Transverse observable resummation at the LHC

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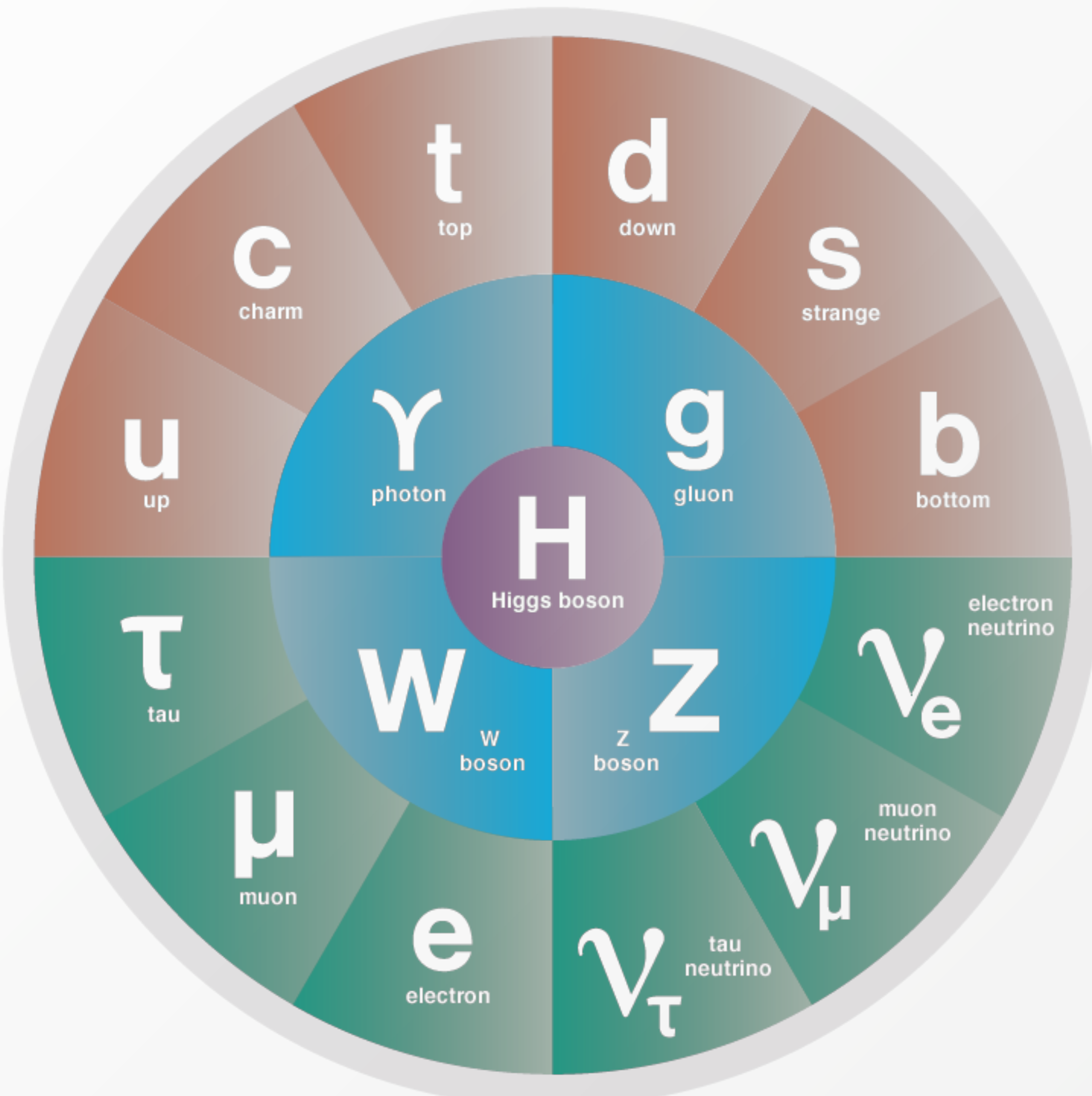
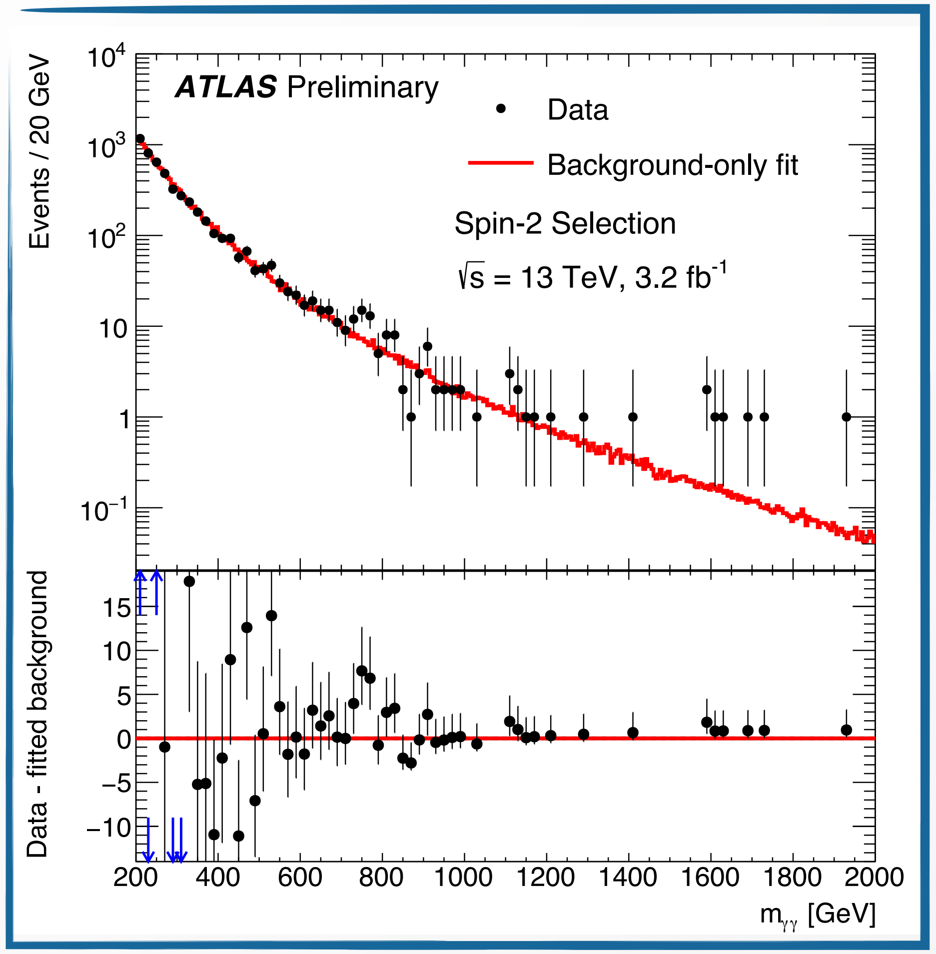
After Higgs



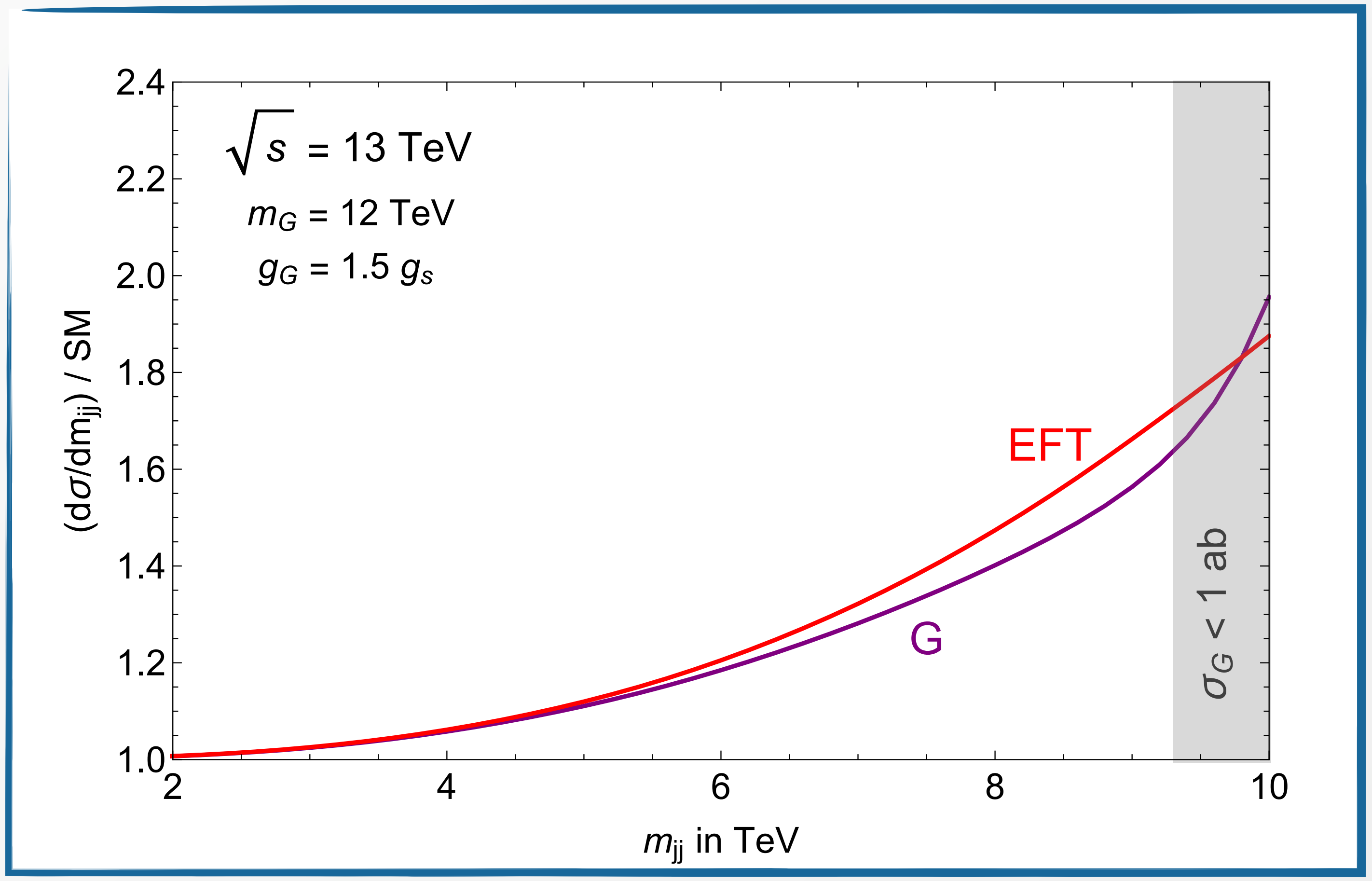
- The Higgs discovery in 2012 completed the Standard Model (SM) puzzle
- We do know that the picture is not yet complete: there are various phenomena which call for **physics beyond the SM** (neutrino masses, dark matter, baryon asymmetry…)
- **Naturalness**: since m_H=125 GeV, to avoid fine-tuning scale of new physics (NP) Λ_{NP} should be O(TeV)
- Ongoing direct and indirect searches for NP signatures at the LHC

BSM searches at the LHC

[ATLAS-CONF-2016-018]



[Alioli et al '17]



Bump hunting

- Little or no theoretical input
- Reconstructed signal over a smooth and well understood background
- Besides Higgs, not very successful…

Deviations from SM predictions

- Look for deviations from SM
- Need an **accurate theoretical description** of the kinematic distributions

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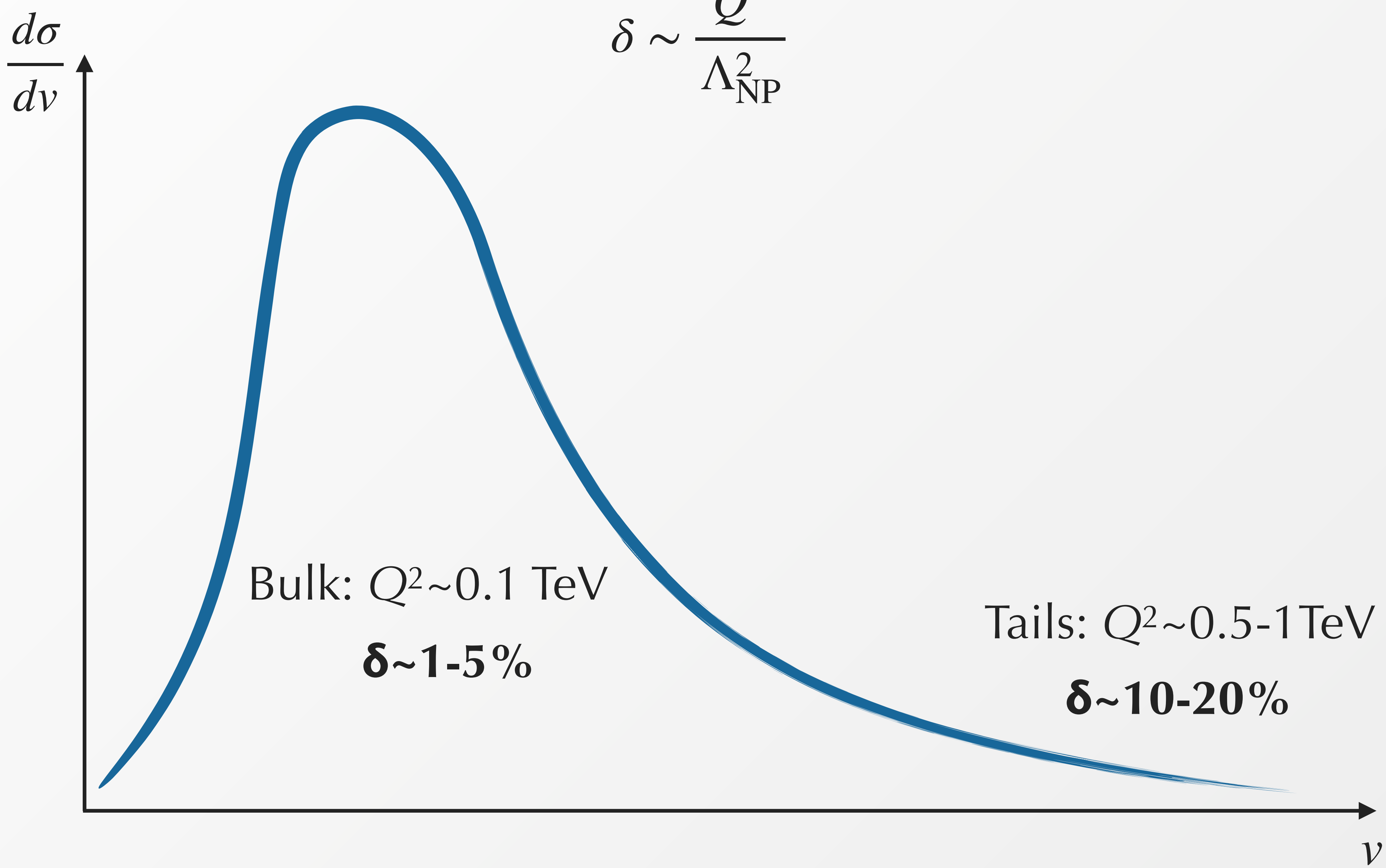
Quest for precision

