Resummation of transverse observables in direct space

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Based on Bizon, Monni, Re, LR, Torrielli '17 - present

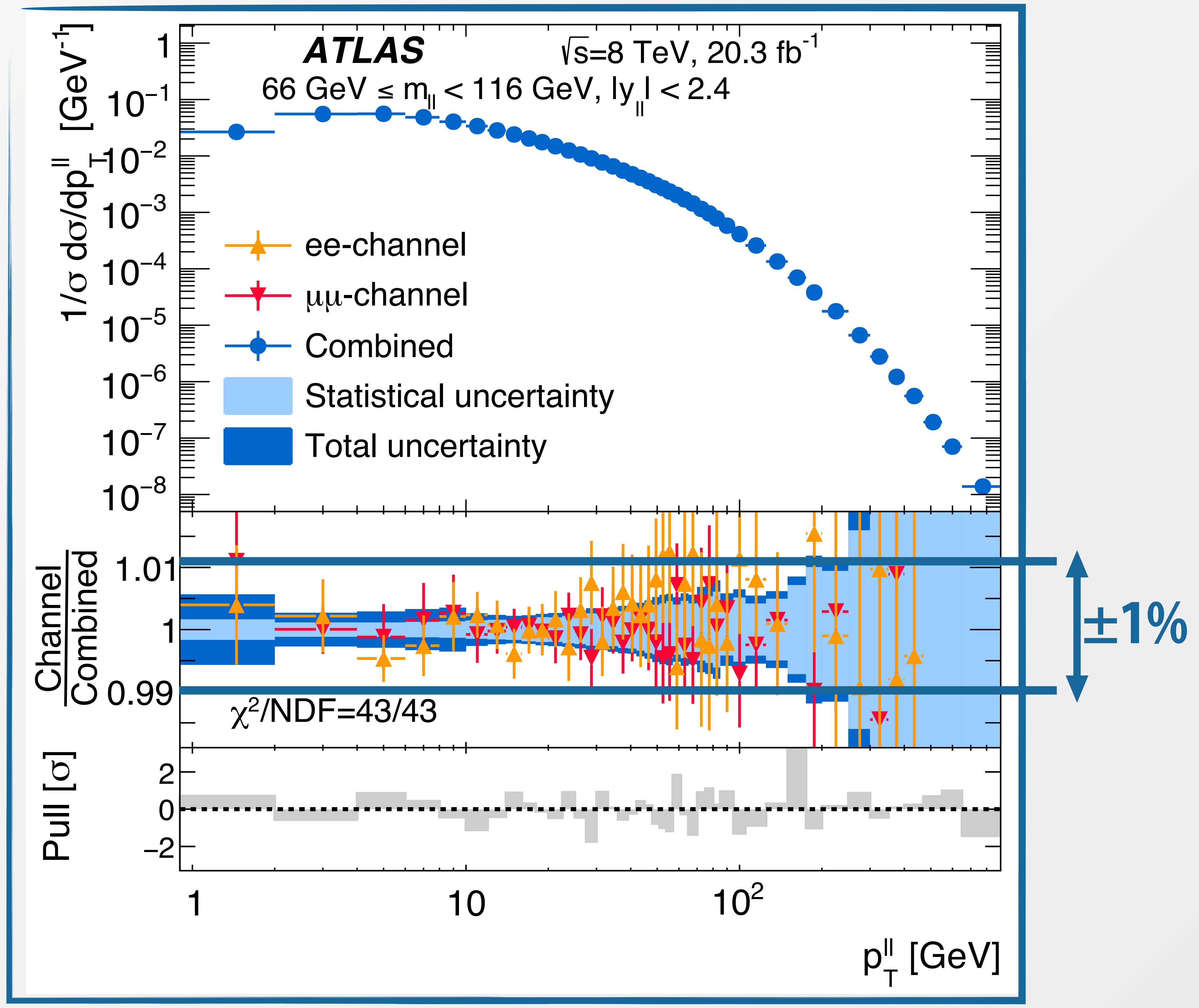
The quest for precision

Transverse observables (i.e. observables which do not depend on the rapidity of the radiation) are a **clean experimental and theoretical environment** for precision physics

Inclusive observables (e.g. transverse momentum p_t) probe directly the kinematics of the colour singlet

$$V(k_1, \ldots k_n) = V(k_1 + \ldots + k_n)$$

- negligible or no sensitivity to multi-parton interactions
- reduced sensitivity to non-perturbative effects
- measured **extremely precisely at experiments**, challenging current theoretical predictions

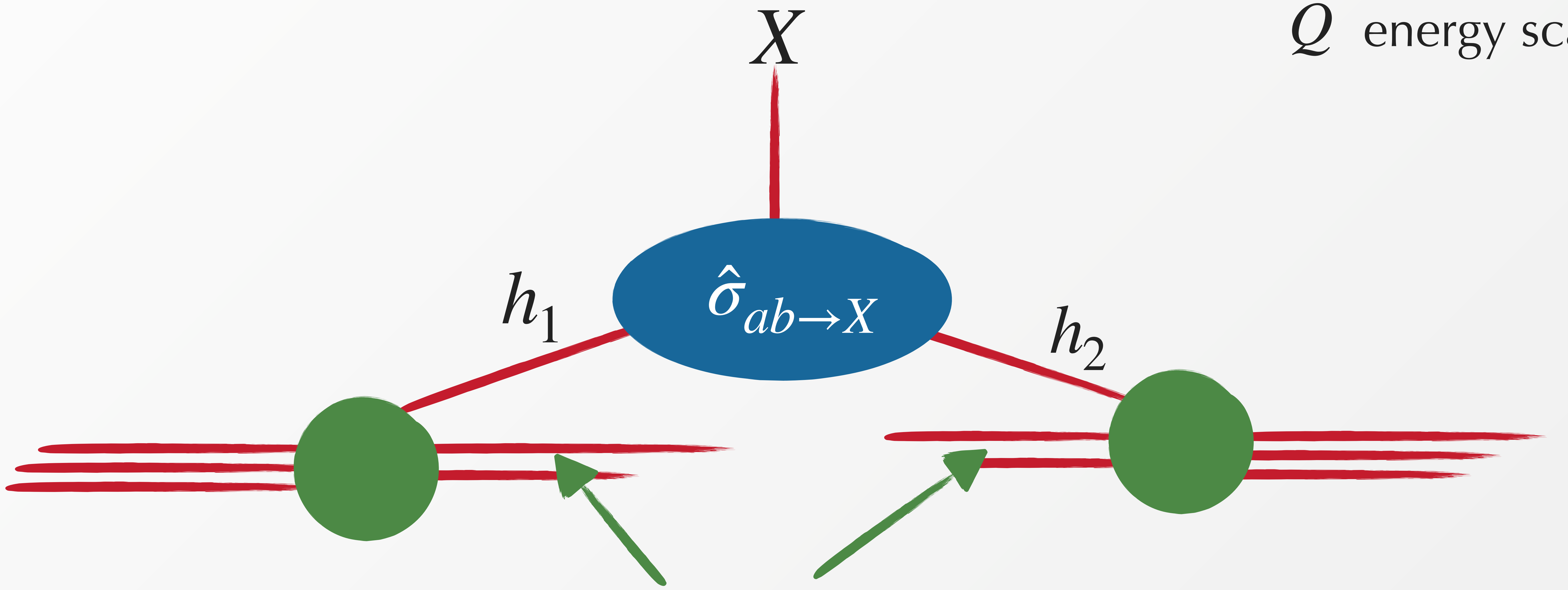


Important implications for extraction of SM parameters (strong coupling and PDF determination, ***W* mass measurements**…)

Precision physics at the LHC: theory

Key concept: **collinear factorization**

$\sqrt{s}$ centre-of-mass energy
Q energy scale of the process



$$\sigma(s,Q^2) = \sum_{a,b} \int dx_1 dx_2 f_{a/h_1}(x_1,Q^2) f_{b/h_2}(x_2,Q^2) \hat{\sigma}_{ab\to X}(Q^2, x_1 x_2 s) + \mathcal{O}(\Lambda^p_{\mathrm{QCD}}/Q^p)$$

Parton Distribution Functions (PDFs)

Long-distance, non-perturbative, universal objects

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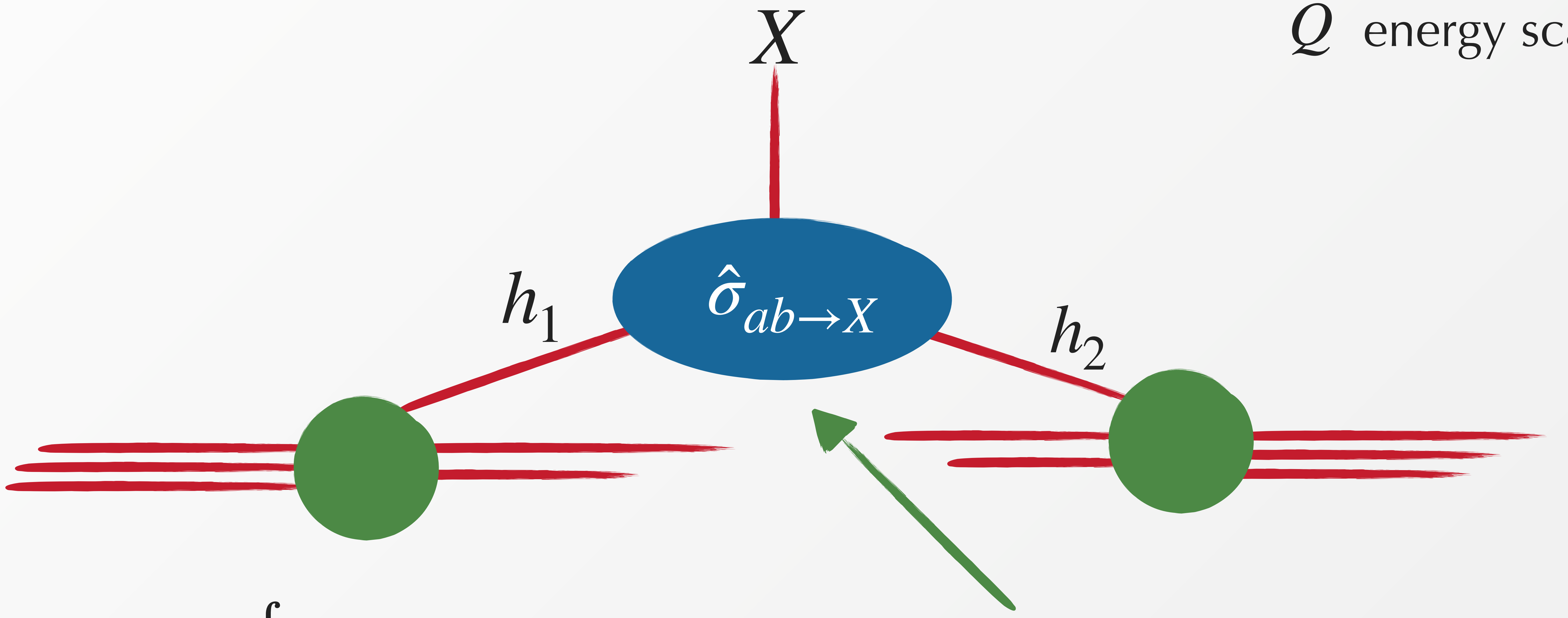

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Hard-scattering matrix element

Short-distance, perturbative, process-dependent

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Input parameters:

strong coupling α_s

PDFs f

few percent uncertainty; improvable

Non-perturbative effects

percent effect; not yet under control

Precision physics at the LHC: theory

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$$\tilde{\sigma} = 1 + \alpha_s \tilde{\sigma}_1 + \alpha_s^2 \tilde{\sigma}_2 + \alpha_s^3 \tilde{\sigma}_3 + \ldots$$

LO **NLO** **NNLO** **N^{3}LO**

$\alpha_s \sim 0.1$

δ~10-20% **NLO**

δ~1-5% **NNLO** (or even N^{3}LO)

NLO now standard and largely automated

NNLO available for an increasing number of processes

N^{3}LO available for few hadron-collider processes (Higgs production in gluon fusion and VBF, DY production…)

QCD beyond fixed order

Perturbative QCD at fixed order

$$\tilde{\sigma} = 1 + \alpha_s \tilde{\sigma}_1 + \alpha_s^2 \tilde{\sigma}_2 + \alpha_s^3 \tilde{\sigma}_3 + \ldots$$

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LO NLO NNLO N^{3}LO

Assumption: perturbative coefficients $\tilde{\sigma}_n$ are well behaved

Many observables studied at the LHC depend on more than one scale; **single** or **double** logs of the ratio of those scales at all orders in perturbation theory

$$(\alpha_s \ln R)^n \qquad (\alpha_s \ln^2 R)^n$$

If the logarithms are large the convergence of the series is spoiled

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LO **NLO** **NNLO** **N³LO**

Assumption: p

Fixed order predictions no longer reliable:
all order resummation of the perturbative series mandatory

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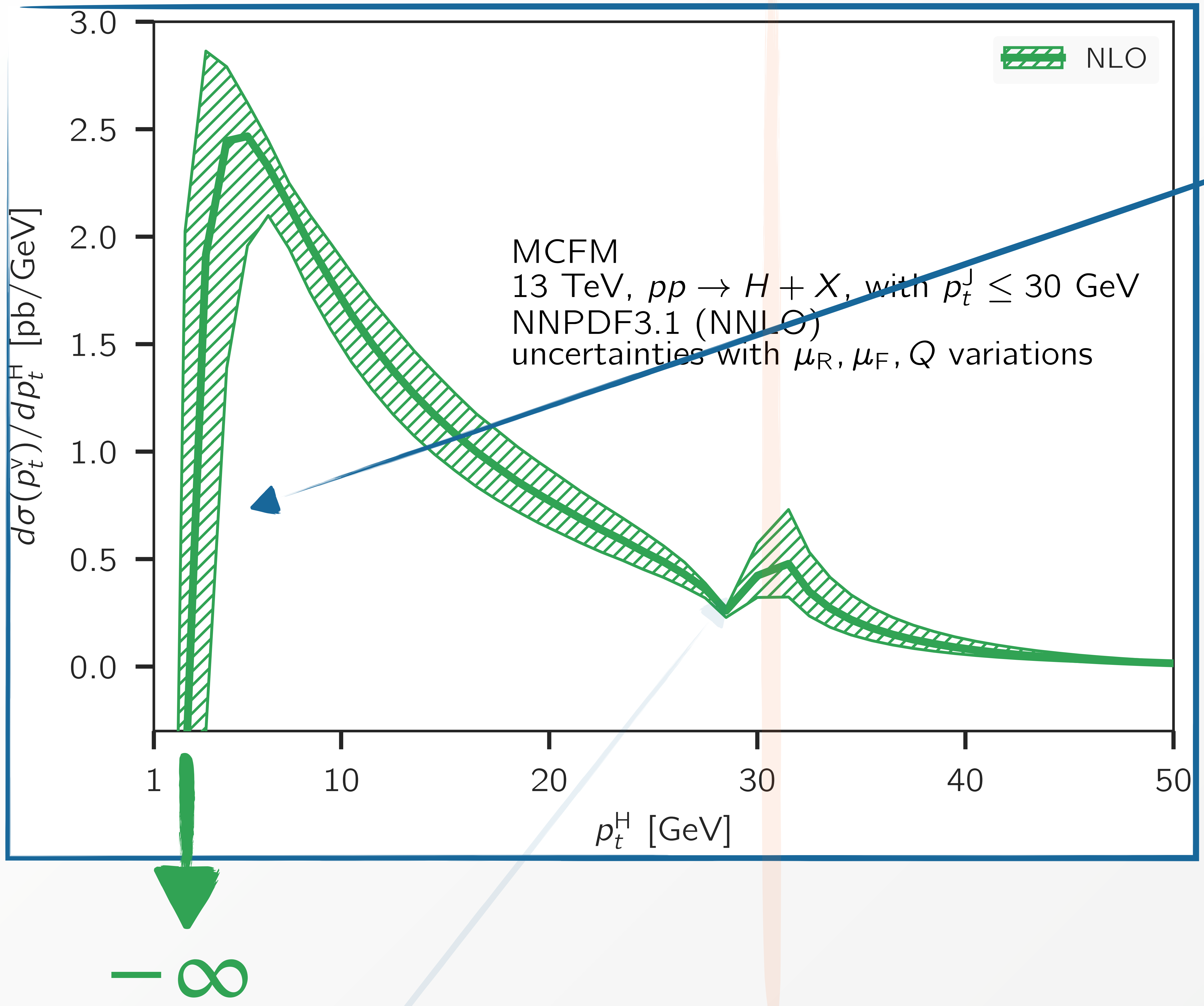
Example: Higgs transverse momentum with a jet veto



Large **transverse momentum** logarithms

$$L = \ln(p_\perp^H / m_H) \qquad p_\perp^H \ll m_H$$

Large(ish) **jet veto** logarithms

$$L = \ln(p_\perp^{J,v} / m_H) \qquad p_\perp^{J,v} \ll m_H$$

Jet veto = 30 GeV

Integrable double logarithms at the shoulder for $p_\perp^{J,v} \sim p_\perp^H$

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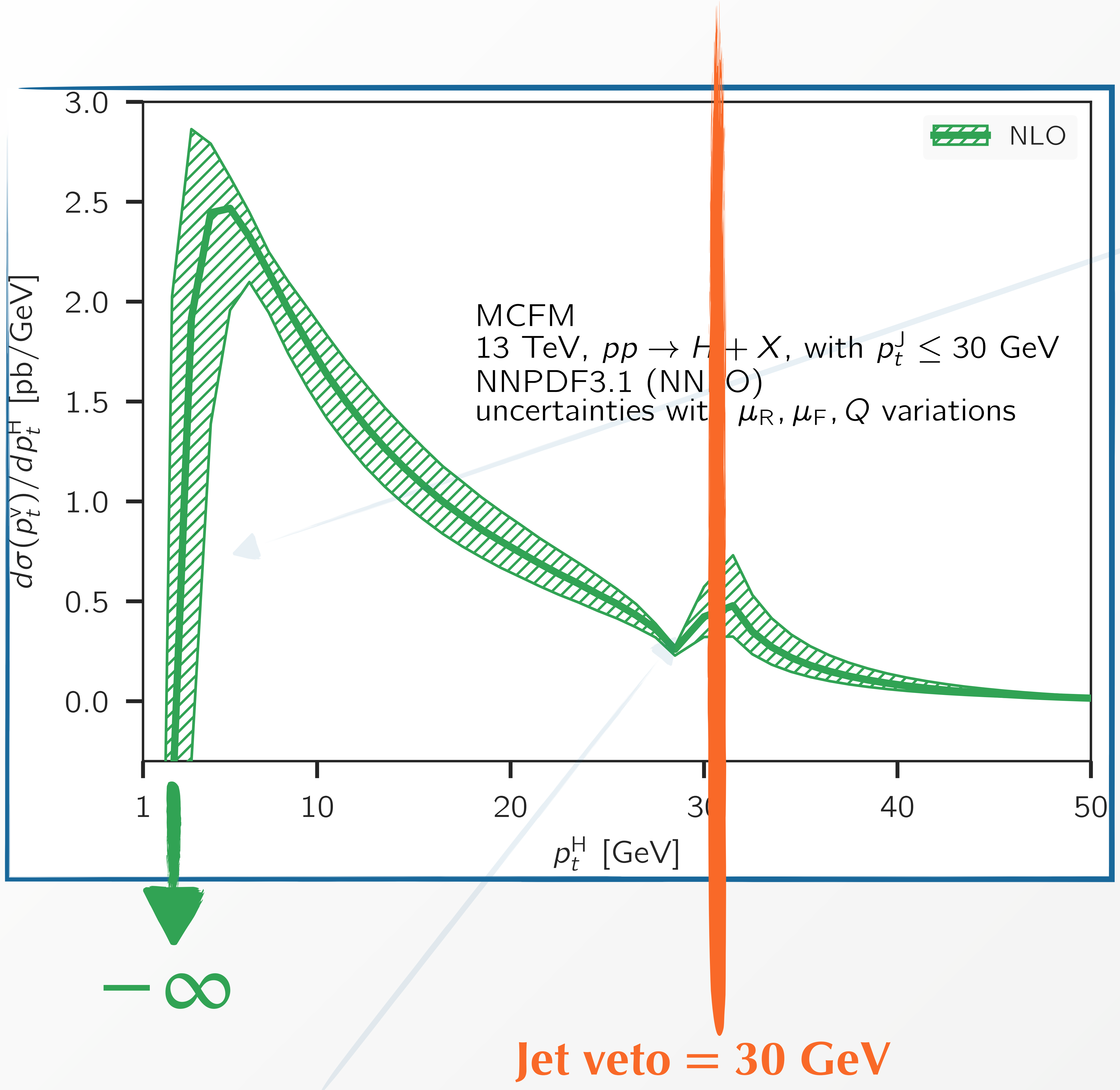

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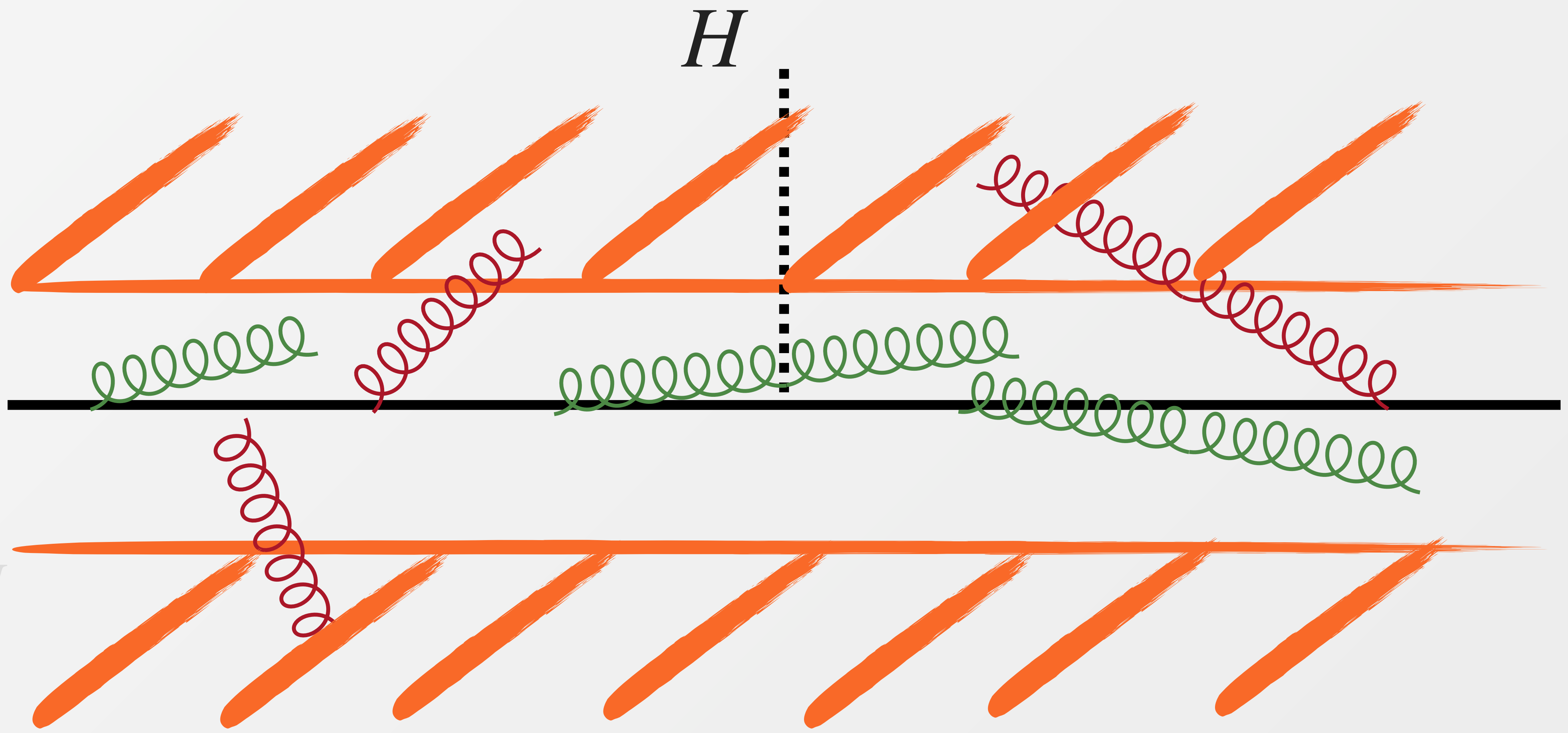

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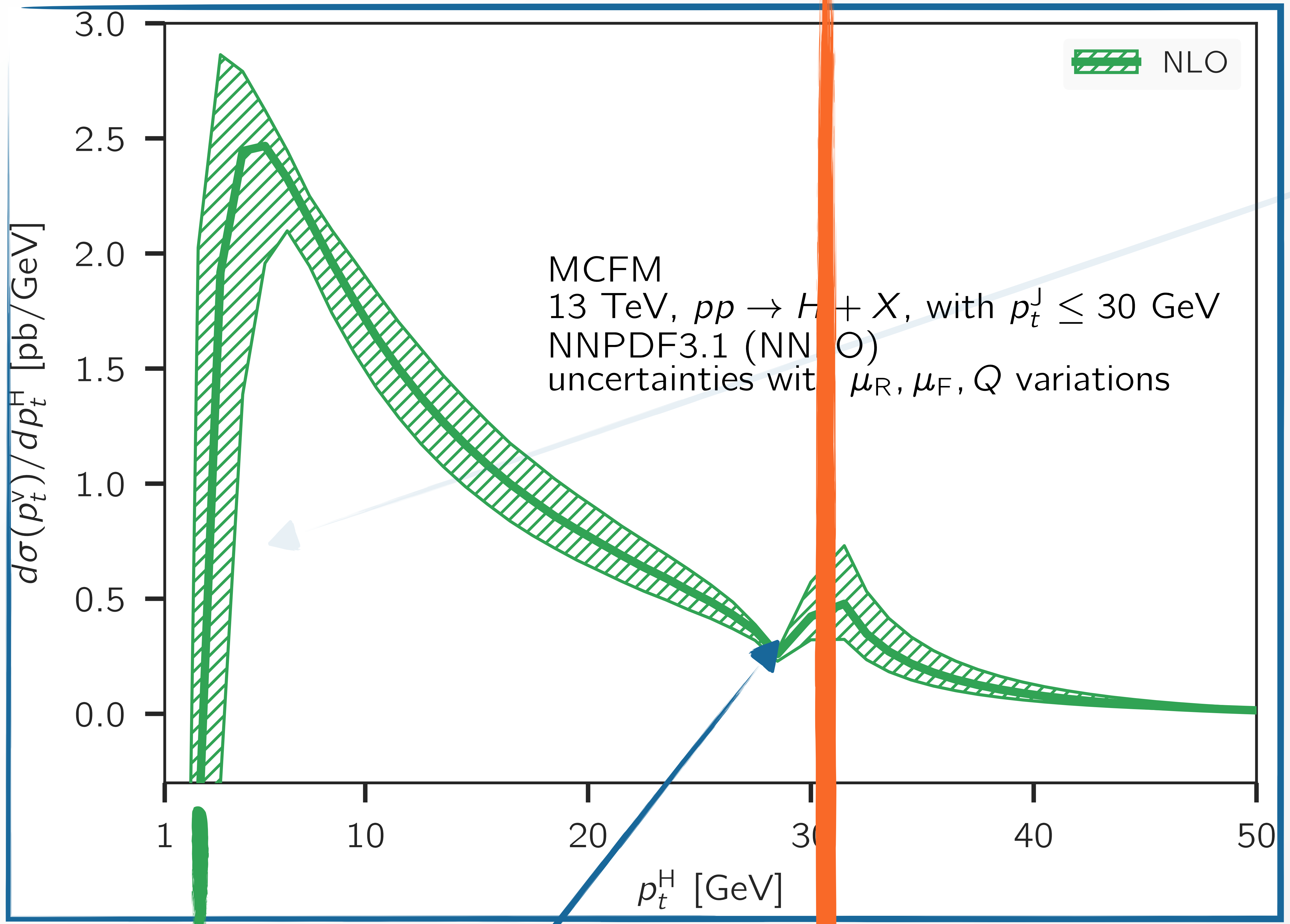

$-\infty$

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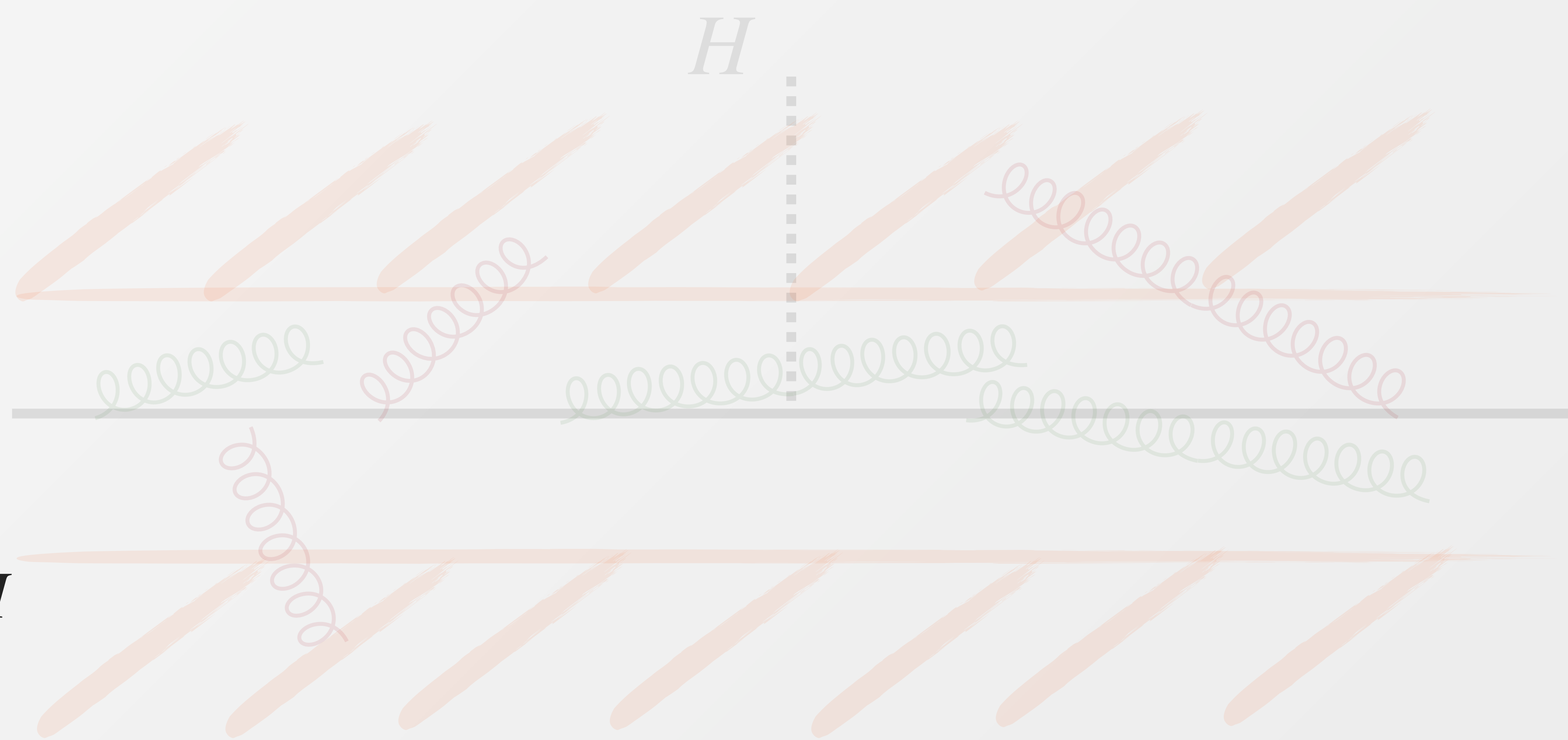


