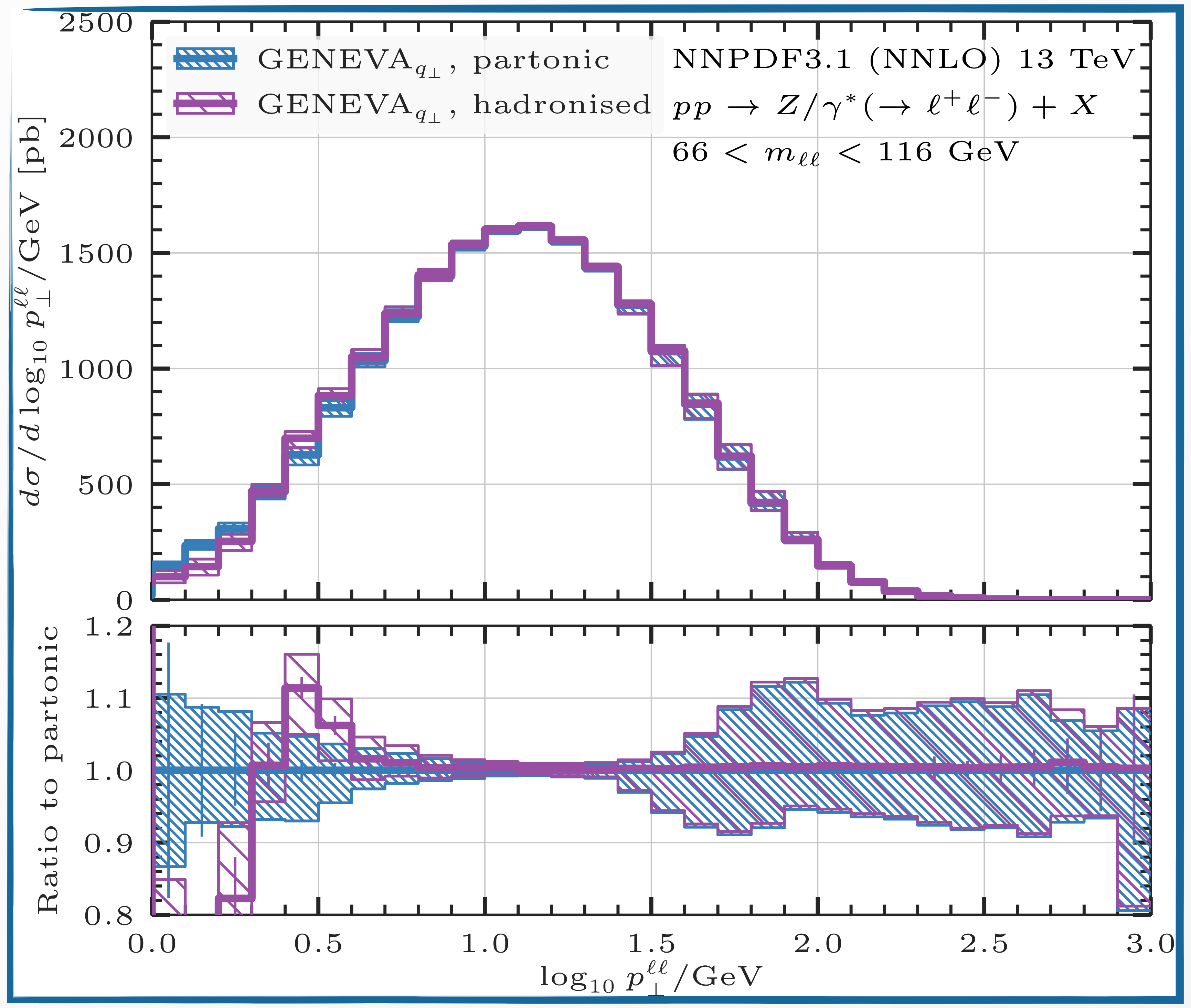
q_T: UNNLOPS, [Höche, Li, Prestel] MiNNLOPS [Nason, Monni, Re, Wiesemann, Zanderighi]

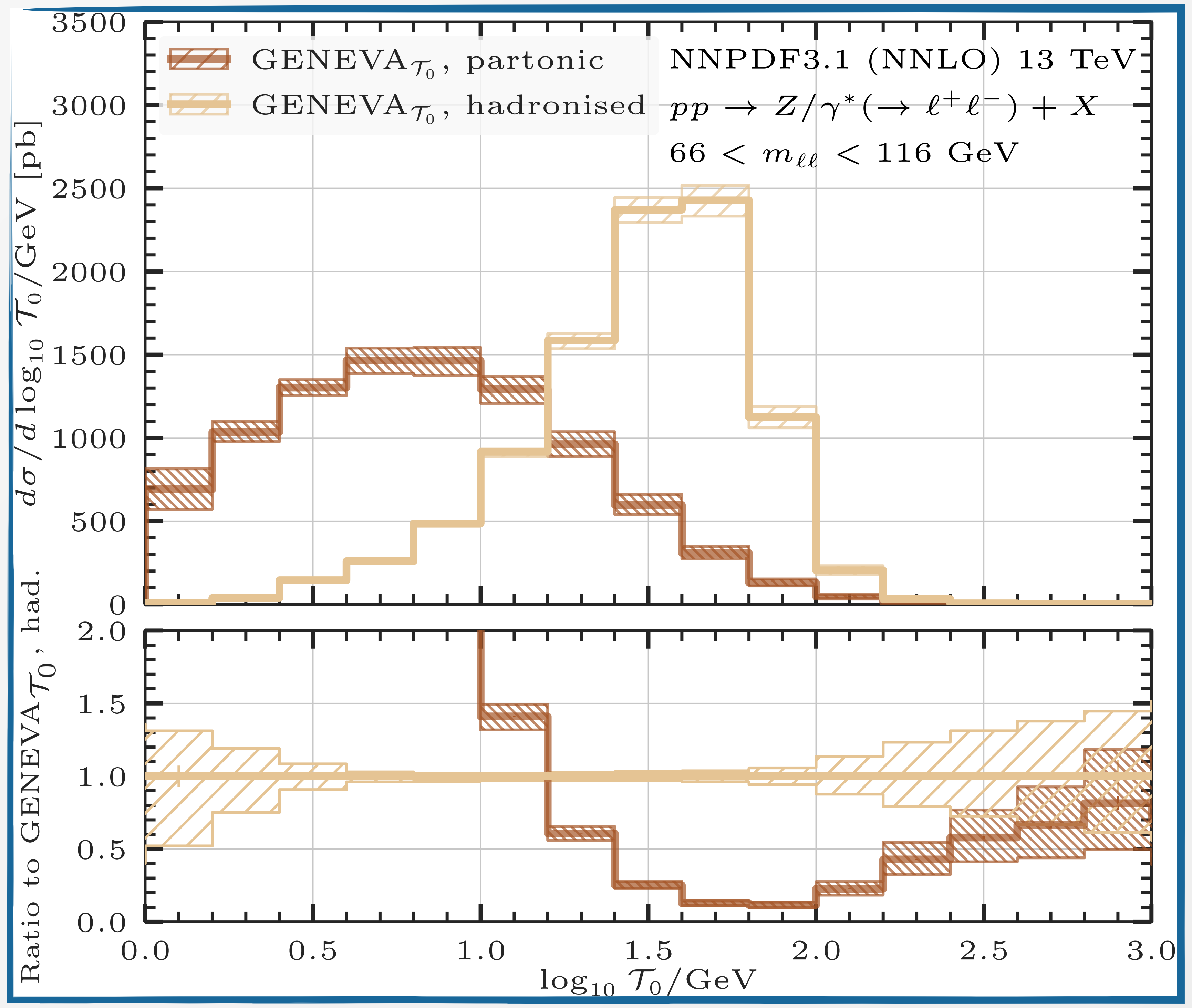
τ_0: GENEVA [Alioli, Bauer, Berggren, Tackmann, Walsh] recently extended to q_T [Alioli, Bauer, Broggio, Gavardi, Kallweit, Lim, Nagar, Napoletano, LR]

q_T and τ_0 resummation

Resummation for both variables known at **high logarithmic accuracy**: NNLL′ for τ_0, N^{3}LL′ for q_T

[Gaunt, Stahlhofen, Tackmann, Walsh][Billis, Dehnadi, Ebert, Michel, Stewart, Tackmann][Re, LR, Torrielli][Camarda, Cieri, Ferrera][Ju, Schönherr][Neumann]





[Alioli, Bauer, Broggio, Gavardi, Kallweit, Lim, Nagar, Napoletano, LR]

Predictiveness of resummed predictions affected by corrections of NP origin (hadronisation, MPI). Spectrum in q_T **mildly affected**, **large corrections** due to MPI in the case of τ_0

The 0 jet case: non-local subtraction and power corrections

General formula for non-local subtraction methods for colour singlet production at NNLO

Virtual correction after subtraction of IR singularities and contribution of soft/collinear origin **(beam, soft, jet functions)**

Real contribution with one additional parton, **divergent** in the limit $r_{\mathrm{cut}} \to 0$, $r_{\mathrm{cut}} \sim q_T/Q, \mathcal{T}_0/Q, \ldots$

$$d\hat{\sigma}^{\mathrm{F+X}}_{\mathrm{NNLO}} = \mathcal{H}^{\mathrm{F}}_{\mathrm{NNLO}} \otimes d\hat{\sigma}^{\mathrm{F}}_{\mathrm{LO}} + \left[d\hat{\sigma}^{\mathrm{F+1\,jet}}_{\mathrm{NLO}} - d\hat{\sigma}^{\mathrm{CT,F}}_{\mathrm{NNLO}} \right] + \mathcal{O}(r^p_{\mathrm{cut}})$$

Counterterm, matches the real calculation in the limit $r \to 0$

Missing **power corrections** below the slicing cut-off

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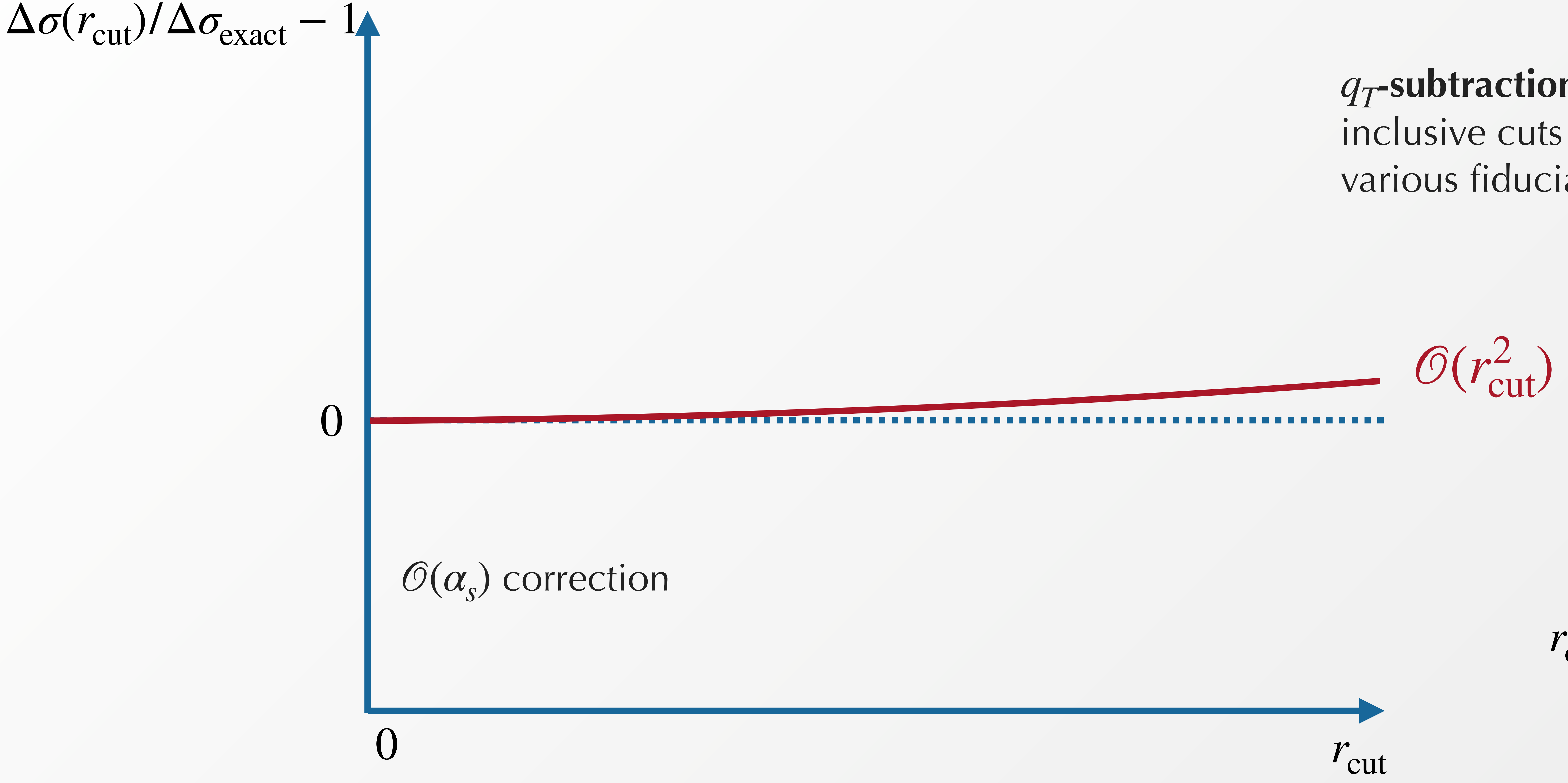
Missing **power corrections** below the slicing cut-off

Sensitivity to **power corrections below the cut-off** generally **depends on the observable** and affects the **performance** of the method

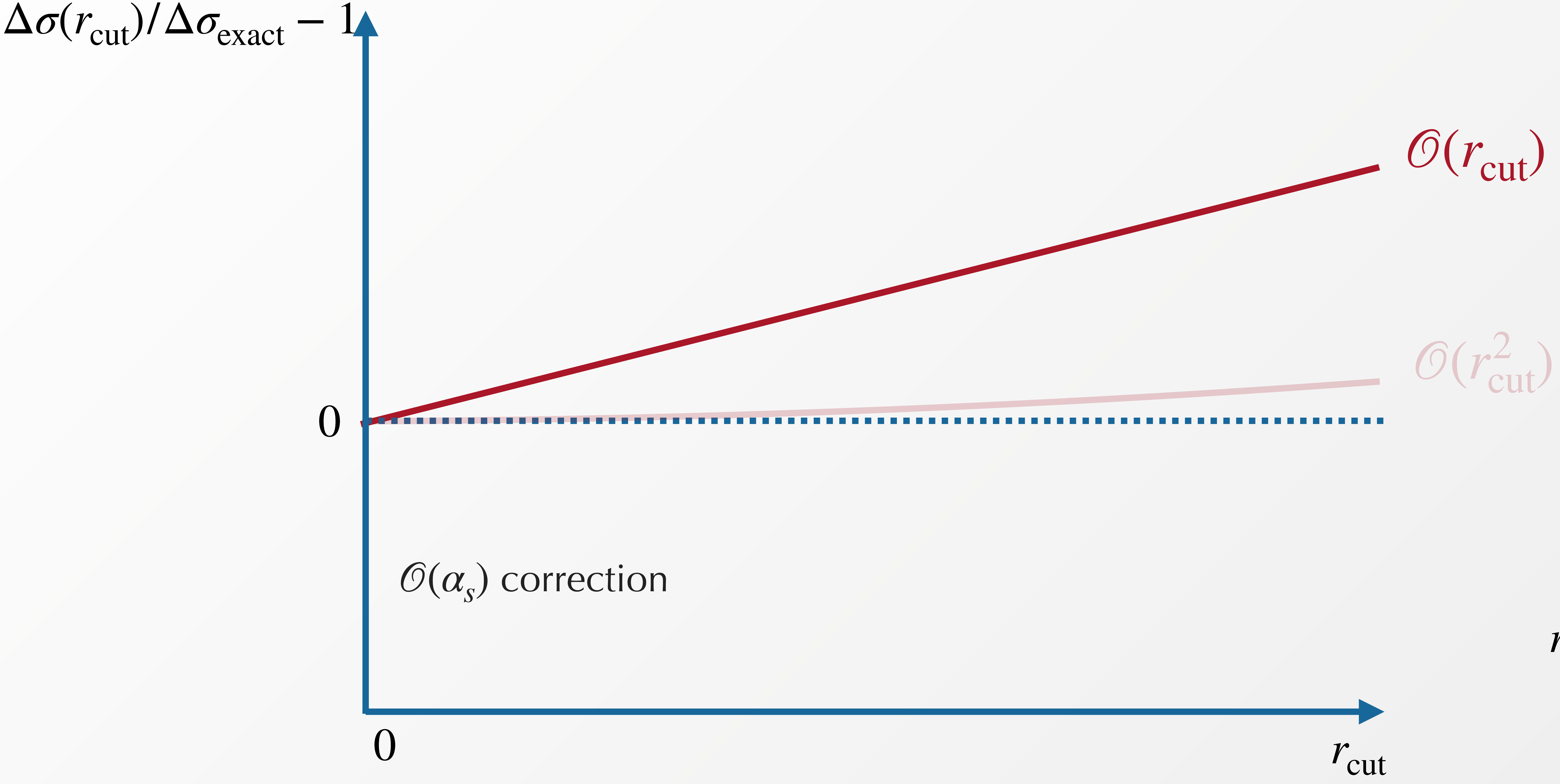
The 0 jet case: non-local subtraction and power corrections



q_T**-subtraction** with inclusive cuts and in various fiducial setups

$r_{\text{cut}} \sim q_T/Q, \mathcal{T}_0/Q, \ldots$

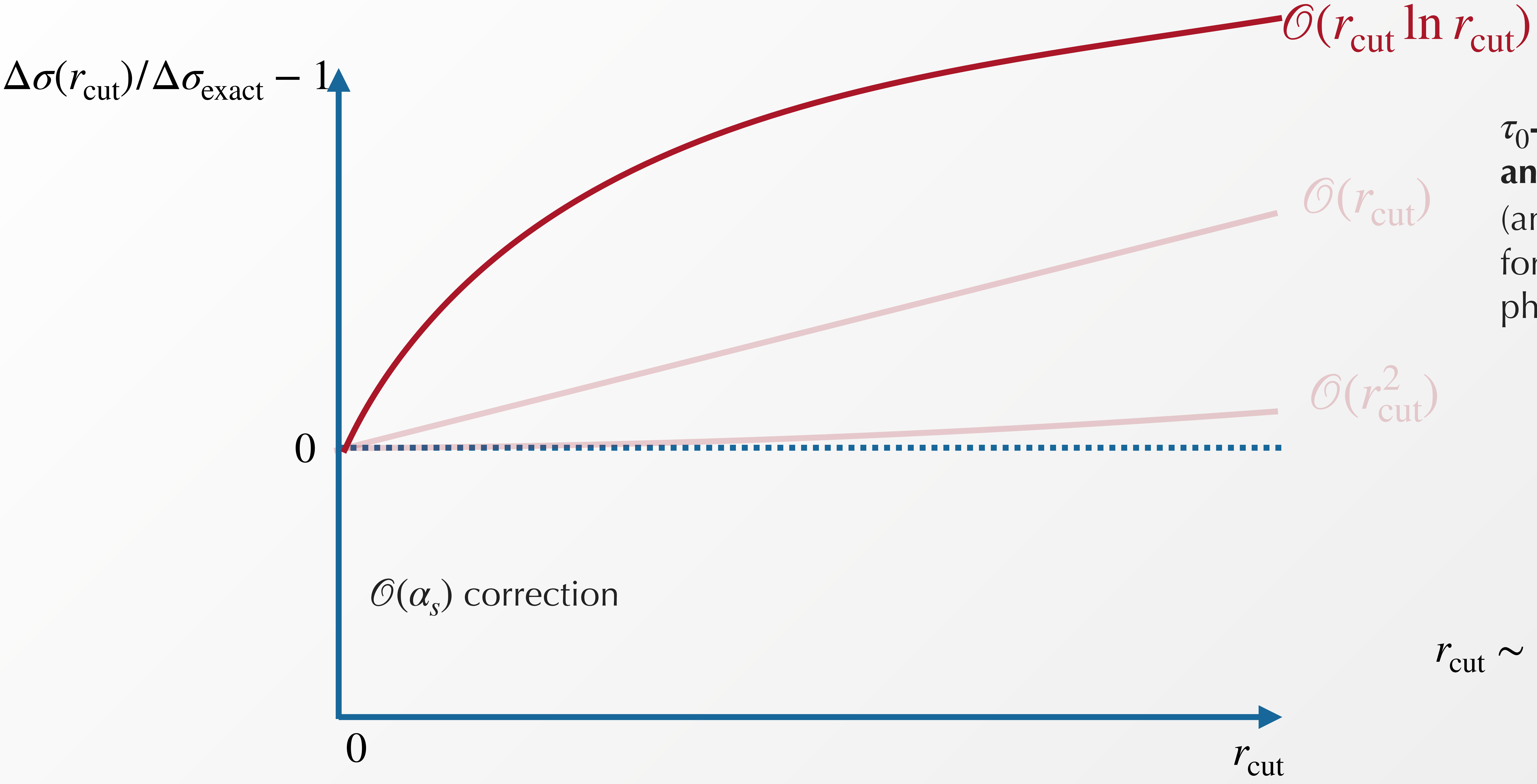
The 0 jet case: non-local subtraction and power corrections