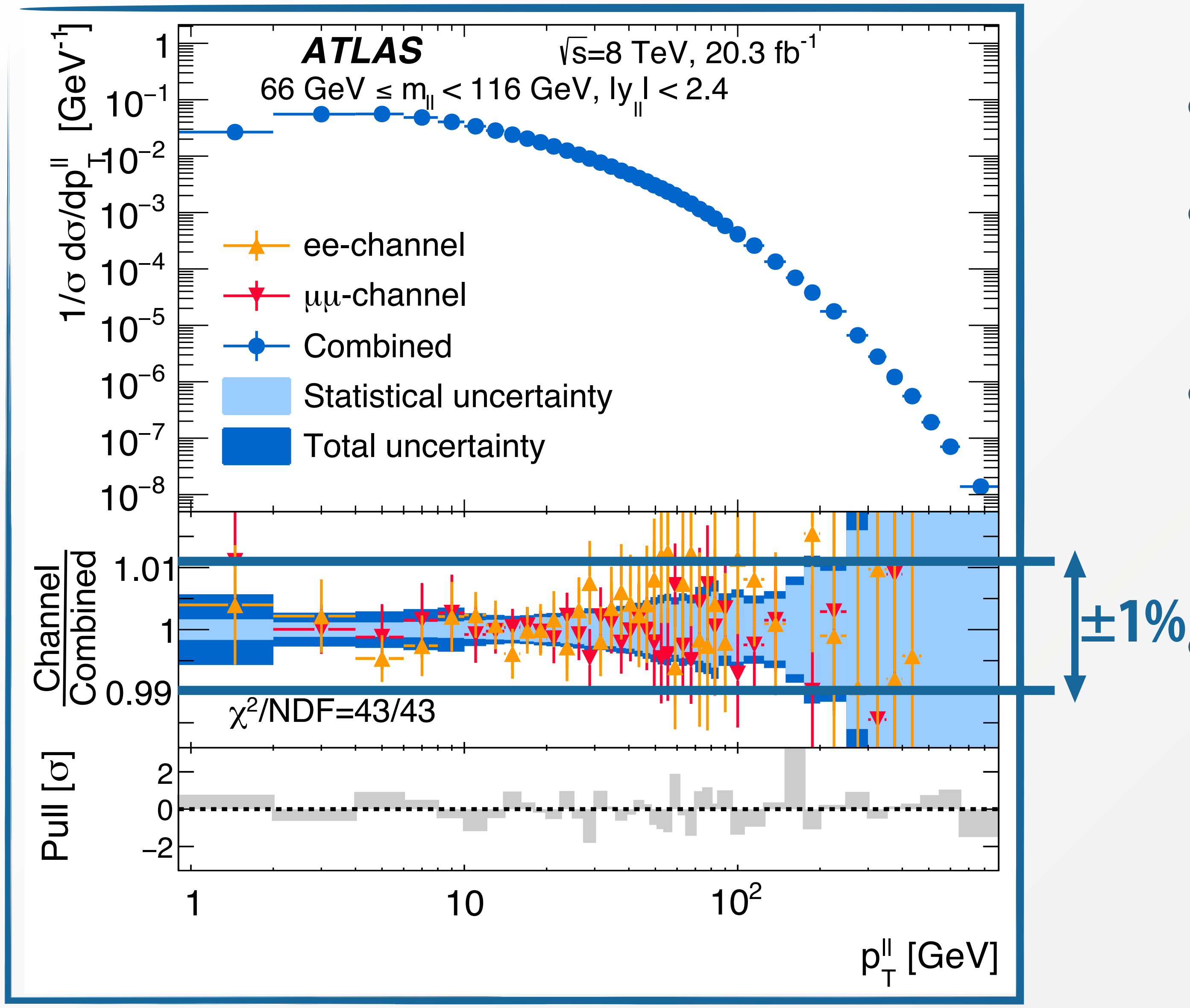
If the scale for NP Λ_{NP} is a few TeV, expected deviations from the SM behaviour are

$$\delta \sim \frac{Q^2}{\Lambda_{NP}^2}$$



LHC in the precision era

1-5% level of precision is within reach at the (HL)-LHC



- Luminosity reached 100 fb^{-1} at 13 TeV
- Increase in statistics enables study of **differential distributions** in detail
- Measurements at % level (or even smaller) are available for several processes
- Astonishing level of precision reached in e.g. Z transverse momentum: luminosity and other systematics are cancelled or reduced if results are normalized by fiducial cross section

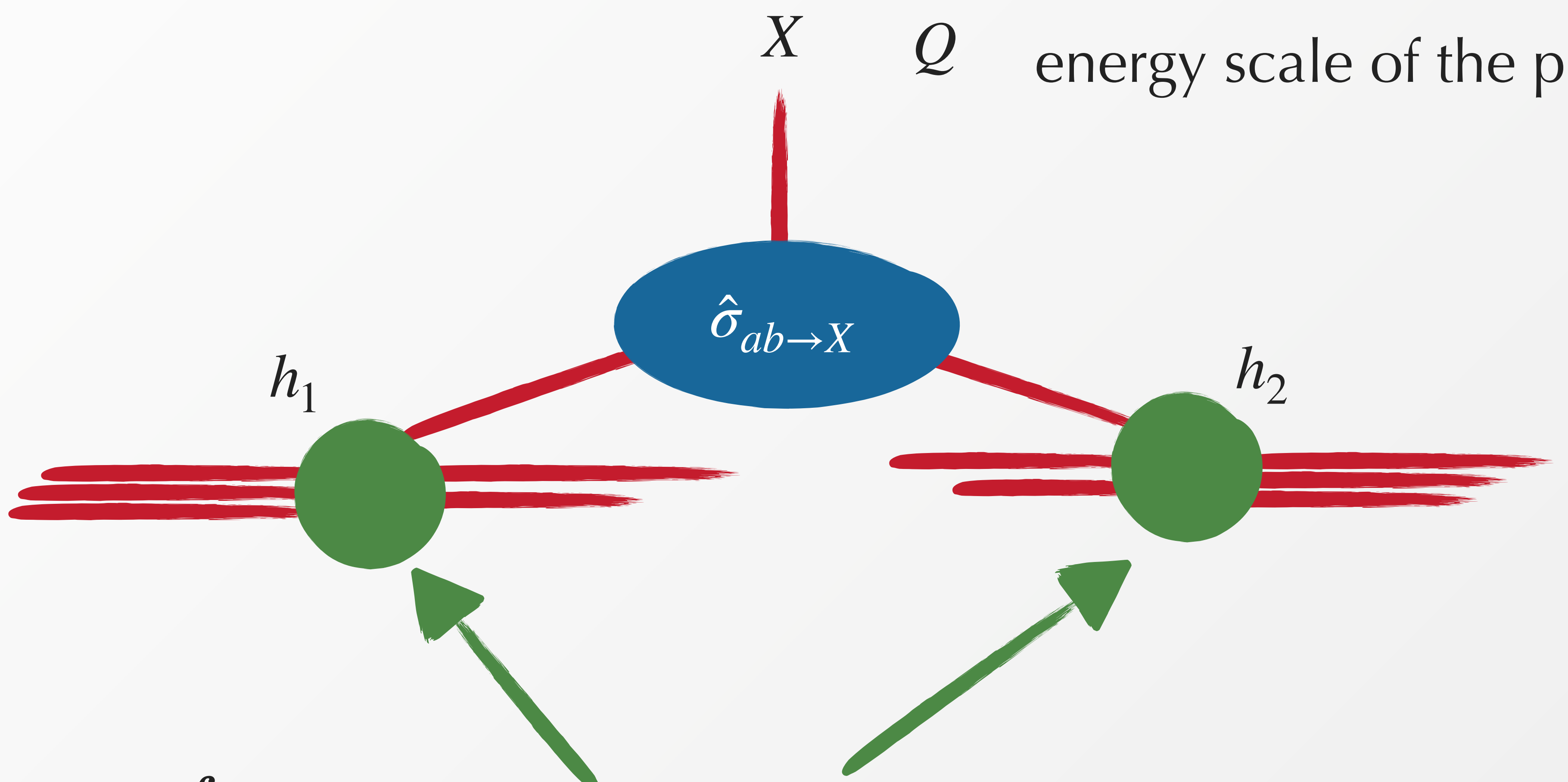
Very accurate theoretical predictions needed

Precision physics at the LHC: theory

Key concept: **collinear factorization**

$\sqrt{s}$ centre-of-mass energy

Q energy scale of the process



$$\sigma(s,Q^2)=\sum_{a,b}\int dx_1 dx_2\, f_{a/h_1}(x_1,Q^2) f_{b/h_2}(x_2,Q^2)\,\hat{\sigma}_{ab\to X}(Q^2,x_1x_2s)+\mathcal{O}(\Lambda^p_{\mathrm{QCD}}/Q^p)$$

Parton Distribution Functions (PDFs)

Long-distance, non-perturbative, universal objects

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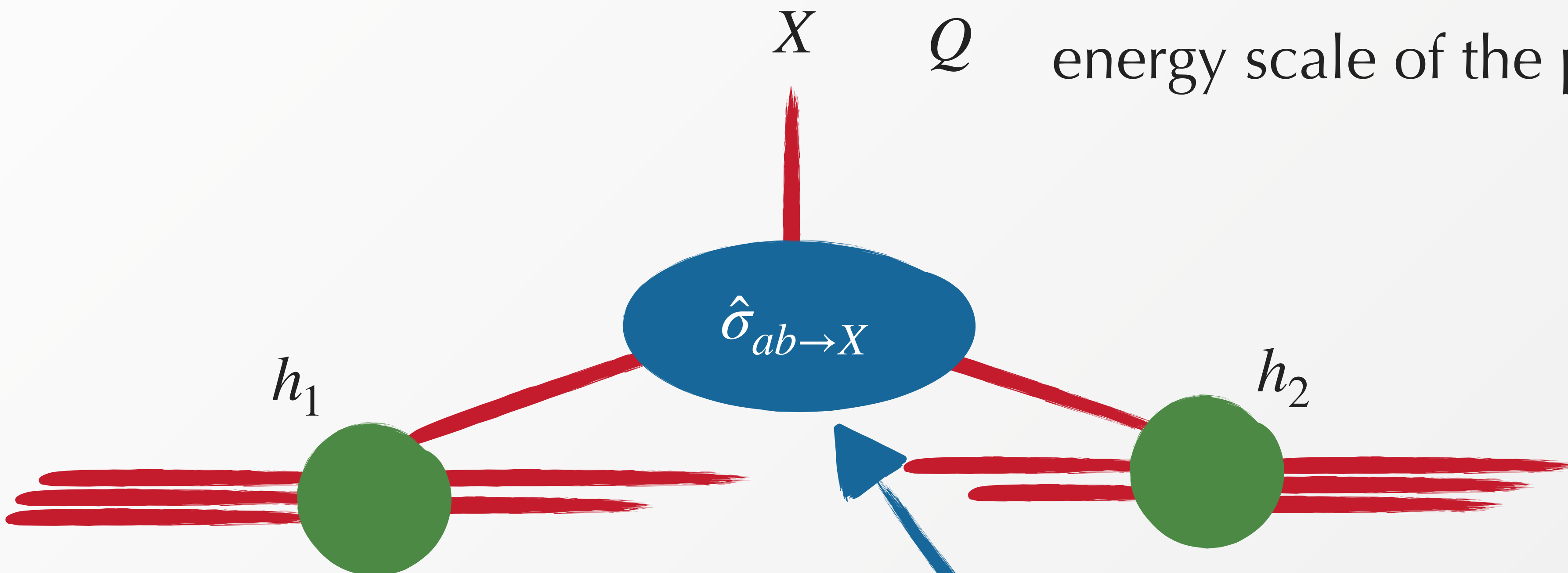

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Hard-scattering matrix element

Short-distance, perturbative, process-dependent

Precision physics at the LHC: theory

$$\sigma(s,Q^2) = \sum_{a,b} \int dx_1 dx_2\, f_{a/h_1}(x_1,Q^2) f_{b/h_2}(x_2,Q^2) \hat{\sigma}_{ab\to X}(Q^2, x_1 x_2 s) + \mathcal{O}(\Lambda^p_{\mathrm{QCD}}/Q^p)$$

Input parameters:

strong coupling α_s

PDFs f

few percent uncertainty; improvable

Non-perturbative effects

percent effect; not yet under control

Precision physics at the LHC: theory

$$\sigma(s, Q^2) = \sum_{a,b} \int dx_1 dx_2\, f_{a/h_1}(x_1, Q^2) f_{b/h_2}(x_2, Q^2) \hat{\sigma}_{ab\to X}(Q^2, x_1 x_2 s) + \mathcal{O}(\Lambda^p_{\mathrm{QCD}}/Q^p)$$

$$\hat{\sigma} = \hat{\sigma}_0(1 + \alpha_s C_1 + \alpha_s^2 C_2 + \alpha_s^3 C_3 + \ldots)$$

LO NLO NNLO N^{3}LO

$\alpha_s \sim 0.1$

δ~10-20% **NLO**

δ~1-5% **NNLO** (or even N^{3}LO)

Transverse observables

Particularly **clean experimental and theoretical environment** for precision physics

Parameterized as

$$V(k) = \left(\frac{k_t}{M}\right)^a f(\phi)$$

for a **single soft** QCD emission *k* **collinear** to incoming leg. Independent of the rapidity of radiation. $V \to 0$ for soft/collinear radiation.

Inclusive observables (e.g. transverse momentum p_t) probe directly the kinematics of the colour singlet

- negligible sensitivity to multi-parton interactions
- reduced sensitivity to non-perturbative effects
- measured extremely precisely at experiments (sub-percent in *Z* differential)