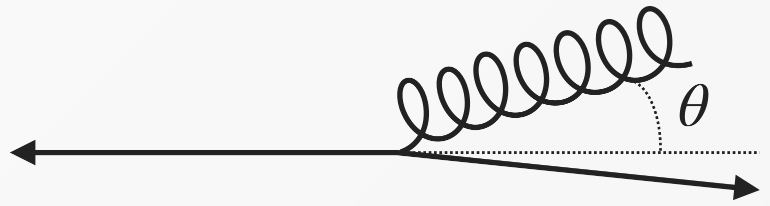
Integrable double logarithms at the shoulder for $p_\perp^{\mathrm{J,v}} \sim p_\perp^H$

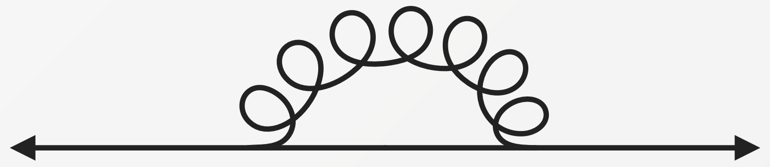
It’s not a bug, it’s a feature

Real emission diagrams singular for **soft/collinear emission**. Singularities are cancelled by virtual counterparts for IRC safe observables

Consider processes where real radiation is **constrained** in a corner of the phase space, (exclusive boundary of the phase space, **restrictive cuts**)

$$\tilde{\sigma}_1(p_\perp) \quad \sim \quad \underbrace{\int \frac{d\theta}{\theta}\frac{dE}{E}\Theta\left(p_\perp - E\theta\right)} \quad - \quad \underbrace{\int \frac{d\theta}{\theta}\frac{dE}{E}}$$





$$\sim \quad -\int \frac{dE}{E}\frac{d\theta}{\theta}\Theta(E\theta - p_\perp)$$

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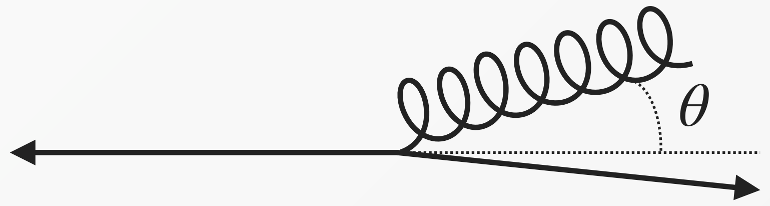
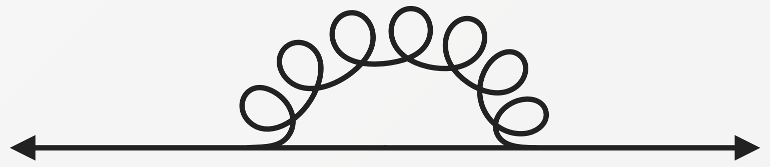




$p_\perp \to 0$: observable can become negative even in the perturbative regime

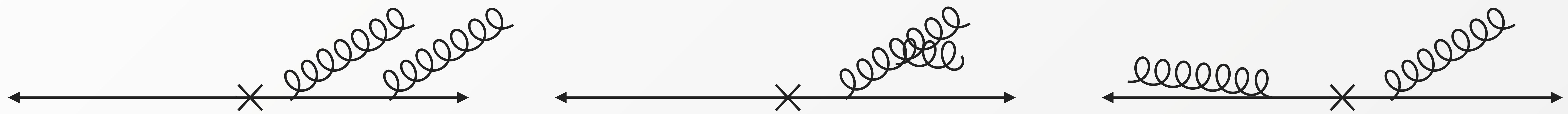
$$\sim \quad -\int \frac{dE}{E}\frac{d\theta}{\theta}\Theta(E\theta - p_\perp) \sim -\frac{1}{2}\ln^2 p_\perp/m_H$$

Sudakov logarithms

Double logarithms **leftovers** of the real-virtual cancellation of IRC divergences

Making pQCD great again: all-order resummation

Soft-collinear emission of two gluons



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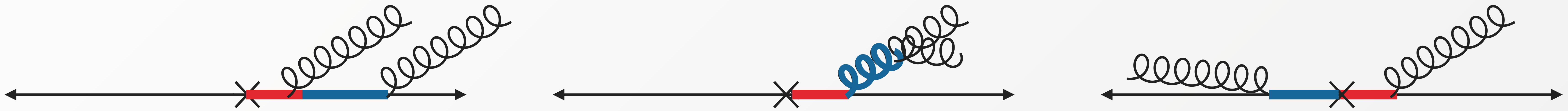

Two propagators nearly on shell, 4 divergences. Diagrams can potentially give $\alpha_s^2 \ln^4 p_\perp / m_H$

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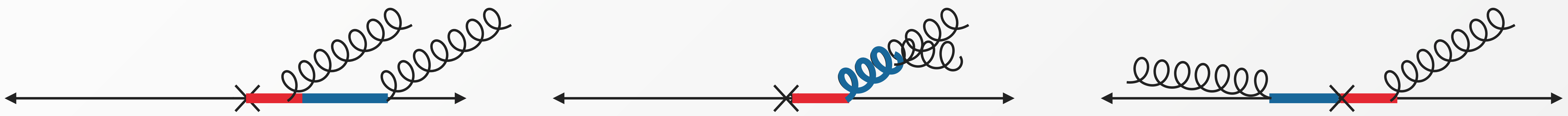

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All order structure

$$\tilde{\sigma}(v) = \sum_{n=0}^{\infty} \alpha_s^n \sum_{m=1}^{2n} c_{nm} L^m + \dots \qquad L = \ln(p_\perp/m_H)$$

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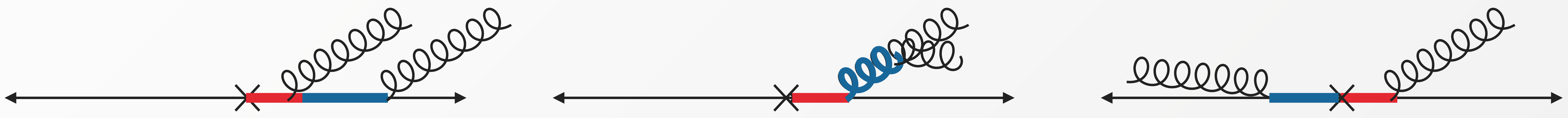

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Origin of the logs is simple. Resum them to all orders by **reorganizing** the series

$$\tilde{\sigma}(v) = f_1(\alpha_s L^2) + \frac{1}{L} f_2(\alpha_s L^2) + \dots$$

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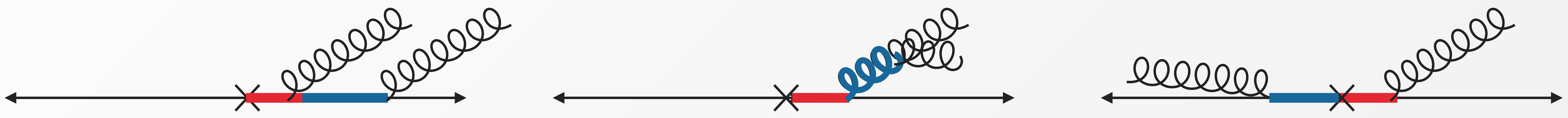

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$$\tilde{\sigma}(v) = \boxed{f_1(\alpha_s L^2)} + \frac{1}{L} f_2(\alpha_s L^2) + \dots$$

Poor man's leading logarithmic (LL) resummation of the perturbative series

Accurate for $L \sim 1/\sqrt{\alpha_s}$

Making pQCD great again: all-order resummation

$$\tilde{\sigma}(v) = f_1(\alpha_s L^2) + \frac{1}{L} f_2(\alpha_s L^2) + \ldots$$

*È la somma che fa il totale

All-order resummation: exponentiation

Independent emissions $k_1, \ldots k_n$ (plus corresponding virtual contributions) in the soft and collinear limit with strong angular ordering

$$d\Phi_n |\mathscr{M}(k_1, \ldots k_n)|^2 \to \frac{1}{n!}\alpha_s^n \prod_{i=1}^{n} \frac{dE_i}{E_i}\frac{d\theta_i}{\theta_i}$$

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Calculate observable with arbitrary number of emissions: **exponentiation**

$$\tilde{\sigma} \simeq \sum_{n=0}^{\infty} \frac{1}{n!} \alpha_s^n \prod_{i=1}^{n} \int \frac{dE_i}{E_i} \frac{d\theta_i}{\theta_i} [\Theta(p_\perp - E_i\theta_i) - 1] \simeq e^{-\alpha_s L^2}$$

Sudakov suppression [Sudakov '54]
Price for constraining real radiation

Exponentiated form allows for a **more powerful reorganization**

$$\tilde{\sigma} = \exp\left[\sum_n \left(\underbrace{\mathcal{O}(\alpha_s^n L^{n+1})}_{\textbf{LL}} + \underbrace{\mathcal{O}(\alpha_s^n L^n)}_{\textbf{NLL}} + \underbrace{\mathcal{O}(\alpha_s^n L^{n-1})}_{\textbf{NNLL}} + \dots \right) \right]$$

Region of applicability now valid up to $L \sim 1/\alpha_s$, successive terms suppressed by $\mathcal{O}(\alpha_s)$

All-order resummation: exponentiation

Independent emissions $k_1, \dots k_n$ (plus corresponding virtual contributions) in the soft and collinear limit with strong angular ordering

Exponentiation in direct space generally not possible.
Phase-space constraints typically do not factorize in direct space

Price for constraining real radiation

$$\tilde{\sigma}(v) \sim \int \prod_i^n [dk_i] \, |\mathcal{M}(k_1, \dots, k_n)|^2 \, \Theta_{\rm PS}(v - V(k_1, \dots k_n))$$

Exponentiated form allows for a **more powerful reorganization**

How to achieve resummation?

Region of applicability now valid up to $L \sim 1/\alpha_s$, successive terms suppressed by $\mathcal{O}(\alpha_s)$

All-order resummation: (re)-factorization

Solution 1: move to **conjugate space** where phase space factorization is manifest

Exponentiation in conjugate space; **inverse transform** to move back to direct space

Extremely successful approach

"direct QCD"

- Catani, Trentadue, Mangano, Marchesini, Webber, Nason, Dokshitzer…
- Collins, Soper, Sterman, Laenen, Magnea…

Emphasis on properties of QCD matrix elements and QCD radiation

SCET

- Manohar, Bauer, Stewart, Becher, Neubert….

+ many others!

Factorization properties in the singular region and associated RGEs (factorization → evolution →resummation)

SCET vs. dQCD **not an issue** [Sterman *et al.* '13, '14][Bonvini, Forte, Ghezzi, Ridolfi, LR '12, '13, '14][Becher, Neubert *et al.* '08, '11, 14]

Limitation: it is **process-dependent**, and must be performed manually and analytically **for each observable** for some complex observable difficult/impossible to derive **factorization theorem**

All-order resummation: CAESAR/ARES approach

Solution 2: Translate the resummability into properties of the observable in the presence of multiple radiation: **recursive infrared and collinear (rIRC) safety**

[Banfi, Salam, Zanderighi '01, '03, '04]
[Banfi, McAslan, Monni, Zanderighi, El-Menoufi '14, '18]

Simple observable easy to calculate

$$\tilde{\sigma} \sim \int \frac{dv_1}{v_1} \Sigma_s(v_1) \mathcal{F}(v, v_1)$$

Transfer function relates the resummation of the full observable to the one of the simple observable.

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i.e. conditional probability

Separation obtained by introducing a **resolution scale** $q_0 = \epsilon k_{t,1}$

$$\tilde{\sigma} \sim \int [dk_1] e^{-R(q_0)}$$

Unresolved emission can be treated as **totally unconstrained** ⟶ **exponentiation**

$$\times |\mathcal{M}(k_1)|^2 \left(\sum_{m=0}^{\infty} \frac{1}{m!} \int \prod_{i=2}^{m+1} [dk_i] |\mathcal{M}(k_i)|^2 \Theta(V(k_i) - q_0) \Theta\left(v - V(k_1, \ldots, k_{m+1})\right) \right)$$

Resolved emission treated exclusively with **Monte Carlo methods.** Integral is finite, can be integrated in d=4 with a computer

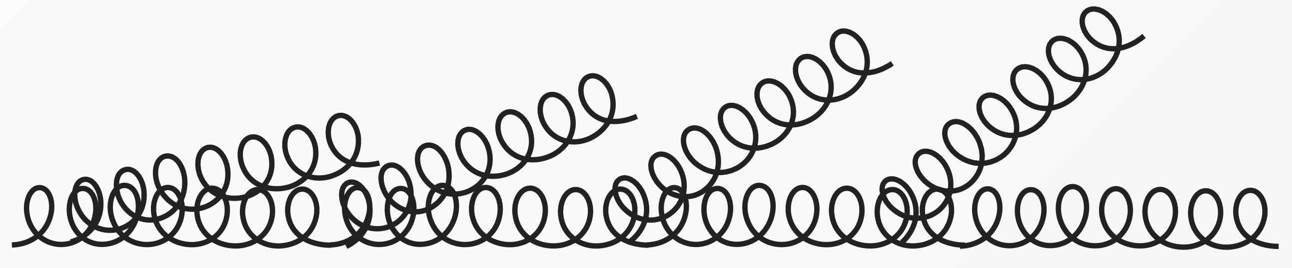
Approach recently formulated within SCET language [Bauer, Monni '18, '19 + ongoing work]

Method entirely formulated in **direct space**

Resummation of the transverse momentum spectrum

Resummation of transverse momentum is particularly delicate because $p_\perp$ is a **vectorial quantity**

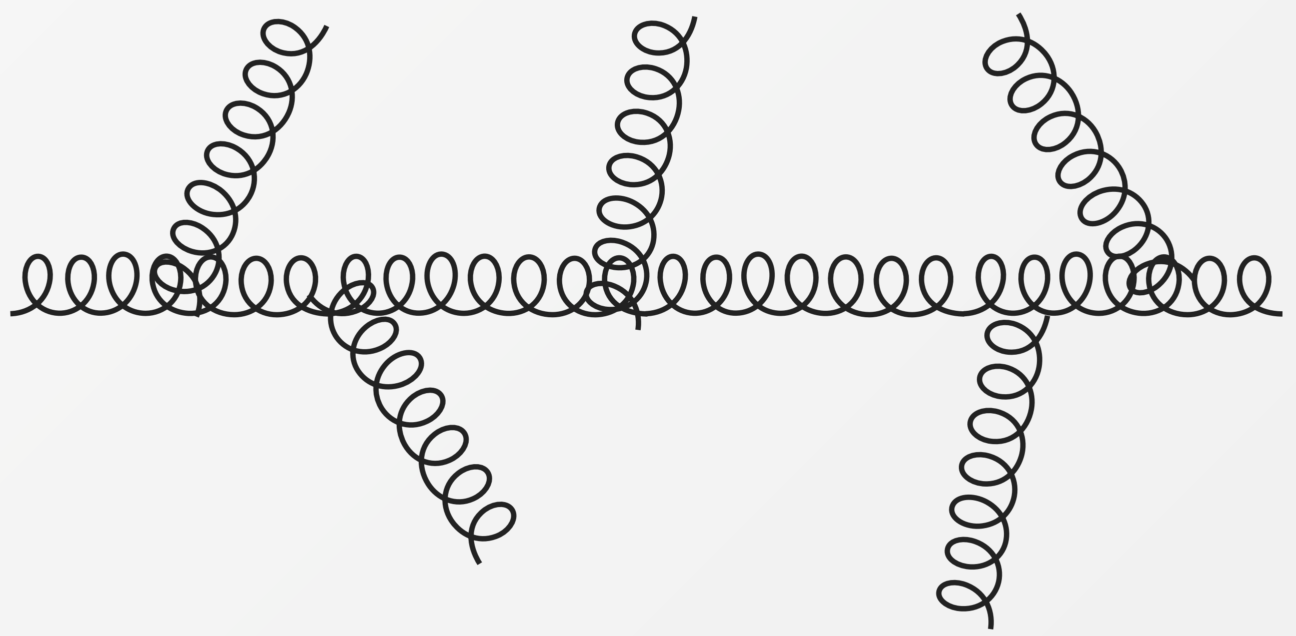
Two concurring mechanisms leading to a system with small $p_\perp$



$$p_\perp^2 \sim k_{t,i}^2 \ll m_H^2$$

cross section naturally suppressed as there is no phase space left for gluon emission (Sudakov limit)

Exponential suppression



$$\sum_{i=1}^{n} \vec{k}_{t,i} \simeq 0$$

Large kinematic cancellations

$p_\perp$ ~0 far from the Sudakov limit

Power suppression

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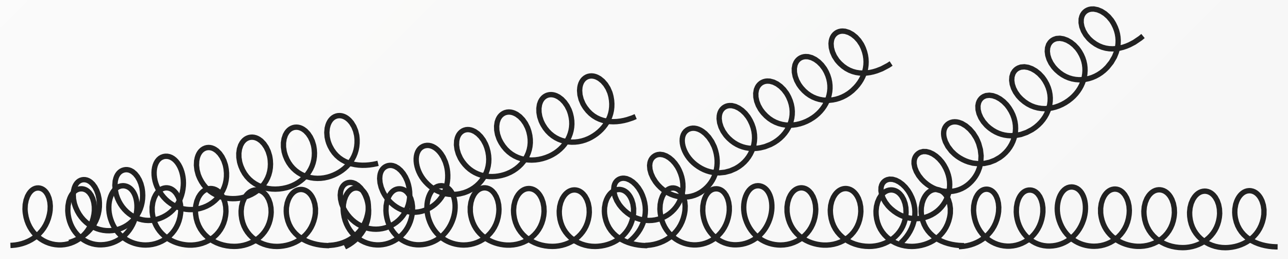

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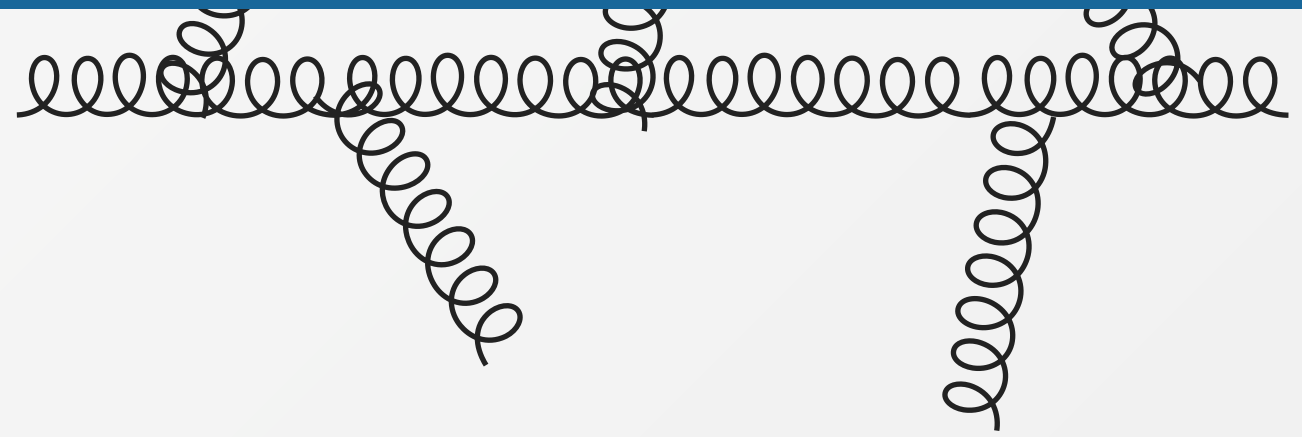
Dominant at small $p_\perp$



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Resummation of the transverse momentum spectrum

Approach 1: impact parameter space

$$\delta^{(2)}\left(\vec{p}_t - \sum_{i=1}^{n} \vec{k}_{t,i}\right) = \int d^2b \frac{1}{4\pi^2} e^{i\vec{b}\cdot\vec{p}_t} \prod_{i=1}^{n} e^{-i\vec{b}\cdot\vec{k}_{t,i}}$$

two-dimensional momentum conservation

[Parisi, Petronzio '79; Collins, Soper, Sterman '85]

Exponentiation in conjugate space

NLL formula with scale-independent PDFs

$$\sigma = \sigma_0 \int d^2\vec{p}_\perp^H \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_\perp^H} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \int [dk_i] |M(k_i)|^2 \left(e^{i\vec{b}\cdot\vec{k}_{t,i}} - 1\right)$$

$$= \sigma_0 \int d^2\vec{p}_\perp^H \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_\perp^H} e^{-R_{\rm NLL}(L)}$$

virtual corrections

$$R_{\rm NLL}(L) = -Lg_1(\alpha_s L) - g_2(\alpha_s L) \qquad L = \ln(m_H b/b_0)$$

Approach 2: momentum space (RadISH)

[Bizon, Monni, Re, LR, Torrielli '16, '17, '18]

Approach exploits factorization properties of the QCD squared amplitudes

NLL formula with scale-independent PDFs

Simple observable

$$v_i = k_{t,i}/m_H, \quad \zeta_i = v_i/v_1$$

$$\sigma(p_\perp) = \sigma_0 \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(v_1)}$$

$$\times\, e^{R'(v_1)} R'(v_1) \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'(v_1)\, \Theta\left(p_\perp - |\vec{k}_{t,i} + \cdots \vec{k}_{t,n+1}|\right)$$

Transfer function

Formula can be evaluated with Monte Carlo method; dependence on ϵ vanishes (as $\mathcal{O}(\epsilon)$) and result is **finite** in four dimensions

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$$= \sigma_0 \int d^2\vec{p}_\perp^H \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_\perp^H} e^{-R_{\rm NLL}(L)}$$

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Approach 2: momentum space (RadISH)

[Bizon, Monni, Re, LR, Torrielli '16, '17, '18]

Approach exploits factorization properties of the QCD squared amplitudes

NLL formula with scale-independent PDFs

Simple observable

$$\sigma(p_\perp) = \sigma_0 \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(v_1)} \qquad v_i = k_{t,i}/m_H, \quad \zeta_i = v_i/v_1$$

$$\times\, e^{R'(v_1)} R'(v_1) \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'(v_1)\, \Theta\left(p_\perp - |\vec{k}_{t,i} + \cdots \vec{k}_{t,n+1}|\right)$$

Transfer function

Formula can be evaluated with Monte Carlo method; dependence on ϵ vanishes (as $\mathcal{O}(\epsilon)$) and result is **finite** in four dimensions

Resummation of the transverse momentum spectrum

Approach 1: impact parameter space

$$\delta^{(2)}\left(\vec{p}_t - \sum_{i=1}^{n} \vec{k}_{t,i}\right) = \int d^2b \frac{1}{4\pi^2} e^{i\vec{b}\cdot\vec{p}_t} \prod_{i=1}^{n} e^{-i\vec{b}\cdot\vec{k}_{t,i}}$$

two-dimensional momentum conservation

[Parisi, Petronzio '79; Collins, Soper, Sterman '85]

Exponentiation in conjugate space

NLL formula with scale-independent PDFs

$$\sigma = \sigma_0 \int d^2\vec{p}_\perp^H \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_\perp^H} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \int [dk_i] |M(k_i)|^2 \left(e^{i\vec{b}\cdot\vec{k}_{t,i}} - 1\right)$$

$$= \sigma_0 \int d^2\vec{p}_\perp^H \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_\perp^H} e^{-R_{\rm NLL}(L)}$$

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$v_i = k_{t,i}/m_H, \quad \zeta_i = v_i/v_1$

$$\times\, \epsilon^{R'(v_1)} R'(v_1) \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'(v_1)\, \Theta\left(p_\perp - |\vec{k}_{t,i} + \cdots \vec{k}_{t,n+1}|\right)$$

Transfer function

Formula can be evaluated with Monte Carlo method; dependence on ϵ vanishes (as $\mathcal{O}(\epsilon)$) and result is **finite** in four dimensions

Direct space formulation

1. Similar in spirit to a **semi-inclusive parton shower**, but with higher-order logarithms, and **full control on the formal accuracy**

2. Thanks to its versatility, the approach can be exploited to formulate the resummation for entire classes of observables in an **unique framework**

3. **More differential description** of the QCD radiation than that usually possible in a conjugate-space formulation

Direct space formulation

Price to pay: less compact formulation

N^{3}LL result

$$\frac{d\Sigma(v)}{d\Phi_B} = \int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}\partial_L\left(-e^{-R(k_{t1})}\mathcal{L}_{\mathrm{N^3LL}}(k_{t1})\right)\int d\mathcal{Z}[\{R',k_i\}]\Theta\left(v - V(\{\tilde{p}\},k_1,\dots,k_{n+1})\right)$$

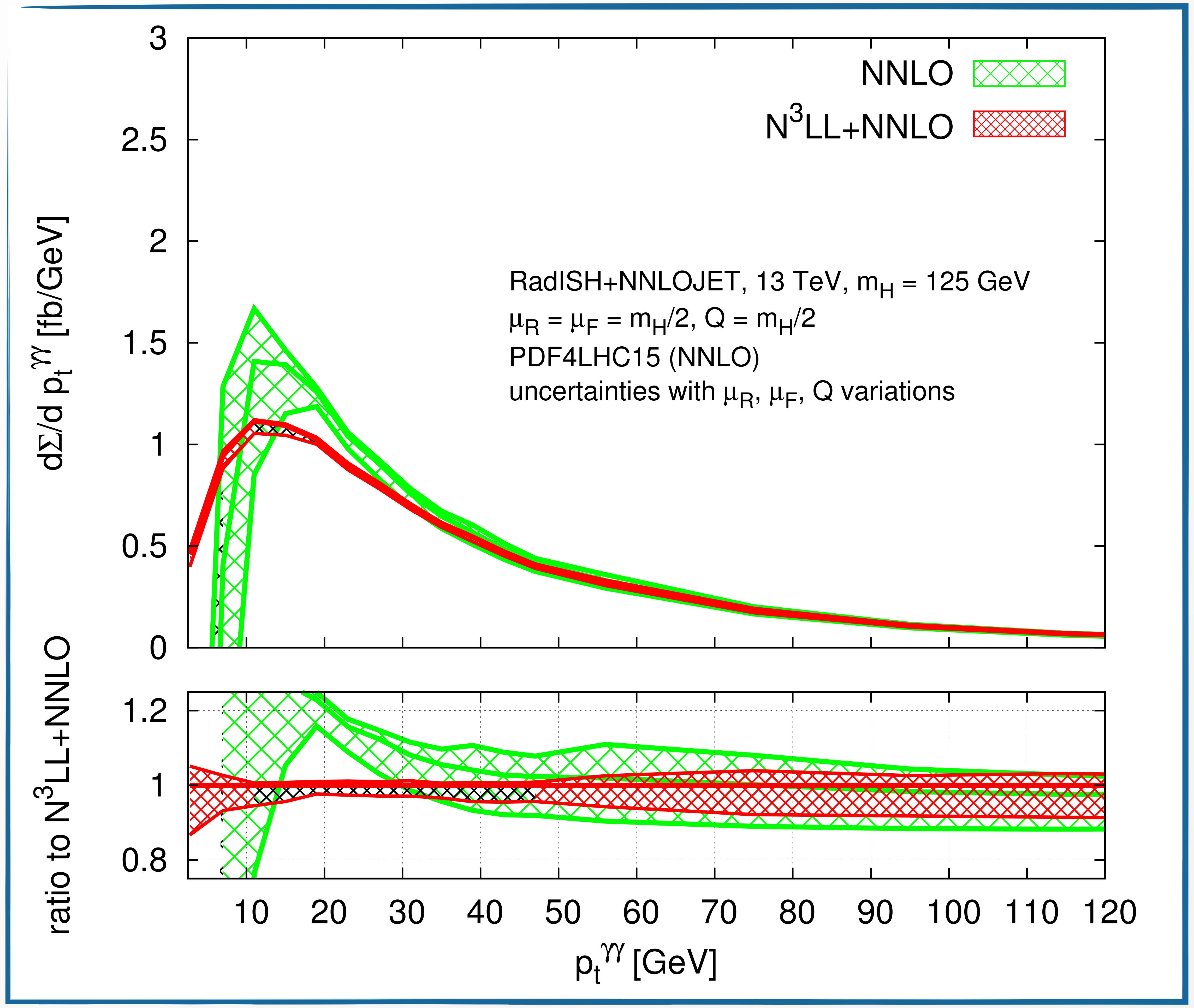
$$+\int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}e^{-R(k_{t1})}\int d\mathcal{Z}[\{R',k_i\}]\int_0^1\frac{d\zeta_s}{\zeta_s}\frac{d\phi_s}{2\pi}\Bigg\{\left(R'(k_{t1})\mathcal{L}_{\mathrm{NNLL}}(k_{t1}) - \partial_L\mathcal{L}_{\mathrm{NNLL}}(k_{t1})\right)$$
$$\times\left(R''(k_{t1})\ln\frac{1}{\zeta_s}+\frac{1}{2}R'''(k_{t1})\ln^2\frac{1}{\zeta_s}\right) - R'(k_{t1})\left(\partial_L\mathcal{L}_{\mathrm{NNLL}}(k_{t1}) - 2\frac{\beta_0}{\pi}\alpha_s^2(k_{t1})\hat{P}^{(0)}\otimes\mathcal{L}_{\mathrm{NLL}}(k_{t1})\ln\frac{1}{\zeta_s}\right)$$
$$+\frac{\alpha_s^2(k_{t1})}{\pi^2}\hat{P}^{(0)}\otimes\hat{P}^{(0)}\otimes\mathcal{L}_{\mathrm{NLL}}(k_{t1})\Bigg\}\Bigg\{\Theta\left(v - V(\{\tilde{p}\},k_1,\dots,k_{n+1},k_s)\right) - \Theta\left(v - V(\{\tilde{p}\},k_1,\dots,k_{n+1})\right)\Bigg\}$$

