m_W determination at hadron colliders

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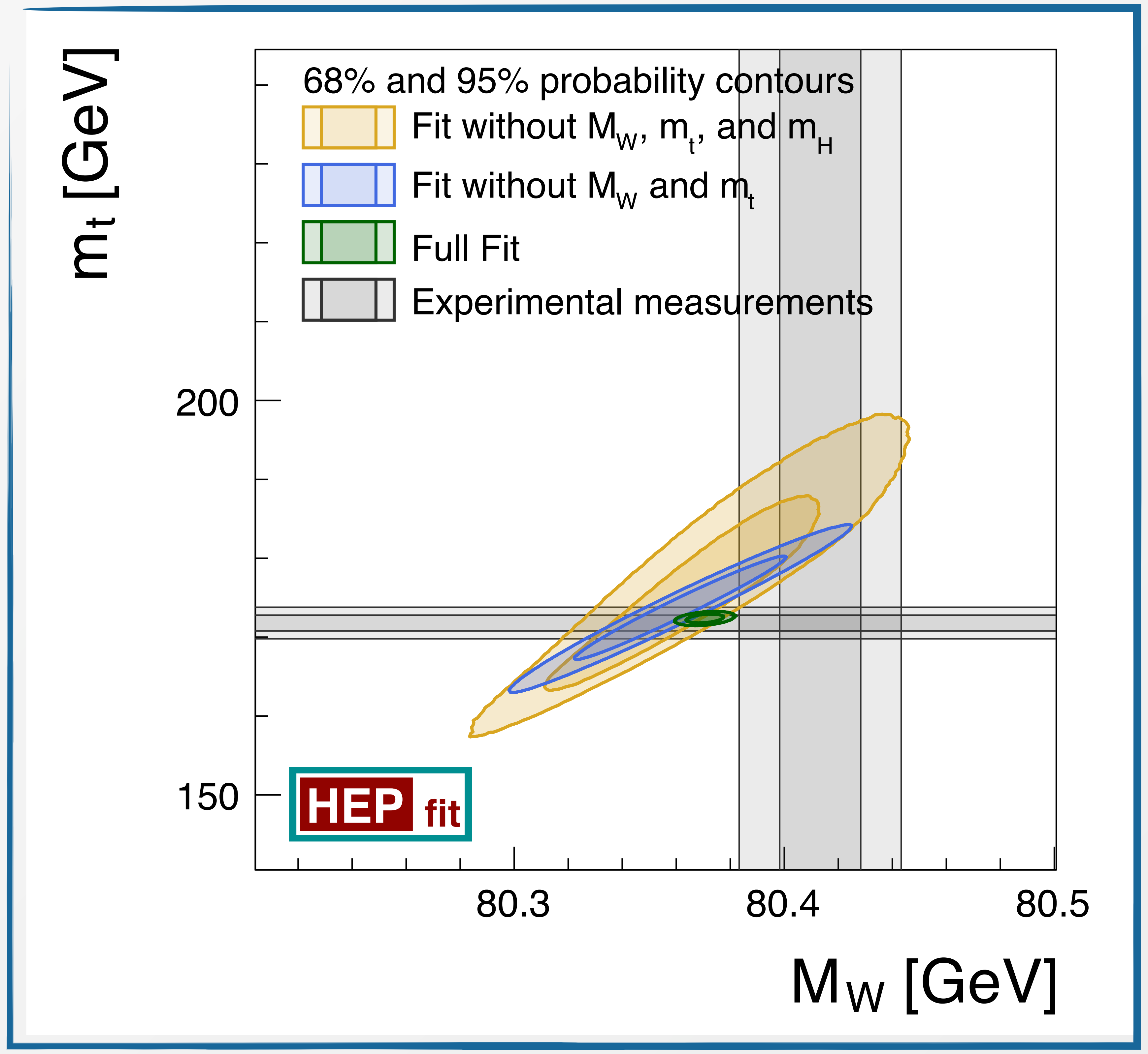
Measurements of m_W at hadron colliders

The discovery of the Higgs boson and the measurement of its mass allow for the prediction of the W mass with high precision

$$m_W = 80.350 \pm 8\,\mathrm{GeV}$$

Which is in a 2σ agreement with the experimental average (pre-CDF II)

$$m_W = 80.385 \pm 15\,\mathrm{GeV}$$



[de Blas, Pierini, Reina, Silvestrini '22]

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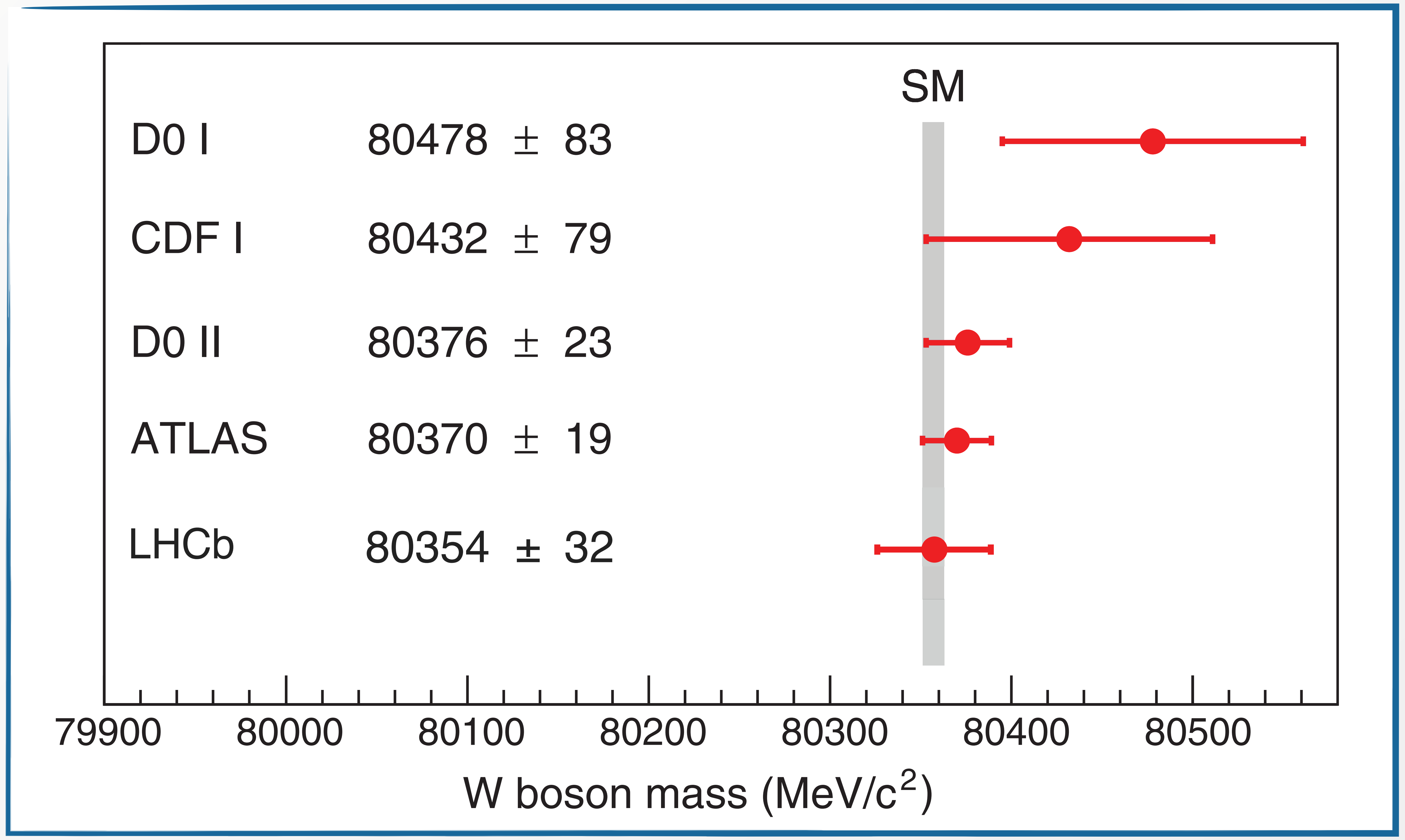

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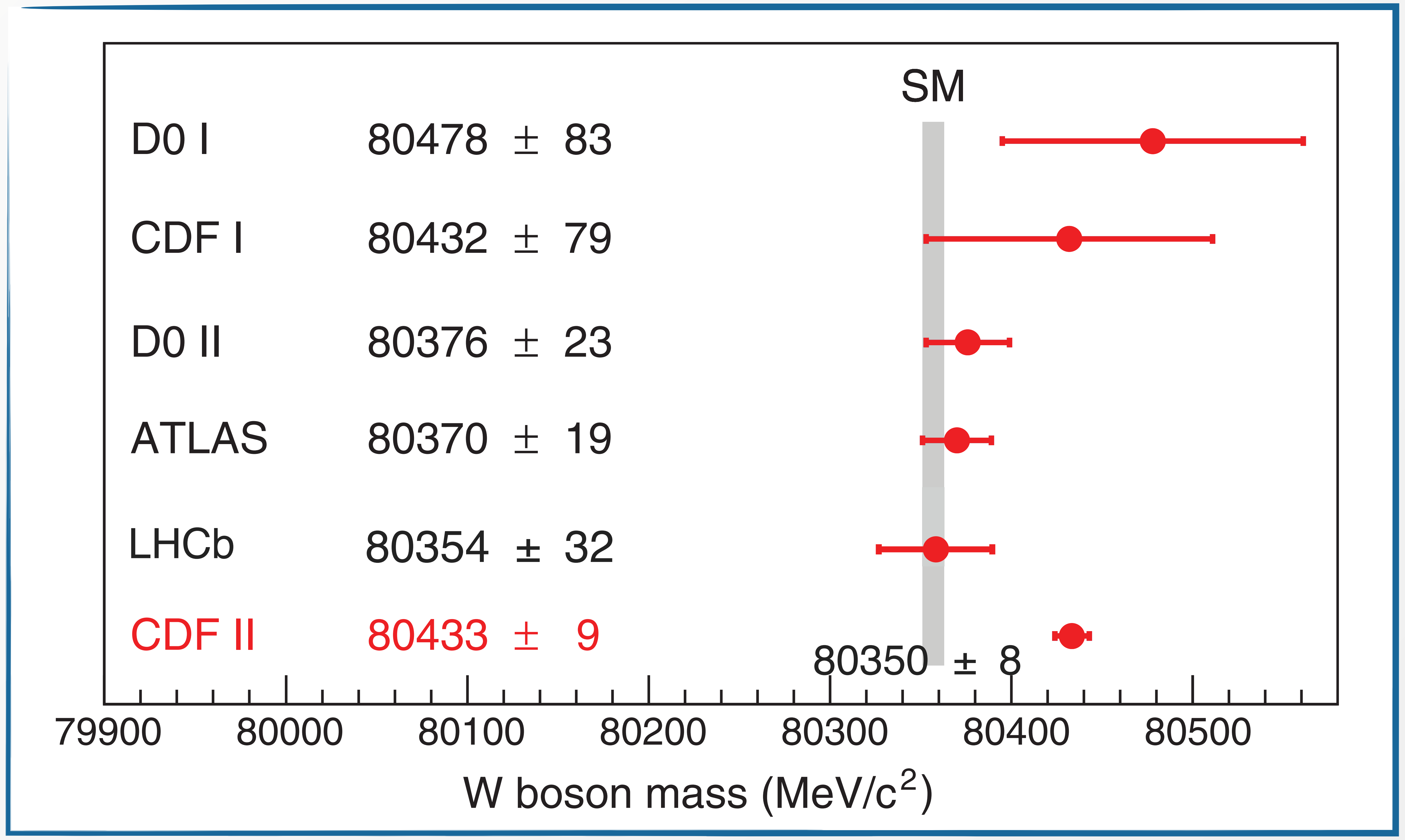


Outline

- The Drell-Yan kinematical distributions and the m_W determination: strategy and challenges
- Recent progress in theoretical computations
- Proposal of a new observable and discussion of the associated theory uncertainties

Measurements of m_W at hadron colliders

Full kinematics of **charged DY production** is not accessible at hadron colliders; in particular, the invariant mass of the neutrino-lepton pair cannot be reconstructed

Reconstruction possible in the **transverse plane** (requires precise measurement of the **hadronic recoil**)

Precise determinations of the W mass exploit observables with high sensitivity to small variations $\mathcal{O}(10^{-4})$ of m_W, such as the lepton transverse momentum p_T^ℓ or the transverse mass $m_T = \sqrt{2p_T^\ell p_T^\nu(1-\cos\Delta\phi_{\ell\nu})}$

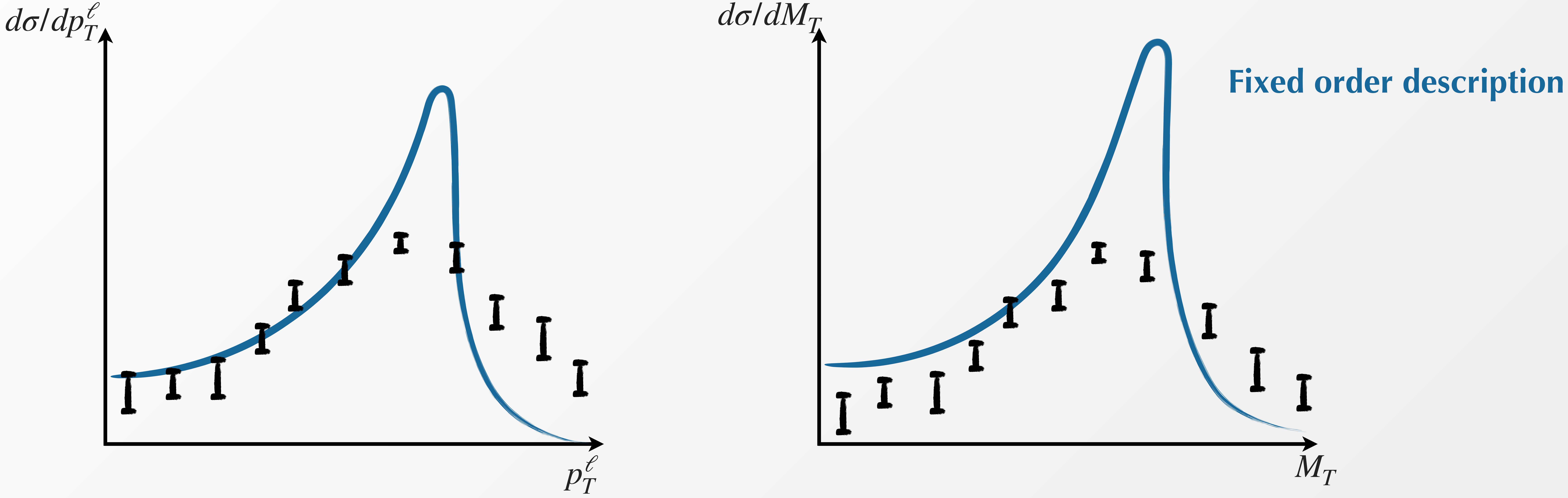
$$\frac{d\sigma}{d|p_T^\ell|^2} \sim \frac{1}{\sqrt{1-4\frac{|p_T^\ell|^2}{\hat{s}}}} \sim \frac{1}{\sqrt{1-4\frac{|p_T^\ell|^2}{m_W^2}}}$$

Jacobian peak at $p_T^\ell \sim m_W/2$

Enhanced sensitivity to m_W in both distributions at the $\mathcal{O}(10^{-3})$—$\mathcal{O}(10^{-2})$ level.

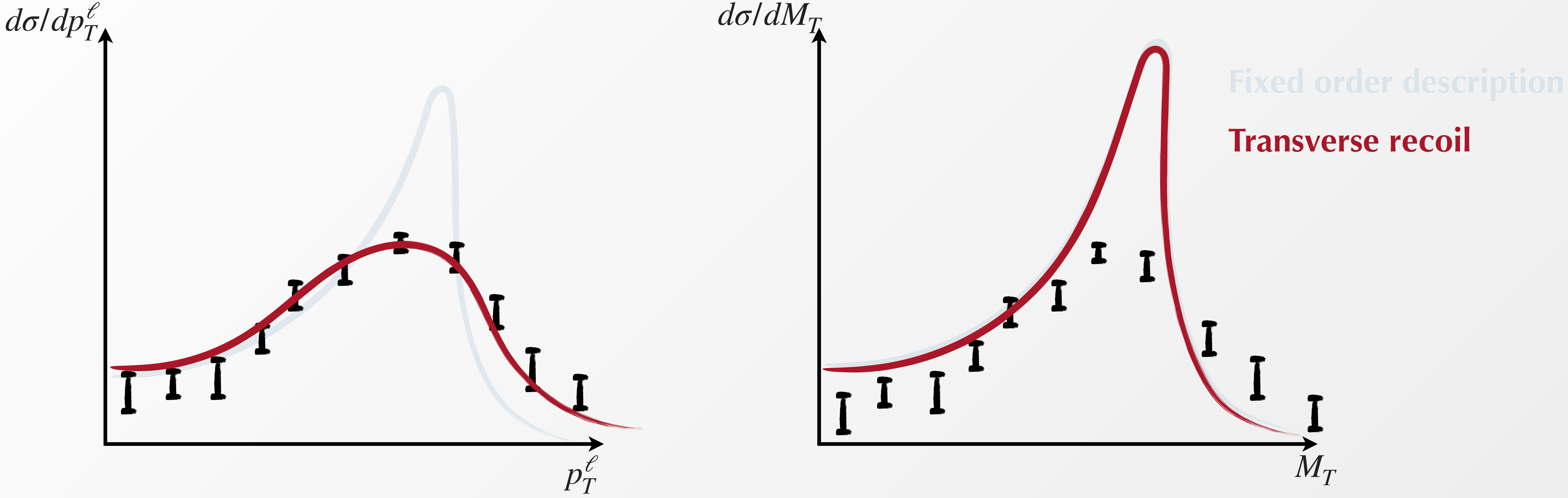
Measurements of m_W at hadron colliders

Different sensitivity to experimental uncertainties and quality of theoretical modelling



Measurements of m_W at hadron colliders

Different sensitivity to experimental uncertainties and quality of theoretical modelling

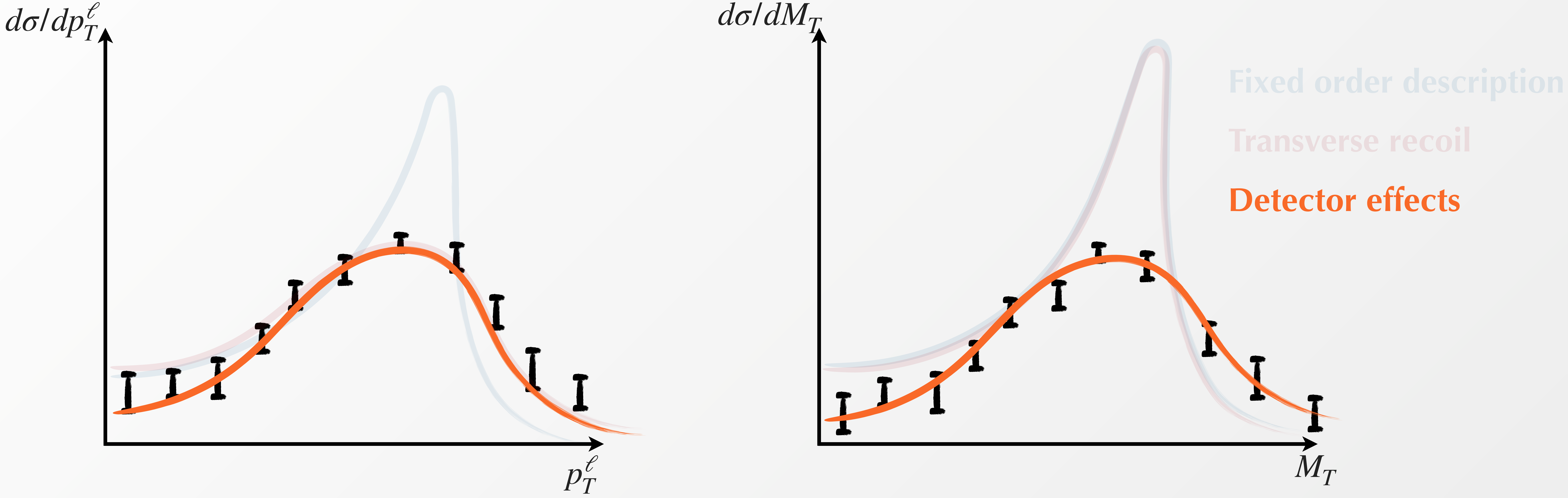


Description of the data requires:

- Modelling of **IS QCD + FS QED radiation**

Measurements of m_W at hadron colliders

Different sensitivity to experimental uncertainties and quality of theoretical modelling

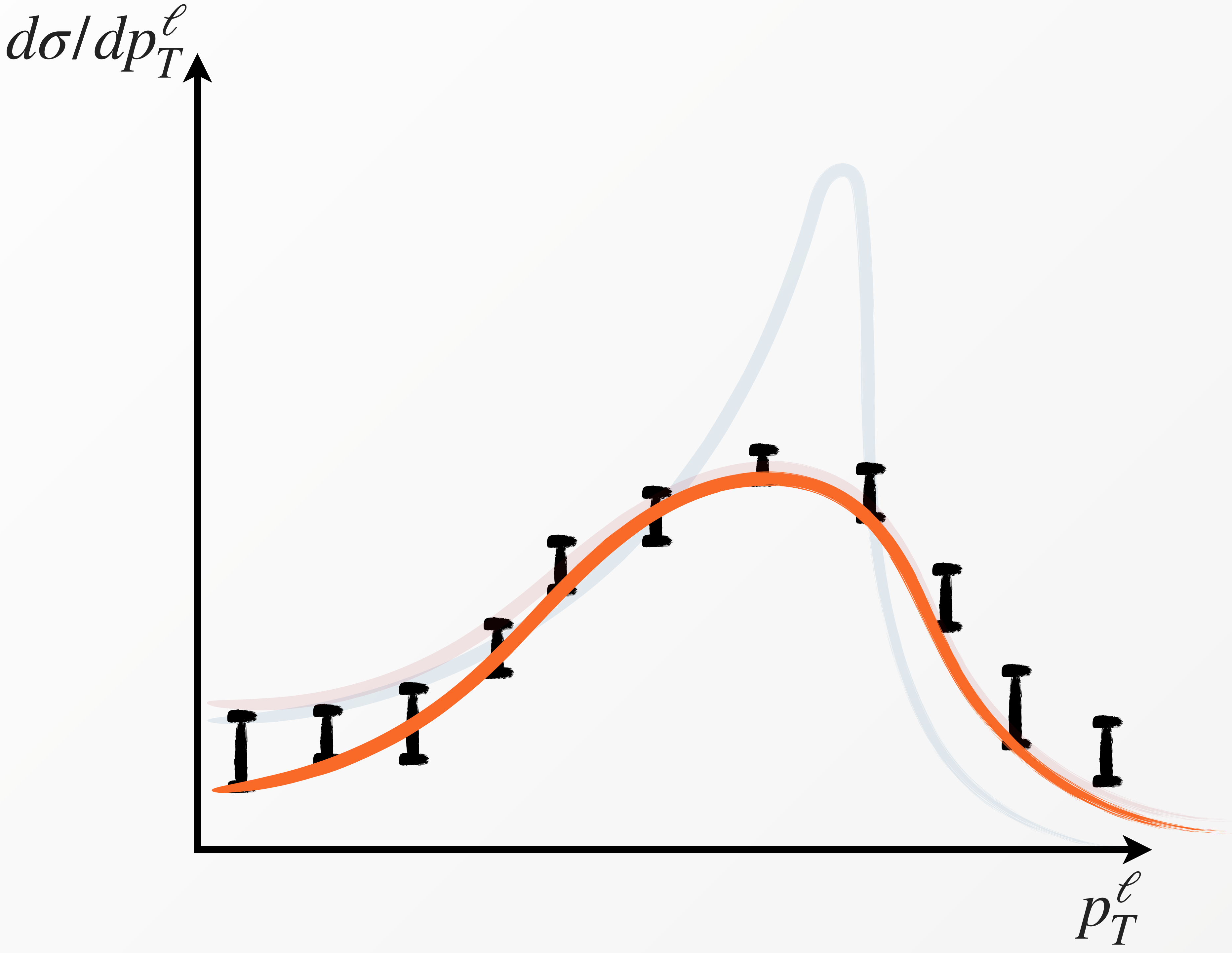


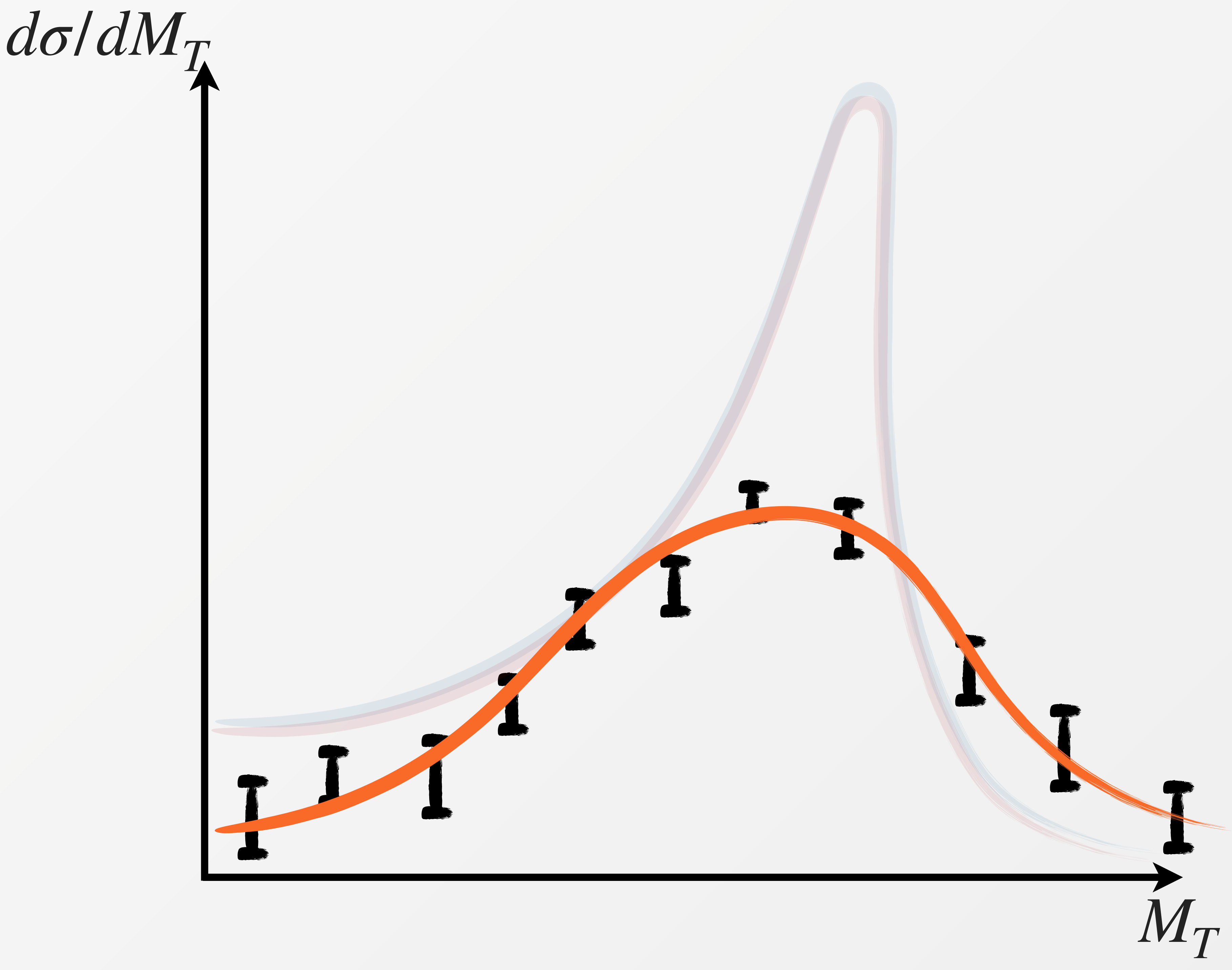
Description of the data requires:

- Modelling of **IS QCD + FS QED radiation**
- Modelling of the **smearing** of the distributions due to the reconstruction of the neutrino in the transverse plane

Measurements of m_W at hadron colliders

Different sensitivity to experimental uncertainties and quality of theoretical modelling





Mostly **QCD + QED radiation**

Mostly **detector effects**

$$m_T = \sqrt{2p_T^\ell p_T^\nu (1 - \cos\Delta\phi_{\ell\nu})}$$

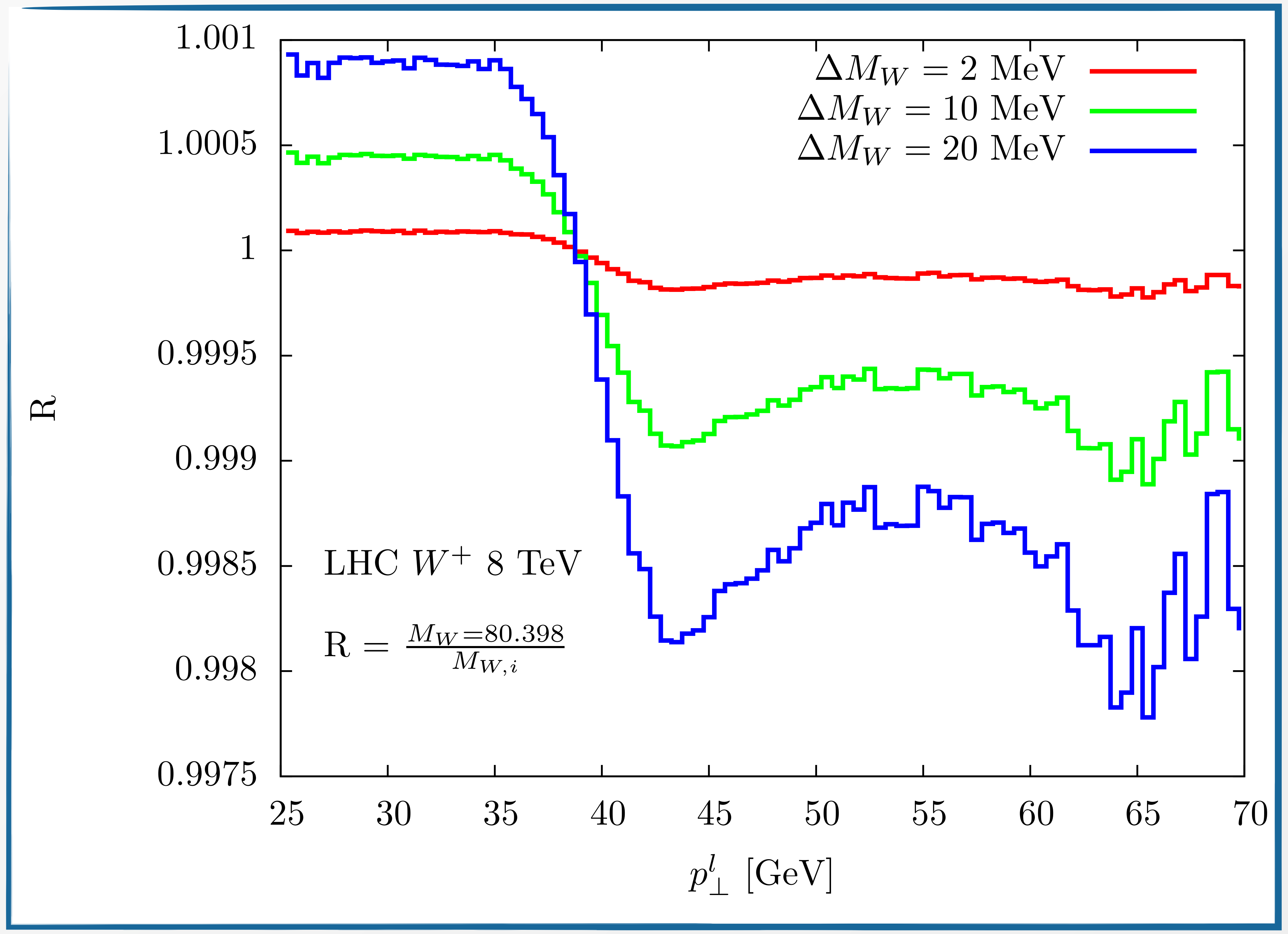
Requires precise determination of the neutrino transverse momentum: **challenging at the LHC** due to worse control of the hadronic recoil

Measurements of m_W at hadron colliders: template fitting

Extraction of m_W performed by template fittings of relevant kinematic observables e.g. lepton transverse momentum $p_\perp^\ell$

1. Compute theoretical distributions for different values of $m_W^{(k)}$
2. For each hypothesis, compute a figure of merit χ_k^2 for a defined window in $p_\perp^\ell$
3. The minimum value of χ_k^2 defines the experimental value of m_W

Permille-level control of the shape is necessary to obtain m_W with 10^{-4} precision