$$V(k_1, \ldots k_n) = V(k_1 + \ldots + k_n)$$

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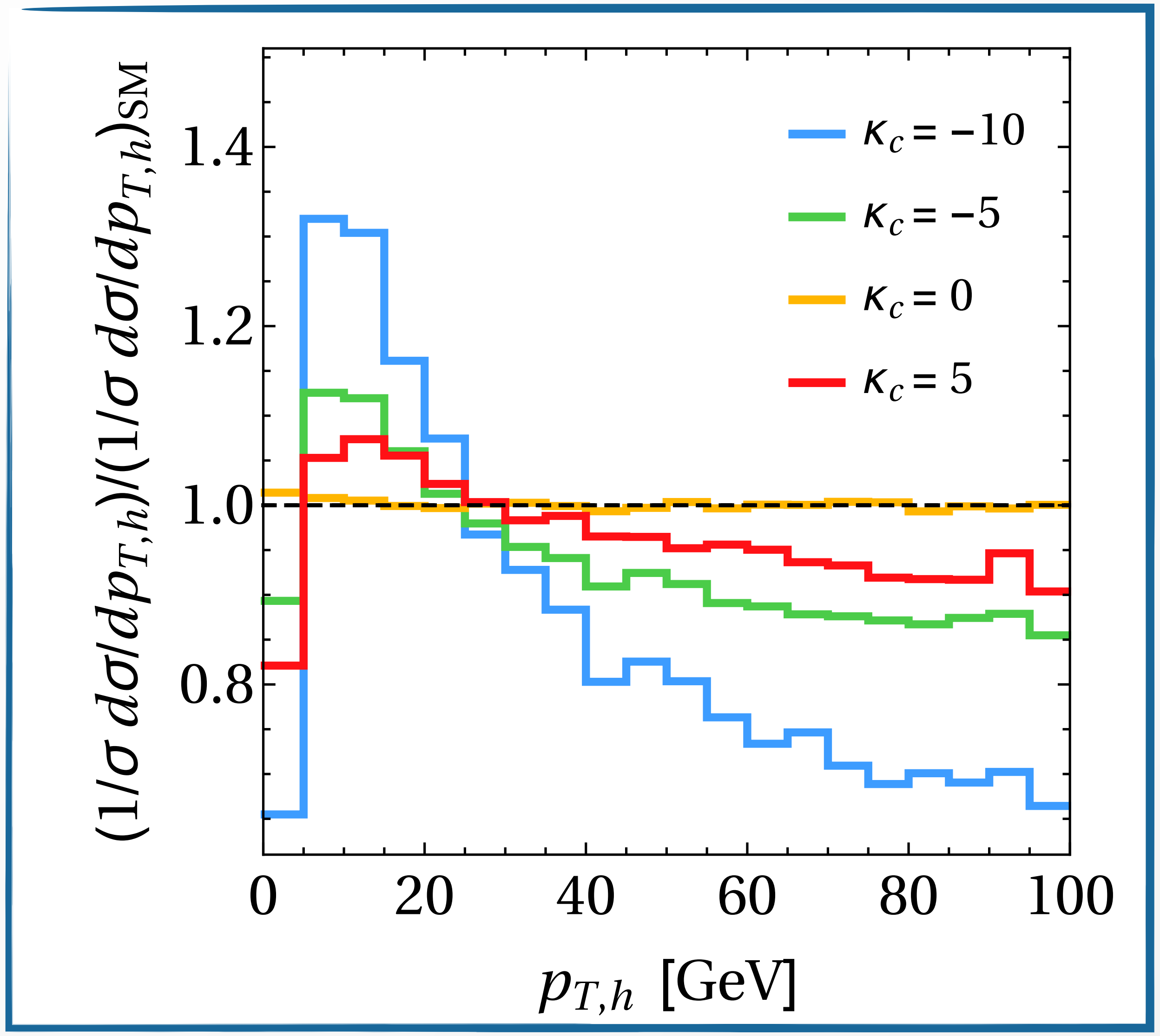
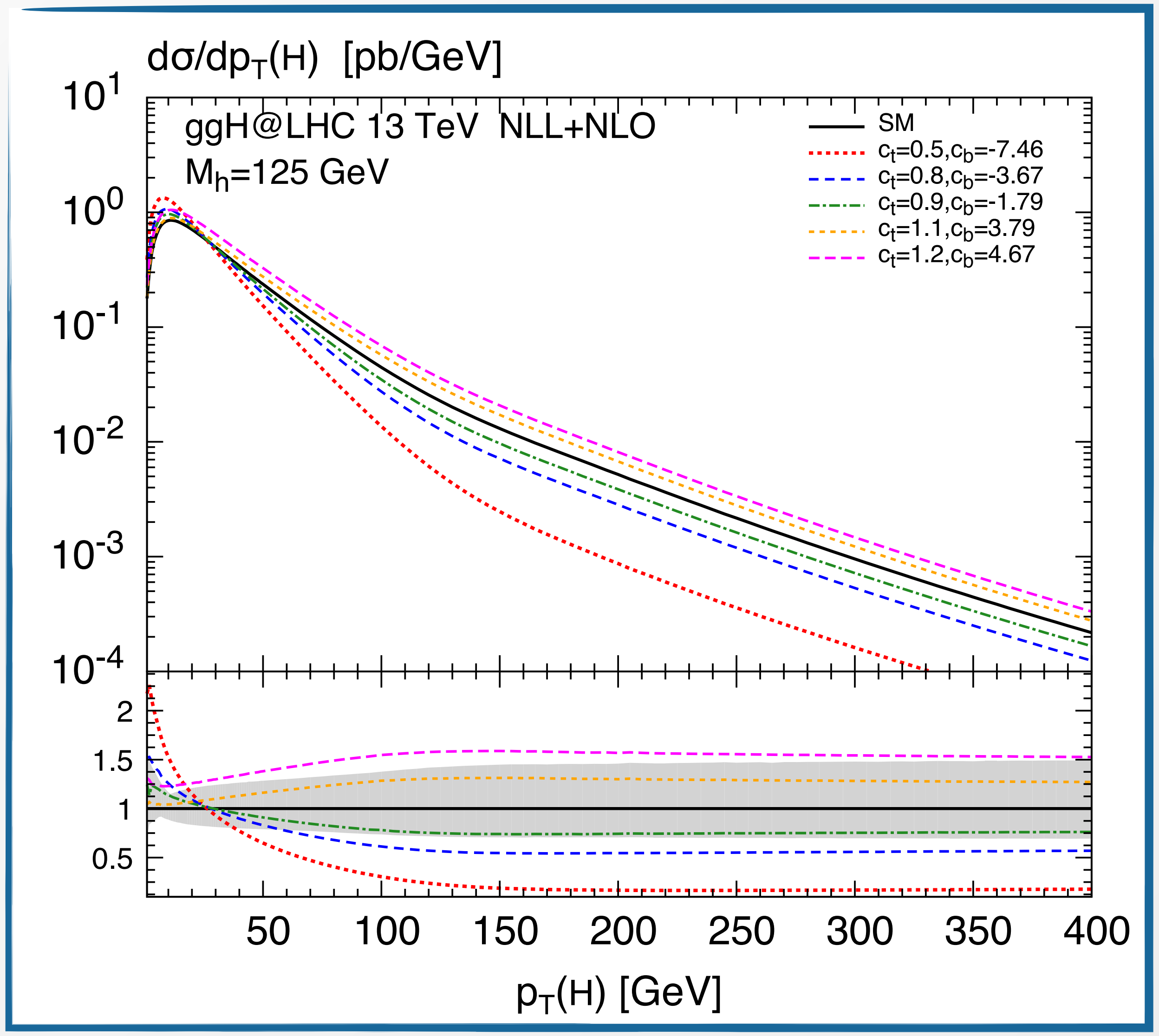
Transverse observables at the LHC

Implications for indirect constraints on BSM physics



[Bishara et al '16]



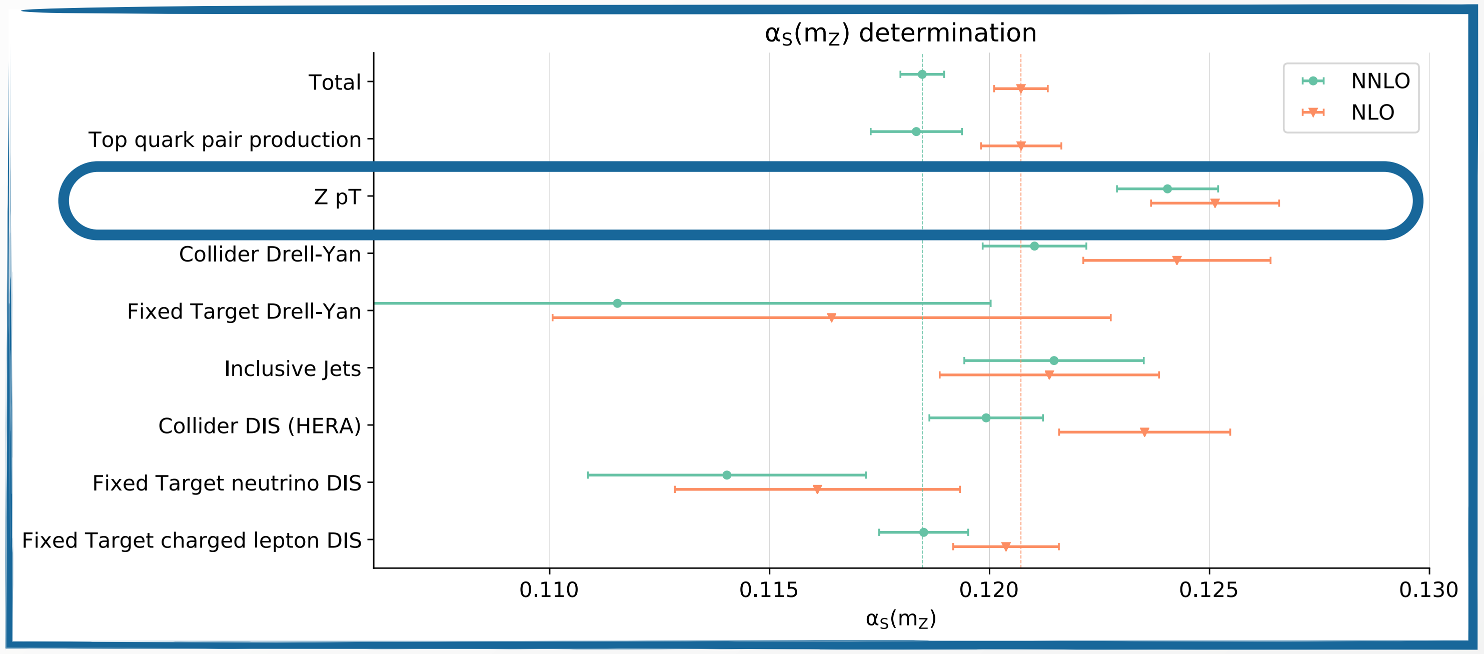
[Grazzini et al '16]

Bound on light Yukawa couplings

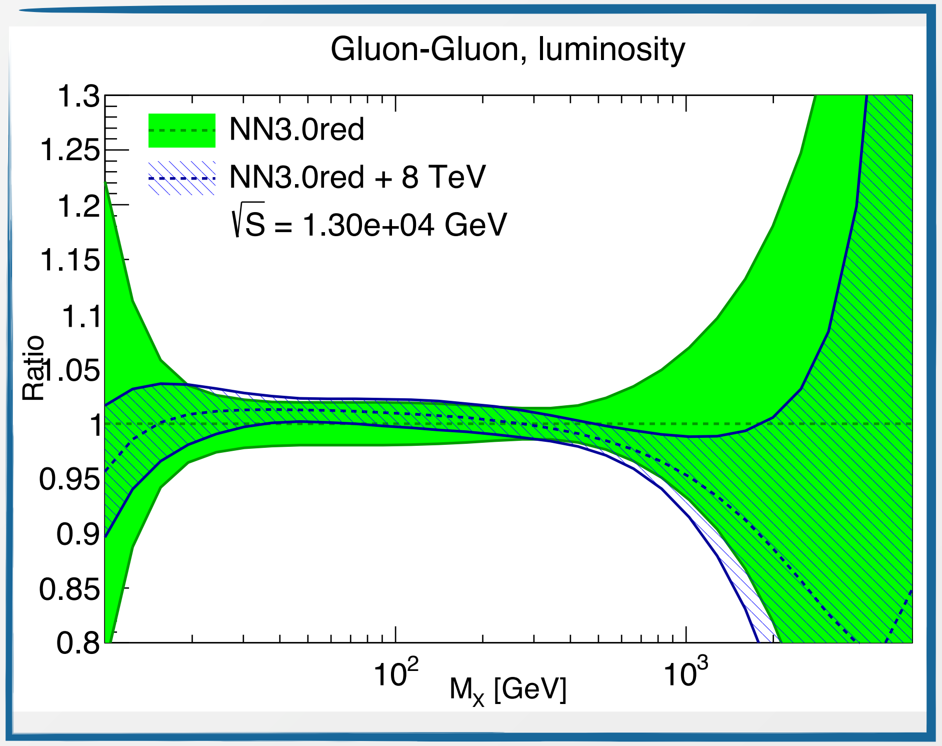
Sensitivity to dimension-6 operators

Transverse observables at the LHC

Also important implications for extraction of SM parameters (strong coupling and PDF determination, *W* mass measurements…)



[NNPDF '18]



[Boughezal et al '17]

All-order resummation

Cumulative cross section

$$\Sigma(v) = \int_0^v dV \frac{d\sigma}{dV} \sim \sigma_0[1 + \alpha_s + \alpha_s^2 + \ldots]$$

Fixed-order prediction: reliable for **inclusive enough** observables and in regions not marred by **soft/collinear radiation**

Real and virtual contributions can become however **highly unbalanced** in processes where the real radiation is strongly constrained by kinematics

Large logarithms appear at **all order** as a left-over of the real-virtual cancellation of IRC divergences

$$\ln \Sigma(v) = \sum_n \{\mathcal{O}(\alpha_s^n L^{n+1}) + \mathcal{O}(\alpha_s^n L^n) + \mathcal{O}(\alpha_s^n L^{n-1}) + \ldots\}$$

LL **NLL** **NNLL**

$$L = \ln R$$

R: ratio of typical scales characterizing the system

Fixed order predictions no longer reliable: all-order resummation of the perturbative series

All-order resummation

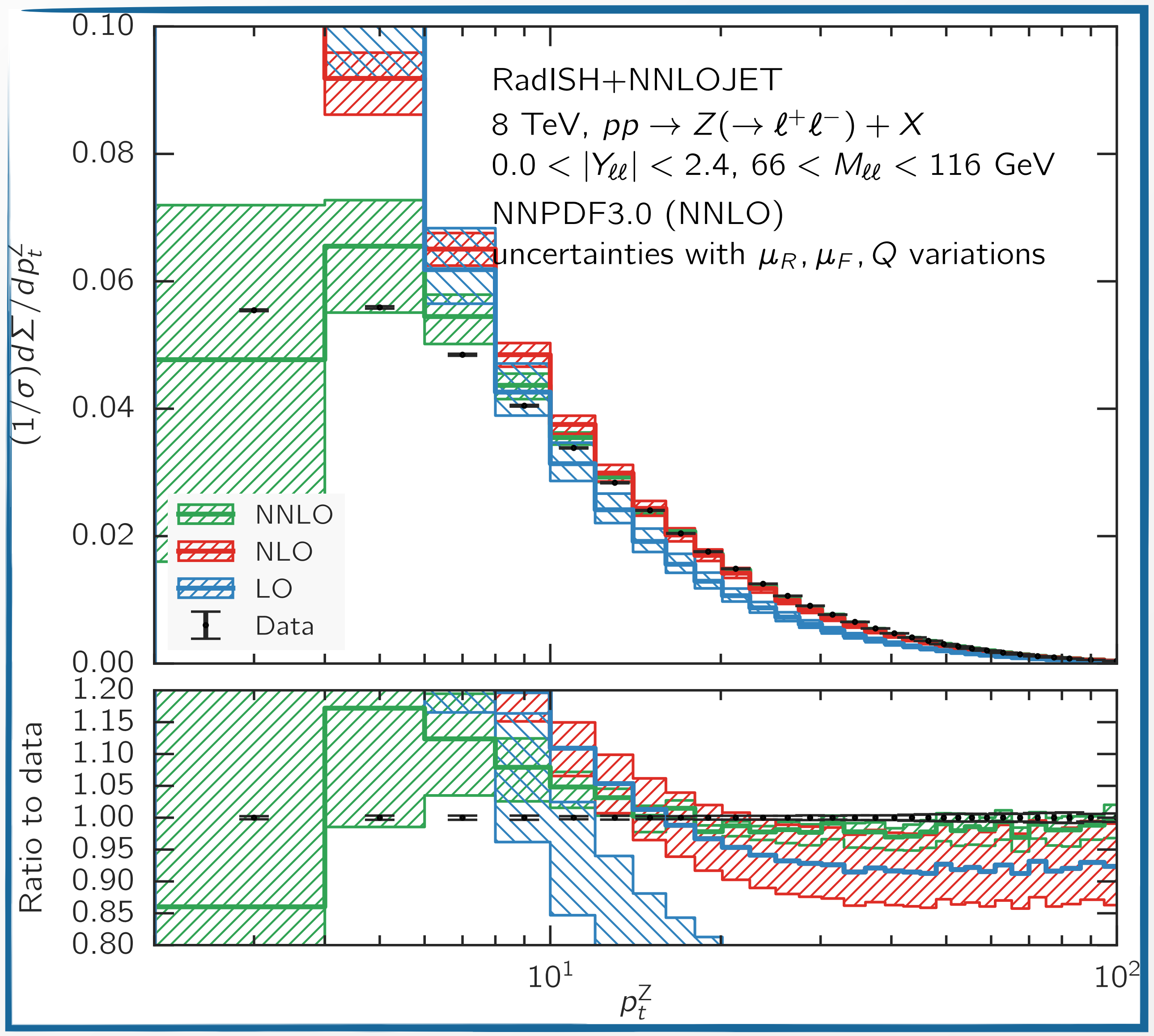
$$\ln \Sigma(v) = \sum_n \{\mathcal{O}(\alpha_s^n L^{n+1}) + \mathcal{O}(\alpha_s^n L^n) + \mathcal{O}(\alpha_s^n L^{n-1}) + \ldots\}$$

*It's the sum that makes the total

Example: transverse momentum spectrum

System with high invariant mass $M \gg p_t$, where the transverse momentum p_t vanishes at Born level

If $p_t \ll M$, the emission of real radiation is strongly suppressed. Double logarithms of p_t/M appear as a leftover of the real/virtual cancellation at all orders and **spoil the perturbative convergence** at small p_t

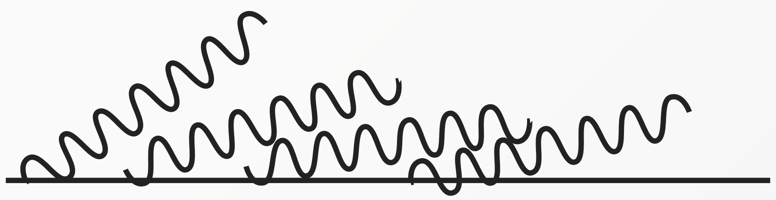


Logarithms of p_t/M must be resummed to reliably describe the **small p_t region**

Case study: transverse momentum p_t

Resummation of transverse momentum is particularly delicate because p_t is a **vectorial quantity**