q_T**-subtraction** for $2 \to 2$ processes with (a)symmetric cuts

$r_{\text{cut}} \sim q_T/Q, \mathcal{T}_0/Q, \ldots$

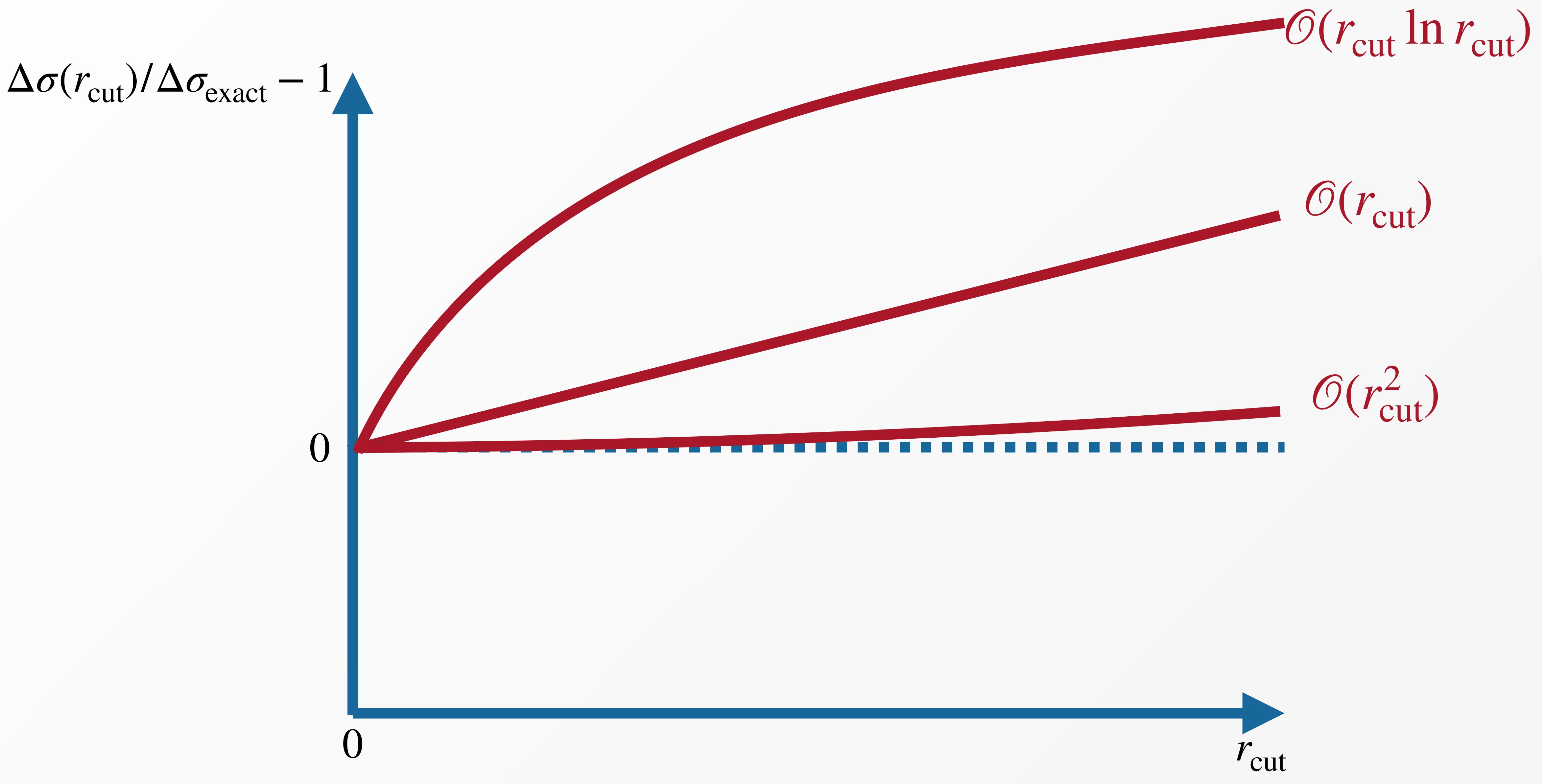
The 0 jet case: non-local subtraction and power corrections



τ_0-subtraction for any colour-singlet (and q_T-subtraction for processes with photon isolation cuts)

$r_\text{cut} \sim q_T/Q, \mathcal{T}_0/Q, \ldots$

The 0 jet case: non-local subtraction and power corrections



Relative size of power corrections affects **stability and performance** of non-local subtraction methods

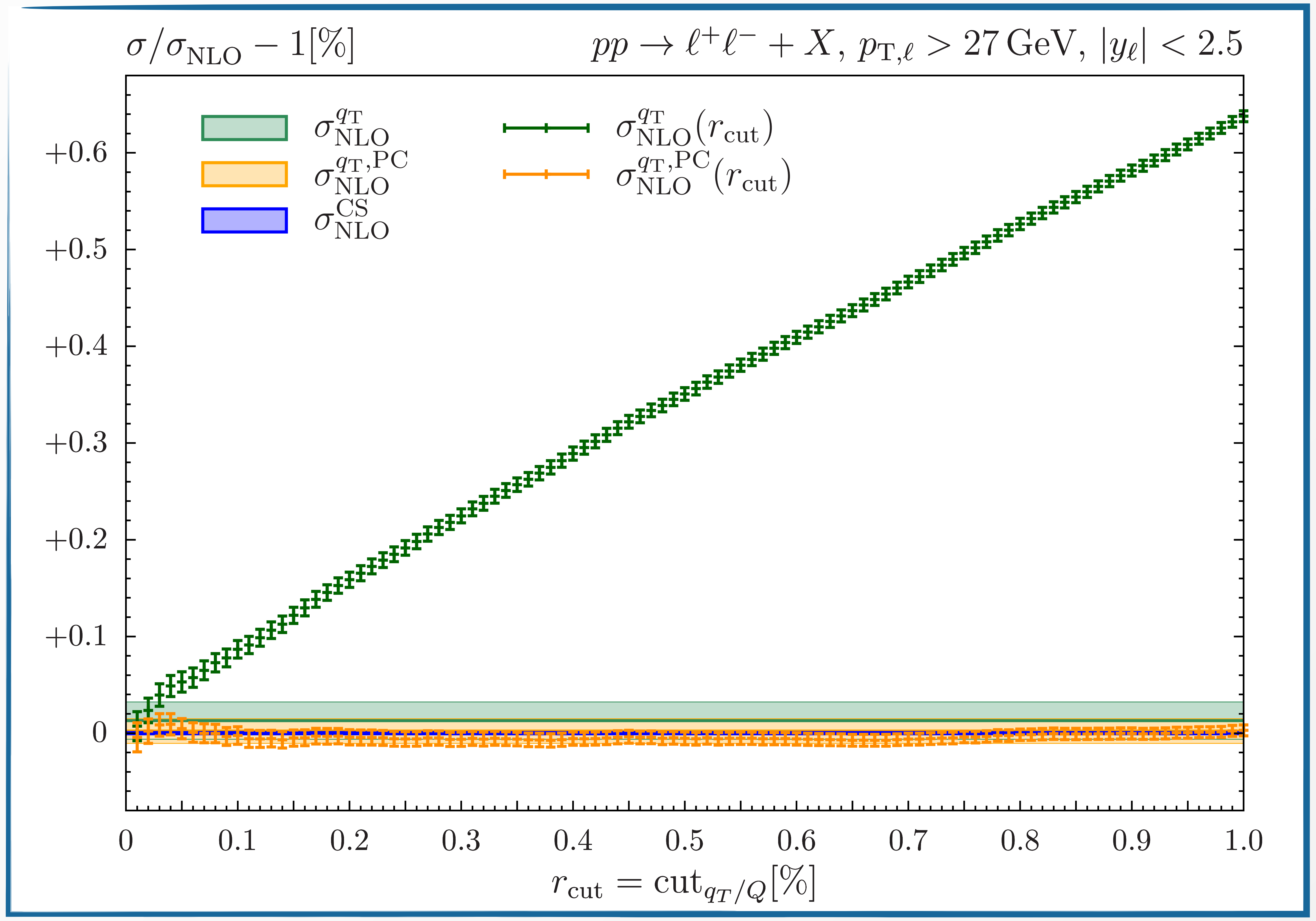
The larger the power corrections, the lower are the values of the slicing parameters needed for extrapolation of correct result (CPU consuming, numerically unstable)

Computation of missing (leading) power corrections helps to tame numerical instabilities, especially in the 0-jettiness case, where power corrections are larger
[Moult, Rothen, Stewart, Tackmann, Zhu, Ebert, Vita][Boughezal, Isgrò, Liu, Petriello]

The 0 jet case: linear power corrections for q_T subtraction

For $2 \to 2$ processes with **(a)symmetric cuts**, **fiducial linear** power corrections for q_T-subtraction can be calculated numerically via a proper treatment of the **transverse recoil**

[Billis, Dehnadi, Ebert, Michel, Stewart, Tackmann][Buonocore, Kallweit, LR, Wiesemann][Camarda, Cieri, Ferrera]



[Buonocore, Kallweit, LR, Wiesemann 2111.13661]

Much improved convergence over linear power correction case

Accurate computation of the NLO correction without the need to push r_{cut} to very low values

Remark: linear power corrections in the **symmetric/asymmetric** case are related to **ambiguities** in the perturbative expansion and can be avoided with different sets of cuts

[Salam, Slade]

Beyond 0 jet: N-jettiness

So far N-jettiness is the most studied resolution variable for the generic $N \to N+1$ transition

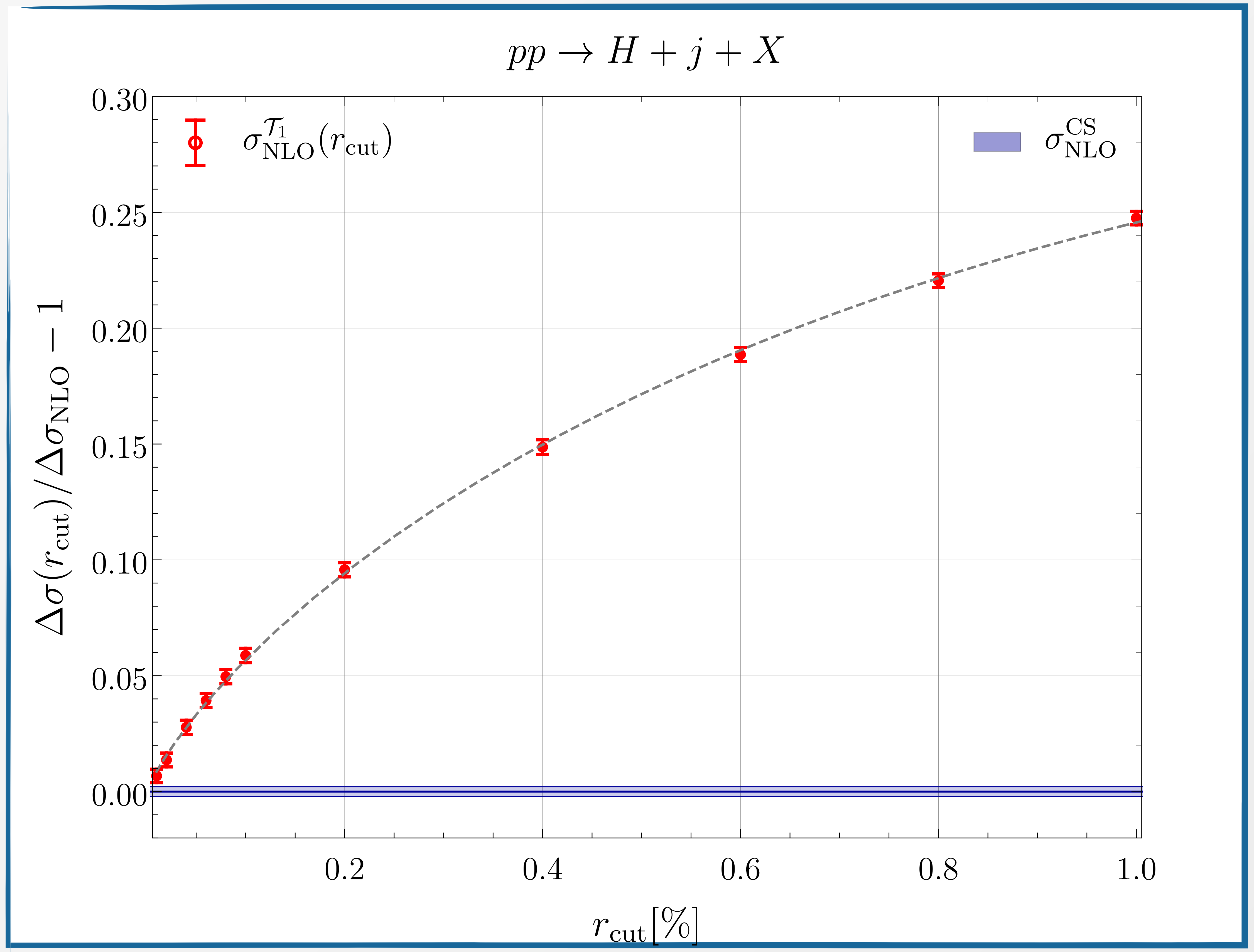
Ingredients for 1-jettiness subtraction at NNLO have been computed, and NNLO calculations for $V+1$ jet using 1-jettines subtraction have been performed [Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello, Williams]

Soft-function for 2-jettiness at NNLO also available, allows for potential computation of dijet at NNLO [Jin, Liu]

Application to $V+1$ processes requires careful estimate of the large missing power corrections which characterise the observable
[Campbell, Ellis, Seth]

$$\mathcal{T}_1 = \sum_i \min_l \left\{ \frac{2q_l \cdot p_i}{Q_l} \right\} \qquad Q_l = 2E_l$$

$$r = \mathcal{T}_1 / \sqrt{m_H^2 + (p_T^j)^2}$$



New resolution variables for $V + 1$ jet

N-jettiness has proved a successful resolution variable for processes with 1 jet, but so far is essentially the only player in the game

It may prove worthwhile to explore other resolution variables which **overcome some of the shortcomings** of jettiness and which could have

- **smaller power corrections** → Applications to NNLO subtraction and beyond
- more **direct experimental relevance** → Comparison of resummed prediction with data
- simpler relation with parton shower ordering variables → Improved NNLO+PS matching

q_T-imbalance for $V + j$ production

[Buonocore, Grazzini, Haag, LR, 2110.06913]

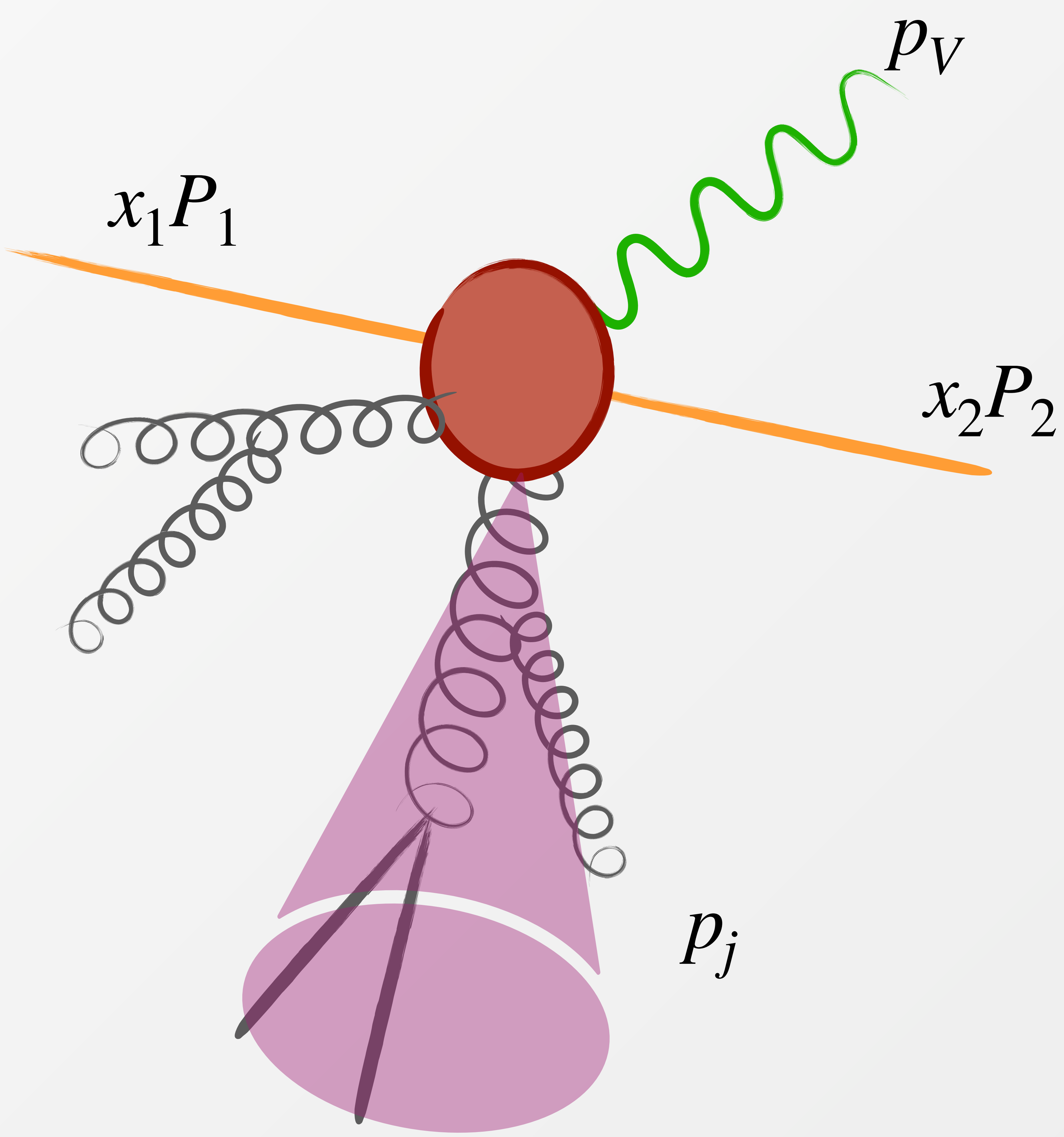
Consider production of boson V in association with a jet

$$h_1(P_1) + h_2(P_2) \to V(p_V) + j(p_j) + X$$

Define q_T**-imbalance** as

$$\vec{q}_T = (\vec{p}_V + \vec{p}_J)_T$$

Variable depends on the **jet definition**: jet defined through anti-k_t algorithm with jet radius R



Fixed-order calculation develops **large logarithms** of $\ln(q_T)^2/Q^2$ in the limit $q_T \to 0$.