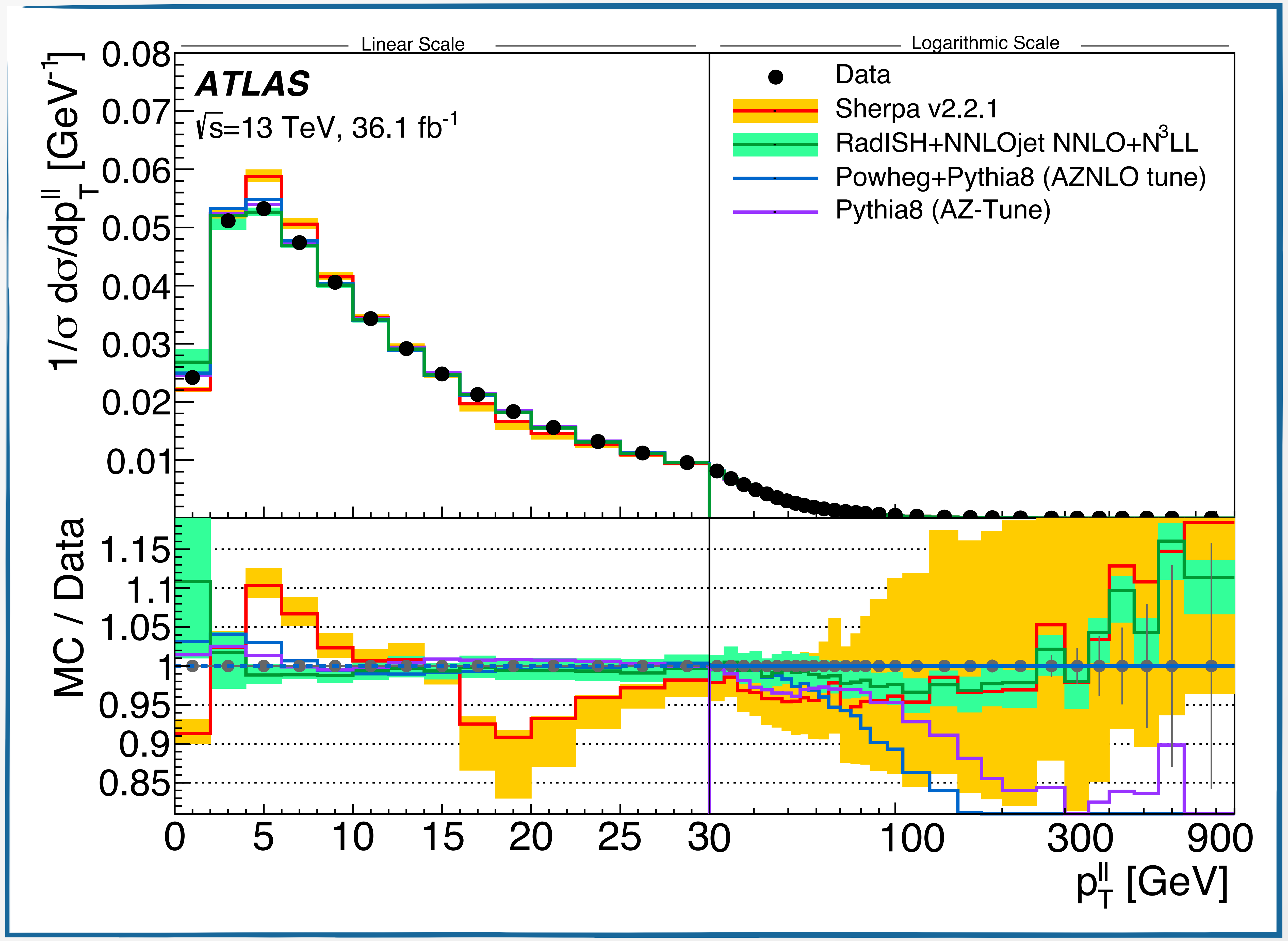
$$+\frac{1}{2}\int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}e^{-R(k_{t1})}\int d\mathcal{Z}[\{R',k_i\}]\int_0^1\frac{d\zeta_{s1}}{\zeta_{s1}}\frac{d\phi_{s1}}{2\pi}\int_0^1\frac{d\zeta_{s2}}{\zeta_{s2}}\frac{d\phi_{s2}}{2\pi}R'(k_{t1})$$
$$\times\Bigg\{\mathcal{L}_{\mathrm{NLL}}(k_{t1})\left(R''(k_{t1})\right)^2\ln\frac{1}{\zeta_{s1}}\ln\frac{1}{\zeta_{s2}} - \partial_L\mathcal{L}_{\mathrm{NLL}}(k_{t1})R''(k_{t1})\left(\ln\frac{1}{\zeta_{s1}}+\ln\frac{1}{\zeta_{s2}}\right)$$
$$+\frac{\alpha_s^2(k_{t1})}{\pi^2}\hat{P}^{(0)}\otimes\hat{P}^{(0)}\otimes\mathcal{L}_{\mathrm{NLL}}(k_{t1})\Bigg\}$$
$$\times\Bigg\{\Theta\left(v - V(\{\tilde{p}\},k_1,\dots,k_{n+1},k_{s1},k_{s2})\right) - \Theta\left(v - V(\{\tilde{p}\},k_1,\dots,k_{n+1},k_{s1})\right) -$$
$$\Theta\left(v - V(\{\tilde{p}\},k_1,\dots,k_{n+1},k_{s2})\right) + \Theta\left(v - V(\{\tilde{p}\},k_1,\dots,k_{n+1})\right)\Bigg\} + \mathcal{O}\left(\alpha_s^n\ln^{2n-6}\frac{1}{v}\right), \qquad (3.18)$$

1. *Similar in spirit to a* ***semi-inclusive parton shower****, but with higher-order logarithms, and* ***full control on the formal accuracy***

Resummation of the transverse momentum spectrum at N³LL+NNLO

N³LL result matched to NNLO H+j, Z+j, W±+j **[Bizon, LR *et al.* '17, '18, '19]**





[ATLAS 1912.02844]

H+j at same accuracy also in SCET **[Chen *et al.* '18]**

Results for Z/W^+ ratio

Z and W production share a similar pattern of QCD radiative corrections

Crucial to understand correlation between Z and W spectra to exploit data-driven predictions

$$\frac{1}{\sigma^W}\frac{d\sigma^W}{p_\perp^W} \sim \frac{1}{\sigma^Z_{\rm data}}\frac{d\sigma^Z_{\rm data}}{p_\perp^Z}\,\frac{\frac{1}{\sigma^W_{\rm theory}}\frac{d\sigma^W_{\rm theory}}{p_\perp^W}}{\frac{1}{\sigma^Z_{\rm theory}}\frac{d\sigma^Z_{\rm theory}}{p_\perp^Z}}$$

Several choices are possible:

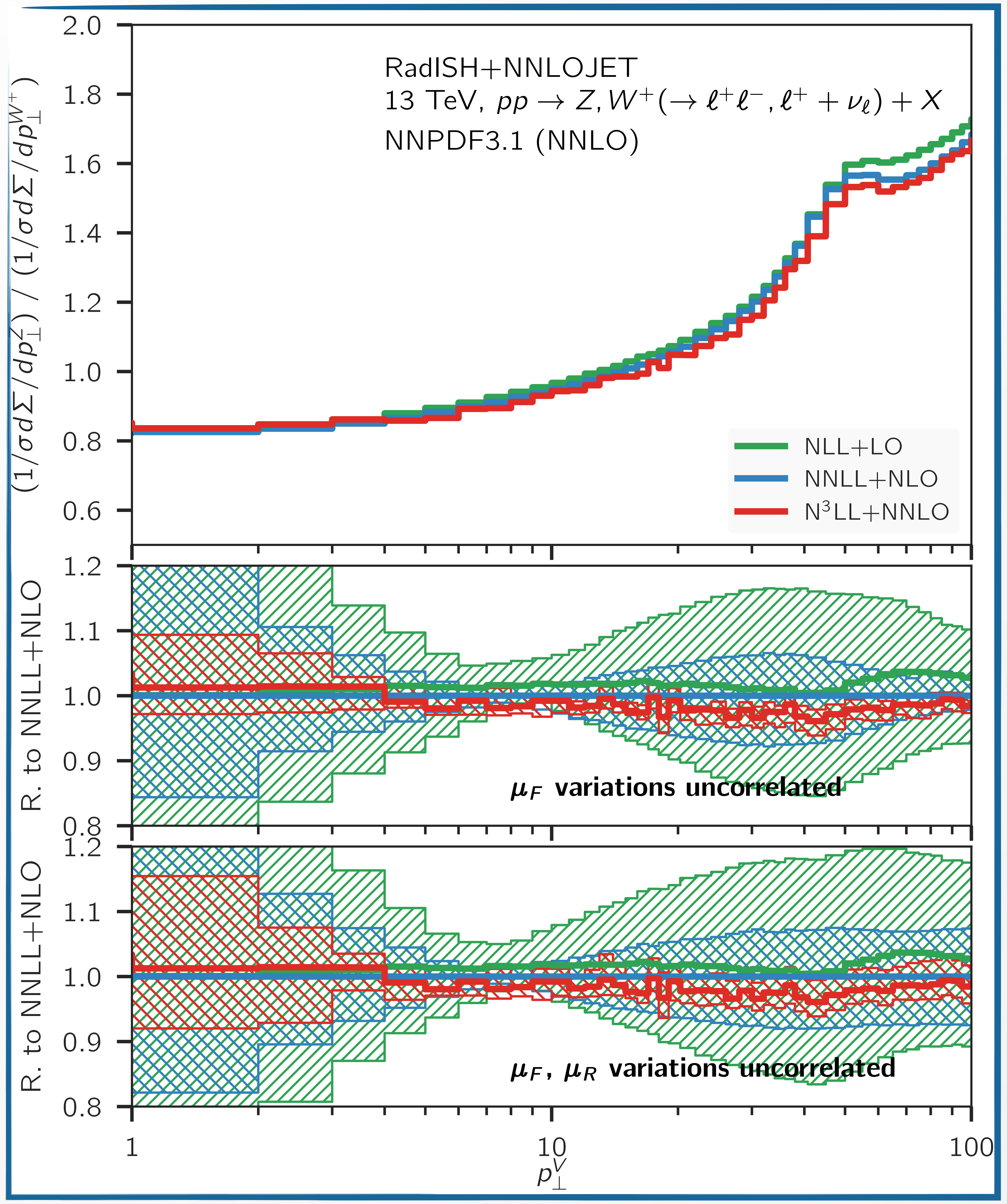
- **Correlate resummation and renormalisation** scale variations, keep **factorisation scale uncorrelated**, while keeping

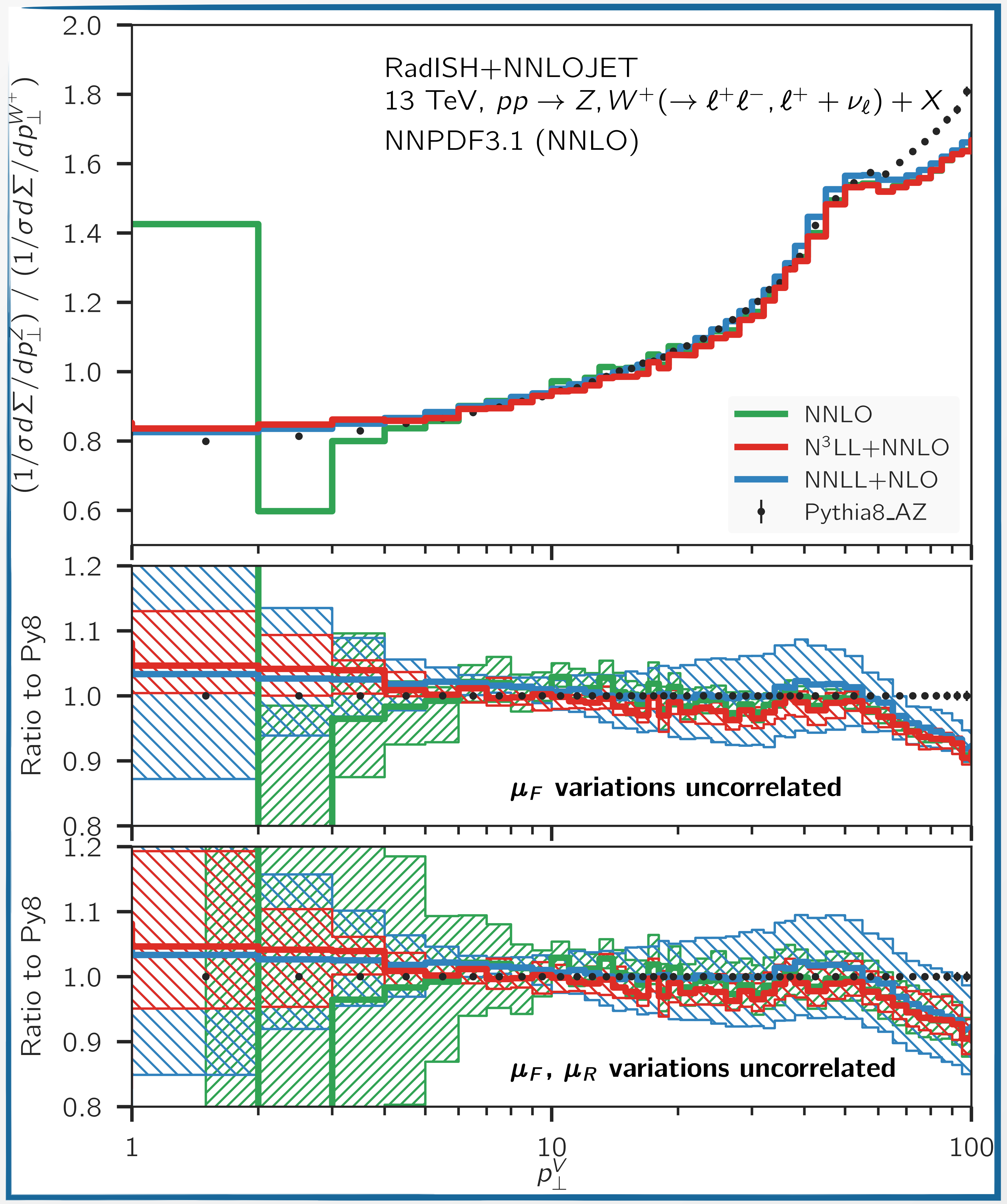
$$\frac{1}{2} \leq \frac{\mu_{\rm F}^{\rm num}}{\mu_{\rm F}^{\rm den}} \leq 2$$

- More **conservative** estimate: vary both **renormalisation and factorisation scales in an uncorrelated** way with

$$\frac{1}{2} \leq \frac{\mu^{\rm num}}{\mu^{\rm den}} \leq 2$$

Results for Z/W^+ ratio





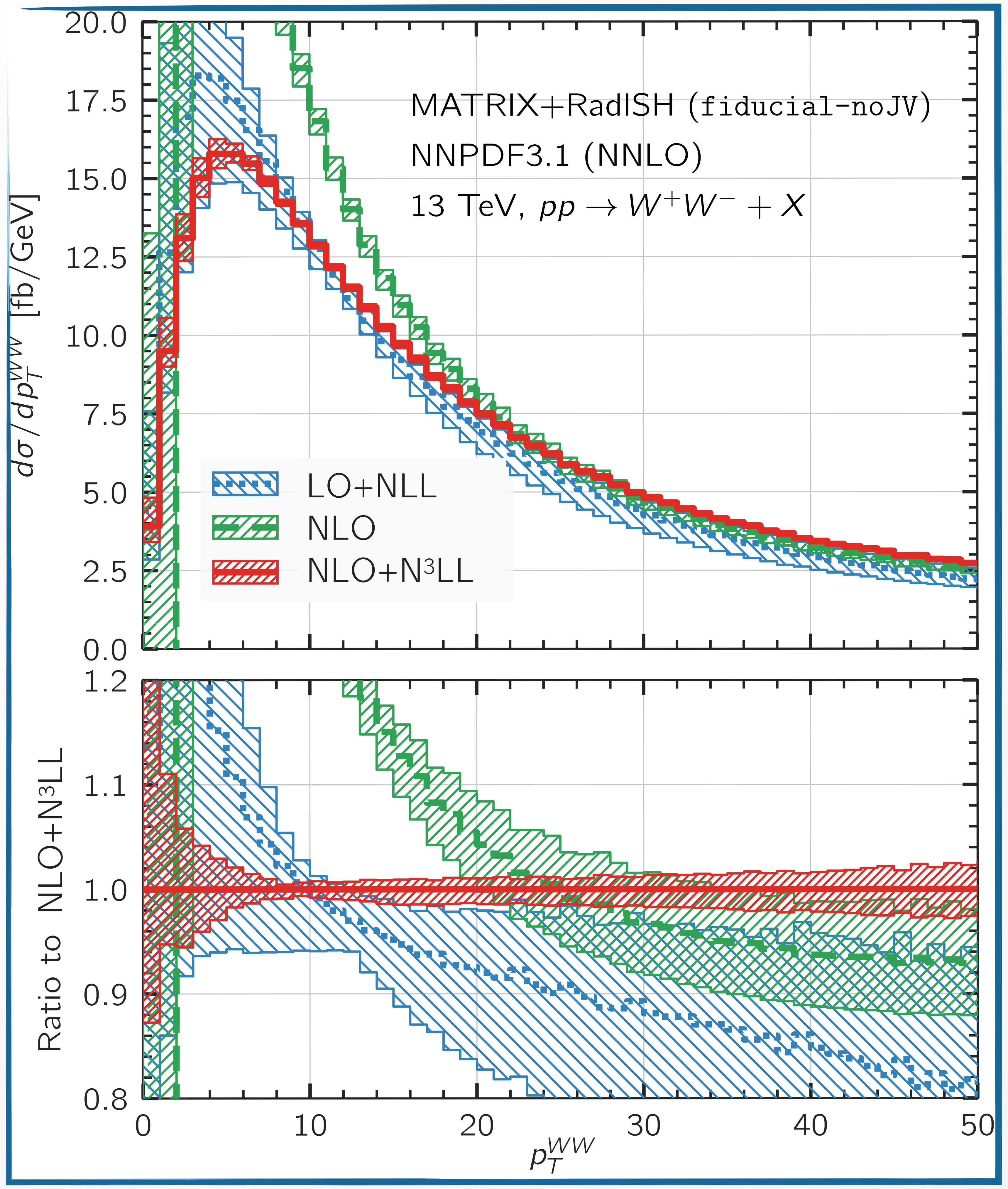
[Bizon, LR *et al.* '19]

LHC results

RadISH+MATRIX fully automated framework for generic 2⟶1 and 2⟶2 colour singlet processes

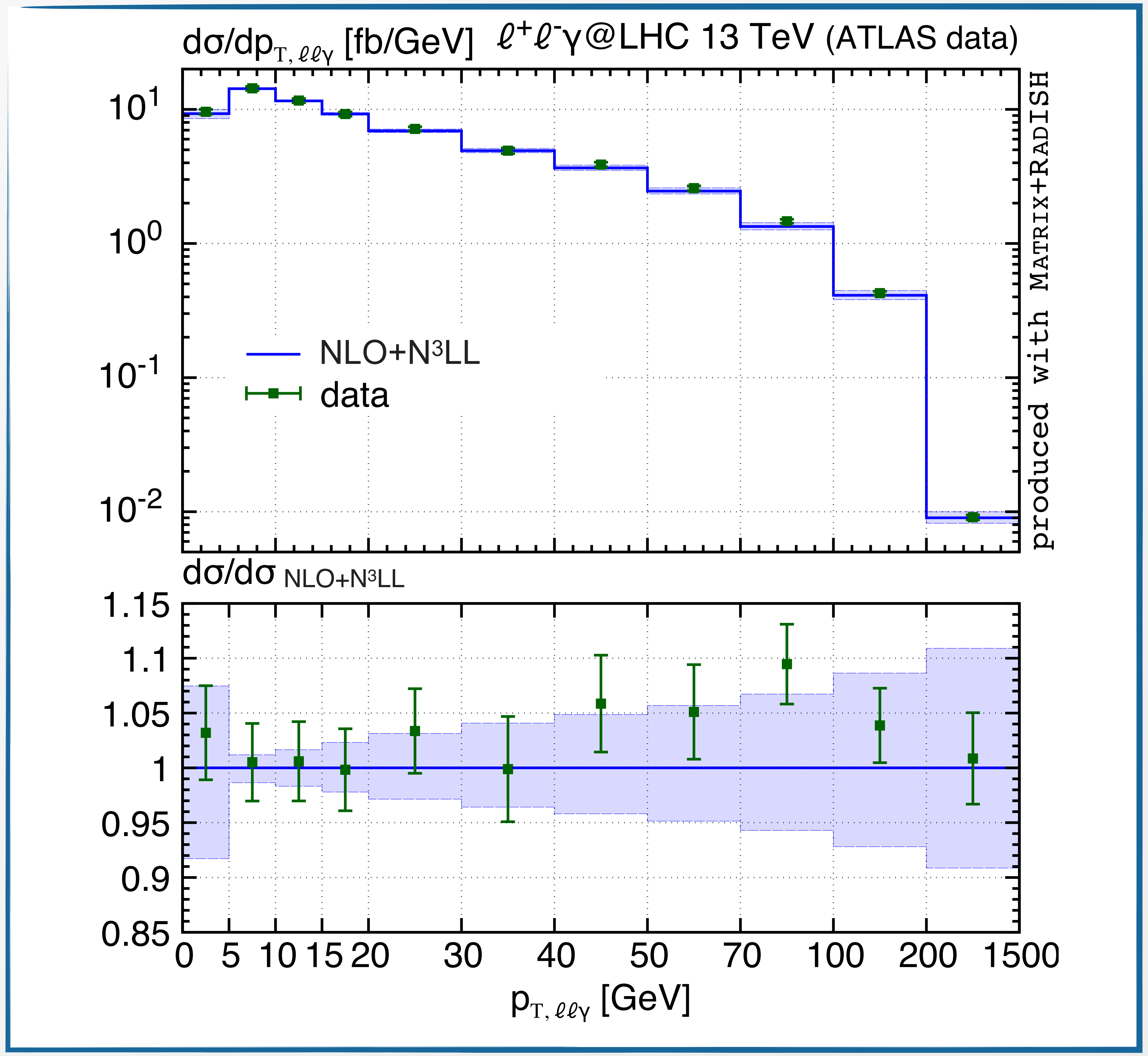
[Grazzini, Kallweit, Rathlev, Wiesemann '15, '17]

[Kallweit, Re, LR, Wiesemann 2004.07720]



[Kallweit, Re, LR, Wiesemann, 2004.07720]

W^+W^- production



$Z\gamma$ production

[Wiesemann, Rottoli, Torrielli 2006.09338]

2. *Thanks to its versatility, the approach can be exploited to formulate the resummation for entire classes of observables in an* ***unique framework***

Direct space formulation: general considerations

NLL result for $p_\perp^H$

$$\sigma(p_\perp^H) = \sigma_0 \int d^2\vec{p}_\perp^H \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_\perp^H} e^{-R_{\rm NLL}(L)}$$

NLL result for $p_\perp^J$

$$\sigma(p_\perp^{\rm J}) = \sigma_0 e^{Lg_1(\alpha_s\beta_0 L)+g_2(\alpha_s\beta_0 L)}$$

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General formula for a generic transverse observable at NLL **[Bizon, Monni, Re, LR, Torrielli '17]**

$$\sigma(v) = \sigma_0 \int \frac{dk_{t,1}}{k_{t,1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(k_{t,1})} R'\left(k_{t,1}\right) d\mathcal{Z}\, \Theta\left(v - V(k_1, \ldots, k_{n+1})\right)$$

$$d\mathcal{Z} = \epsilon^{R'(k_{t,1})} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'\left(k_{t,1}\right)$$

$$R_{\rm NLL}(L) = -Lg_1(\alpha_s L) - g_2(\alpha_s L)$$

$$L = \ln(k_{t,1}/M)$$

$$R'_{\rm NLL}(k_t) = 4\left(\frac{\alpha_s^{\rm CMW}(k_t)}{\pi} C_A \ln\frac{m_H}{k_t} - \alpha_s(k_t)\beta_0\right)$$

CMW scheme

(inclusion of 2-loop cusp anomalous dimension)

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understanding of the structure in momentum space provides guidance to **double-differential resummation**

General

$$\sigma(v) = \sigma_0 \int \frac{dk_{t,1}}{k_{t,1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(k_{t,1})} R'\left(k_{t,1}\right) d\mathcal{Z}\, \Theta\left(p_T^H - |\vec{k}_{t,1} + \cdots \vec{k}_{t,n+1}|\right)$$

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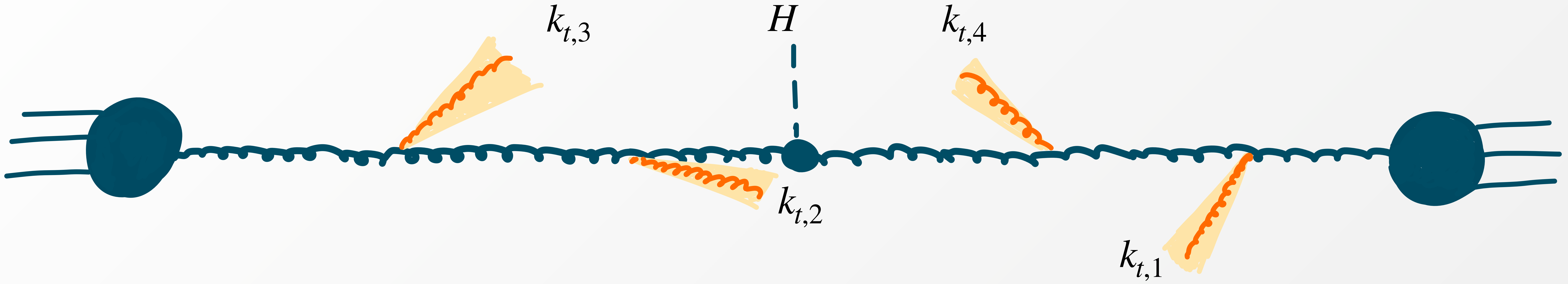
Double-differential resummation at NLL in b space

At NLL, emissions are **strongly ordered** in angle. k_t-type algorithms will associate **each emission** to a **different** jet



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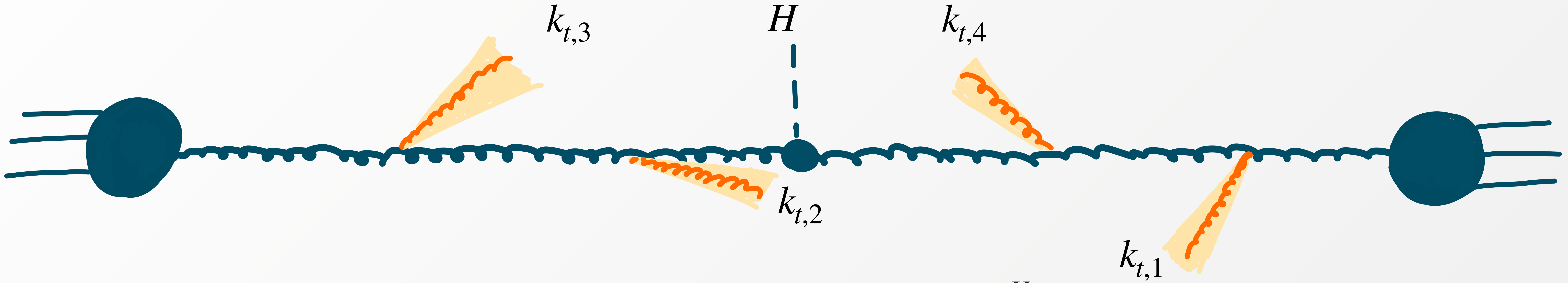

Additional constraint on **real radiation**

$$\Theta(p_\perp^{\mathrm{J,v}} - \max\{k_{t,1}, \ldots, k_{t,n}\}) = \prod_{i=1}^{n} \Theta(p_\perp^{\mathrm{J,v}} - k_{t,i})$$

$p_\perp^H$ resummation formula

$$\frac{d\sigma}{d^2\vec{p}_\perp^H} = \sigma_0 \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_\perp^H} e^{-R_{\mathrm{NLL}}(L)}$$

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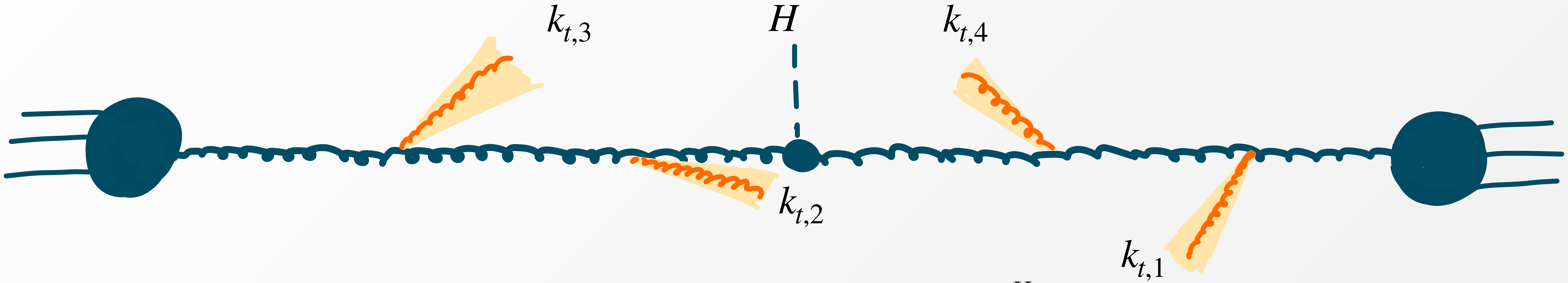

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$$L = \ln(m_H b/b_0)$$

Joint $p_\perp^H$, $p_\perp^{\mathrm{J,v}}$ resummation formula

$$\frac{d\sigma(p_\perp^H, p_\perp^{\mathrm{J,v}})}{d^2\vec{p}_\perp^H} = \sigma_0 \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_\perp^H} e^{-S_{\mathrm{NLL}}(L)}$$

$$S_{\mathrm{NLL}}(L) = -Lg_1(\alpha_s L) - g_2(\alpha_s L) + \int_0^{m_H} \frac{dk_t}{k_t} R'_{\mathrm{NLL}}(k_t) J_0(bk_t) \Theta(k_t - p_\perp^{\mathrm{J,v}})$$

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Double-differential resummation at NNLL in b space

Crucial observation: in b space the phase space constraints entirely factorize → $e^{i\vec{b}\cdot\vec{k}_{t,i}}$

The jet veto constraint can be included by implementing the jet veto resummation at the b-space integrand level **directly in impact-parameter space**

Inclusive contribution: phase space constraint of the form

$$\Theta(p_\perp^{\mathrm{J,v}} - \max\{k_{t,1}, \ldots, k_{t,n}\}) = \prod_{i=1}^{n} \Theta(p_\perp^{\mathrm{J,v}} - k_{t,i})$$

Promote radiator at NNLL

$$\frac{d\sigma(p_\perp^H, p_\perp^{\mathrm{J,v}})}{d^2\vec{p}_\perp^H} = \sigma_0 \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_\perp^H} e^{-S_{\mathrm{NLL}}(L)} \longrightarrow \frac{d\sigma(p_\perp^H, p_\perp^{\mathrm{J,v}})}{d^2\vec{p}_\perp^H} = \sigma_0 \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_\perp^H} e^{-S_{\mathrm{NNLL}}(L)}$$

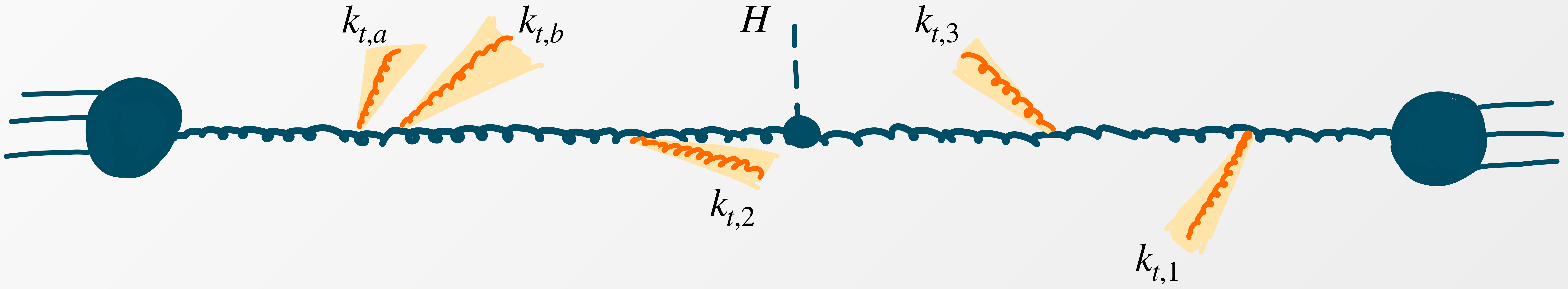
$$S_{\mathrm{NNLL}} = -Lg_1(\alpha_s L) - g_2(\alpha_s L) - \alpha_s g_3(\alpha_s L) + \int_0^{m_H} \frac{dk_t}{k_t} R'_{\mathrm{NNLL}}(k_t) J_0(bk_t) \Theta(k_t - p_\perp^{\mathrm{J,v}})$$

