Higgs and Drell-Yan transverse observables at N^{3}LL+NNLO

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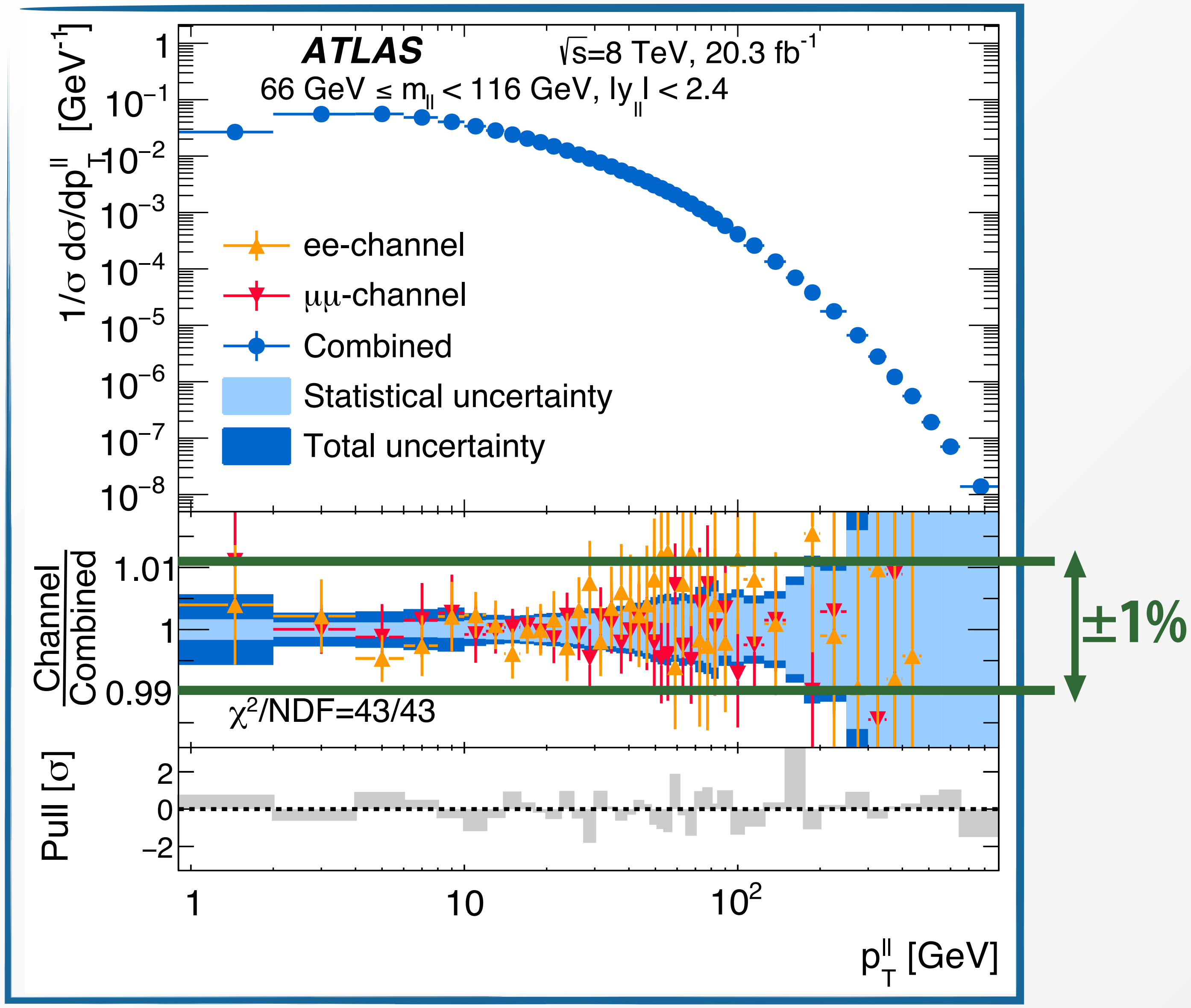
Monni, Re, Torrielli '16; Bizon, Monni, Re, LR, Torrielli '17; Bizon et al, in preparation

LHC in the precision era



- LHC is delivering a wealth of very precise data: measurements at % level (or even smaller) are available for several processes
- ~40 fb^{-1} delivered in 2016, ~50 fb^{-1} in 2017
- Increase in statistics enables study of **differential distributions** in detail
- Astonishing level of precision reached in e.g. Z transverse momentum: luminosity and other systematics are cancelled or reduced if results are normalized by fiducial cross section

Transverse observables in colour-singlet production

Transverse observables offer a particularly **clean experimental and theoretical environment** for **precision physics**

Parameterized as

$$V(k) = \left(\frac{k_t}{M}\right)^a f(\phi) \qquad M \sim \text{ singlet scale}$$

for a single soft QCD emission k collinear to incoming leg. Independent of the rapidity of radiation. $V \to 0$ for soft/collinear radiation.

Inclusive observables (p_T, φ^*) probe directly the kinematics of the colour singlet

$$V(k_1, \ldots k_n) = V(k_1 + \ldots + k_n)$$

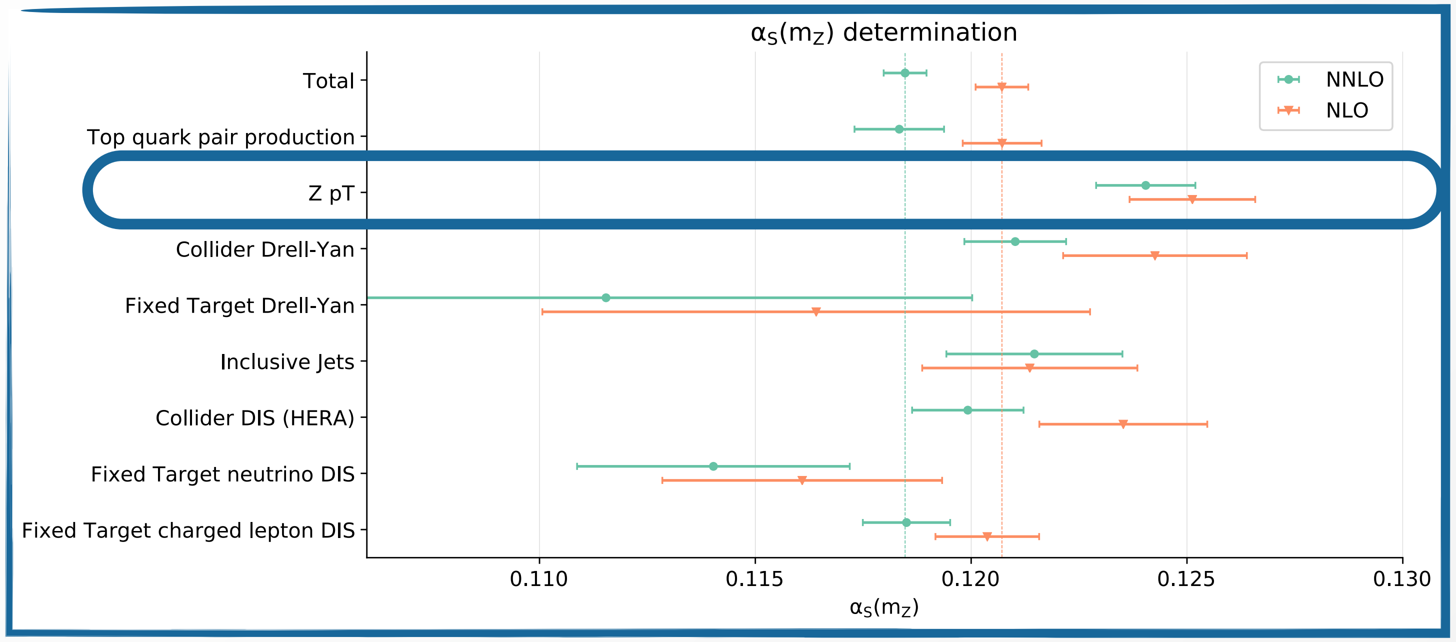
- negligible sensitivity to multi-parton interactions
- reduced sensitivity to non-perturbative effects
- measured extremely precisely at experiments (sub-percent in Z differential)