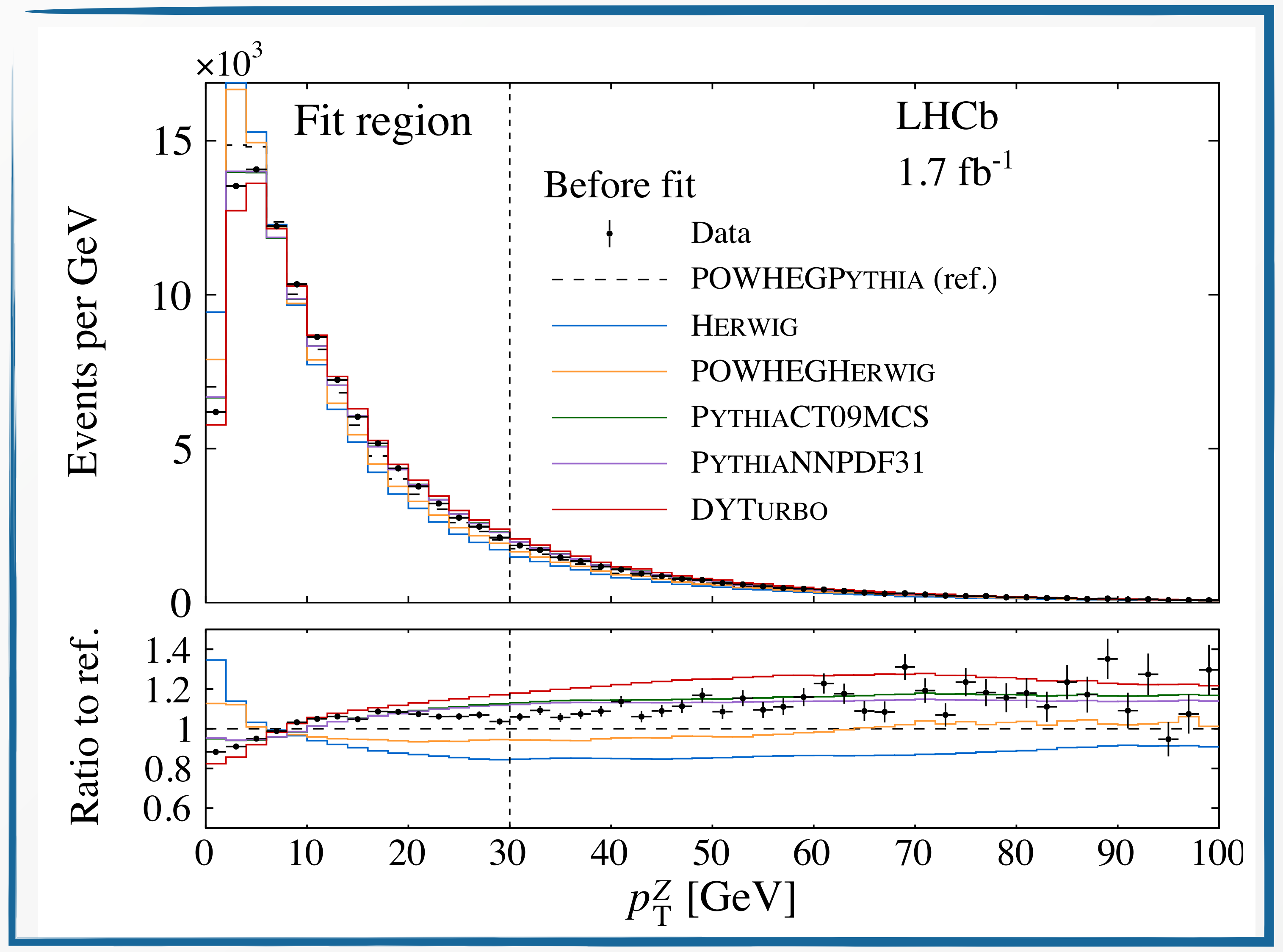
The **description of experimental data** plays a crucial role

Precise control of the **associated theoretical uncertainties** needed to assess the **theoretical systematic error** on m_W

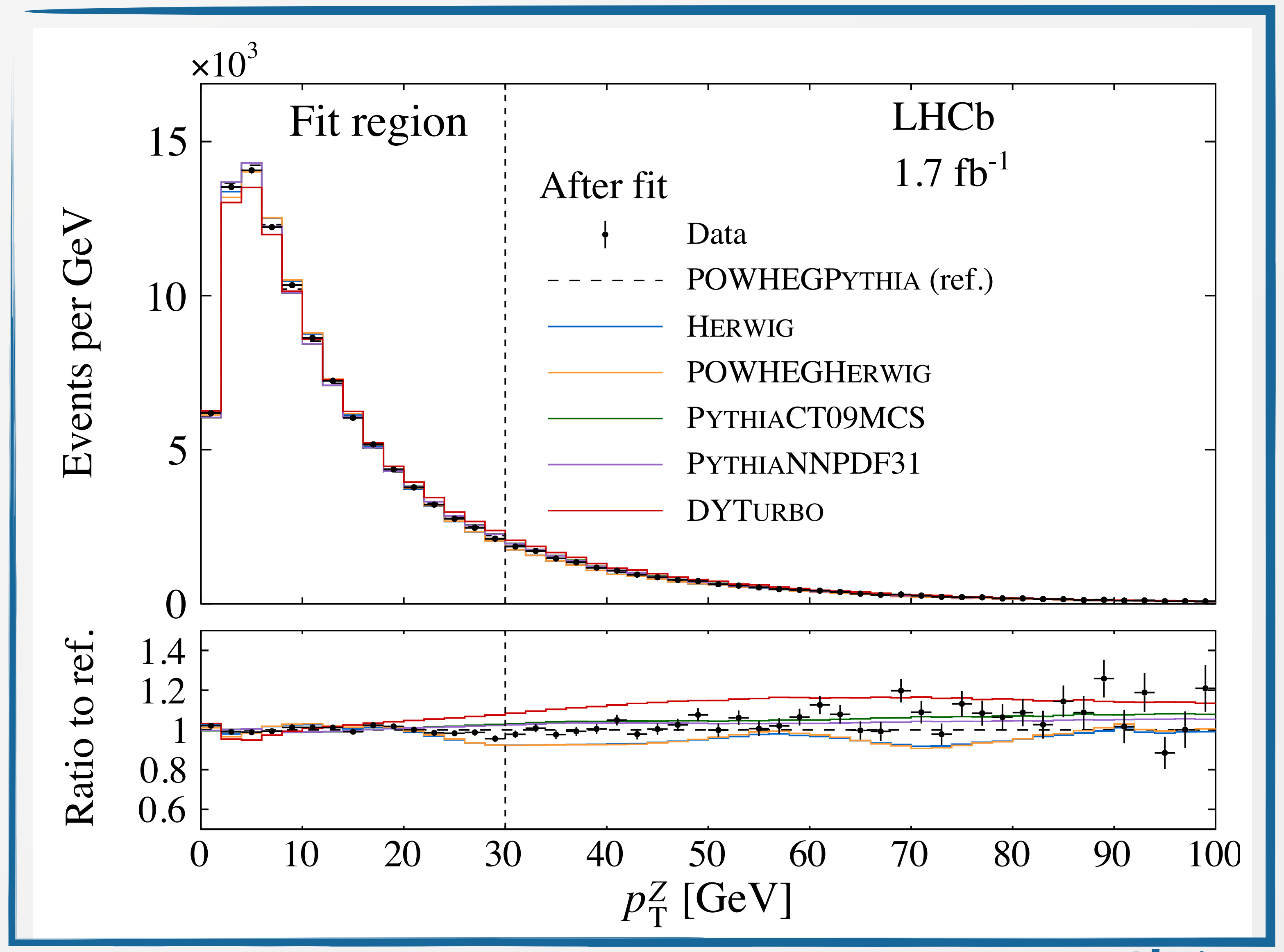
Template fitting and tuning

Template fitting procedure requires that the theoretical distribution can describe the data with high quality

Data-driven approach: Monte Carlo event generators tuned with NCDY data exploiting astonishing precision of $p_\perp^{\ell\ell}$ spectrum

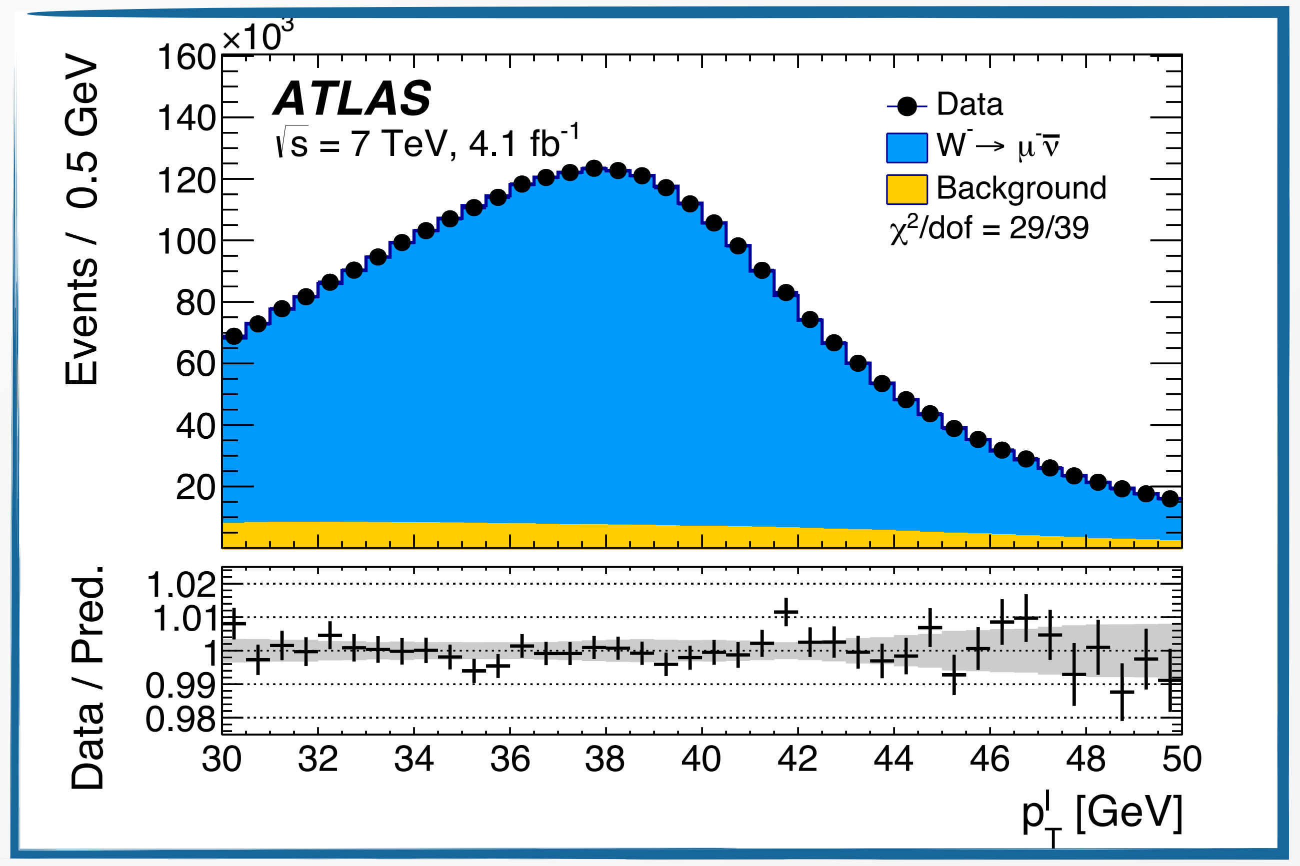


[LHCb '21]

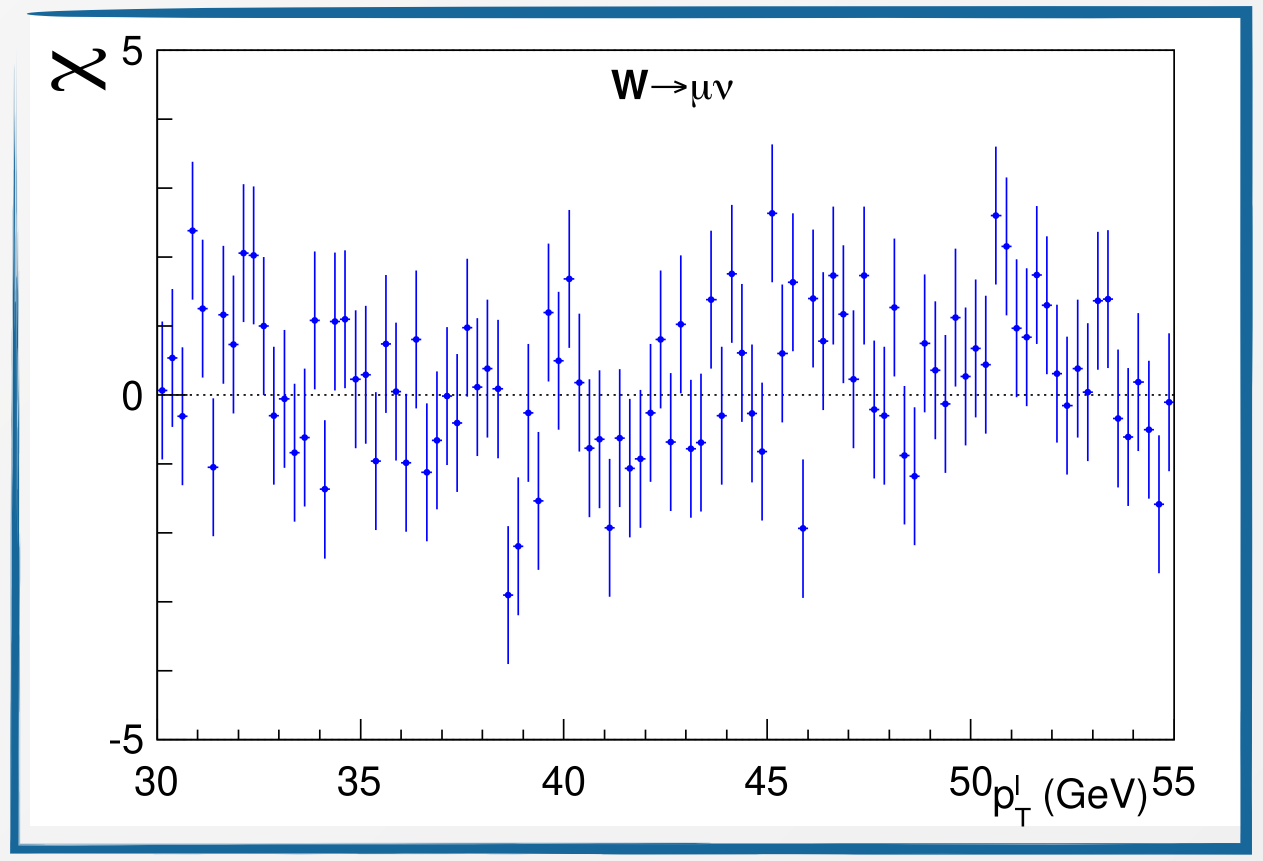


[LHCb '21]

Templates prepared for CCDY using the same tuned parameters



[ATLAS '17]



[CDF '22]

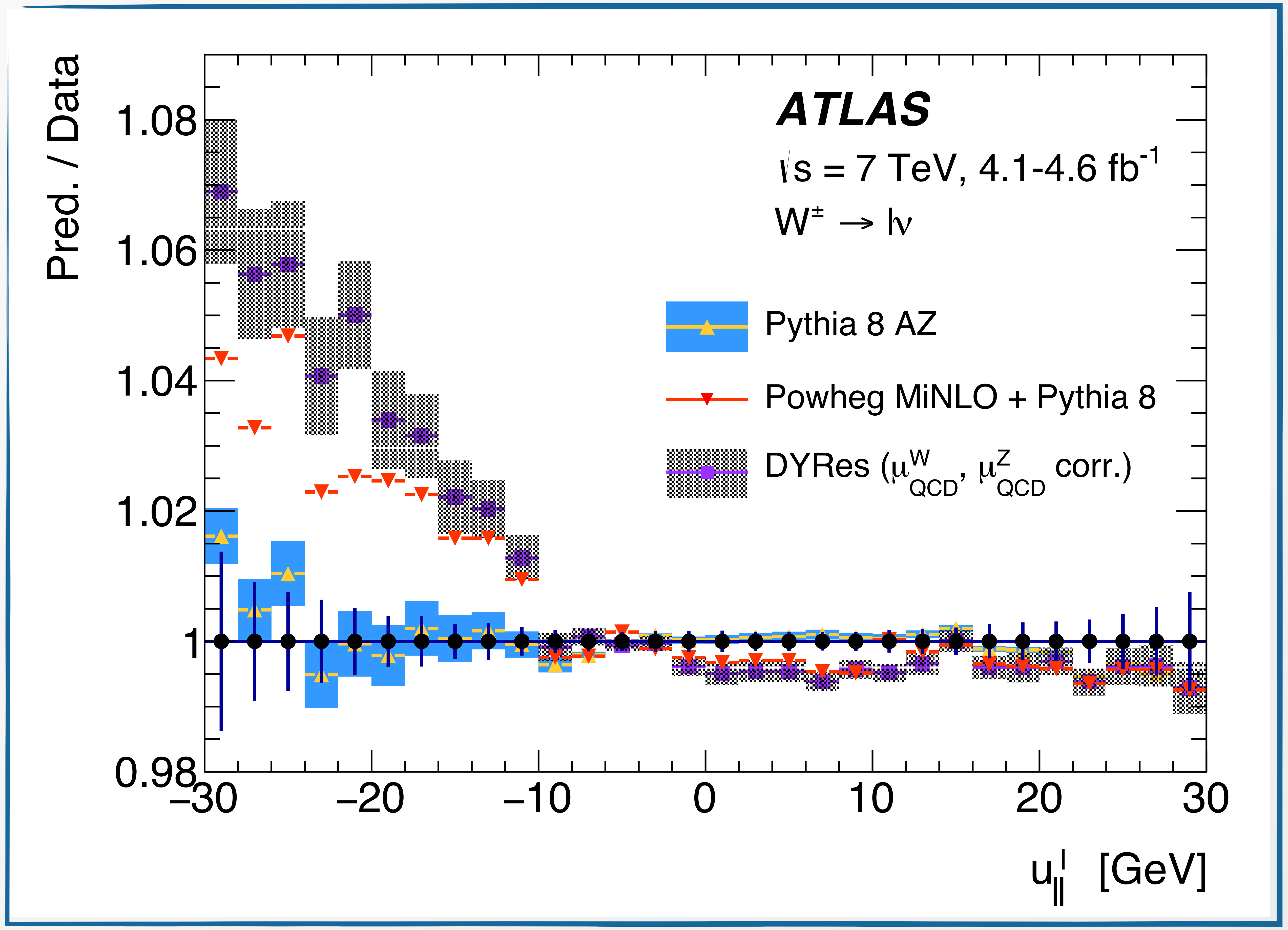
Procedure heavily relies on the similarities between NC and CC DY, and assumes that the information obtained from the data is fully correlated between the two processes

Description of experimental data

Predictions with formal higher accuracy could still be outperformed by **tuned** parton shower simulation

E.g. ATLAS 2017 analysis found that (tuned) PYTHIA8 provided a better description of relevant kinematic distribution with respect to codes formally more accurate

$u_{||}^{\ell}$: projection of hadronic recoil on the axis parallel to reconstructed charged-lepton transverse momentum



[ATLAS '17]

Template fits and their limitations

- Interpretation of the fitted m_W value as Lagrangian parameter can be subtle
- Possible that BSM effects are absorbed in the tuning
- Tuning assumes universality of CCDY and NCDY, although **several differences** play a role (experimental acceptances, different phase spaces, different QED corrections, PDFs and heavy quark effects…)
- Tuning performed at the level of the $p_\perp^{\ell\ell}$ spectrum: **uncertainty related to the transfer of information** to other kinematic distribution in W production ($p_\perp^{\ell}$, $M_\perp^{\ell\nu}$)
- Template fitting procedure relies on tools with **low formal accuracy**; missing higher order information captured only within some approximation (e.g. reweighing)
- Minimisation procedure sensible when $\chi^2/N_{\text{dat}} \sim 1$

$$\chi^2 = (D-T)^T \cdot C^{-1} \cdot (D-T)$$

$$C = \Sigma_{\text{stat}} + \Sigma_{\text{syst}} + \Sigma_{\text{MC}} + \Sigma_{\text{PDF}}$$

Inclusion of Σ_{th} contribution to the covariance matrix not possible due to **non-statistical nature** of theory uncertainty

Data-driven approach allows the possibility to determine m_W via template fits at the **price** of

1. losing the possibility to **assess robustly the theoretical uncertainties** on the modelling
2. incapability to **fully exploit recent progress in theoretical calculations** for candle LHC processes

4.

Precision physics at the LHC: theory

$$\sigma(s, Q^2) = \sum_{a,b} \int dx_1 dx_2\, f_{a/h_1}(x_1, Q^2) f_{b/h_2}(x_2, Q^2) \hat{\sigma}_{ab\to X}(Q^2, x_1 x_2 s) + \mathcal{O}(\Lambda^p_{\rm QCD}/Q^p)$$

Input parameters:

strong coupling α_s

PDFs f

few percent uncertainty; improvable

Non-perturbative effects

percent effect; not yet under full control

Precision physics at the LHC: fixed order computations

$$\sigma(s,Q^2) = \sum_{a,b} \int dx_1 dx_2\, f_{a/h_1}(x_1,Q^2) f_{b/h_2}(x_2,Q^2) \hat{\sigma}_{ab\to X}(Q^2, x_1 x_2 s) + \mathcal{O}(\Lambda^p_{\mathrm{QCD}}/Q^p)$$

$$\tilde{\sigma} = 1 + \alpha_s \tilde{\sigma}_1 + \alpha_s^2 \tilde{\sigma}_2 + \alpha_s^3 \tilde{\sigma}_3 + \dots$$

LO_{QCD} NLO_{QCD} $NNLO_{QCD}$ N^3LO_{QCD}

$\alpha_s \sim 0.1$

δ~10-20% NLO_{QCD}

δ~1-5% $NNLO_{QCD}$ (or even N^3LO_{QCD})

$NNLO_{QCD}$ available for a larger number of processes, 2→3 computations current frontier

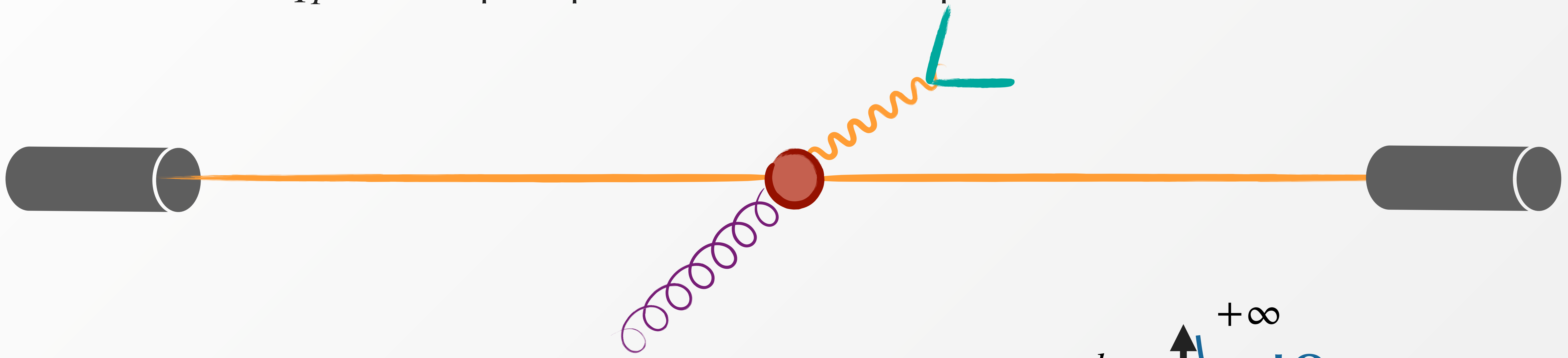
N^3LO_{QCD} available for few LHC processes

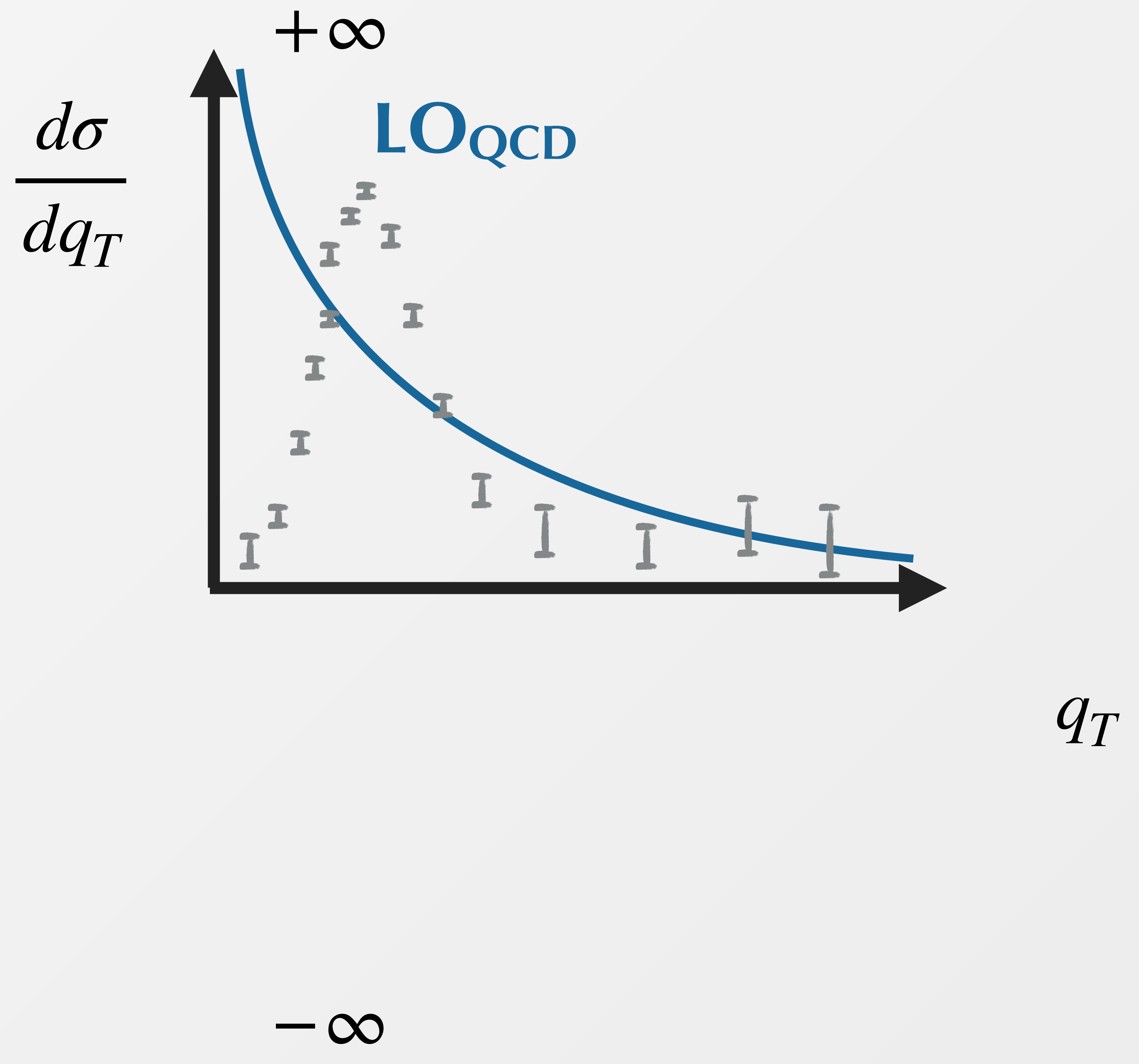
Calculation of NLO_{EW} and of mixed $NNLO_{QCD\text{-}EW}$ corrections relevant for precise phenomenology (especially for candle processes such as DY production) $\alpha \sim 0.01$

Precision physics at the LHC: all-order resummation

Fixed-order description not sufficient for observable sensitive to soft / collinear radiation

E.g transverse momentum q_T of the lepton pair in NC Drell-Yan production





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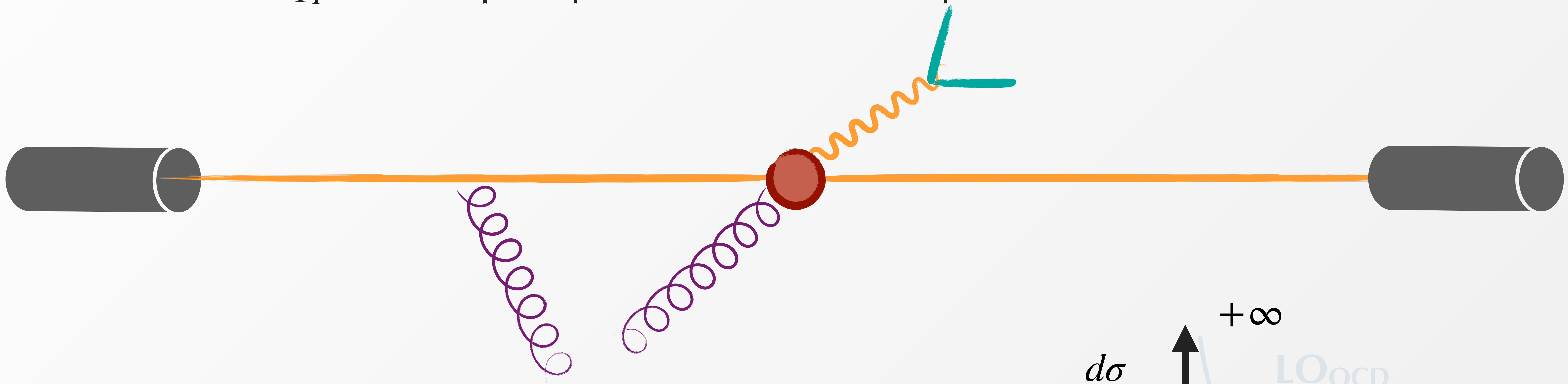

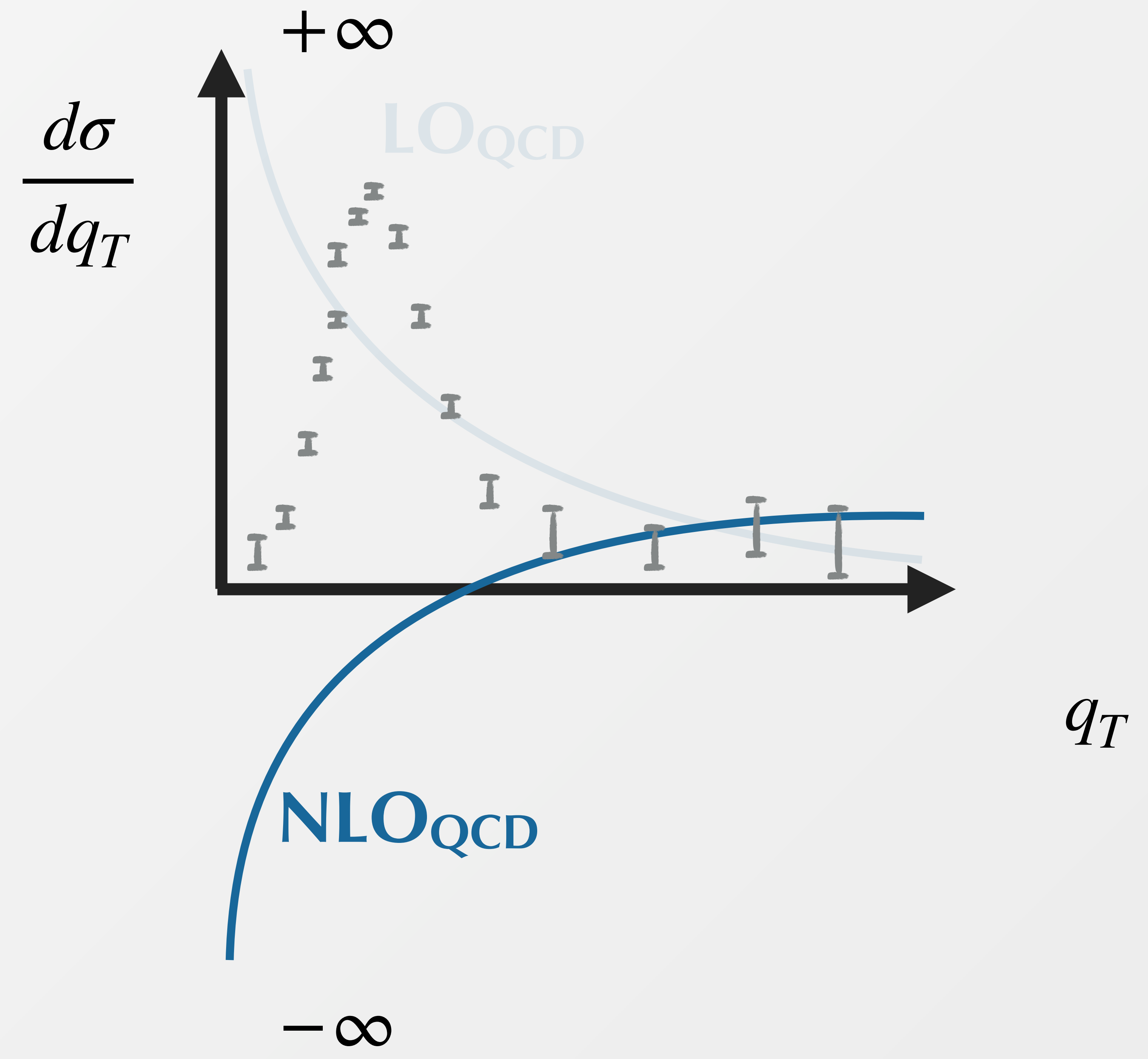

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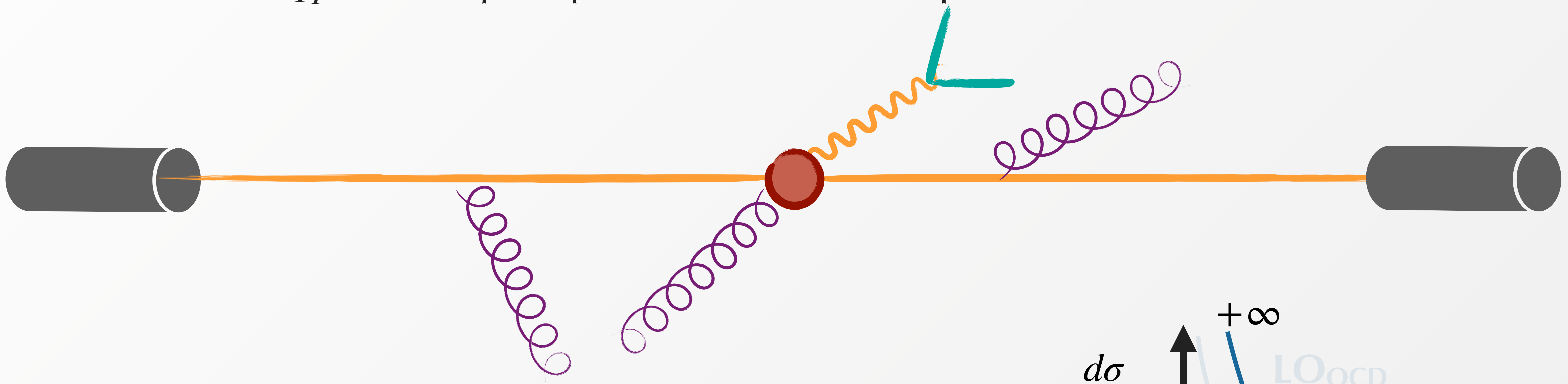