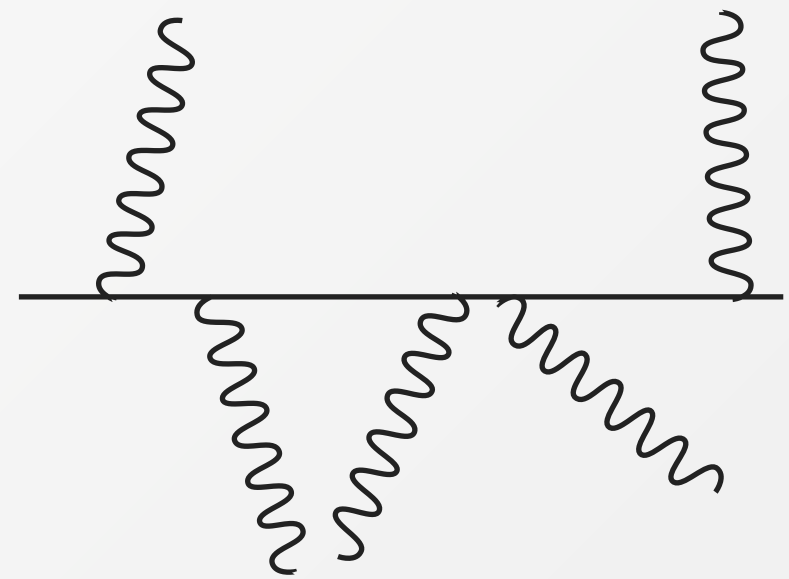
Two concurring mechanisms leading to a system with small p_t



$$p_t^2 \sim k_{t,i}^2 \ll M^2$$

cross section naturally suppressed as there is no phase space left for gluon emission (Sudakov limit)

Exponential suppression



$$\sum_{i=1}^{n} \vec{k}_{t,i} \simeq 0$$

Large kinematic cancellations

$p_t \sim 0$ far from the Sudakov limit

Power suppression

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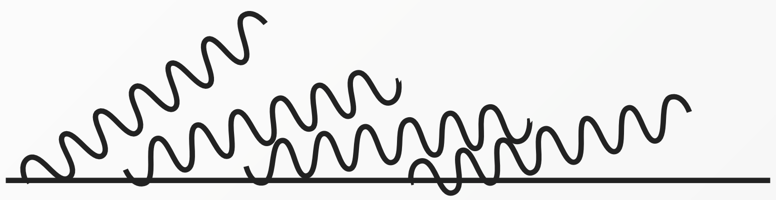

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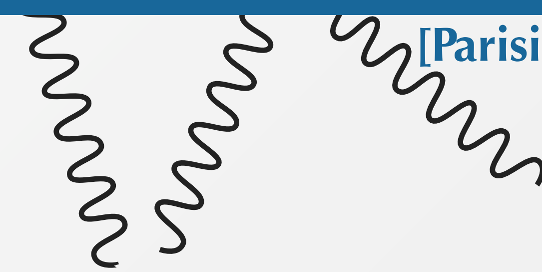
Exponential suppression

Dominant at small p_t

[Parisi, Petronzio '78]



$$\sum_{i=1}^{n} \vec{k}_{t,i} \simeq 0$$

Large kinematic cancellations

p_t ~0 far from the Sudakov limit

Power suppression

Resummation in conjugate space

Resummation usually performed in impact-parameter (b) space where the two competing mechanisms are handled trough a **Fourier transform**

$$\delta\left(\vec{p}_t - \sum_{i=1}^{n} \vec{k}_{t,i}\right) = \int d^2b \frac{1}{4\pi^2} e^{i\vec{b}\cdot\vec{p}_t} \prod_{i=1}^{n} e^{-i\vec{b}\cdot\vec{k}_{t,i}}$$

[Parisi, Petronzio '78; Collins, Soper, Sterman '85]

Transverse-momentum conservation is respected

All-order result

coefficient functions [Catani, Grazzini '11][Catani et al. '12] [Gehrmann,Luebbert, Yang '14]

$$\frac{d^2\Sigma(v)}{d\Phi_B dp_t} = \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \int b\, db\, p_t J_0(p_t b)\, \mathbf{f}^T(b_0/b) \mathbf{C}^{c_1;T}_{N_1}(\alpha_s(b_0/b)) H_{\mathrm{CSS}}(M) \mathbf{C}^{c_2}_{N_2}(\alpha_s(b_0/b)) \mathbf{f}(b_0/b)$$

hard-virtual corrections

$$\times \exp\left\{-\sum_{\ell=1}^{2} \int_0^M \frac{dk_t}{k_t} \mathbf{R}'_{\mathrm{CSS},\ell}(k_t)\, \Theta(k_t - \frac{b_0}{b})\right\}$$

$$R_{\mathrm{CSS}}(b) = \sum_{l=1}^{2} \int_{b_0/b}^{M} \frac{dk_T}{k_T} R'_{\mathrm{CSS},l}(k_T) = \sum_{l=1}^{2} \int_{b_0/b}^{M} \frac{dk_T}{k_T} \left(A_{\mathrm{CSS},\ell}(\alpha_s(k_T)) \ln\frac{M^2}{k_T^2} + B_{\mathrm{CSS},\ell}(\alpha_s(k_T)) \right)$$

anomalous dimensions

[Davies, Stirling '84] [De Florian, Grazzini '01] [Becher, Neubert '10][Li, Zhu '16][Vladimirov '16]

Resummation & factorization

All-order resummation based on **factorization properties**

- Of the **amplitudes**: when radiation becomes **soft and/or collinear** amplitudes factorize up to regular terms

  Necessary condition to establish an all-order formulation since the same structures **must** appear at all-orders

- Of the **observable**: in the presence of multiple emissions k_i, the observable is related to the radiation through phase-space constraints

$$\Sigma(v) \sim \int [dk_i] \mathcal{M}(k_1, \ldots, k_n) \Theta(v - V(k_1, \ldots k_n))$$

  Factorization seems required to disentangle the phase-space constraints

  Kinematic factorization is however process-dependent, and must be performed separately for each observable. Typically performed in a conjugate space where factorization is manifest, like for the p_t case

Resummation & factorization

Resummation techniques based on observable factorization **very successful** for various observables

However, approach have some limitations

- only observables for which a factorization theorem is known can be resummed
- since factorization is usually achieved in a conjugate space, one has to compute an inverse transform, which sometime causes numerical instabilities.

Is it possible to achieve resummation without the need to establish factorization properties on a case-by-case basis?

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Yes!

The CAESAR/ARES method: resummation in direct space

Translate the resummability of the observable into properties of the observable in the presence of multiple radiation: **recursive infrared and collinear safety** (rIRC)

[Banfi, Salam, Zanderighi '01, '03, '04]

a) in the presence of multiple soft and/or collinear emissions the observable has the **same scaling properties** as with just one of them

b) there exists a **resolution scale** q_0, **independent of the observable**, such that emissions below q_0 do not contribute significantly to the observable's value.

Unresolved emission can be treated as totally uncorrelated

Conjugate space unnecessary as resolved emission can be treated exclusively in momentum space with Monte Carlo methods

Only $N^{k-1}LL$ ingredients enter in $\mathcal{F}$ thanks to rIRC safety

$$\Sigma(v) = v \int \frac{dv_s}{v_s} \Sigma(v_s) \mathcal{F}(v, v_s)$$

Observable to resum at N^kLL accuracy

"Simple" observable whose resummation is known analytically

Resummation in direct space: the p_t case

Non-trivial problem: not possible to find a closed analytic expression in direct space which is both

a) free of logarithmically subleading corrections

b) free of singularities at finite p_t values

[Frixione, Nason, Ridolfi '98]

A naive logarithmic counting at small p_t is not sensible, as one loses the **correct power-suppressed scaling** if only logarithms are retained

It is not possible to reproduce a power-like behaviour with logs of p_t/M

Can we apply the CAESAR method to transverse-momentum resummation?

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Yes!

[Monni, Re, Torrielli '16]
[Bizon, Monni, Re, LR, Torrielli '17]

(alternative approaches for p_t resummation in direct space: [Ebert, Tackmann '16][Kang,Lee,Vaidya '17])

All-order structure of the matrix element

All-order cumulative cross section can be written as $v = p_t/M$

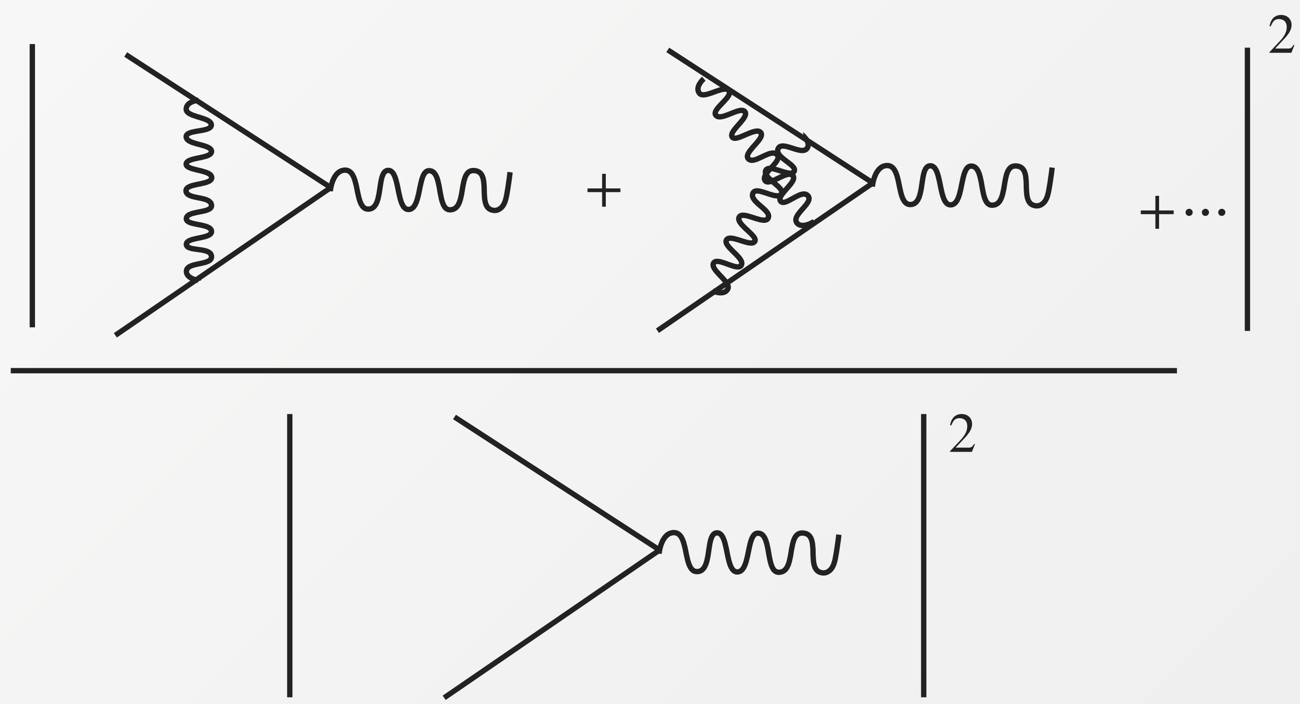
single-particle phase space

matrix element for *n* real emissions

$$\Sigma(v) = \int d\Phi_B \mathscr{V}(\Phi_B) \sum_{n=0}^{\infty} \int \prod_{i=1}^{n} [dk_i] \, |\mathscr{M}(\Phi_B, k_1, \ldots k_n)|^2 \, \Theta(v - V(\{\Phi_B\}, k_1, \ldots k_n))$$

all-order form factor

e.g. [Dixon, Magnea, Sterman '08]



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All-order structure of the matrix element

To find resummed expression one needs to establish an **explicit logarithmic counting** for the squared matrix element $|\mathcal{M}(\Phi_B, k_1, \ldots k_n)|^2$

Possibile to do that by decomposing the squared amplitude in terms of ***n*-particle correlated blocks,** denoted by $|\tilde{\mathcal{M}}(k_1, \ldots, k_n)|^2 \quad (|\tilde{\mathcal{M}}(k_1)|^2 = |\mathcal{M}(k_1)|^2)$

*expression valid for inclusive observables

$$\sum_{n=0}^{\infty} |\mathcal{M}(\Phi_B, k_1, \ldots, k_n)|^2 = |\mathcal{M}_B(\Phi_B|^2$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \left\{ \prod_{i=1}^{n} \left(|\mathcal{M}(k_i)|^2 + \int [dk_a][dk_b] |\tilde{\mathcal{M}}(k_a, k_b)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} - \vec{k}_{ti}) \delta(Y_{ab} - Y_i) \right.\right.$$

$$\left.\left. + \int [dk_a][dk_b][dk_c] |\tilde{\mathcal{M}}(k_a, k_b, k_c)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} + \vec{k}_{tc} - \vec{k}_{ti}) \delta(Y_{abc} - Y_i) + \ldots \right) \right\}$$

$$\equiv |\mathcal{M}_B(\Phi_B)|^2 \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} |\mathcal{M}(k_i)|^2_{\text{inc}}$$

In the soft-collinear limit, $\prod_{i=1}^{n} |\mathcal{M}(k_i)|$ comes with a factor $\alpha_s^n \ln^{2n}(v)$ whereas correlated blocks with *n* emissions $|\tilde{\mathcal{M}}(k_1, \ldots k_n)|$ contribute at most with $\alpha_s^n \ln^{n+1}(v)$ thanks to rIRC safety

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LL NLL NNLL

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$$\left.\left. + \int [dk_a][dk_b][dk_c] \underbrace{|\tilde{\mathcal{M}}(k_a, k_b, k_c)|^2}_{\text{NNLL}} \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} + \vec{k}_{tc} - \vec{k}_{ti}) \delta(Y_{abc} - Y_i) + \ldots \right) \right\}$$

$$\equiv |\mathcal{M}_B(\Phi_B)|^2 \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} |\mathcal{M}(k_i)|^2_{\text{inc}}$$

Upon integration over the phase space, the expansion can be put in a **one to one correspondence** with the logarithmic structure

$$\ln |\mathcal{M}(\Phi_B, k_1, \ldots, k_n)|^2 \to \mathcal{O}(\alpha_s^n \ln(v)^{n+1}) + \mathcal{O}(\alpha_s^n \ln(v)^n) + \mathcal{O}(\alpha_s^n \ln(v)^{n-1}) \ldots$$

Systematic recipe to include terms up to the desired logarithmic accuracy

Cancellation of the IRC singularities

Exploit rIRC safety of the observable to single out the IRC singularities of the real matrix element and achieve the **cancellation of the exponentiated divergences** of virtual origin

Introduce a slicing parameter ϵ ≪ 1 such that all inclusive blocks with $k_{t,i} < k_{t,1}$, $k_{t,1}$ hardest emission, can be neglected in the computation of the observable

$$\Sigma(v) = \int d\Phi_B |\mathcal{M}_B(\Phi_B)|^2 \mathcal{V}(\Phi_B)$$
$$\times \int [dk_1] |\mathcal{M}(k_1)|^2_{\text{inc}} \left(\sum_{l=0}^{\infty} \frac{1}{l!} \int \prod_{j=2}^{l+1} [dk_j] |\mathcal{M}(k_j)|^2_{\text{inc}} \Theta(\epsilon V(k_1) - V(k_j)) \right)$$
$$\times \left(\sum_{m=0}^{\infty} \frac{1}{m!} \int \prod_{i=2}^{m+1} [dk_i] |\mathcal{M}(k_i)|^2_{\text{inc}} \Theta(V(k_i) - \epsilon V(k_1)) \Theta\left(v - V(\Phi_B, k_1, \ldots, k_{m+1})\right) \right)$$

resolved emissions

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unresolved emissions

$$\Sigma(v) = \int d\Phi_B |\mathcal{M}_B(\Phi_B)|^2 \mathcal{V}(\Phi_B)$$
$$\times \int [dk_1] |\mathcal{M}(k_1)|^2_{\rm inc} \left(\sum_{l=0}^{\infty} \frac{1}{l!} \int \prod_{j=2}^{l+1} [dk_j] |\mathcal{M}(k_j)|^2_{\rm inc} \Theta(\epsilon V(k_1) - V(k_j)) \right.$$
$$\left. \times \left(\sum_{m=0}^{\infty} \frac{1}{m!} \int \prod_{i=2}^{m+1} [dk_i] |\mathcal{M}(k_i)|^2_{\rm inc} \Theta(V(k_i) - \epsilon V(k_1)) \Theta\left(v - V(\Phi_B, k_1, \ldots, k_{m+1})\right) \right) \right)$$

Unresolved emission doesn't contribute to the evaluation of the observable: it can be exponentiated directly and employed to cancel the virtual divergences, giving rise to a Sudakov radiator

$$\mathcal{V}(\Phi_B) \exp\left\{ \int [dk] |\mathcal{M}(k)|^2_{\rm inc} \Theta(\epsilon V(k_1) - V(k)) \right\} \simeq e^{-R(\epsilon V(k_1))}$$