Perturbative expansion rescued by the **all-order resummation** of logarithmically enhanced terms

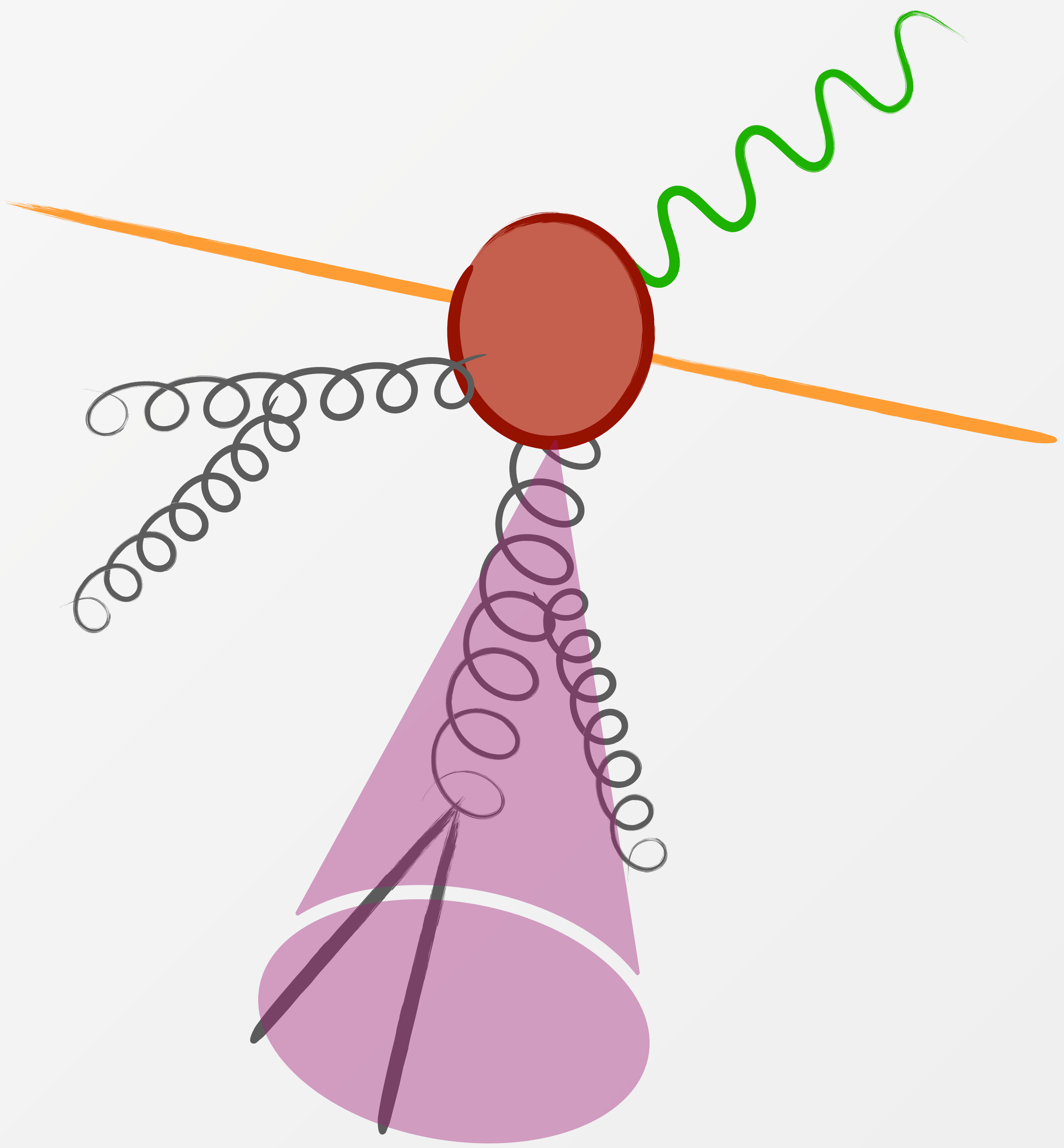
q_T-imbalance for $V + j$ production

[Buonocore, Grazzini, Haag, LR, 2110.06913]

Resummation already considered both in direct QCD and in SCET [Sung, Yan, Yuan, Yuan][Chien, Shao, Wu]

In both cases, anomalous dimensions computed in the **narrow jet approximation** (valid only in the small-R limit)

In view of potential applications for e.g. subtraction scheme, it is important to assess the impact of such an approximation



In our calculation:

- Full R **dependence** in the anomalous dimensions
- Full **azimuthal dependence**
- Inclusion of all finite contributions (**NLL′ accuracy**)

Singularity structure and factorisation

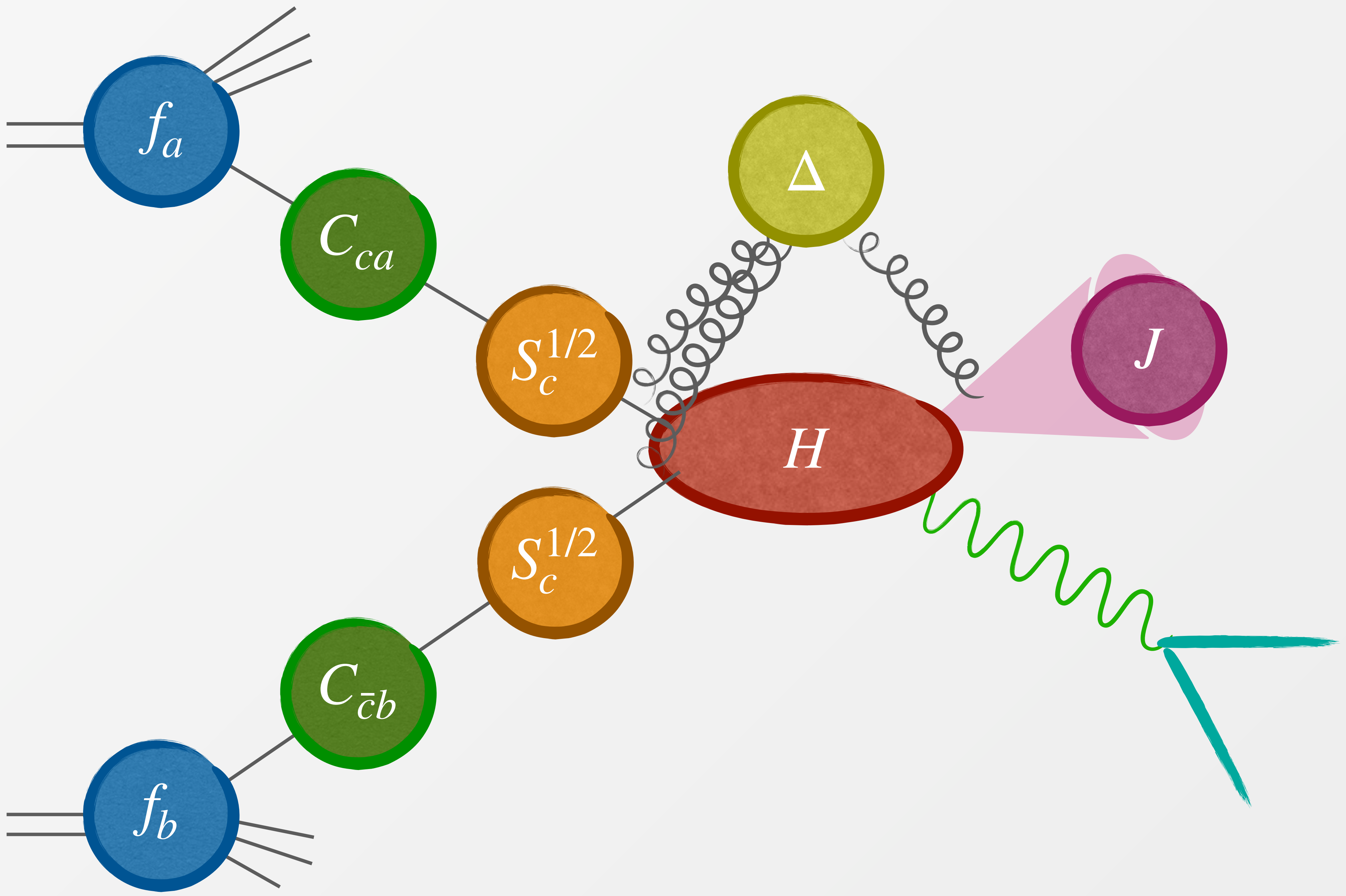
Richer singularity structure since the final state parton radiates

Singularities of **soft/collinear** origin from **initial state partons**

Singularities of **soft origin** due to the emission of soft gluons at **wide angle** connecting the three emitters

Final state collinear singularity regulated by finite jet radius

Presence of finite jet radius induces harsh boundary in the phase space - **non global logarithms**



Resummation formula at NLL

[Buonocore, Grazzini, Haag, LR, 2110.06913]

Observable factorizes in impact parameter (**b**) space like transverse momentum in colour-singlet production
Resummation akin to the resummation of transverse momentum in $t\bar{t}$ production

Fully differential resummation formula at NLL (for **global** contribution)

$$\frac{d\sigma}{d^2\mathbf{q_T}dQ^2dy\,d\mathbf{\Omega}} = \frac{Q^2}{2P_1\cdot P_2}\sum_{(a,c)\in\mathcal{I}}[d\sigma_{ac}^{(0)}]\int\frac{d^2\mathbf{b}}{(2\pi)^2}e^{i\mathbf{b}\cdot\mathbf{q_T}}\mathcal{S}_{ac}(Q,b)$$

$$\times\sum_{a_1,a_2}\int_{x_1}^{1}\frac{dz_1}{z_1}\int_{x_2}^{1}\frac{dz_2}{z_2}[(\mathbf{H\Delta})C_1C_2]_{ac;a_1a_2}f_{a_1/h_1}(x_1/z_1,b_0^2/b^2)f_{a_2/h_2}(x_2/z_2,b_0^2/b^2)$$

$$\mathcal{S}_{ac}(Q,b) = \exp\left\{-\int_{b_0^2/b^2}^{Q^2}\frac{dq^2}{q^2}\left[A_{ac}(\alpha_s(q^2))\ln\frac{Q^2}{q^2}+B_{ac}(\alpha_s(q^2))\right]\right\}$$

Sudakov exponent is the same as for colourless case

$$[(\mathbf{H\Delta})C_1C_2]_{ac;a_1a_2}$$

Contains additional contribution which starts at NLL accuracy and describes QCD radiation of **soft-wide angle radiation** (colour singlet: $\mathbf{\Delta}=1$)

Same beam function as q_T

The soft-wide angle contribution

$$[(\mathbf{H\Delta})C_1C_2]_{ac;a_1a_2}$$

The factor $(\mathbf{H\Delta})$ depends on $\mathbf{b}$, Q and on the underlying Born. It also contains an explicit dependence on the **jet definition**

$\mathbf{H}$: **process-dependent** hard factor, independent on $\mathbf{b}$

$(\mathbf{H\Delta}) = \mathrm{Tr}[\mathbf{H\Delta}]$: non-trivial dependence on the colour structure of the partonic process (can be worked out simply in $V + j$ production)

Explicit **azimuthal** dependence (azimuthal correlations)

[Catani, Grazzini, Sargsyan, Torre]

All-order structure of $\mathbf{\Delta}$

$$\mathbf{\Delta}(\mathbf{b}, Q; t/u, \phi_{Jb}) = \mathbf{V}^\dagger(\mathbf{b}, Q, t/u, R)\, \mathbf{D}(\alpha_s(b_0^2/b^2), t/u, R; \phi_{Jb})\mathbf{V}(\mathbf{b}, Q, t/u, R)\,.$$

$$\mathbf{V}(\mathbf{b}, Q, t/u, R) = \overline{P}_q \exp\left\{-\int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2}\mathbf{\Gamma}(\alpha_s(q^2), t/u, R)\right\}$$

Evolution operator resumming logs stemming from soft-wide angle radiation

Calculation of NLL′ coefficients

Resummation formula at NLL′ requires the computation of 1-loop resummation coefficients

$$\mathbf{\Gamma}(\alpha_s, t/u, R) = \frac{\alpha_s}{\pi}\mathbf{\Gamma}^{(1)}(t/u, R) + \sum_{n>1}\left(\frac{\alpha_s}{\pi}\right)^n \mathbf{\Gamma}^{(n)}(t/u, R) \qquad \mathbf{D}(\alpha_s, t/u, R) = \frac{\alpha_s}{\pi}\mathbf{D}^{(1)}(t/u, R) + \sum_{n>1}\left(\frac{\alpha_s}{\pi}\right)^n \mathbf{D}^{(n)}(t/u, R)$$

Calculation performed by defining the NLO eikonal current associated to the emission of a soft gluon

$$\mathbf{J}^2(\{p_i\}, k; R) = \left(\mathbf{T}_1 \cdot \mathbf{T}_2 \frac{p_1 \cdot p_2}{p_1 \cdot k \;\; p_2 \cdot k} + \mathbf{T}_1 \cdot \mathbf{T}_3 \frac{p_1 \cdot p_3}{p_1 \cdot k \;\; p_3 \cdot k} + \mathbf{T}_2 \cdot \mathbf{T}_3 \frac{p_2 \cdot p_3}{p_2 \cdot k \;\; p_3 \cdot k}\right) \times \Theta(R_{3k}^2 > R^2)$$

And subtracting the **double counting** (contributions of soft/collinear origin from the initial state legs)

$$\mathbf{J}^2_{\mathrm{sub}}(\{p_i\}, k; R) = \mathbf{J}^2 - \sum_{i=1,2}\left(-\mathbf{T}_i^2 \frac{p_1 \cdot p_2}{p_i \cdot k \;\; (p_2 + p_2) \cdot k}\right) \times 1$$

The resummation coefficients can be calculated via

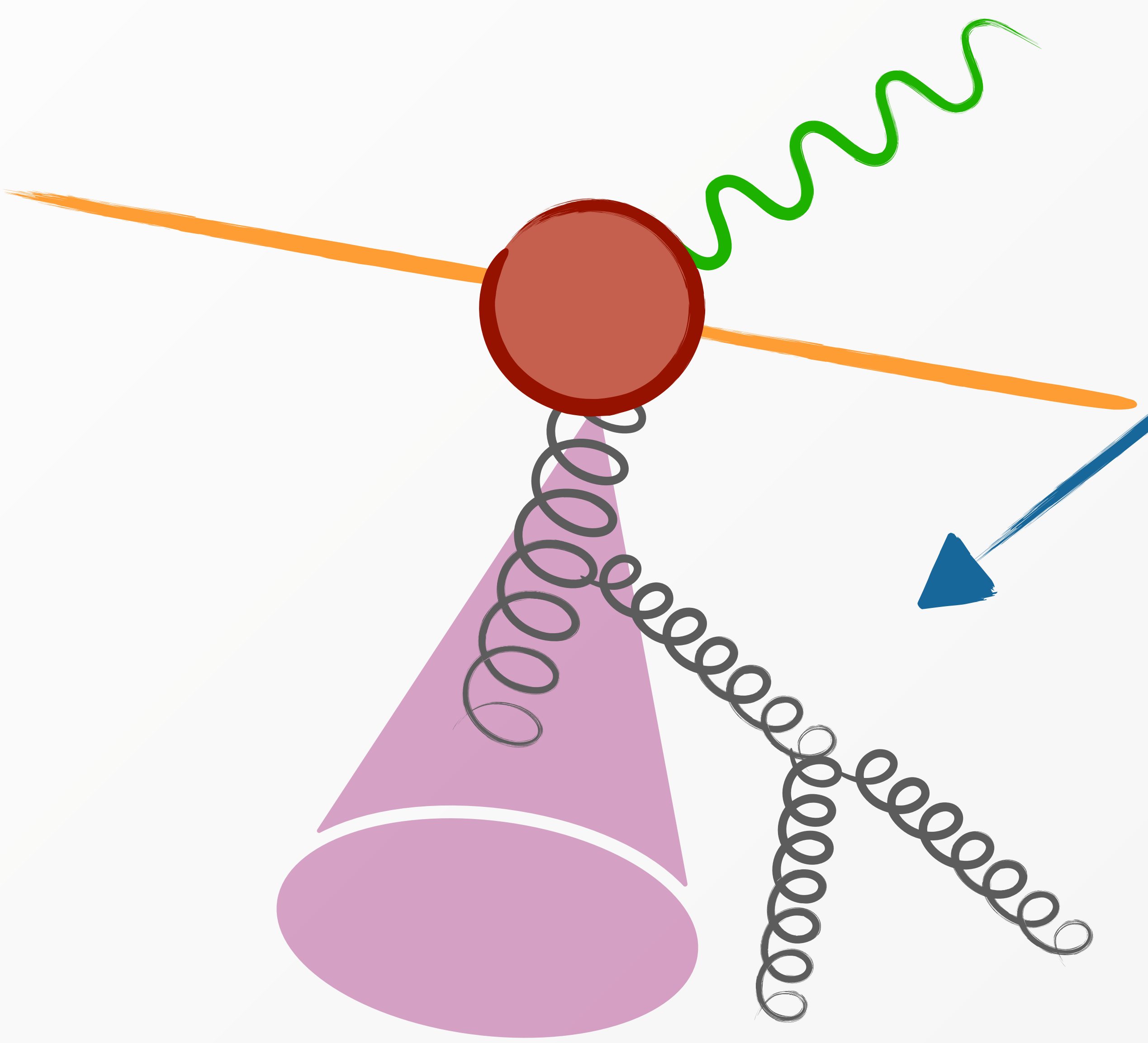
$$\tilde{\mathbf{J}}_{\mathrm{sub}}(\mathbf{b}, t/u; R) = \mu^{2\epsilon}\int d^d k \delta_+(k^2) e^{i\mathbf{b}\cdot\mathbf{k}_\perp} \mathbf{J}^2_{\mathrm{sub}}(\{p_i\}, k; R) = \frac{1}{4}\left(\frac{\mu^2 b^2}{4}\right)^\epsilon \Gamma(1-\epsilon)^2 \Omega_{2-2\epsilon}\left(\frac{4}{\epsilon}\mathbf{\Gamma}^{(1)}(t/u; R) - 2\mathbf{R}^{(1)}(\hat{\mathbf{b}}, t/u; R) + \ldots\right) \qquad \mathbf{D}^{(1)} = \mathbf{R}^{(1)} - \langle \mathbf{R}^{(1)} \rangle\,.$$

Hard factor $\mathbf{H}$: contains finite contributions of virtual origin, the finite jet function $J(R)$, and a finite contribution of soft origin $\mathbf{F}^{(1)}(R) = -2\langle \mathbf{R}^{(1)} \rangle (R)$

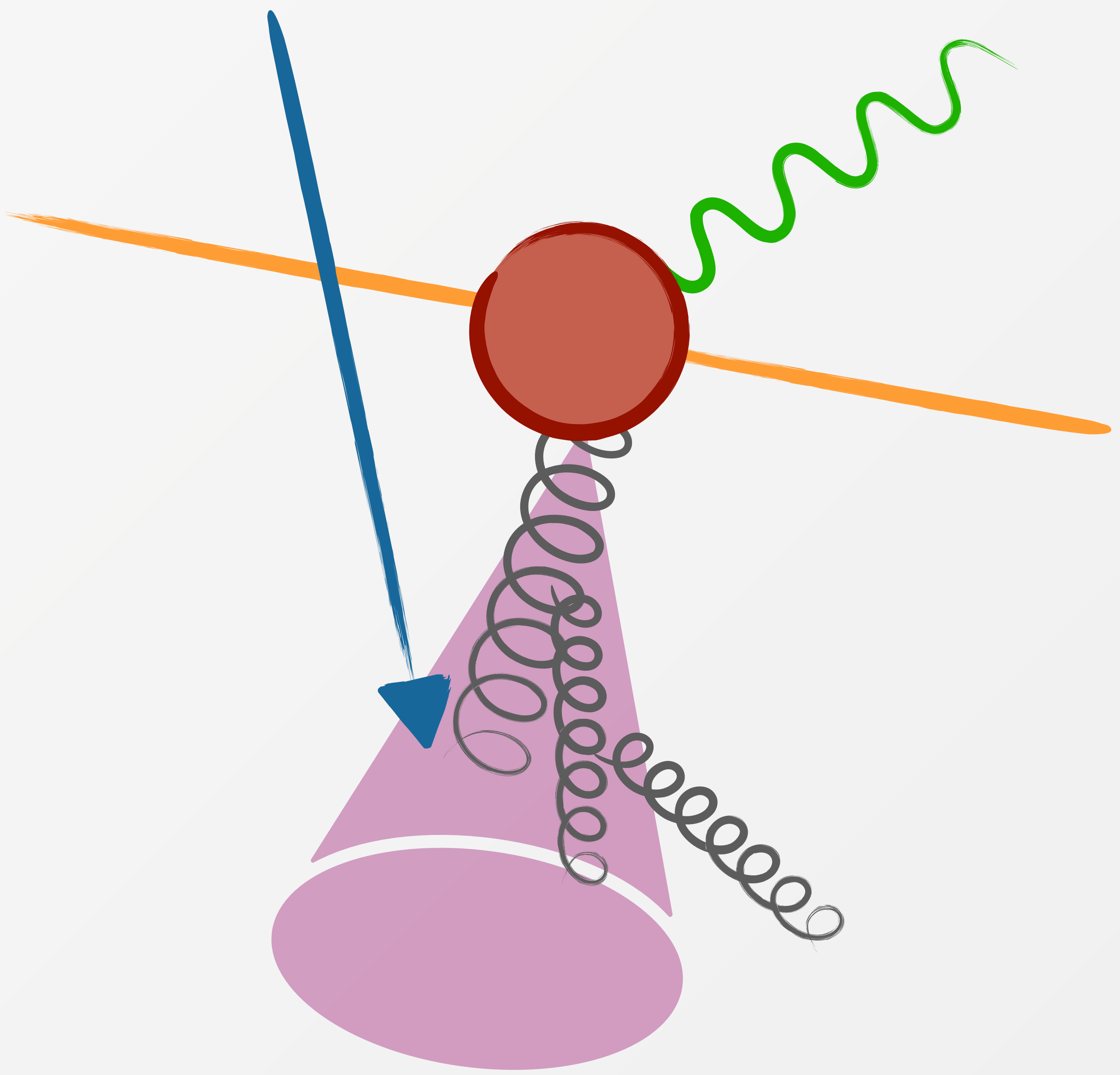
Non global logarithms

NLL accuracy requires the inclusion of **non-global logarithms** [Dasgupta, Salam]

In the strongly ordered soft limit at two loops there are a **global** and a **non-global** contributions at $\alpha_s^2 \ln q_t^2/Q^2$



$+$ Virt.