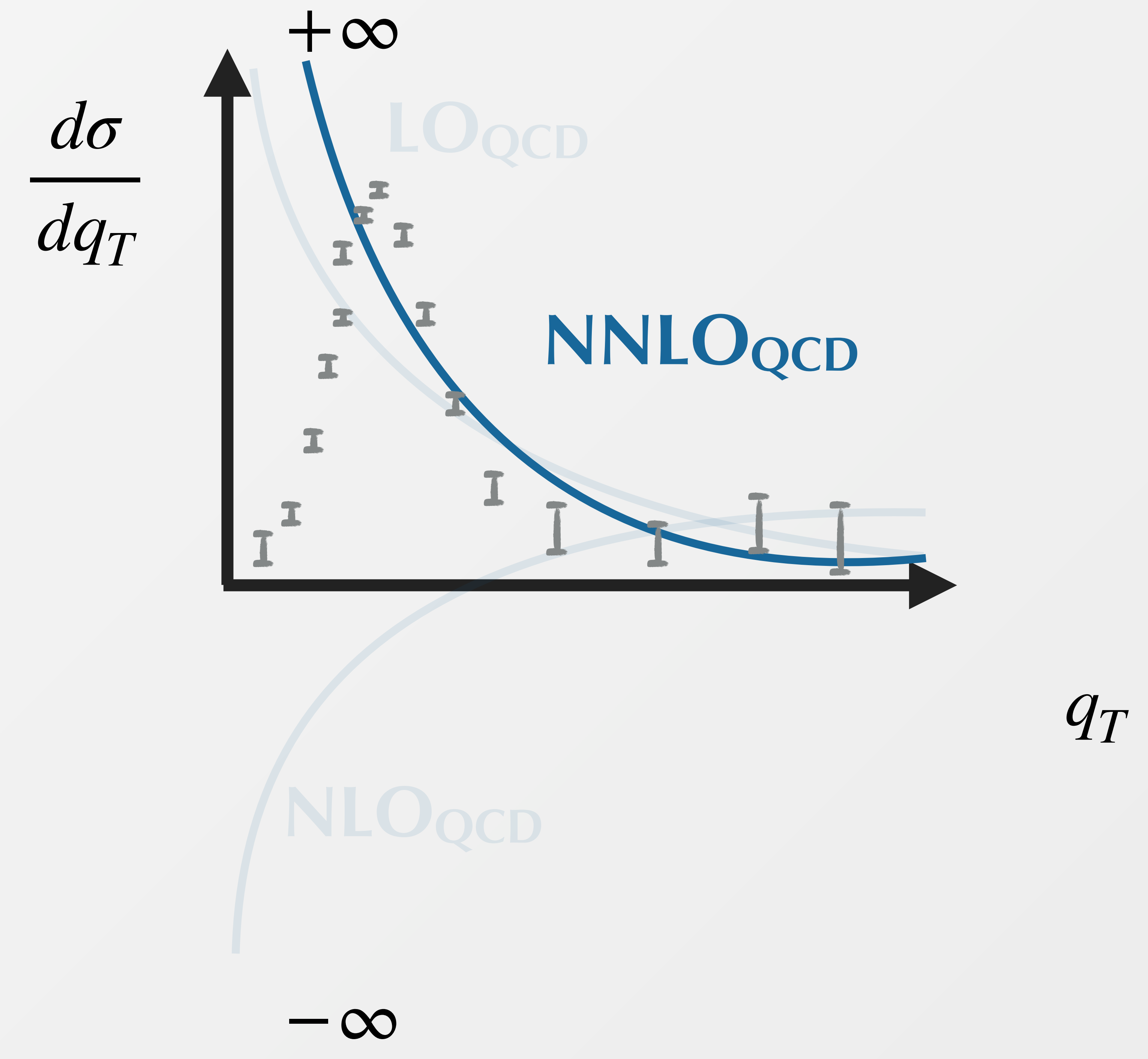

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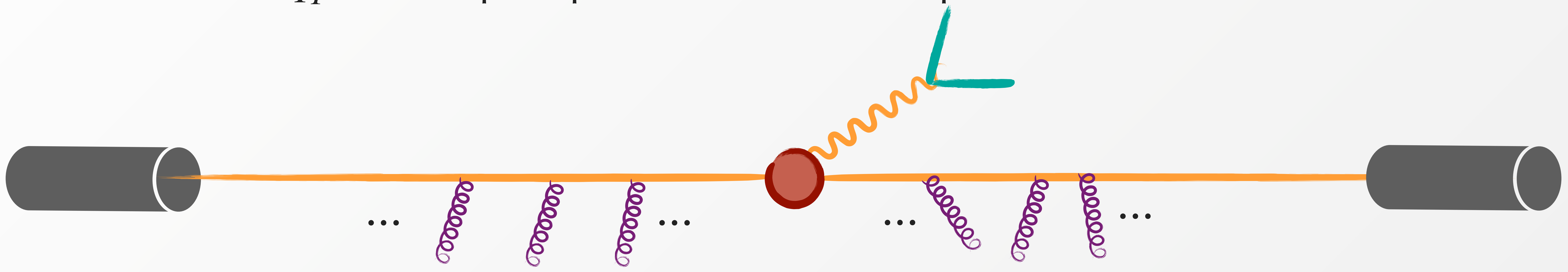
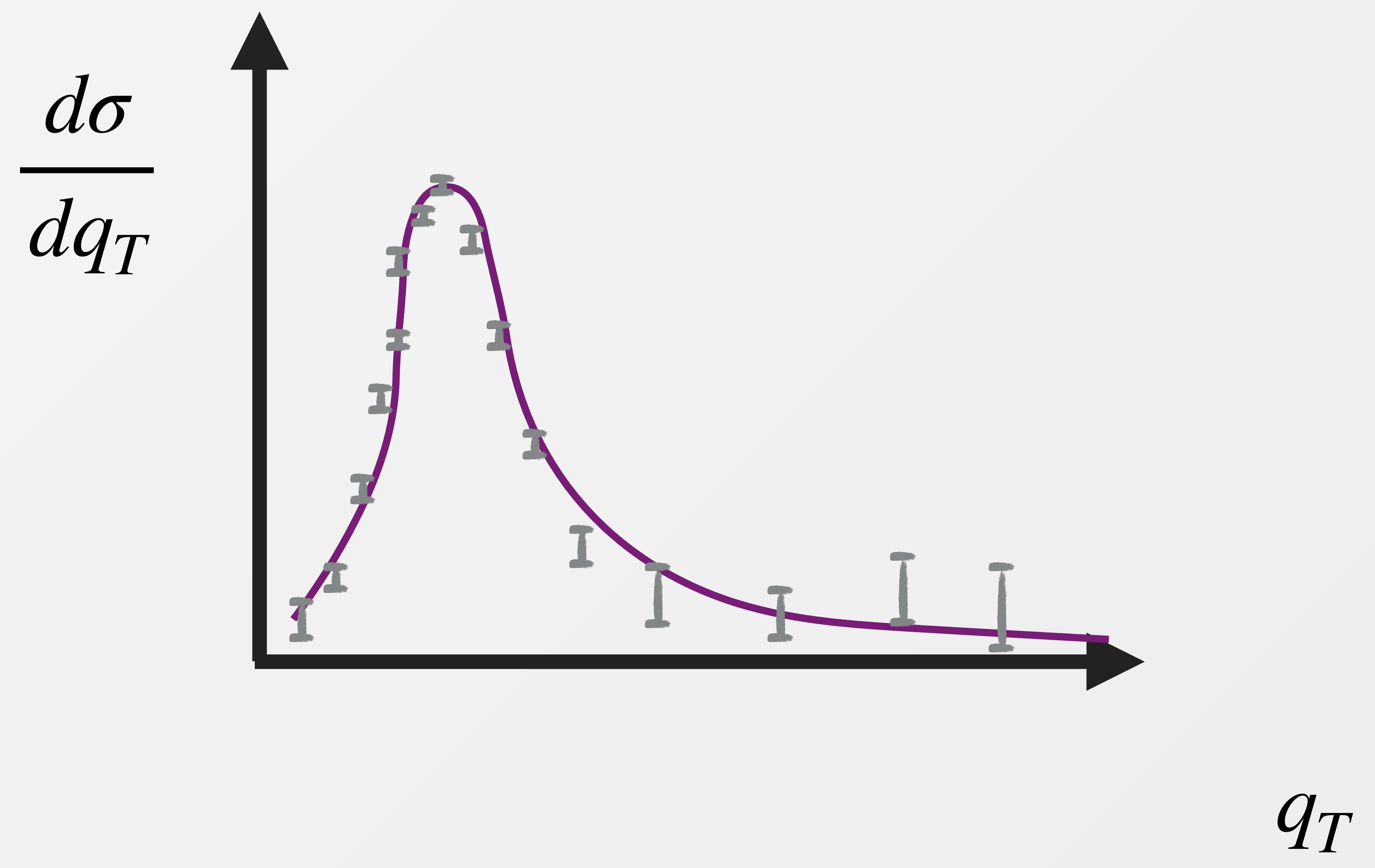


Need to consider infinite number of soft / collinear emissions



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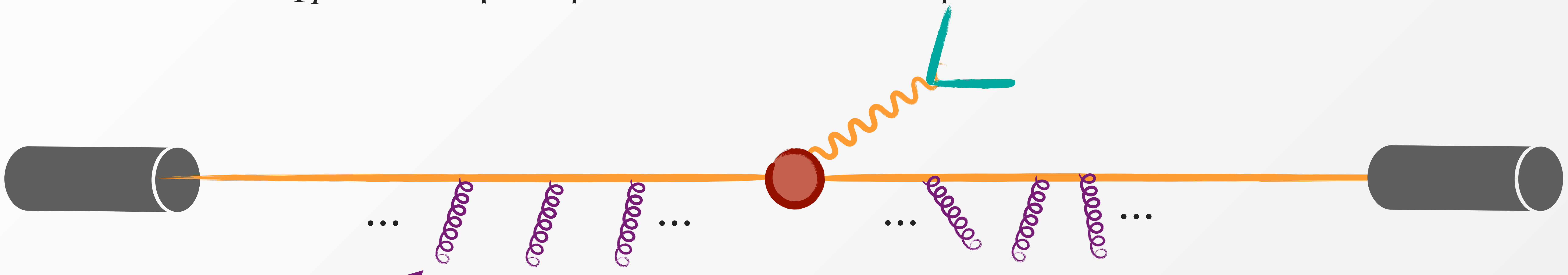

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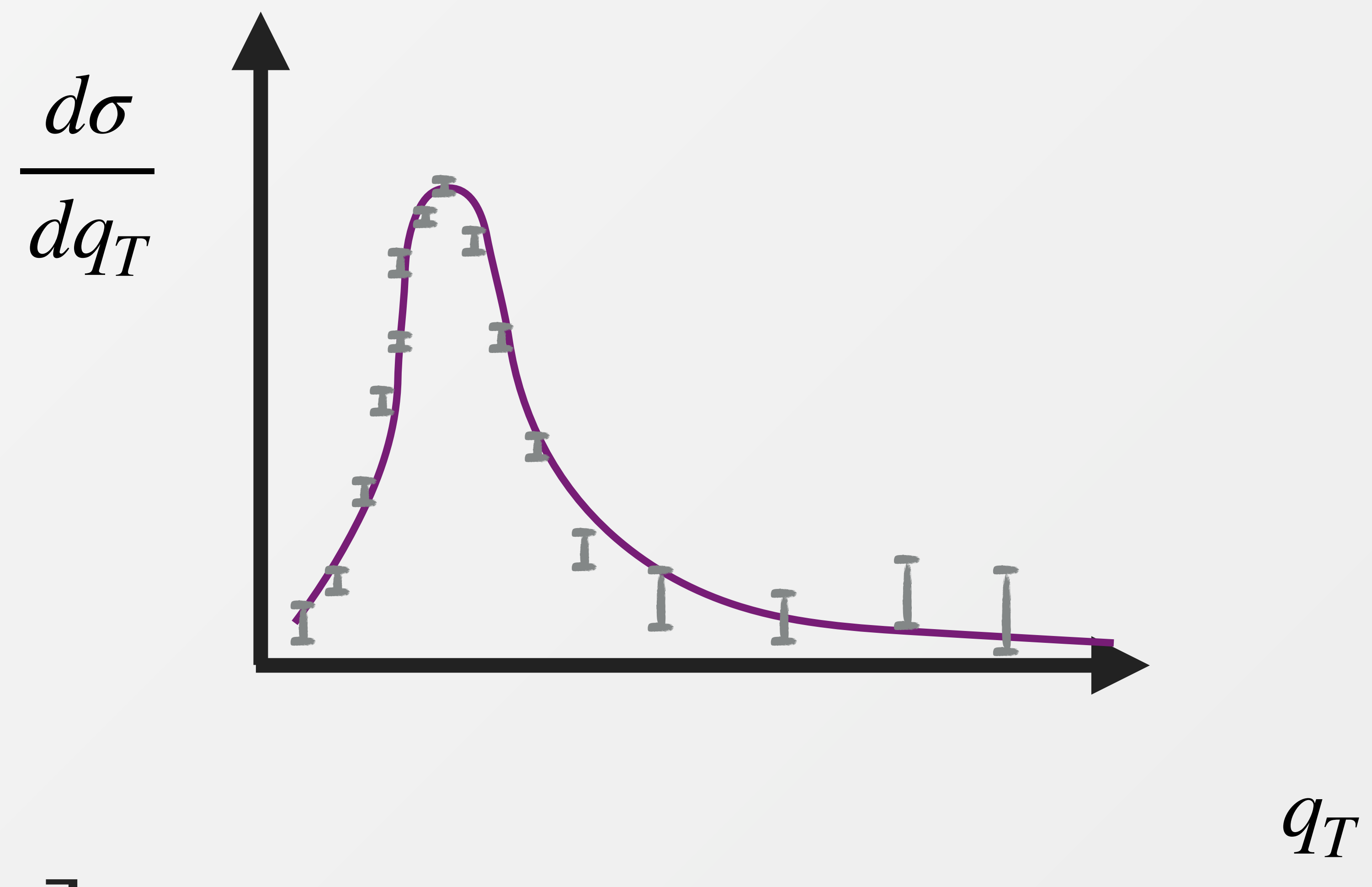

Many independent **soft-collinear gluons** with comparable angles and transverse momenta

$v \to 0$

$L = \ln(1/v)$

LL$_{QCD}$ NLL$_{QCD}$ NNLL$_{QCD}$

$$\tilde{\sigma}(v) = \exp\left[\sum_n \left(\mathcal{O}(\alpha_s^n L^{n+1}) + \mathcal{O}(\alpha_s^n L^n) + \mathcal{O}(\alpha_s^n L^{n-1}) + \ldots\right)\right]$$

Precision physics at the LHC: all-order resummation

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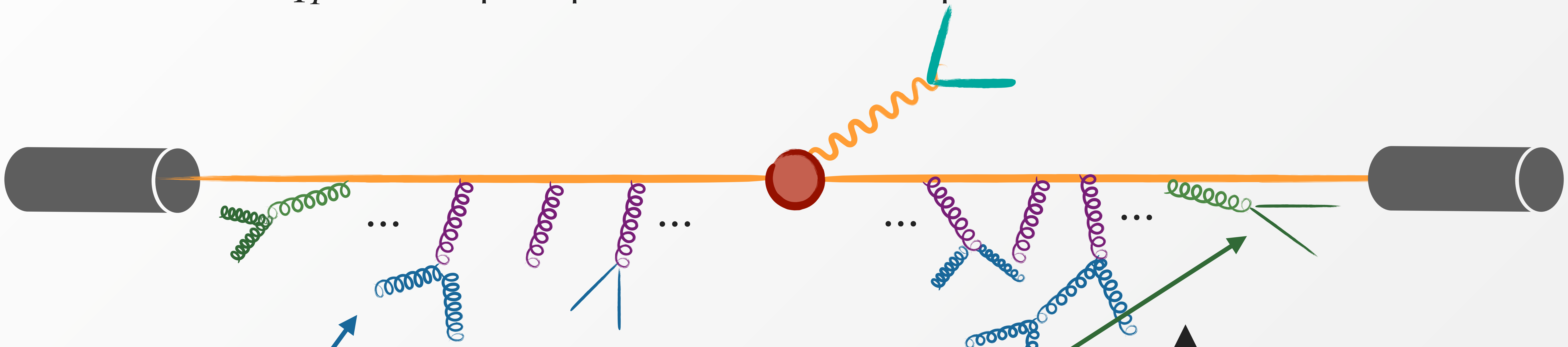

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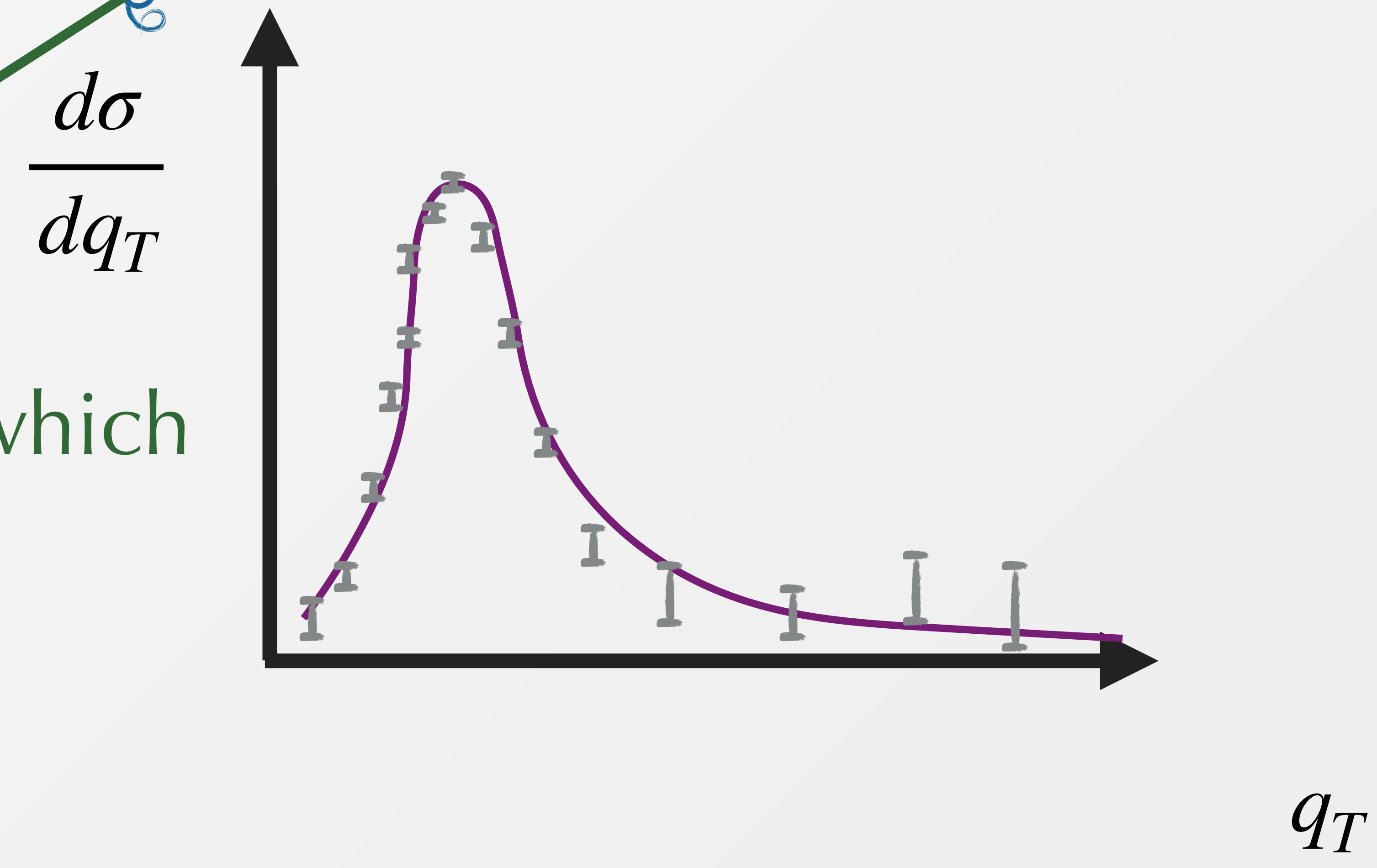


Further, less singular, soft splittings

Hard-collinear emission which can also split

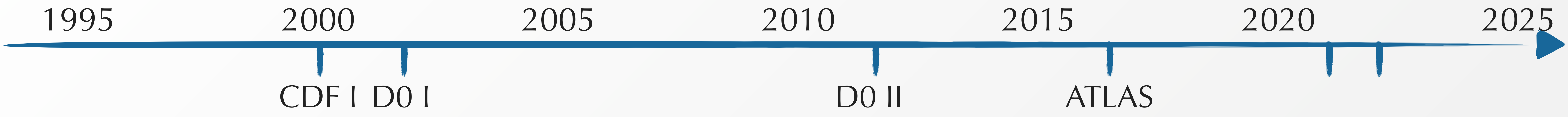
LL$_{QCD}$ NLL$_{QCD}$ NNLL$_{QCD}$

$v \to 0$

$L = \ln(1/v)$

$$\tilde{\sigma}(v) = \exp\left[\sum_n \left(\mathcal{O}(\alpha_s^n L^{n+1}) + \mathcal{O}(\alpha_s^n L^n) + \mathcal{O}(\alpha_s^n L^{n-1}) + \ldots\right)\right]$$

Progress in theoretical calculations



(N)NLL$_{QCD}$+NLO$_{QCD}$
[Balasz, Yuan '97]

NNLL'$_{QCD}$+NNLO$_{QCD}$
[Bozzi, Catani, Ferrera, De Florian, Grazzini '10]

Huge progress in the theoretical description of NC and CC Drell-Yan processes in the last few years

Progress in theoretical calculations

(N)NLL$_{QCD}$+NLO$_{QCD}$
[Balasz, Yuan '97]

NNLL′$_{QCD}$+NNLO$_{QCD}$
[Bozzi, Catani, Ferrera, De Florian, Grazzini '10]

N^{3}LL$_{QCD}$+NNLO$_{QCD}$
[Bizon, Monni, Re, LR, Torrielli '17]

N^{3}LL′$_{QCD}$+N^{3}LO$_{QCD}$
[Camarda, Cieri, Ferrera '21]
[Re, LR, Torrielli '21]
[Chen, Gehrmann, Glover, Huss, Monni, Re, LR, Torrielli '22]
[Neumann, Campbell '22]

NNLO$_{QCD-EW}$
[Armadillo, Bonciani, Buonocore, Devoto, Grazzini, Kallweit, Rana, Savoini, Tramontano, Vicini '21, '22]
[Buccioni, Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, Röntsch, Signorile-Signorile '22]

Huge progress in the theoretical description of NC and CC Drell-Yan processes in the last few years

Fixed-order description now reaches $\mathcal{O}(\alpha_s^3)$ (N^{3}LO$_{QCD}$)

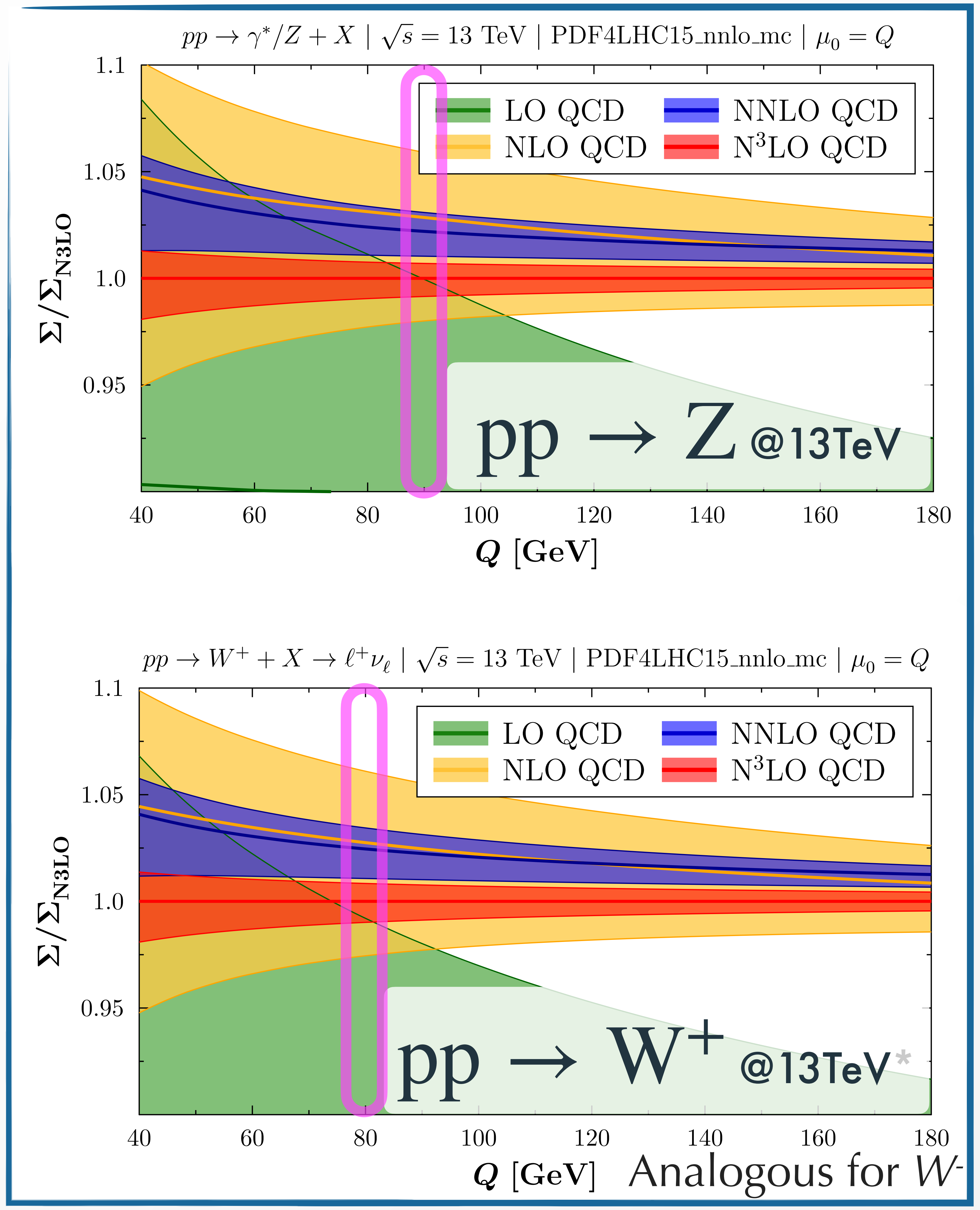
All-order resummation up to N^{3}LL′$_{QCD}$

QCD-EW correction at order $\mathcal{O}(\alpha_s \alpha)$ NNLO$_{QCD-EW}$

Drell-Yan production at N^{3}LO$_{QCD}$

Inclusive Drell-Yan cross section known analytically at N^{3}LO

[Duhr, Mistlberger '21]



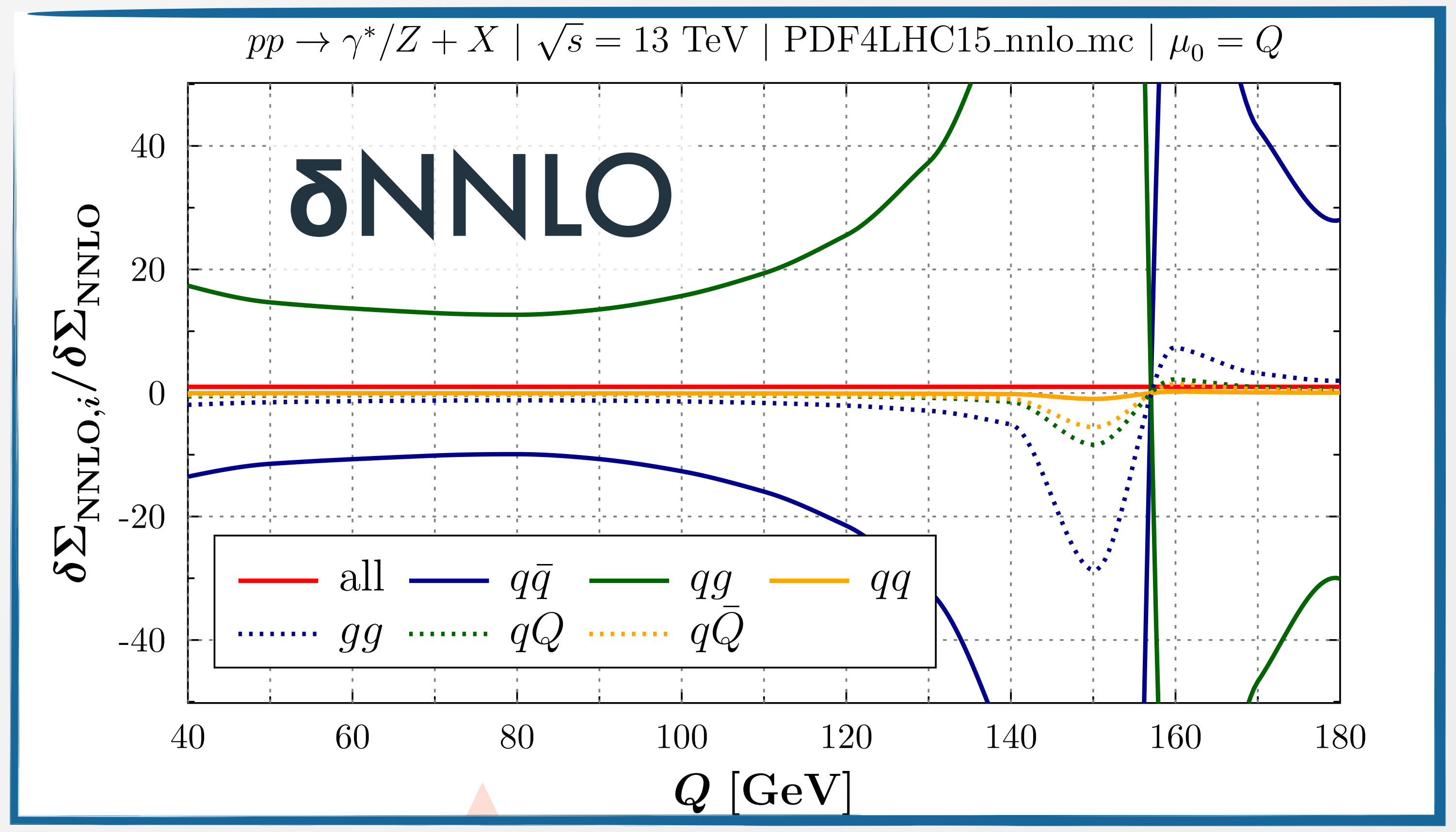
Analogous for W^-

[Baglio, Duhr, Mistlberger, Szafron '22]