Necessary to push perturbation theory to its limit

Transverse observables at the LHC

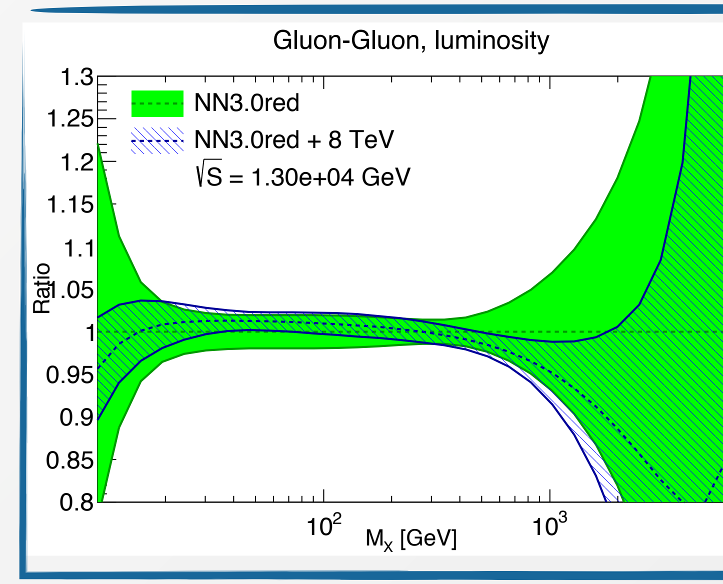
- Implications both for **SM measurements**…

Strong coupling



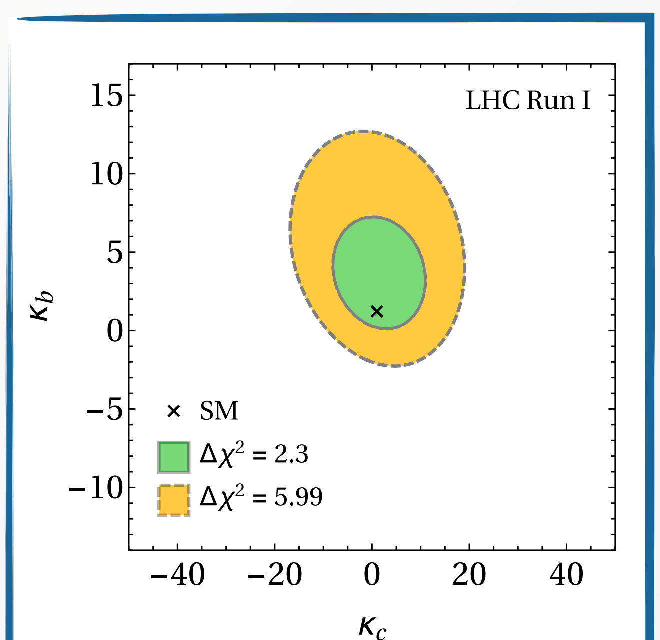
[NNPDF collab., 1802.03398]

PDF

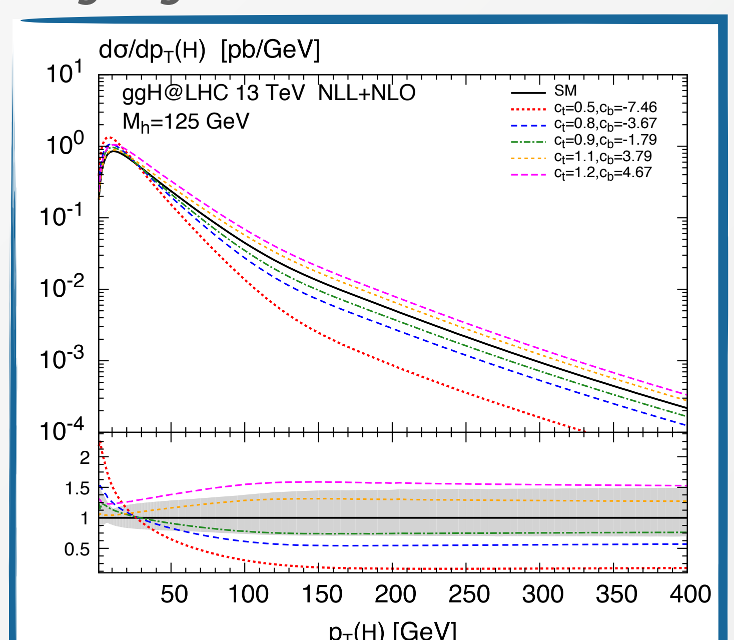


[Boughezal et al.,1705.00343]

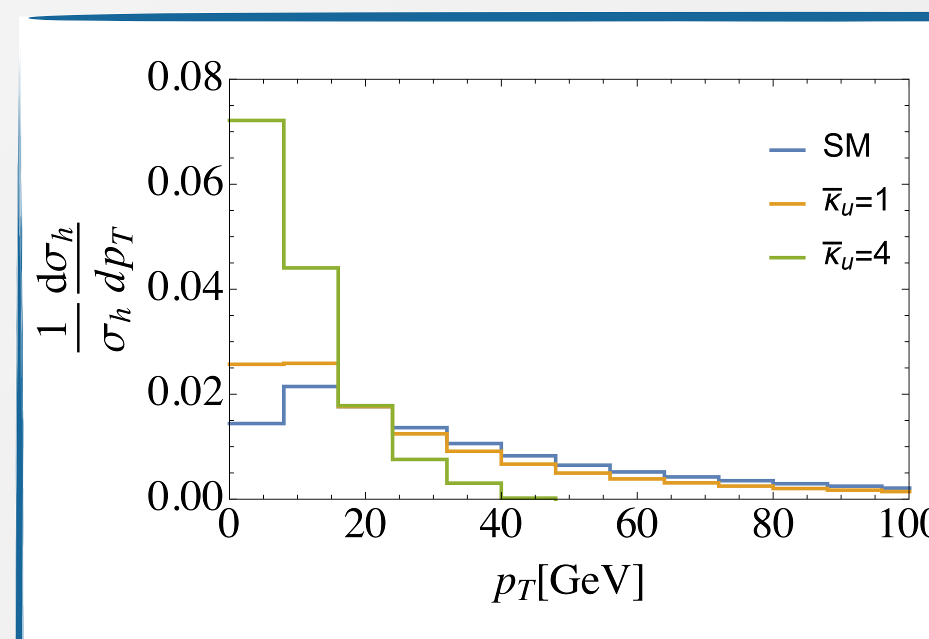
- …and **BSM measurements** (e.g. light Yukawa)