Double-differential resummation at NNLL in *b* space

Additional corrections must be included at NNLL[Banfi et al. '12][Becher et al. '12 ,'13][Stewart et al. '13]

$$\frac{d\sigma(p_\perp^H, p_\perp^{\rm J,v})}{d^2\vec{p}_\perp^H} = \sigma_0 \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_\perp^H} e^{-S_{\rm NNLL}(L)} (1 + \mathcal{F}_{\rm clust} + \mathcal{F}_{\rm correl})$$

clustering correction: jet algorithm can cluster two emissions into the same jet



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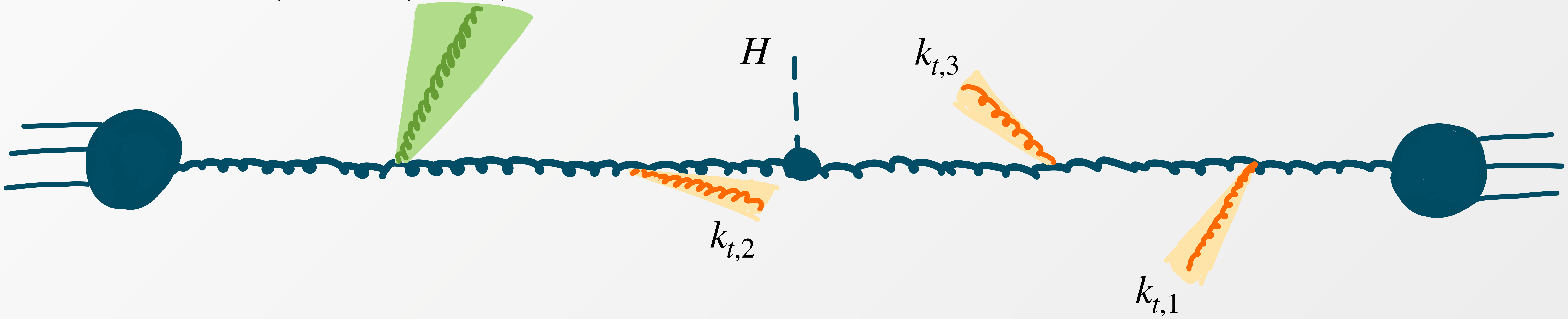
clustering correction: jet algorithm can cluster two emissions into the same jet

$$\mathcal{F}_{\rm clust} = \frac{1}{2!} \int [dk_a][dk_b] M^2(k_a) M^2(k_b) J_{ab}(R)\, e^{i\vec{b}\cdot\vec{k}_{t,ab}} \left[\Theta(p_\perp^{\rm J,v} - k_{t,ab}) - \Theta(p_\perp^{\rm J,v} - \max\{k_{t,a}, k_{t,b}\})\right]$$

$$J_{ab}(R) = \Theta\left(R^2 - \Delta\eta_{ab}^2 - \Delta\phi_{ab}^2\right)$$

$$k_{t,ab} = |\vec{k}_{t,a} + \vec{k}_{t,b}|$$

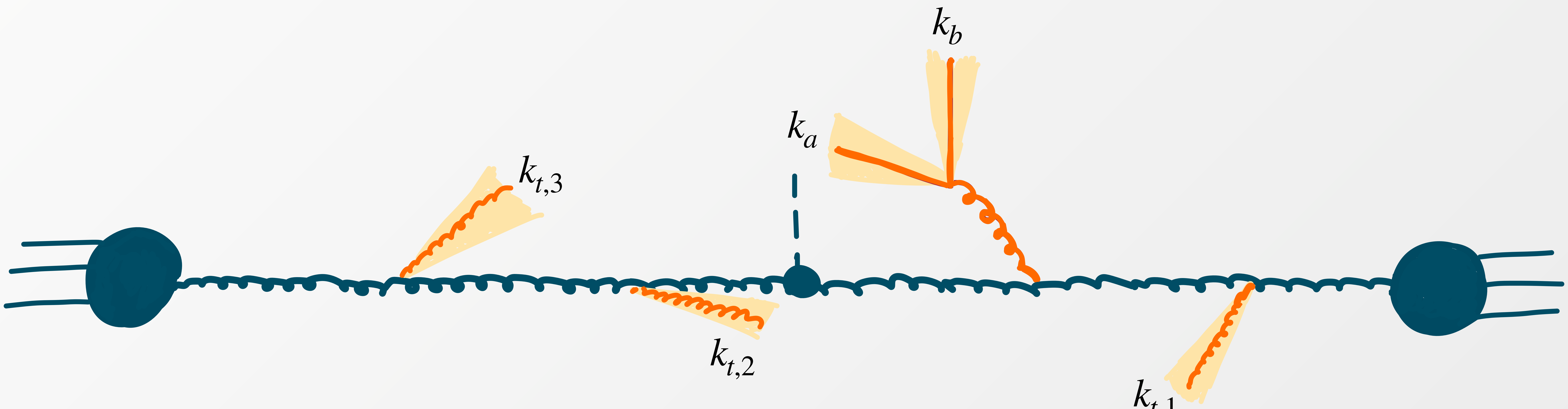


Double-differential resummation at NNLL in b space

Additional corrections must be included at NNLL [Banfi et al. '12][Becher et al. '12 ,'13][Stewart et al. '13]

$$\frac{d\sigma(p_\perp^H, p_\perp^{\rm J,v})}{d^2\vec{p}_\perp^H} = \sigma_0 \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_\perp^H} e^{-S_{\rm NNLL}(L)} (1 + \mathcal{F}_{\rm clust} + \mathcal{F}_{\rm correl})$$

correlated correction: amends the inclusive treatment of the **correlated squared amplitude** for two emission accounting for configurations where the two correlated emissions are not clustered in the same jet



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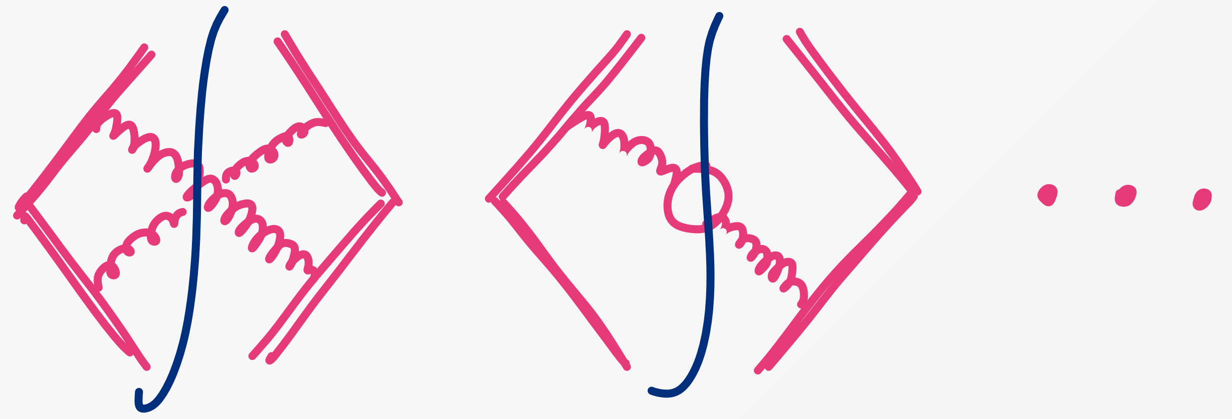
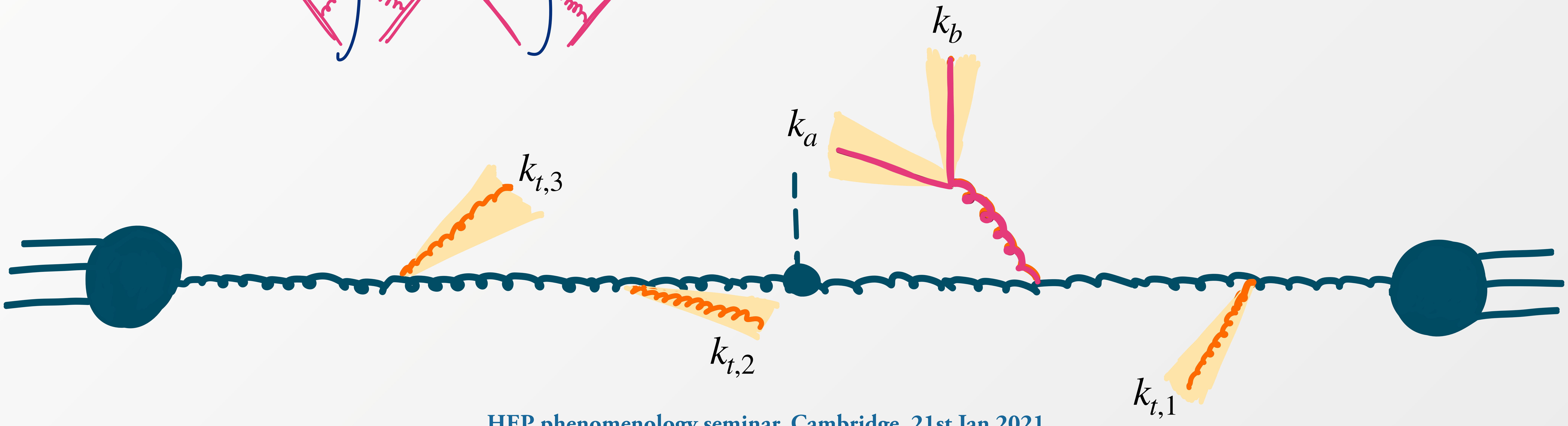
correlated correction: amends the inclusive treatment of the **correlated squared amplitude** for two emission accounting for configurations where the two correlated emissions are not clustered in the same jet

$$\mathscr{F}_{\mathrm{correl}} = \frac{1}{2!}\int [dk_a][dk_b]\tilde{M}^2(k_a, k_b)(1 - J_{ab}(R))e^{i\vec{b}\cdot\vec{k}_{t,ab}} \times \left[\Theta(p_\perp^{\mathrm{J,v}} - \max\{k_{t,a}, k_{t,b}\}) - \Theta(p_\perp^{\mathrm{J,v}} - k_{t,ab})\right]$$





Double-differential resummation at NNLL in b space

NNLL prediction finally requires the consistent treatment of non-soft collinear emissions off the initial state particles

Soft and non-soft emission cannot be clustered by a k_t-type jet algorithm. Non-soft collinear radiation can be handled by taking a Mellin transform of the resummed cross section, giving rise to **scale evolution of PDFs** and of the $\mathcal{O}(\alpha_s)$ **collinear coefficient functions**

Final result at NNLL, including **hard-virtual corrections** at and $\mathcal{O}(\alpha_s)$ **collinear coefficient functions**

$$\frac{d\sigma(p_\perp^H, p_\perp^{\mathrm{J,v}})}{dy_H d^2\vec{p}_\perp^H} = \frac{2\pi}{s} M^2_{\mathrm{gg}\to\mathrm{H}} \mathcal{H}(\alpha_s(m_H)) \int_{\mathcal{C}_1} \frac{d\nu_1}{2\pi i} \int_{\mathcal{C}_2} \frac{d\nu_2}{2\pi i} x_1^{-\nu_1} x_2^{-\nu_2} \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_\perp^H} e^{-S_{\mathrm{NNLL}}} \left(1 + \mathcal{F}_{\mathrm{clust}} + \mathcal{F}_{\mathrm{correl}}\right)$$
$$\times \left[\mathcal{P} e^{\int_0^{m_H} \frac{d\mu}{\mu} \mathbf{\Gamma}_{\nu_1}(\alpha_s(\mu))(\Theta(p_t^{\mathrm{J,v}}-\mu)J_0(b\mu)-1)}\right]_{c_1 a_1} \left[\mathcal{P} e^{\int_0^{m_H} \frac{d\mu}{\mu} \mathbf{\Gamma}_{\nu_2}(\alpha_s(\mu))(\Theta(p_t^{\mathrm{J,v}}-\mu)J_0(b\mu)-1)}\right]_{c_2 a_1} f_{\nu_1,a_1}(m_H) f_{\nu_2 a_2}(m_H)$$
$$\times \left[e^{\int_0^{m_H} \frac{d\mu}{\mu}[\Gamma^{(C)}_{\nu_1}(\alpha_s(\mu))]_{gc_1}(\Theta(p_t^{\mathrm{J,v}}-\mu)J_0(b\mu)-1)}\right]\left[e^{\int_0^{m_H} \frac{d\mu}{\mu}[\Gamma^{(C)}_{\nu_2}(\alpha_s(\mu))]_{gc_2}(\Theta(p_t^{\mathrm{J,v}}-\mu)J_0(b\mu)-1)}\right] C_{\nu_1,gc_1}(\alpha_s(m_h))\, C_{\nu_2,gc_2}(\alpha_s(m_H))$$

Mellin moments

Flavour indices

Asymptotic limits reproduce $p_\perp^{\mathrm{J,v}}$ ($p_\perp^{\mathrm{H}}$) canonical resummation when $p_\perp^H \gg p_\perp^{\mathrm{J,v}}$ ($p_\perp^{\mathrm{J,v}} \gg p_\perp^H$)

Double-differential resummation in direct space

Just need to **combine measurement functions**!

At NLL

$$\sigma(p_\perp^H) = \sigma_0 \int \frac{dk_{t,1}}{k_{t,1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(k_{t,1})} R'\left(k_{t,1}\right) d\mathcal{Z} \Theta\left(p_\perp^H - |\vec{k}_{t,1} + \cdots \vec{k}_{t,n+1}|\right)$$

Double-differential resummation in direct space

Just need to **combine measurement functions**!

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$$\sigma(p_\perp^{\mathrm{J,v}}) = \sigma_0 \int \frac{dk_{t,1}}{k_{t,1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(k_{t,1})} R'\left(k_{t,1}\right) d\mathcal{Z} \Theta \left(p_\perp^{\mathrm{J,v}} - \max\left\{k_{t,1}, \ldots k_{t,n+1}\right\}\right)$$

Double-differential resummation in direct space

Just need to **combine measurement functions**!

At NLL

$$\sigma(p_\perp^{\mathrm{J,v}}, p_\perp^{H}) = \sigma_0 \int \frac{dk_{t,1}}{k_{t,1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(k_{t,1})} R'\left(k_{t,1}\right) d\mathcal{Z}\, \Theta\left(p_\perp^{\mathrm{J,v}} - \max\left\{k_{t,1}, \ldots k_{t,n+1}\right\}\right) \Theta\left(p_\perp^{H} - |\vec{k}_{t,1} + \cdots \vec{k}_{t,n+1}|\right)$$

Double-differential resummation in direct space

Just need to **combine measurement functions**!

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Same philosophy at NNLL

$$\sigma^{\mathrm{NNLL}}(p_\perp^{\mathrm{J,v}}) = \sigma^{\mathrm{NNLL}}_{\mathrm{incl}}(p_\perp^{\mathrm{J,v}}) + \sigma^{\mathrm{NNLL}}_{\mathrm{clust}}(p_\perp^{\mathrm{J,v}}) + \sigma^{\mathrm{NNLL}}_{\mathrm{corr}}(p_\perp^{\mathrm{J,v}})$$

where e.g.

$$\sigma^{\mathrm{NNLL}}_{\mathrm{clust}}(p_\perp^{\mathrm{J,v}}) \simeq \int_0^\infty \frac{dk_{t,1}}{k_{t,1}} \frac{d\phi_1}{2\pi} \int d\mathcal{Z}\, e^{-R(k_{t,1})}\, 8\, C_A^2 \frac{\alpha_s^2(k_{t,1})}{\pi^2} \Theta\left(p_\perp^{\mathrm{J,v}} - \max_{i>1}\{k_{t,i}\}\right)$$
$$\times \int_0^{k_{t,1}} \frac{dk_{t,s_1}}{k_{t,s_1}} \frac{d\phi_{s_1}}{2\pi} \int_{-\infty}^{\infty} d\Delta\eta_{1s_1} J_{1s_1}(R) \left[\Theta\left(p_\perp^{\mathrm{J,v}} - |\vec{k}_{t,1} + \vec{k}_{t,s_1}|\right) - \Theta\left(p_\perp^{\mathrm{J,v}} - k_{t,1}\right)\right]$$

Double-differential resummation in direct space

Just need to **combine measurement functions**!

At NLL

$$\sigma(p_\perp^{\mathrm{J,v}}, p_\perp^{H}) = \sigma_0 \int \frac{dk_{t,1}}{k_{t,1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(k_{t,1})} R'\left(k_{t,1}\right) d\mathcal{Z}\, \Theta\left(p_\perp^{J,v} - \max\left\{k_{t,1}, \ldots k_{t,n+1}\right\}\right) \Theta\left(p_\perp^{H} - |\vec{k}_{t,1} + \cdots \vec{k}_{t,n+1}|\right)$$

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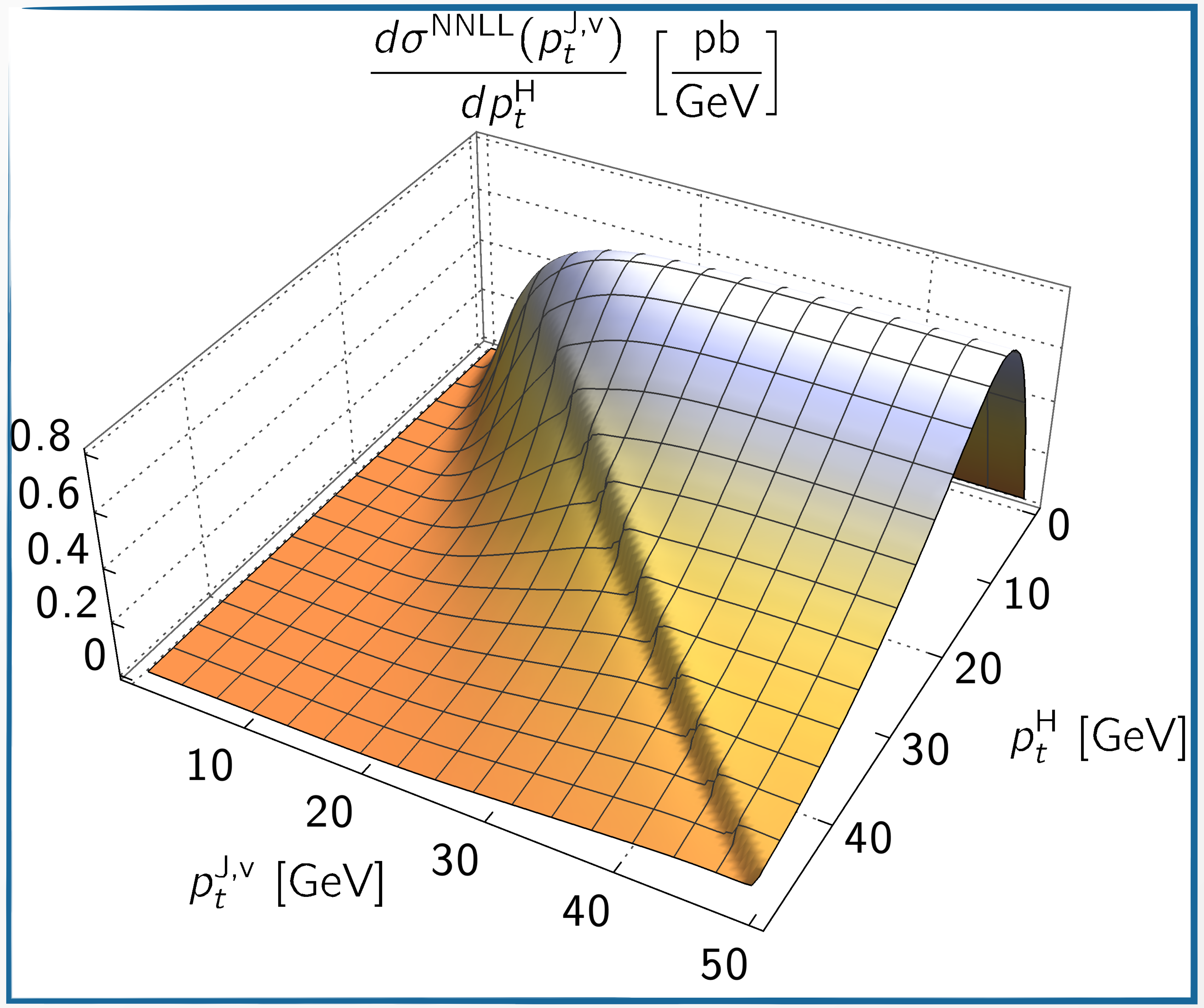
$$\times\, \Theta\left(p_\perp^{H} - |\vec{k}_{t,1} + \ldots + \vec{k}_{t,n+1} + \vec{k}_{t,s_1}|\right)$$

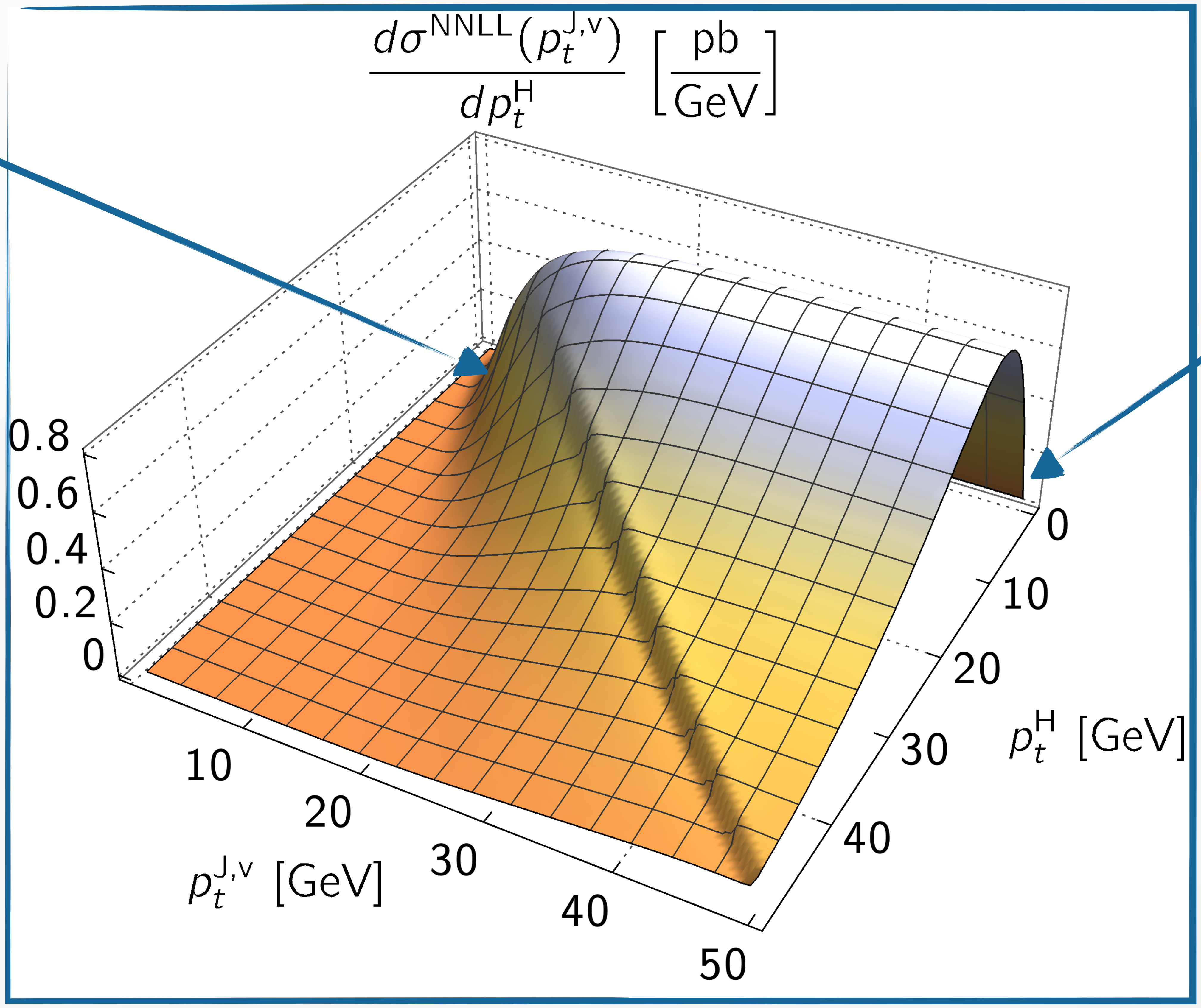
And analogously for other contributions

NNLL cross section differential in $p_\perp^H$, cumulative in $p_\perp^J \leq p_\perp^{J,v}$



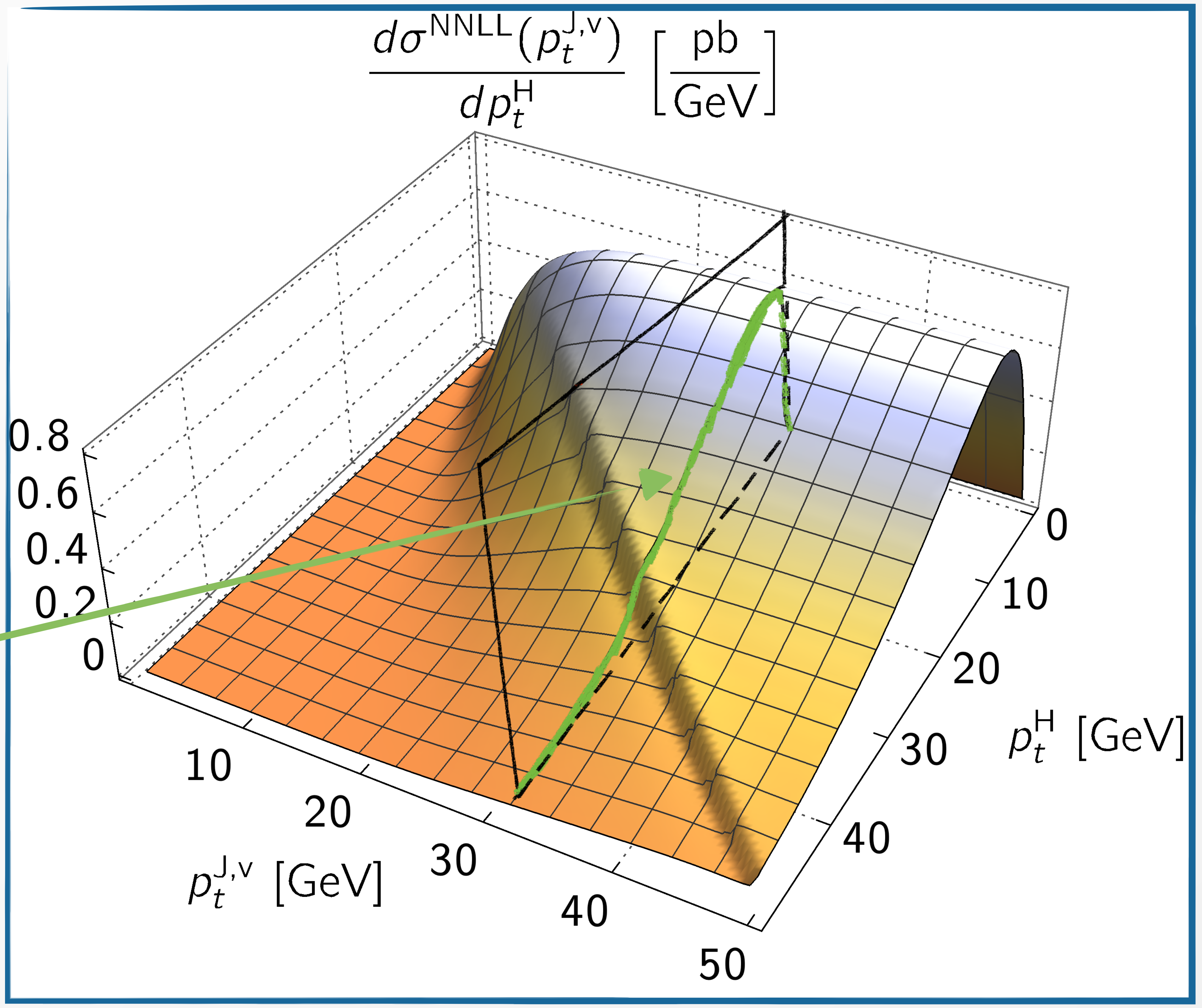
NNLL cross section differential in $p_\perp^H$, cumulative in $p_\perp^J \leq p_\perp^{J,v}$