Resonance region: non-overlapping bands

Scale uncertainty does not reduce at N^{3}LO $\quad \Delta^{\mathrm{scale}}_{\mathrm{NNLO}} \simeq \Delta^{\mathrm{scale}}_{\mathrm{N^3LO}}$

Large cancellations between different partonic channels



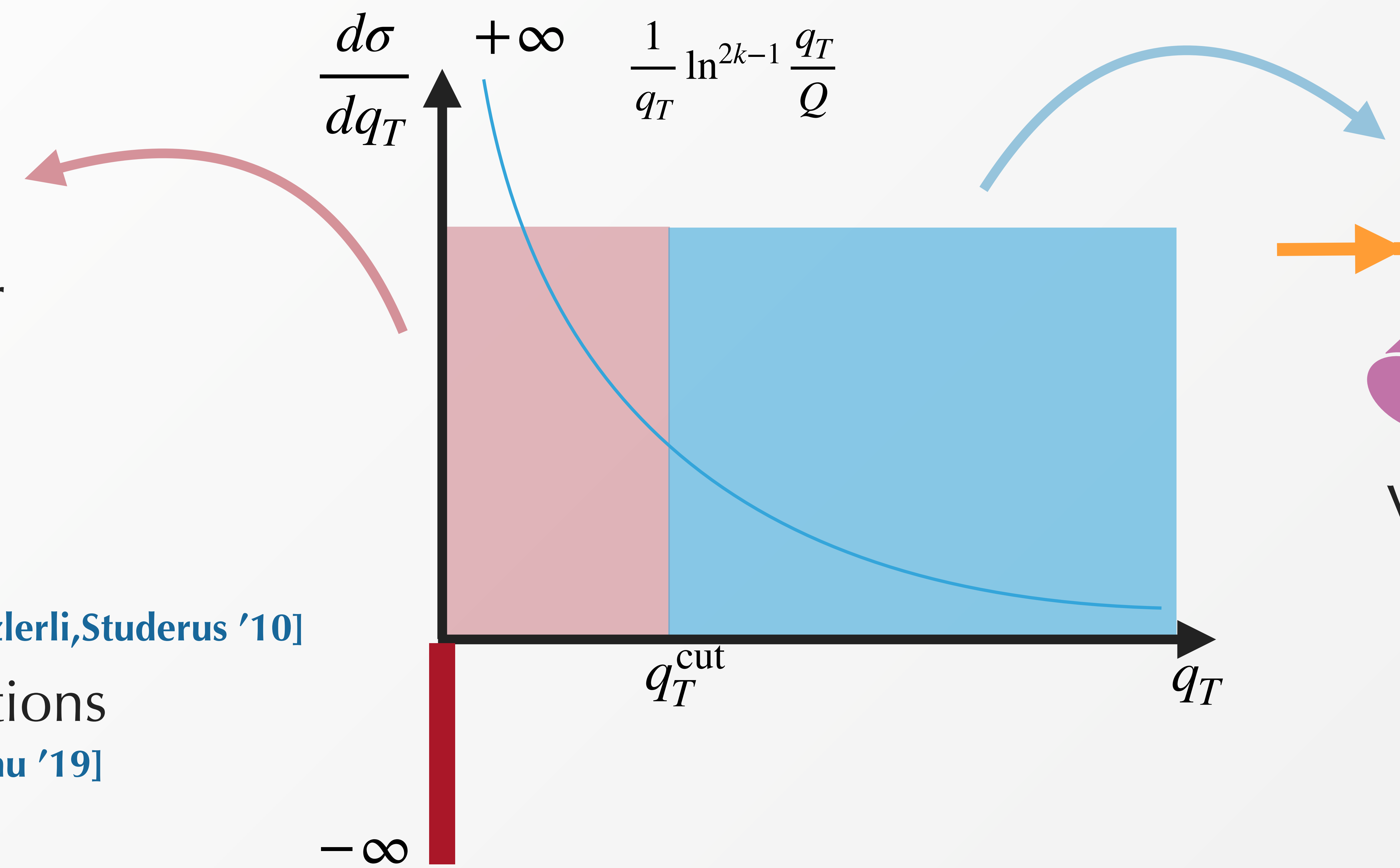
Δ(N^{3}LO-NNLO) ~ 2% **Necessary for % level target**

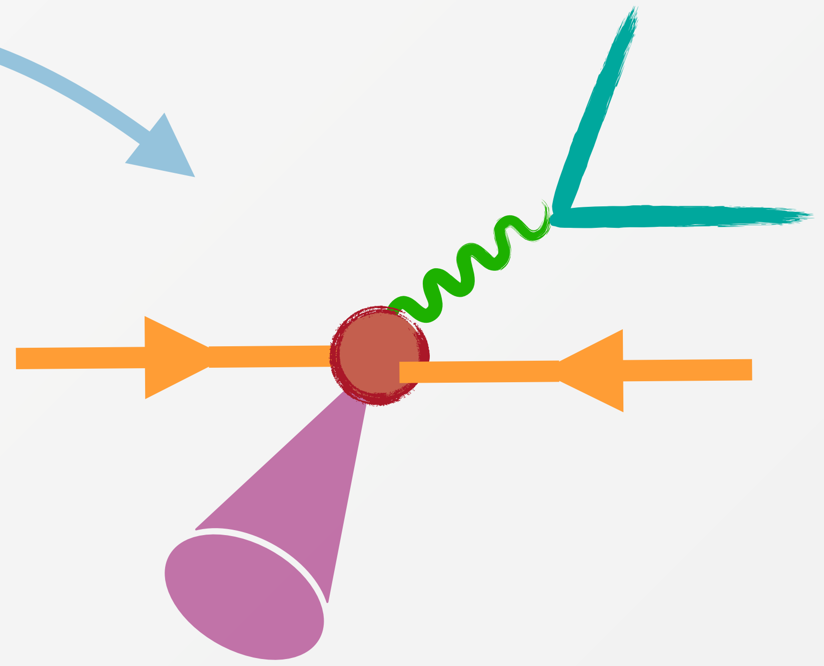
Calculation at the fiducial level crucial step for the LHC precision programme

Fiducial Drell-Yan production at N^3LO_{QCD}

q_T resummation

- Expand to fixed order
- $\mathcal{O}(\alpha_s^3)$ ingredients
  - Hard function
    [Gehrmann,Glover,Huber,Ikizlerli,Studerus '10]
  - Beam and soft functions
    [Li, Zhu '16][Luo,Yang,Zhu,Zhu '19]
    [Ebert,Mistlberger,Vita '20]





V+j @ $N^{k-1}LO$

q_T-subtraction formalism
[Catani, Grazzini '08]

$$d\sigma_V^{N^kLO} \equiv d\sigma_V^{N^kLO}\Big|_{q_T<q_T^{cut}} + d\sigma_V^{N^kLO}\Big|_{q_T>q_T^{cut}}$$

$$d\sigma_V^{N^kLO} \equiv \mathscr{H}_V^{N^kLO} \otimes d\sigma_V^{LO} + \left[d\sigma_{V+jet}^{N^{k-1}LO} - \left[d\sigma_V^{N^kLL}\right]_{\mathcal{O}(\alpha_s^k)}\right]_{q_T>q_t^{cut}} + \mathcal{O}((q_T^{cut}/M)^n)$$

Fiducial Drell-Yan production at N^{3}LO$_{\rm QCD}$

$$d\sigma_V^{\rm N^kLO} \equiv \mathscr{H}_{\rm V}^{\rm N^kLO} \otimes d\sigma_{\rm V}^{\rm LO} + \left[d\sigma_{\rm V+jet}^{\rm N^{k-1}LO} - \left[d\sigma_V^{\rm N^kLL} \right]_{\mathcal{O}(\alpha_s^k)} \right]_{q_T > q_t^{\rm cut}} + \mathcal{O}((q_T^{\rm cut}/M)^n)$$

Competing interests: $q_T^{\rm cut}$ as large as possible $\leftrightarrow$ $q_T^{\rm cut}$ as small as possible

Numerical stability

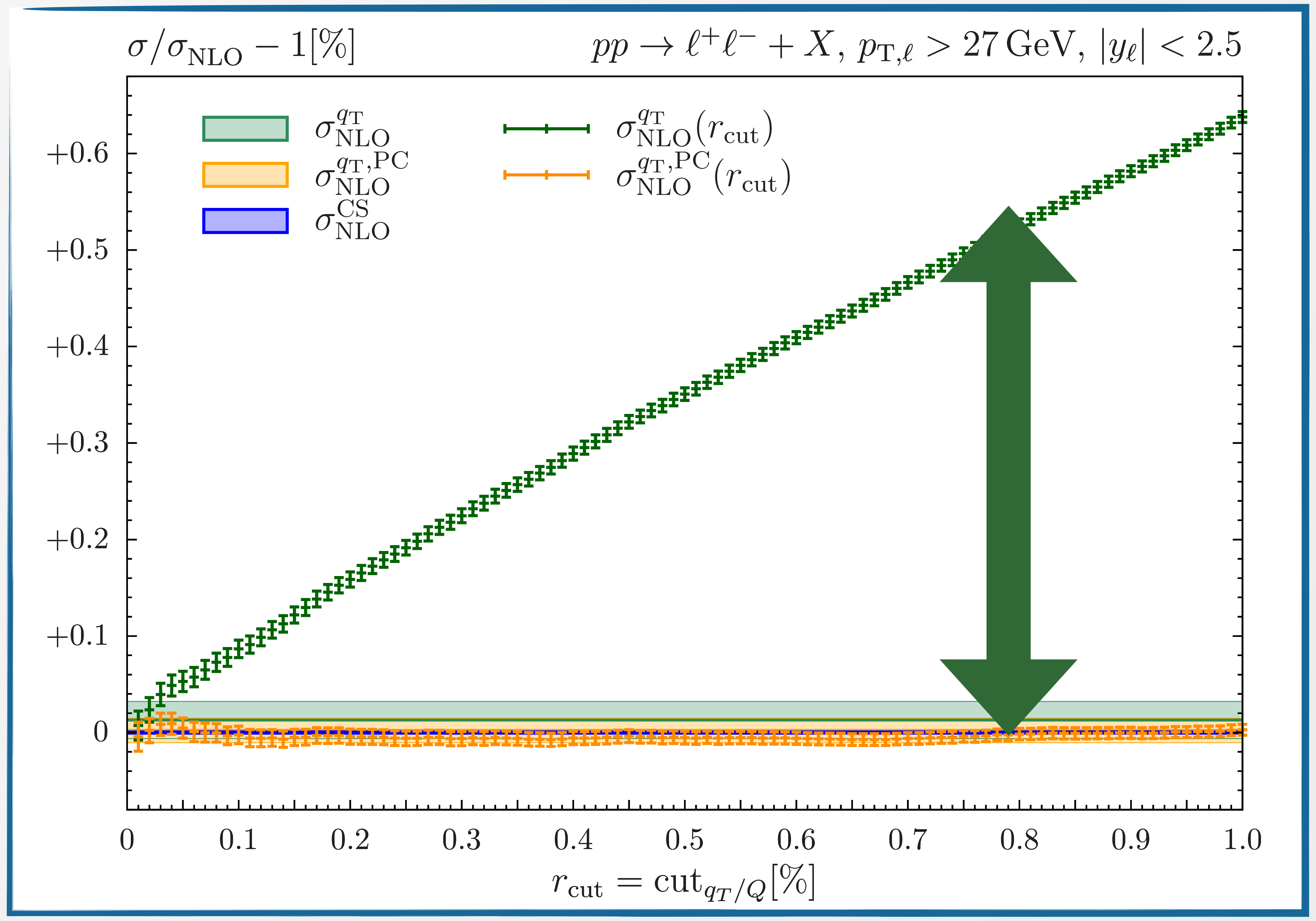
Suppress **power corrections**

Power corrections larger when **symmetric cuts** applied on final state leptons due to enhanced sensitivity to soft radiation in back-to-back configurations [Salam, Slade '21]

Control of fiducial linear power corrections improves dramatically the efficiency of the non-local subtraction

[Kallweit, Buonocore, LR, Wiesemann '21][Camarda, Cieri, Ferrera '21]

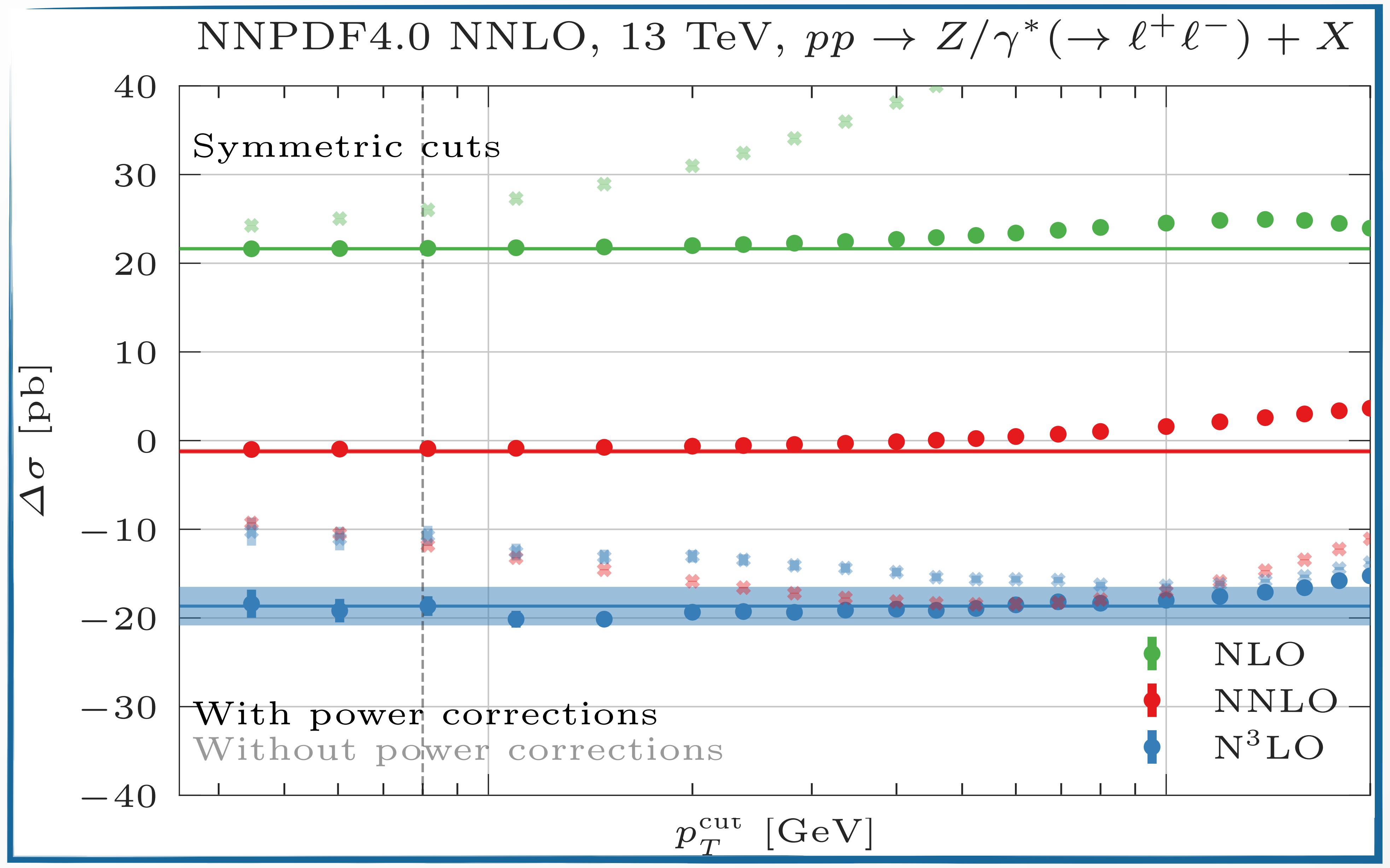
Necessary to reach N^{3}LO accuracy for **fiducial setup**



[Kallweit, Buonocore, LR, Wiesemann '21]

Fiducial Drell-Yan production at N^3LO_{QCD}

$q_T^{cut} = 0.8\,\text{GeV}$



Order	σ [pb] Symmetric cuts	
k	N^kLO	N^kLO+N^kLL
0	$721.16^{+12.2\%}_{-13.2\%}$	—
1	$742.80(1)^{+2.7\%}_{-3.9\%}$	$748.58(3)^{+3.1\%}_{-10.2\%}$
2	$741.59(8)^{+0.42\%}_{-0.71\%}$	$740.75(5)^{+1.15\%}_{-2.66\%}$
3	$722.9(1.1)^{+0.68\%}_{-1.09\%} \pm 0.9$	$726.2(1.1)^{+1.07\%}_{-0.77\%}$

Includes resummation of linear power corrections

[Chen, Gehrmann, Glover, Huss, Monni, Re, LR, Torrielli]

- 2.5% negative correction at N^3LO in the ATLAS fiducial region. N^3LO larger than the NNLO correction and outside its error band
- More robust estimate of the theory uncertainty when **resummation effects are included**

Resummation in direct space: RadISH in a nutshell

RadISH performs resummation in direct space - similar in spirit to a parton shower, with control on formal accuracy

Result at NLL accuracy with scale-independent PDFs

$$\sigma(p_T) = \sigma_0 \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(\epsilon v_1)}$$

$$\times R'\left(v_1\right) \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^{1} \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'\left(\zeta_i v_1\right) \Theta\left(p_T - |\vec{k}_{t,i} + \cdots \vec{k}_{t,n+1}|\right)$$

$$v_i = k_{t,i}/M, \quad \zeta_i = v_i/v_1$$

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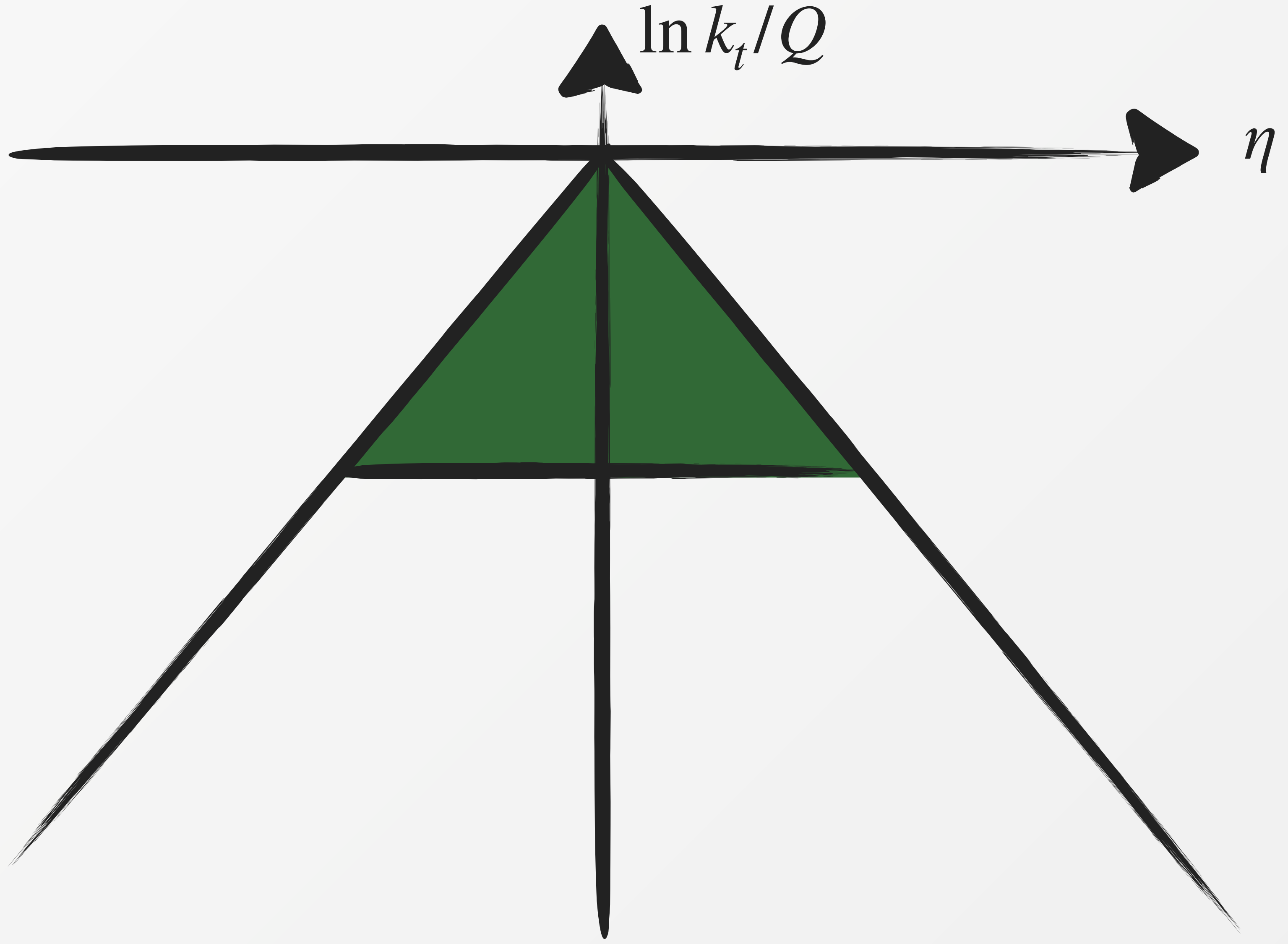
Simple observable

$$v_i = k_{t,i}/M, \quad \zeta_i = v_i/v_1$$

$$\times R'(v_1) \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'(\zeta_i v_1)\, \Theta\left(p_T - |\vec{k}_{t,i} + \cdots \vec{k}_{t,n+1}|\right)$$



Resummation in direct space: RadISH in a nutshell

RadISH performs resummation in direct space - similar in spirit to a parton shower, with control on formal accuracy

Result at NLL accuracy with scale-independent PDFs

$$\sigma(p_T) = \sigma_0 \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} \boxed{e^{-R(\epsilon v_1)}}$$

Simple observable

$$v_i = k_{t,i}/M, \quad \zeta_i = v_i/v_1$$

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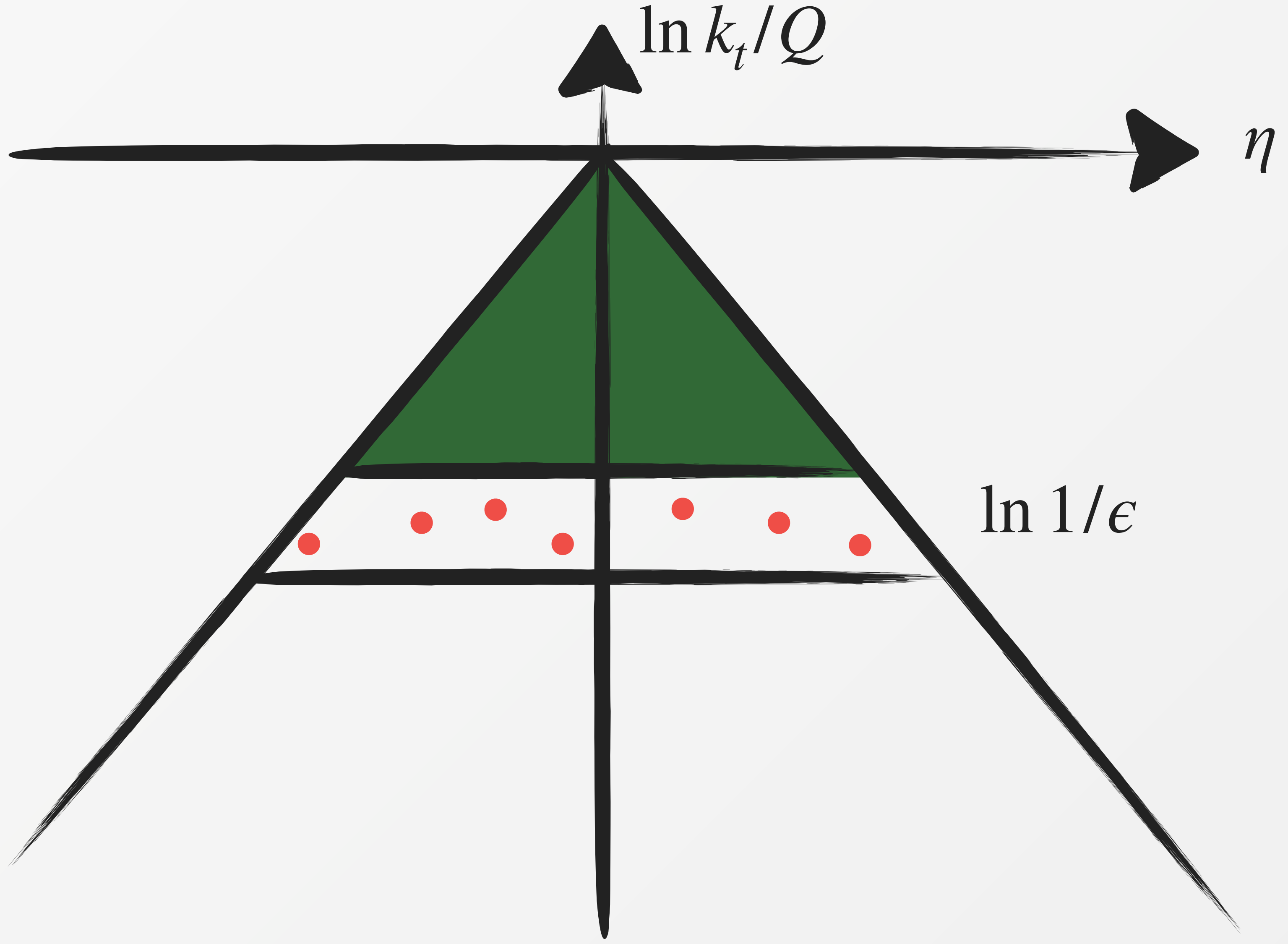
Transfer function



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$$v_i = k_{t,i}/M, \quad \zeta_i = v_i/v_1$$

Formula can be evaluated with Monte Carlo methods; dependence on ϵ vanishes (as $\mathcal{O}(\epsilon)$) and result is **finite** in four dimensions

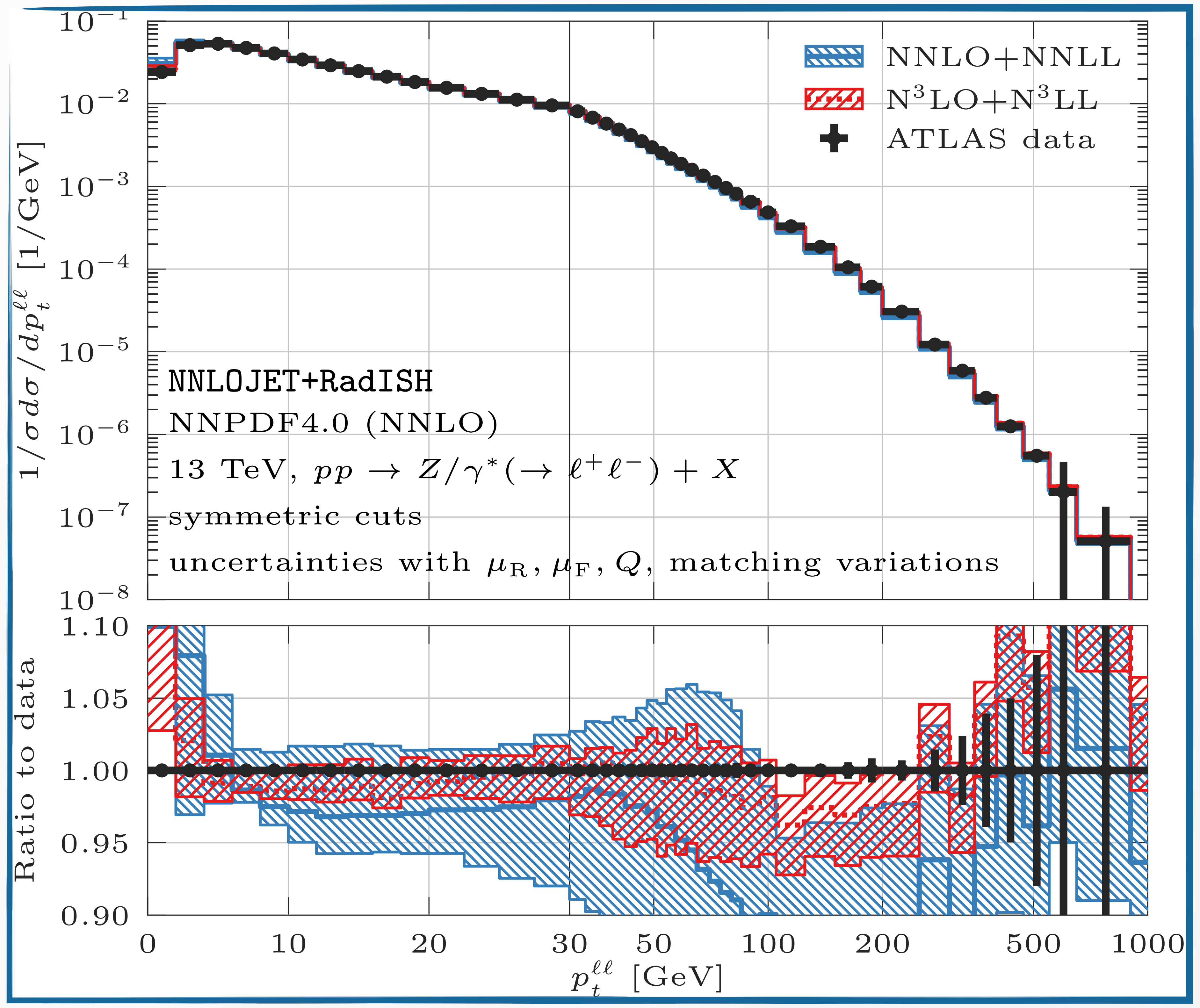
Logarithmic accuracy defined in terms of $\ln(M/k_{t1})$

Result formally equivalent to the b-space formulation [Bizon, Monni, Re, LR, Torrielli '17]

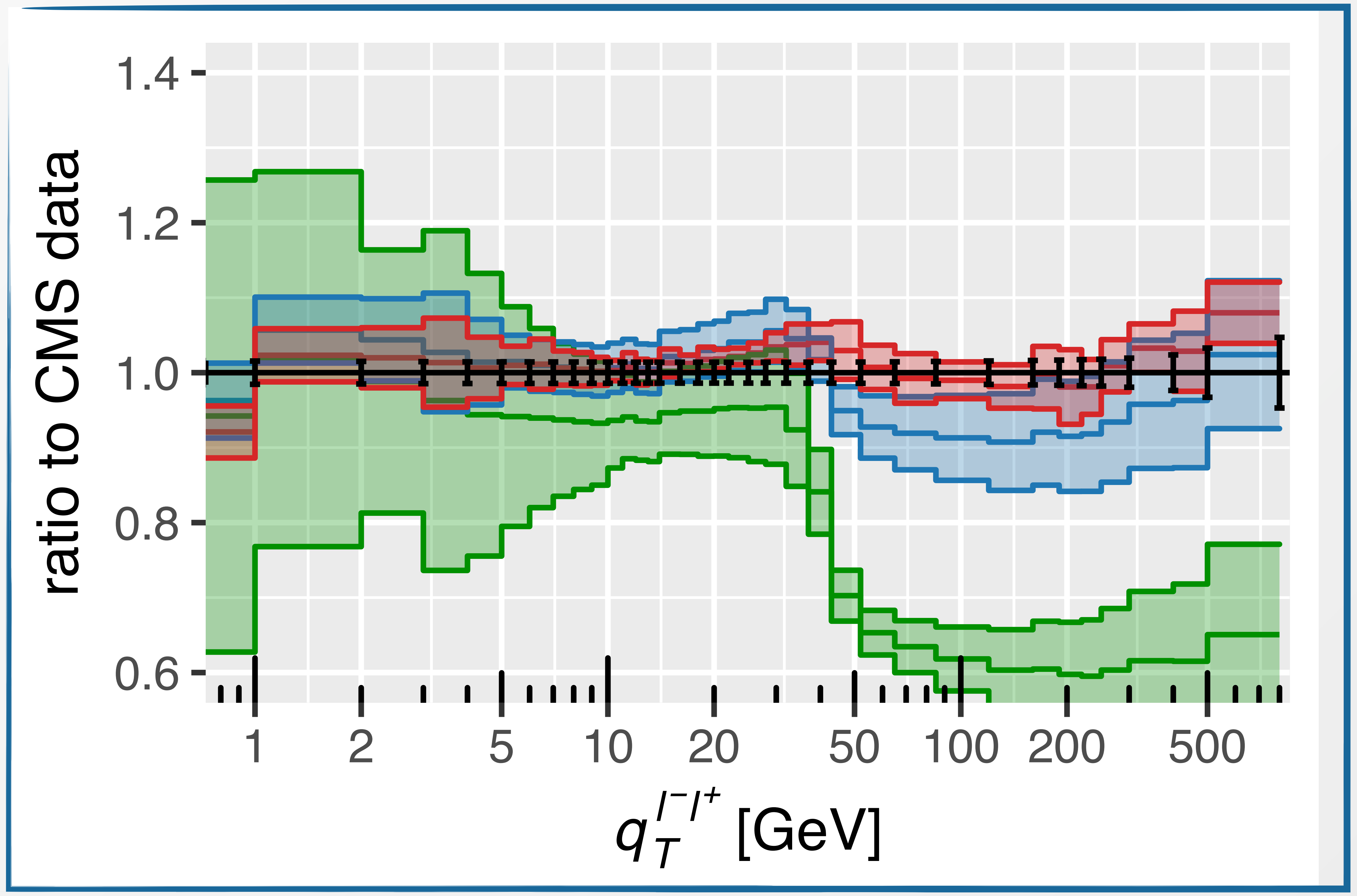
Resummation formalism can be extended at higher accuracies; resummation at N^{3}LL′ available [Re, LR, Torrielli '21]