[Bishara et al.,1606.09253]



[Grazzini et al, 1612.00283]



[Soreq et al, 1606.09621]

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Resummation of transverse observables

In regions dominated by soft and collinear radiation, the perturbative expansion of the ***cumulative*** cross section

$$\Sigma(v) = \int_0^v dV \frac{d\sigma}{dV} \sim \alpha_s^{\mathrm{B}}[1 + \alpha_s + \alpha_s^2 + \dots]$$

LO NLO NNLO

is spoiled as large logarithms appear at all orders in perturbation theory

$$\frac{d\sigma}{dv} \sim \frac{1}{v}\alpha_s^n L^k, \qquad k \leq 2n-1, \qquad L \equiv \ln v$$

Resummation of enhanced terms to all orders in perturbation theory

$$\ln \Sigma(v) = \sum_n \{\mathcal{O}(\alpha_s^n L^{n+1}) + \mathcal{O}(\alpha_s^n L^n) + \mathcal{O}(\alpha_s^n L^{n-1}) + \dots\}$$

LL NLL NNLL

Resummation usually performed in a conjugate space where the observable factorizes: log-enhanced contributions built starting from simpler blocks, then transform back to physical space

Not always possible/convenient: observables may not factorize at all, or need several nested transforms, making resummation cumbersome.

Resummation of transverse observables

Factorization of the observable is however **not necessary**: resummability of the observable can be translated into scaling properties of the observable in presence of multiple emissions: **recursive Infrared and Collinear (rIRC) Safety**

[Banfi, Salam, Zanderighi, 0112156, 0304148, 0407286]

"[. . .] when there are emissions on multiple widely separated scales, it should always be possible to remove the ***softer/more collinear ones*** *without affecting the value of the observable"*

- Scaling properties of V are the same for any number of soft/collinear emissions.
- Properties unchanged if one adds infinitely soft/collinear emission: the more soft/collinear, the less it contributes to the value of the observable.

'CAESAR/ARES' approach: resummation of rIRC observables **performed in direct space**!

Momentum space formulation

[Monni, Re, Torrielli '16,Bizon, Monni, Re, LR, Torrielli '17]
(SCET: [Ebert, Tackmann '16;Kang,Lee,Vaidya '17])

All-order cumulative cross section can be written as

single-particle phase space

$$\Sigma(v) = \int d\Phi_B \mathcal{V}(\Phi_B) \sum_{n=0}^{\infty} \int \prod_{i=1}^{n} [dk_i] |M(\tilde{p}_1, \tilde{p}_2, k_1, \ldots k_n)|^2 \Theta(v - V(\{\tilde{p}\}, k_1, \ldots k_n))$$

Born phase space

all-order form factor
e.g. [Dixon, Magnea, Sterman '08]

all-order real amplitude squared

rIRC safety allows to

- exponentiate *unresolved* radiation (smaller than fraction ϵ of the hardest emission k_{T1})

divergences contained in $V(\Phi_B)$ are cancelled at all orders

Hard-virtual corrections

Sudakov Radiator

$$\mathcal{V}(\Phi_B) \rightarrow \sim H \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(\epsilon k_{t1})}$$

- establish a well defined logarithmic counting

Possibile to do that by decomposing the squared amplitude in terms of *n*-particle correlated blocks: correlated blocks with *n* particles start contributing one logarithmic order higher than those with *n-1* particles

$$\sum_{n=0}^{\infty} |M(\tilde{p}_1, \tilde{p}_2, k_1, \ldots, k_n)|^2 = |M_B(\tilde{p}_1, \tilde{p}_2)|^2$$

LL **NLL**

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \left\{ \prod_{i=1}^{n} \left(|M(k_i)|^2 + \int [dk_a][dk_b] |\tilde{M}(k_a, k_b)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} - \vec{k}_{ti}) \delta(Y_{ab} - Y_i) \right.\right.$$

NNLL

$$\left.\left. + \int [dk_a][dk_b][dk_c] |\tilde{M}(k_a, k_b, k_c)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} + \vec{k}_{tc} - \vec{k}_{ti}) \delta(Y_{abc} - Y_i) + \ldots \right) \right\}$$

for inclusive observables

$$\equiv |M_B(\tilde{p}_1, \tilde{p}_2)|^2 \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} |M(k_i)|^2_{\text{inc}}$$

Result at NLL accuracy

Integrands can be expanded about k_{Ti}~k_{T1} to the desired accuracy: more efficient

The divergences cancel with the terms contained in the resolved real radiation

$$= e^{-R'(k_{T1})\ln\frac{1}{\epsilon}} \qquad R' = \frac{d}{d\ln(M/k_{t1})}R$$

$$\frac{d\Sigma(v)}{d\Phi_B} = \int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}\partial_L\left(-e^{-R(k_{t1})}\mathcal{L}_{\rm NLL}(k_{t1})\right)\epsilon^{R'(k_{t1})}\sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=2}^{n+1}\int_{\epsilon}^{1}\frac{d\zeta_i}{\zeta_i}\int_0^{2\pi}\frac{d\phi_i}{2\pi}R'(k_{t1})\Theta(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1}))$$

$$\zeta_i = k_{ti}/k_{t1}$$

resolved emission

emission of *n* identical independent blocks

parton luminosity at NLL reads

$$\mathcal{L}_{\rm NLL}(k_{t1}) = \sum_{c,c'}\frac{d|M_B|^2_{cc'}}{d\Phi_B}f_c(k_{t1},x_1)\,f_{c'}(k_{t1},x_2)$$

At higher logarithmic accuracy, it includes coefficient functions and hard-virtual corrections