Result at NLL accuracy

Result at NLL accuracy can be written as

$$\Sigma(v) = \sigma^{(0)} \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(\epsilon v_1)} R'\left(v_1\right) \qquad v_i = V(k_i), \quad \zeta_i = v_i/v_1$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'\left(\zeta_i v_1\right) \Theta\left(v - V(\Phi_B, k_1, \ldots, k_{n+1})\right)$$

Formula can be evaluated with Monte Carlo method; dependence on ϵ vanishes exactly and result is finite in four dimensions

It contains **subleading effect** which in the original CAESAR approach are disposed of by expanding R and R' around v

$$R(\epsilon v_1) = R(v) + \frac{dR(v)}{d\ln(1/v)} \ln\frac{v}{\epsilon v_1} + \mathcal{O}\left(\ln^2 \frac{v}{\epsilon v_1}\right)$$

$$R'\left(v_i\right) = R'(v) + \mathcal{O}\left(\ln\frac{v}{v_i}\right)$$

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$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^{1} \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'(\zeta_i v_1)\, \Theta\left(v - V(\Phi_B, k_1, \ldots, k_{n+1})\right)$$

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$$R'(v_i) = R'(v) + \mathcal{O}\left(\ln\frac{v}{v_i}\right)$$

Not possible! valid only if the ratio v_i/v remains of order one in the whole emission phase space, but for observables which feature kinematic cancellations there are configurations with $v_i \gg v$. **Subleading effects necessary**

Result at NLL accuracy

Result at NLL accuracy can be written as

$$\Sigma(v) = \sigma^{(0)} \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(\epsilon v_1)} R'\left(v_1\right) \qquad v_i = V(k_i), \quad \zeta_i = v_i/v_1$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^{1} \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'\left(\zeta_i v_1\right) \Theta\left(v - V(\Phi_B, k_1, \ldots, k_{n+1})\right)$$

Formula can be evaluated with Monte Carlo method; dependence on ϵ vanishes exactly and result is finite in four dimensions

Convenient to perform an expansion around v_1 (more efficient and simpler implementation)

$$R(\epsilon v_1) = R(v_1) + \frac{dR(v_1)}{d\ln(1/v_1)} \ln\frac{1}{\epsilon} + \mathcal{O}\left(\ln^2\frac{1}{\epsilon}\right)$$

$$R'\left(v_i\right) = R'(v_1) + \mathcal{O}\left(\ln\frac{v_1}{v_i}\right)$$

✓

Subleading effects retained: no divergence at small v, power-like behaviour respected

Result at NLL accuracy

Final result including **parton luminosity**

$$\frac{d\Sigma(v)}{d\Phi_B} = \int \frac{dk_{t,1}}{k_{t,1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(k_{t,1})} \epsilon^{R'(k_{t,1})} \mathscr{L}_{\mathrm{NLL}}(k_{t,1}) R'\left(k_{t,1}\right)$$
$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^{1} \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'\left(\zeta_i k_{t,1}\right) \Theta\left(v - V(\Phi_B, k_1, \ldots, k_{n+1})\right)$$

Parton luminosity at NLL reads

$$\mathscr{L}_{\mathrm{NLL}}(k_{t,1}) = \sum_c \frac{d|M_B|^2_{c\bar{c}}}{d\Phi_B} f_c\left(x_1, k_{t,1}^2\right) f_{\bar{c}}\left(x_2, k_{t,1}^2\right)$$

At higher logarithmic accuracy, it includes **coefficient functions** and **hard-virtual** corrections

This formula can be evaluated by means of fast Monte Carlo methods

RadISH (Radiation off Initial State Hadrons)

Result at N³LL accuracy

$$\frac{d\Sigma(v)}{d\Phi_B} = \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \partial_L \left(-e^{-R(k_{t1})} \mathcal{L}_{\mathrm{N^3LL}}(k_{t1})\right) \int d\mathcal{Z}[\{R', k_i\}] \Theta\left(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1})\right)$$

$$+ \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(k_{t1})} \int d\mathcal{Z}[\{R', k_i\}] \int_0^1 \frac{d\zeta_s}{\zeta_s} \frac{d\phi_s}{2\pi} \left\{ \left(R'(k_{t1}) \mathcal{L}_{\mathrm{NNLL}}(k_{t1}) - \partial_L \mathcal{L}_{\mathrm{NNLL}}(k_{t1}) \right) \right.$$
$$\times \left(R''(k_{t1}) \ln \frac{1}{\zeta_s} + \frac{1}{2} R'''(k_{t1}) \ln^2 \frac{1}{\zeta_s} \right) - R'(k_{t1}) \left(\partial_L \mathcal{L}_{\mathrm{NNLL}}(k_{t1}) - 2\frac{\beta_0}{\pi} \alpha_s^2(k_{t1}) \hat{P}^{(0)} \otimes \mathcal{L}_{\mathrm{NLL}}(k_{t1}) \ln \frac{1}{\zeta_s} \right)$$
$$\left. + \frac{\alpha_s^2(k_{t1})}{\pi^2} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathcal{L}_{\mathrm{NLL}}(k_{t1}) \right\} \left\{ \Theta\left(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1}, k_s)\right) - \Theta\left(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1})\right) \right\}$$

$$+ \frac{1}{2} \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(k_{t1})} \int d\mathcal{Z}[\{R', k_i\}] \int_0^1 \frac{d\zeta_{s1}}{\zeta_{s1}} \frac{d\phi_{s1}}{2\pi} \int_0^1 \frac{d\zeta_{s2}}{\zeta_{s2}} \frac{d\phi_{s2}}{2\pi} R'(k_{t1})$$
$$\times \left\{ \mathcal{L}_{\mathrm{NLL}}(k_{t1}) \left(R''(k_{t1})\right)^2 \ln \frac{1}{\zeta_{s1}} \ln \frac{1}{\zeta_{s2}} - \partial_L \mathcal{L}_{\mathrm{NLL}}(k_{t1}) R''(k_{t1}) \left(\ln \frac{1}{\zeta_{s1}} + \ln \frac{1}{\zeta_{s2}} \right) \right.$$
$$\left. + \frac{\alpha_s^2(k_{t1})}{\pi^2} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathcal{L}_{\mathrm{NLL}}(k_{t1}) \right\}$$
$$\times \left\{ \Theta\left(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1}, k_{s1}, k_{s2})\right) - \Theta\left(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1}, k_{s1})\right) - \right.$$
$$\left. \Theta\left(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1}, k_{s2})\right) + \Theta\left(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1})\right) \right\} + \mathcal{O}\left(\alpha_s^n \ln^{2n-6} \frac{1}{v}\right), \qquad (3.18)$$

Result formally equivalent to the *b*-space formulation

[Parisi, Petronzio '78][Collins, Soper, Sterman '85]

Implementation: matching to fixed order

$$\Sigma(v,\phi_B) = \int_0^v dv' \frac{d\sigma}{dv' d\phi_B}$$

Cumulative cross section should reduce to the fixed order at large v

$\rightarrow \Sigma_{\rm res}$ $\quad p_t \ll M_B$

$\rightarrow \Sigma_{\rm f.o.}$ $\quad p_t \gtrsim M_B$

Additive matching

$$\Sigma^{\rm matched}_{\rm add}(v) = \Sigma^{\rm res}(v) + \Sigma^{\rm f.o.}(v) - \Sigma^{\rm expanded}(v)$$

- perhaps more natural, simpler
- numerically delicate in the very small v limit as f.o. can be unstable

Multiplicative matching

$$\Sigma^{\rm mult}_{\rm matched}(v) = \Sigma_{\rm res}(v)\left[\frac{\Sigma_{\rm f.o.}(v)}{\Sigma_{\rm res}(v)}\right]_{\rm expanded}$$

- it allows one to extract the relative $O(\alpha^3)$ constant terms from the fixed-order whenever the N^{3}LO total cross section is known, e.g. Higgs
- only viable solution till constant terms are not known analytically to consistently match to NNLO
- numerically more stable as the physical suppression at small v cures potential instabilities

Implementation: matching to fixed order

$$\Sigma(v, \phi_B) = \int_0^v dv' \frac{d\sigma}{dv' d\phi_B}$$

Cumulative cross section should reduce to the fixed order at large v

$\rightarrow \Sigma_{\text{res}}$ $\quad p_t \ll M_B$

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Additive matching

$$\Sigma_{\text{add}}^{\text{matched}}(v) = \Sigma^{\text{res}}(v) + \Sigma^{\text{f.o.}}(v) - \Sigma^{\text{expanded}}(v)$$

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Multiplicative matching

$$\Sigma_{\text{matched}}^{\text{mult}}(v) = \Sigma_{\text{res}}(v) \left[\frac{\Sigma_{\text{f.o.}}(v)}{\Sigma_{\text{res}}(v)} \right]_{\text{expanded}}$$

- it allows one to extract the relative $O(\alpha^3)$ constant terms from the fixed-order whenever the N^{3}LO total cross section is known, e.g. Higgs
- only viable solution till constant terms are not known analytically to consistently match to NNLO
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Multiplicative matching

[Bizon, Monni, Re, LR, Torrielli + NNLOJET '18]

$$\Sigma^{\text{mult}}_{\text{matched}}(v) = \Sigma_{\text{res}}(v) \left[\frac{\Sigma_{\text{f.o.}}(v)}{\Sigma_{\text{res}}(v)} \right]_{\text{expanded}}$$

Drawback: the fixed-order result at large v receives **spurious contributions**; e.g. at N^3LO

$$\Sigma^{\text{matched}}_{\text{mult}}(v) \sim \Sigma^{\text{N}^3\text{LO}}(v)\left(1 + \mathcal{O}(\alpha_s^4)\right)$$

Reason: when logarithms L tend to zero, $\Sigma_{\text{res}}(v)$ tends to

$$\Sigma^{\text{res}}_{\text{asym.}} = \int_{\text{with cuts}} d\Phi_B \quad \left(\lim_{L\to 0} \mathcal{L}_{\text{N}^k\text{LL}} \right)$$

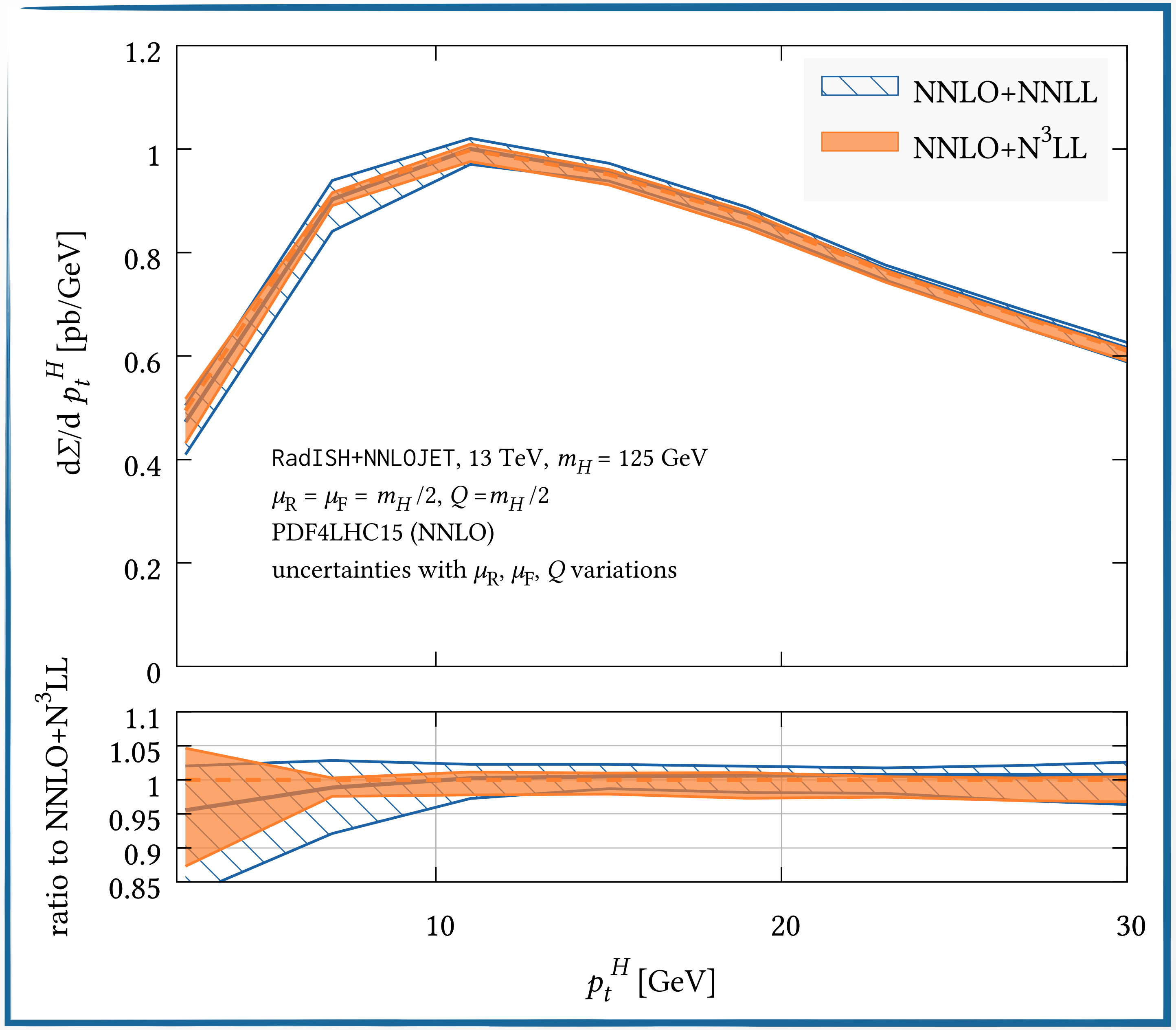
Solution: normalize to the **asymptotic value**

$$\Sigma^{\text{matched}}_{\text{mult}}(v) = \frac{\Sigma^{\text{res}}(v)}{\Sigma^{\text{res}}_{\text{asym.}}} \left[\Sigma^{\text{res}}_{\text{asym.}} \frac{\Sigma^{\text{f.o.}}(v)}{\Sigma^{\text{exp}}(v)} \right]_{\text{expanded}}$$

Higgs transverse momentum at NNLO+N^{3}LL: inclusive

[Bizon, Monni, Re, LR, Torrielli + NNLOJET '18]