Description of experimental data at N³LO$_{QCD}$+N³LL'$_{QCD}$

The theoretical progress made in the the past 5 years has **significantly improved** the description of the experimental data, pinning down the theoretical uncertainties to the **few percent level** in the description of differential spectra



[Gehrmann, Glover, Huss, Chen, Monni, Re, LR, Torrielli, 2203.01565]



[Neumann, Campbell '22]

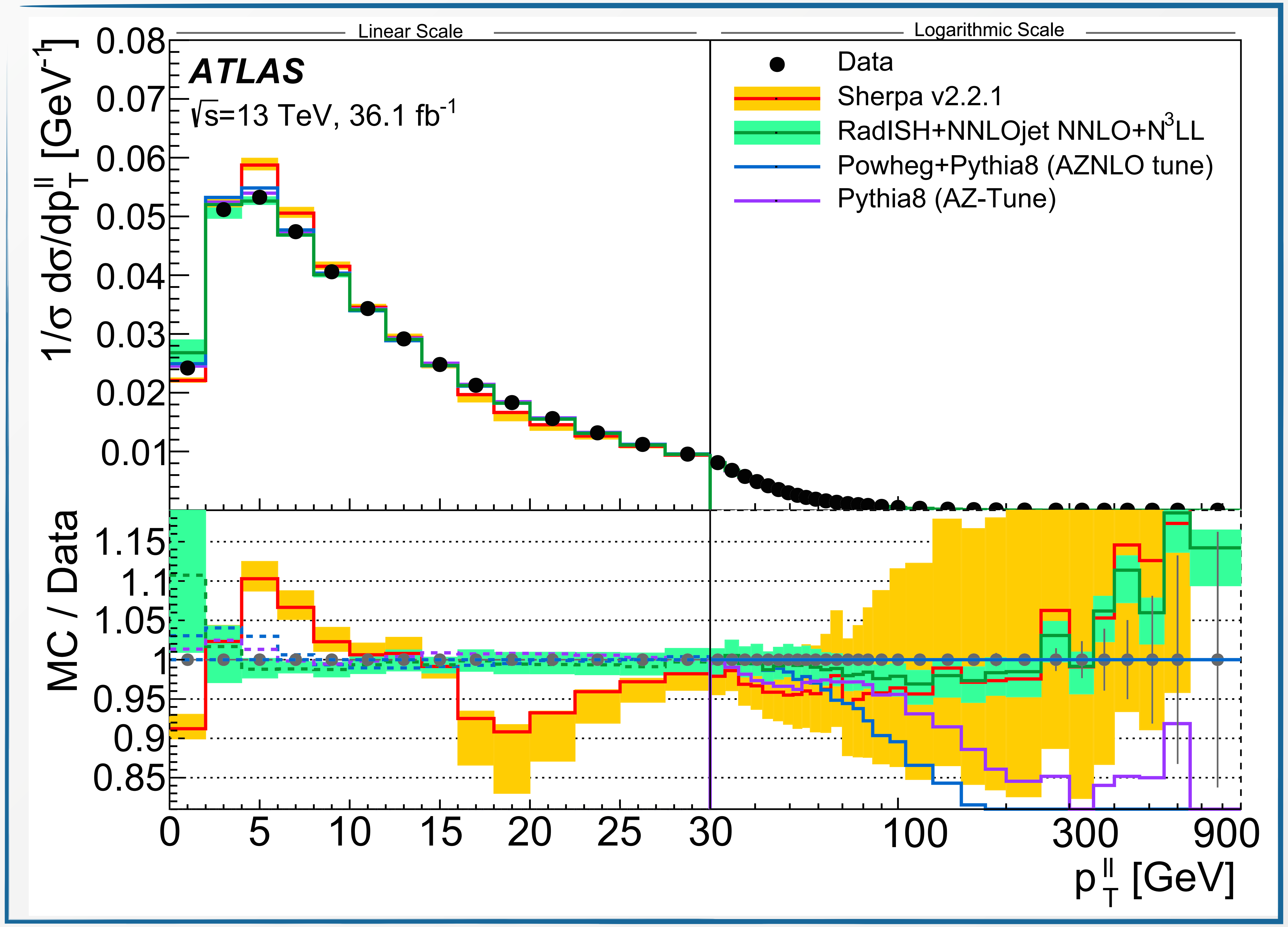
blue: NNLL'$_{QCD}$+NNLO$_{QCD}$

red: N³LL'$_{QCD}$+N³LO$_{QCD}$

Description of experimental data at N^3LO_{QCD}+$N^3LL'_{QCD}$

Theoretical predictions now are capable of describing the data **precisely** across a wide range of scales

green: N^3LL_{QCD}+N^3LO_{QCD}



[ATLAS '20]

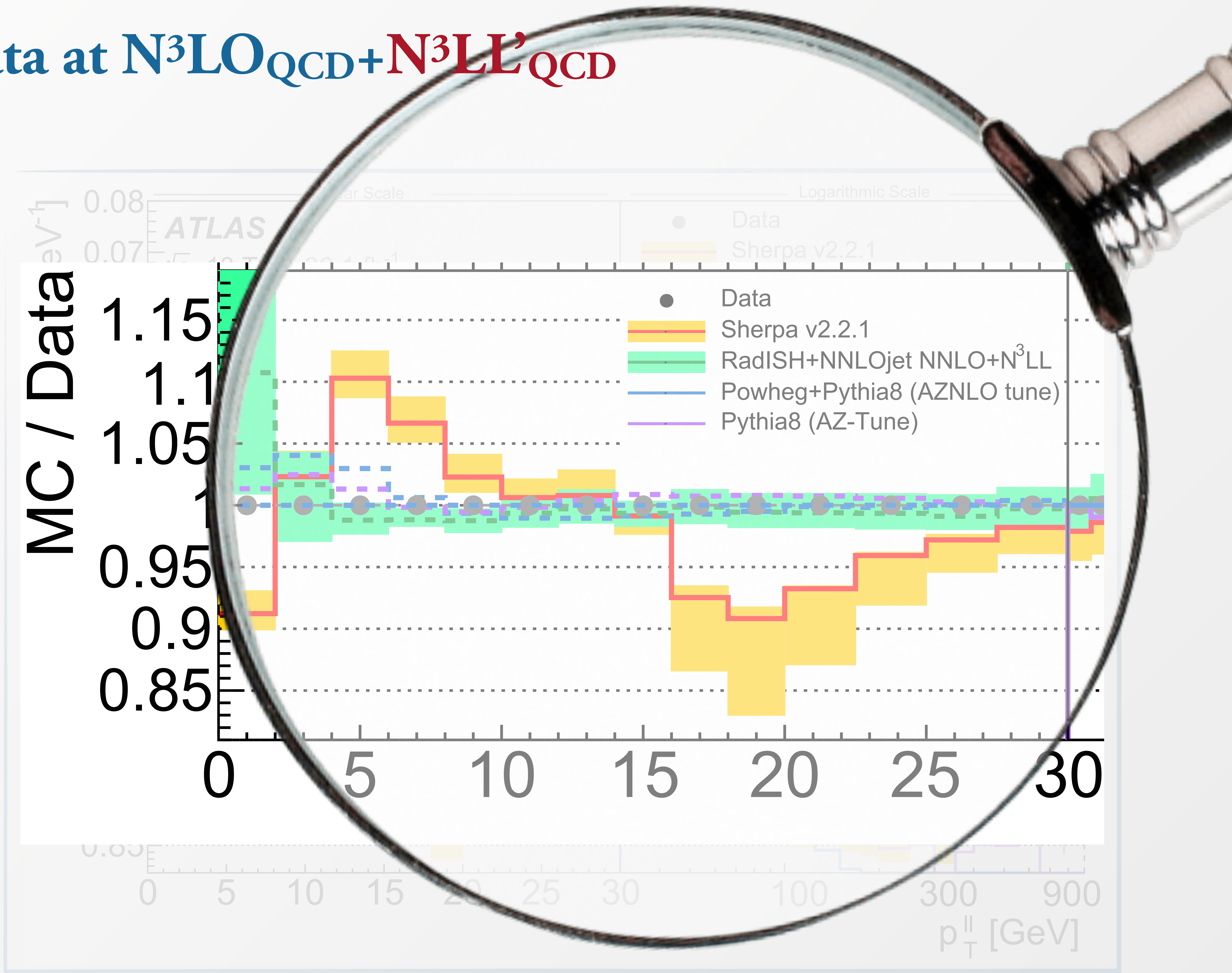
Description of experimental data at N^3LO_{QCD}+$N^3LL'_{QCD}$

Theoretical predictions now are capable of describing the data **precisely** across a wide range of scales

green: N^3LL_{QCD}+N^3LO_{QCD}

And are on par, if not better, than parton showers predictions that have been **tuned to experimental data**

N.B. : `RadISH+NNLOJET` predictions **do not** include any non-perturbative modelling at low q_T



[ATLAS '20]

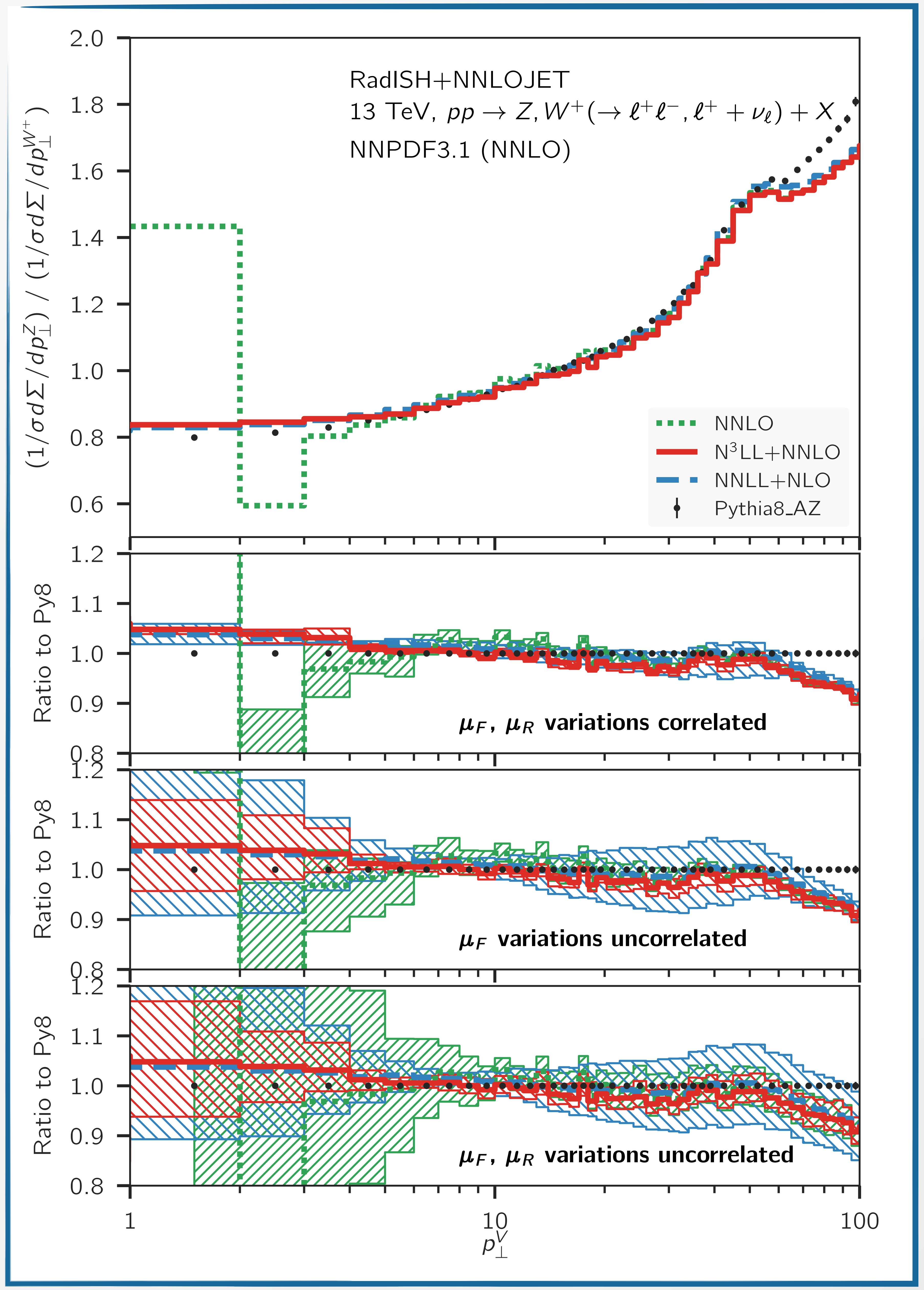
Understanding the Z and W correlations

Thanks to the availability of theoretical prediction at high accuracy, it is possible to assess reliably the behaviour of the perturbative series for crucial observables such as p_T^Z/p_T^W ratio

$$\frac{1}{\sigma^W}\frac{d\sigma^W}{p_\perp^W} \sim \frac{1}{\sigma^Z_{\text{data}}}\frac{d\sigma^Z_{\text{data}}}{p_\perp^Z}\,\frac{\frac{1}{\sigma^W_{\text{theory}}}\frac{d\sigma^W_{\text{theory}}}{p_\perp^W}}{\frac{1}{\sigma^Z_{\text{theory}}}\frac{d\sigma^Z_{\text{theory}}}{p_\perp^Z}}$$

Stability of the ratio indicates **high level of correlation** between the two spectra

Comparison with tuned event generator such as PYTHIA* however indicates that full correlation might be too strong an assumption



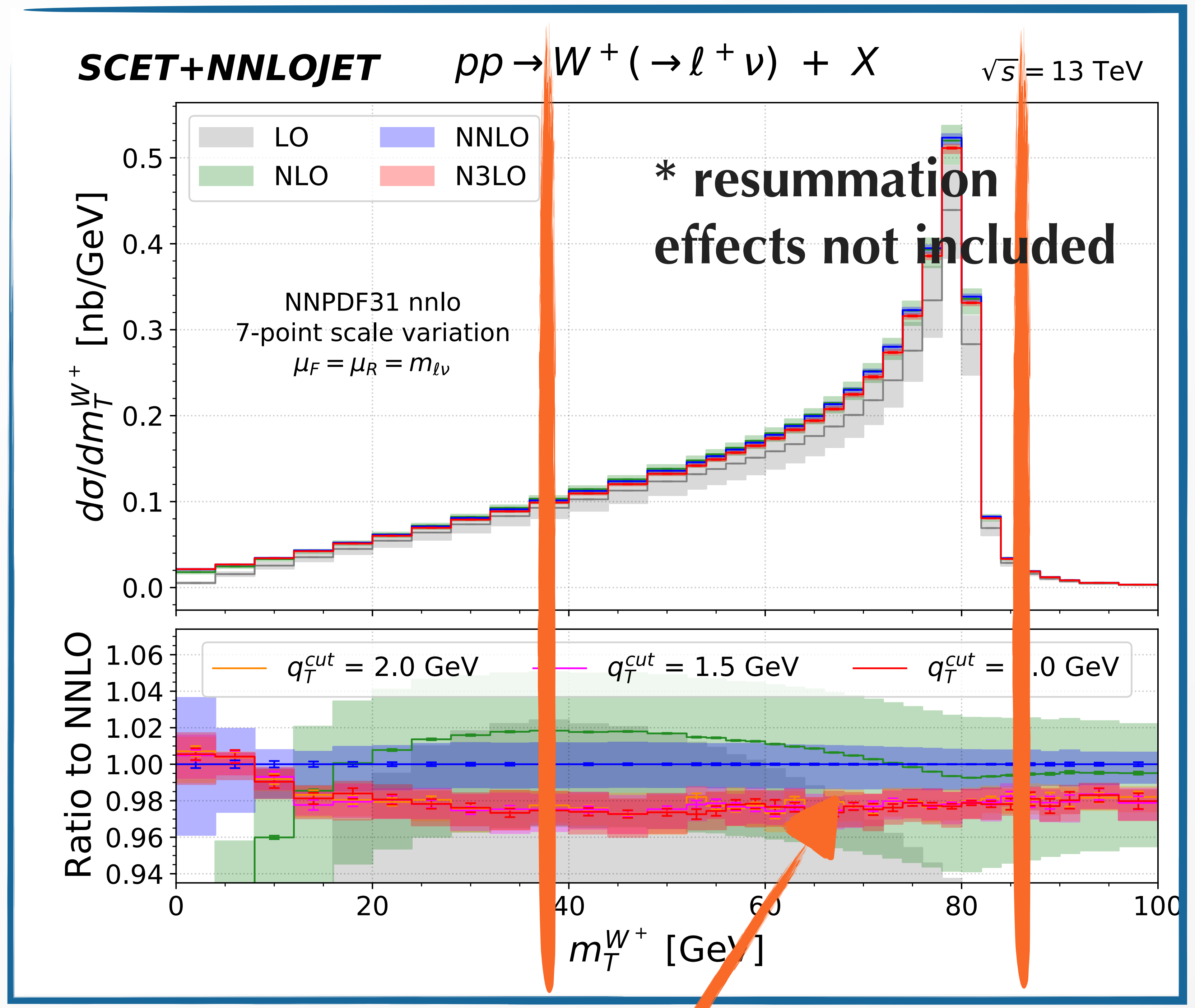
* *"PYTHIA is not QCD"*

[Kirill Melnikov, QCD@LHC 2016]

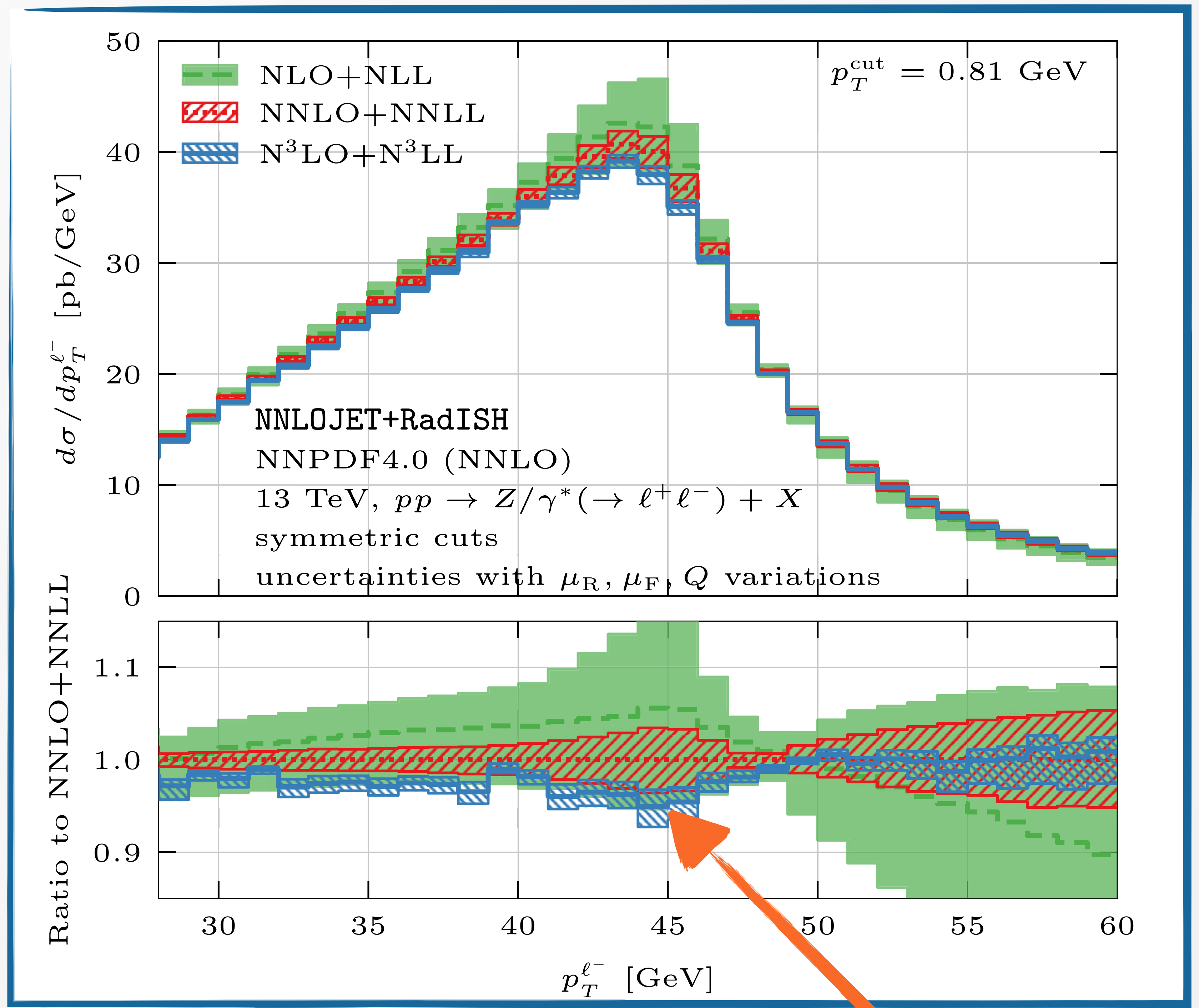
[Bizon, de Ridder, Gehrmann, Glover, Huss, Chen, Monni, Re, LR, Walker, 1905.05171]

Control of the differential distributions in DY production

Shape of differential spectra is affected by higher order predictions



[Gehrmann, Glover, Huss, Chen, Yang, Zhu 2205.11426]



[Gehrmann, Glover, Huss, Chen, Monni, Re, LR, Torrielli, 2203.01565]

Residual uncertainties at **N³LO$_{QCD}$** are at the $\mathcal{O}(1-2\%)$ level

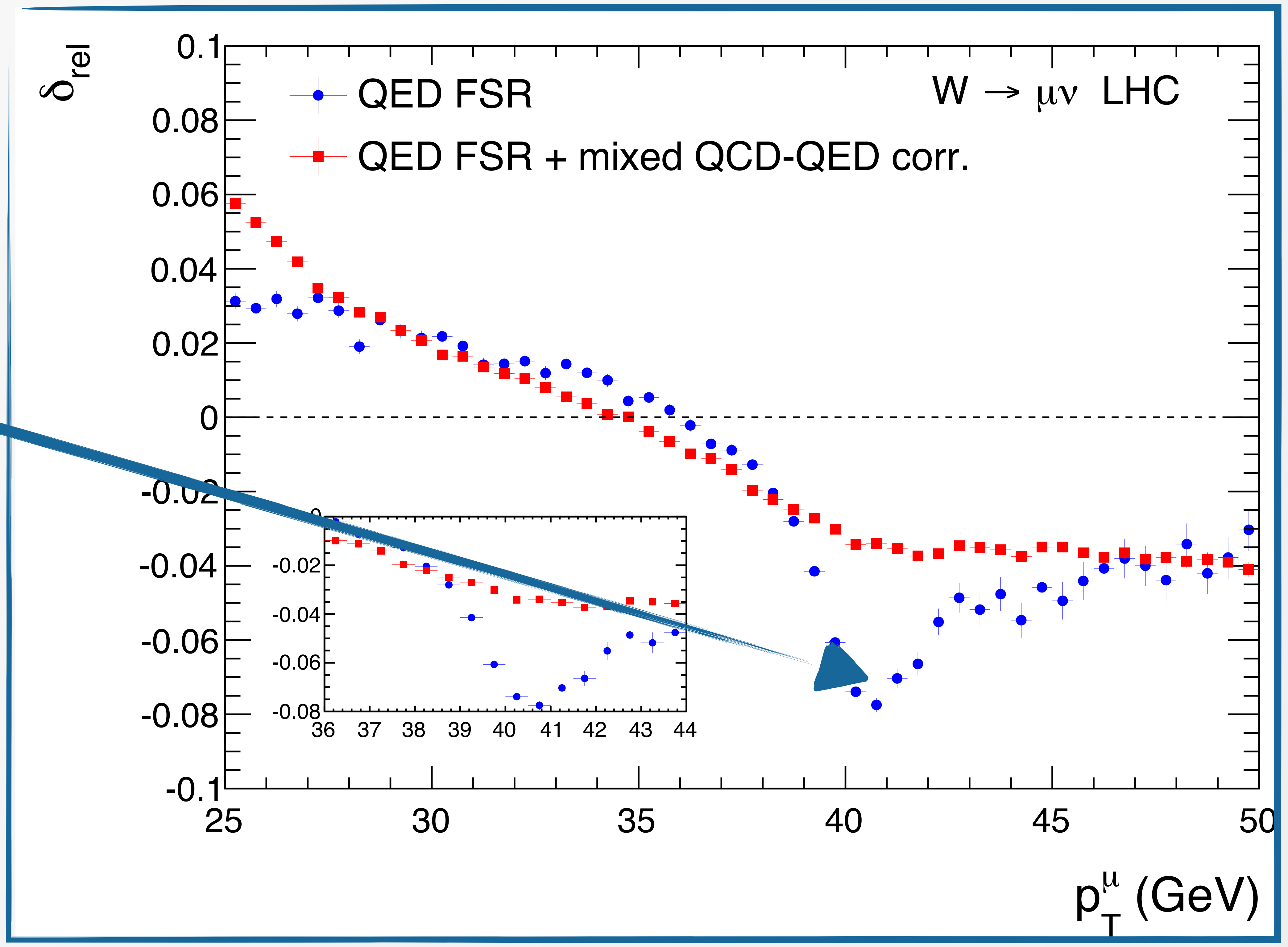
Impact of **N³LO$_{QCD}$** corrections relatively flat in the fit window for m_T

N³LL′$_{QCD}$+N³LO$_{QCD}$ modifies the shape after the Jacobian peak for p_T^{ℓ}

Interplay of QCD and EW corrections further modify the shape of the differential distributions

Impact of QED and mixed QCD×QED corrections

Both p_T^ℓ and m_T features large radiative corrections due to **QED final state radiation** at the **jacobian peak**



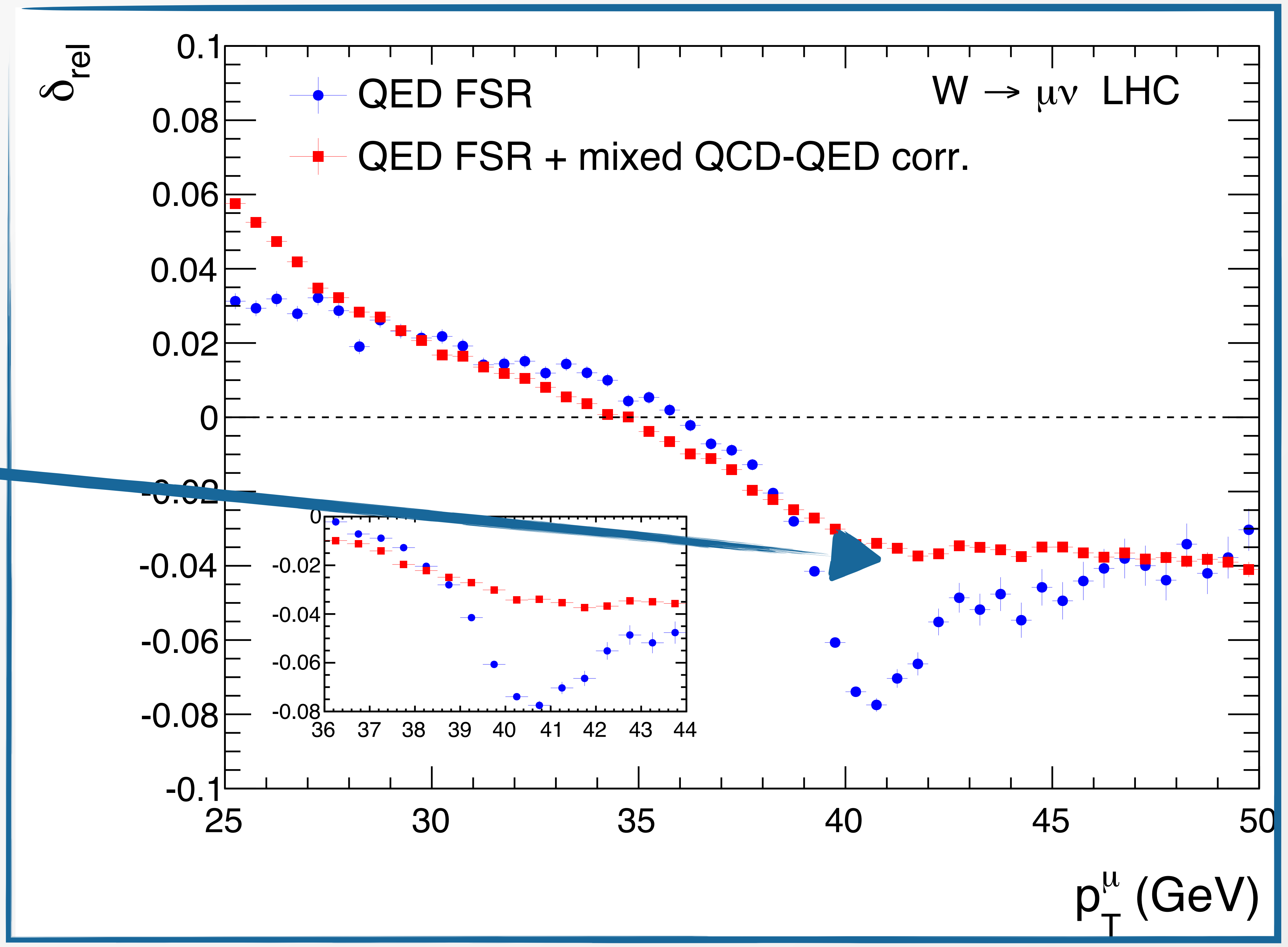
[Carloni Calame, Chiesa, Martinez, Montagna, Nicrosini, Piccinini, Vicini 1612.02841]

Impact of QED and mixed QCD×QED corrections

Both p_T^ℓ and m_T features large radiative corrections due to **QED final state radiation** at the **jacobian peak**

The precise shape of p_T^ℓ at the Jacobian peak is determined by the **interplay of QCD and QED corrections**

Impact of mixed corrections minor for m_T



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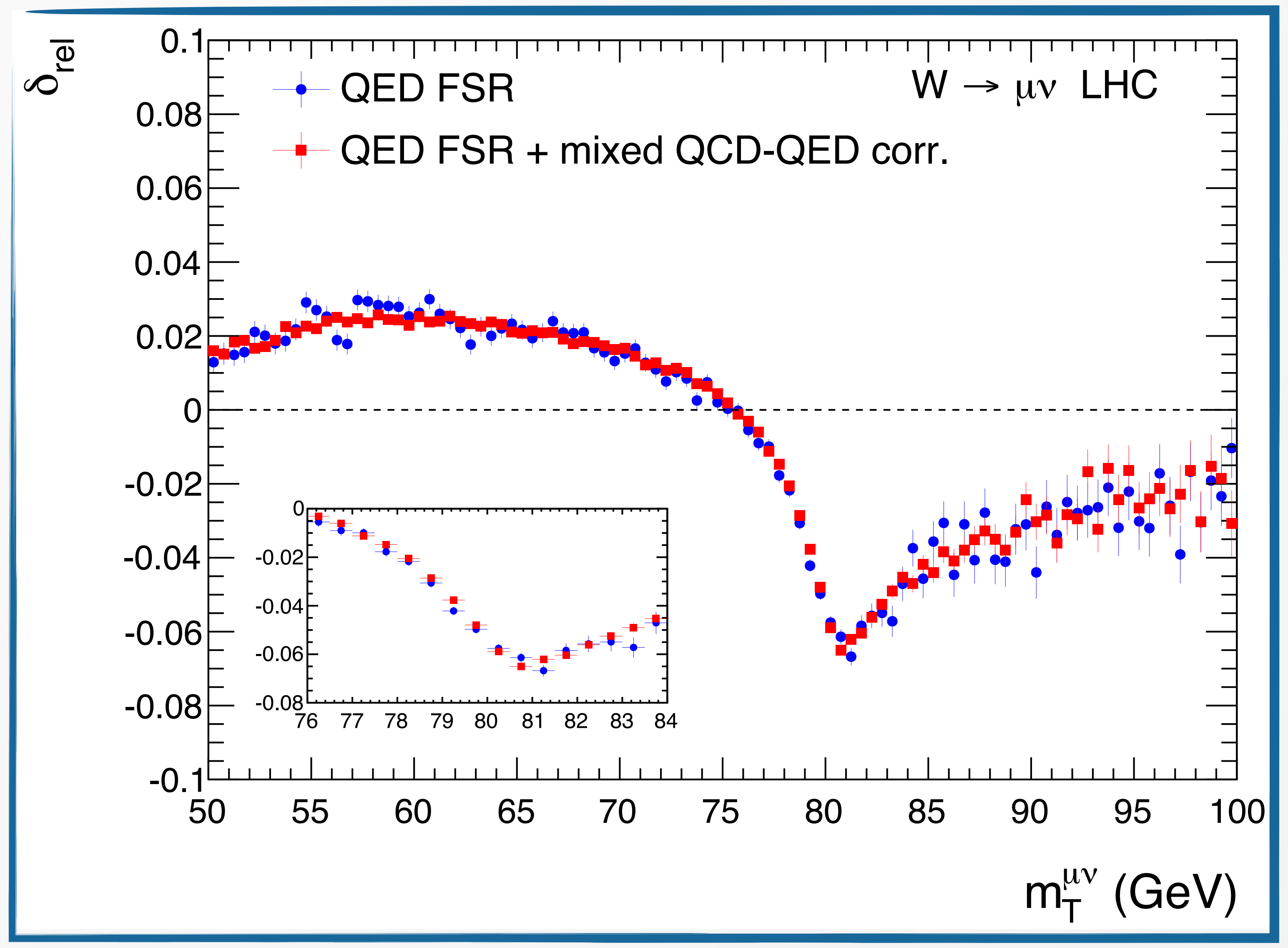