This formula can be evaluated by means of fast **Monte Carlo methods**

`RadISH (Radiation off Initial State Hadrons)`

Result at N^{3}LL accuracy

$$
\begin{aligned}
\frac{d\Sigma(v)}{d\Phi_B} &= \int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}\partial_L\left(-e^{-R(k_{t1})}\mathcal{L}_{\mathrm{N^3LL}}(k_{t1})\right)\int d\mathcal{Z}[\{R',k_i\}]\Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1})\right)\\
&+\int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}e^{-R(k_{t1})}\int d\mathcal{Z}[\{R',k_i\}]\int_0^1\frac{d\zeta_s}{\zeta_s}\frac{d\phi_s}{2\pi}\Bigg\{\Big(R'(k_{t1})\mathcal{L}_{\mathrm{NNLL}}(k_{t1})-\partial_L\mathcal{L}_{\mathrm{NNLL}}(k_{t1})\Big)\\
&\times\left(R''(k_{t1})\ln\frac{1}{\zeta_s}+\frac{1}{2}R'''(k_{t1})\ln^2\frac{1}{\zeta_s}\right)-R'(k_{t1})\left(\partial_L\mathcal{L}_{\mathrm{NNLL}}(k_{t1})-2\frac{\beta_0}{\pi}\alpha_s^2(k_{t1})\hat{P}^{(0)}\otimes\mathcal{L}_{\mathrm{NLL}}(k_{t1})\ln\frac{1}{\zeta_s}\right)\\
&+\frac{\alpha_s^2(k_{t1})}{\pi^2}\hat{P}^{(0)}\otimes\hat{P}^{(0)}\otimes\mathcal{L}_{\mathrm{NLL}}(k_{t1})\Bigg\}\Big\{\Theta\left(v-V(\{\tilde{p}\},k_1,\ldots,k_{n+1},k_s)\right)-\Theta\left(v-V(\{\tilde{p}\},k_1,\ldots,k_{n+1})\right)\Big\}\\
&+\frac{1}{2}\int\frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}e^{-R(k_{t1})}\int d\mathcal{Z}[\{R',k_i\}]\int_0^1\frac{d\zeta_{s1}}{\zeta_{s1}}\frac{d\phi_{s1}}{2\pi}\int_0^1\frac{d\zeta_{s2}}{\zeta_{s2}}\frac{d\phi_{s2}}{2\pi}R'(k_{t1})\\
&\times\Bigg\{\mathcal{L}_{\mathrm{NLL}}(k_{t1})\left(R''(k_{t1})\right)^2\ln\frac{1}{\zeta_{s1}}\ln\frac{1}{\zeta_{s2}}-\partial_L\mathcal{L}_{\mathrm{NLL}}(k_{t1})R''(k_{t1})\left(\ln\frac{1}{\zeta_{s1}}+\ln\frac{1}{\zeta_{s2}}\right)\\
&+\frac{\alpha_s^2(k_{t1})}{\pi^2}\hat{P}^{(0)}\otimes\hat{P}^{(0)}\otimes\mathcal{L}_{\mathrm{NLL}}(k_{t1})\Bigg\}\\
&\times\Big\{\Theta\left(v-V(\{\tilde{p}\},k_1,\ldots,k_{n+1},k_{s1},k_{s2})\right)-\Theta\left(v-V(\{\tilde{p}\},k_1,\ldots,k_{n+1},k_{s1})\right)-\\
&\Theta\left(v-V(\{\tilde{p}\},k_1,\ldots,k_{n+1},k_{s2})\right)+\Theta\left(v-V(\{\tilde{p}\},k_1,\ldots,k_{n+1})\right)\Big\}+\mathcal{O}\left(\alpha_s^n\ln^{2n-6}\frac{1}{v}\right),
\end{aligned}
\tag{3.18}
$$

Result formally equivalent to the *b*-space formulation

[Parisi, Petronzio '78; Collins, Soper, Sterman '85; Li,Zhu '16]

Implementation: matching to fixed order

$$\Sigma(v,\phi_B) = \int_0^v dv' \frac{d\sigma}{dv' d\phi_B}$$

Cumulative cross section should reduce to the fixed order at large *v*

$\rightarrow \Sigma_{\text{res}}$ $\quad p_t \ll M_B$

$\rightarrow \Sigma_{\text{f.o.}}$ $\quad p_t \gtrsim M_B$

Additive matching

$$\Sigma^{\text{add}}_{\text{matched}}(v) = \Sigma_{\text{res}}(v) + \Sigma_{\text{f.o.}}(v) - \Sigma_{\text{res,exp}}(v)$$

- perhaps more natural, simpler
- numerically delicate in the very small p_T limit as f.o. can be unstable

Multiplicative matching *

$$\Sigma^{\text{mult}}_{\text{matched}}(v) = \Sigma_{\text{res}}(v)\frac{\Sigma_{\text{f.o.}}(v)}{\Sigma_{\text{res,exp}}(v)}$$

- N^3LO **constant terms** (formally N^4LL) can be included from fixed order
- only viable solution to consistently match to the NNLO differential distribution
- numerically more stable as the physical suppression at small *v* cures potential instabilities

*version implemented slightly more involved

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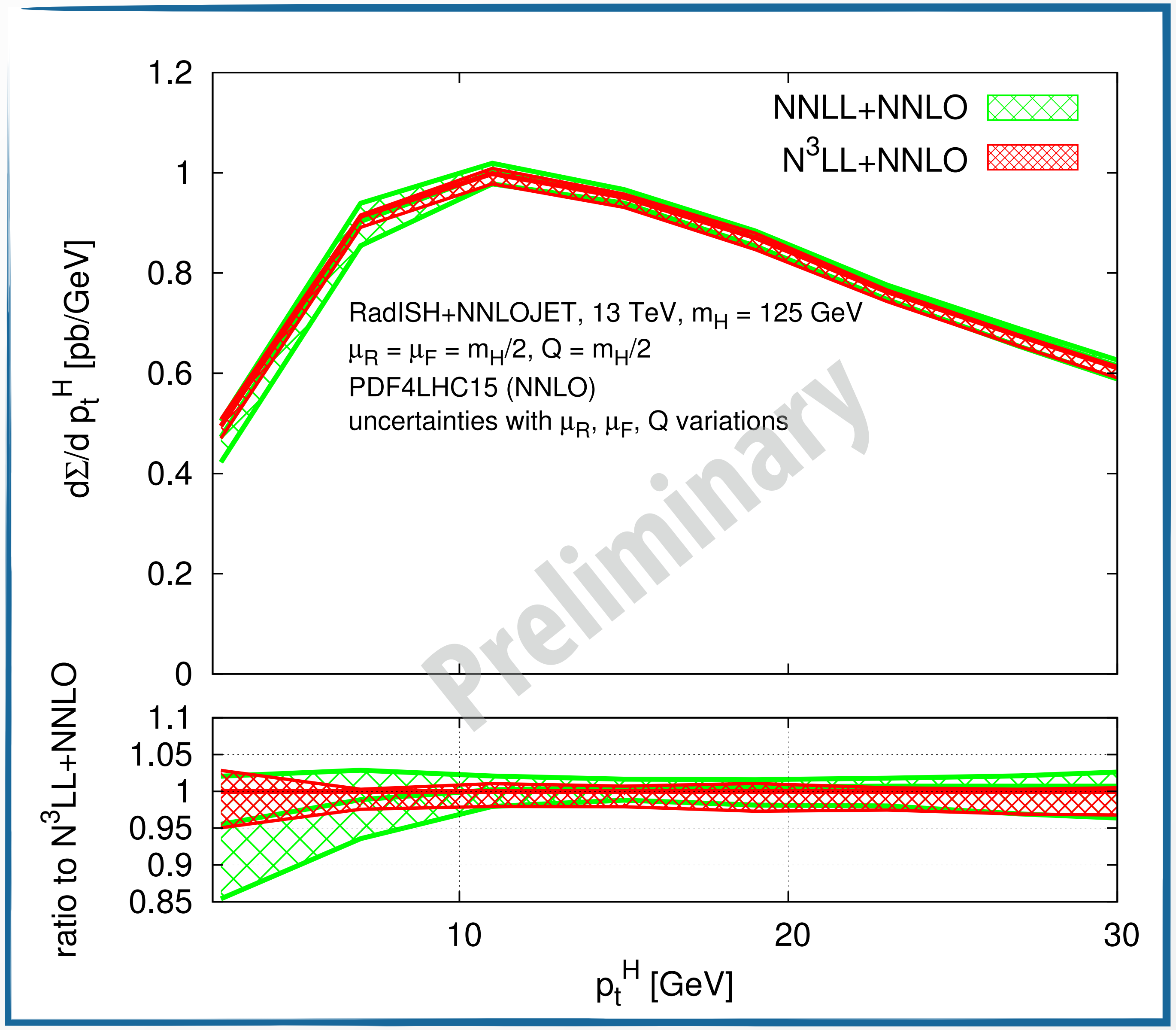
Higgs p_T

Results at N³LL+NNLO

[Bizon et al, ongoing]