Sudakov suppression at small $p_\perp^J$

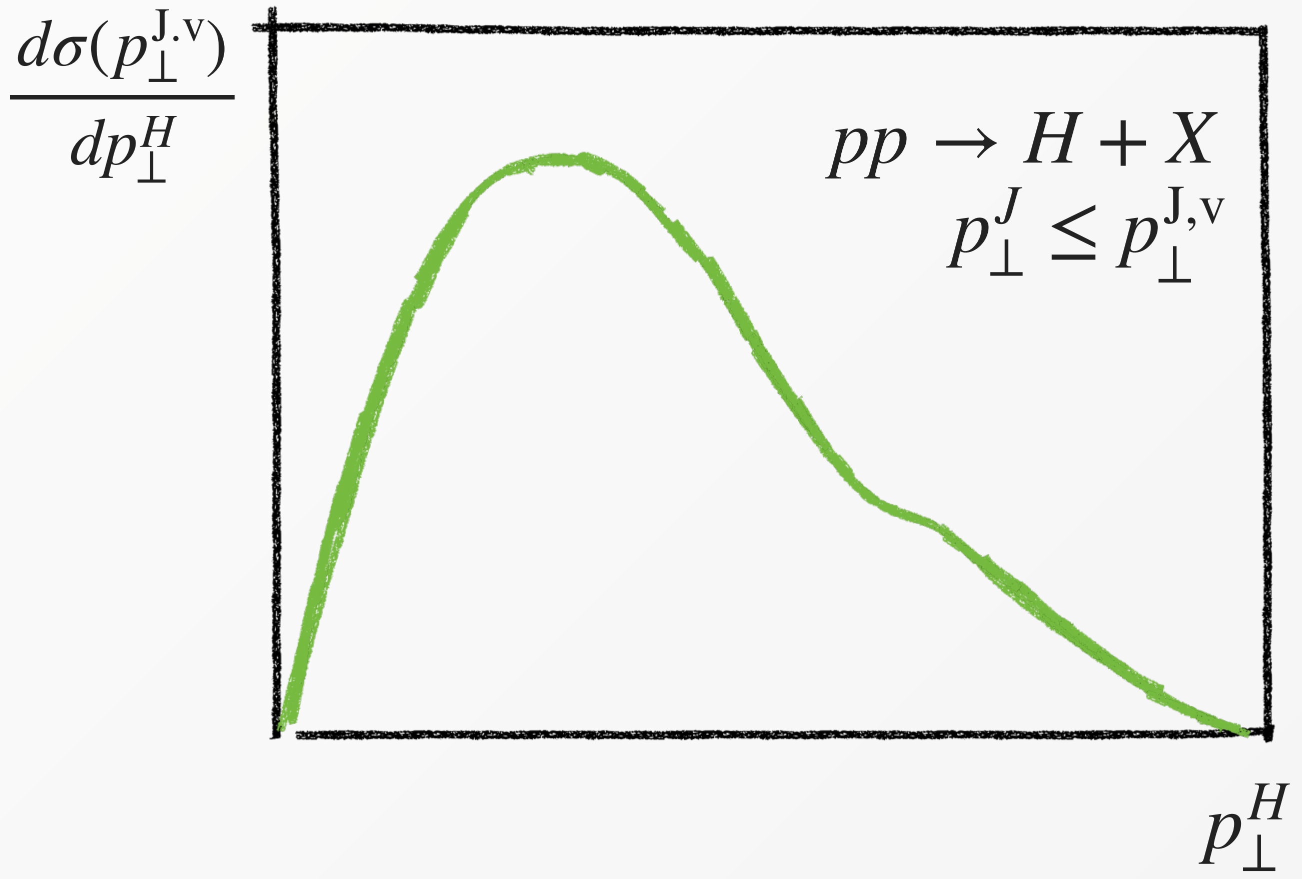


Peaked structure (Sudakov) + power-like suppression at very small $p_\perp^H$

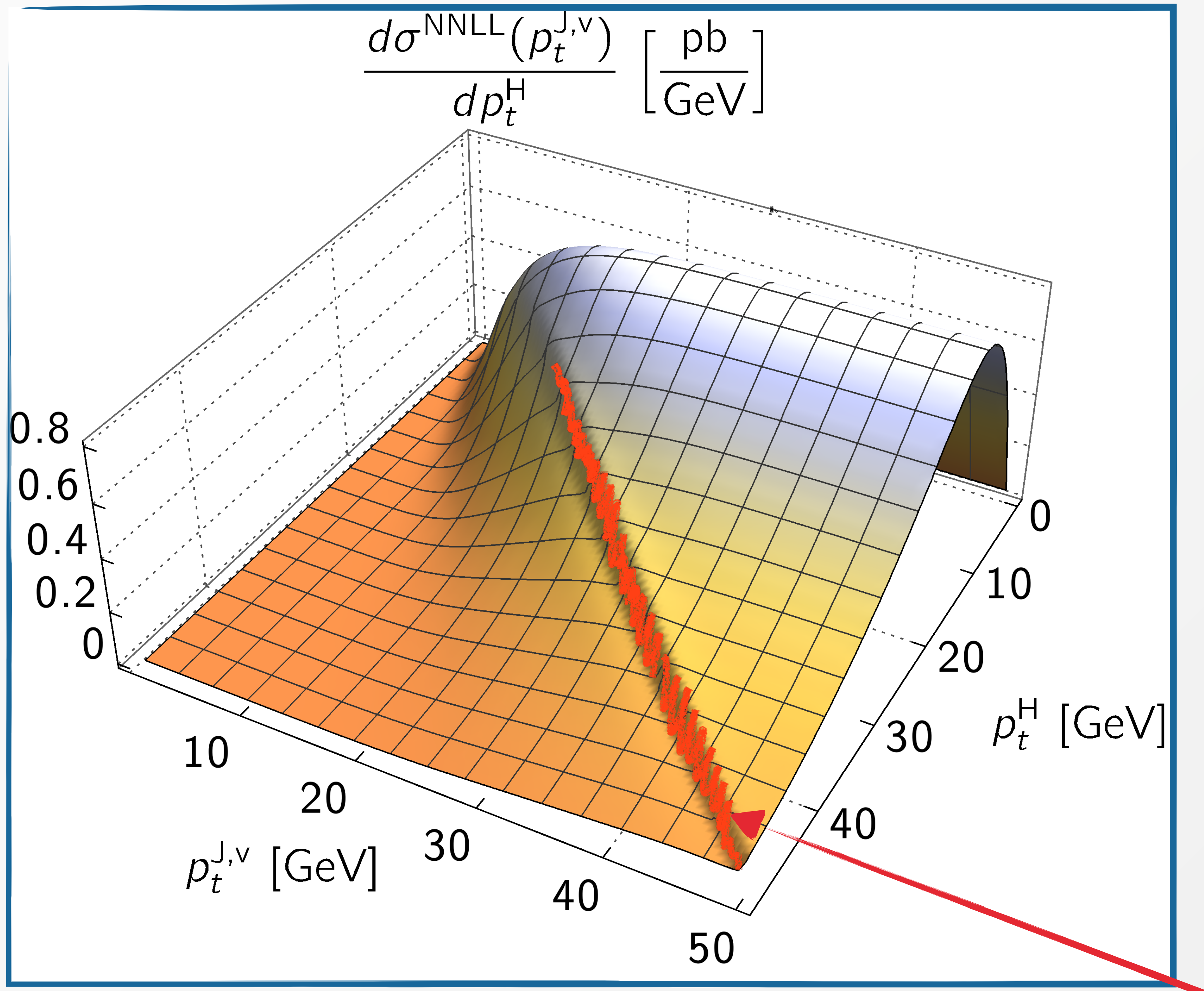
NNLL cross section differential in $p_\perp^H$, cumulative in $p_\perp^{\rm J} \leq p_\perp^{\rm J,v}$



At a given value of $p_\perp^{\rm J,v}$ it corresponds to the $p_\perp^H$ cross section in the 0-jet bin



NNLL cross section differential in $p_\perp^H$, cumulative in $p_\perp^{\rm J} \leq p_\perp^{\rm J,v}$

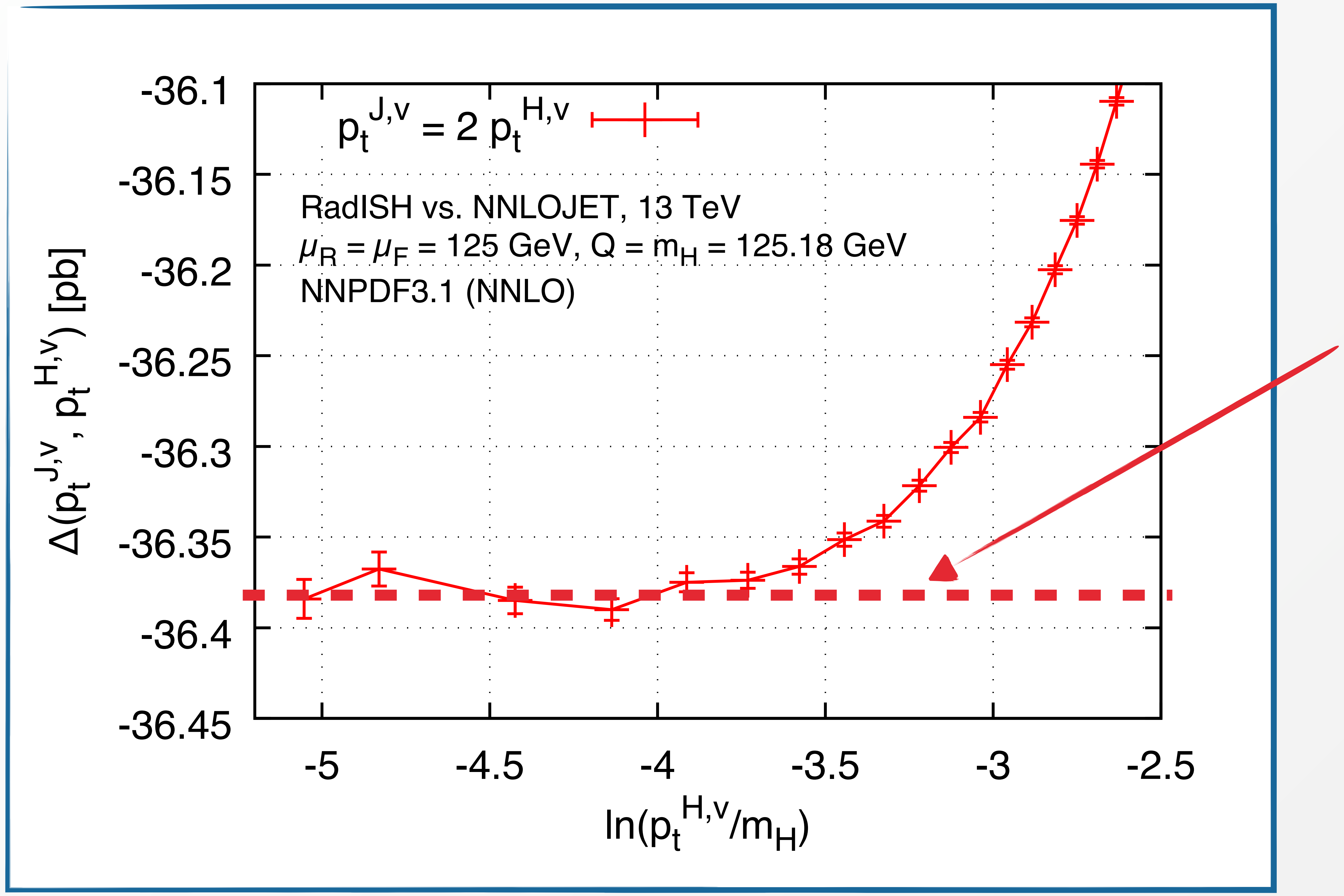


[Catani, Webber '97]

Sudakov shoulder: integrable singularity beyond LO at $p_\perp^H \simeq p_\perp^{\rm J,v}$

Logarithms associated to the Shoulder are resummed in the limit $p_\perp^H \sim p_\perp^{\rm J,v} \ll m_H$

Accuracy check at $\mathcal{O}(\alpha_s^2)$



Comparison of the expansion of the resummed result with the fixed order at $\mathcal{O}(\alpha_s^2)$ in the limit $p_\perp^H \sim p_\perp^{J,v} \ll m_H$

Difference at the double-cumulative level goes to a **constant** (all logarithmic terms correctly predicted)

Very strong check: **NNLL resummation** of the logarithms associated to the shoulder

Analogous checks performed in the limits $p_\perp^H \ll p_\perp^{J,v} < m_H$ and $p_\perp^{J,v} \ll p_\perp^H < m_H$

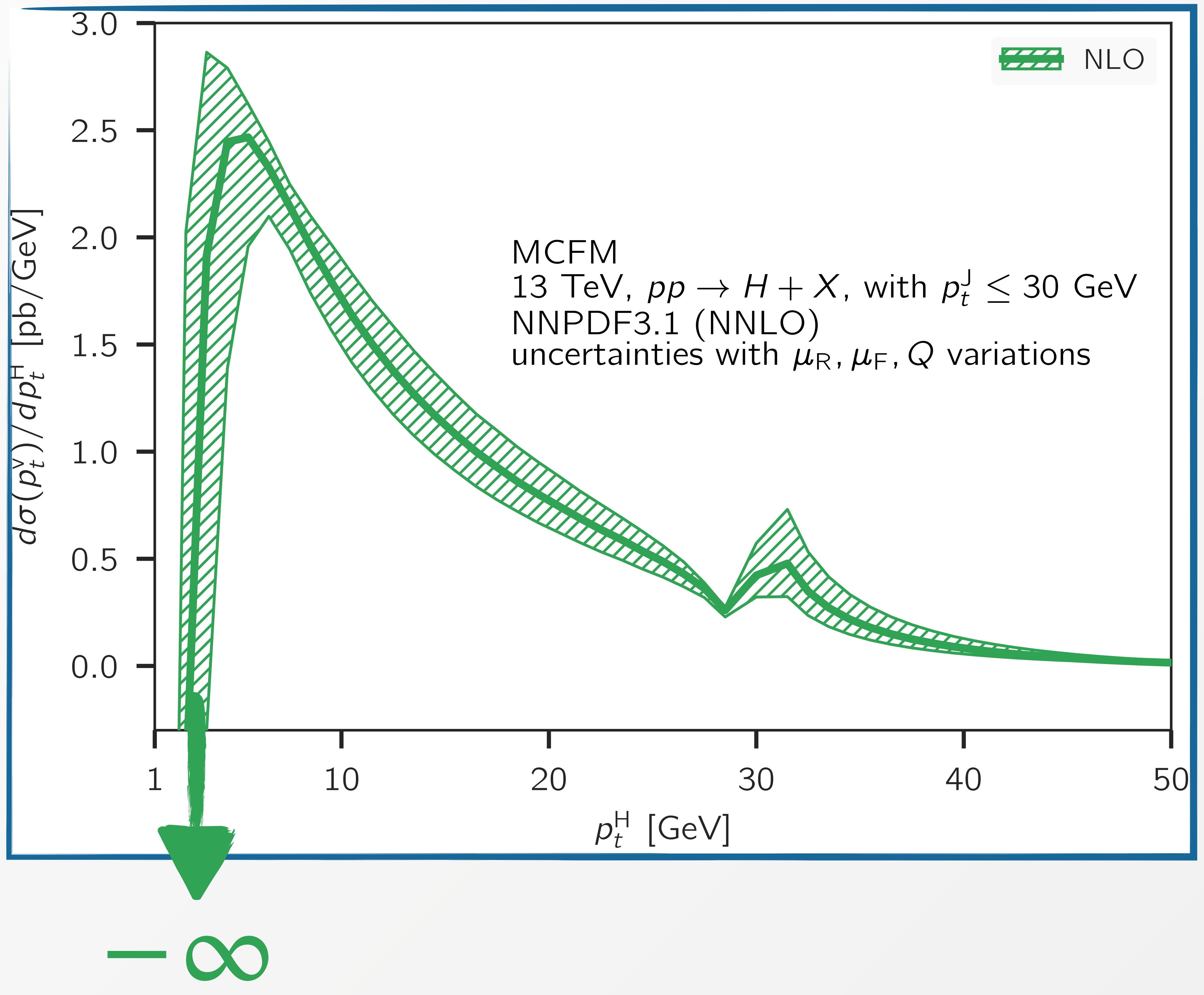
$$\Delta(p_\perp^{J,v}, p_\perp^{H,v}) = \sigma^{NNLO}(p_\perp^{J,v}, p_\perp^{H,v}) - \sigma^{NNLL}_{exp.}(p_\perp^{J,v}, p_\perp^{H,v})$$

$$\sigma^{NNLO}(p_\perp^H < p_\perp^{H,v}, p_\perp^J < p_\perp^{J,v}) = \sigma^{NNLO} - \int \Theta(p_\perp^H > p_\perp^{H,v}) \vee \Theta(p_\perp^J > p_\perp^{J,v}) d\sigma^{NLO}_{H+J}$$

LHC results: Higgs transverse momentum with a jet veto

Multiplicative matching to fixed order (NLO H+j from MCFM, NNLO H from ggHiggs)

[Campbell, Ellis, Giele,'15] [Bonvini et al '13]



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Multiplicative matching to fixed order (NLO H+j from MCFM, NNLO H from ggHiggs)

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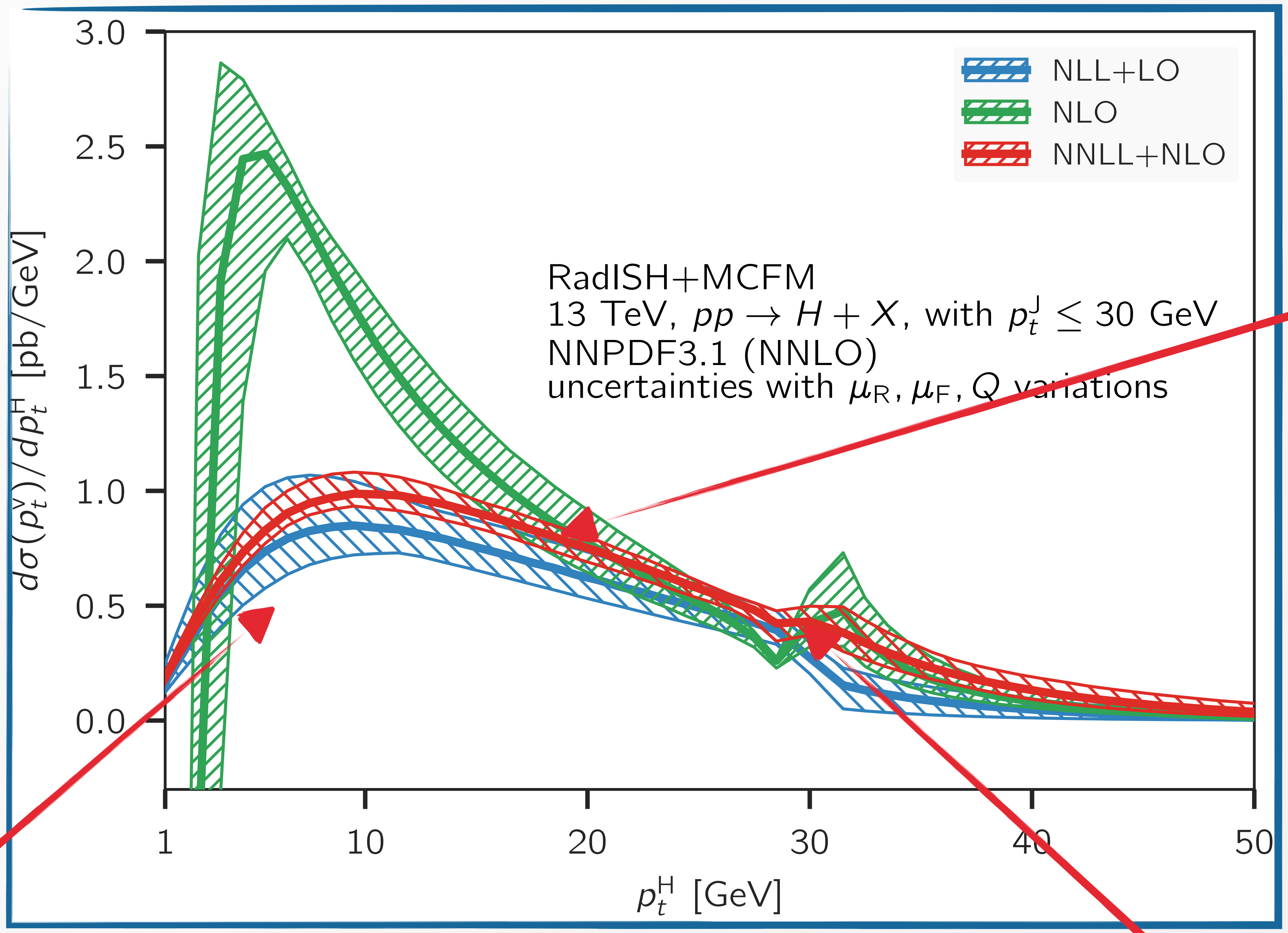
residual uncertainties at **NNLL+NLO** at the 10% level



large K-factor becomes relevant at larger $p_\perp^H$

good perturbative convergence below 10 GeV

much reduced sensitivity to the Sudakov shoulder with respect to NLO spectrum

LHC applications to more complex processes: W^+W^- production

Jet vetoed analyses commonly enforced in LHC searches

For instance, W^+W^- channel, which is relevant for BSM searches into leptons, missing energy and/or jets and Higgs measurements, suffers from a signal contamination due to large top-quark background

Fiducial region defined by a rather stringent jet veto

$$p_{T,\ell} > 27\,\mathrm{GeV}, \quad |\eta_\ell| < 2.5, \quad m_{\ell^-\ell^+} > 55\,\mathrm{GeV}, \quad p_{T,\ell^-\ell^+} > 30\,\mathrm{GeV}$$

$$p_T^{\mathrm{miss}} > 20\,\mathrm{GeV}$$

$$\text{anti-}k_T \text{ jets with } R = 0.4;$$

$$N_{\mathrm{jet}} = 0 \text{ for } p_T^J > 35\,\mathrm{GeV}$$

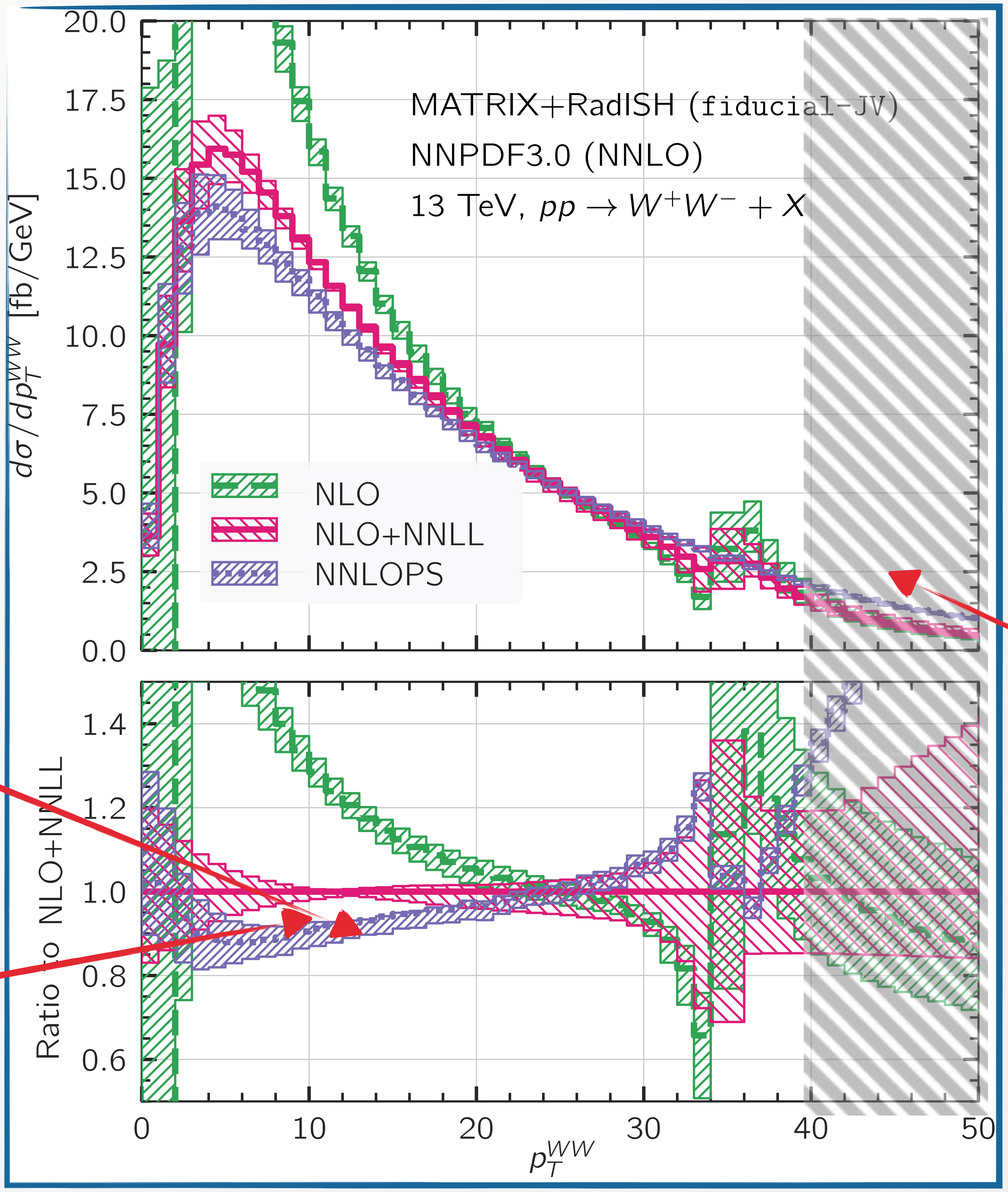
LHC applications: W^+W^- production

NNLL+NLO spectrum obtained by interfacing RadISH with MATRIX [Grazzini, Kallweit, Rathlev, Wiesemann '15, '17]

[Wiesemann, Re, Zanderighi '18]

Comparison with NNLOPS result (much lower log accuracy) shows differences at the $\mathcal{O}(10\%)$ level

Accidental cancellation of perturbative uncertainties; more conservative prescriptions can be considered

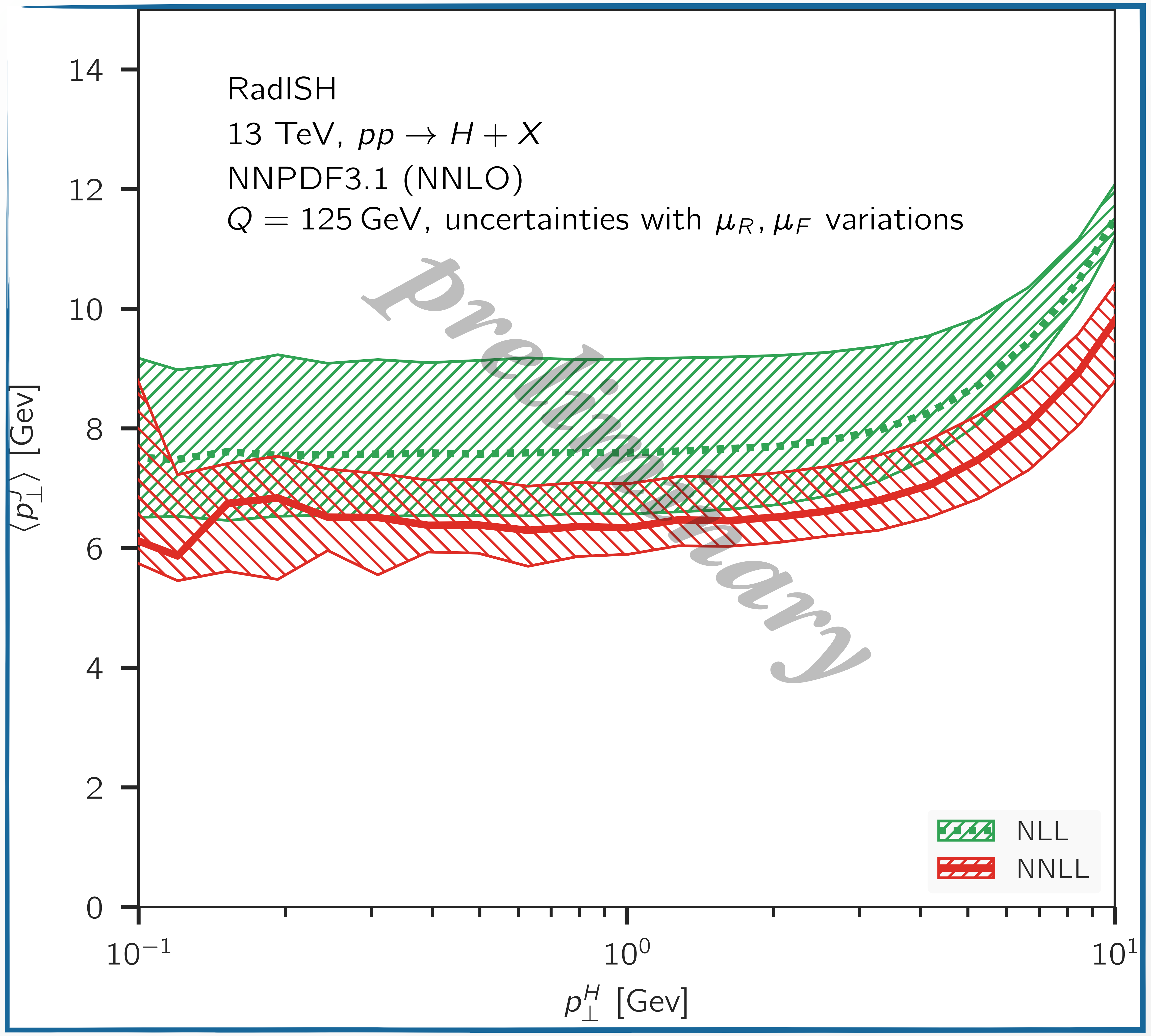


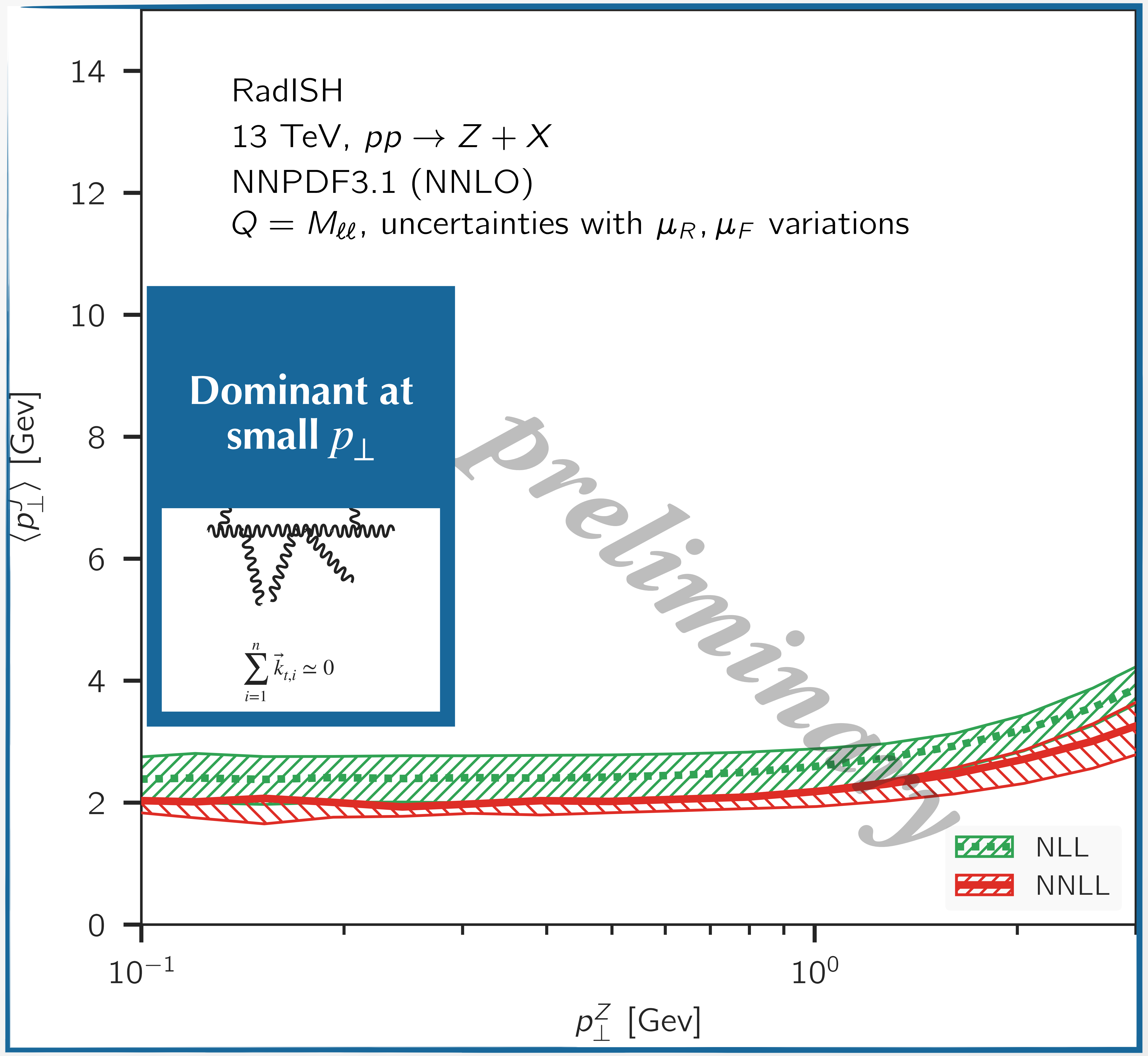
Multi-parton configurations become relevant above the shoulder

[Kallweit, Wiesemann, Re, LR '2004.07720]

*3. **More differential description** of the QCD radiation than that usually possible in a conjugate-space formulation*

Direct space: access to differential information and underlying dynamics





Possible access to subleading jets and higher moments

Summary

- Precision of the data demands an increasing theoretical accuracy at the **multi-differential level** to fully exploit LHC potential

- New formalism formulated in **direct space** for all-order resummation up to **N^{3}LL accuracy** for inclusive, transverse observables.