Impact of QED and mixed QCD×QED corrections

Largest shifts induced by **QED FSR**

Subleading EW effects induce **few MeV** shifts

$pp \to W^+$, $\sqrt{s} = 14$ TeV Templates accuracy: LO Pseudo–data accuracy	M_W shifts (MeV) $W^+ \to \mu^+\nu$ M_T	 p_T^ℓ	 $W^+ \to e^+\nu$ M_T	 p_T^ℓ
1 HORACE only FSR-LL at $\mathcal{O}(\alpha)$	-94±1	-104±1	-204±1	-230±2
2 HORACE FSR-LL	-89±1	-97±1	-179±1	-195±1
3 HORACE NLO-EW with QED shower	-90±1	-94±1	-177±1	-190±2
4 HORACE FSR-LL + Pairs	-94±1	-102±1	-182±2	-199±1
5 PHOTOS FSR-LL	-92±1	-100±2	-182±1	-199±2

$p\bar{p} \to W^+$, $\sqrt{s} = 1.96$ TeV Templates accuracy: NLO-QCD+QCD$_{PS}$ Pseudodata accuracy	 QED FSR	M_W shifts (MeV) $W^+ \to \mu^+\nu$ M_T	 p_T^ℓ	 $W^+ \to e^+\nu$(dres) M_T	 p_T^ℓ
1 NLO-QCD+(QCD+QED)$_{PS}$	PYTHIA	-91±1	-308±4	-37±1	-116±4
2 NLO-QCD+(QCD+QED)$_{PS}$	PHOTOS	-83±1	-282±4	-36±1	-114±3
3 NLO-(QCD+EW)-two-rad+(QCD+QED)$_{PS}$	PYTHIA	-86±1	-291±3	-38±1	-115±3
4 NLO-(QCD+EW)-two-rad+(QCD+QED)$_{PS}$	PHOTOS	-85±1	-290±4	-37±2	-113±3

[Carloni Calame, Chiesa, Martinez, Montagna, Nicrosini, Piccinini, Vicini 1612.02841]

Analyses do include the bulk of the QCD×QED corrections

The impact on the m_W shifts of the mixed QCD×QED corrections strongly depends on the underlying QCD model

Note: in this approach non-factorizable contributions are neglected

Progress in mixed QCD×EW corrections

Complete set of corrections to neutral and charged current Drell-Yan production recently obtained by two groups

NNLO QCD-EW corrections to charged-current DY (2-loop contributions in pole approximation).
[Buonocore, Grazzini, Kallweit, Savoini, Tramontano 2102.12539]

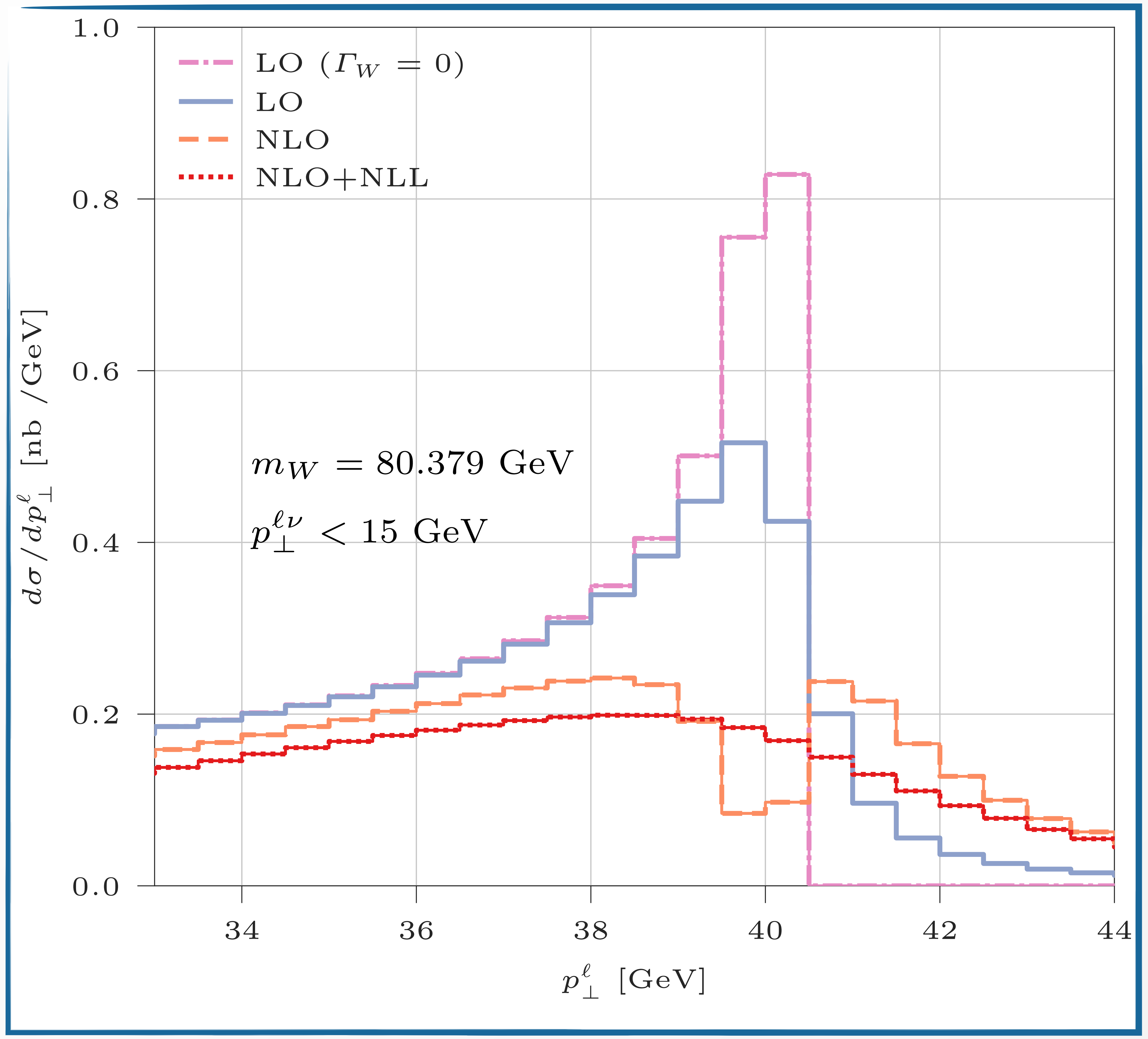
exact NNLO QCD-EW corrections to neutral-current DY
[Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini, 2106.11953]
[Armadillo, Bonciani, Devoto, Rana, Vicini 2201.01754]
[Buccioni, Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, Röntsch, Signorile-Signorile 2203.11237]

Impact of mixed $\mathcal{O}(\alpha_s\alpha)$ corrections estimated to be potentially relevant for $\mathcal{O}(10\,\mathrm{MeV})$ extraction at the LHC
[Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch 2103.02671]

Matching of such corrections to **QCD and QED all-order resummation** of high relevance for accurate and precise analysis of the p_T^ℓ distribution

Combination of QCD+QED resummation so far available only for Z/W production without decays
[Autieri, Cieri, Ferrera, Sborlini ’18, ’23]

The lepton transverse momentum distribution in CC DY



The lepton transverse momentum distribution features a **Jacobian peak** at $p_T^\ell \sim m_W/2$

At **LO**, in the narrow width approx., the distribution features a kinematical endpoint at $m_W/2$

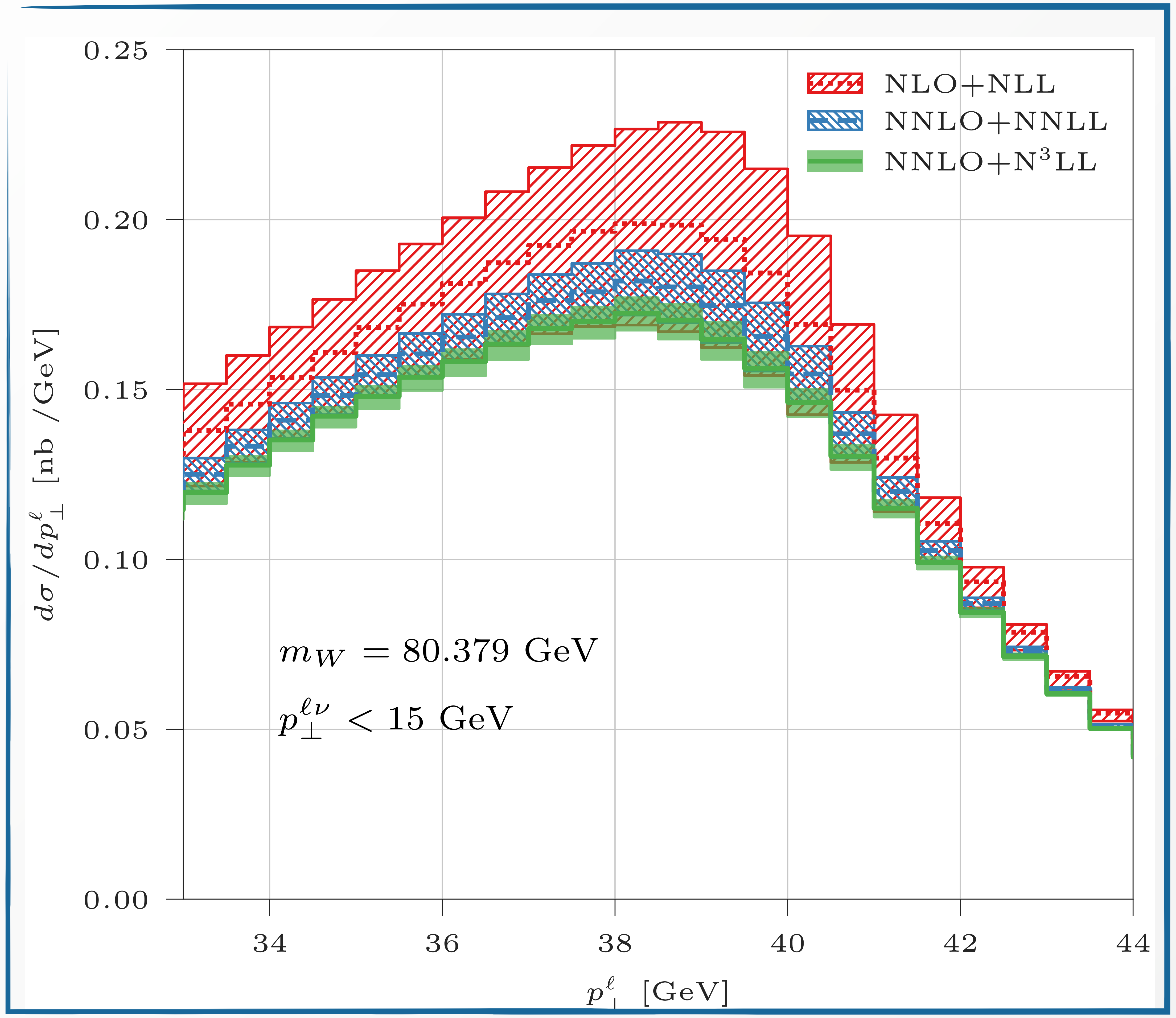
Width effects broaden the distribution above $m_W/2$

Beyond LO, sensitivity to **soft radiation** creates unphysical instabilities around $m_W/2$ in fixed-order computations [Catani, Webber '97]

All-order resummation effects cure such instabilities ad provide **physical prediction**

Presence of the endpoint makes the distribution particularly sensitive to m_W

The lepton transverse momentum distribution in CC DY



[LR, P. Torrielli, A. Vicini '23]

Progress in theoretical computations allows for a **precise theoretical description** of the distribution, with $\mathcal{O}(2\%)$ residual uncertainties

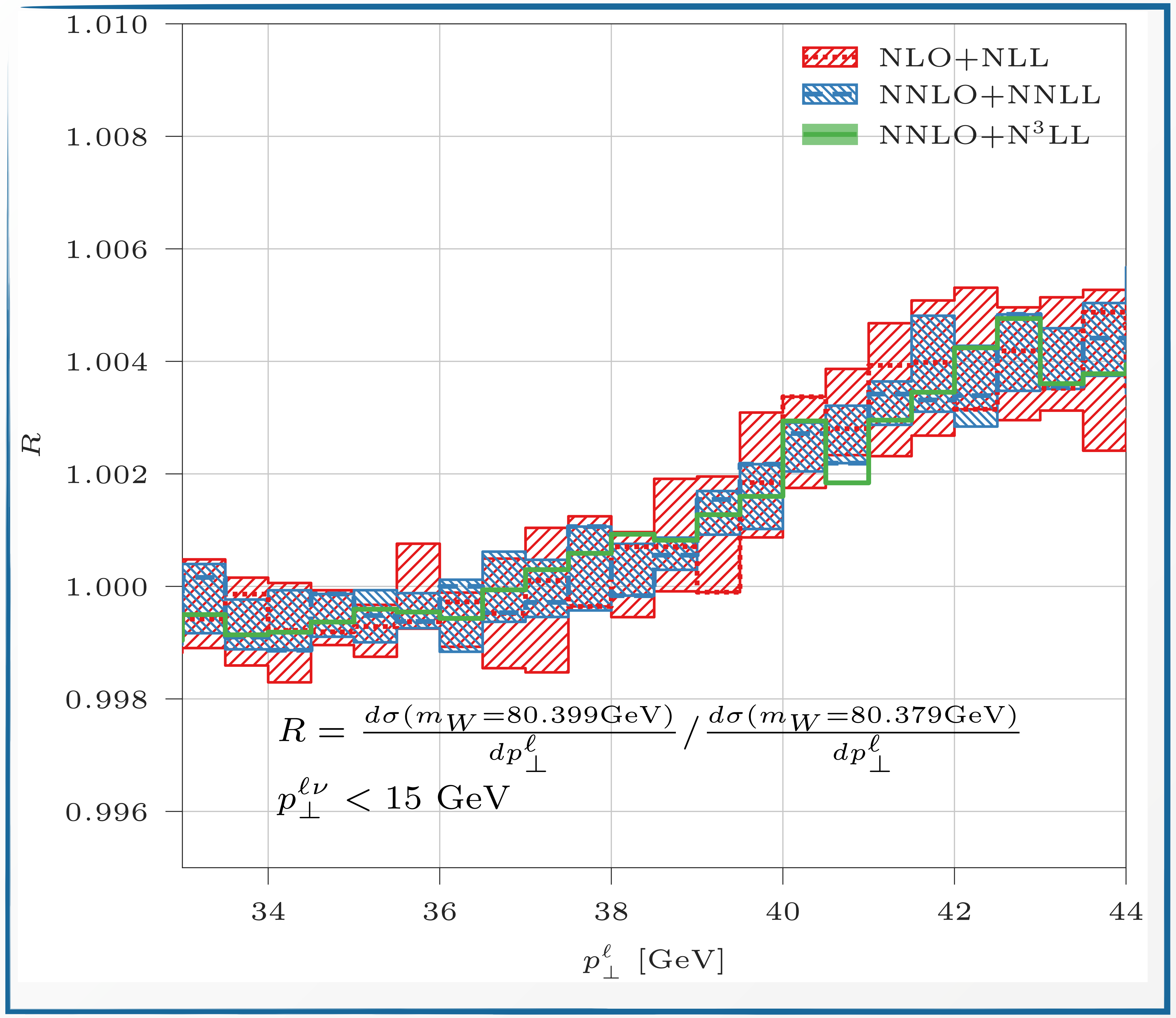
Uncertainty band encodes canonical 7-point scale variation envelope and resummation scale variation for central scales (total: 9-point envelope)

$$\mu_{F,R} = \xi_{F,R}\sqrt{m_{\ell\nu}^2 + p_{\perp,\ell\nu}^2} \qquad Q = \xi_Q m_{\ell\nu}$$

$$\xi_{F,R} \in \{1/2,1,2\}, \qquad \xi_Q \in \{1/4,1/2,1\}$$

When width and resummation effects are included, the peak is located at ~38.5 GeV

The lepton transverse momentum distribution in CC DY



[LR, P. Torrielli, A. Vicini '23]

Determination of m_W with $\mathcal{O}(10\,\text{MeV})$ precision requires control of the shape at the few permille level

Distortion of the shape largely independent of the accuracy or scale choice in pure QCD

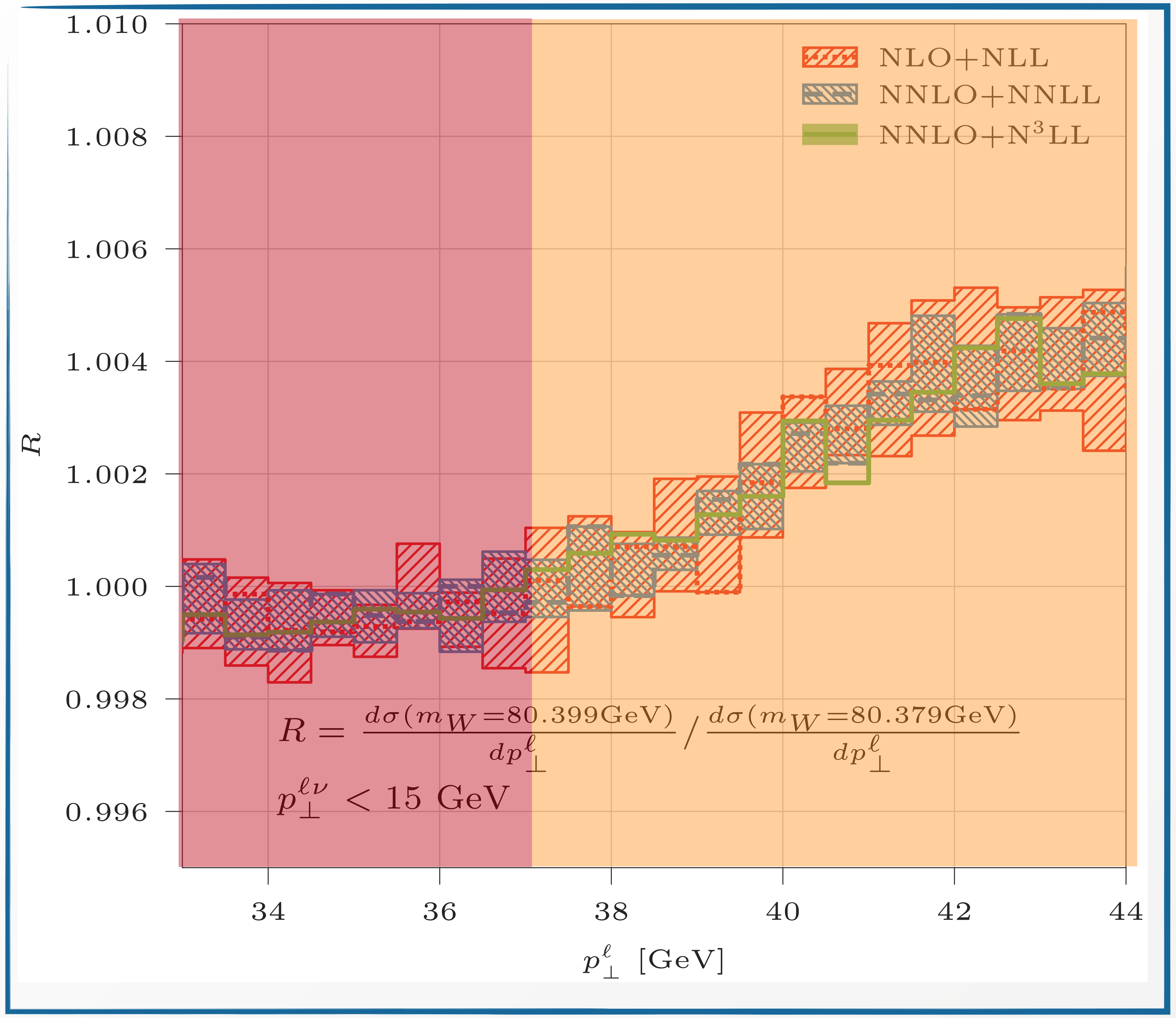
Sensitivity to m_W related to propagation and decay of the W boson

Consequence of the factorisation between production (subject to QCD effect) and propagation and decay

Sensitivity to m_W **independent** on the QCD approximation

Uncertainty on m_W instead related to the QCD approximation

The lepton transverse momentum distribution in CC DY



[LR, P. Torrielli, A. Vicini '23]

Sensitivity on m_W of the bins of the p_T^ℓ distribution can be quantified by means of the **covariance matrix** with respect to m_W variations

$$C_{ij} = \langle \sigma_i \sigma_j \rangle - \langle \sigma_i \rangle \langle \sigma_j \rangle$$

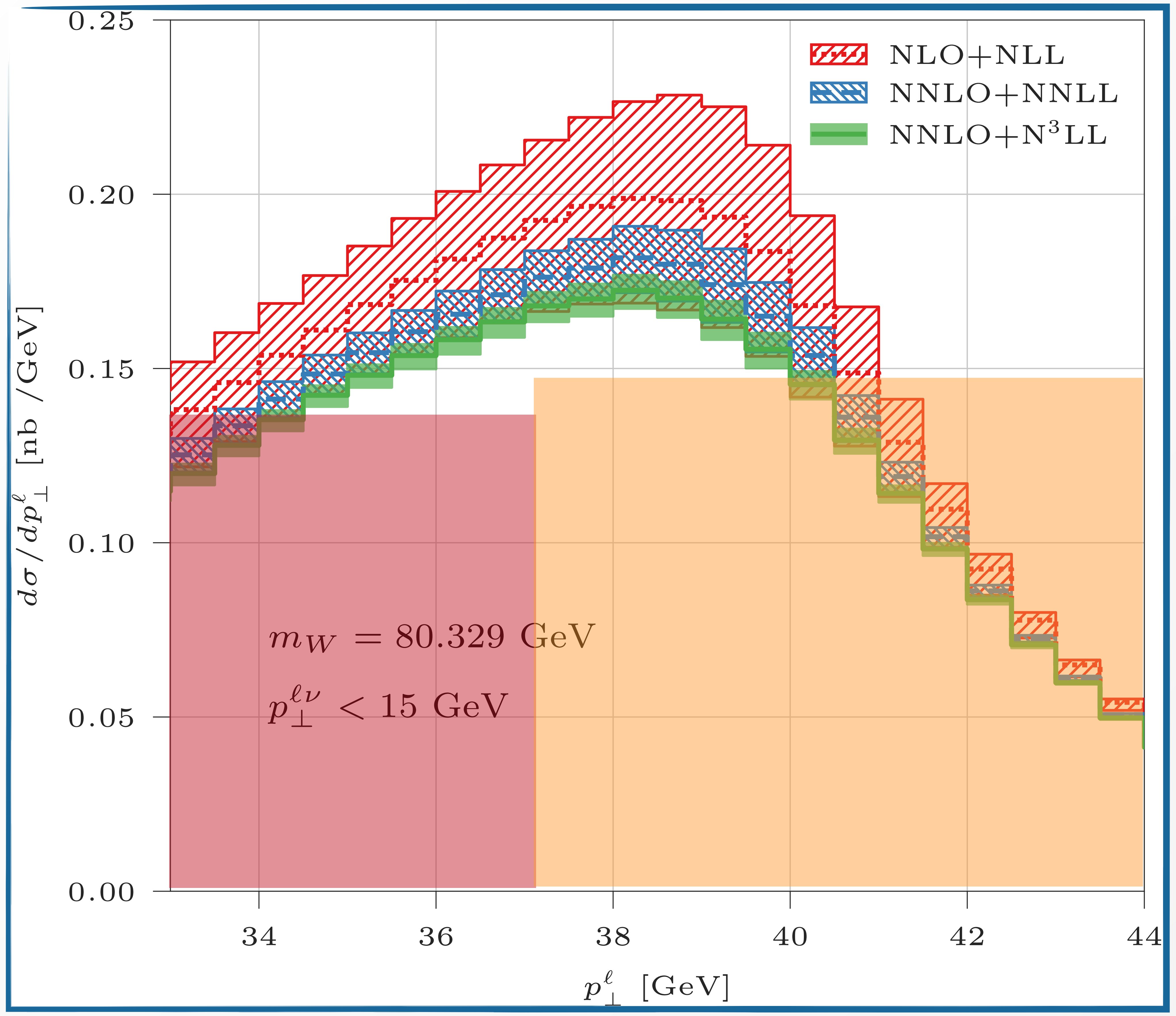
The eigenvalues of this matrix express sensitivity on m_W on **linear combinations of bins** of the distribution

Large hierarchy between the first eigenvalue and the others, suggesting that the majority of the sensitivity is captured by the largest eigenvalue

Coefficients **changes sign** around $p_T^\ell \simeq 37$ GeV

Following this indication, we design a **new observable**

The jacobian asymmetry $\mathscr{A}_{p_\perp^\ell}$



[LR, P. Torrielli, A. Vicini '23]

$$L = \int_{p^\ell_{\perp,\text{min}}}^{p^\ell_{\perp,\text{mid}}} dp_\perp^\ell \frac{d\sigma}{dp_\perp^\ell}$$

$$U = \int_{p^\ell_{\perp,\text{mid}}}^{p^\ell_{\perp,\text{max}}} dp_\perp^\ell \frac{d\sigma}{dp_\perp^\ell}$$

$$\mathscr{A}(p^\ell_{\perp,\text{min}}, p^\ell_{\perp,\text{mid}}, p^\ell_{\perp,\text{max}}) = \frac{L_{p_\perp^\ell} - U_{p_\perp^\ell}}{L_{p_\perp^\ell} + U_{p_\perp^\ell}}$$

Scalar observable (i.e. it is measurable via counting) which depends only on the edges of the two defining bins

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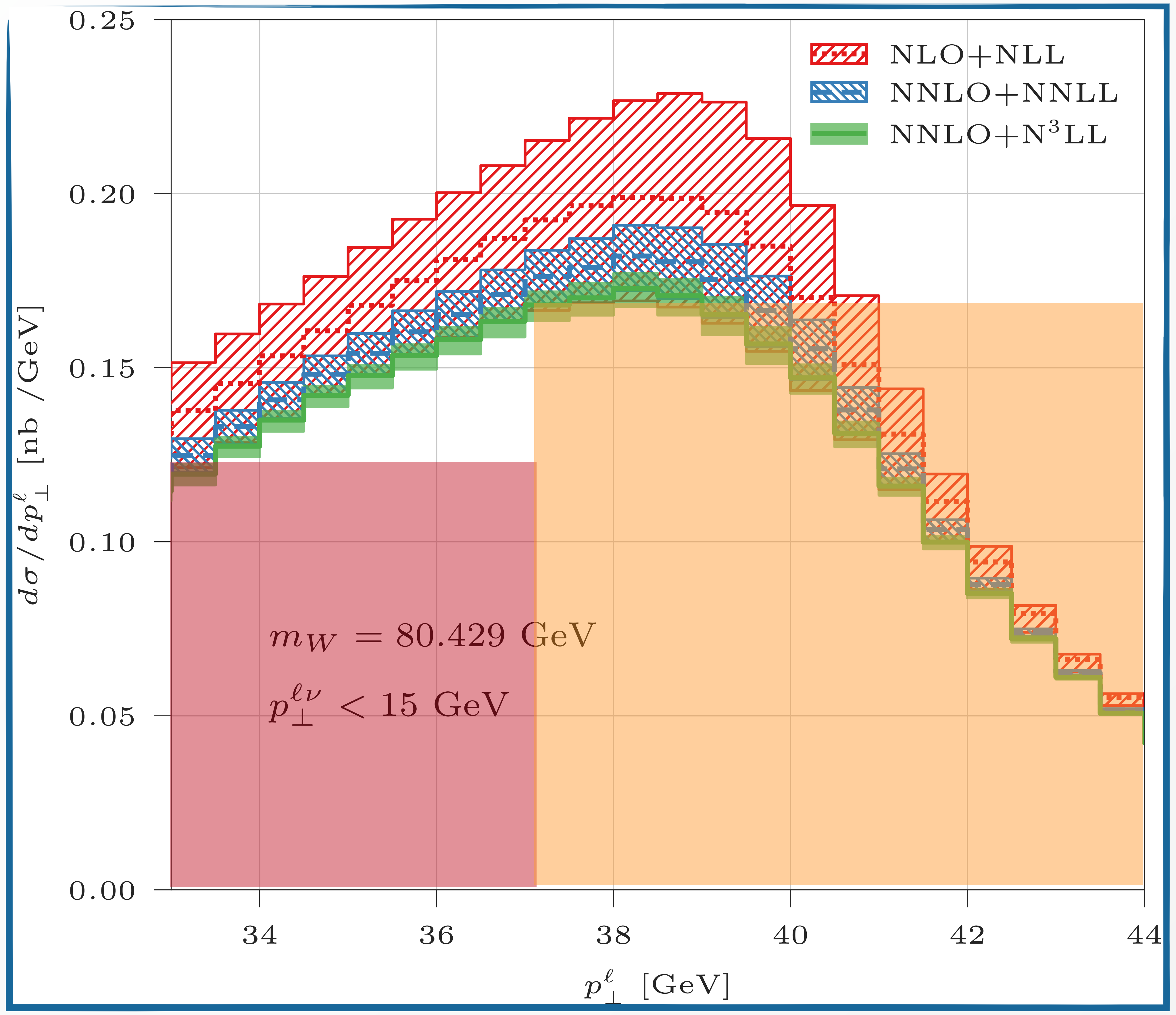

[LR, P. Torrielli, A. Vicini '23]