N³LL vs NNLL



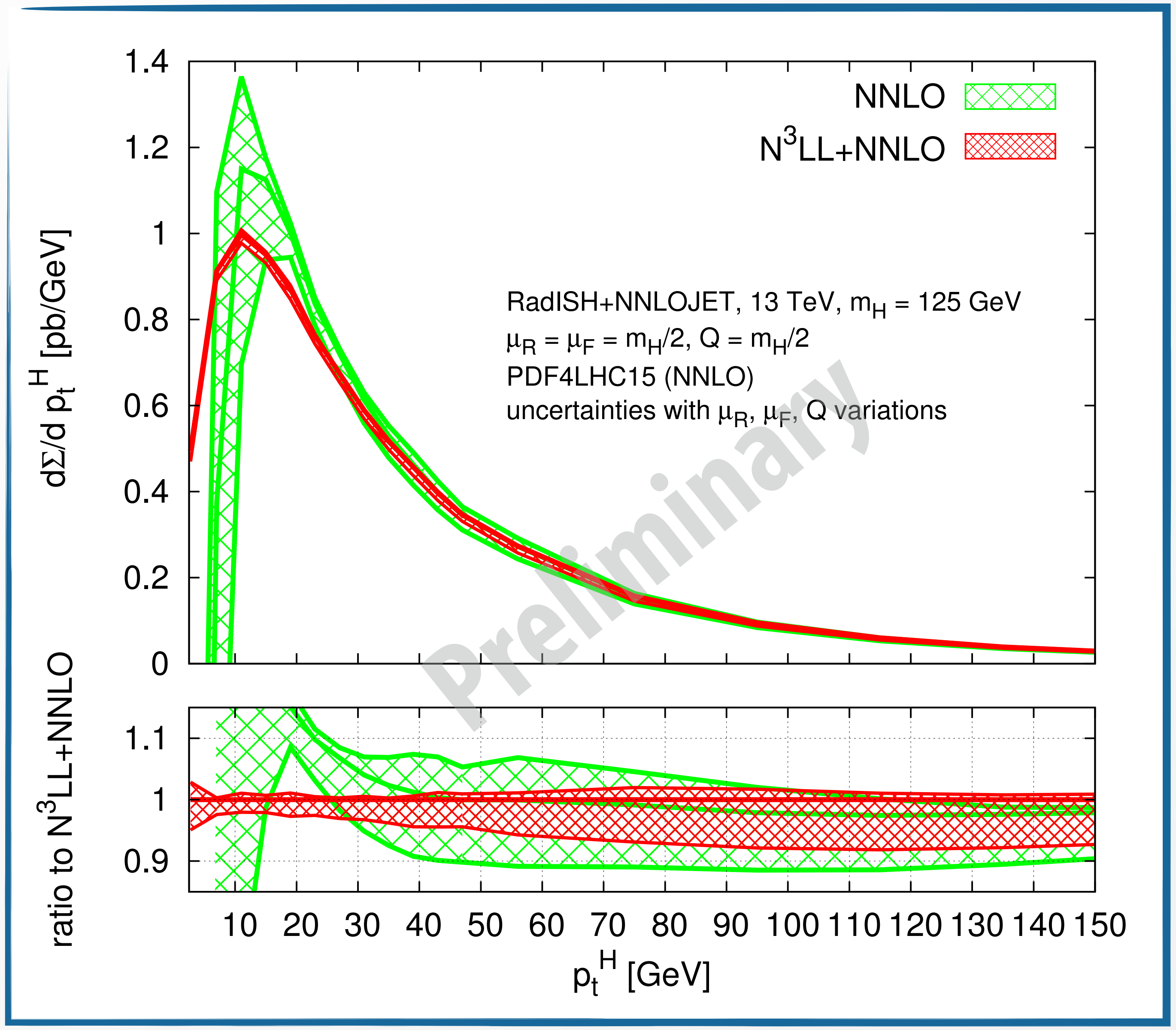
- N³LL corrections moderate in size (~ 5% at low p_T) and entirely contained in the NNLO+NNLL band
- Reduction of the perturbative uncertainty by a factor of 2 for $p_T \lesssim 10$ GeV

n.b. thanks to multiplicative scheme, NNLO+NNLL follows resummation scaling at low p_T

Results at N³LL+NNLO

[Bizon et al, ongoing]

Impact of resummation

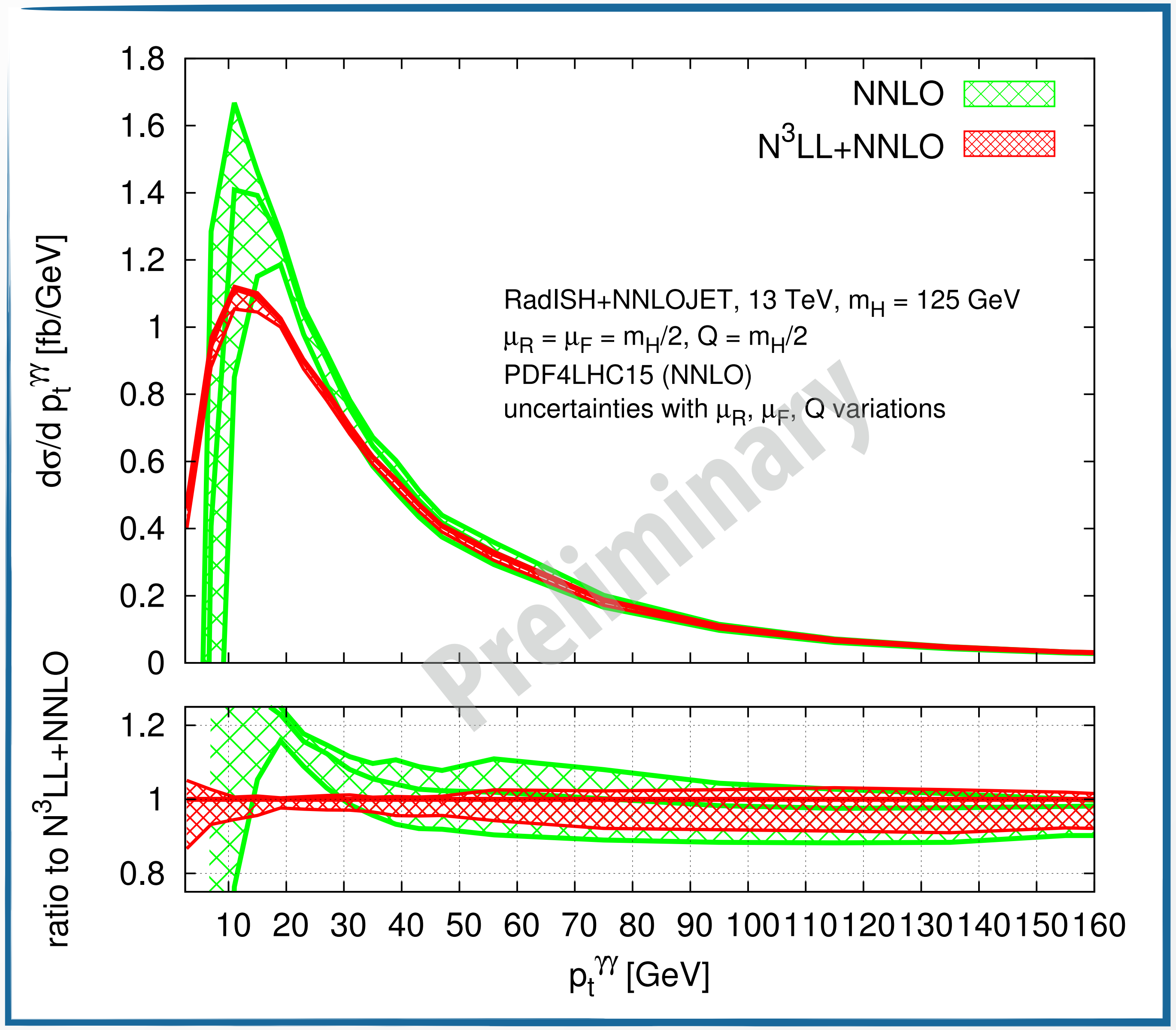


- effect of resummation starts to be increasingly important for $p_T \lesssim 40$ GeV
- Resummation effects are progressively less important above *50* GeV