- Direct space formulation (RadISH) provides guidance to obtain **elegant and compact formulation in *b*-space** for **joint resummation** for a **double-differential** kinematic observable involving a **jet algorithm** at NNLL accuracy and offers access to underlying dynamics

- Formalism can be readily extended to **more complex final states**; 2$\longrightarrow$1 and 2$\longrightarrow$2 colour singlet processes available via MATRIX+RadISH framework

Backup

All-order structure of the matrix element

$$v = p_t/M$$

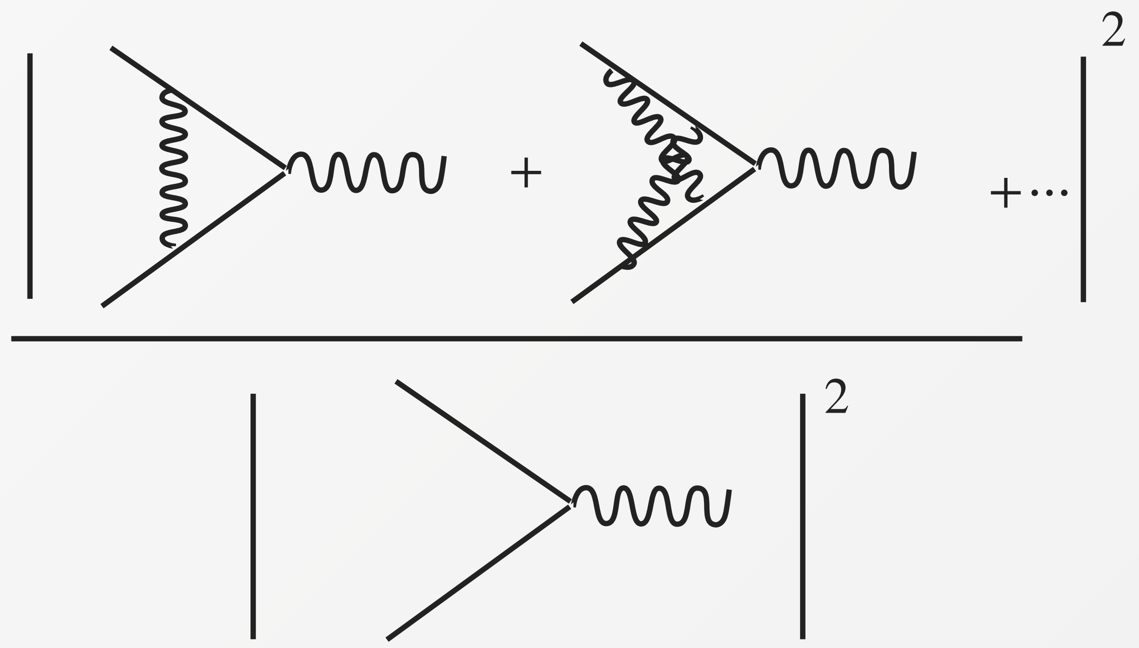
single-particle phase space

matrix element for *n* real emissions

$$\Sigma(v) = \int d\Phi_B \mathcal{V}(\Phi_B) \sum_{n=0}^{\infty} \int \prod_{i=1}^{n} [dk_i] \, |\mathcal{M}(\Phi_B, k_1, \ldots k_n)|^2 \, \Theta(v - V(\{\Phi_B\}, k_1, \ldots k_n))$$

all-order form factor
(virtuals)

e.g. [Dixon, Magnea, Sterman '08]



Transverse observable resummation with RadISH

1. Establish a **logarithmic counting** for the squared matrix element $|\mathcal{M}(\Phi_B, k_1, \ldots k_n)|^2$

Decompose the squared amplitude in terms of ***n*-particle correlated blocks,** denoted by $|\tilde{\mathcal{M}}(k_1, \ldots, k_n)|^2$ $(|\tilde{\mathcal{M}}(k_1)|^2 = |\mathcal{M}(k_1)|^2)$

$$\sum_{n=0}^{\infty} |\mathcal{M}(\Phi_B, k_1, \ldots, k_n)|^2 = |\mathcal{M}_B(\Phi_B|^2$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \left\{ \prod_{i=1}^{n} \left(\underbrace{|\mathcal{M}(k_i)|^2}_{\text{LL}} + \int [dk_a][dk_b] \underbrace{|\tilde{\mathcal{M}}(k_a, k_b)|^2}_{\text{NLL}} \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} - \vec{k}_{ti}) \delta(Y_{ab} - Y_i) \right.\right.$$

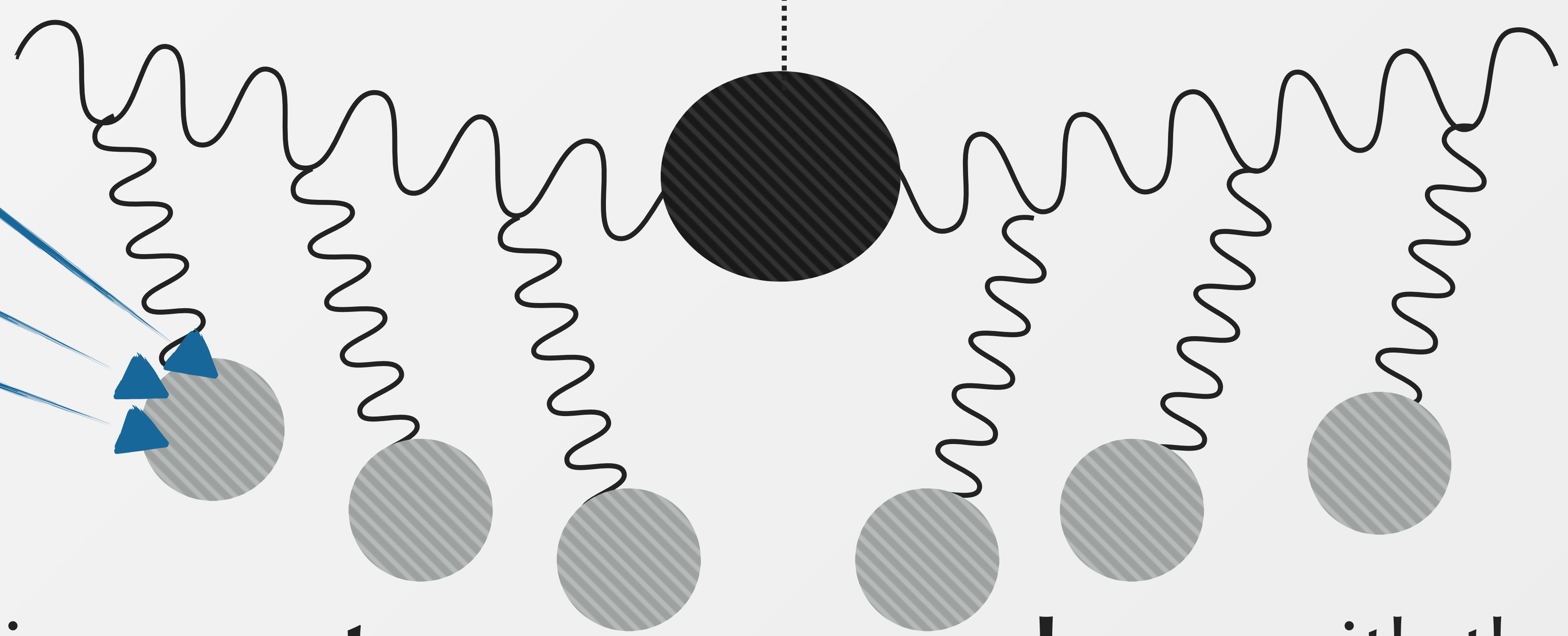
$$\left.\left. + \int [dk_a][dk_b][dk_c] \underbrace{|\tilde{\mathcal{M}}(k_a, k_b, k_c)|^2}_{\text{NNLL}} \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} + \vec{k}_{tc} - \vec{k}_{ti}) \delta(Y_{abc} - Y_i) + \ldots \right) \right\}$$

$$\equiv |\mathcal{M}_B(\Phi_B)|^2 \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} |\mathcal{M}(k_i)|^2_{\text{inc}}$$

*expression valid for inclusive observables

$$|\tilde{M}(k_1)|^2 = \frac{|M(k_1)|^2}{|M_B|^2} = |M(k_1)|^2$$

$$|\tilde{M}(k_1, k_2)|^2 = \frac{|M(k_1, k_2)|^2}{|M_B|^2} - \frac{1}{2!} |M(k_1)|^2 M|(k_2)|^2$$



Upon integration over the phase space, the expansion can be put in a **one to one correspondence** with the logarithmic structure

Systematic recipe to include terms up to the desired logarithmic accuracy

Resummation in direct space: the p_t case

2. Exploit rIRC safety to single out the IRC singularities of the real matrix element and achieve the **cancellation of the exponentiated divergences** of virtual origin

Introduce a slicing parameter $\epsilon \ll 1$ such that all inclusive blocks with $k_{t,i} < \epsilon k_{t,1}$, with $k_{t,1}$ hardest emission, can be neglected in the computation of the observable

$$\Sigma(v) = \int d\Phi_B |\mathcal{M}_B(\Phi_B)|^2 \mathcal{V}(\Phi_B)$$

unresolved emissions

$$\times \int [dk_1] |\mathcal{M}(k_1)|^2_{\text{inc}} \left(\sum_{l=0}^{\infty} \frac{1}{l!} \int \prod_{j=2}^{l+1} [dk_j] |\mathcal{M}(k_j)|^2_{\text{inc}} \Theta(\epsilon V(k_1) - V(k_j)) \right)$$

$$\times \left(\sum_{m=0}^{\infty} \frac{1}{m!} \int \prod_{i=2}^{m+1} [dk_i] |\mathcal{M}(k_i)|^2_{\text{inc}} \Theta(V(k_i) - \epsilon V(k_1)) \Theta\left(v - V(\Phi_B, k_1, \ldots, k_{m+1})\right) \right)$$

resolved emissions

Unresolved emission doesn't contribute to the evaluation of the observable: it can be exponentiated directly and employed to cancel the virtual divergences, giving rise to a Sudakov radiator

$$\mathcal{V}(\Phi_B) \exp\left\{ \int [dk] |\mathcal{M}(k)|^2_{\text{inc}} \Theta(\epsilon V(k_1) - V(k)) \right\} \simeq e^{-R(\epsilon V(k_1))}$$

Resummation in direct space: the p_t case

Result at NLL accuracy can be written as

$$\Sigma(v) = \sigma^{(0)} \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(\epsilon v_1)} R'\left(v_1\right) \qquad v_i = V(k_i), \quad \zeta_i = v_i / v_1$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^{1} \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'\left(\zeta_i v_1\right)\, \Theta\left(v - V(\Phi_B, k_1, \ldots, k_{n+1})\right)$$

Formula can be evaluated with Monte Carlo method; dependence on ϵ vanishes exactly and result is finite in four dimensions

It contains **subleading effect** which in the original CAESAR approach are disposed of by expanding R and R' around v

$$R(\epsilon v_1) = R(v) + \frac{dR(v)}{d\ln(1/v)} \ln \frac{v}{\epsilon v_1} + \mathcal{O}\left(\ln^2 \frac{v}{\epsilon v_1}\right)$$

$$R'\left(v_i\right) = R'(v) + \mathcal{O}\left(\ln \frac{v}{v_i}\right)$$

Not possible! valid only if the ratio v_i/v remains of order one in the whole emission phase space, but for observables which feature kinematic cancellations there are configurations with $v_i \gg v$. **Subleading effects necessary**

Resummation in direct space: the p_t case

Result at NLL accuracy can be written as

$$\Sigma(v) = \sigma^{(0)} \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(\epsilon v_1)} R'\left(v_1\right) \qquad v_i = V(k_i), \quad \zeta_i = v_i/v_1$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^{1} \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'\left(\zeta_i v_1\right) \, \Theta\left(v - V(\Phi_B, k_1, \ldots, k_{n+1})\right)$$

Formula can be evaluated with Monte Carlo method; dependence on ϵ vanishes exactly and result is finite in four dimensions

Convenient to perform an expansion around k_{t1} (more efficient and simpler implementation)

$$R(\epsilon k_{t1}) = R(k_{t1}) + \frac{dR(k_{t1})}{d\ln(1/k_{t1})} \ln\frac{1}{\epsilon} + \mathcal{O}\left(\ln^2\frac{1}{\epsilon}\right)$$

$$R'\left(k_{ti}\right) = R'(k_{t1}) + \mathcal{O}\left(\ln\frac{k_{t1}}{k_{ti}}\right)$$

Subleading effects retained: no divergence at small v, power-like behaviour respected

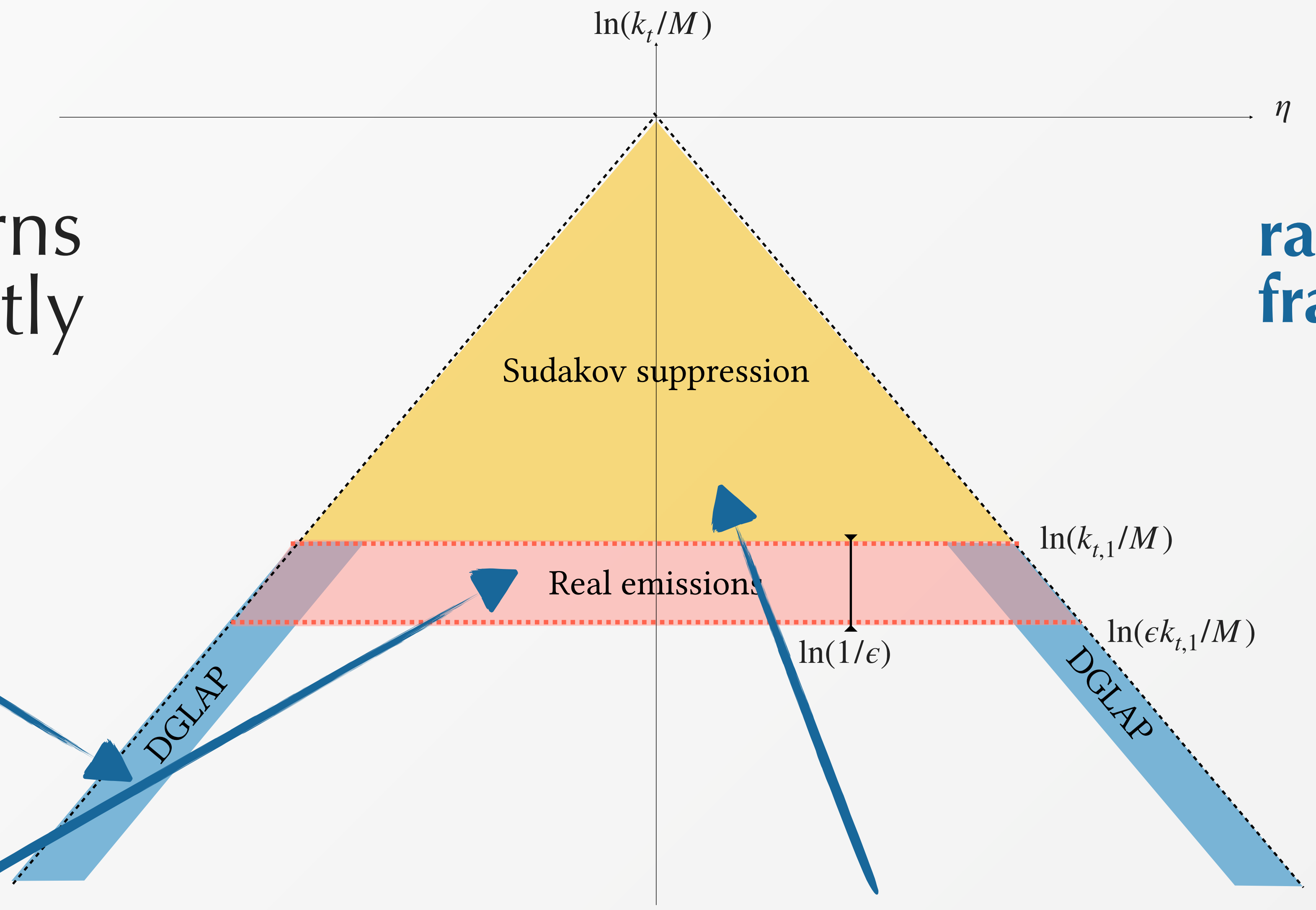
Logarithmic accuracy defined in terms of $\ln(M/k_{t1})$

Result formally equivalent to the b-space formulation

Parton luminosities

Consider configurations in which emissions are ordered in $k_{t,i}$, $k_{t,1}$ hardest emission

Phase space for each secondary emission can be depicted in the Lund diagram



DGLAP evolution governs the radiation in the strictly collinear limit

rapidity in the centre-of-mass frame of the incoming partons

resolved emissions live in this strip

remaining **unresolved real emissions** are combined with the virtual corrections to give rise to Sudakov suppression

- DGLAP evolution can be performed **inclusively** up to $\epsilon k_{t,1}$ thanks to rIRC safety
- In the **overlapping region** hard-collinear emissions modify the observable's value: the evolution should be performed exclusively (unintegrated in k_t)
- At NLL the real radiation can be approximated with its soft limit: DGLAP can be performed inclusively up to $k_{t,1}$ (i.e. one can evaluate $\mu_F = k_{t,1}$)

Beyond NLL

Extension to NNLL and beyond requires the systematic inclusion of the correlated blocks necessary to achieve the desired logarithmic accuracy

Moreover, one needs to **relax a series of assumptions** which give rise to subleading corrections neglected at NLL (for instance, exact rapidity bounds). These corrections can be included systematically by including additional terms in the expansion

$$R(\epsilon v_1) = R(v_1) + \frac{dR(v_1)}{d\ln(1/v_1)} \ln\frac{1}{\epsilon} + \mathcal{O}\left(\ln^2\frac{1}{\epsilon}\right)$$

Finally, one needs to specify a complete treatment for **hard-collinear radiation.** Starting at NNLL one or more real emissions can be hard and collinear to the emitting leg, and the available phase space for subsequent real emissions changes

Two classes of contributions:

- one soft by construction and which is analogous to the R′ contribution

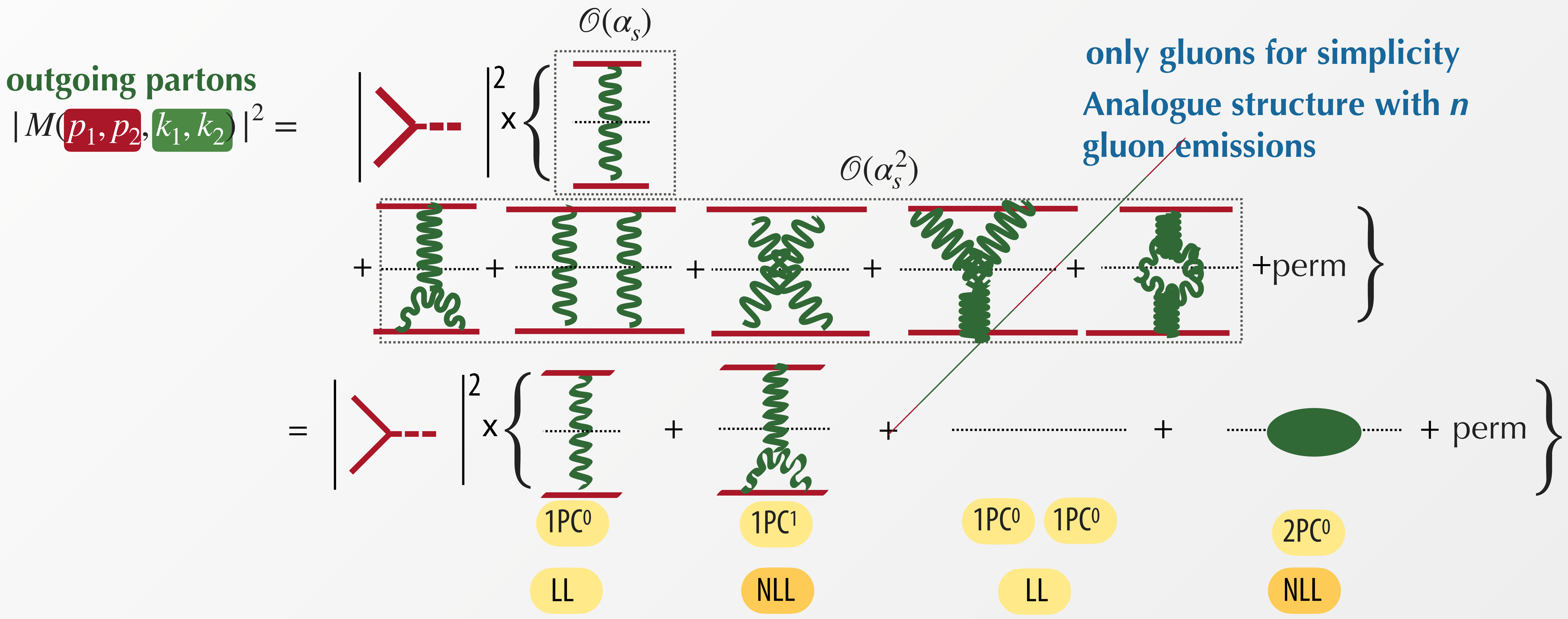
$$R'\left(v_i\right) = R'(v_1) + \mathcal{O}\left(\ln\frac{v_1}{v_i}\right)$$

- another hard and collinear (exclusive DGLAP step): last step of DGLAP evolution must be performed unintegrated in k_t

Logarithmic counting

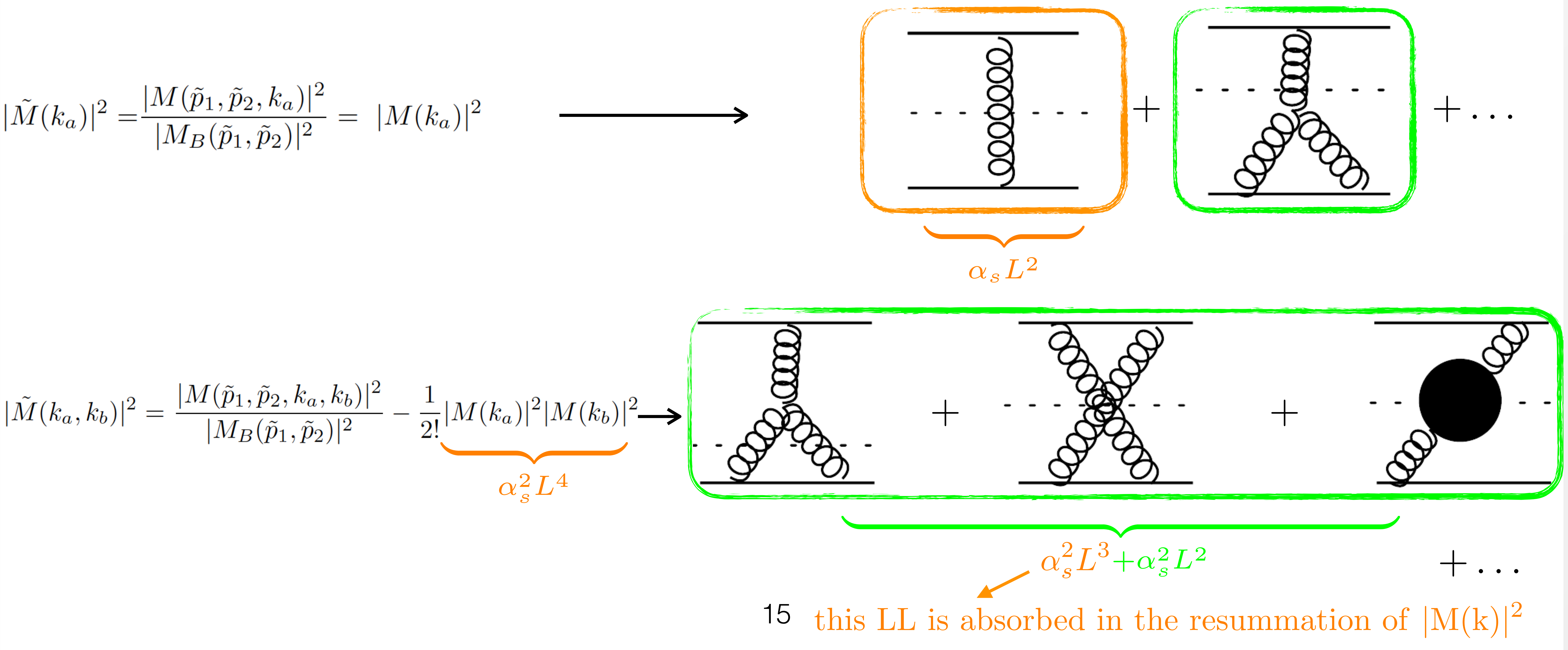
Necessary to establish a **well defined logarithmic counting**: possibile to do that by decomposing the squared amplitude in terms of n-particle correlated blocks (**nPC**)

e.g. $pp \to H$ + emission of up to 2 (soft) gluons $O(\alpha_s^2)$



Logarithmic counting defined in terms of **nPC blocks** (owing to rIRC safety of the observable)

Logarithmic counting: correlated blocks



Thanks to P. Monni

Resummation at NLL accuracy

Final result at NLL

$$\frac{d\Sigma(v)}{d\Phi_B} = \int \frac{dk_{t,1}}{k_{t,1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(k_{t,1})} \epsilon^{R'(k_{t,1})} \mathscr{L}_{\mathrm{NLL}}(k_{t,1}) R'\left(k_{t,1}\right)$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^{1} \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'\left(\zeta_i k_{t,1}\right) \Theta\left(v - V(\Phi_B, k_1, \ldots, k_{n+1})\right)$$

This formula can be evaluated by means of fast Monte Carlo methods `RadISH (Radiation off Initial State Hadrons)`

Parton luminosity at NLL reads

$$\mathscr{L}_{\mathrm{NLL}}(k_{t,1}) = \sum_c \frac{d|M_B|^2_{c\bar{c}}}{d\Phi_B} f_c\left(x_1, k_{t,1}^2\right) f_{\bar{c}}\left(x_2, k_{t,1}^2\right)$$

At higher logarithmic accuracy, it includes **coefficient functions** and **hard-virtual** corrections

Result at N^{3}LL accuracy

$$
\begin{aligned}
\frac{d\Sigma(v)}{d\Phi_B} &= \int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}\partial_L\left(-e^{-R(k_{t1})}\mathcal{L}_{\mathrm{N^3LL}}(k_{t1})\right)\int d\mathcal{Z}[\{R', k_i\}]\Theta\left(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1})\right)\\
&+ \int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}e^{-R(k_{t1})}\int d\mathcal{Z}[\{R', k_i\}]\int_0^1 \frac{d\zeta_s}{\zeta_s}\frac{d\phi_s}{2\pi}\Bigg\{\left(R'(k_{t1})\mathcal{L}_{\mathrm{NNLL}}(k_{t1}) - \partial_L\mathcal{L}_{\mathrm{NNLL}}(k_{t1})\right)\\
&\times\left(R''(k_{t1})\ln\frac{1}{\zeta_s} + \frac{1}{2}R'''(k_{t1})\ln^2\frac{1}{\zeta_s}\right) - R'(k_{t1})\left(\partial_L\mathcal{L}_{\mathrm{NNLL}}(k_{t1}) - 2\frac{\beta_0}{\pi}\alpha_s^2(k_{t1})\hat{P}^{(0)}\otimes\mathcal{L}_{\mathrm{NLL}}(k_{t1})\ln\frac{1}{\zeta_s}\right)\\
&+ \frac{\alpha_s^2(k_{t1})}{\pi^2}\hat{P}^{(0)}\otimes\hat{P}^{(0)}\otimes\mathcal{L}_{\mathrm{NLL}}(k_{t1})\Bigg\}\Big\{\Theta\left(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1}, k_s)\right) - \Theta\left(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1})\right)\Big\}\\
&+ \frac{1}{2}\int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}e^{-R(k_{t1})}\int d\mathcal{Z}[\{R', k_i\}]\int_0^1 \frac{d\zeta_{s1}}{\zeta_{s1}}\frac{d\phi_{s1}}{2\pi}\int_0^1 \frac{d\zeta_{s2}}{\zeta_{s2}}\frac{d\phi_{s2}}{2\pi}R'(k_{t1})\\
&\times\Bigg\{\mathcal{L}_{\mathrm{NLL}}(k_{t1})\left(R''(k_{t1})\right)^2\ln\frac{1}{\zeta_{s1}}\ln\frac{1}{\zeta_{s2}} - \partial_L\mathcal{L}_{\mathrm{NLL}}(k_{t1})R''(k_{t1})\left(\ln\frac{1}{\zeta_{s1}} + \ln\frac{1}{\zeta_{s2}}\right)\\
&+ \frac{\alpha_s^2(k_{t1})}{\pi^2}\hat{P}^{(0)}\otimes\hat{P}^{(0)}\otimes\mathcal{L}_{\mathrm{NLL}}(k_{t1})\Bigg\}\\
&\times\Big\{\Theta\left(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1}, k_{s1}, k_{s2})\right) - \Theta\left(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1}, k_{s1})\right) -\\
&\Theta\left(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1}, k_{s2})\right) + \Theta\left(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1})\right)\Big\} + \mathcal{O}\left(\alpha_s^n\ln^{2n-6}\frac{1}{v}\right), \qquad (3.18)
\end{aligned}
$$

[Bizon, Monni, Re, LR, Torrielli '17]

All ingredients to perform resummation at **N^{3}LL accuracy** are now available

[Catani *et al.* '11, '12][Gehrmann *et al.* '14][Li, Zhu '16, Vladimirov '16][Moch *et al.* '18, Lee *et al.* '19]

Fixed-order predictions now available at **NNLO**

[A. Gehrmann-De Ridder *et al.* '15, 16, '17][Boughezal *et al.* '15, 16]

Matching with fixed order

Multiplicative matching performed at the **double-cumulant level**

fixed-order double-cumulative result at NNLO

double-cumulative result at NNLL

$$\sigma_{\mathrm{NNLO}}(p_\perp^H < p_\perp^{H,\mathrm{v}}, p_\perp^{\mathrm{J}} < p_\perp^{\mathrm{J,v}}) = \sigma_{\mathrm{NNLO}} - \int \Theta(p_\perp^H > p_\perp^{H,\mathrm{v}}) \vee \Theta(p_\perp^{\mathrm{J}} > p_\perp^{\mathrm{J,v}}) d\sigma_{H+J,\mathrm{NLO}}$$

$$\sigma_{\mathrm{match}}(p_\perp^H < p_\perp^{H,\mathrm{v}}, p_\perp^{\mathrm{J}} < p_\perp^{\mathrm{J,v}}) = \frac{\sigma_{\mathrm{NNLL}}(p_\perp^H < p_\perp^{H,\mathrm{v}}, p_\perp^{\mathrm{J}} < p_\perp^{\mathrm{J,v}})}{\sigma_{\mathrm{NNLL}}(\{p_\perp^{\mathrm{J,v}}, p_\perp^{H,\mathrm{v}}\} \to \infty)} \left[\sigma_{\mathrm{NNLL}}(\{p_\perp^{\mathrm{J,v}}, p_\perp^{H,\mathrm{v}}\} \to \infty) \frac{\sigma_{\mathrm{NNLO}}(p_\perp^H < p_\perp^{H,\mathrm{v}}, p_\perp^{\mathrm{J}} < p_\perp^{\mathrm{J,v}})}{\sigma_{\mathrm{NNLL,exp}}(p_\perp^H < p_\perp^{H,\mathrm{v}}, p_\perp^{\mathrm{J}} < p_\perp^{\mathrm{J,v}})} \right]_{\mathcal{O}(\alpha_s^2)}$$

asymptotic limit of the NNLL result

expansion of the double-cumulative result at NNLL

- NNLL+NNLO result for $p_\perp^{\mathrm{J,v}}$ recovered for $p_\perp^{\mathrm{H,v}} \to \infty$
- **NNLO constant** included through multiplicative matching (NNLL′ accuracy)

Matching to fixed order: multiplicative matching

Cumulative cross section should reduce to the fixed order at large v

$$\Sigma^{\text{mult}}_{\text{matched}}(v) \sim \Sigma_{\text{res}}(v) \left[\frac{\Sigma_{\text{f.o.}}(v)}{\Sigma_{\text{res}}(v)} \right]_{\text{expanded}}$$

$$\Sigma_{\text{f.o}}(v) = \sigma_{f.o.} - \int_v^\infty \frac{d\sigma}{dv} dv$$

- allows to include constant terms from NNLO (if N3LO total xs available)
- physical suppression at small v cures potential instabilities

To ensure that resummation does not affect the hard region of the spectrum when the matching is performed we introduce **modified logarithms**