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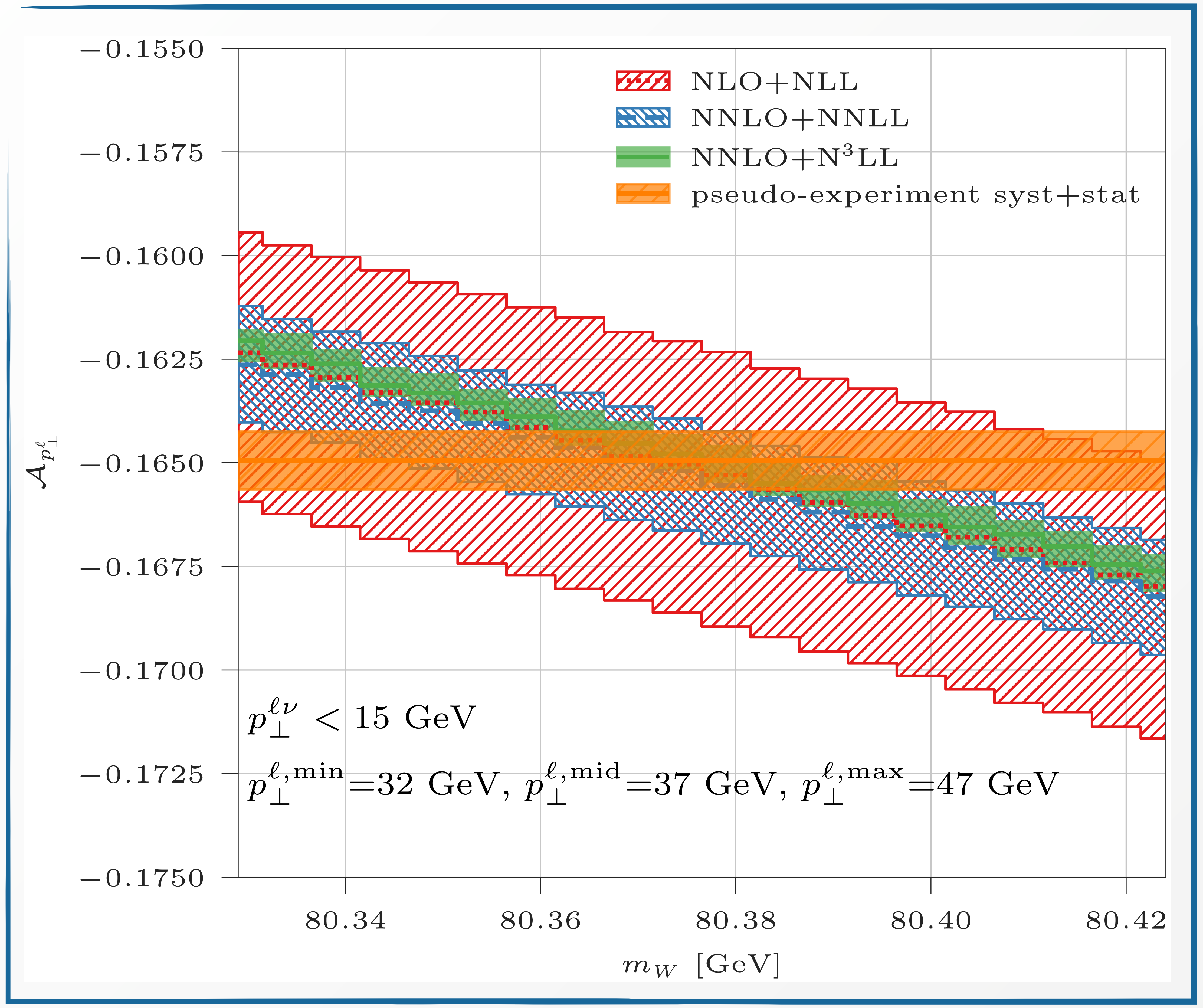
Scalar observable (i.e. it is measurable via counting) which depends only on the edges of the two defining bins

Increasing m_W shifts the peak to the right

Orange bin gets more populated → asymmetry decreases

The jacobian asymmetry $\mathcal{A}_{p_\perp^\ell}$ and m_W



[LR, P. Torrielli, A. Vicini '23]

The asymmetry features a linear dependence on m_W, which stems from the linear dependence of the end-point position in the $p_\perp^\ell$ distribution

Sensitivity to m_W expressed through the slope in each $(p_{\perp,\min}^\ell, p_{\perp,\text{mid}}^\ell, p_{\perp,\max}^\ell)$ window

Slope **independent** on the QCD approximation

Bin size $\mathcal{O}(10\,\text{GeV})$ has threefold advantage

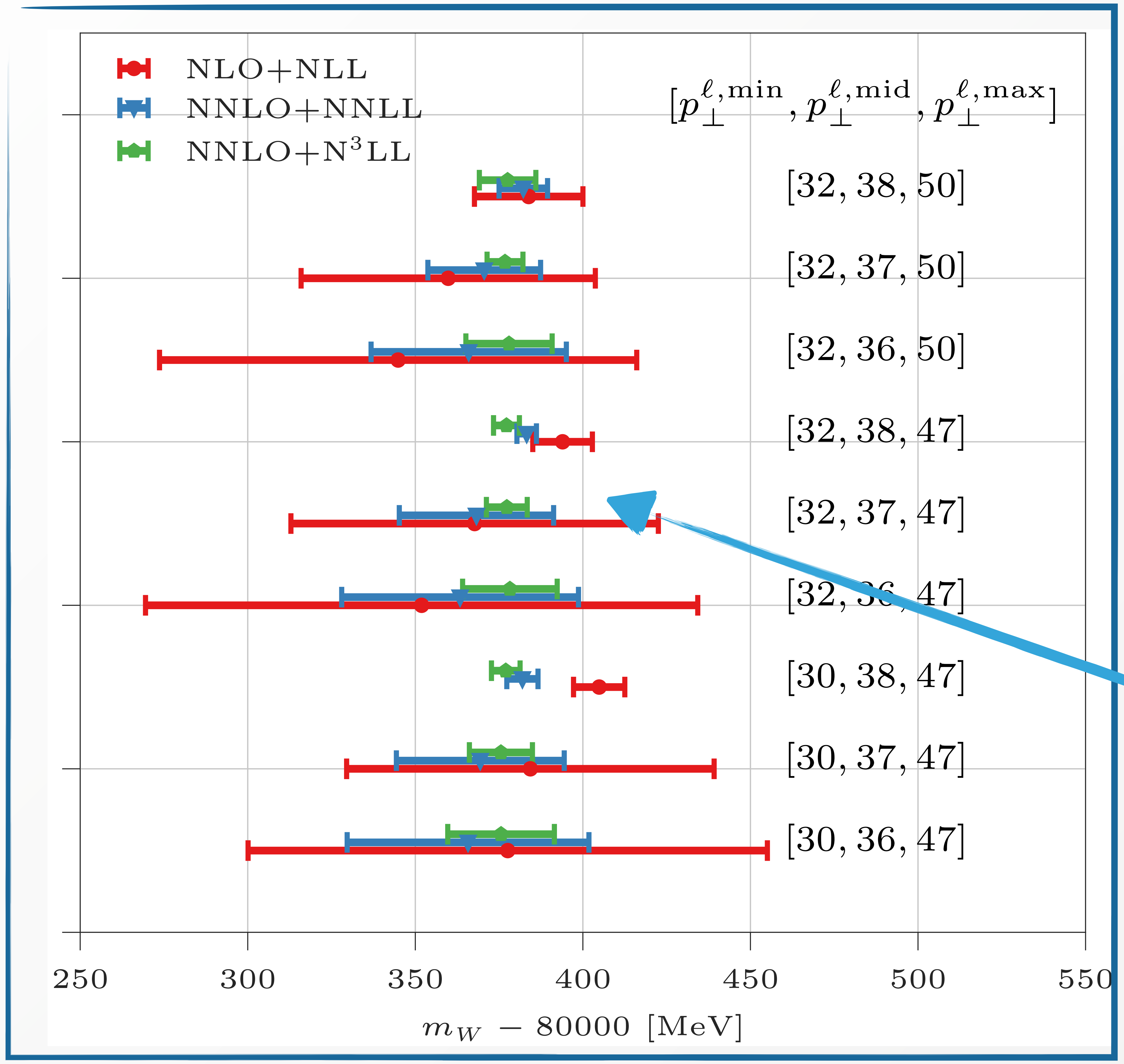
1. Small statistical error
2. Perturbative stability of the QCD result
3. Unfolding to particle level viable

Experimental result and theoretical predictions can be directly compared by looking at the intersection between the lines

The main systematics on the two fiducial cross sections is related to the lepton momentum scale resolution

Determination at the ±15 MeV level from the experimental side seems possible ($\delta A_{p_\perp^\ell}$=0.0007 with 140 fb^{-1} and 0.001 systematic error on U, L)

The jacobian asymmetry $\mathscr{A}_{p_\perp^\ell}$ and its theoretical uncertainty



[LR, P. Torrielli, A. Vicini '23]

For each interval choice the QCD scale-variation band determines a given m_W interval

N^{3}LL corrections play an important role in reducing uncertainty band

We check the convergence order-by-order. If we observe convergence the size of the m_W interval provides an estimate of the QCD uncertainty

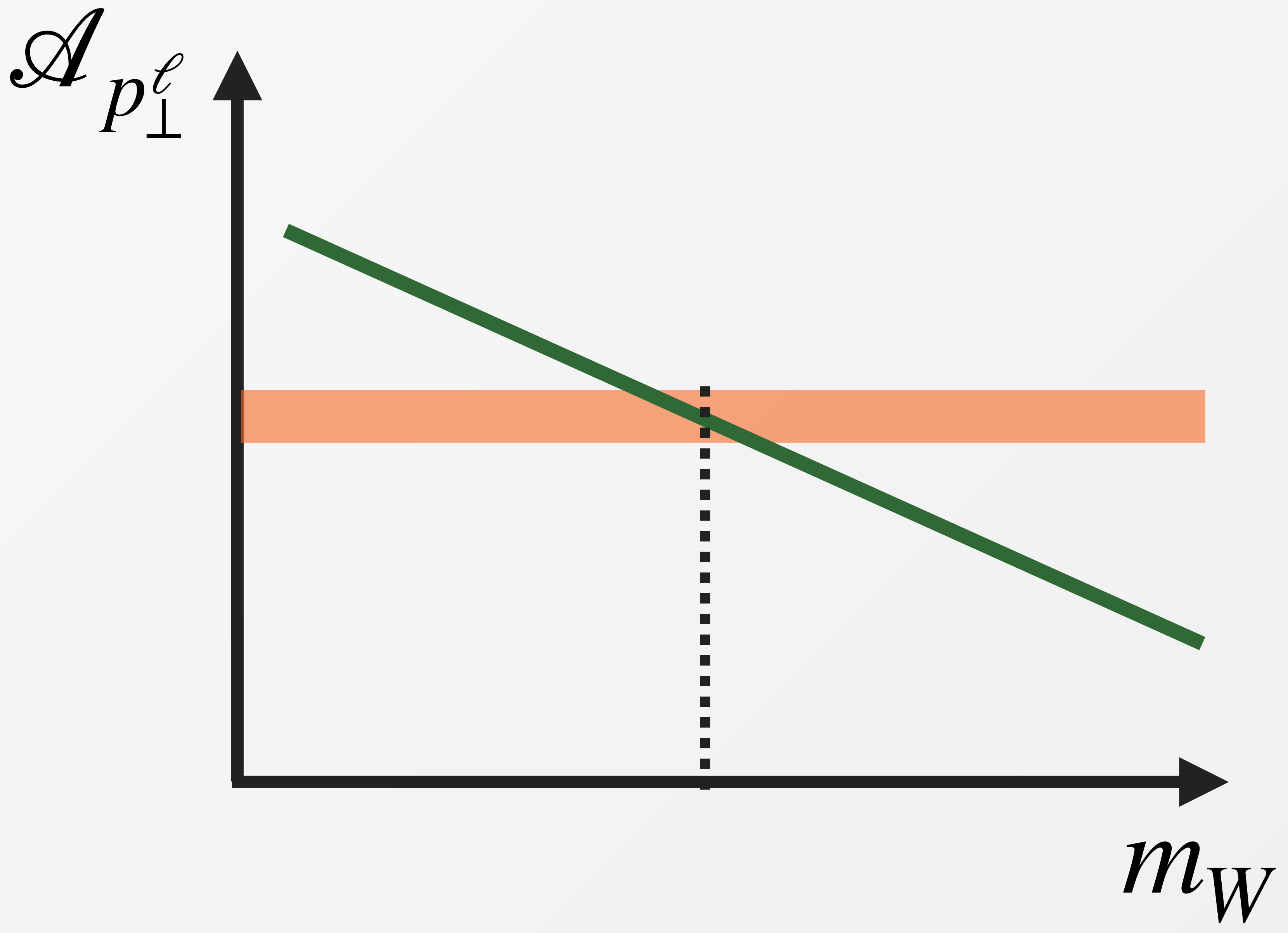
A perturbative QCD uncertainty at the ± 5 MeV level is achievable **using CC DY data alone**

The choice of the midpoint is important to identify two regions with excellent QCD convergence (see regions with $p^\ell_{\perp,\mathrm{mid}} = 38$ GeV)

The jacobian asymmetry $\mathscr{A}_{p_\perp^\ell}$: additional effects and uncertainties

Excellent convergence properties of the asymmetry in perturbative QCD are a good starting point to discuss additional effects we did not include:

Impact on the central m_W value of
- missing perturbative corrections (QED, QCDxEW)
- non-perturbative effects



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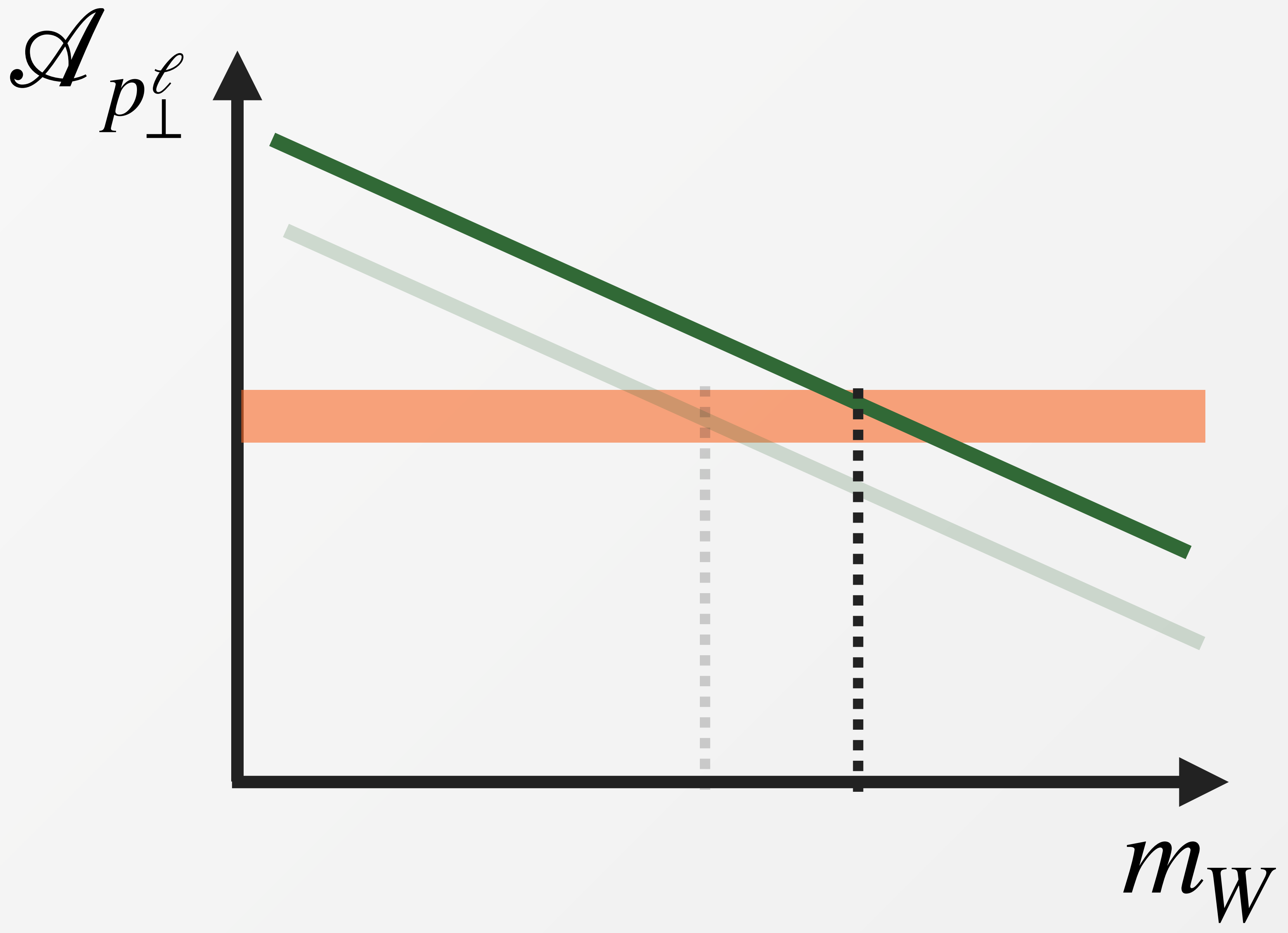
Each effect yields a vertical offset on the asymmetry



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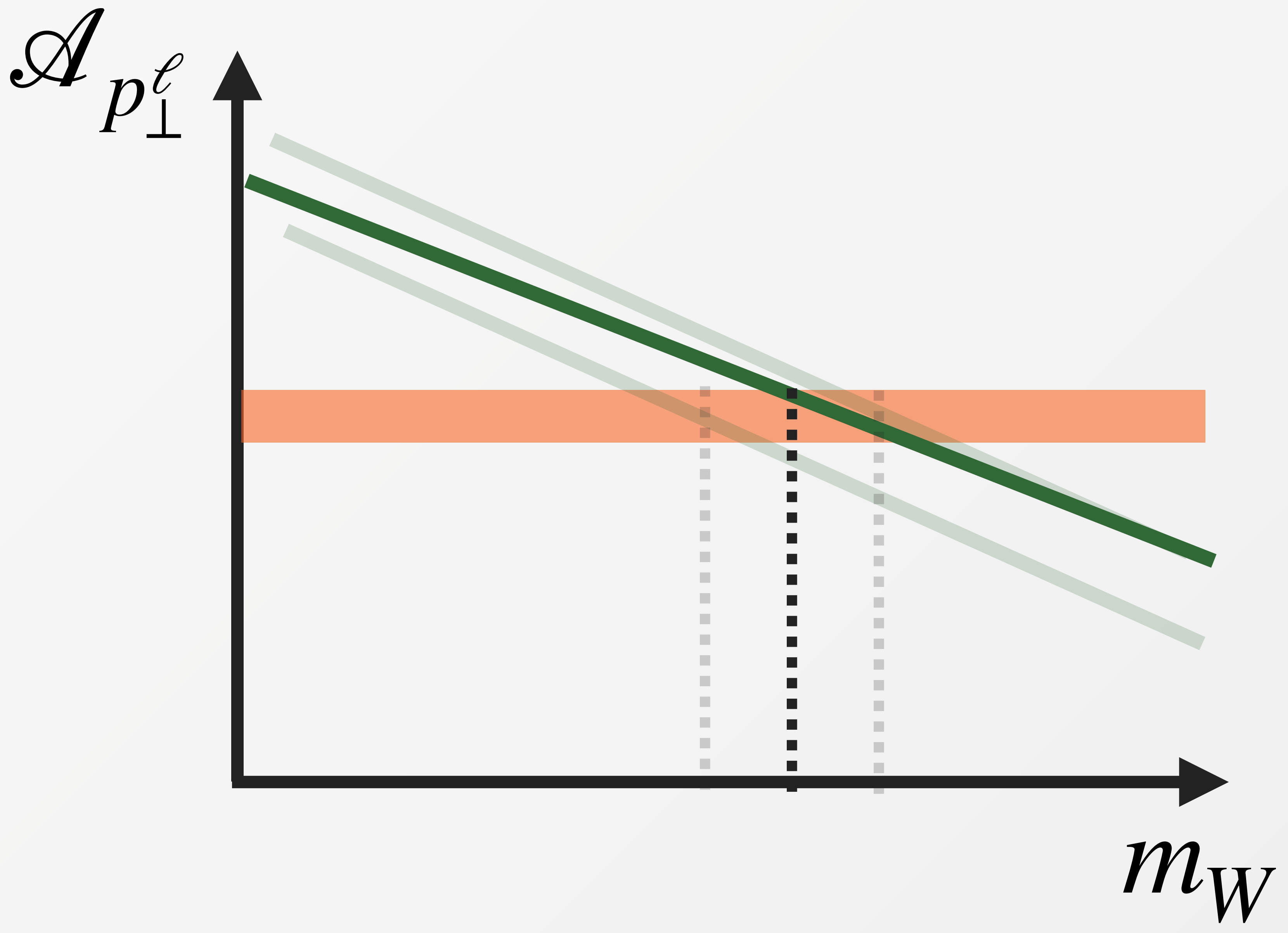
QED corrections might also change the shape

$\rightarrow$ shift on m_W



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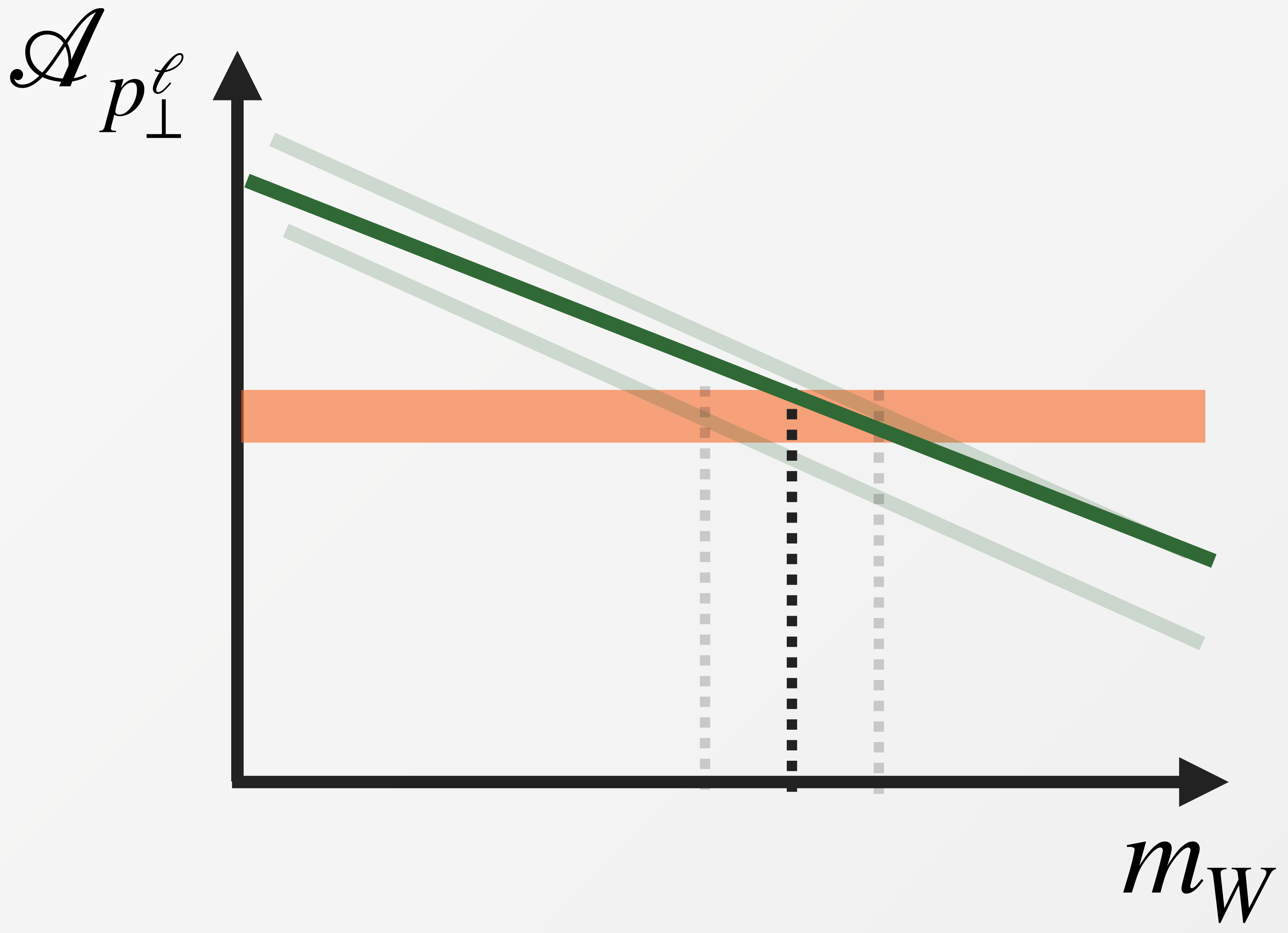

Impact on the central m_W value of

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- non-perturbative effects

Each effect yields a vertical offset on the asymmetry

QED corrections might also change the shape

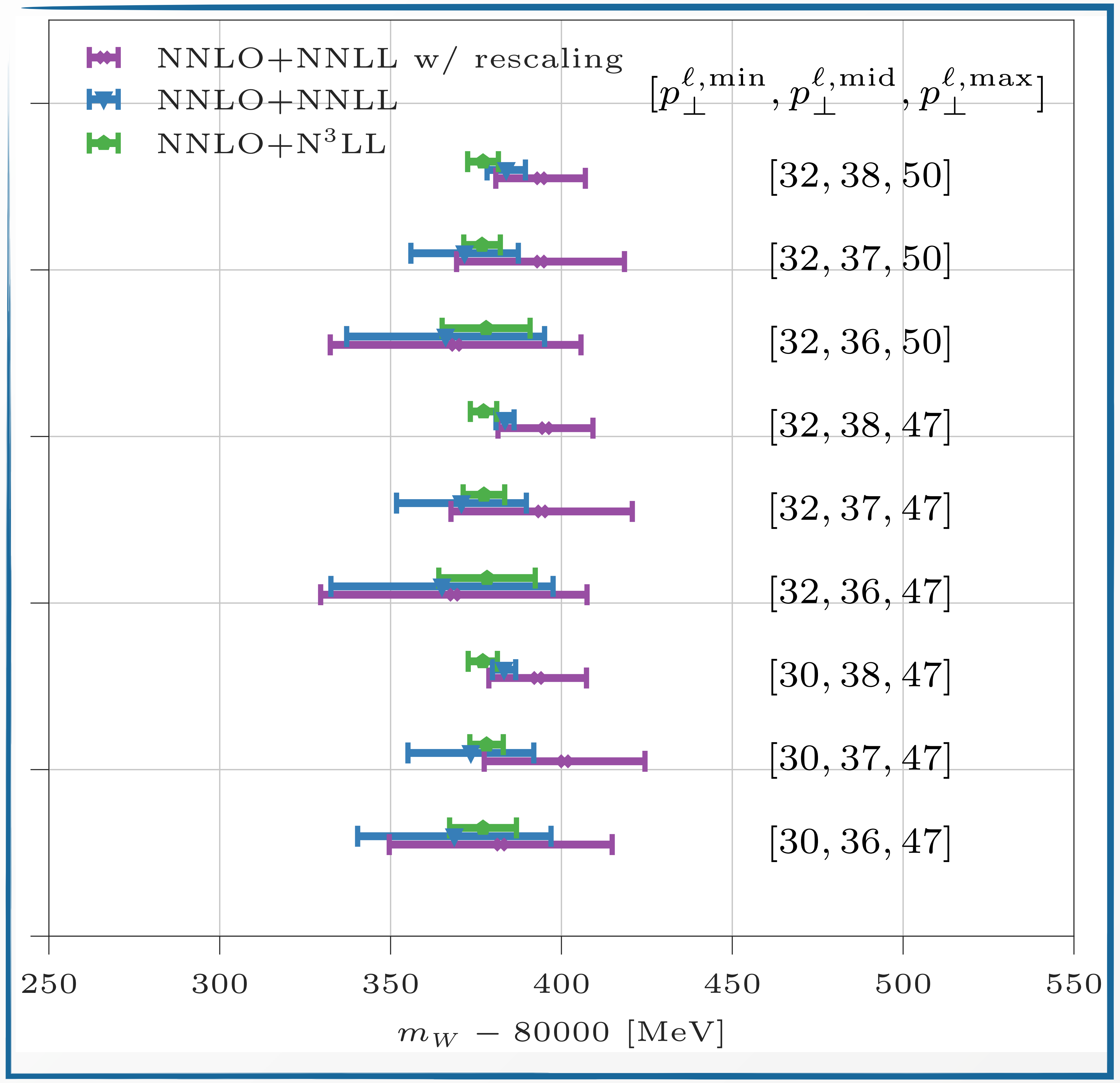
→ shift on m_W

Impact of non-perturbative corrections expected to reduce when using NNLO+N^{3}LL predictions with respect to results with lower accuracy: **interplay of NP QCD model and perturbative accuracy**

Parton distribution functions are an additional source of theoretical uncertainty

Linearity of the dependence on m_W allows an easy propagation of each uncertainty source

Information transfer from NCDY to CCDY



NNLO+NNLL taken as our theory model

NNLO+N^{3}LL with central scales as our MC truth
- pseudo-data generated both for NCDY and CCDY
- reweighting function computed from NNLO+NNLL to the pseudo-data in NC
- same reweighing function applied in CC DY

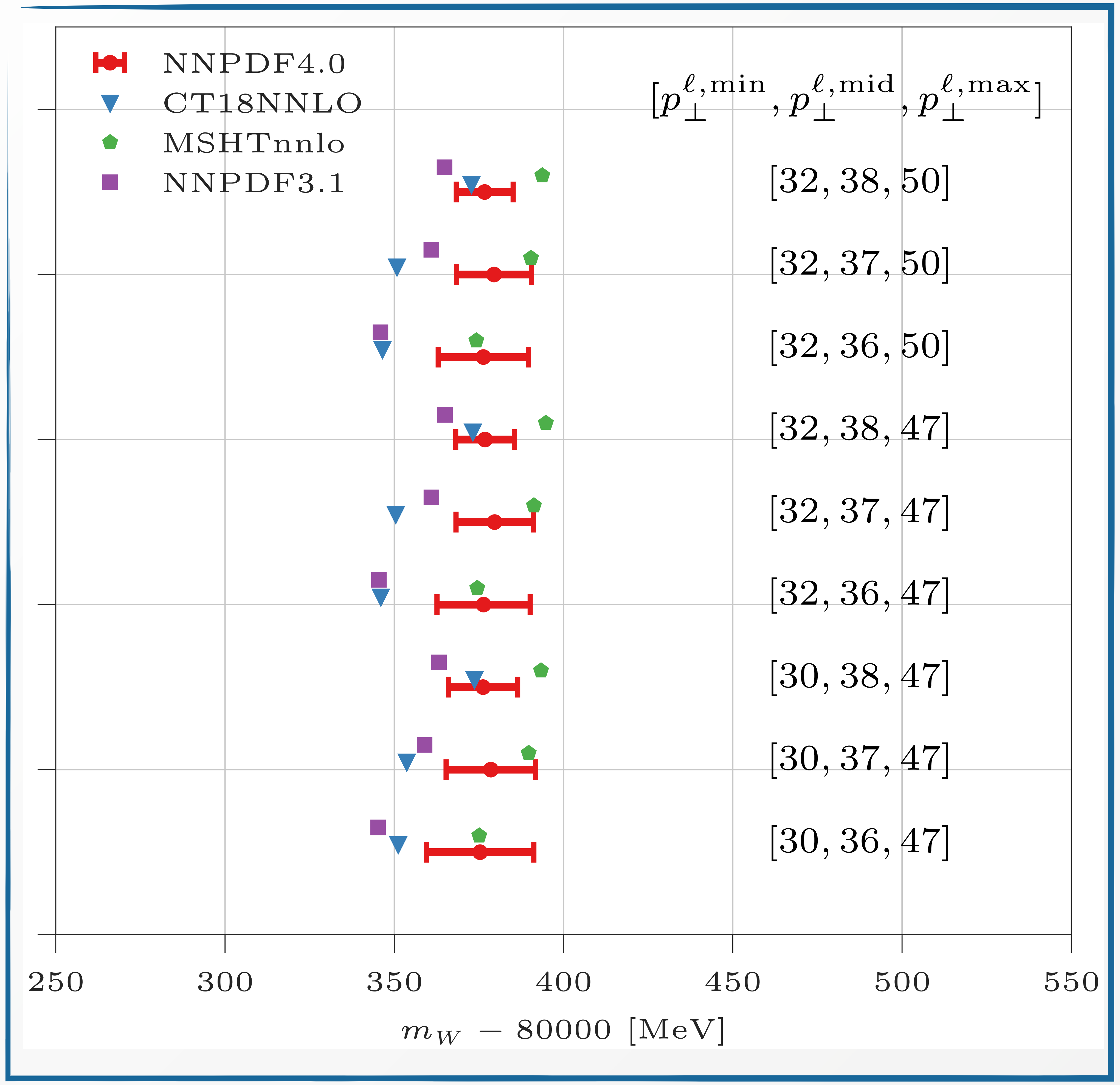
The $p_\perp^{\ell\nu}$ and the $p_\perp^{\ell}$ distributions get closer to the CCDY pseudodata but still maintain some shape differences

→delicate to assume that $p_\perp^{\ell\nu}$ rescaling applies equally well to $p_\perp^{\ell}$

Perturbative QCD uncertainty on m_W estimated with or without reweighing is of similar size

Usage of the highest available perturbative order is recommended to minimize the systematics in the transfer from Z to W

The lepton transverse momentum distribution in CC DY



PDF uncertainties on m_W evaluated conservatively using the 100 replicas of the NNPDF4.0 set at NLO+NLL

$$\delta m_W = \pm\, 11\,\mathrm{MeV}$$

Spread of the central values of CT18NNLO, MSHTnnlo, NNPDF4.0 of $\sim 30\,\mathrm{MeV}$

Size of the uncertainty expected, as the asymmetry is a single scalar observable particularly sensitive to PDF variations

More information needed to mitigate PDF uncertainty, e.g. profiling using additional bins of the $p_\perp^\ell$ distribution

PDF uncertainty can be reduced to the **few MeV level** thanks to the **strong anti correlated behaviour** of the two tails of $p_\perp^\ell$

Conclusion and outlook

- **Huge amount of theoretical work** in the last few years in the computation of **higher-order corrections** (**QCD resummation, mixed QCD and QED corrections**), which now allow for a precise and accurate description of neutral and charged DY production

- Future measurement of m_W should exploit these computation for a **reliable estimates** of the theoretical uncertainties

- Shape of the p_T^ℓ distribution and presence of the Jacobian peak motivates the definition of a **scalar observable** which **maximises the sensitivity** on m_W and has several advantages