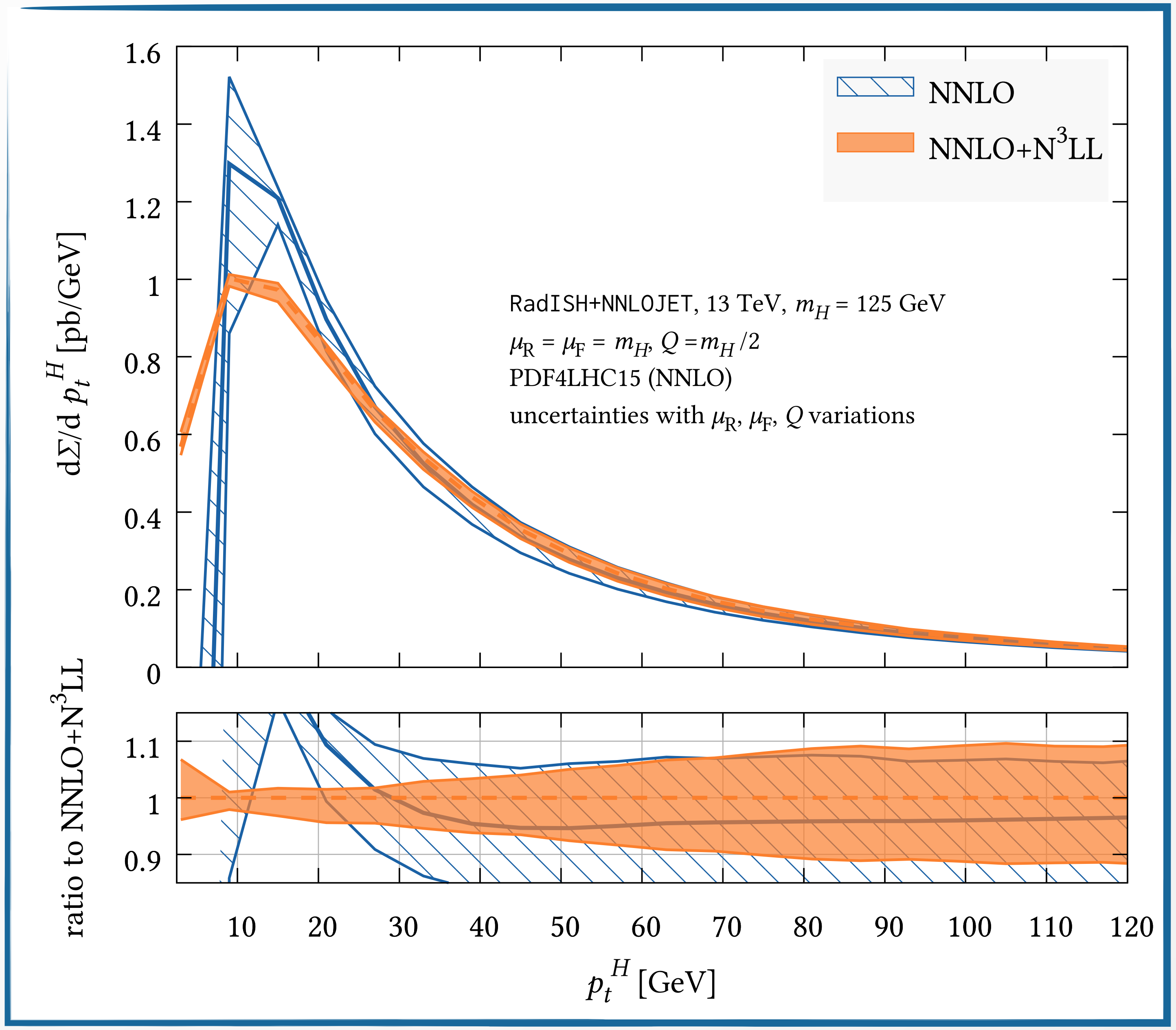
N^{3}LL vs NNLL



- N^{3}LL corrections moderate in size (~ 5% at low p_T) and contained in the NNLO+NNLL band
- Reduction of the perturbative uncertainty by a factor of 2 for $p_t \lesssim 10$ GeV

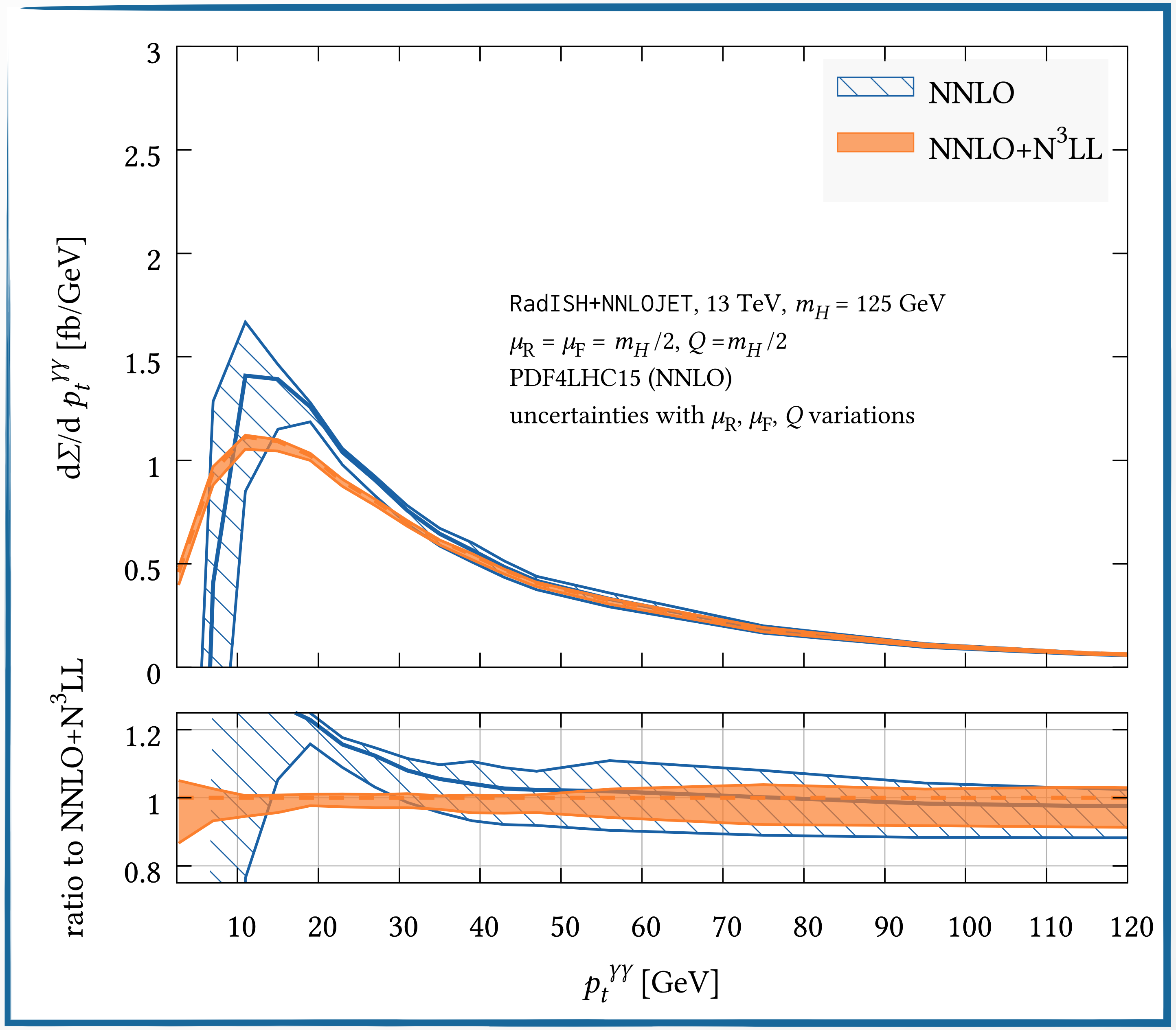
n.b. thanks to multiplicative scheme, NNLO+NNLL follows resummation scaling at low p_t

Higgs transverse momentum at NNLO+N³LL: inclusive



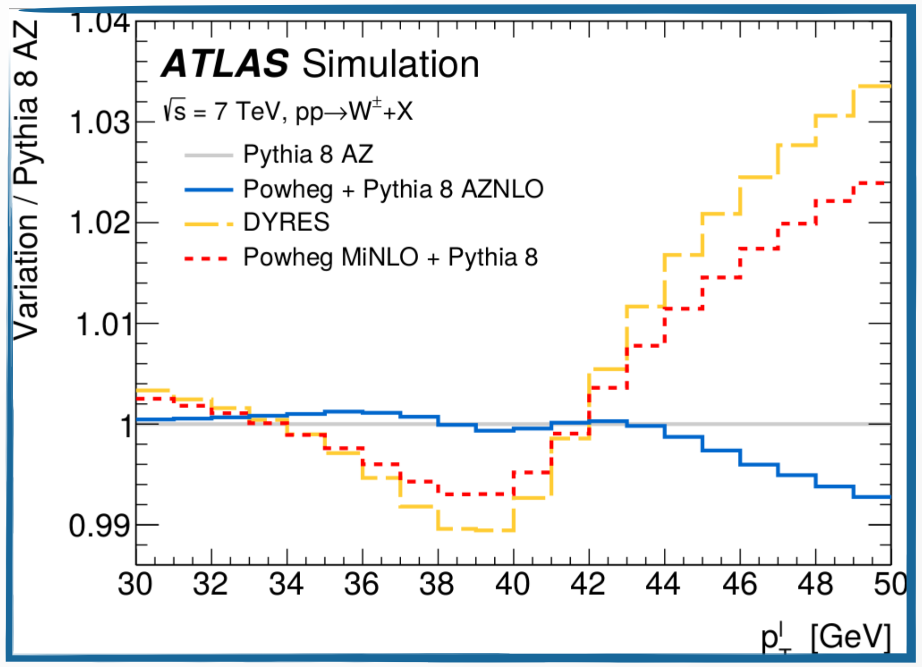
- Effect of resummation starts to be increasingly important for $p_t \lesssim 40$ GeV
- Resummation effects are progressively less important above 50 GeV
- Heavy-quark mass effects start to be relevant at this level of precision

Higgs transverse momentum at NNLO+N^{3}LL: fiducial cuts



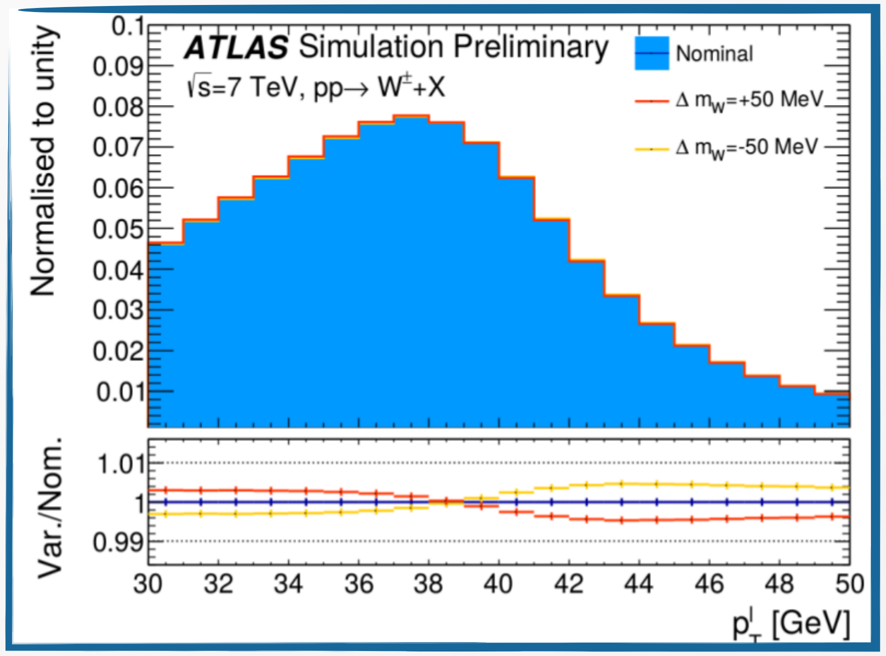
- Effect of resummation starts to be increasingly important for $p_t \lesssim 40$ GeV
- Resummation effects are progressively less important above 50 GeV
- Heavy-quark mass effects start to be relevant at this level of precision
- Similar results for fiducial region

Drell-Yan



- State-of-the-art QCD prediction do not match the precision of the data
- LO MC are used, tuned on *Z* data
- Would be preferable to use more accurate theoretical predictions

Extreme precision is needed for *W* mass extraction

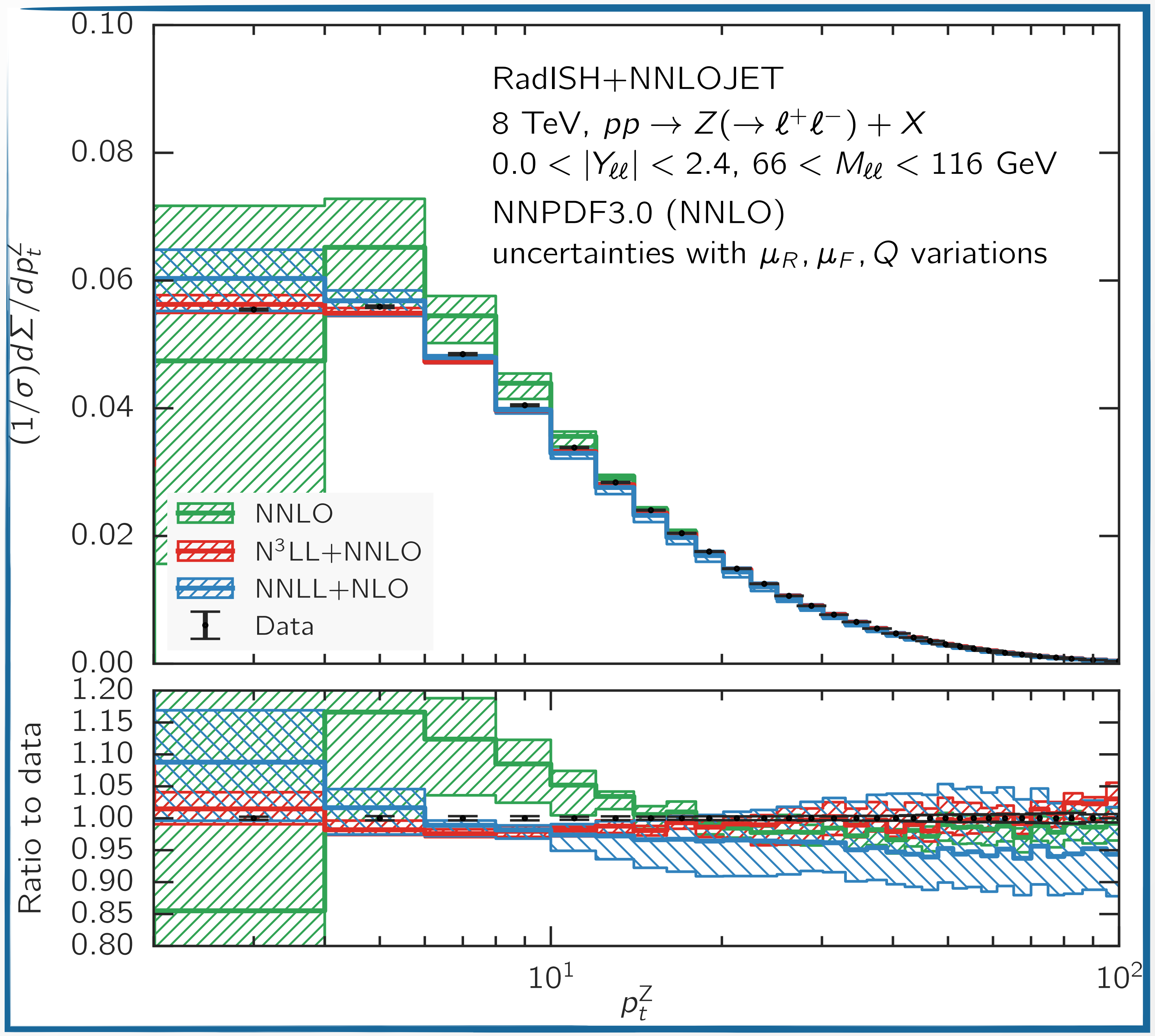


- Template fits to lepton observables
- Modelling of $p_{t,W}$ is crucial. Fit predictions to *Z* data, apply to *W*

Drell-Yan transverse momentum

[Bizon, Monni, Re, LR, Torrielli + NNLOJET '18]

Comparison with ATLAS data @ 8 TeV [1512.02192]



- Matched results offer a good description of the data in the low-medium p_T range, in all fiducial regions
- Perturbative uncertainty at the few percent level, still does not match the precision of the data
- Estimate of non-perturbative effects may start to be relevant