$+$ Virt.

Resummation formula to be supplemented by the factor $\mathcal{U}^f_{\mathrm{NGL}}$ embedding the resummation of NGL [Dasgupta, Salam]

$$\frac{d\sigma}{d^2\mathbf{q_T}dQ^2dy\,d\mathbf{\Omega}} = \frac{Q^2}{2P_1\cdot P_2}\sum_{(a,c)\in\mathcal{I}}[d\sigma^{(0)}_{ac}]\int\frac{d^2\mathbf{b}}{(2\pi)^2}e^{i\mathbf{b}\cdot\mathbf{q_T}}\mathcal{S}_{ac}(Q,b)$$

$$\times\sum_{a_1,a_2}\int_{x_1}^1\frac{dz_1}{z_1}\int_{x_2}^1\frac{dz_2}{z_2}[(\mathbf{H\Delta})C_1C_2]_{ac;a_1a_2}f_{a_1/h_1}(x_1/z_1,b_0^2/b^2)f_{a_2/h_2}(x_2/z_2,b_0^2/b^2)\mathcal{U}^f_{\mathrm{NGL}}$$

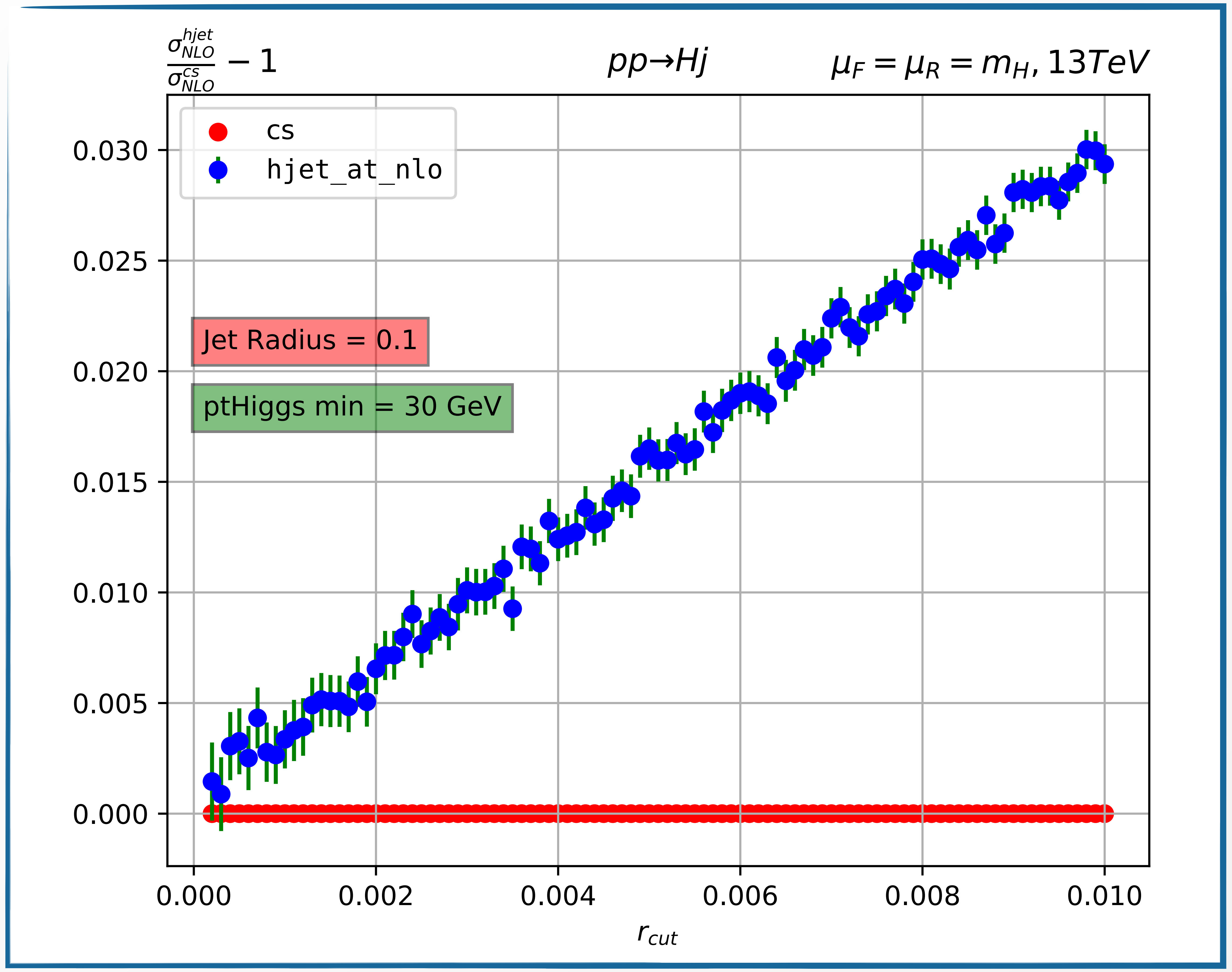
$$\mathcal{U}^f_{\mathrm{NGL}} \sim \exp\left\{-C_AC_f\lambda^2 f(\lambda,R)\right\}$$

$$\lambda = \frac{\alpha_s(Q^2)}{2\pi}\ln\frac{Qb}{b_0}$$

Non-local subtraction at NLO for H+j

[M. Costantini Master's thesis, UZH]

The expansion of the NLL′ formula at fixed order allows us to construct a non-local subtraction scheme using q_T -imbalance as resolution variable



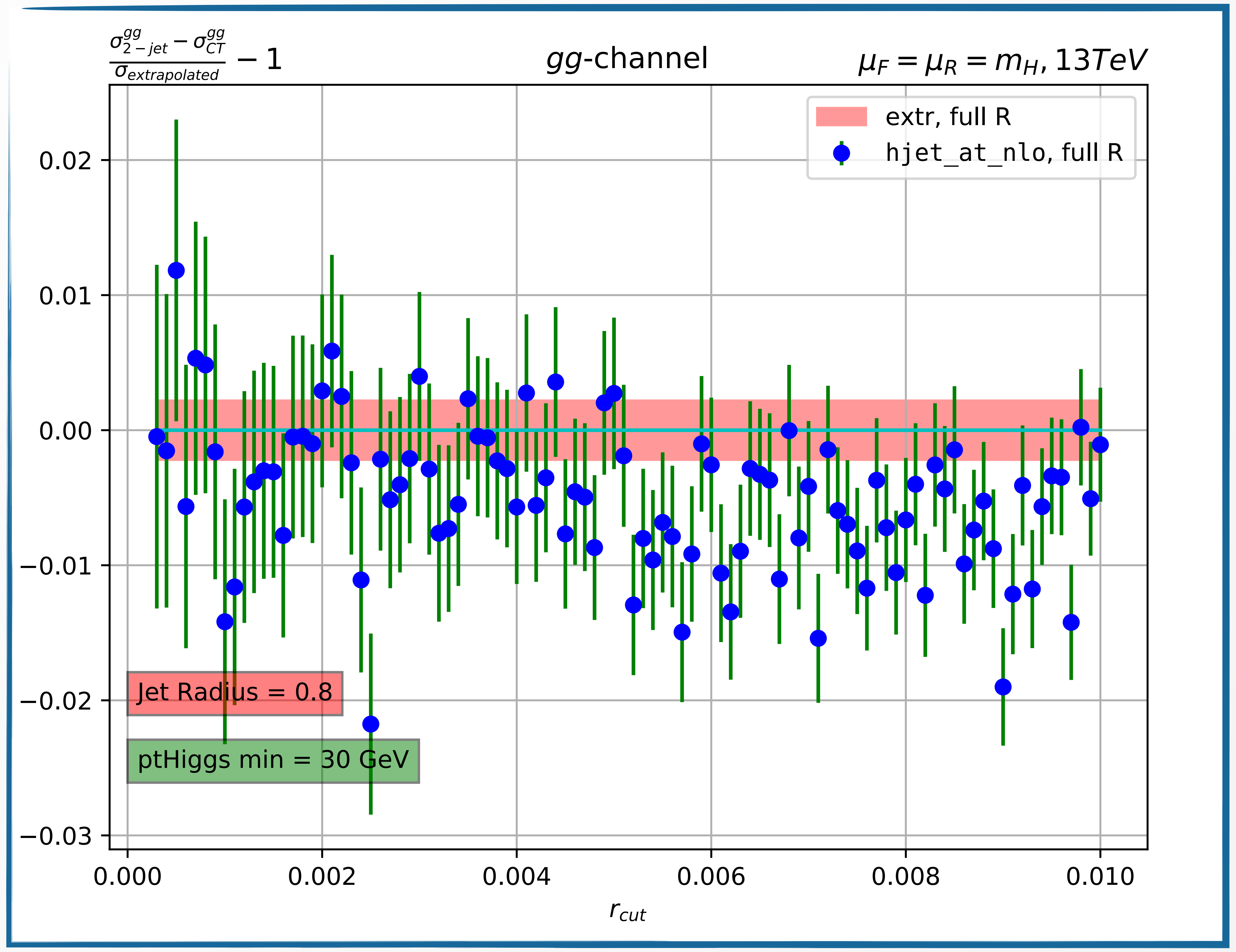
Linear scaling observed, good convergence towards the exact result

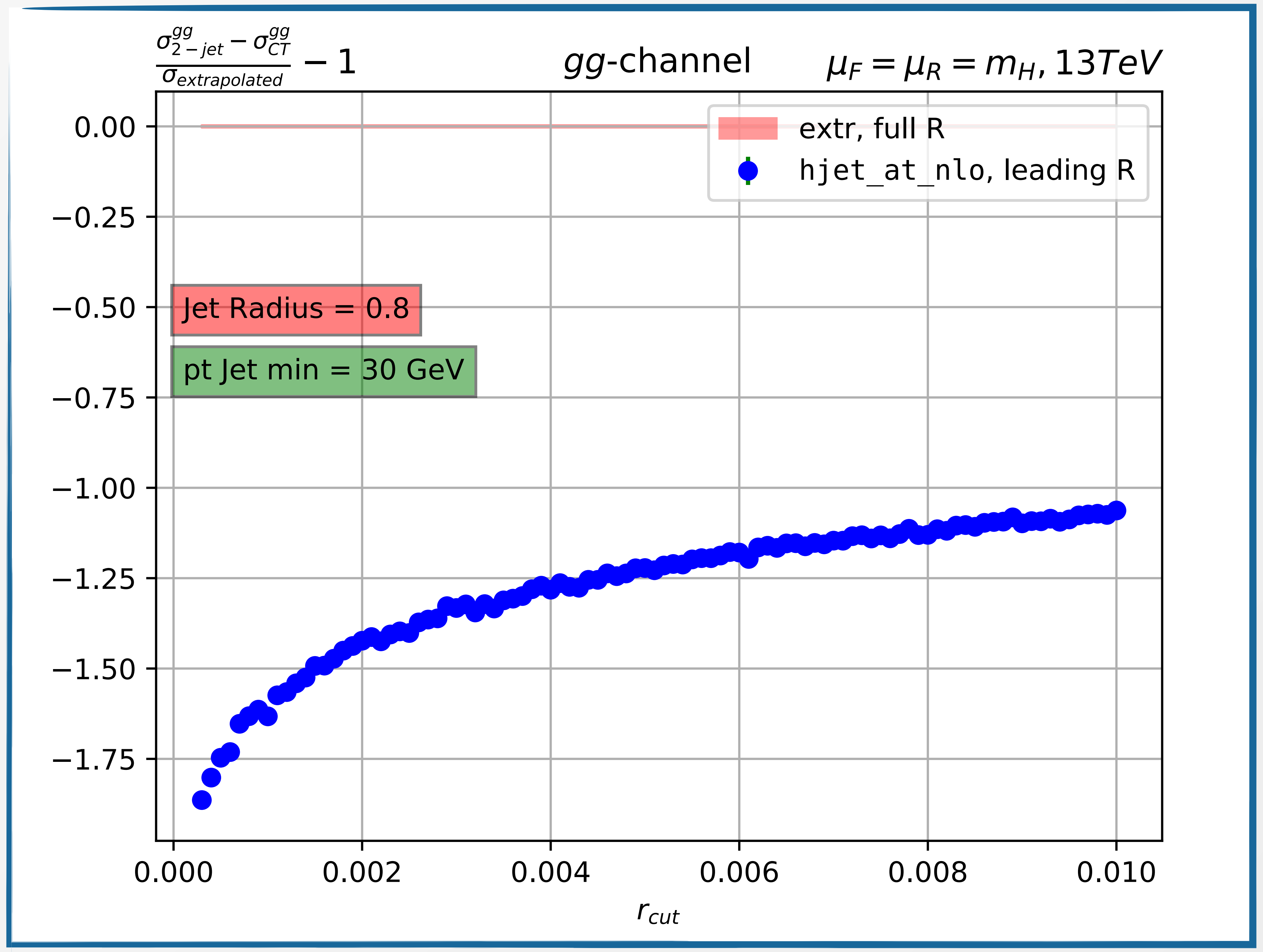
NLO [pb]	$\mu_F = \mu_R = m_H$
q_T subtraction	13.256 ± 0.034
mcfm	13.250 ± 0.007
LO [pb]	7.758 ± 0.007

Non-local subtraction at NLO for H+j: dependence on the jet radius

[M. Costantini Master's thesis, UZH]

Exact dependence on the jet radius crucial to ensure proper cancellation of logarithmic enhanced terms





The quest for novel resolution variables

q_T-imbalance has nice convergence properties but has some limitations, which makes the extension at higher orders more complex:

- The observable is defined through a jet algorithm, which induces a dependence on an additional cutting variable (the jet radius R)
- The resummation of q_T-imbalance involves additional difficulties such as NGL entering at $\mathcal{O}(\alpha_s^2)$

A variable which does not suffer from these problems in $V+j$ production is the difference between the transverse energy and the transverse momentum of the vector boson

$$\Delta E_T = \sum_{i=1}^{n} |\vec{p}_{T,i}| - |\vec{p}_{T,V}|$$

ΔE_T as a resolution variable: challenges

The variable has however a **more convoluted structure** than q_T-imbalance due the different scalings in each singular region. Parametrising the emission with FKS variables,

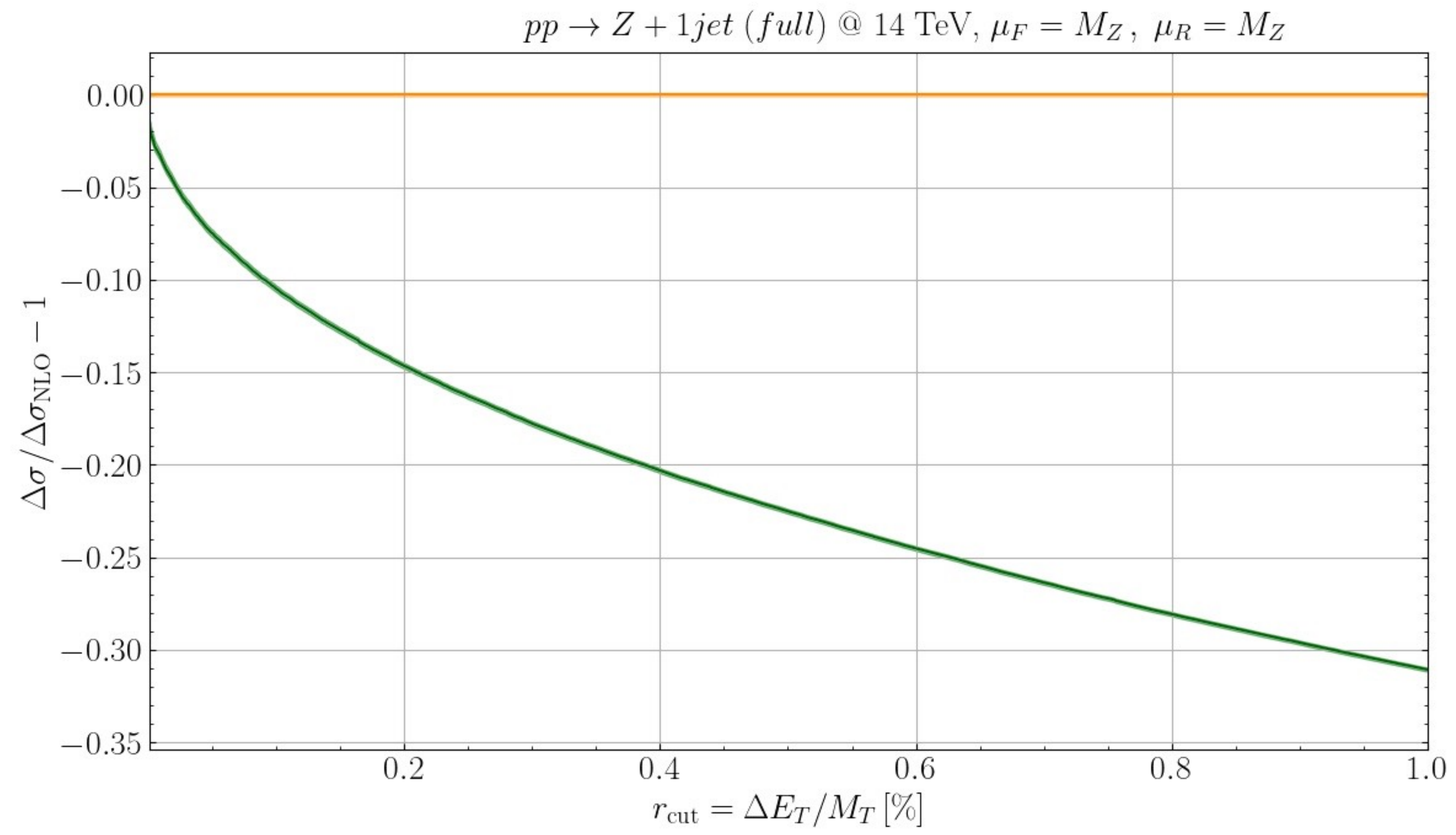
IS $$\Delta E_T \sim k_T(1+\cos\phi)$$

FS $$\Delta E_T \sim k_T\theta\sin(\phi)^2$$

The non-trivial dependence on ϕ leads to different beam functions with respect to q_T and makes their computation more delicate (need to take into account **polarised splitting kernels**)

Structure of the subtracted soft current also more involved (collinear singularity of final state no longer screened by a finite jet radius), also due to the different scaling of the observable in each region

ΔE_T as a resolution variable: results



Power corrections rather large, **logarithmic enhancement** makes the convergence problematic

Same behaviour as 1-jettiness. Perhaps related to the scaling of the observable?