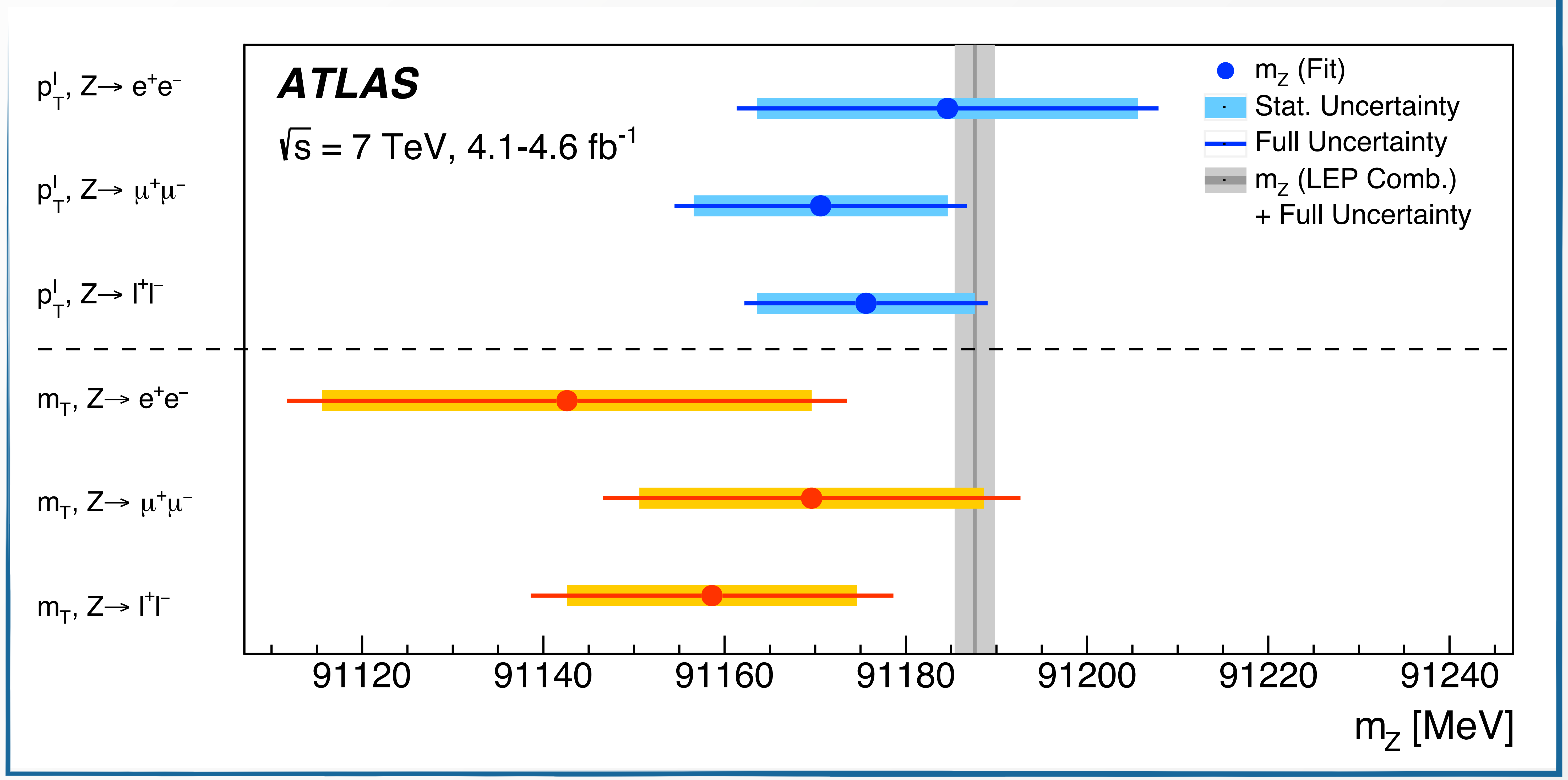
  - excellent pQCD convergence
  - large linear dependence on m_W→ sensitivity for a precision measurement
  - possibility to unfold the data to particle level → simplicity in a global combination

- Determination at the $\pm 15\,\mathrm{MeV}$ level from the experimental side seems possible; perturbative QCD uncertainty at the $\pm 5\,\mathrm{MeV}$ level is achievable using CC DY data alone

- For the future: thorough phenomenological study, including all the available SM radiative corrections

Backup

Controlling systematics



[ATLAS '17]

Robust check of many underlying systematics (although not sensitive to modelling of p_T^Z/p_T^W ratio) can be performed by extracting the Z mass using template fit technique

"Quia vidisti me, credidisti; beati, qui non viderunt et crediderunt"

Johannes 20, 29

[because thou hast seen me, thou hast believed: blessed *are* they that have not seen, and *yet* have believed]

PDFs and their uncertainties

Uncertainties related to PDFs can have different origin:

- Uncertainty propagated from the statistical and systematic errors on the measurements used in their determination (canonical "**PDF uncertainty**")
- Theoretical uncertainties of the predictions used in PDF fits, such as missing higher order uncertainty: these are starting to be addressed only recently, and are typically not included in the nominal PDF uncertainty [Abdul Khalek, Ball, LR, et al, (NNPDF Coll.), 1905.04311] [J. McGowan, T. Cridge, L. Harland-Lang, R. Thorne (MSHT Coll.) 2207.04739]

Comparisons between different groups used to assess sources of **methodological uncertainty** in the PDF extraction

m_W measurements typically include the nominal PDF uncertainty and, more conservatively, they also assess that it encompasses the envelope of various PDF sets

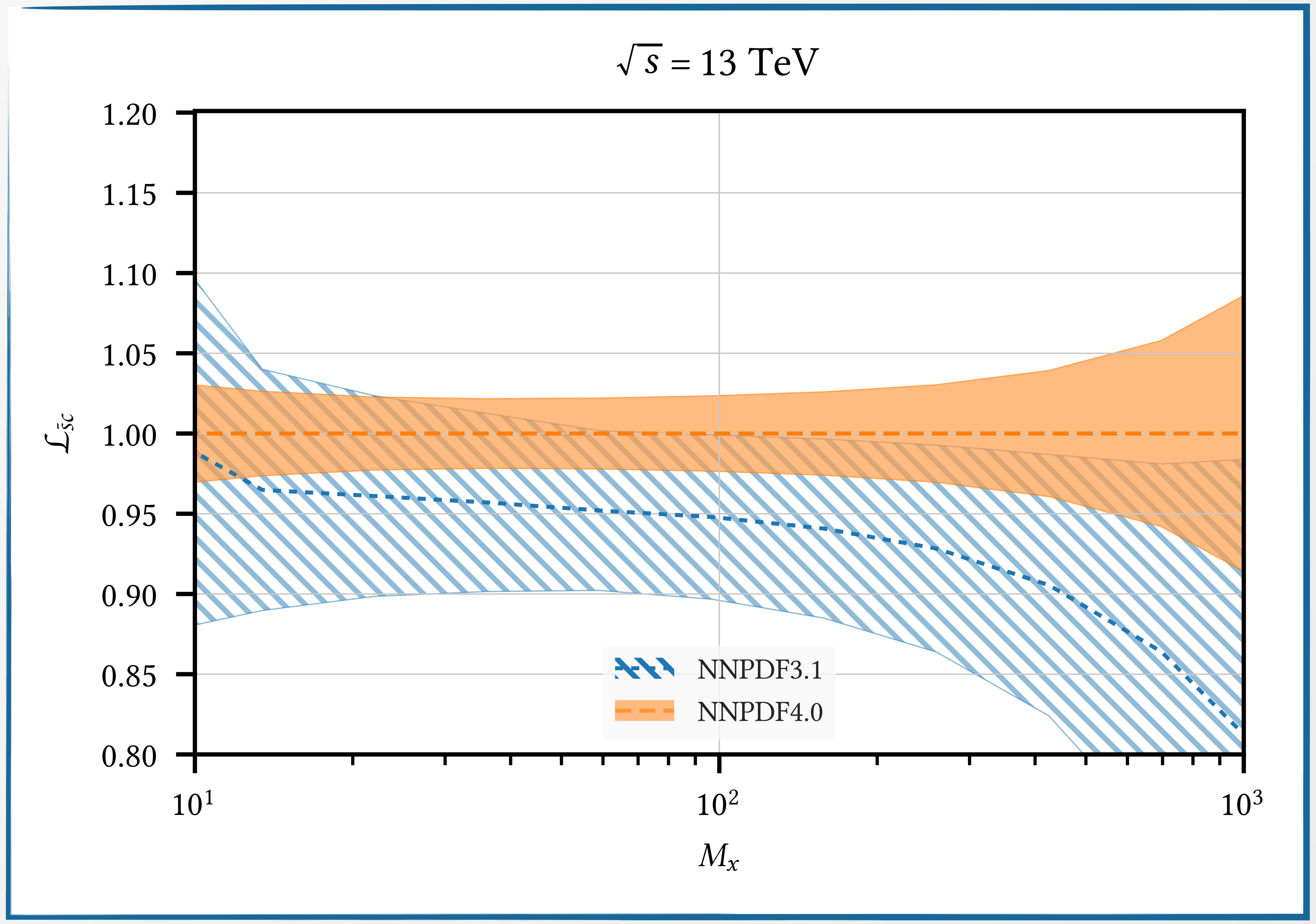
PDFs and their uncertainties

Numerous studies on the impact of PDF uncertainties have been performed at both colliders
[Tevatron 0707.0085,0708.3642,0908.0766,1203.0275,1203.0293,1307.7627] [Bozzi, Citelli, Rojo, Vesterinen, Vicini 1104.2056, 1501.05587, 1508.06954]
[ATLAS 1701.07240] [Kotwal PRD 98, 033008] [Manca, Cerri, Foppiani, Rolandi 1707.09344] [Bianchini, Rolandi 1902.03028] [Farry, Lupton, Pili, Vesterinen, 1902.04323]
[Bagnaschi, Vicini 1910.04726] [Hussein, Isaacson, Huston 1905.00110] [Gao, Liu, Xie 2205.03942]

The relative size of PDFs uncertainties at the Tevatron and at the LHC is affected by the different **centre-of-mass** energy of the collision and the **different initial states**

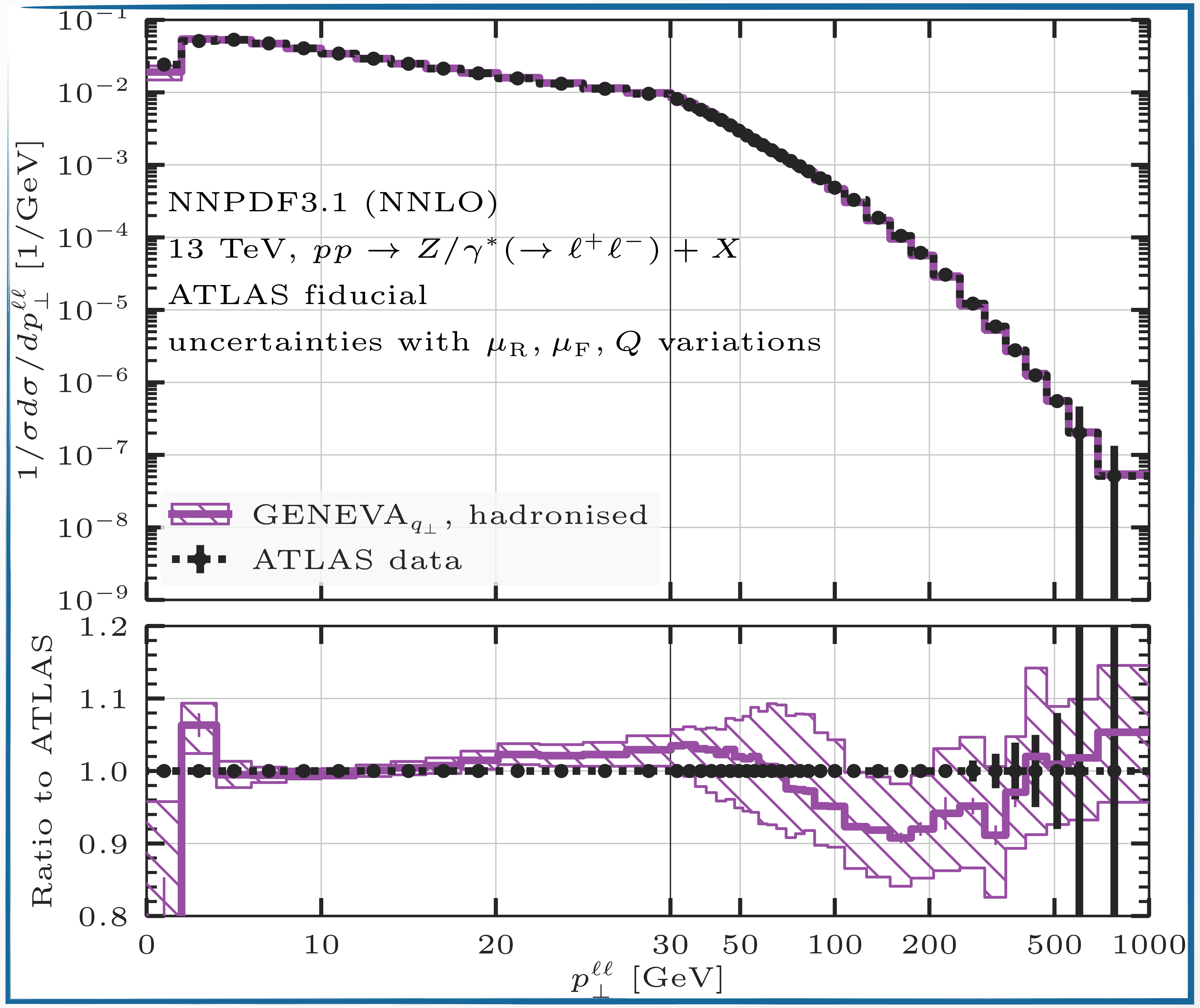
PDFs uncertainties **not an obstacle at Tevatron**; they have long been considered a **limiting factor at the LHC** due to the smaller values of the partonic x probed (higher collider energy) and the larger contribution from the second quark generation

Latest generation of NNPDF parton densities (large number of LHC datasets included, new machine-learning based methodology) achieves **substantial reduction** of PDF uncertainty

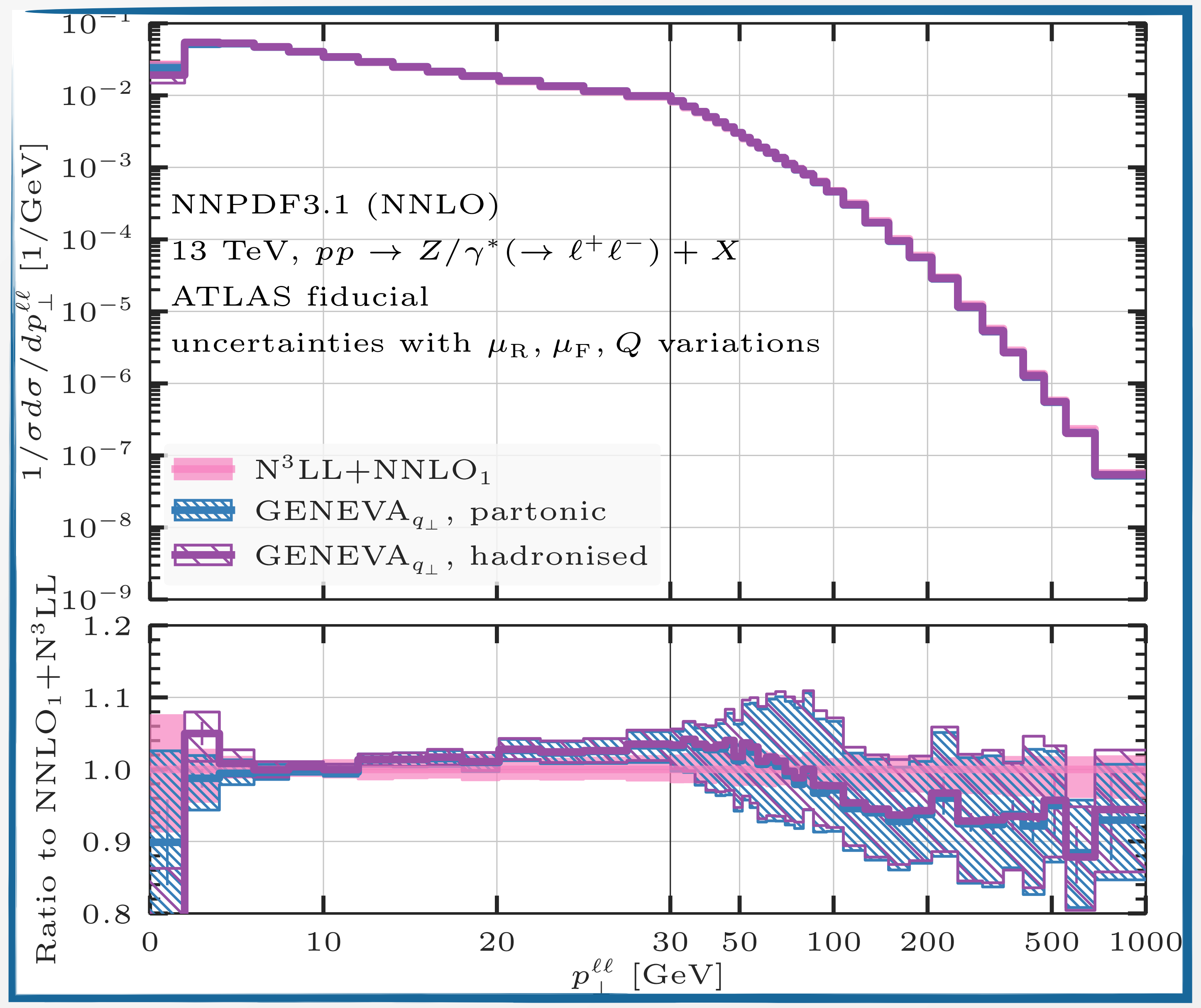


Control of the differential distributions in DY production: NNLO+PS

Knowledge of transverse-momentum resummation at high accuracy exploited to develop NNLO+PS Monte Carlo event generators (MiNNLO$_{PS}$, GENEVA)



[Alioli, Bauer, Broggio, Gavardi, Kallweit, Lim, Nagar, Napoletano, LR '21]



Comparison with parton-level results indicates excellent numerical agreement between NNLO+PS results and **N^{3}LL$_{QCD}$** resummed results

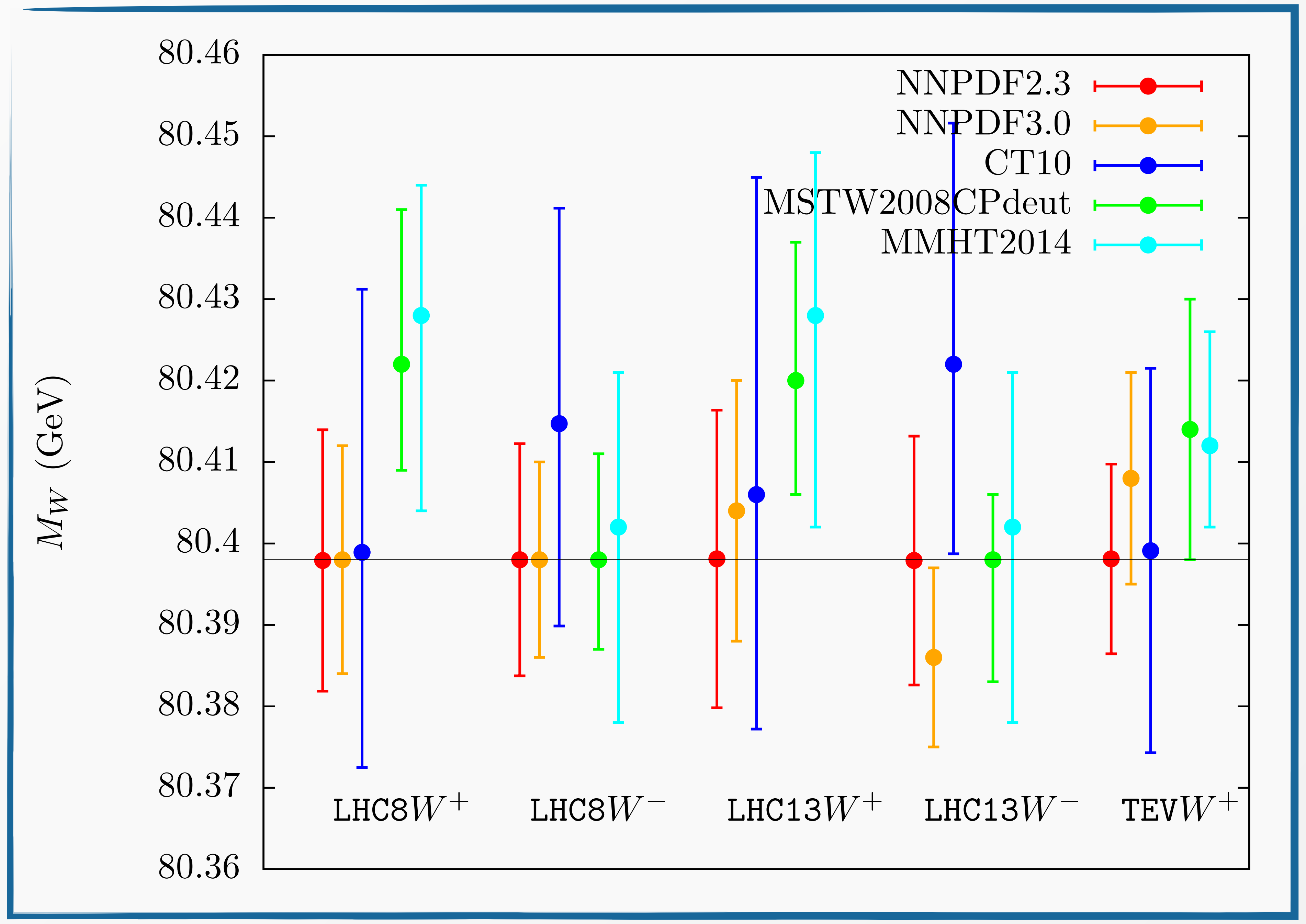
Very good description of experimental data — effects of hadronisation and MPI reduced for transverse momentum spectrum

PDFs and their uncertainties: template fits

PDF-induced uncertainty typically computed by generating templates with a given PDF member *i* for various values of m_W, and subsequently fitting all other members *j* defining a proper figure of merit

$$\chi^2_{i,j} = \sum_{k \in \mathrm{bins}} \frac{(T^j_k - D^i_k)^2}{\sigma^2_k}$$

Once the preferred value for m_W for each member has been determined by minimising the figure of merit, compute PDF-induced uncertainty



[Bozzi, Citelli, Vicini 1501.05587]

PDF uncertainties with this strategy are **relatively large at the LHC**, with a resulting uncertainty larger than 10 MeV and considerably large spreads between different PDF sets

Cfr. ~ 4 MeV quoted by CDF II with NNLO PDFs

4 MeV also claimed by CDF II to be the shift between NNPDF3.1 NNLO and ~**15 years old** NLO CTEQ6.6 PDFs

THE STANDARD MODEL AND MISSING E_T

OR

THE MANY ROADS TO PARADISE

Stephen D. Ellis[‡]

CERN - Geneva

cuts. We are thus able to sum the contributions of MANY SMALL sources which were absent in previous studies and which can, in the sum, yield a sizeable result. These are the "many roads" of the title. [This concern, that many small numbers can yield a large sum, was colourfully voiced at the meeting by G. Altarelli who described his vision of a mixture of small effects -- the Altarelli cocktail -- leading to the observed signal.] Before proceeding to the

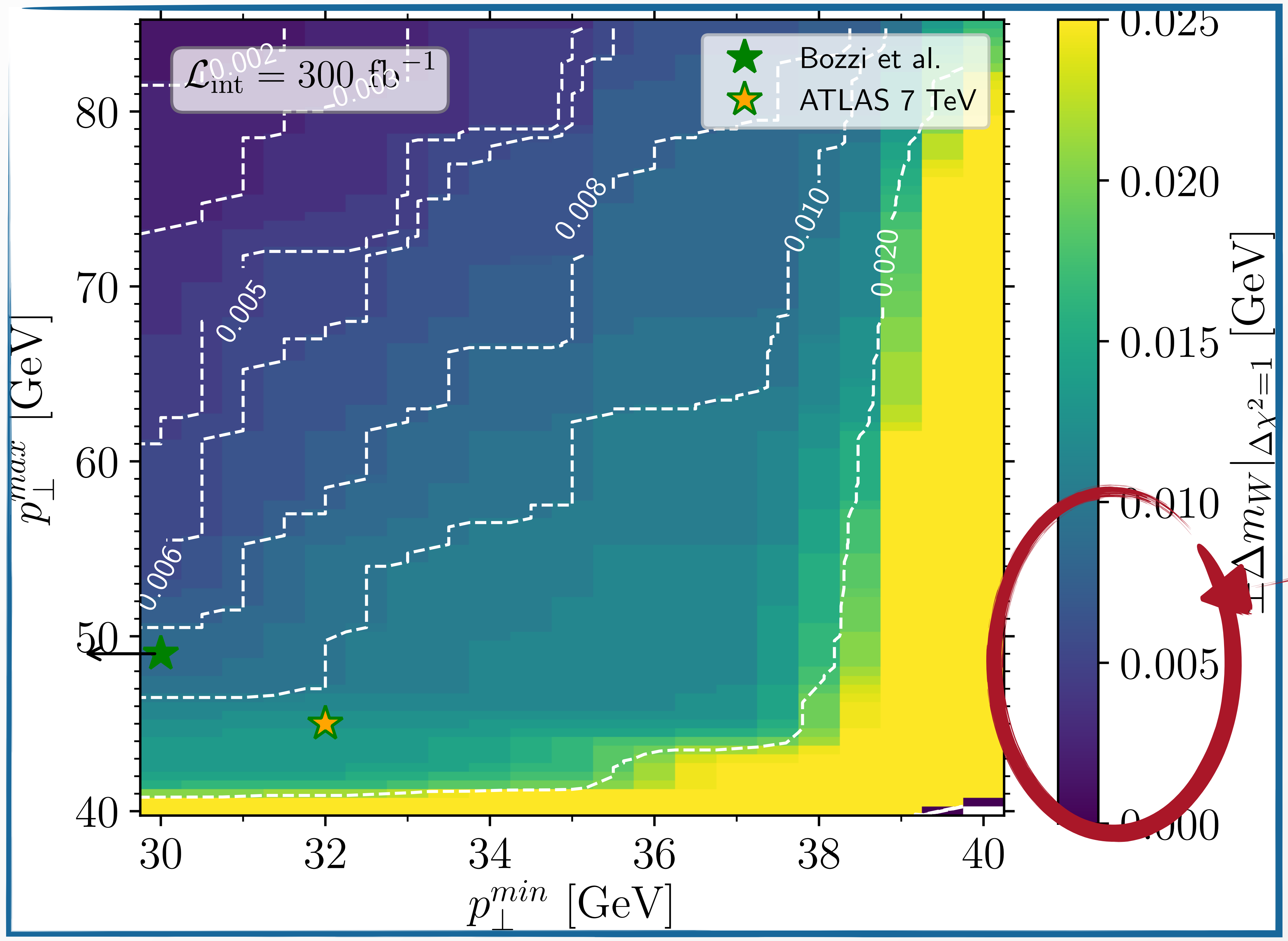
PDFs and their uncertainties: bin-by-bin correlations

Bin-by-bin correlations between PDF replicas can be taken into account inserting the information about PDFs in the covariance matrix

$$(\Sigma_{\rm PDF})_{ij} = \langle (\mathcal{T} - \langle \mathcal{T} \rangle_{\rm PDF})_i (\mathcal{T} - \langle \mathcal{T} \rangle_{\rm PDF})_j \rangle_{\rm PDFs}$$

Compute χ^2 using full covariance matrix in the definition

$$\chi^2_{i,\min} = \sum_{k,l \in \text{bins}} (T_{0,i} - D)_k (C^{-1})_k (T_{0,i} - D)_l \qquad \forall m_{W,i} \qquad C = \Sigma_{\rm PDF} + \Sigma_{\rm MC} + \Sigma_{\rm stat} + \Sigma_{\rm exp,syst}$$

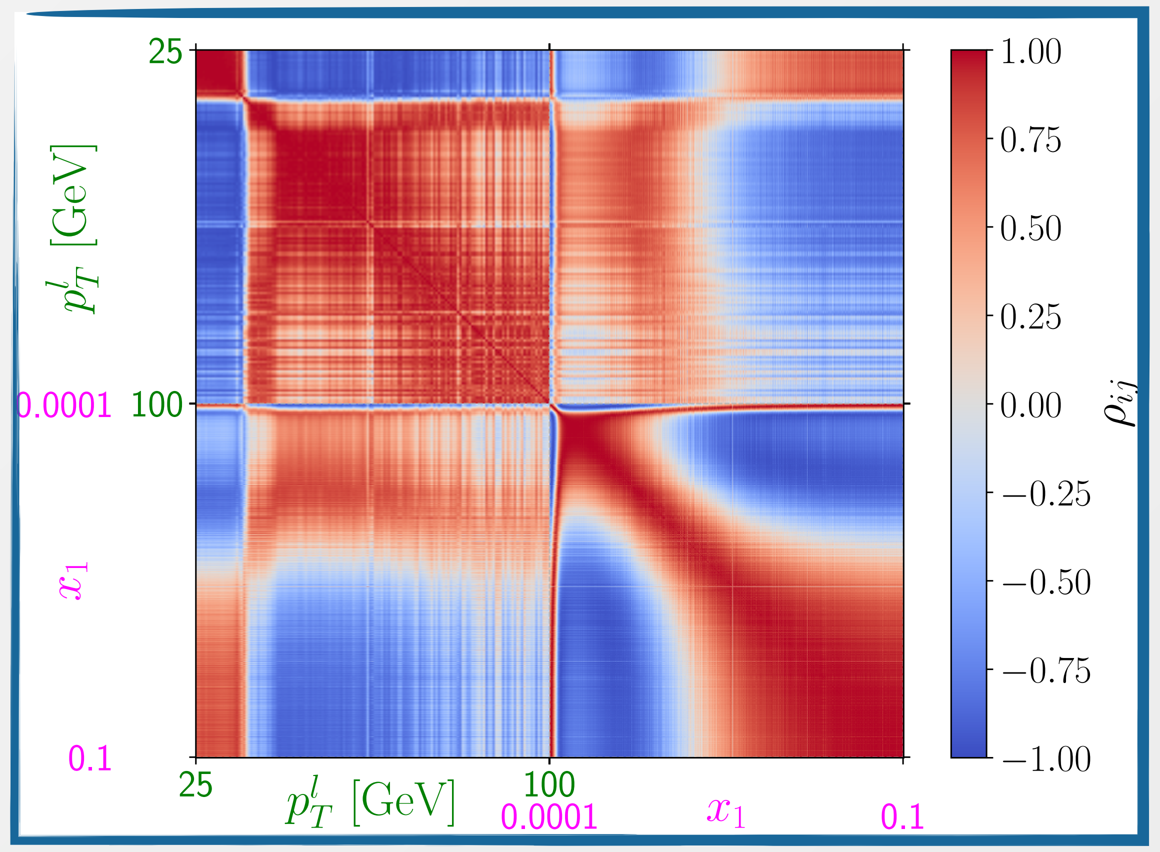


[Bagnaschi, Vicini 1910.04726]

Inserting the information about PDFs in the covariance matrix leads to a profiling action given by the data

PDF uncertainty can be reduced to the **few MeV level** thanks to the **strong anti correlated behaviour** of the two tails of $p_\perp^\ell$

$$\rho_{ij} = \frac{\langle (\mathcal{O}_i - \langle \mathcal{O} \rangle)(\mathcal{O}_j - \langle \mathcal{O} \rangle) \rangle}{\sigma_i \sigma_j}$$



Data-driven approach to m_W extraction

A theory-agnostic extraction of m_W

[E. Manca, PhD Thesis 2016; V. Bertacchi, Tesi di Perfezionamento 2021]

Exploit statistics collected by CMS during Run II at the LHC to extract the value of m_W simultaneously with q_T^W, y_W and polarization spectra to obtain a statistically-dominated measurement of m_W

Decoupling of the (unknown) production physics from the (known) decay physics

unpolarised cross section

W and lepton variables

$$\frac{d\sigma}{dq_{T,W}^2 dy_W d\cos\theta_\mu d\phi_\mu dm_W} = \frac{3}{16\pi} \frac{d\sigma^{U+L}}{dq_{T,W}^2 dY_W dm_W} \left[(1+\cos^2\theta_\mu) + \sum_{i=0}^{7} A_i P_i(\cos\theta_\mu, \phi_\mu) \right]$$

angular coefficients

1. Decompose inclusive $\eta^\mu \times p_T^\mu$ distribution in bins of m_W, Y_W, q_T^W for each P_i
2. Fit $\eta^\mu \times p_T^\mu$ distribution measured on data
3. Unfolding from the sole lepton kinematics to the underling boson kinematics