Drell-Yan φ^*

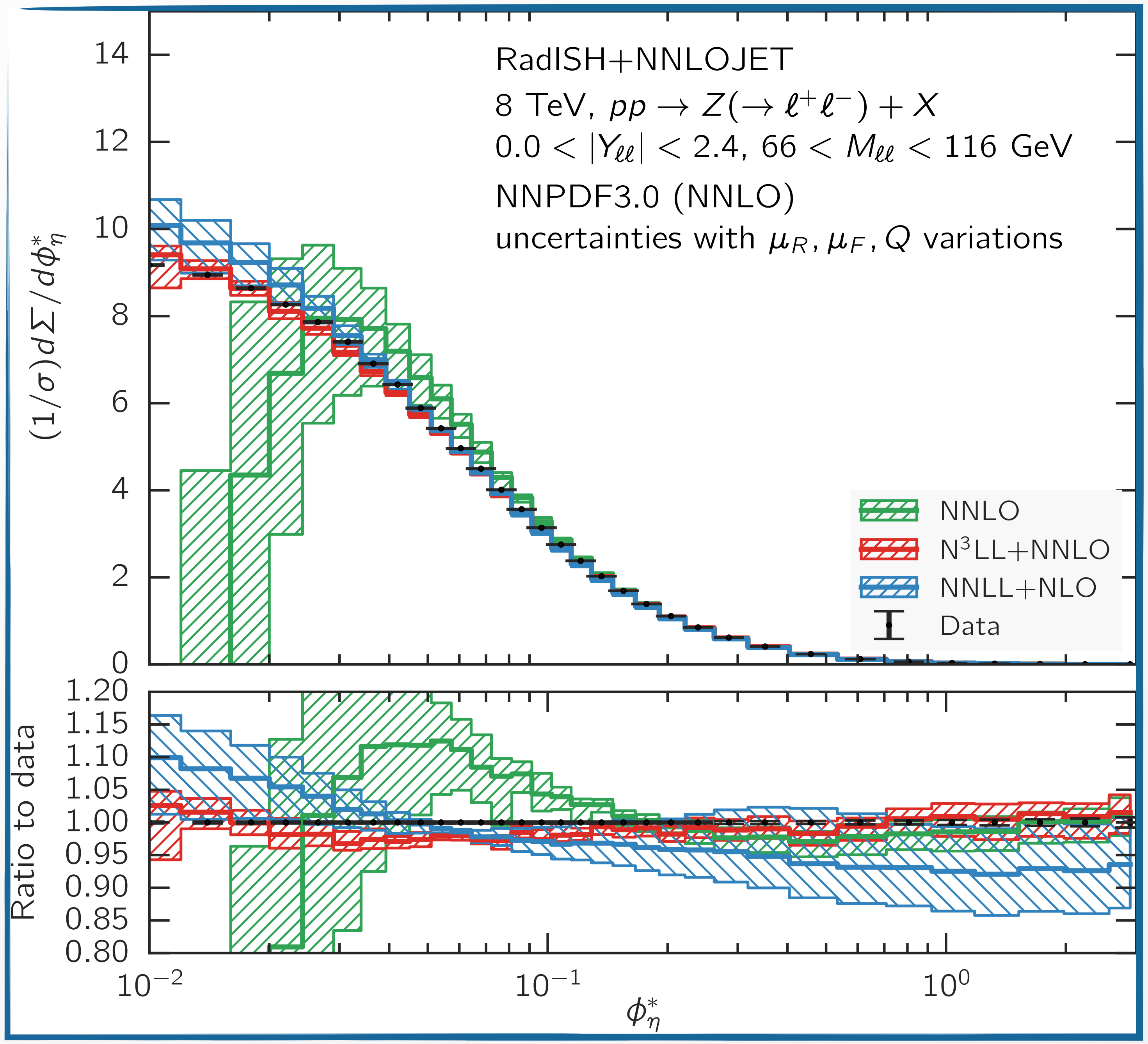
[Bizon, Monni, Re, LR, Torrielli + NNLOJET '18]

Approach can be used for resumming other transverse observables; e.g φ^*

$$\phi^* = \tan\left(\frac{\pi - \Delta\phi}{2}\right)\sin\theta^* \qquad \phi^* \sim \frac{p_t}{2M}$$

angle between electron and beam axis, in Z boson rest frame

Comparison with ATLAS data @ 8 TeV [1512.02192]



- Similar situation as p_t, with perturbative uncertainty at the few percent level but with experimental errors at the sub-percent level

Conclusion

- No sign of NP at the LHC so far - necessary to perform detailed theory/ experimental comparisons, to look for deviations from SM. Perturbation theory must be pushed at its limit

- New formalism formulated in **momentum space** for all-order resummation up to **N^3LL accuracy** for inclusive, transverse observables.

- Access to multi-differential information. This is effectively similar to a semi-inclusive parton shower, but with higher-order logarithms, and control on formal accuracy

- Method allows for an **efficient implementation in a computer code**. Towards a single generator able to resum entire classes of observables at high accuracy.

- Results at NNLO+N^3LL for Higgs and DY differential distributions

- Good description of the data in the fiducial distributions, with uncertainties at the few percent level

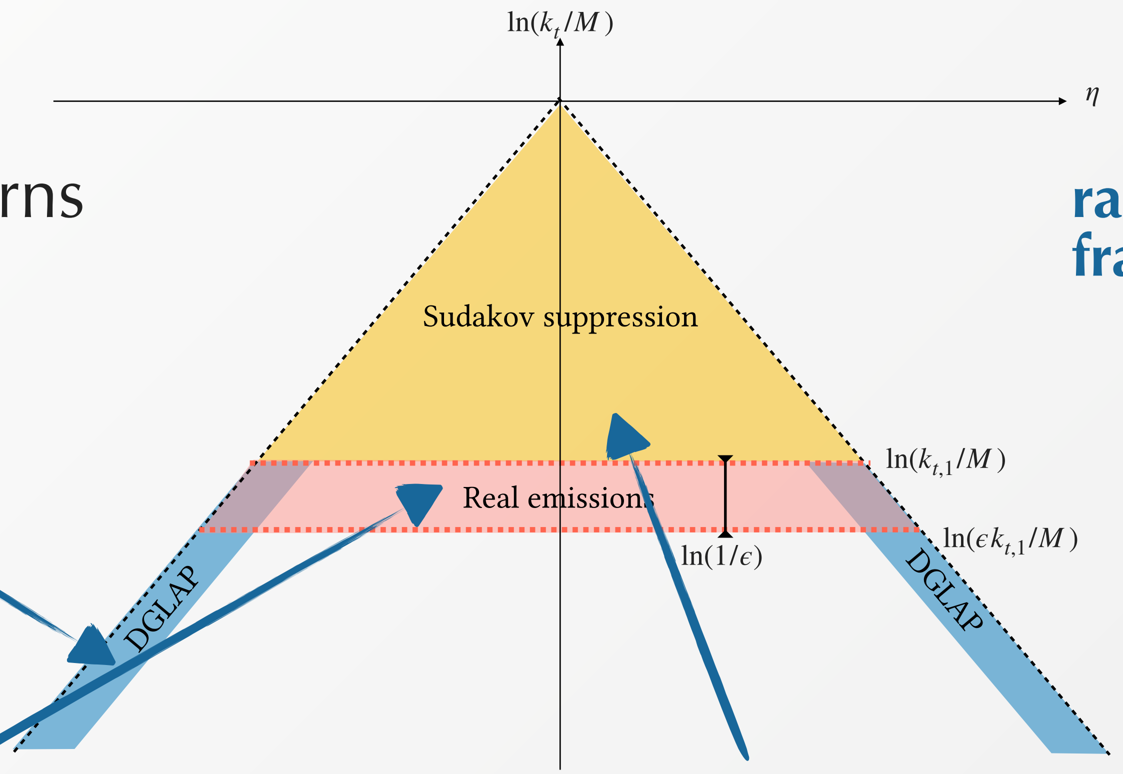
Backup

Parton luminosities

Consider configurations in which emissions are ordered in $k_{t,i}$, $k_{t,1}$ hardest emission

Phase space for each secondary emission can be depicted in the Lund diagram

DGLAP evolution governs the radiation in the strictly collinear limit



rapidity in the centre-of-mass frame of the incoming partons

resolved emissions live in this strip

remaining **unresolved real emissions** are combined with the virtual corrections to give rise to Sudakov suppression

- DGLAP evolution can be performed **inclusively** up to $\epsilon k_{t,1}$ thanks to rIRC safety
- In the **overlapping region** hard-collinear emissions modify the observable's value: the evolution should be performed exclusively (unintegrated in k_t)
- At NLL the real radiation can be approximated with its soft limit: DGLAP can be performed inclusively up to $k_{t,1}$ (i.e. one can evaluate $\mu_F = k_{t,1}$)

Beyond NLL

Extension to NNLL and beyond requires the systematic inclusion of the correlated blocks necessary to achieve the desired logarithmic accuracy

Moreover, one needs to **relax a series of assumptions** which give rise to subleading corrections neglected at NLL (for instance, exact rapidity bounds). These corrections can be included systematically by including additional terms in the expansion

$$R(\epsilon v_1) = R(v_1) + \frac{dR(v_1)}{d\ln(1/v_1)} \ln\frac{1}{\epsilon} + \mathcal{O}\left(\ln^2\frac{1}{\epsilon}\right)$$

Finally, one needs to specify a complete treatment for **hard-collinear radiation.** Starting at NNLL one or more real emissions can be hard and collinear to the emitting leg, and the available phase space for subsequent real emissions changes

Two classes of contributions:

- one soft by construction and which is analogous to the R′ contribution

$$R'\left(v_i\right) = R'(v_1) + \mathcal{O}\left(\ln\frac{v_1}{v_i}\right)$$

- another hard and collinear (exclusive DGLAP step): last step of DGLAP evolution must be performed unintegrated in k_t

Logarithmic counting

Necessary to establish a **well defined logarithmic counting**: possibile to do that by decomposing the squared amplitude in terms of n-particle correlated blocks (**nPC**)

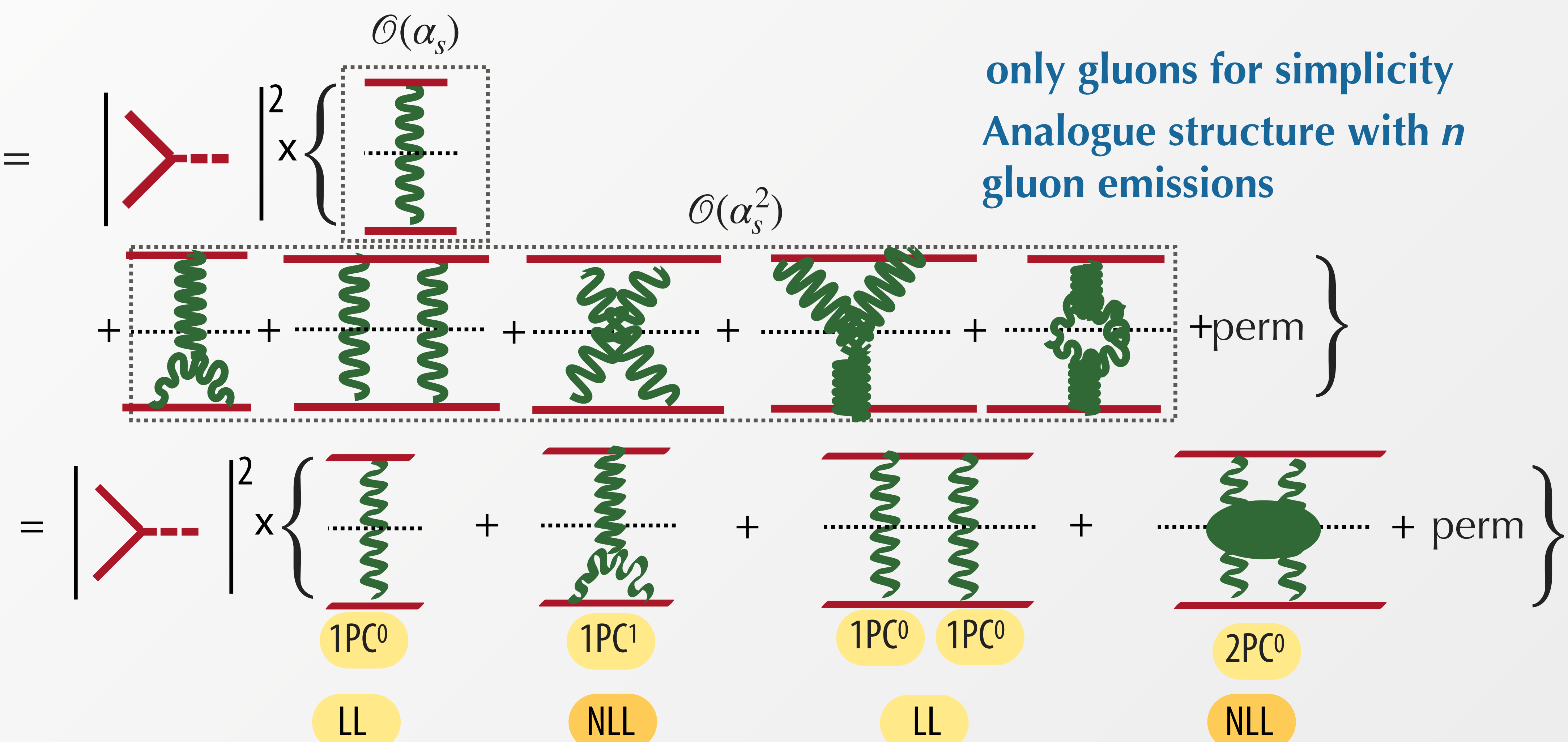
e.g. $pp \to H$ + emission of up to 2 (soft) gluons $O(\alpha_s^2)$

outgoing partons

$|M(p_1, p_2, k_1, k_2)|^2 =$

only gluons for simplicity

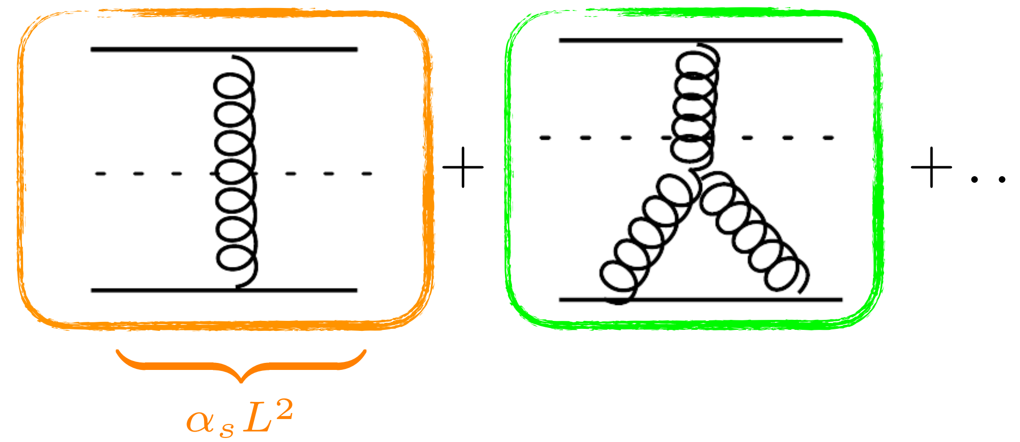
Analogue structure with *n* gluon emissions



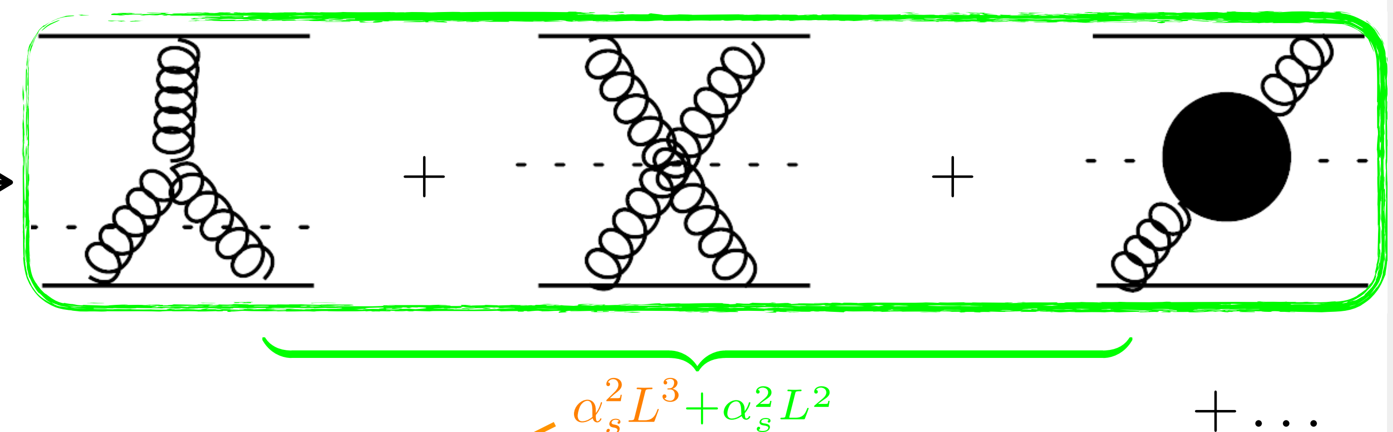
Logarithmic counting defined in terms of **nPC blocks** (owing to rIRC safety of the observable)