TAKE 2

~~The quest~~ The long and winding road for novel resolution variables

q_T-imbalance has nice convergence properties but has some limitations, which makes the extension at higher orders more complex:

- The observable is defined through a jet algorithm, which induces a dependence on an additional cutting variable (the jet radius R)

- The resummation of q_T-imbalance involves additional difficulties such as NGL entering at $\mathcal{O}(\alpha_s^2)$

We look for a variable which has:

- Same convergence properties of q_T-imbalance: **linear scaling** (or better)
- Does not feature NGL
- Can be easily extended to an **arbitrary number of jets**

~~The quest~~ The long and winding road for novel resolution variables

q_T-imbalance … higher
orders more

We look for a

Our proposal: k_T^{ness}

[Buonocore, Grazzini, Haag, LR, Savoini 2201.11519]

Global dimensionful variable capable of capturing the $N \to N+1$ jet transition

Physically, the variable represents an **effective transverse momentum** in which the additional jet is unresolved:

- When the unresolved radiation is close to the colliding beams, k_T^{ness} coincides with the transverse momentum of the final state system.
- When the unresolved radiation is emitted close to one of the final-state jets, k_T^{ness} describes the relative transverse-momentum with respect to the jet direction

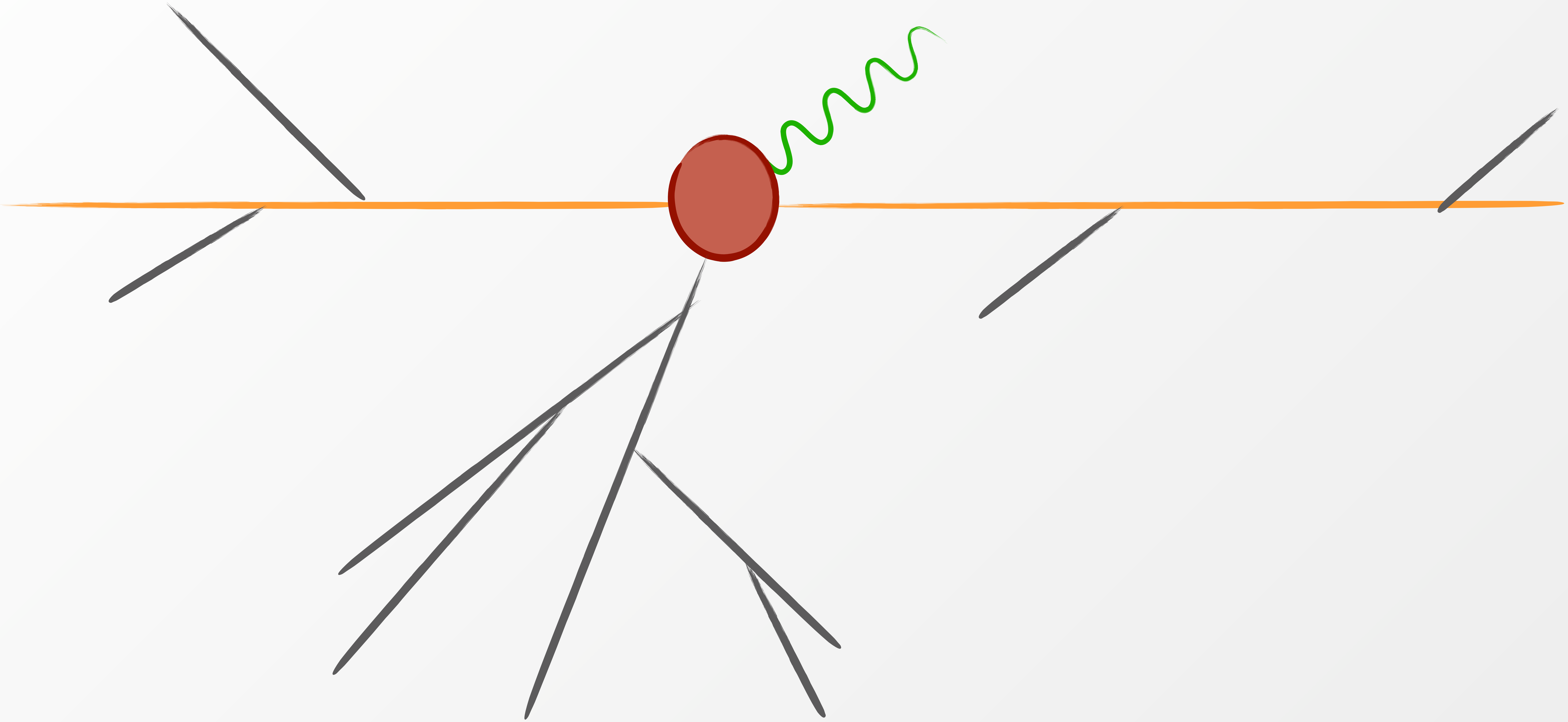
The variable takes its name from the k_T clustering algorithm and is defined via a **recursive procedure**

Definition of N-k_T^{ness}

Run the k_T clustering algorithm till $N+1$ proto-jets are left

[Catani, Dokshitzer, Seymour,Webber][Ellis, Soper]

$$d_{ij} = \min(p_{Ti}, p_{Tj})\Delta R_{ij}/D, \qquad d_{iB} = p_{Ti}$$



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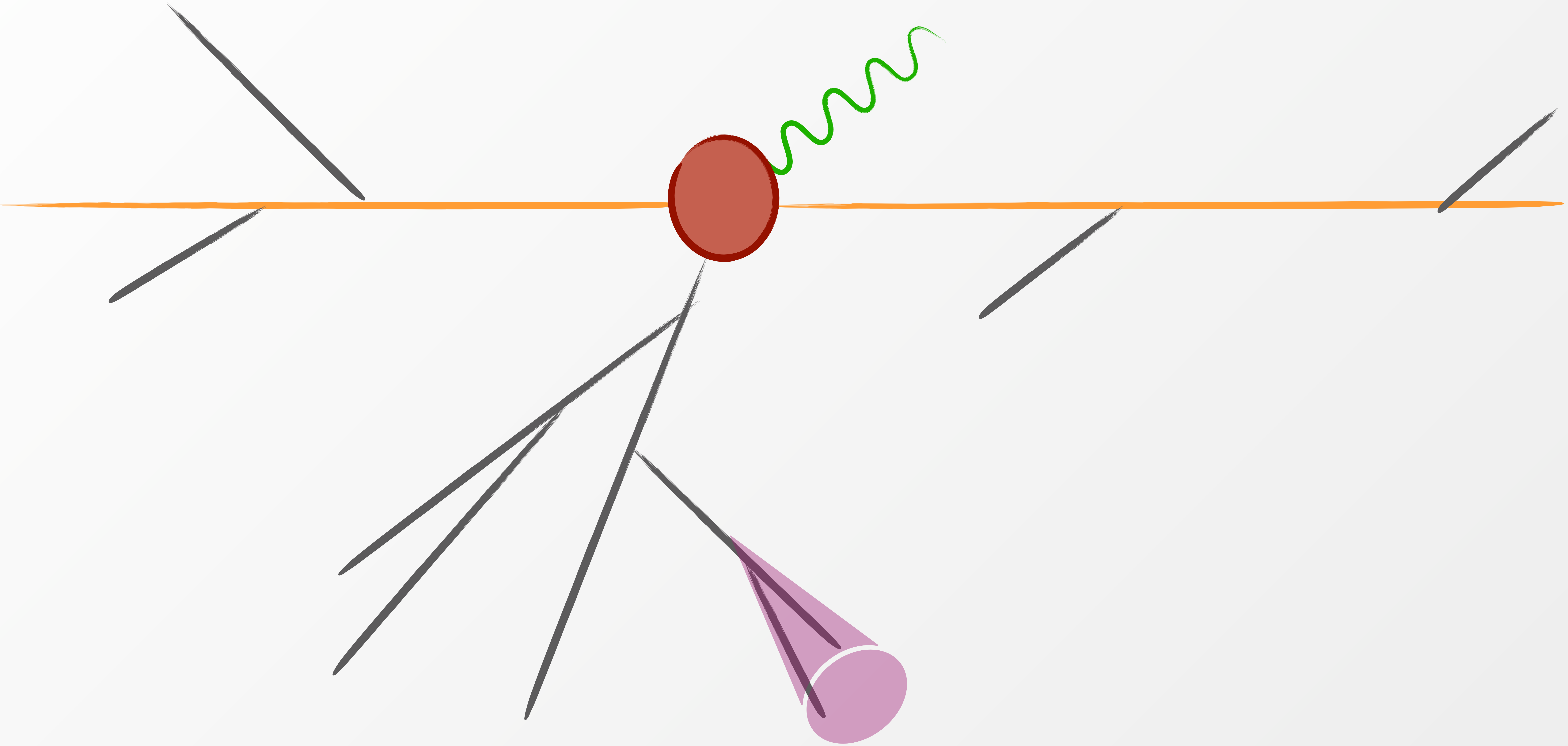

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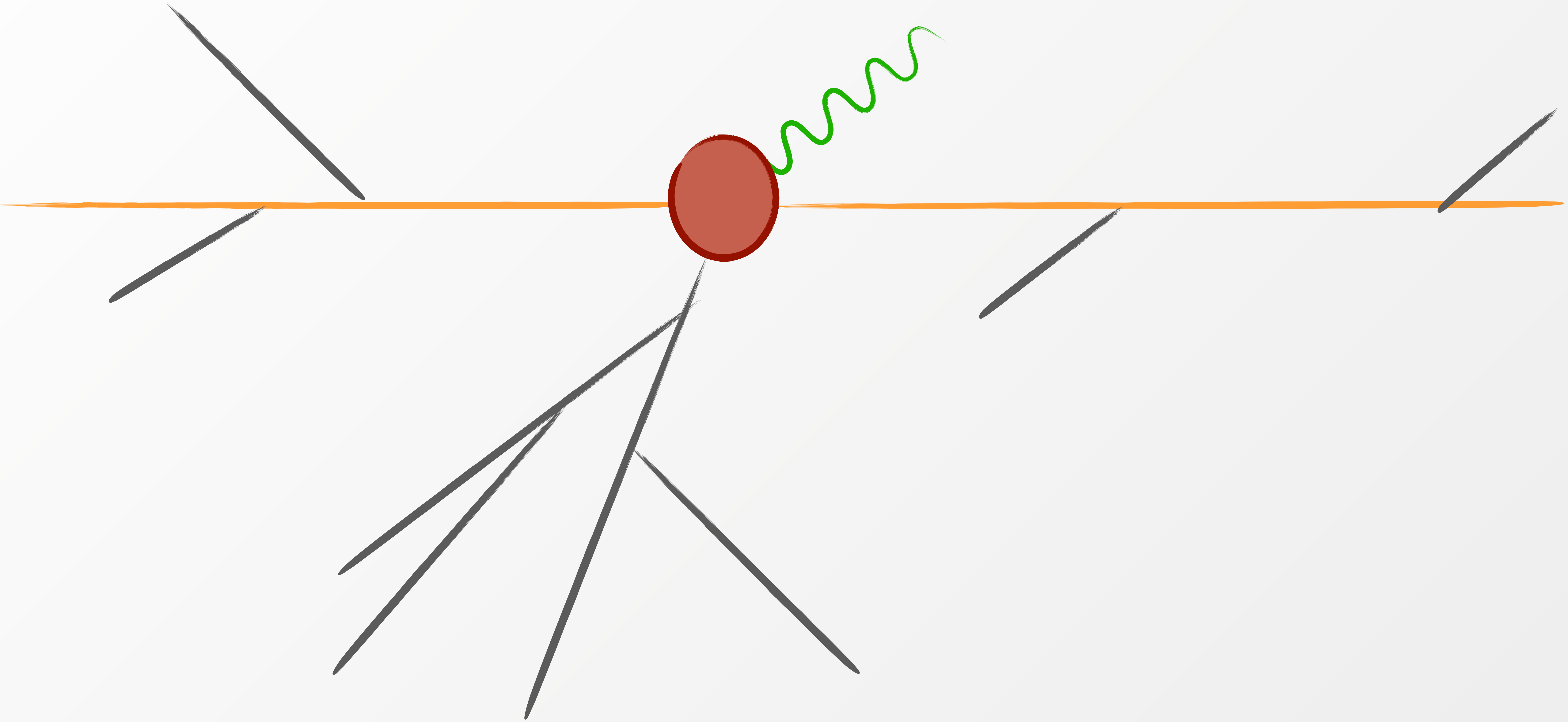

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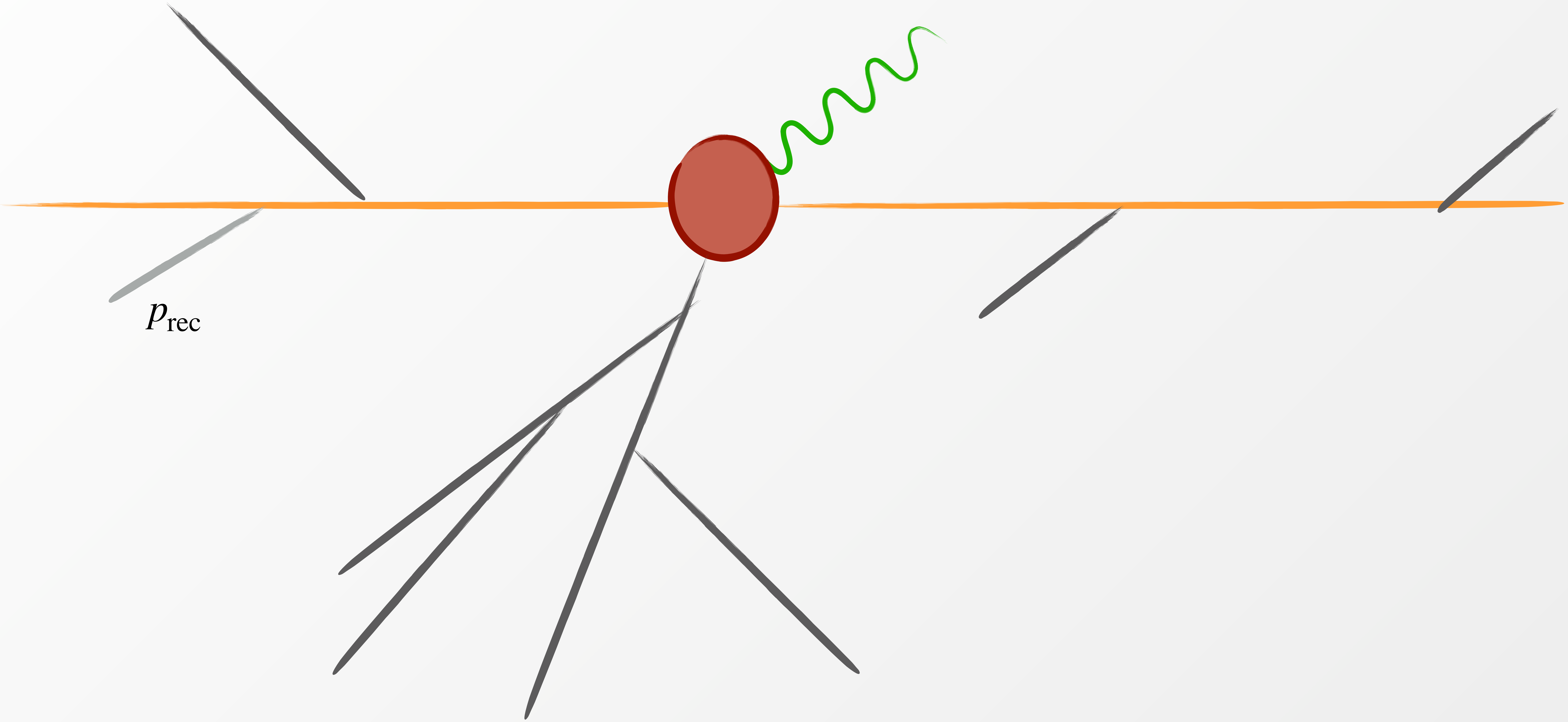

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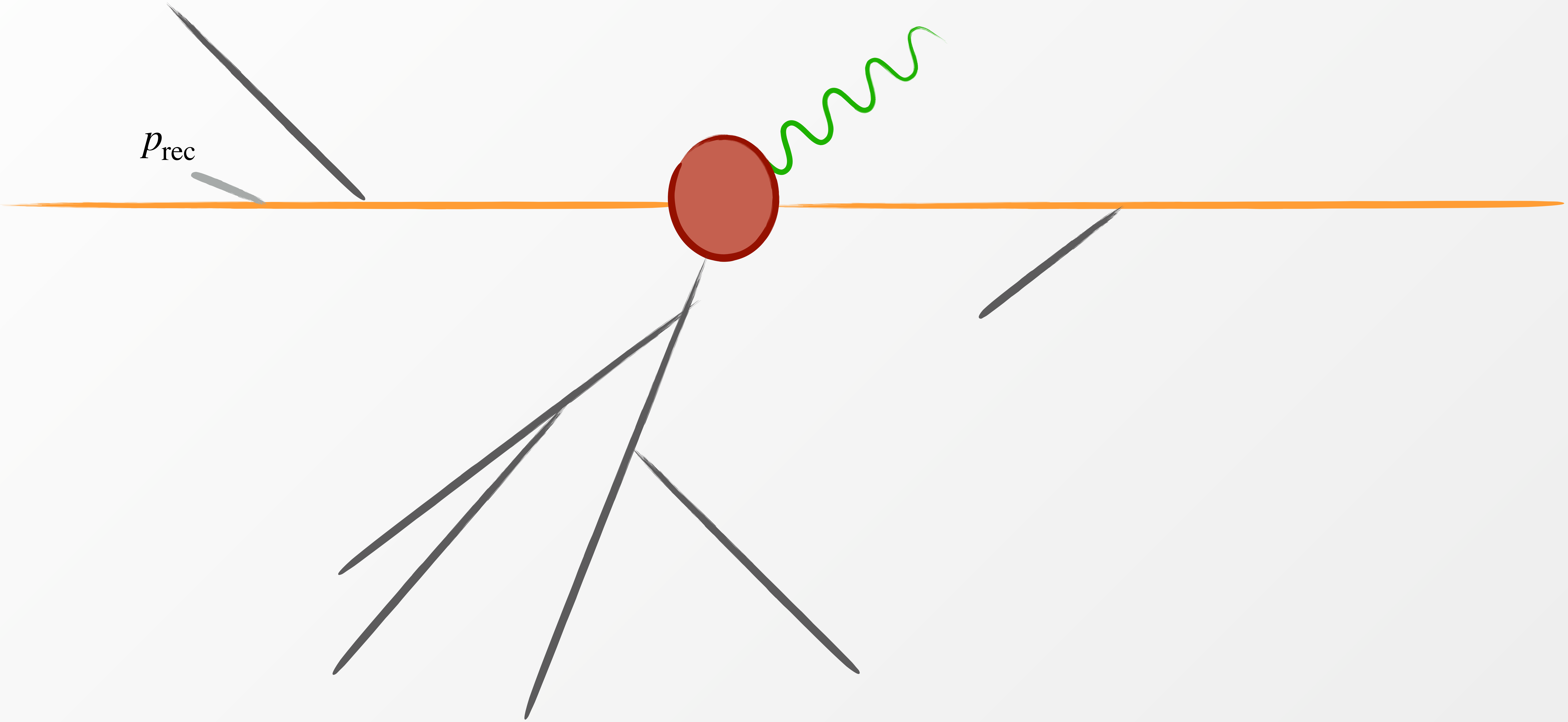