Results at N³LL+NNLO

[Bizon et al, ongoing]

Results within fiducial cuts



- Effect of resummation starts to be increasingly important for $p_T \lesssim 40$ GeV
- Resummation effects are progressively less important above *50* GeV
- Similar results for fiducial region

Drell-Yan $p_{T,ll}$ and φ^*

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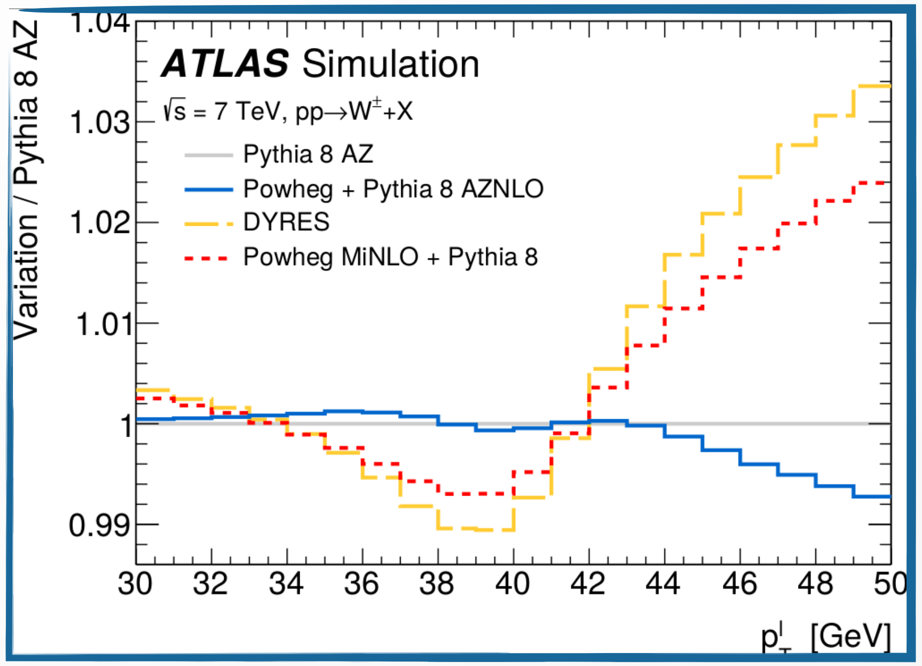
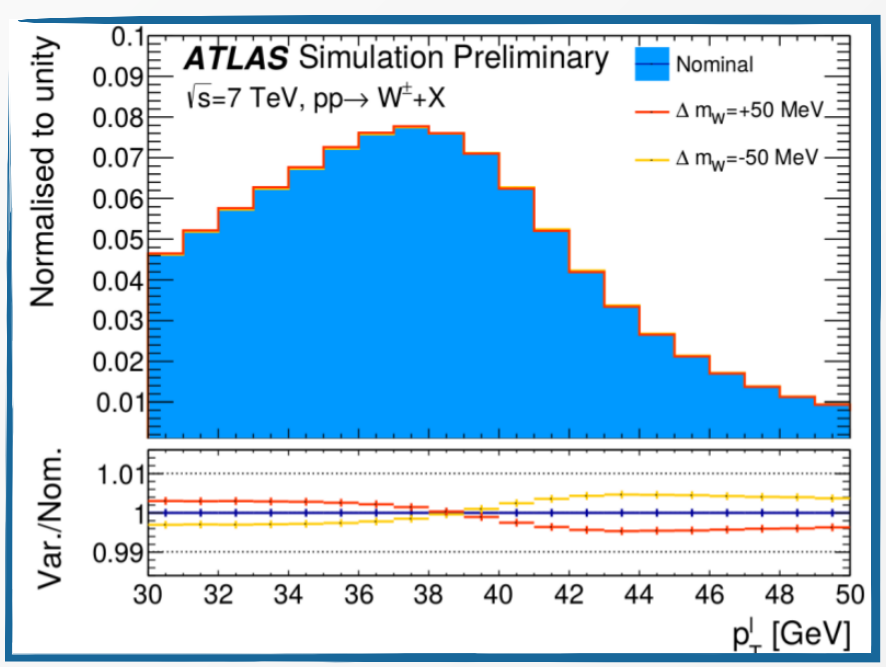
Drell-Yan



- State-of-the-art QCD prediction do not match the precision of the data
- LO MC are used, tuned on *Z* data
- Would be preferable to use more accurate theoretical predictions

Extreme precision is needed e.g. for *W* mass extraction



- Template fits to lepton observables
- Modelling of $p_{T,W}$ is crucial. Fit predictions to *Z* data, apply to *W*