This corresponds to restrict the rapidity phase space at large k_t

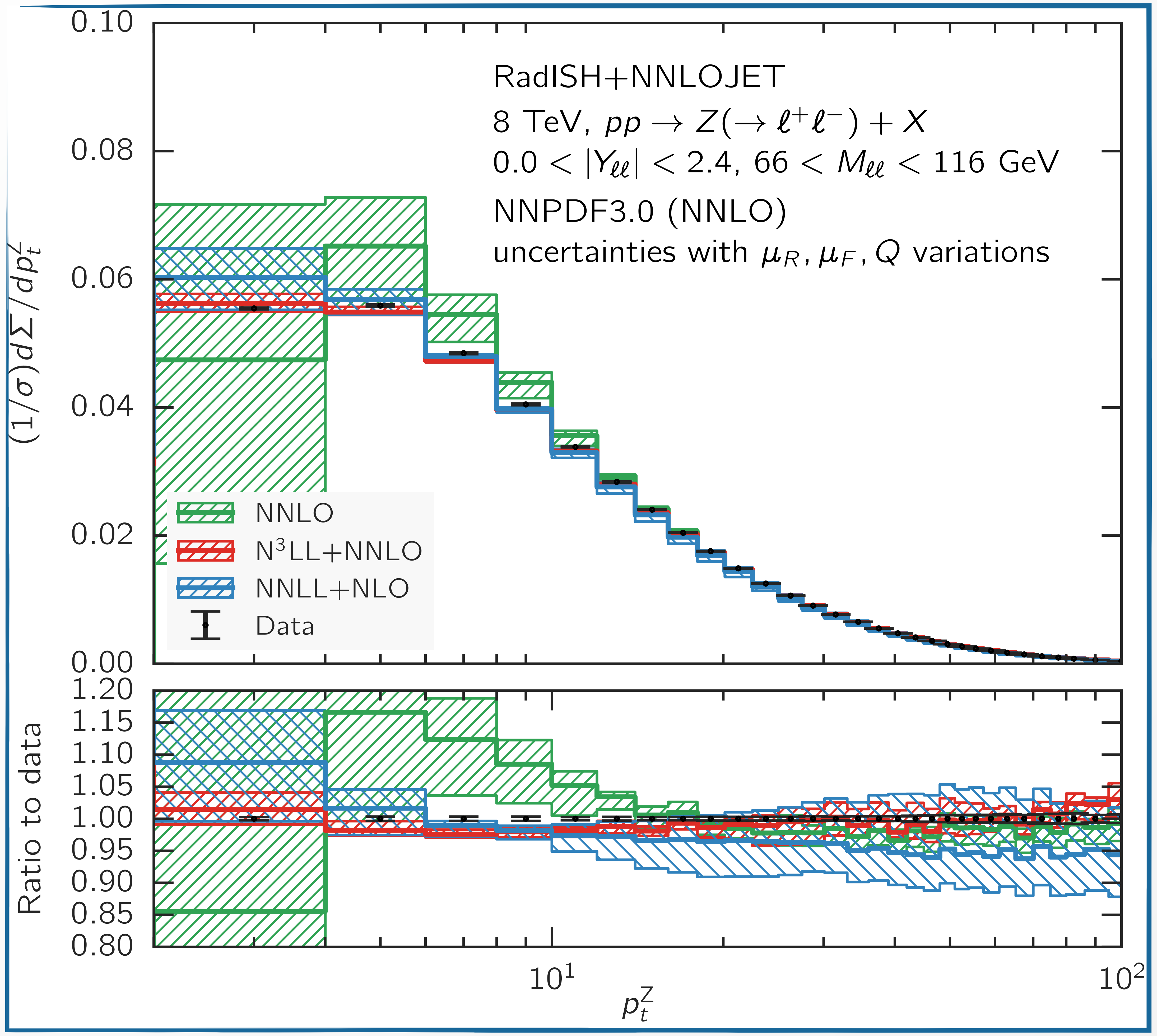
$$\int_{-\ln Q/k_{t,i}}^{\ln Q/k_{t,i}} d\eta \to \int_{-\ln Q/k_{t,1}}^{\ln Q/k_{t,1}} d\eta \to \int_{-\epsilon}^{\epsilon} d\eta \to 0$$

$$\ln(Q/k_{t1}) \to \frac{1}{p} \ln\left(1 + \left(\frac{Q}{k_{t1}}\right)^p\right)$$

Q : **perturbative resummation scale** used to probe the size of subleading logarithmic corrections

p : arbitrary matching parameter

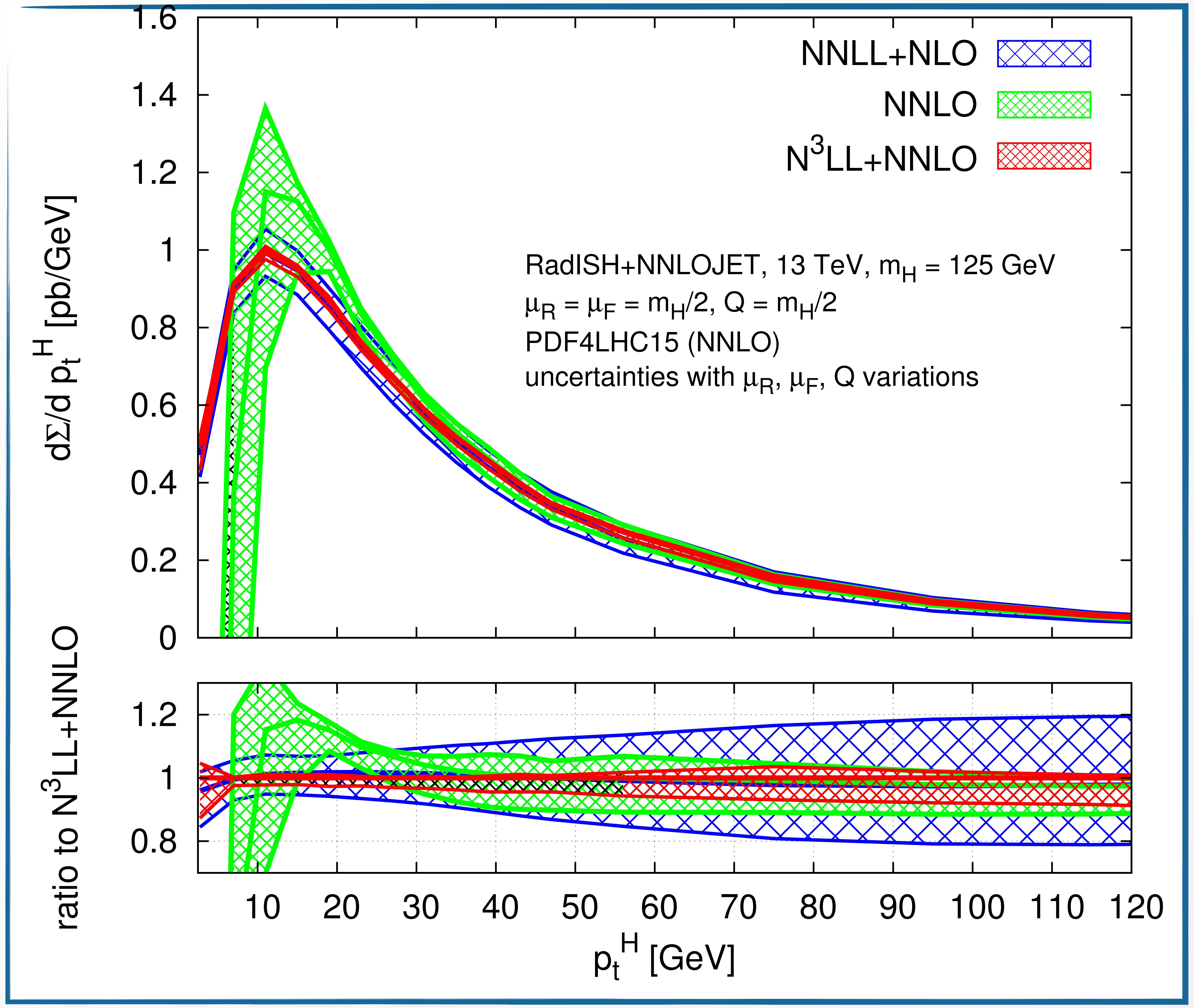
Predictions for the Z spectrum at 8 TeV

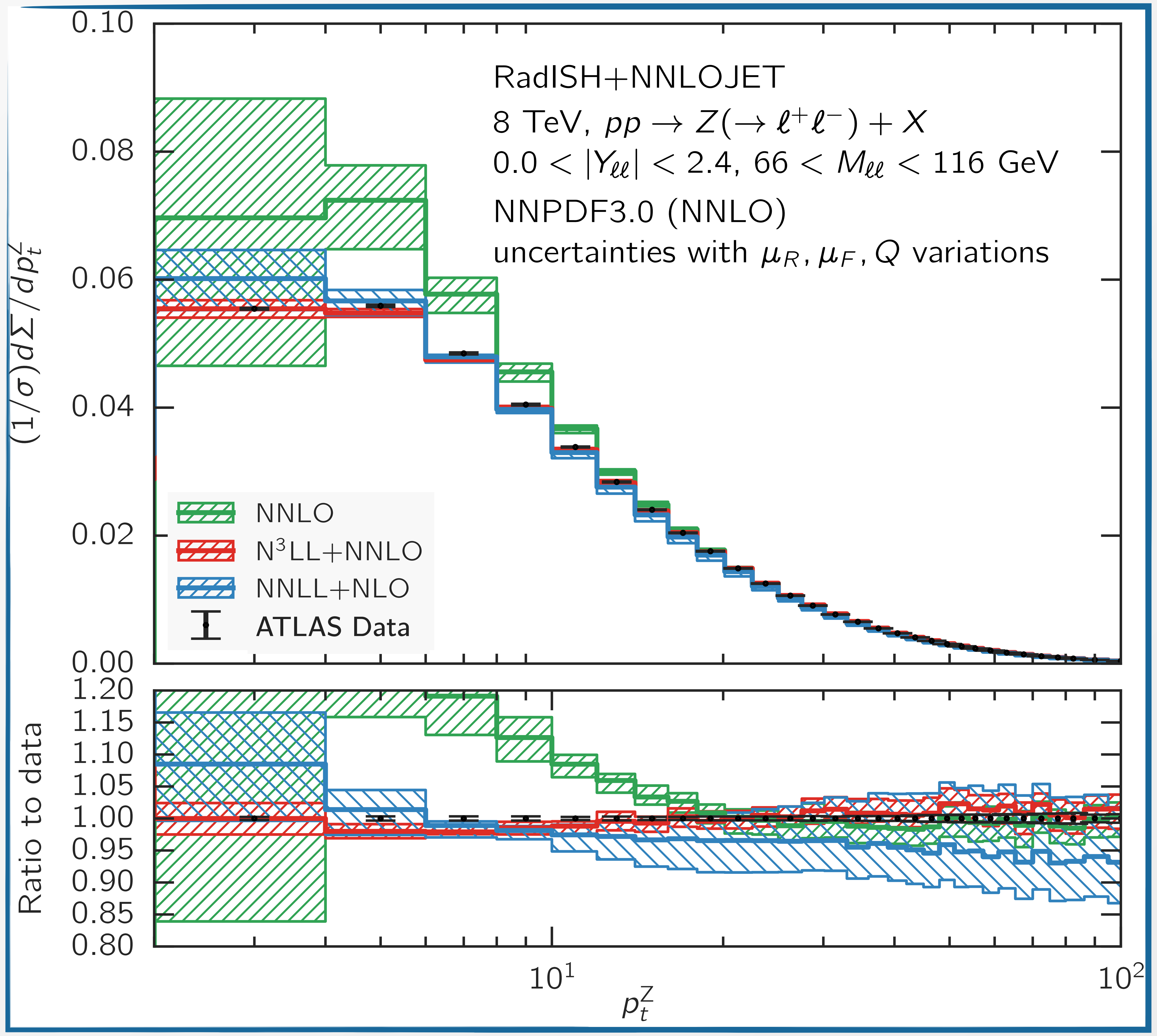


- Good description of the data in all fiducial regions
- Perturbative uncertainty at the few percent level, still does not match the precision of the ATLAS data

Resummation of the transverse momentum spectrum at N³LL+NNLO

N³LL result matched to NNLO H+j, Z+j, W±+j **[Bizon, LR *et al.* '18, '19]**





Theoretical predictions for Z and W observables at 13 TeV

Bizon, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Monni, Re, LR, Walker, 190x.xxxx

Results obtained using the following fiducial cuts (agreed with ATLAS)

$$p_t^{\ell^\pm} > 25\,\text{GeV}, \quad |\eta^{\ell^\pm}| < 2.5, \quad 66\,\text{GeV} < M_{\ell\ell} < 116\,\text{GeV}$$

$$p_t^{\ell} > 25\,\text{GeV}, \quad |\eta^{\ell}| < 2.5, \quad \not{E}_T^{\nu_\ell} > 25\,\text{GeV}, \quad m_T > 50\,\text{GeV}$$

using NNPDF3.1 with $\alpha_s(M_Z)$=0.118 and setting the central scales to

$$\mu_R = \mu_F = M_T = \sqrt{M_{\ell\ell'}^2 + p_T^2}, \quad Q = \frac{M_{\ell\ell'}}{2}$$

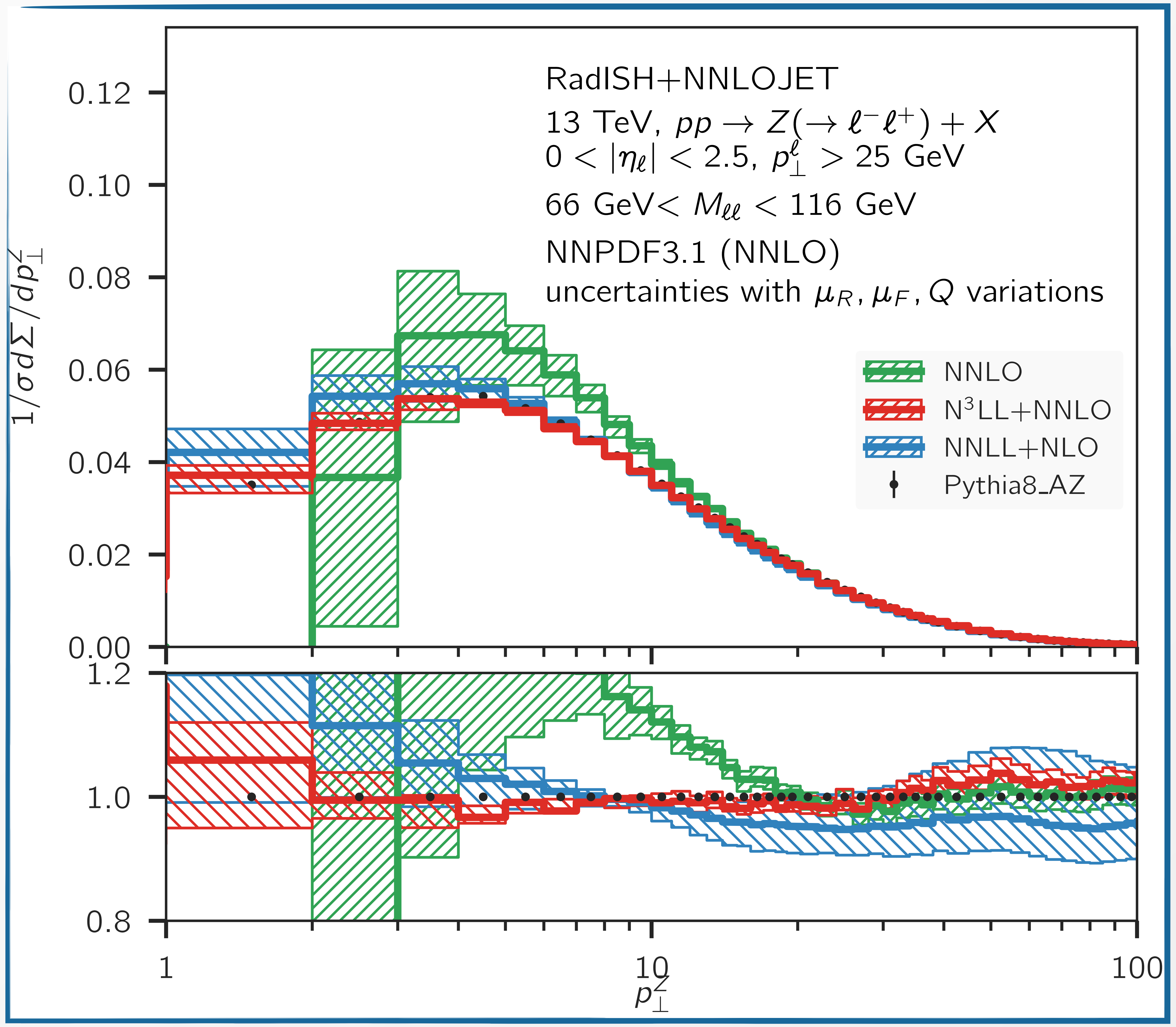
5 flavour (massless) scheme: no HQ effects, LHAPDF PDF thresholds

Scale uncertainties estimated by varying **renormalization** and **factorization** scale by a factor of two around their central value (**7 point variation**) and varying the **resummation** scale by a factor of 2 around its central value for factorization and renormalization scales set to their central value: **9 point envelope**

Matching parameter p set to 4 as a default

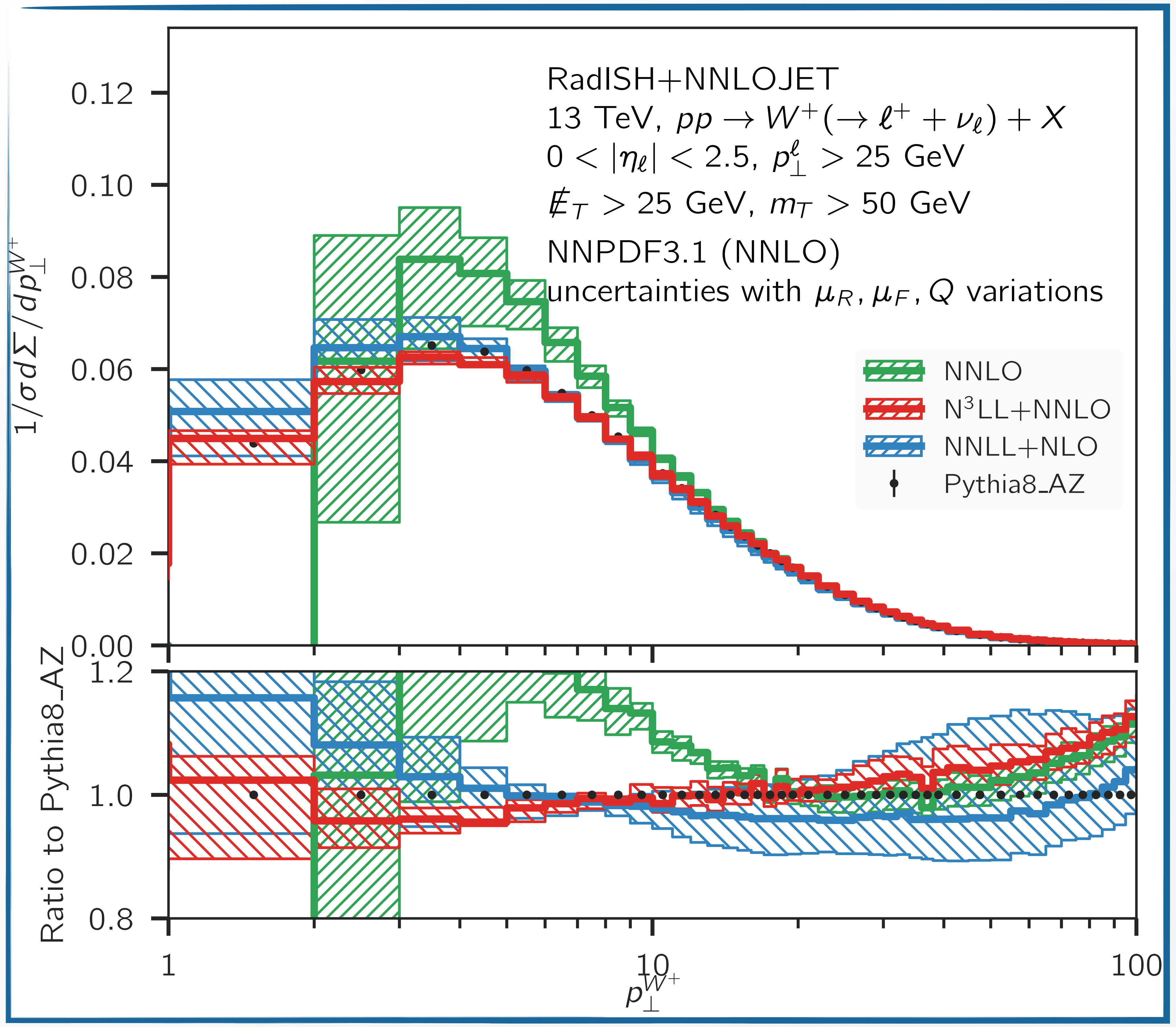
No non perturbative parameters included in the following

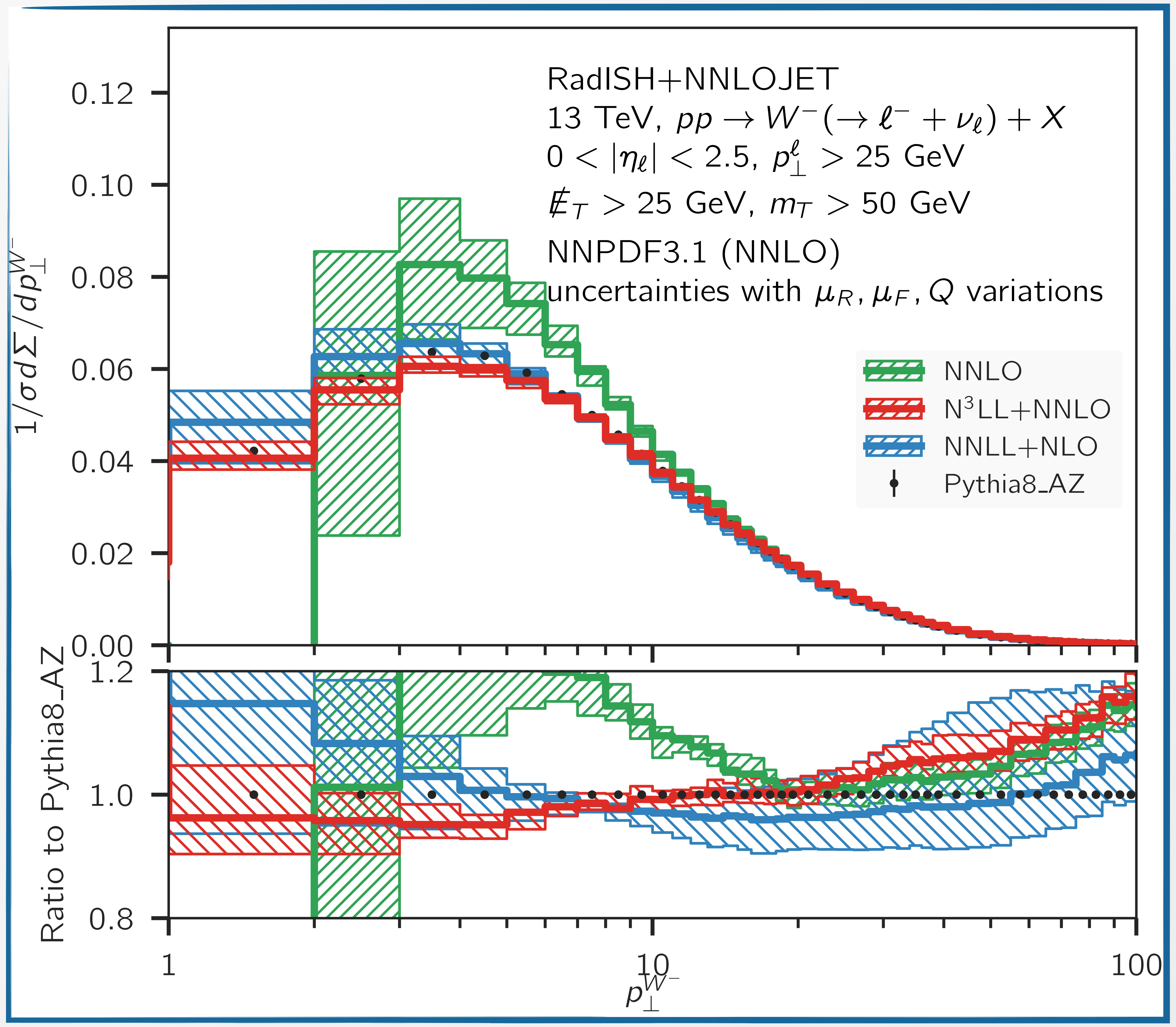
Predictions for the Z spectrum



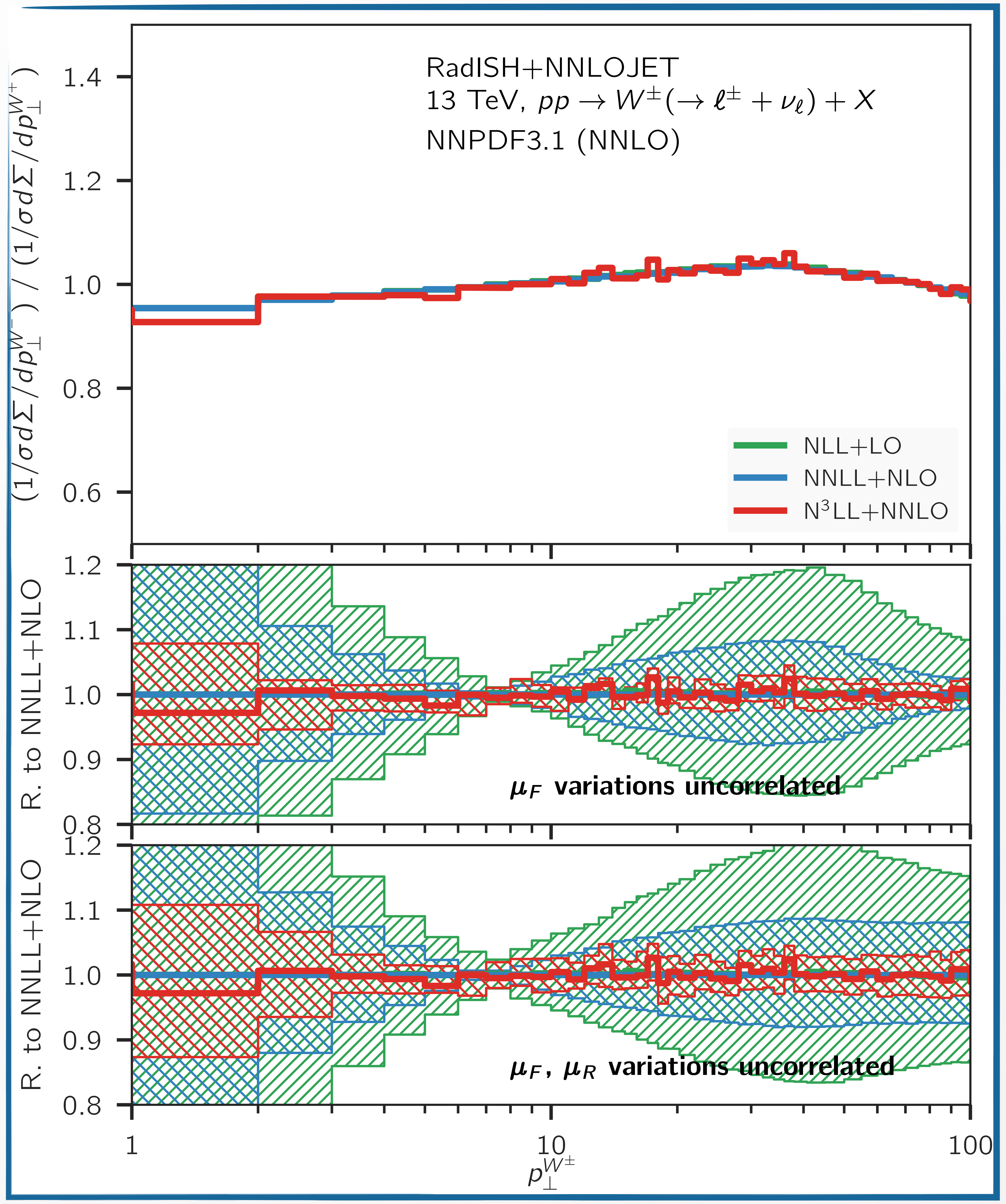
Thanks to Jan Kretzschmar for providing the PYTHIA8 AZ tune results