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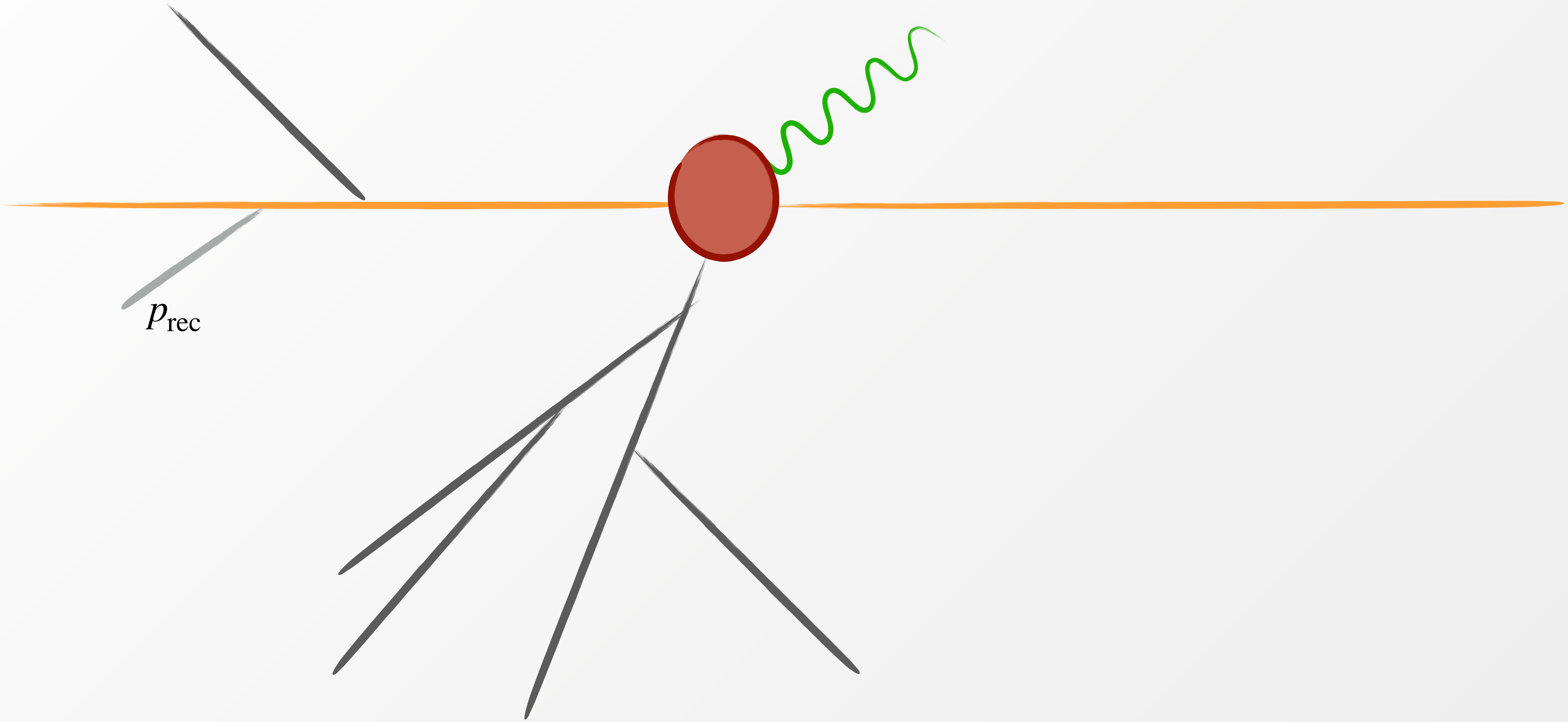

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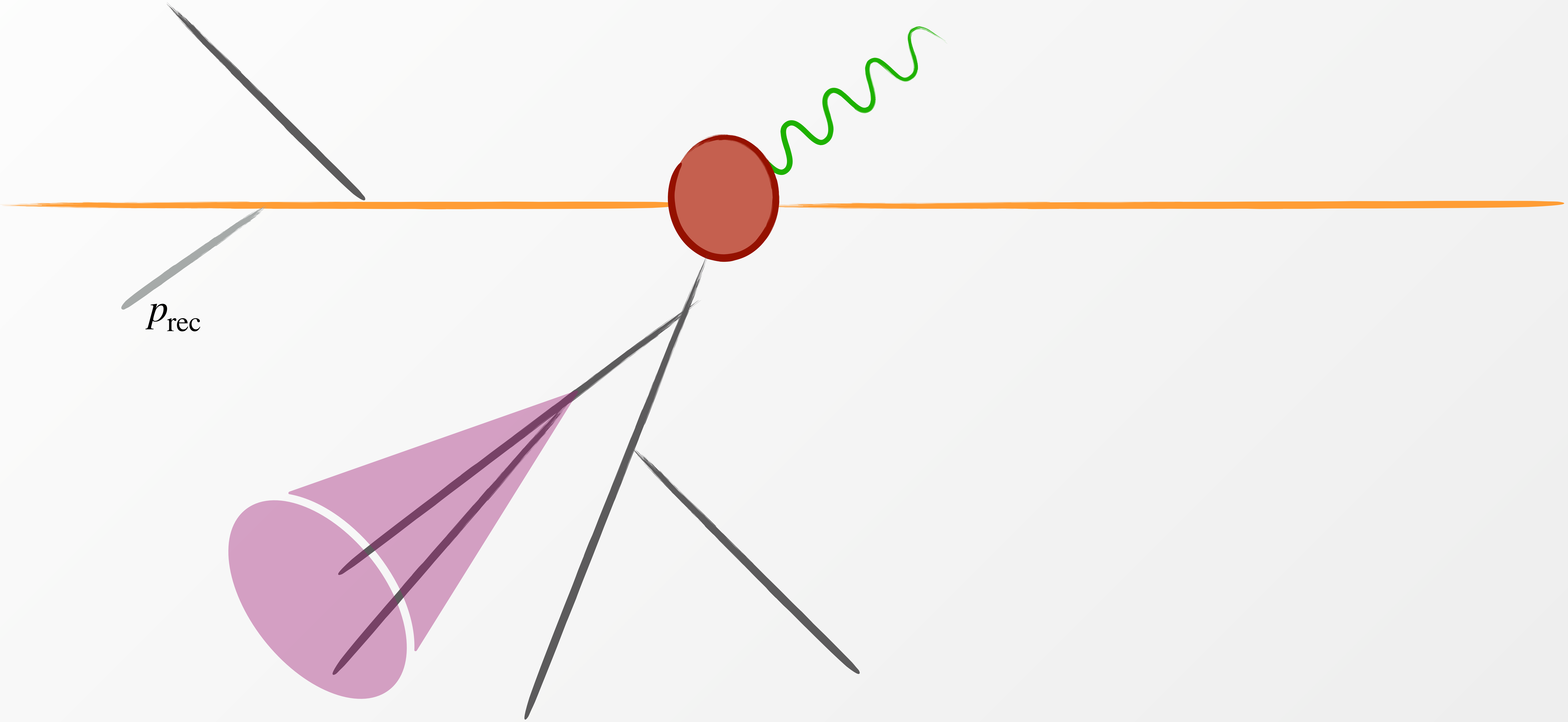

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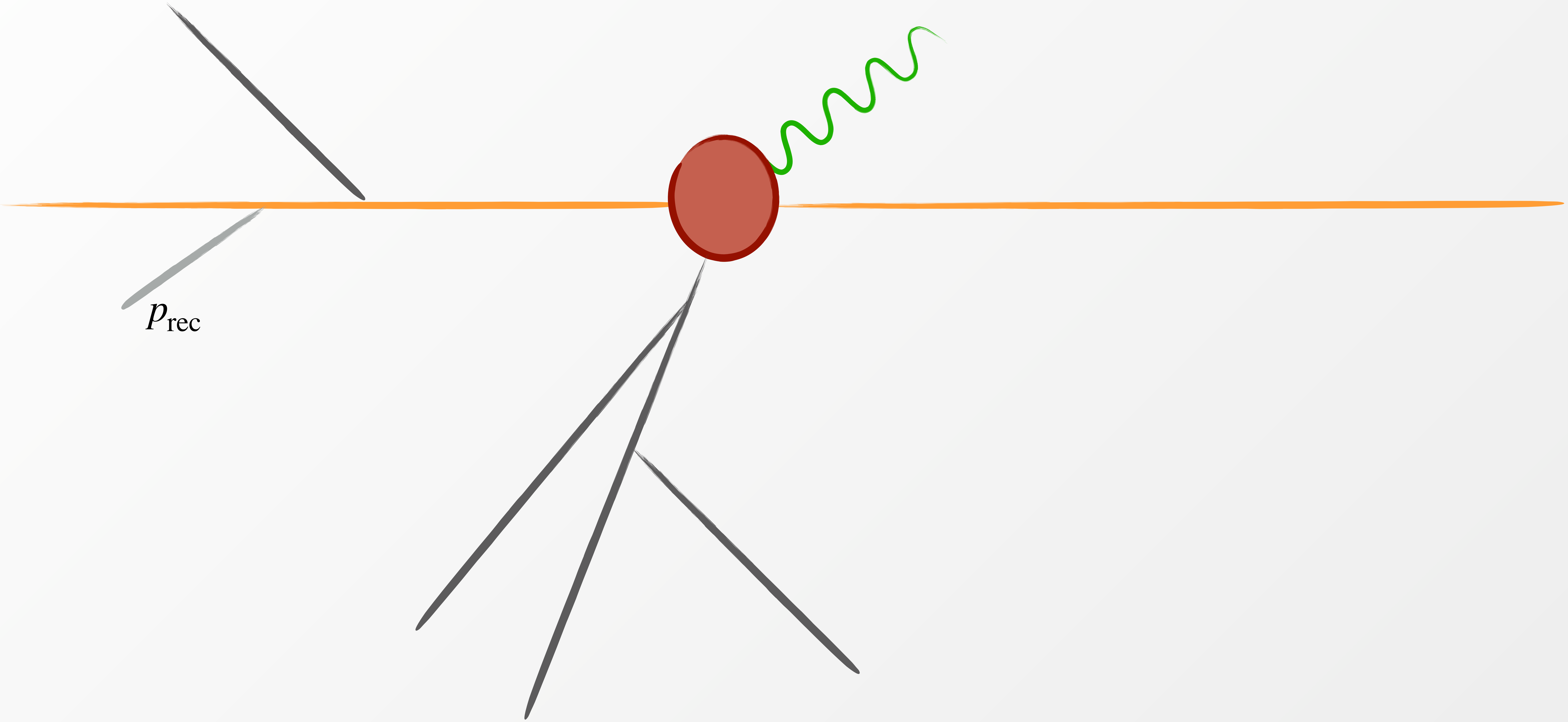

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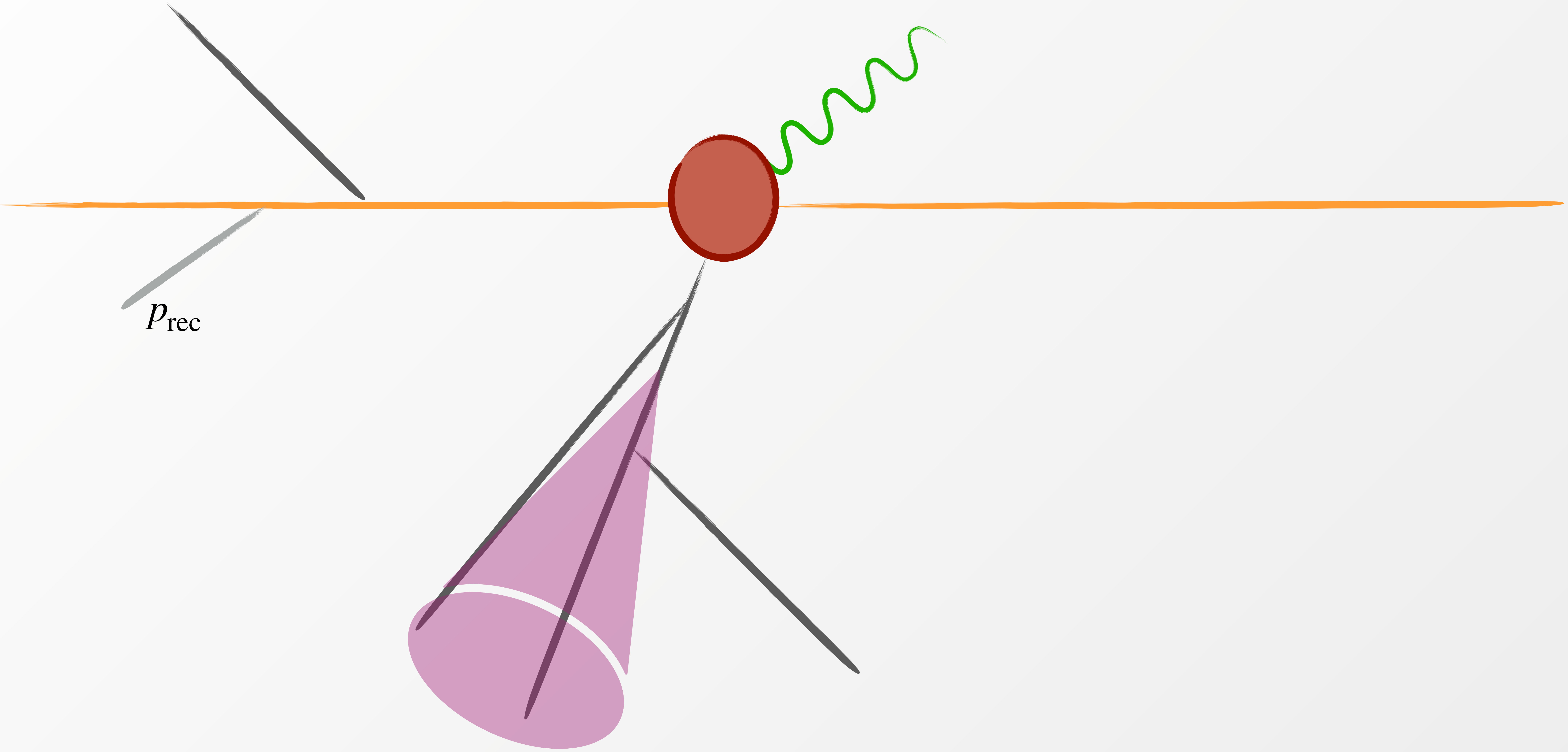

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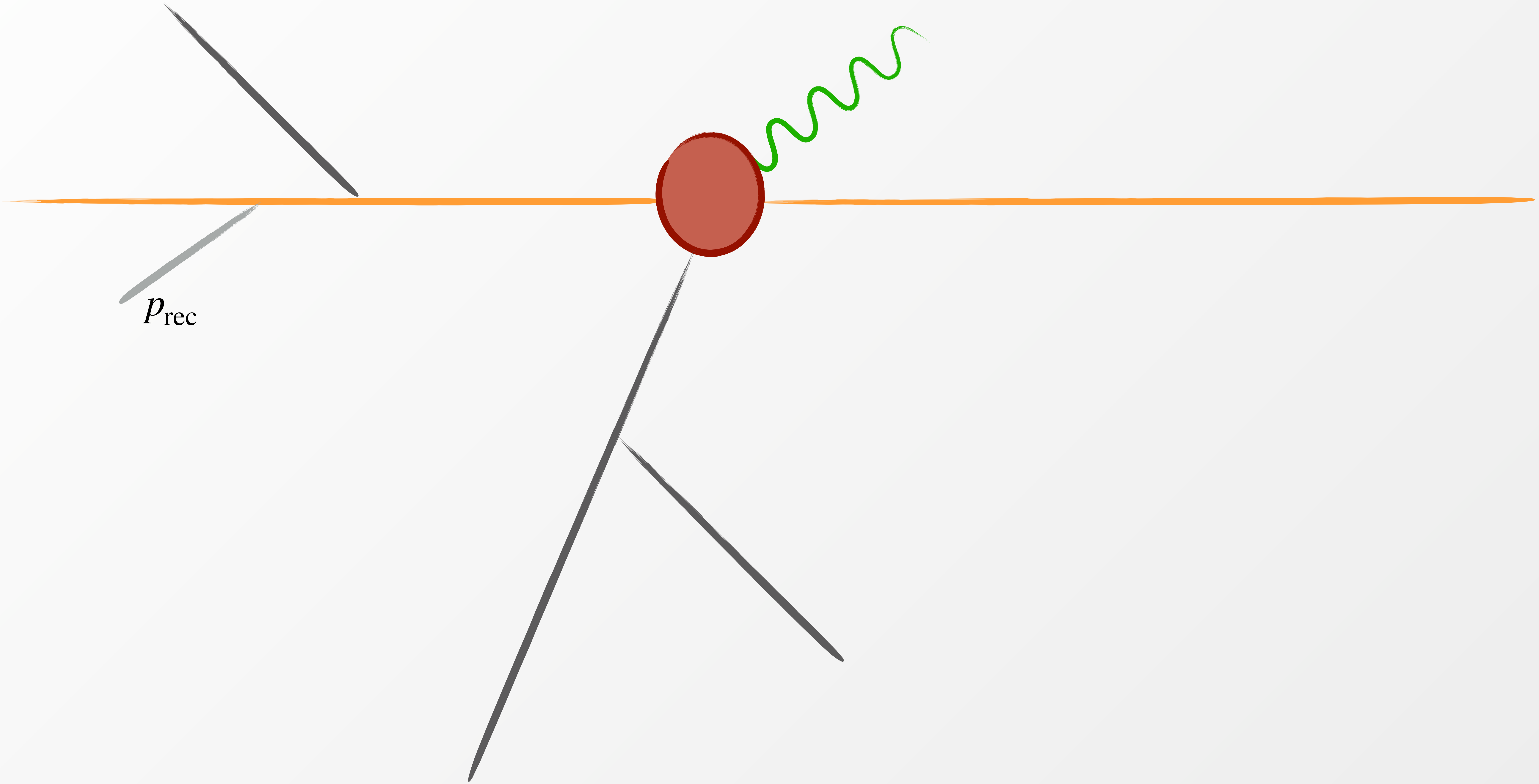

Definition of N-$k_T^{\rm ness}$

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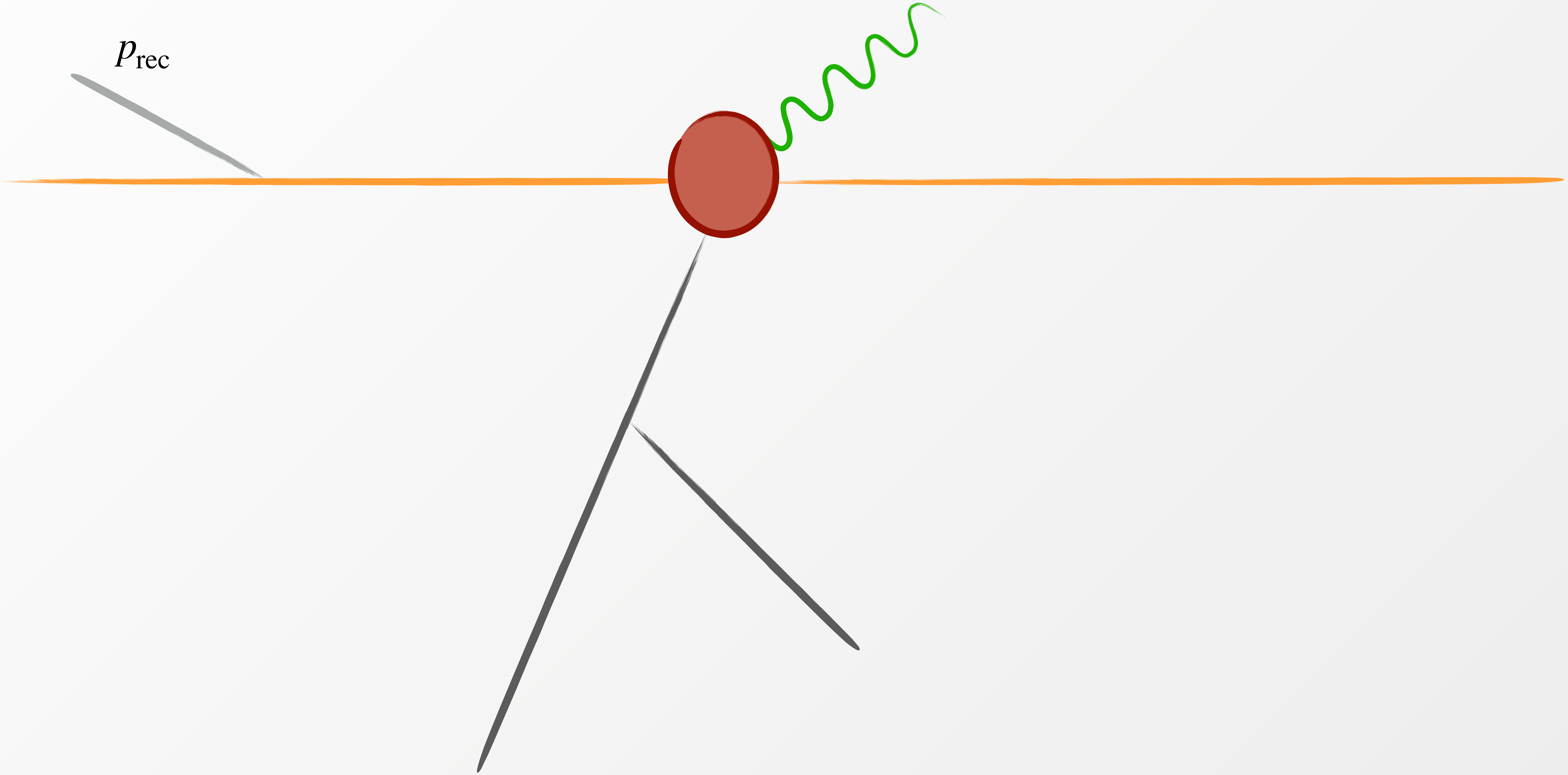
[Catani, Dokshitzer, Seymour,Webber][Ellis, Soper]