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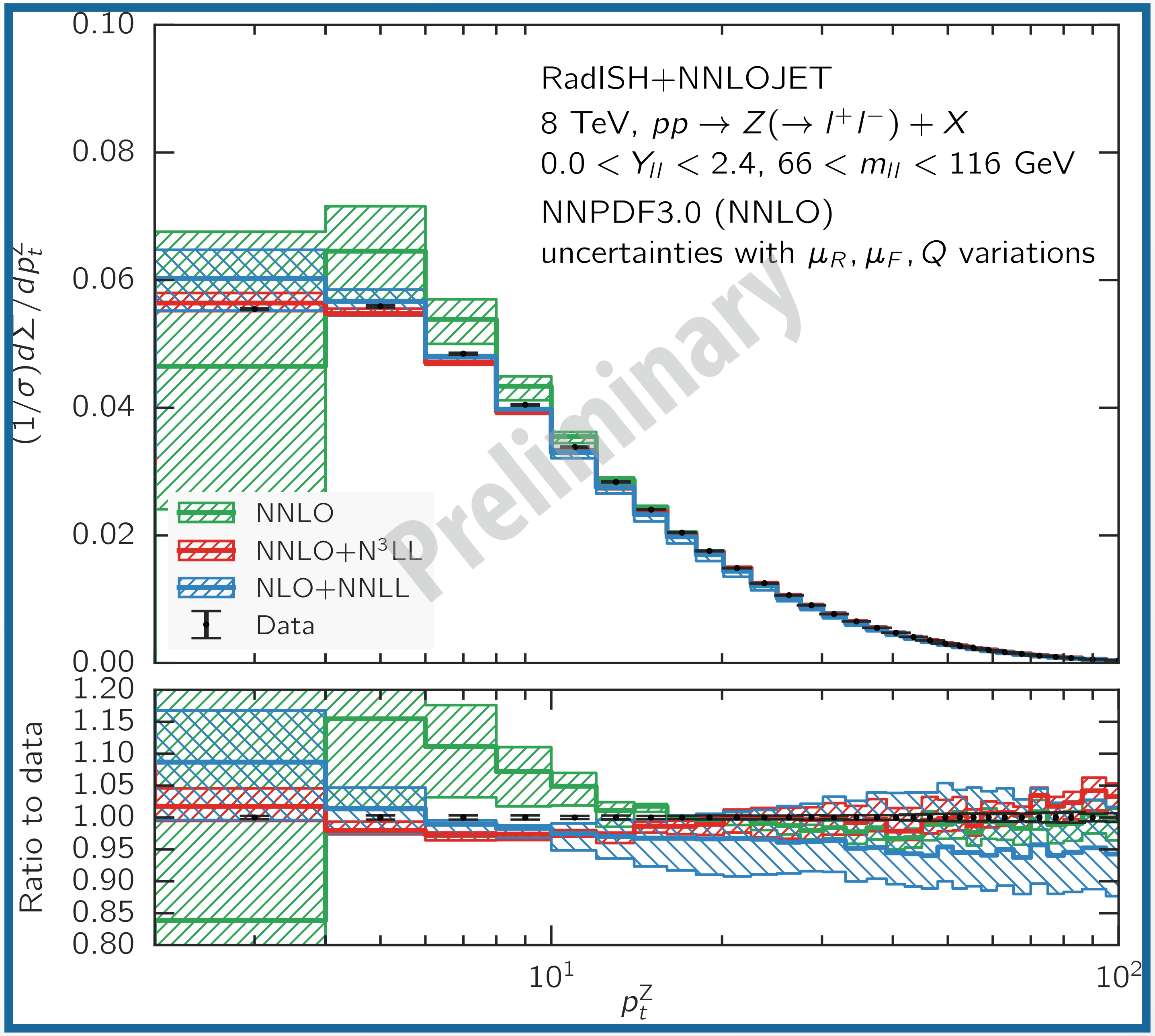
Drell-Yan: $p_{T,ll}$

[Bizon et al, ongoing]

Comparison with ATLAS data @ 8 TeV [1512.02192]



- Matched results offer a good description of the data in the low-medium p_T range, in all fiducial regions
- Perturbative uncertainty at the few percent level, still does not match the precision of the data
- Estimate of non-perturbative effects may start to be relevant

Drell-Yan: φ^*

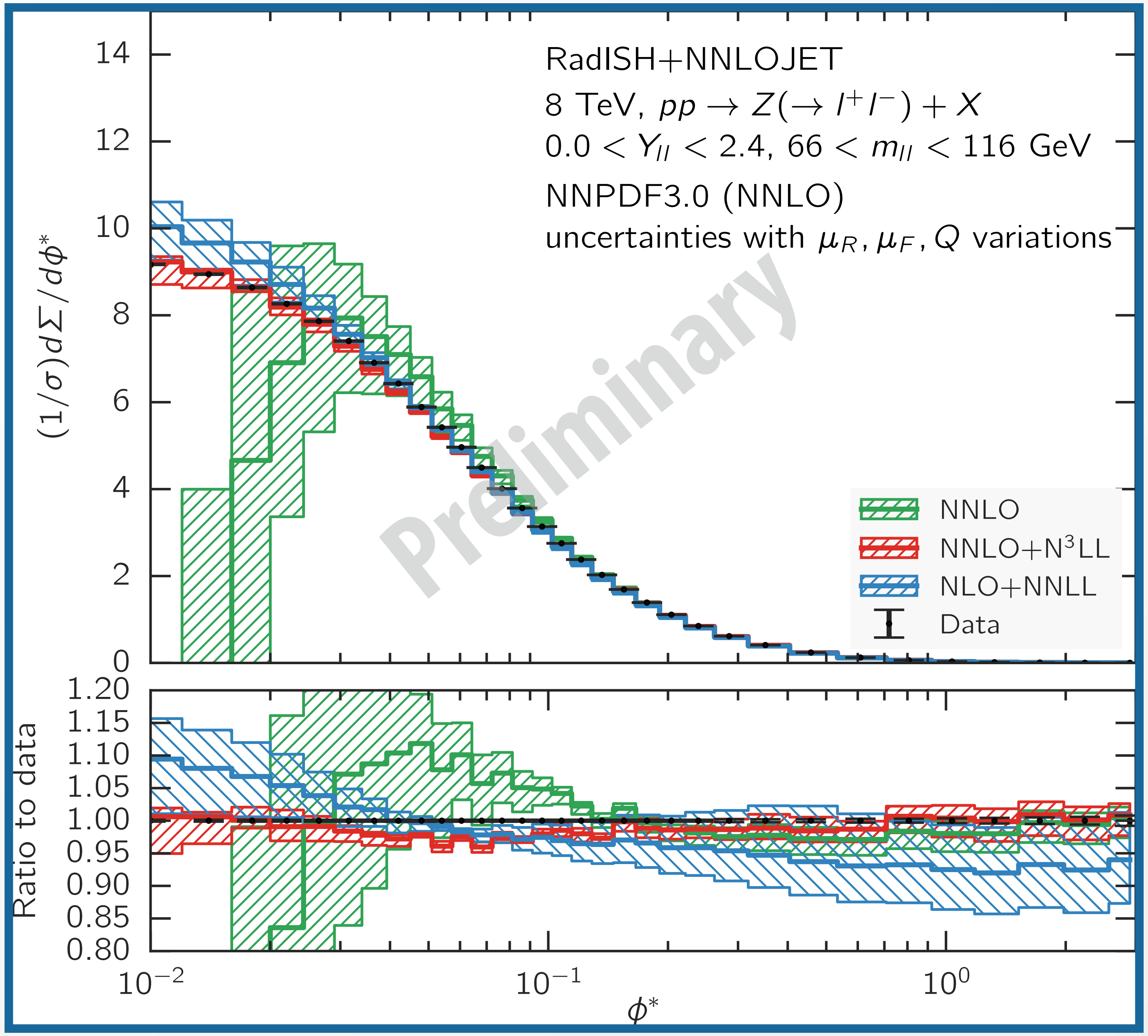
[Bizon et al, ongoing]

Our approach can be used for resumming other transverse observables; e.g φ^*

$$\phi^* = \tan\left(\frac{\pi - \Delta\phi}{2}\right)\sin\theta^* \qquad \phi^* \sim \frac{p_T}{2M}$$

angle between electron and beam axis, in Z boson rest frame

Comparison with ATLAS data @ 8 TeV [1512.02192]



- Similar situation as p_T, with perturbative uncertainty at the few percent level but with experimental errors at the sub-percent level

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Conclusions

- New formalism for all-order resummation up to **N^3LL accuracy** for inclusive, transverse observables.

- Method formulated in **momentum space**, formally equivalent to the standard *b*-space formalism

- Access to multi-differential information. As in parton showers, but with higher-order logarithms, and control on formal accuracy

- Method allows for an **efficient implementation in a computer code**. Towards a single generator able to resum entire classes of observables at high accuracy.