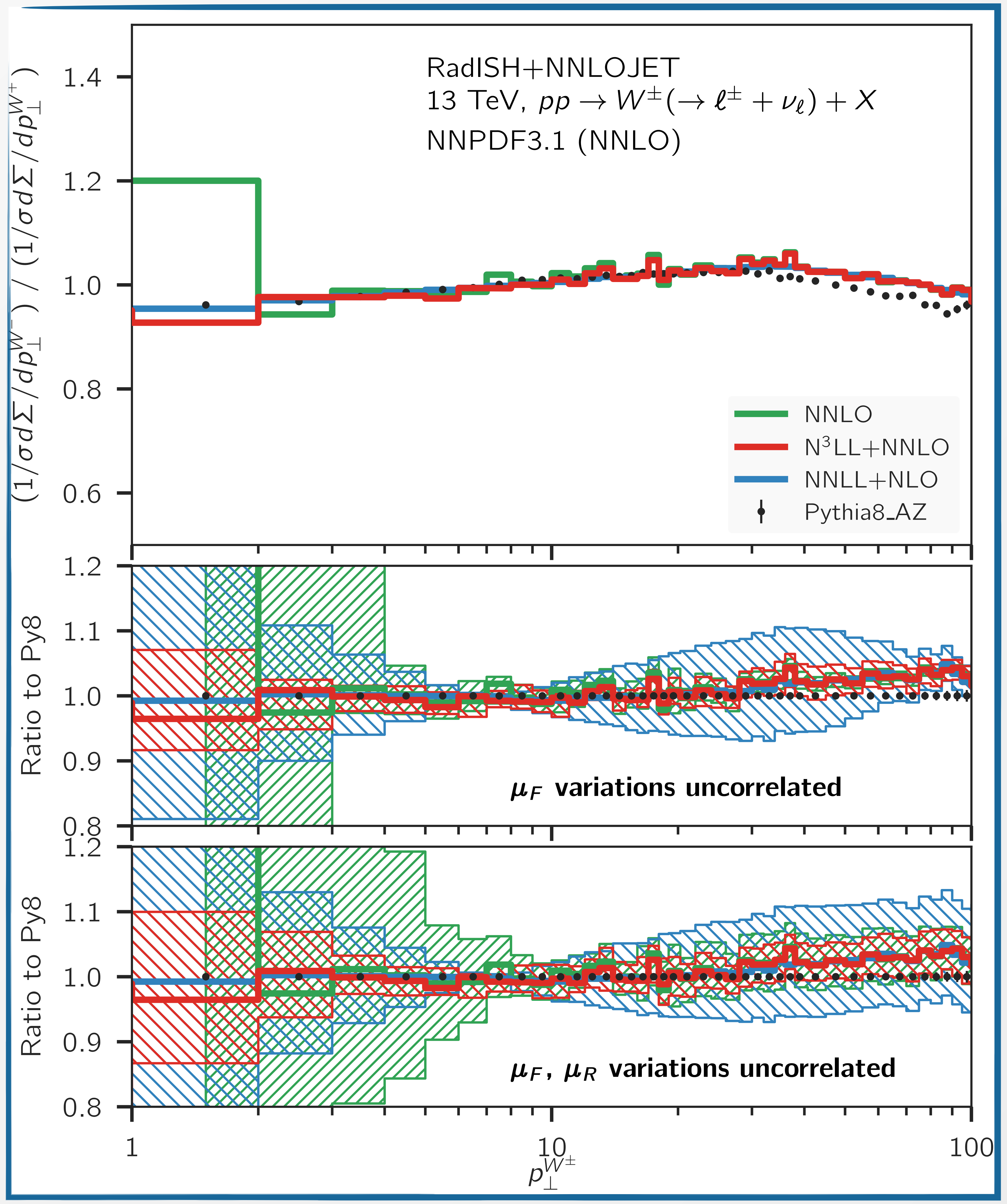
Predictions for the W^+ and W^- spectra



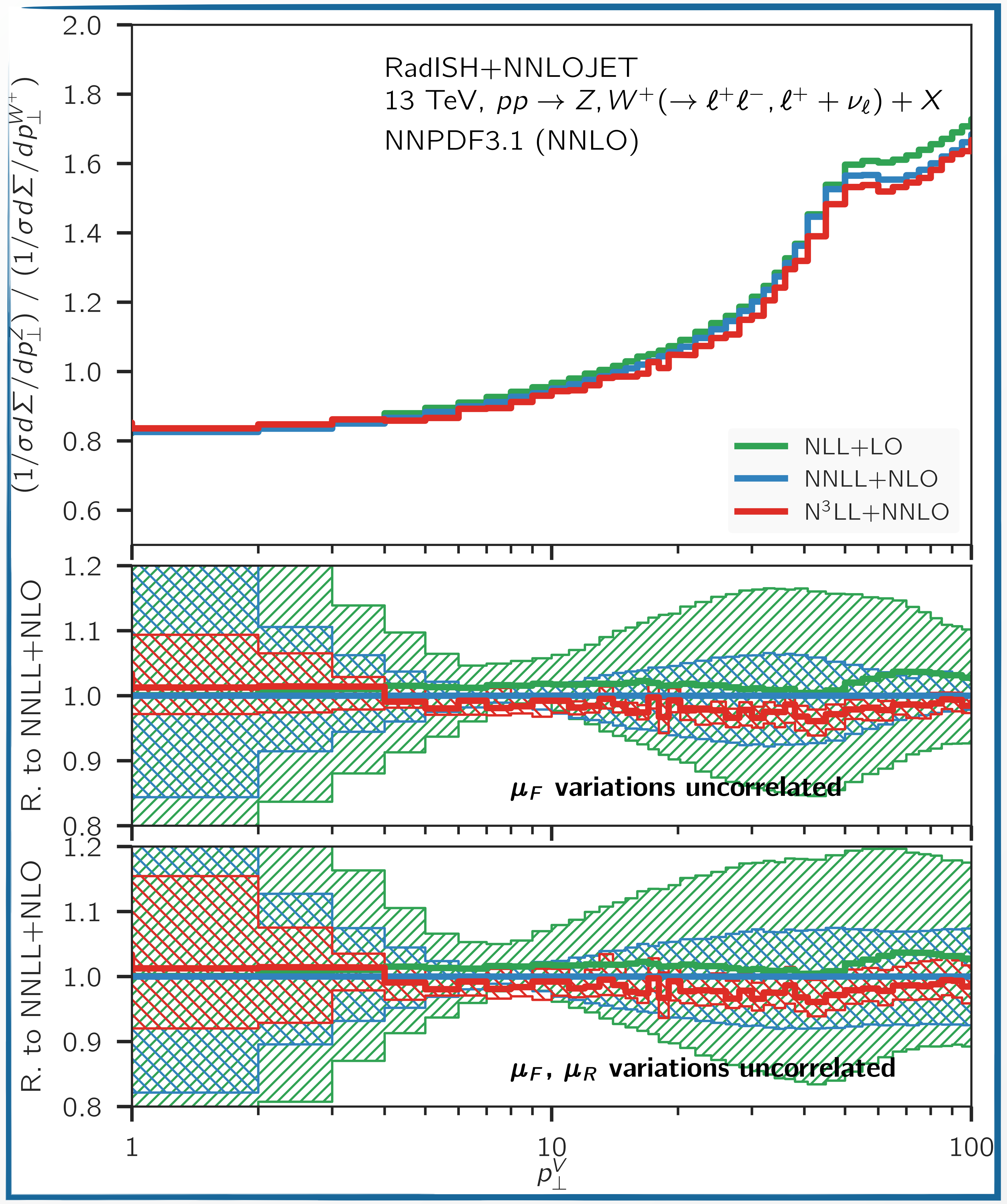


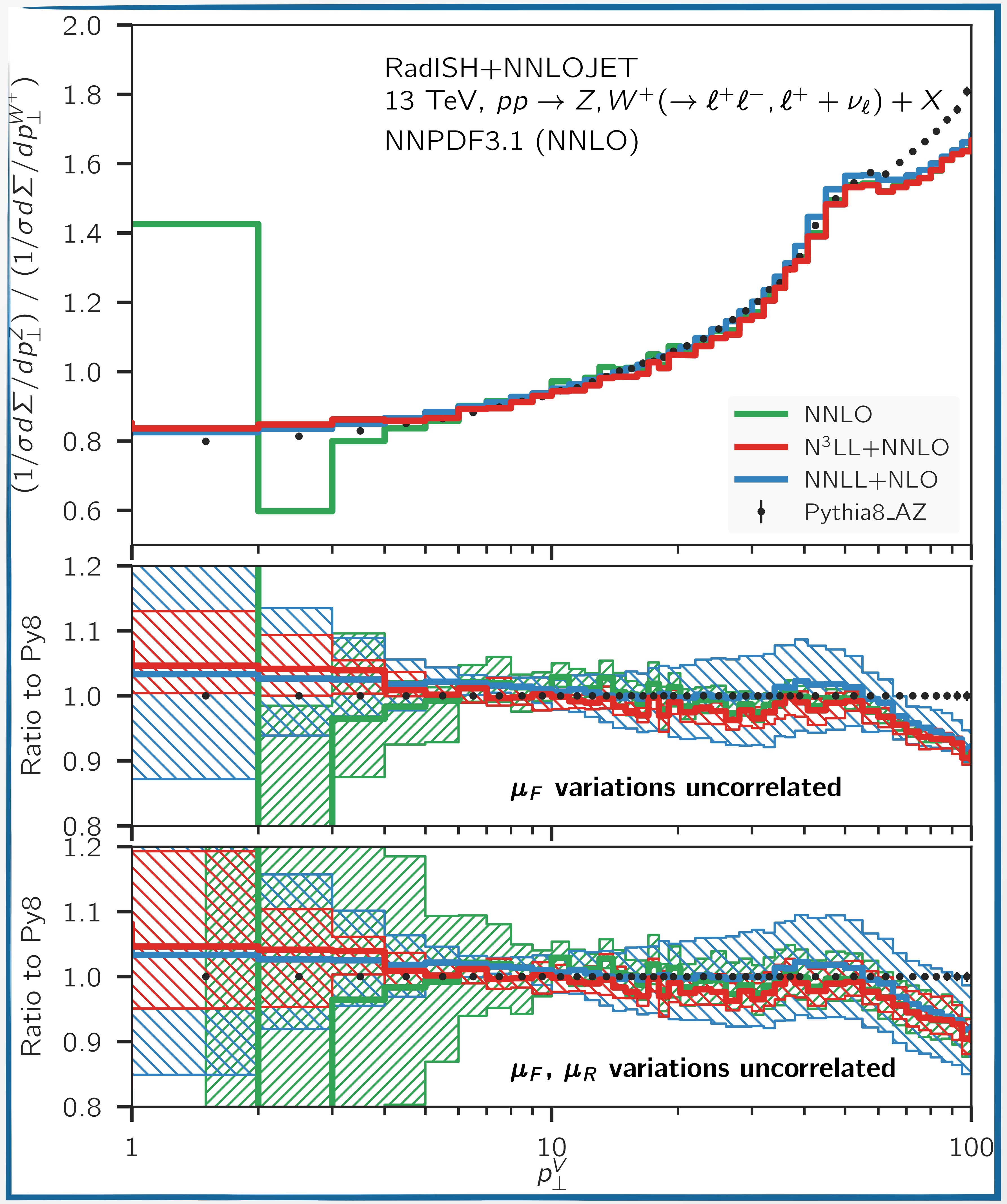
Results for W^-/W^+ ratio





Results for Z/W^+ ratio





Equivalence with *b*-space formulation

$$\frac{d\Sigma(v)}{d\Phi_B} = \int_{\mathscr{C}_1}\frac{dN_1}{2\pi i}\int_{\mathscr{C}_2}\frac{dN_2}{2\pi i}x_1^{-N_1}x_2^{-N_2}\sum_{c_1,c_2}\frac{d|M_B|^2_{c_1c_2}}{d\Phi_B}\mathbf{f}^T_{N_1}(\mu_0)\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v)\mathbf{f}_{N_2}(\mu_0)$$

unresolved emission + virtual corrections

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0))H(\mu_R)\mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right]\int_0^M\frac{dk_{t1}}{k_{t1}}\int_0^{2\pi}\frac{d\phi_1}{2\pi}$$

$$\times e^{-\mathbf{R}(\epsilon k_{t1})}\exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\frac{\alpha_s(k_t)}{\pi}\mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$

Result valid for all inclusive observables (e.g. p_t, φ^*)

resolved emission

$$\sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi}\mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$

$$\times\sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=2}^{n+1}\int_\epsilon^1\frac{d\zeta_i}{\zeta_i}\int_0^{2\pi}\frac{d\phi_i}{2\pi}\sum_{\ell_i=1}^{2}\left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi}\mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right)$$

$$\times\Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1})\right)$$

Formulation **equivalent to *b*-space** result (up to a **scheme change** in the anomalous dimensions)

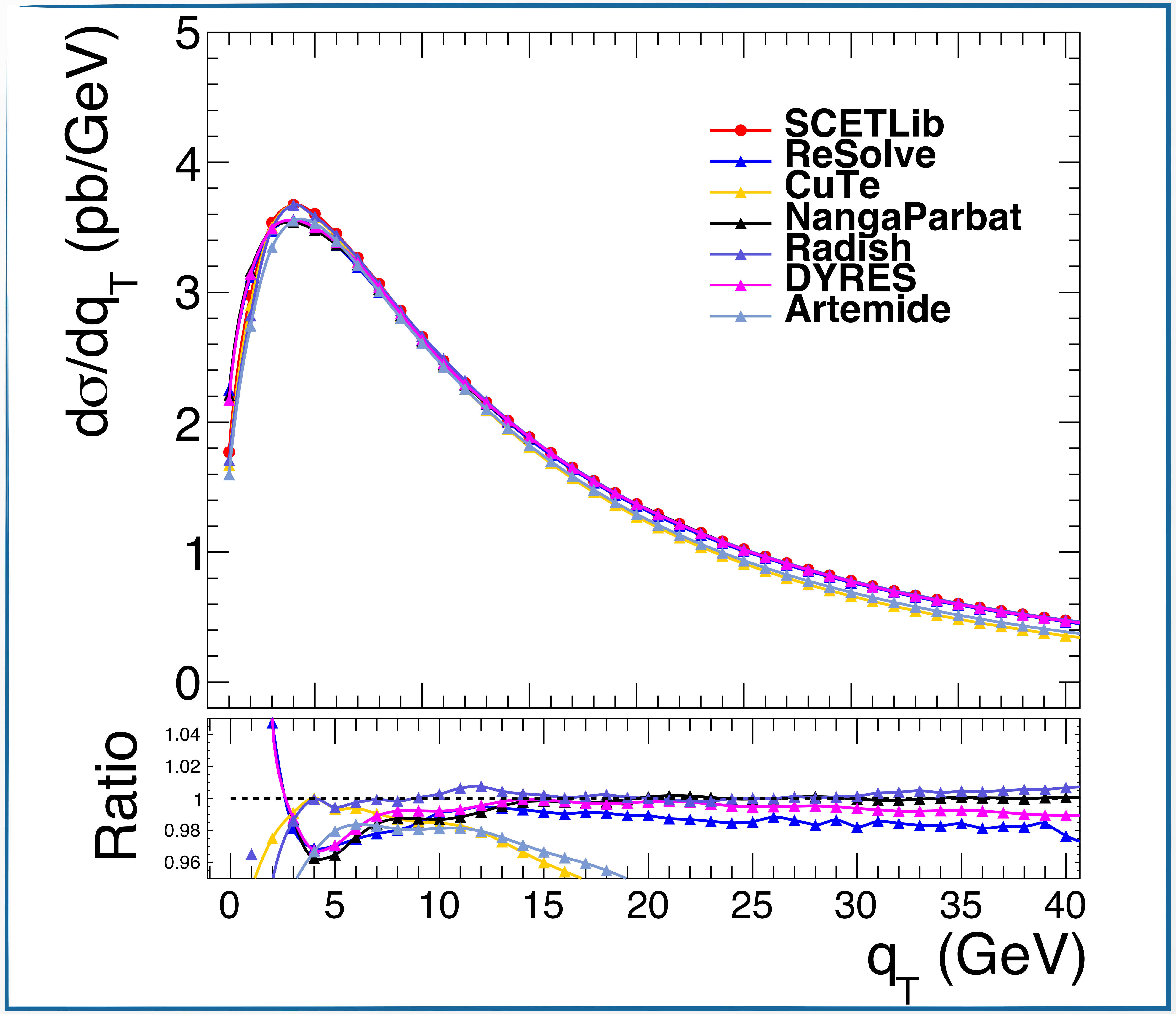
$$\frac{d^2\Sigma(v)}{d\Phi_B dp_t} = \sum_{c_1,c_2}\frac{d|M_B|^2_{c_1c_2}}{d\Phi_B}\int b\,db\,p_tJ_0(p_tb)\,\mathbf{f}^T(b_0/b)\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(b_0/b))H(M)\mathbf{C}^{c_2}_{N_2}(\alpha_s(b_0/b))\mathbf{f}(b_0/b)$$

$$\times\exp\left\{-\sum_{\ell=1}^{2}\int_0^M\frac{dk_t}{k_t}\mathbf{R}'_\ell(k_t)(1-J_0(bk_t))\right\}$$

$$(1-J_0(bk_t)) \simeq \Theta(k_t - \frac{b_0}{b}) + \frac{\zeta_3}{12}\frac{\partial^3}{\partial\ln(Mb/b_0)^3}\Theta(k_t - \frac{b_0}{b})$$

N^{3}LL effect: absorbed in the definition of H_2, B_3, A_4 coefficients wrt to CSS

Equivalence with *b*-space formulation

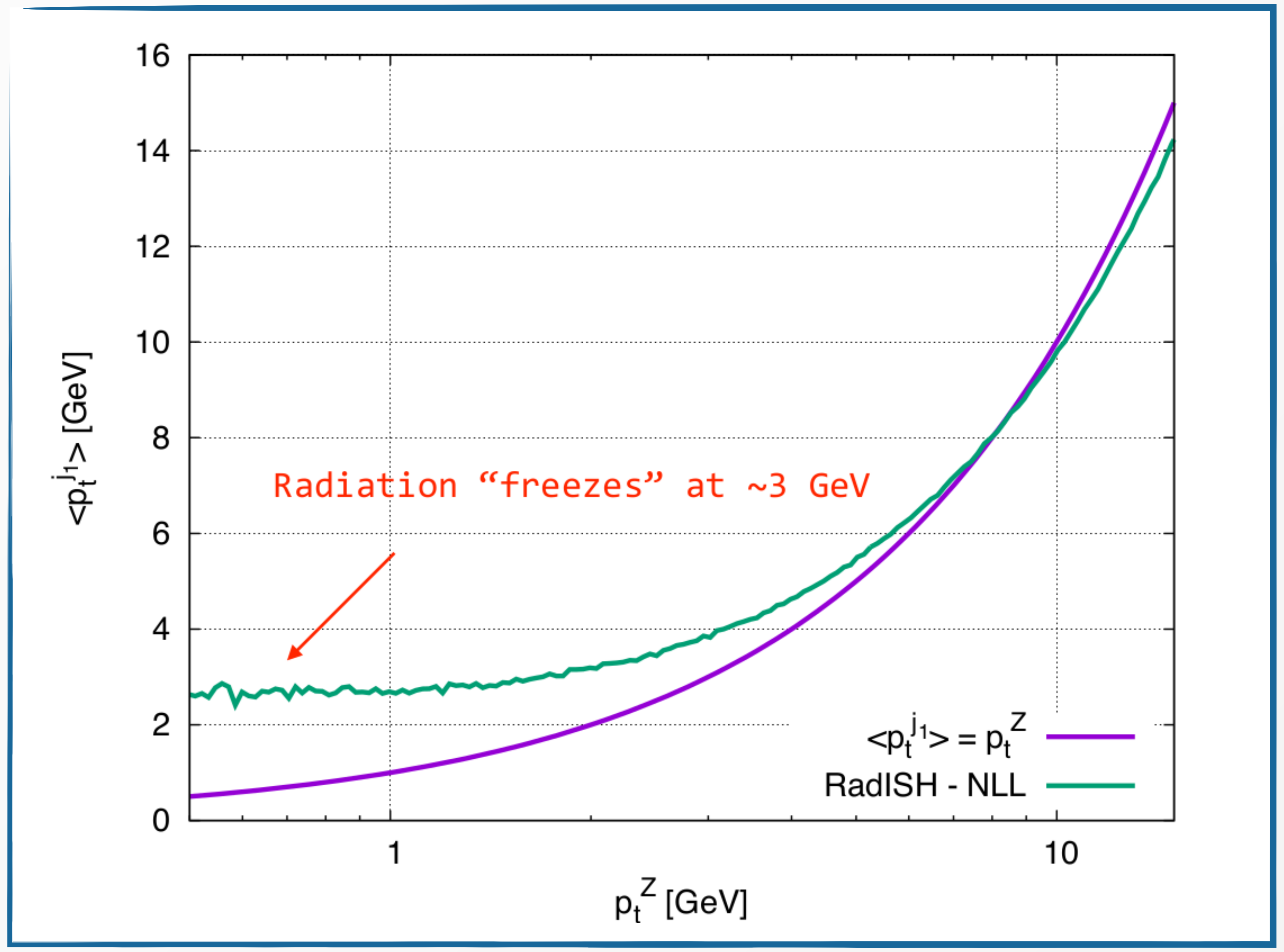


The Landau pole and the small p_T limit

Running coupling $\alpha_s(k_{t1}{}^2)$ and Sudakov radiator hit Landau pole at

$$\alpha_s(\mu_R^2)\beta_0 \ln Q/k_{t1} = \frac{1}{2} \qquad k_{t1} \sim 0.01\,\mathrm{GeV}, \quad \mu_R = Q = m_Z$$

Only real cutoff in the calculation: emission probability is set to zero below this scale and parton densities are frozen.



At small p_t the large azimuthal cancellations dominate over the Sudakov suppression: the cutoff is never an issue in practice

$$\frac{d^2\Sigma(v)}{dp_t d\Phi_B} \simeq 2\sigma^{(0)}(\Phi_B)p_t \left(\frac{\Lambda^2_{\mathrm{QCD}}}{M^2}\right)^{\frac{16}{25}\ln\frac{41}{16}}$$

Thanks to P. Monni

Behaviour at small p_t

Explicit evaluation shows that the Parisi-Petronzio perturbative scaling at small p_t is reproduced. At NLL, Drell-Yan pair production, n_f=4

$$\frac{d^2\Sigma(v)}{dp_t d\Phi_B} = 4\,\sigma^{(0)}(\Phi_B)\,p_t \int_{\Lambda_{\rm QCD}}^{M} \frac{dk_{t1}}{k_{t1}^3} e^{-R(k_{t1})} \simeq 2\sigma^{(0)}(\Phi_B) p_t \left(\frac{\Lambda_{\rm QCD}^2}{M^2}\right)^{\frac{16}{25}\ln\frac{41}{16}}$$

As now higher logarithmic terms (up to N^{3}LL) are under control, the coefficient of this scaling can be systematically improved in *perturbation* theory (non-perturbative effects – of the same order – not considered)

N^{3}LL calculation allows one to have control over the terms of relative order $O(\alpha_s^2)$. Scaling $L \sim 1/\alpha_s$ valid in the deep infrared regime.

Numerical implementation

$$\frac{d\Sigma(p_t)}{d\Phi_B} = \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} \partial_L \left(-e^{-R'(k_{t1})} \mathcal{L}_{\rm NLL}(k_{t1})\right) \times$$
$$\times \underbrace{\epsilon^{R'(k_{t1})} \sum_{n=0}^{\infty} \frac{1}{n!} \left(\prod_{i=2}^{n+1} \int_{\epsilon k_{t1}}^{k_{t1}} \frac{dk_{ti}}{k_{ti}} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'(k_{t1}) \right) \Theta(p_t - |\vec{k}_{t1} + \ldots + \vec{k}_{t(n+1)}|)}_{\equiv \int d\mathcal{Z}[\{R', k_i\}] \Theta(p_t - |\vec{k}_{t1} + \ldots + \vec{k}_{t(n+1)}|)} .$$

- $L = \ln(M/k_{t1})$; luminosity $\mathcal{L}_{\rm NLL}(k_{t1}) = \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1 c_2}}{d\Phi_B} f_{c_1}(x_1, k_{t1}) f_{c_2}(x_2, k_{t1})$.
- $\int d\mathcal{Z}[\{R', k_i\}]\Theta$ finite as $\epsilon \to 0$:

$$\epsilon^{R'(k_{t1})} = 1 - R'(k_{t1}) \ln(1/\epsilon) + \ldots = 1 - \int_{\epsilon k_{t1}}^{k_{t1}} R'(k_{t1}) + \ldots,$$

$$\int d\mathcal{Z}[\{R', k_i\}]\Theta = \left[1 - \int_{\epsilon k_{t1}}^{k_{t1}} R'(k_{t1}) + \ldots\right] \left[\Theta(p_t - |\vec{k}_{t1}|) + \int_{\epsilon k_{t1}}^{k_{t1}} R'(k_{t1}) \Theta(p_t - |\vec{k}_{t1} + \vec{k}_{t2}|) + \ldots\right]$$
$$= \Theta(p_t - |\vec{k}_{t1}|) + \underbrace{\int_0^{k_{t1}}}_{\epsilon \to 0} R'(k_{t1}) \underbrace{\left[\Theta(p_t - |\vec{k}_{t1} + \vec{k}_{t2}|) - \Theta(p_t - |\vec{k}_{t1}|)\right]}_{\text{finite: real-virtual cancellation}} + \ldots$$

- Evaluated with Monte Carlo techniques: $\int d\mathcal{Z}[\{R', k_i\}]$ is generated as a parton shower over secondary emissions.

Thanks to P. Torrielli

Numerical implementation

- Secondary radiation:

$$
\begin{aligned}
d\mathcal{Z}[\{R', k_i\}] &= \sum_{n=0}^{\infty} \frac{1}{n!} \left(\prod_{i=2}^{n+1} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \int_{\epsilon k_{t1}}^{k_{t1}} \frac{dk_{ti}}{k_{ti}} R'(k_{t1}) \right) \epsilon^{R'(k_{t1})} \\
&= \sum_{n=0}^{\infty} \left(\prod_{i=2}^{n+1} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \int_{\epsilon k_{t1}}^{k_{t(i-1)}} \frac{dk_{ti}}{k_{ti}} R'(k_{t1}) \right) \epsilon^{R'(k_{t1})}, \\
\epsilon^{R'(k_{t1})} &= e^{-R'(k_{t1}) \ln 1/\epsilon} = \prod_{i=2}^{n+2} e^{-R'(k_{t1}) \ln k_{t(i-1)}/k_{ti}},
\end{aligned}
$$

with $k_{t(n+2)} = \epsilon k_{t1}$.

- Each secondary emissions has differential probability

$$
dw_i = \frac{d\phi_i}{2\pi} \frac{dk_{ti}}{k_{ti}} R'(k_{t1}) e^{-R'(k_{t1}) \ln k_{t(i-1)}/k_{ti}} = \frac{d\phi_i}{2\pi} d\left(e^{-R'(k_{t1}) \ln k_{t(i-1)}/k_{ti}} \right).
$$

- $k_{t(i-1)} \geq k_{ti}$. Scale k_{ti} extracted by solving $e^{-R'(k_{t1}) \ln k_{t(i-1)}/k_{ti}} = r$, with r random number extracted uniformly in $[0, 1]$. Shower ordered in k_{ti}.

- Extract ϕ_i randomly in $[0, 2\pi]$.

Thanks to P. Torrielli

Joint resummation in direct space

$$\sigma_{\rm incl}^{\rm NNLL}(p_t^{\rm J,v}, p_t^{\rm H,v}) = \int_0^{p_t^{\rm J,v}} \frac{dk_{t,1}}{k_{t,1}} \frac{d\phi_1}{2\pi} \int d\mathcal{Z} \Bigg\{ \frac{d}{d\,L_{t,1}} \left[-e^{-R_{\rm NNLL}(L_{t,1})} \mathcal{L}_{\rm NNLL}\left(\mu_{\rm F} e^{-L_{t,1}}\right) \right] \Theta\Big(p_t^{\rm H,v} - |\vec{k}_{t,1} + \cdots + \vec{k}_{t,n+1}| \Big)$$
$$+ e^{-R_{\rm NLL}(L_{t,1})} \hat{R}'(k_{t,1}) \int_0^{k_{t,1}} \frac{dk_{t,s_1}}{k_{t,s_1}} \frac{d\phi_{s_1}}{2\pi} \left[\left(\delta\hat{R}'(k_{t,1}) + \hat{R}''(k_{t,1}) \ln\frac{k_{t,1}}{k_{t,s_1}} \right) \mathcal{L}_{\rm NLL}\left(\mu_{\rm F} e^{-L_{t,1}}\right) - \frac{d}{d\,L_{t,1}} \mathcal{L}_{\rm NLL}\left(\mu_{\rm F} e^{-L_{t,1}}\right) \right]$$
$$\times \left[\Theta\Big(p_t^{\rm H,v} - |\vec{k}_{t,1} + \cdots + \vec{k}_{t,n+1} + \vec{k}_{t,s_1}| \Big) - \Theta\Big(p_t^{\rm H,v} - |\vec{k}_{t,1} + \cdots + \vec{k}_{t,n+1}| \Big) \right] \Bigg\}, \qquad (38)$$

$$\sigma_{\rm clust}^{\rm NNLL}(p_t^{\rm J,v}, p_t^{\rm H,v}) = \int_0^{\infty} \frac{dk_{t,1}}{k_{t,1}} \frac{d\phi_1}{2\pi} \int d\mathcal{Z}\, e^{-R_{\rm NLL}(L_{t,1})} \mathcal{L}_{\rm NLL}\left(\mu_{\rm F} e^{-L_{t,1}}\right) 8\, C_A^2 \frac{\alpha_s^2}{\pi^2} \frac{L_{t,1}}{(1-2\beta_0\alpha_s L_{t,1})^2} \Theta\left(p_t^{\rm J,v} - \max_{i>1}\{k_{t,i}\} \right)$$
$$\times \Bigg\{ \int_0^{k_{t,1}} \frac{dk_{t,s_1}}{k_{t,s_1}} \frac{d\phi_{s_1}}{2\pi} \int_{-\infty}^{\infty} d\Delta\eta_{1s_1}\, J_{1s_1}(R) \left[\Theta\Big(p_t^{\rm J,v} - |\vec{k}_{t,1} + \vec{k}_{t,s_1}| \Big) - \Theta\Big(p_t^{\rm J,v} - k_{t,1} \Big) \right] \Theta\Big(p_t^{\rm H,v} - |\vec{k}_{t,1} + \cdots + \vec{k}_{t,n+1} + \vec{k}_{t,s_1}| \Big)$$
$$+ \frac{1}{2!} \hat{R}'(k_{t,1}) \int_0^{k_{t,1}} \frac{dk_{t,s_1}}{k_{t,s_1}} \frac{dk_{t,s_2}}{k_{t,s_2}} \frac{d\phi_{s_1}}{2\pi} \frac{d\phi_{s_2}}{2\pi} \int_{-\infty}^{\infty} d\Delta\eta_{s_1 s_2}\, J_{s_1 s_2}(R) \left[\Theta\Big(p_t^{\rm J,v} - |\vec{k}_{t,s_1} + \vec{k}_{t,s_2}| \Big) - \Theta\Big(p_t^{\rm J,v} - \max\{k_{t,s_1}, k_{t,s_2}\} \Big) \right]$$
$$\times \Theta\Big(p_t^{\rm H,v} - |\vec{k}_{t,1} + \cdots + \vec{k}_{t,n+1} + \vec{k}_{t,s_1} + \vec{k}_{t,s_2}| \Big) \Theta\left(p_t^{\rm J,v} - k_{t,1} \right) \Bigg\}, \qquad (42)$$

$$\sigma_{\rm correl}^{\rm NNLL}(p_t^{\rm J,v}, p_t^{\rm H,v}) = \int_0^{\infty} \frac{dk_{t,1}}{k_{t,1}} \frac{d\phi_1}{2\pi} \int d\mathcal{Z}\, e^{-R_{\rm NLL}(L_{t,1})} \mathcal{L}_{\rm NLL}\left(\mu_{\rm F} e^{-L_{t,1}}\right) 8\, C_A^2 \frac{\alpha_s^2}{\pi^2} \frac{L_{t,1}}{(1-2\beta_0\alpha_s L_{t,1})^2} \Theta\left(p_t^{\rm J,v} - \max_{i>1}\{k_{t,i}\} \right)$$
$$\times \Bigg\{ \int_0^{k_{t,1}} \frac{dk_{t,s_1}}{k_{t,s_1}} \frac{d\phi_{s_1}}{2\pi} \int_{-\infty}^{\infty} d\Delta\eta_{1s_1}\, \mathcal{C}\left(\Delta\eta_{1s_1}, \Delta\phi_{1s_1}, \frac{k_{t,1}}{k_{t,s_1}} \right) (1 - J_{1s_1}(R))$$
$$\times \left[\Theta\Big(p_t^{\rm J,v} - k_{t,1} \Big) - \Theta\Big(p_t^{\rm J,v} - |\vec{k}_{t,1} + \vec{k}_{t,s_1}| \Big) \right] \Theta\Big(p_t^{\rm H,v} - |\vec{k}_{t,1} + \cdots + \vec{k}_{t,n+1} + \vec{k}_{t,s_1}| \Big)$$
$$+ \frac{1}{2!} \hat{R}'(k_{t,1}) \int_0^{k_{t,1}} \frac{dk_{t,s_1}}{k_{t,s_1}} \frac{dk_{t,s_2}}{k_{t,s_2}} \frac{d\phi_{s_1}}{2\pi} \frac{d\phi_{s_2}}{2\pi} \int_{-\infty}^{\infty} d\Delta\eta_{s_1 s_2}\, \mathcal{C}\left(\Delta\eta_{s_1 s_2}, \Delta\phi_{s_1 s_2}, \frac{k_{t,s_2}}{k_{t,s_1}} \right) (1 - J_{s_1 s_2}(R))\, \Theta\left(p_t^{\rm J,v} - k_{t,1} \right)$$
$$\times \left[\Theta\Big(p_t^{\rm J,v} - \max\{k_{t,s_1}, k_{t,s_2}\} \Big) - \Theta\Big(p_t^{\rm J,v} - |\vec{k}_{t,s_1} + \vec{k}_{t,s_2}| \Big) \right] \Theta\Big(p_t^{\rm H,v} - |\vec{k}_{t,1} + \cdots + \vec{k}_{t,n+1} + \vec{k}_{t,s_1} + \vec{k}_{t,s_2}| \Big) \Bigg\}. \qquad (43)$$