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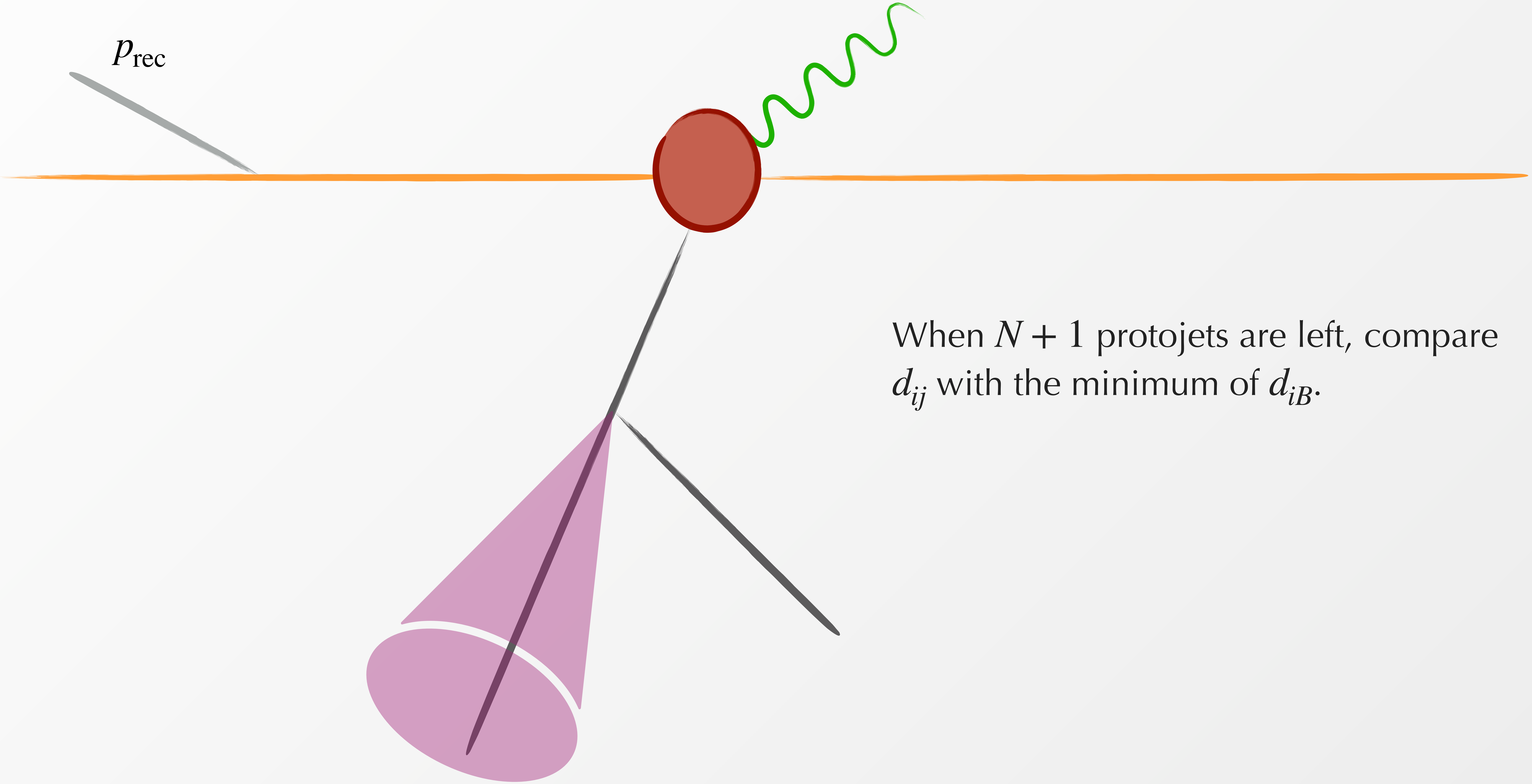

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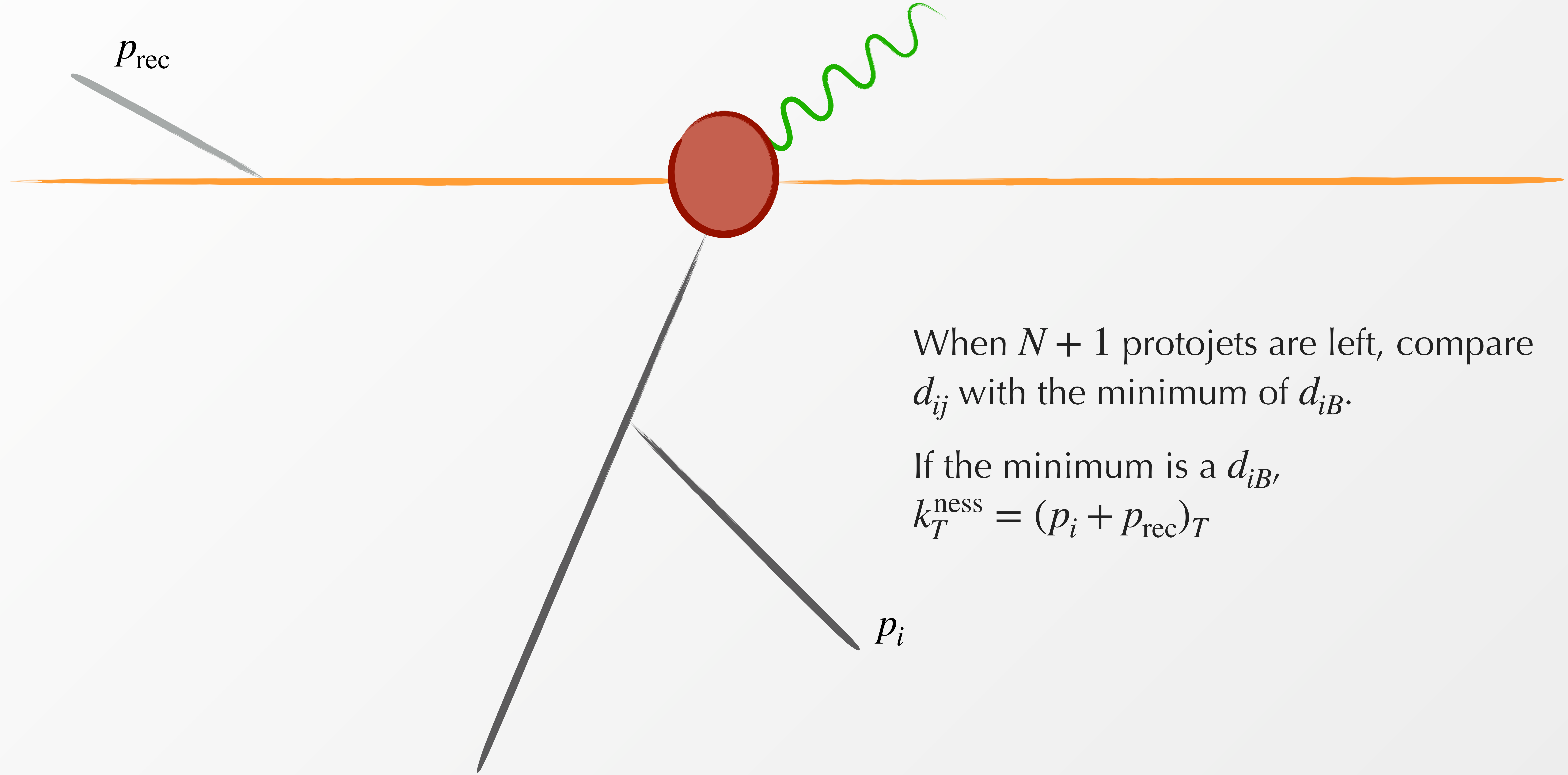

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If the minimum is a d_{iB},
$k_T^{\text{ness}} = (p_i + p_{\text{rec}})_T$

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Run the k_T clustering algorithm till $N+1$ proto-jets are left
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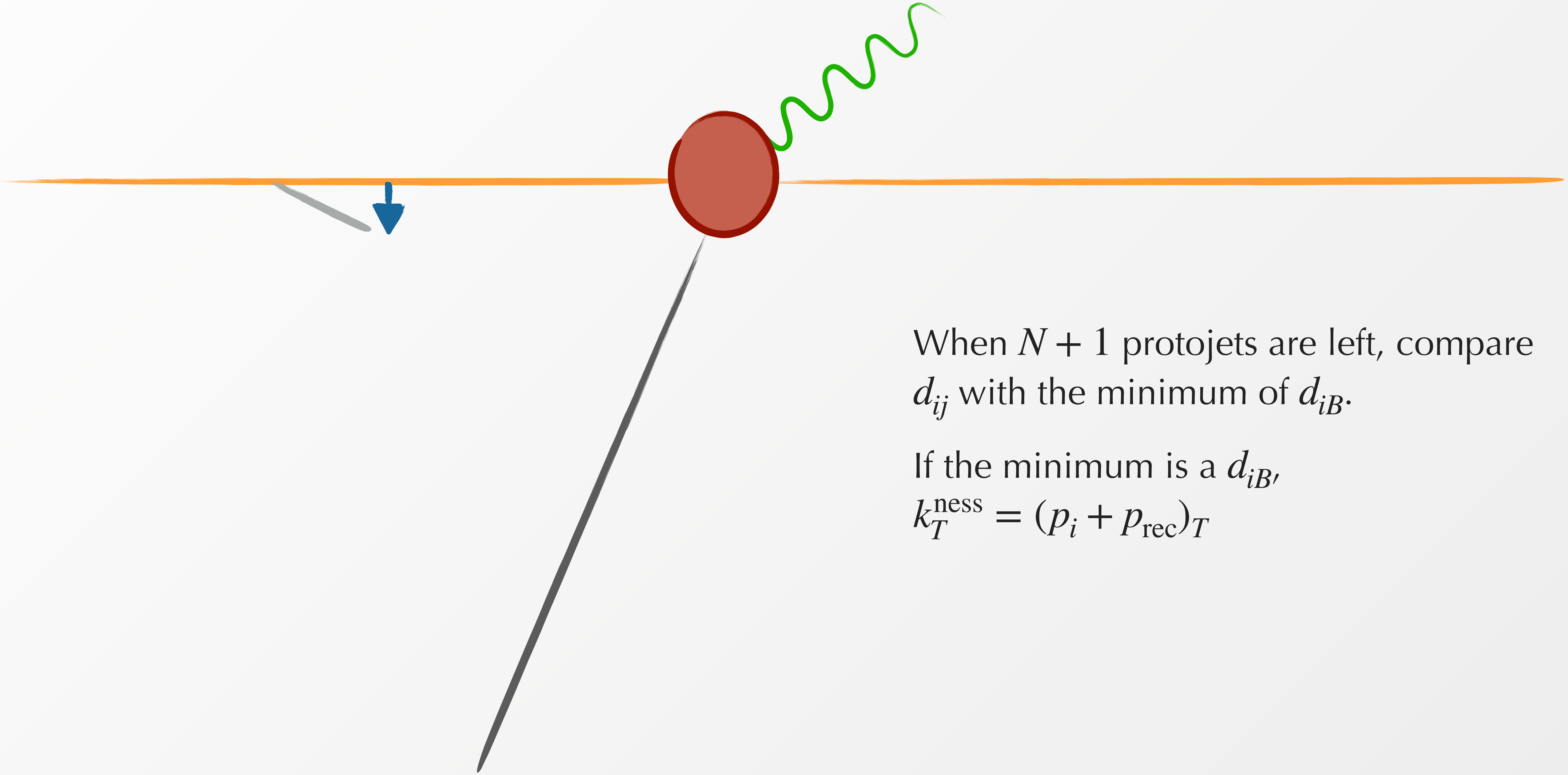

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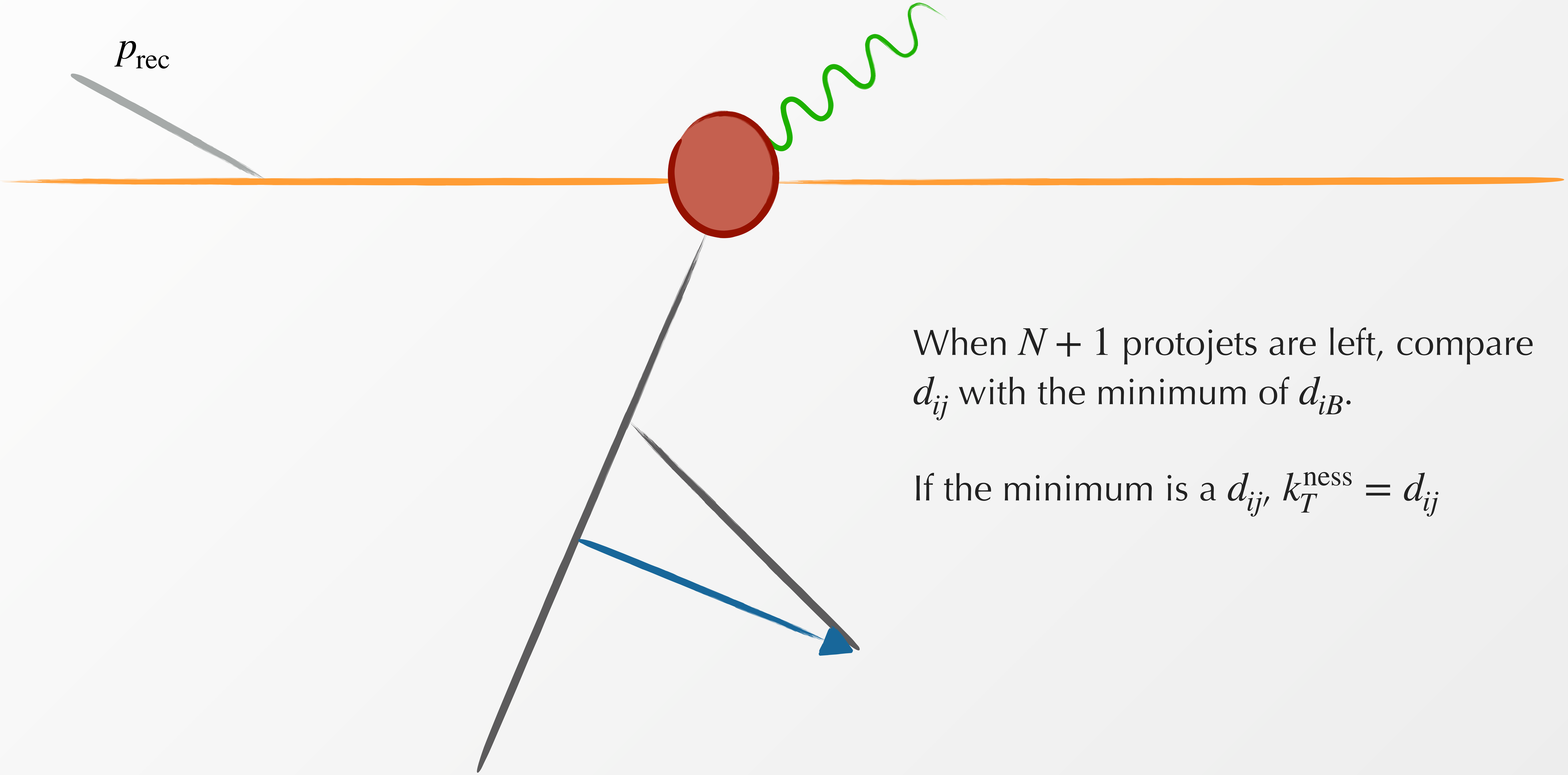


When $N+1$ protojets are left, compare d_{ij} with the minimum of d_{iB}.

If the minimum is a d_{ij}, $k_T^{\rm ness} = d_{ij}$

k_T^{ness}-subtraction

We have computed the singular structure in the limit $k_T^{\text{ness}} \to 0$ at NLO to construct a **non-local subtraction**

$$d\hat{\sigma}_{\text{NLO}}^{\text{F+N jets+X}} = \mathscr{H}_{\text{NLO}}^{\text{F+N jets}} \otimes d\hat{\sigma}_{\text{LO}}^{\text{F+N jets}} + \left[d\hat{\sigma}_{\text{LO}}^{\text{F+(N+1) jets}} - d\hat{\sigma}_{\text{NLO}}^{\text{CT,F+Njets}} \right]$$

Computation of the relevant coefficients proceeds by identifying singular regions and removing the double counting

Structure of the counterterm **remarkably simple**

$$\hat{\sigma}_{\text{NLO}\,ab}^{\text{CT,F+Njets}} = \frac{\alpha_s}{\pi} \frac{dk_t^{\text{ness}}}{k_t^{\text{ness}}} \Bigg\{ \left[\ln \frac{Q^2}{(k_t^{\text{ness}})^2} \sum_\alpha C_\alpha - \sum_\alpha \gamma_\alpha - \sum_i C_i \ln\left(D^2\right) - \sum_{\alpha\neq\beta} \mathbf{T}_\alpha \cdot \mathbf{T}_\beta \ln\left(\frac{2p_\alpha \cdot p_\beta}{Q^2} \right) \right] \times$$

$$\delta_{ac}\delta_{bd}\delta(1-z_1)\delta(1-z_2) + 2\delta(1-z_2)\delta_{bd}P_{ca}^{(1)}(z_1) + 2\delta(1-z_1)\delta_{ac}P_{db}^{(1)}(z_2) \Bigg\} \otimes d\hat{\sigma}_{\text{LO}\,cd}^{\text{F+N jets}}$$

$$\gamma_q = 3C_F/2$$

$$\gamma_g = (11C_A - 2n_F)/6$$

$\mathscr{H}$ contains the finite remainder from the cancellation of singularities of real and virtual origin, and the finite contributions embedded in beam (same as those of q_T), jet and soft functions (which we computed)

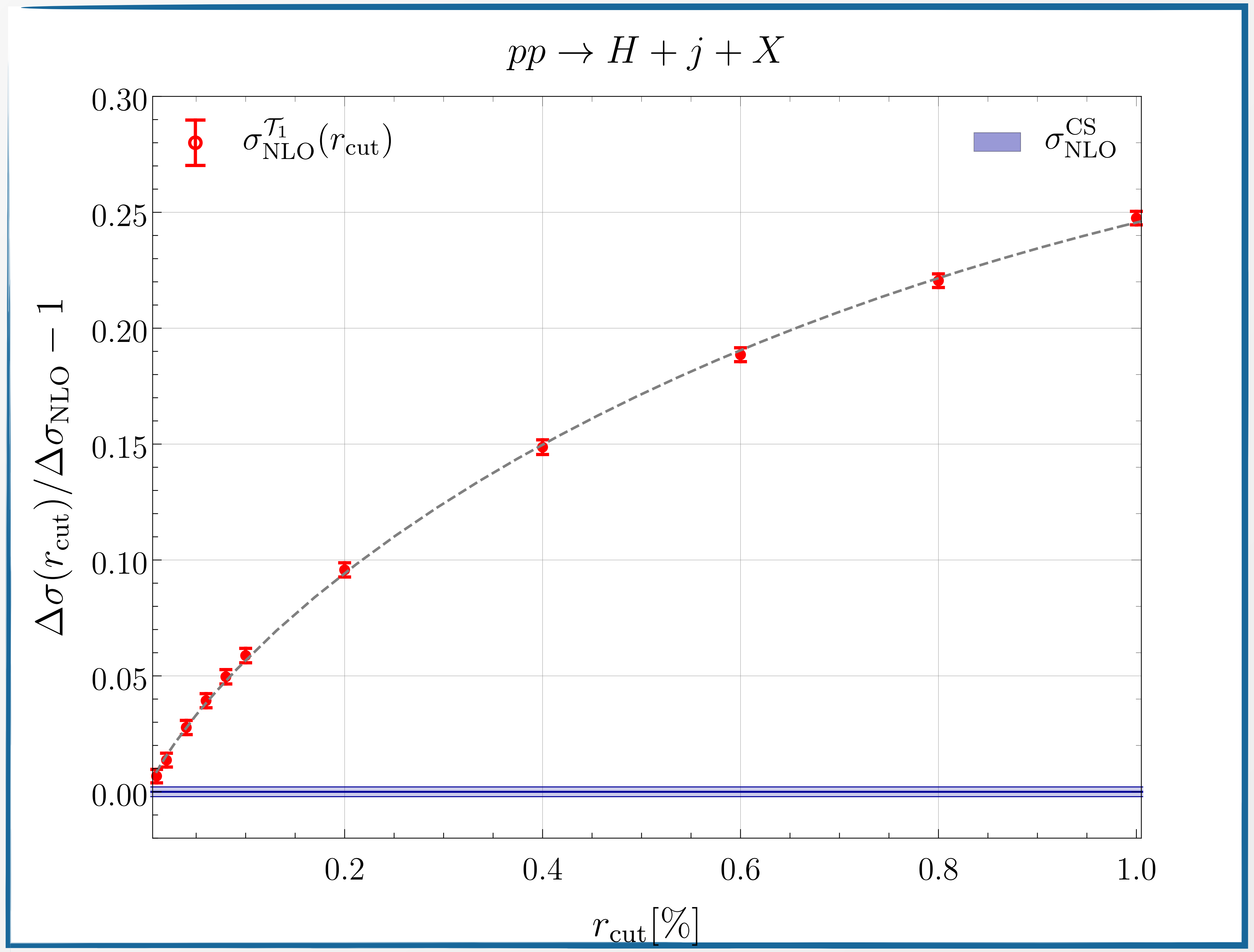
Phenomenological application: $H + j$ production

We have implemented our calculation first to $H + j$ production. Amplitudes from MCFM

We set the parameter D=1 and we require $p_T^j > 30$ GeV.