Logarithmic counting: correlated blocks

$$|\tilde{M}(k_a)|^2 = \frac{|M(\tilde{p}_1, \tilde{p}_2, k_a)|^2}{|M_B(\tilde{p}_1, \tilde{p}_2)|^2} = |M(k_a)|^2 \longrightarrow$$



$$|\tilde{M}(k_a, k_b)|^2 = \frac{|M(\tilde{p}_1, \tilde{p}_2, k_a, k_b)|^2}{|M_B(\tilde{p}_1, \tilde{p}_2)|^2} - \frac{1}{2!}\underbrace{|M(k_a)|^2 |M(k_b)|^2}_{\alpha_s^2 L^4} \longrightarrow$$



15 this LL is absorbed in the resummation of $|\mathrm{M(k)}|^2$

Thanks to P. Monni

Equivalence with *b*-space formulation

$$\frac{d\Sigma(v)}{d\Phi_B} = \int_{\mathscr{C}_1} \frac{dN_1}{2\pi i} \int_{\mathscr{C}_2} \frac{dN_2}{2\pi i} x_1^{-N_1} x_2^{-N_2} \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \mathbf{f}^T_{N_1}(\mu_0) \hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) \mathbf{f}_{N_2}(\mu_0)$$

unresolved emission + virtual corrections

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi}$$

$$\times\, e^{-\mathbf{R}(\epsilon k_{t1})} \exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t}\frac{\alpha_s(k_t)}{\pi}\mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$

Result valid for all inclusive observables (e.g. p_t, φ^*)

resolved emission

$$\sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi}\mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$

$$\times \sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=2}^{n+1}\int_\epsilon^1 \frac{d\zeta_i}{\zeta_i}\int_0^{2\pi}\frac{d\phi_i}{2\pi}\sum_{\ell_i=1}^{2}\left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi}\mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right)$$

$$\times\, \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right)$$

Formulation **equivalent to *b*-space** result (up to a **scheme change** in the anomalous dimensions)

$$\frac{d^2\Sigma(v)}{d\Phi_B dp_t} = \sum_{c_1,c_2}\frac{d|M_B|^2_{c_1c_2}}{d\Phi_B}\int b\, db\, p_t J_0(p_t b)\, \mathbf{f}^T(b_0/b)\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(b_0/b))H(M)\mathbf{C}^{c_2}_{N_2}(\alpha_s(b_0/b))\mathbf{f}(b_0/b)$$

$$\times \exp\left\{-\sum_{\ell=1}^{2}\int_0^M \frac{dk_t}{k_t}\mathbf{R}'_\ell(k_t)(1-J_0(bk_t))\right\}$$

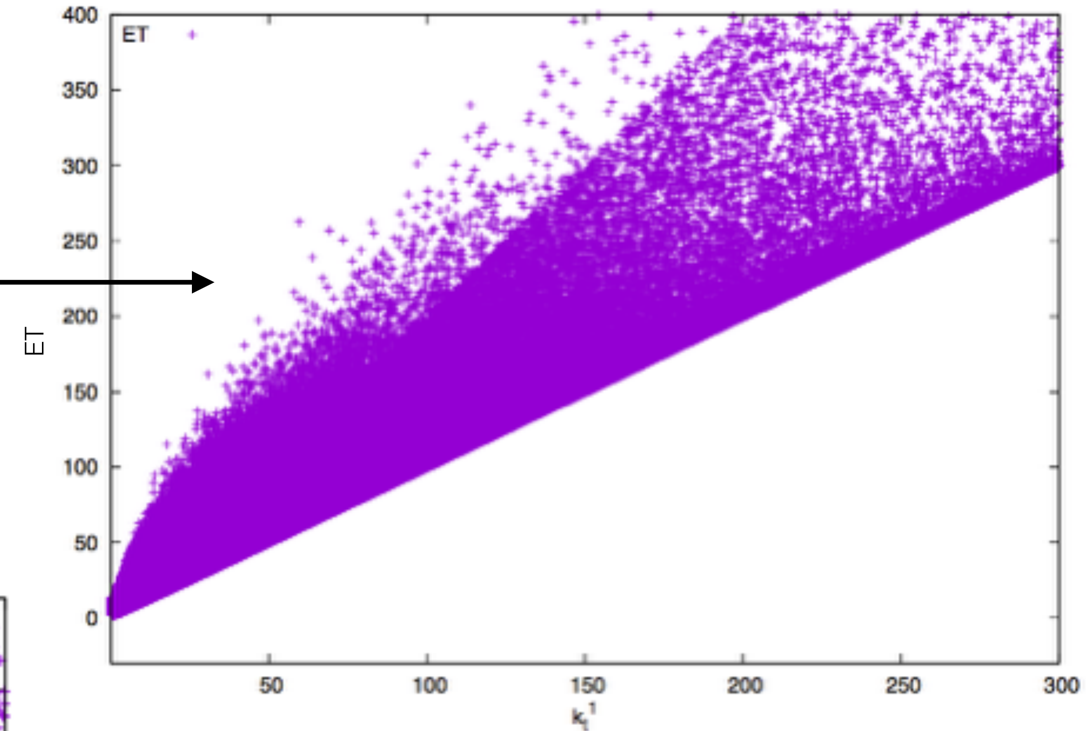
$$(1-J_0(bk_t)) \simeq \Theta(k_t - \frac{b_0}{b}) + \frac{\zeta_3}{12}\frac{\partial^3}{\partial \ln(Mb/b_0)^3}\Theta(k_t - \frac{b_0}{b})$$

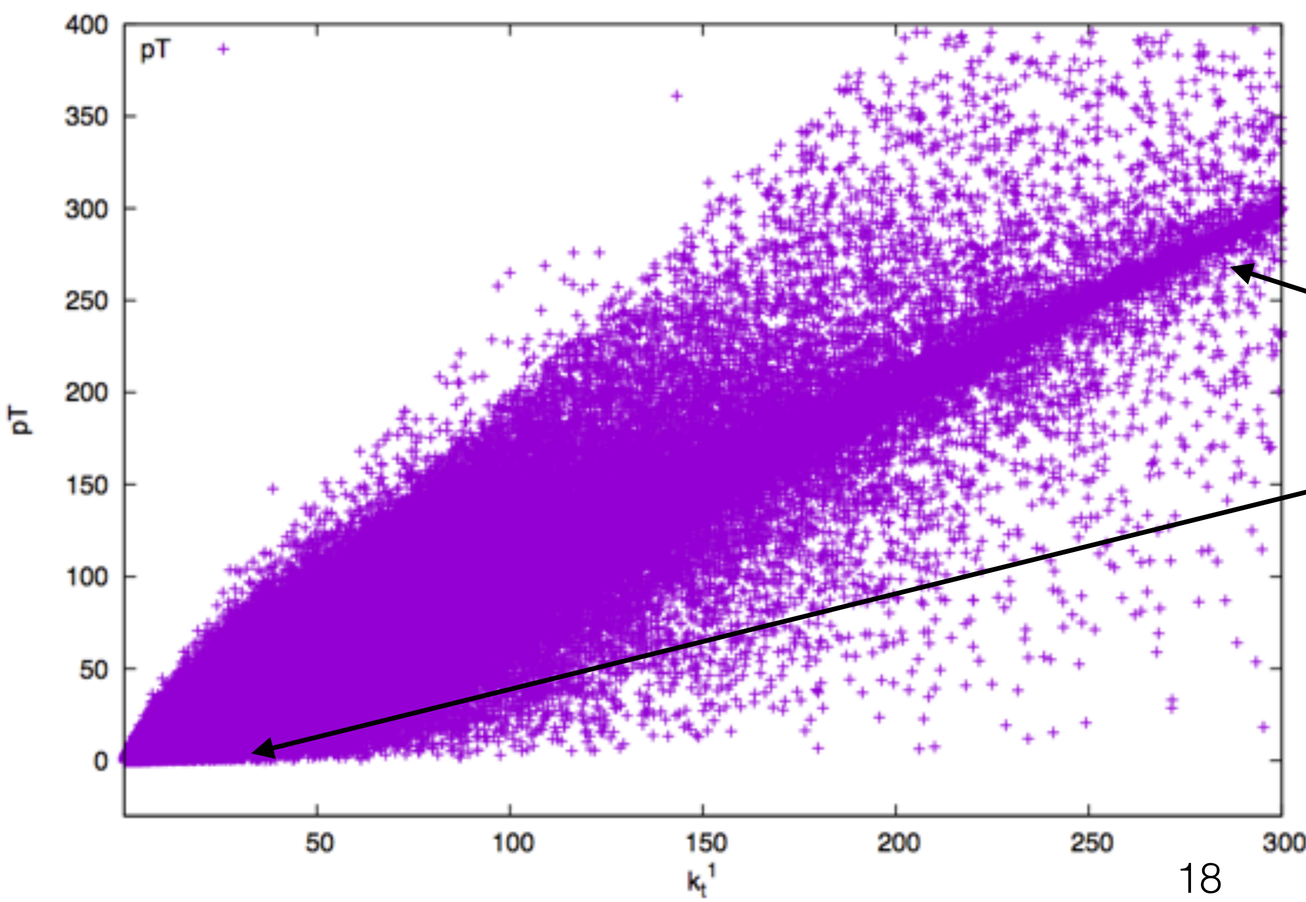
N^{3}LL effect: absorbed in the definition of H_2, B_3, A_4 coefficients wrt to CSS

Behaviour at small p_t

pT vs. ET: dependence on the first emission

Transverse Energy: single (Sudakov) suppression mechanism for all values of kt1





Transverse Momentum:

$R'(k_{t1}) \ll 1$: few emissions $\rightarrow p_T \sim k_{t1}$

$R'(k_{t1}) \geq 2$: many emissions $\rightarrow$ azimuthal cancel.

At some value of $R'(k_{t1})$ a transition takes place and the more likely way to get pT->0 becomes the second mechanism

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Thanks to P. Monni

Behaviour at small p_t

Explicit evaluation shows that the Parisi-Petronzio perturbative scaling at small p_t is reproduced. At NLL, Drell-Yan pair production, n_f=4

$$\frac{d^2\Sigma(v)}{dp_t d\Phi_B} = 4\,\sigma^{(0)}(\Phi_B)\,p_t \int_{\Lambda_{\rm QCD}}^{M} \frac{dk_{t1}}{k_{t1}^3} e^{-R(k_{t1})} \simeq 2\sigma^{(0)}(\Phi_B) p_t \left(\frac{\Lambda_{\rm QCD}^2}{M^2}\right)^{\frac{16}{25}\ln\frac{41}{16}}$$

As now higher logarithmic terms (up to N^3LL) are under control, the coefficient of this scaling can be systematically improved in *perturbation* theory (non-perturbative effects – of the same order – not considered)

N^3LL calculation allows one to have control over the terms of relative order $O(\alpha_s^2)$.
Scaling $L \sim 1/\alpha_s$ valid in the deep infrared regime.

Numerical implementation

$$\frac{d\Sigma(p_t)}{d\Phi_B} = \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} \partial_L \left(-e^{-R'(k_{t1})} \mathcal{L}_{\mathrm{NLL}}(k_{t1})\right) \times$$
$$\times \underbrace{\epsilon^{R'(k_{t1})} \sum_{n=0}^{\infty} \frac{1}{n!} \left(\prod_{i=2}^{n+1} \int_{\epsilon k_{t1}}^{k_{t1}} \frac{dk_{ti}}{k_{ti}} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'(k_{t1}) \right) \Theta(p_t - |\vec{k}_{t1} + \ldots + \vec{k}_{t(n+1)}|)}_{\equiv \int d\mathcal{Z}[\{R', k_i\}] \Theta(p_t - |\vec{k}_{t1} + \ldots + \vec{k}_{t(n+1)}|)} .$$

- $L = \ln(M/k_{t1})$; luminosity $\mathcal{L}_{\mathrm{NLL}}(k_{t1}) = \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1 c_2}}{d\Phi_B} f_{c_1}(x_1, k_{t1}) f_{c_2}(x_2, k_{t1})$.
- $\int d\mathcal{Z}[\{R', k_i\}]\Theta$ finite as $\epsilon \to 0$:

$$\epsilon^{R'(k_{t1})} = 1 - R'(k_{t1}) \ln(1/\epsilon) + \ldots = 1 - \int_{\epsilon k_{t1}}^{k_{t1}} R'(k_{t1}) + \ldots,$$

$$\int d\mathcal{Z}[\{R', k_i\}]\Theta = \left[1 - \int_{\epsilon k_{t1}}^{k_{t1}} R'(k_{t1}) + \ldots\right] \left[\Theta(p_t - |\vec{k}_{t1}|) + \int_{\epsilon k_{t1}}^{k_{t1}} R'(k_{t1}) \Theta(p_t - |\vec{k}_{t1} + \vec{k}_{t2}|) + \ldots\right]$$
$$= \Theta(p_t - |\vec{k}_{t1}|) + \underbrace{\int_0^{k_{t1}}}_{\epsilon \to 0} R'(k_{t1}) \underbrace{\left[\Theta(p_t - |\vec{k}_{t1} + \vec{k}_{t2}|) - \Theta(p_t - |\vec{k}_{t1}|)\right]}_{\text{finite: real-virtual cancellation}} + \ldots$$

- Evaluated with Monte Carlo techniques: $\int d\mathcal{Z}[\{R', k_i\}]$ is generated as a parton shower over secondary emissions.

Thanks to P. Torielli

Numerical implementation

- Secondary radiation:

$$d\mathcal{Z}[\{R', k_i\}] = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\prod_{i=2}^{n+1} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \int_{\epsilon k_{t1}}^{k_{t1}} \frac{dk_{ti}}{k_{ti}} R'(k_{t1}) \right) \epsilon^{R'(k_{t1})}$$

$$= \sum_{n=0}^{\infty} \left(\prod_{i=2}^{n+1} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \int_{\epsilon k_{t1}}^{k_{t(i-1)}} \frac{dk_{ti}}{k_{ti}} R'(k_{t1}) \right) \epsilon^{R'(k_{t1})},$$

$$\epsilon^{R'(k_{t1})} = e^{-R'(k_{t1}) \ln 1/\epsilon} = \prod_{i=2}^{n+2} e^{-R'(k_{t1}) \ln k_{t(i-1)}/k_{ti}},$$

  with $k_{t(n+2)} = \epsilon k_{t1}$.

- Each secondary emissions has differential probability

$$dw_i = \frac{d\phi_i}{2\pi} \frac{dk_{ti}}{k_{ti}} R'(k_{t1}) e^{-R'(k_{t1}) \ln k_{t(i-1)}/k_{ti}} = \frac{d\phi_i}{2\pi} d\left(e^{-R'(k_{t1}) \ln k_{t(i-1)}/k_{ti}} \right).$$

- $k_{t(i-1)} \geq k_{ti}$. Scale k_{ti} extracted by solving $e^{-R'(k_{t1}) \ln k_{t(i-1)}/k_{ti}} = r$, with r random number extracted uniformly in $[0, 1]$. Shower ordered in k_{ti}.

- Extract ϕ_i randomly in $[0, 2\pi]$.

Thanks to P. Torielli

Factorization and resummation in conjugate space

Phase-space constraints do not usually factorize in **direct space**

Resummation usually performed in impact-parameter (b) space where the two competing mechanisms are handled trough a **Fourier transform. Transverse-momentum conservation** is respected

$$\delta\left(\vec{p}_t - \sum_{i=1}^{n} \vec{k}_{t,i}\right) = \int d^2b \frac{1}{4\pi^2} e^{i\vec{b}\cdot\vec{p}_t} \prod_{i=1}^{n} e^{-i\vec{b}\cdot\vec{k}_{t,i}}$$

[Parisi, Petronzio '78; Collins, Soper, Sterman '85]

Extremely successful approach

Some limitations

- resummation tied to the existence of a resummation theorem for the observable
- process-dependent, must be performed manually and analytically in each case (error prone)
- inverse transform sometime causes numerical instabilities

Resumming in direct space: the p_t case

Non-trivial problem: not possible to find a closed analytic expression in direct space which is both

a) free of logarithmically subleading corrections

b) free of singularities at finite p_t values

[Frixione, Nason, Ridolfi '98]

A naive logarithmic counting at small p_t is not sensible, as one loses the **correct power-suppressed scaling** if only logarithms are retained

Resummation in direct space now possible

[Monni, Re, Torrielli '16, Bizon, Monni, Re, LR, Torielli '17]
[Ebert, Tackmann '16]
see also [Kang,Lee,Vaidya '17]