We compare our result with a **1-jettiness** calculation for the same process, which we implemented in MCFM

$$r = \mathcal{T}_1 / \sqrt{m_H^2 + (p_T^j)^2} \qquad r = k_T^{\text{ness}} / \sqrt{m_H^2 + (p_T^j)^2}$$



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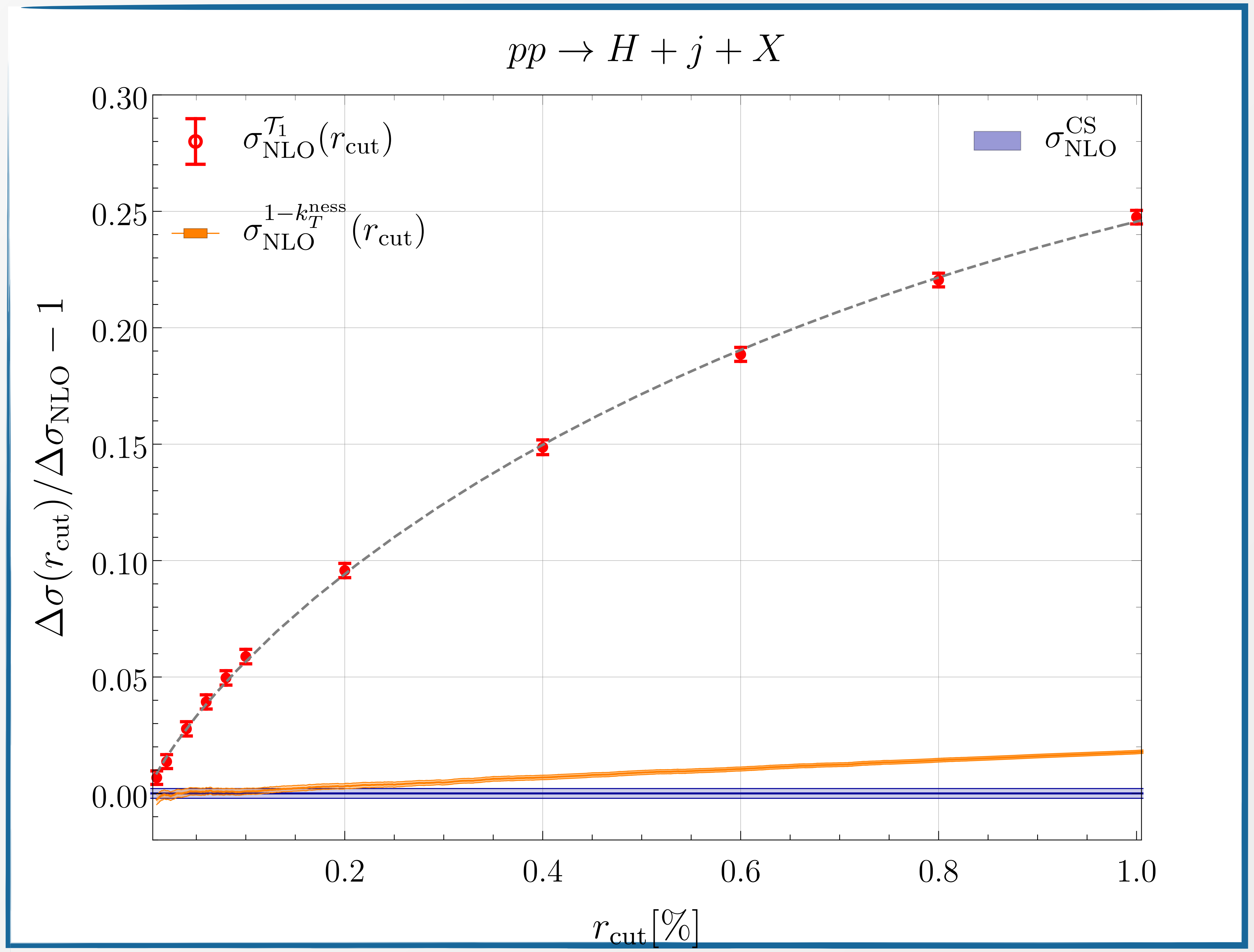


Faster convergence, power corrections compatible with **purely linear behaviour**

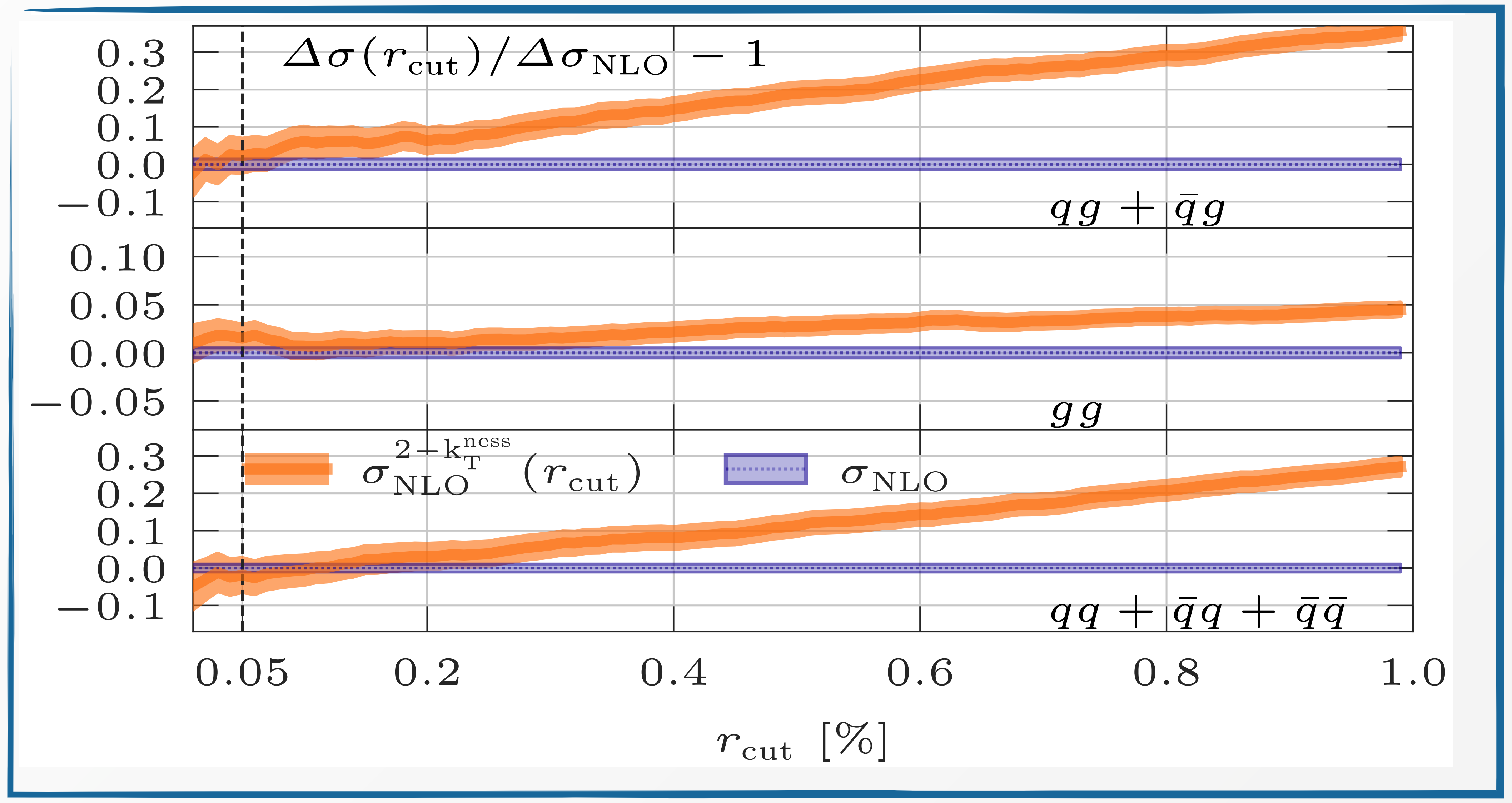
Excellent control of the NLO correction

Phenomenological application: $Z + 2j$ production

We also considered a process with a more complex final state with a non-trivial colour structure

Our implementation uses colour-correlated amplitudes from OL [Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller]

In this case we set the parameter D=0.1 and we require $p_T^j > 30$ GeV.



Power corrections exhibit **linear behaviour** in all partonic channels

Control of the NLO correction at the few **percent level**

Phenomenological application: $Z + 2j$ production

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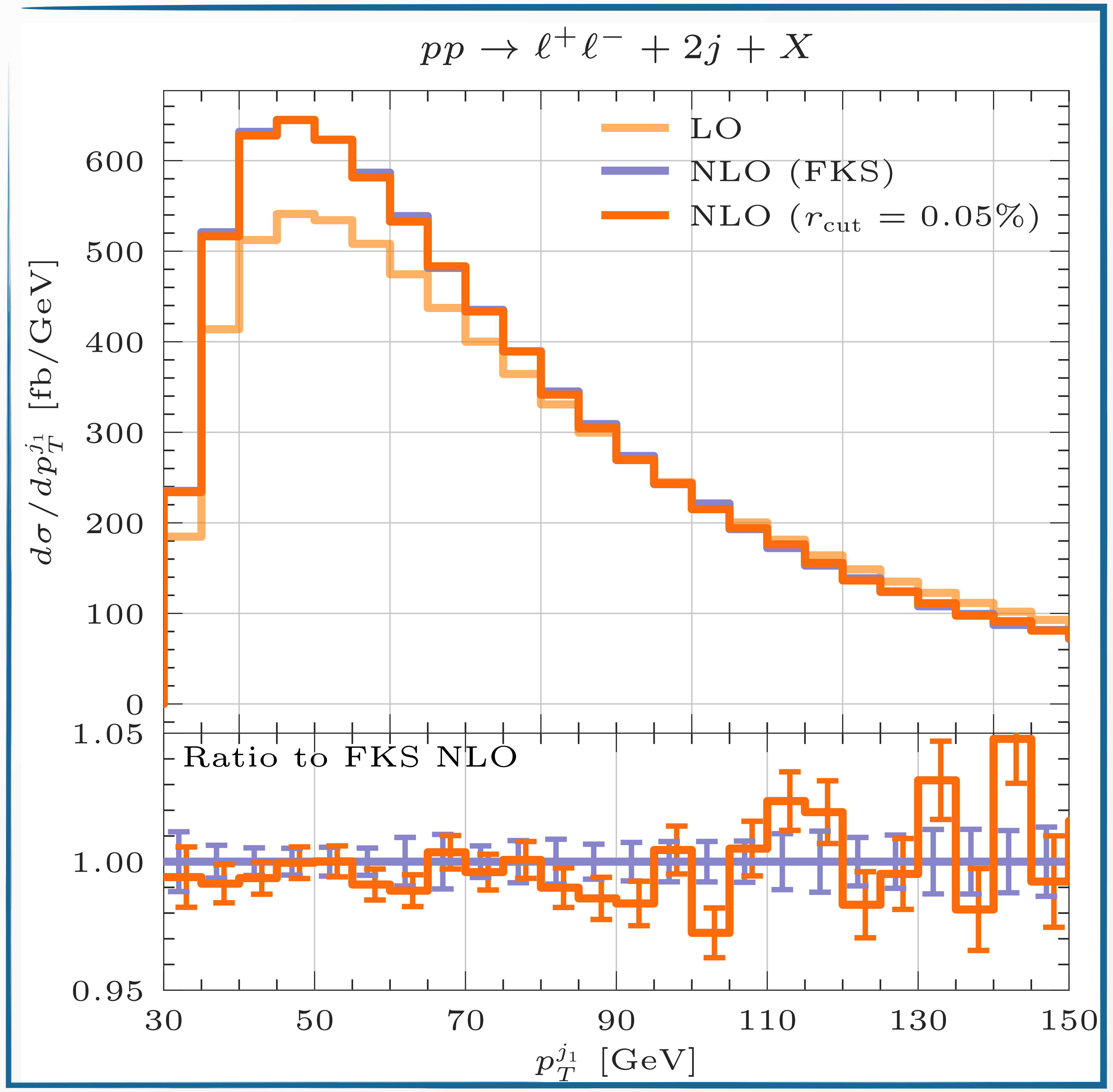
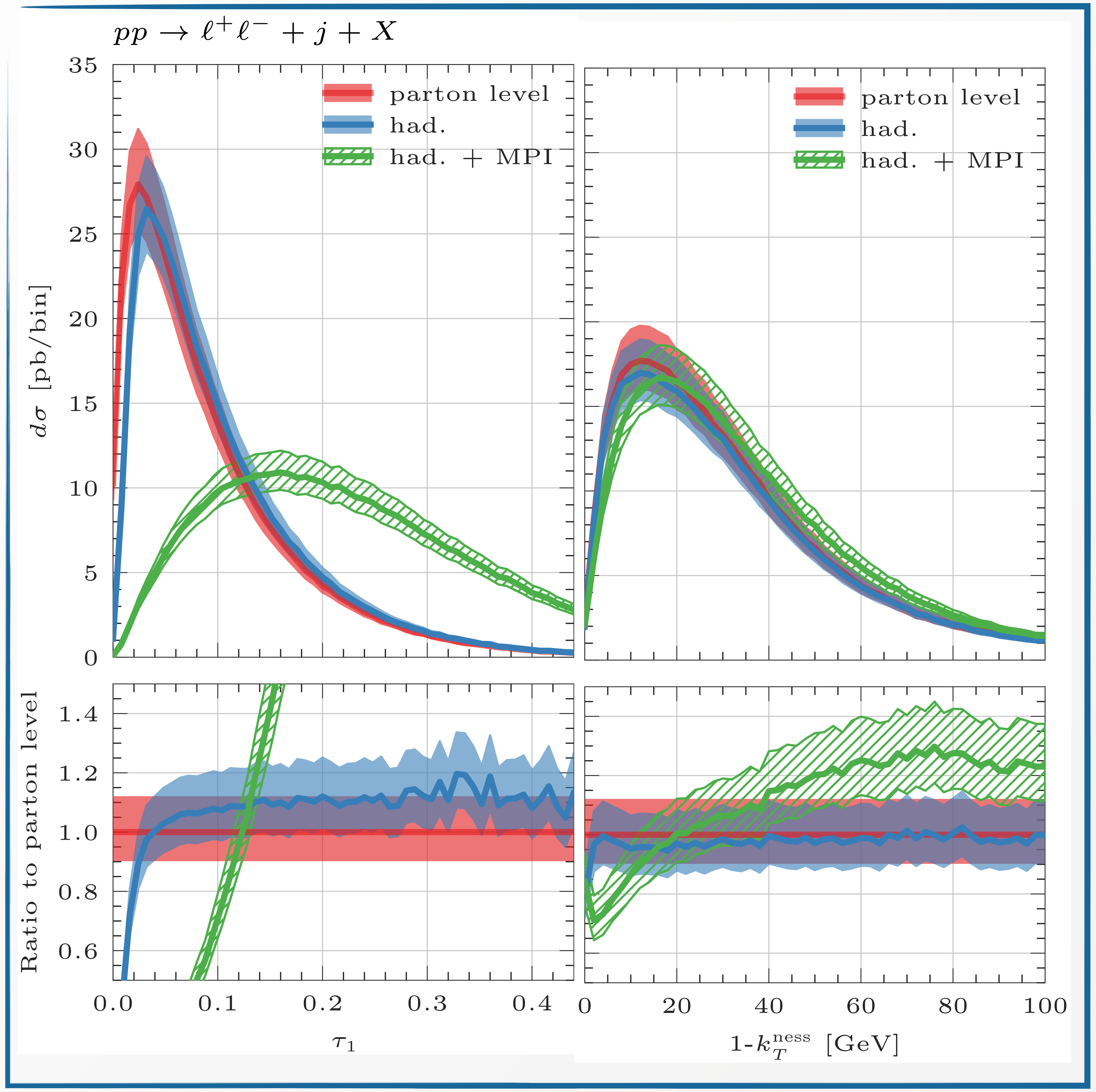


Nice agreement with results obtained with FKS subtraction (from POWHEG) for a variety of observables

Stability with respect to hadronisation and MPI



We have generated a sample of LO events for $Z + j$ with the POWHEG and showered them with PYTHIA8

We compare the impact of **hadronisation** and **MPI** on $k_T^{\rm ness}$

The distribution has a peak at ~ 15 GeV, which remain **stable** upon hadronisation and MPI

Effect of hadronisation marginal, MPI makes the distribution somewhat harder

Compared to 1-jettiness, effects are much reduced

Outlook and conclusion

- Exploration of novel variables in jet processes have a number of applications (resummation, non-local subtraction methods, matching with parton showers…)

- Resummation structure for variables defined in jet processes may involve additional theoretical challenges (non global logarithms, clustering effects, dependence on the jet algorithm, etc)

- We studied the resummation for q_T-imbalance at NLL′ keeping the dependence on the jet radius R with full azimuthal dependence

- We explored new variables in multi jet production. We defined a new variables, k_T^{ness}, which captures the singular structure of processes with N jets

- We computed the relevant ingredients to construct a subtraction at NLO and we tested it for processes with 1 and 2 jets

- The variable shows promising properties: it has mild power corrections, which make it a good candidate for an extension of the subtraction to NNLO; it is relatively stable under hadronisation and MPI; being an effective transverse momentum can prove useful as resolution variable in matching NNLO calculations to k_T-ordered parton shower