Phenomenological results

- Results at NNLO+N^3LL for Higgs and DY differential distributions

- N^3LL corrections moderate in size, but appreciable reduction of the perturbative uncertainty

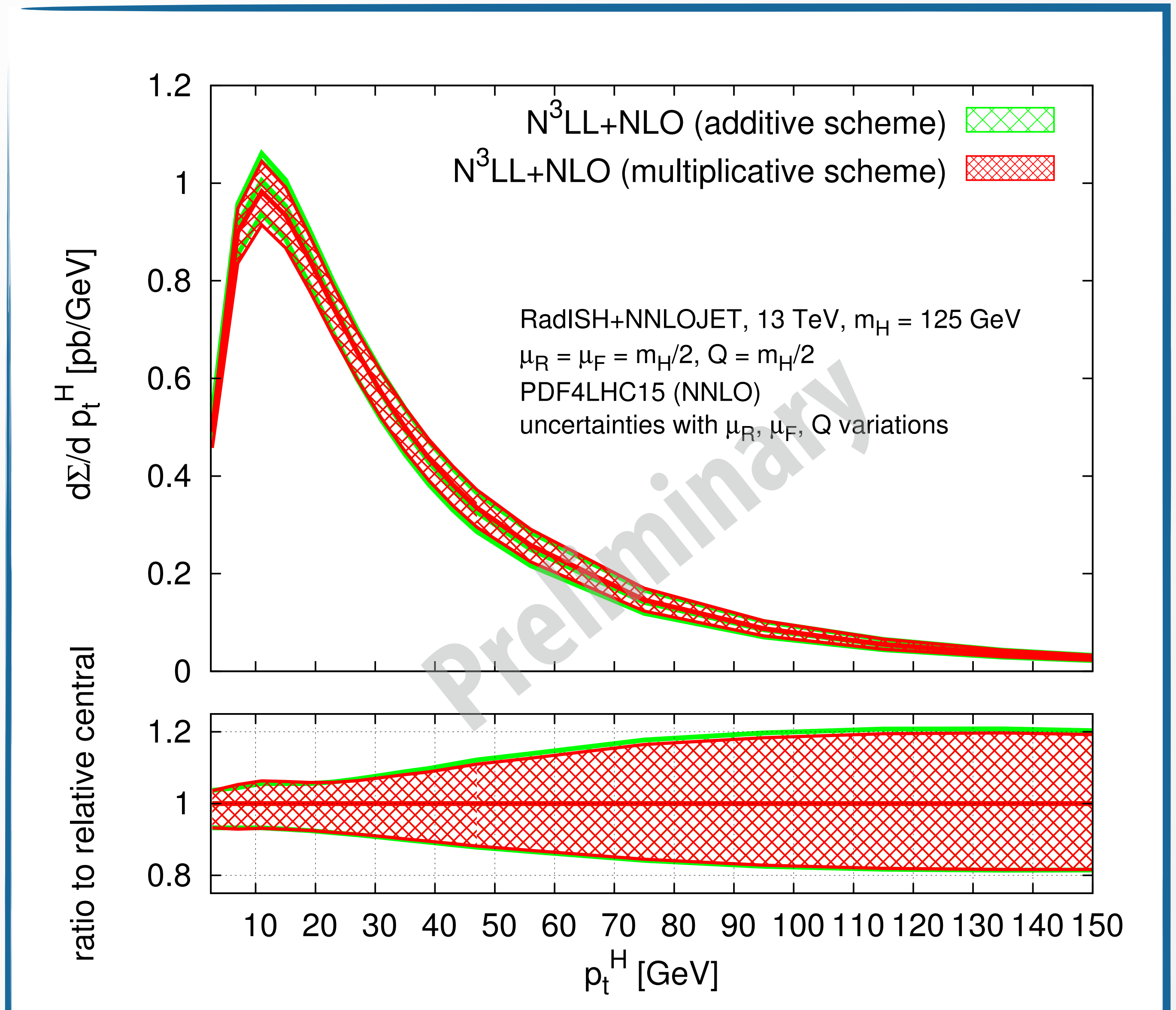
- Good description of the data in the fiducial distributions, with uncertainties at the few percent level

Backup

Results at N³LL+NNLO

[Bizon et al, ongoing]

Multiplicative vs additive matching



Multiplicative scheme: normalization for the resummed prefactor to its asymptotic value for $L \to 0$

$$\Sigma^{\rm mult}_{\rm matched}(v) = \frac{\Sigma_{\rm res}(v)}{\Sigma^{\rm asym.}_{\rm res}}\left[\Sigma^{\rm asym.}_{\rm res}\frac{\Sigma_{\rm f.o.}(v)}{\Sigma_{\rm res,exp}(v)}\right]_{\rm exp}$$

$$\Sigma^{\rm asym.}_{\rm res} = \int_{\rm with\,cuts} d\Phi_B \left(\lim_{L\to 0}\mathcal{L}\right)$$

In the $v \gg Q/M$ limit no large spurious, higher-order, corrections arise and fixed-order result is reproduced by construction

Formulation in momentum space

Is it possible to obtain a formulation in momentum space?

Not possible to find a closed analytic expression in direct space which is both a) free of logarithmically subleading corrections and b) free of singularities at finite p_T values [Frixione, Nason, Ridolfi '98]

Why? A naive logarithmic counting at small p_T is not sensible, as one loses the correct power-suppressed scaling if only logarithms are retained: it's not possible to reproduce a power behaviour with logs of p_T/M

(logarithms of b do not correspond to logarithms of p_T)

Necessary to establish a well defined logarithmic counting in momentum space in order to reproduce the correct behaviour of the observable at small p_T

Zeros in the small-p_T region and b-space formulation

Two different mechanisms give a contribution in the small p_T region

- configurations where the transverse momenta of the radiated partons is small (**Sudakov limit**) — Exponential suppression — Sudakov peak region
- configurations where p_T tends to zero because of cancellations of non-zero transverse momenta of the emissions (**azimuthal cancellations**) — Power suppression — $p_T \to 0$ limit

Power-law scaling at very small p_T

For inclusive observables the vectorial nature of the cancellations can be handled via a **Fourier transform**

[Parisi, Petronzio '78; Collins, Soper, Sterman '85]

[Catani, Grazzini '11][Catani et al. '12,Gehrmann][Luebbert, Yang '14]

coefficient functions

$$\frac{d^2\Sigma(v)}{d\Phi_B dp_t} = \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \int b\, db\, p_t J_0(p_t b)\, \mathbf{f}^T(b_0/b)\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(b_0/b))H_{\rm CSS}(M)\mathbf{C}^{c_2}_{N_2}(\alpha_s(b_0/b))\mathbf{f}(b_0/b)$$

hard-virtual corrections

$$\times \exp\left\{-\sum_{\ell=1}^{2}\int_0^M \frac{dk_t}{k_t}\mathbf{R}'_{{\rm CSS},\ell}(k_t)\,\Theta(k_t - \frac{b_0}{b})\right\}$$

$$R_{\rm CSS}(b) = \sum_{l=1}^{2}\int_{b0/b}^{M}\frac{dk_T}{k_T}R'_{{\rm CSS},l}(k_T) = \sum_{l=1}^{2}\int_{b_0/b}^{M}\frac{dk_T}{k_T}\left(A_{{\rm CSS},\ell}(\alpha_s(k_T))\ln\frac{M^2}{k_T^2} + B_{{\rm CSS},\ell}(\alpha_s(k_T))\right)$$

anomalous dimensions

[Davies, Stirling '84] [De Florian, Grazzini '01] [Becher, Neubert '10][Li, Zhu '16][Vladimirov '16]

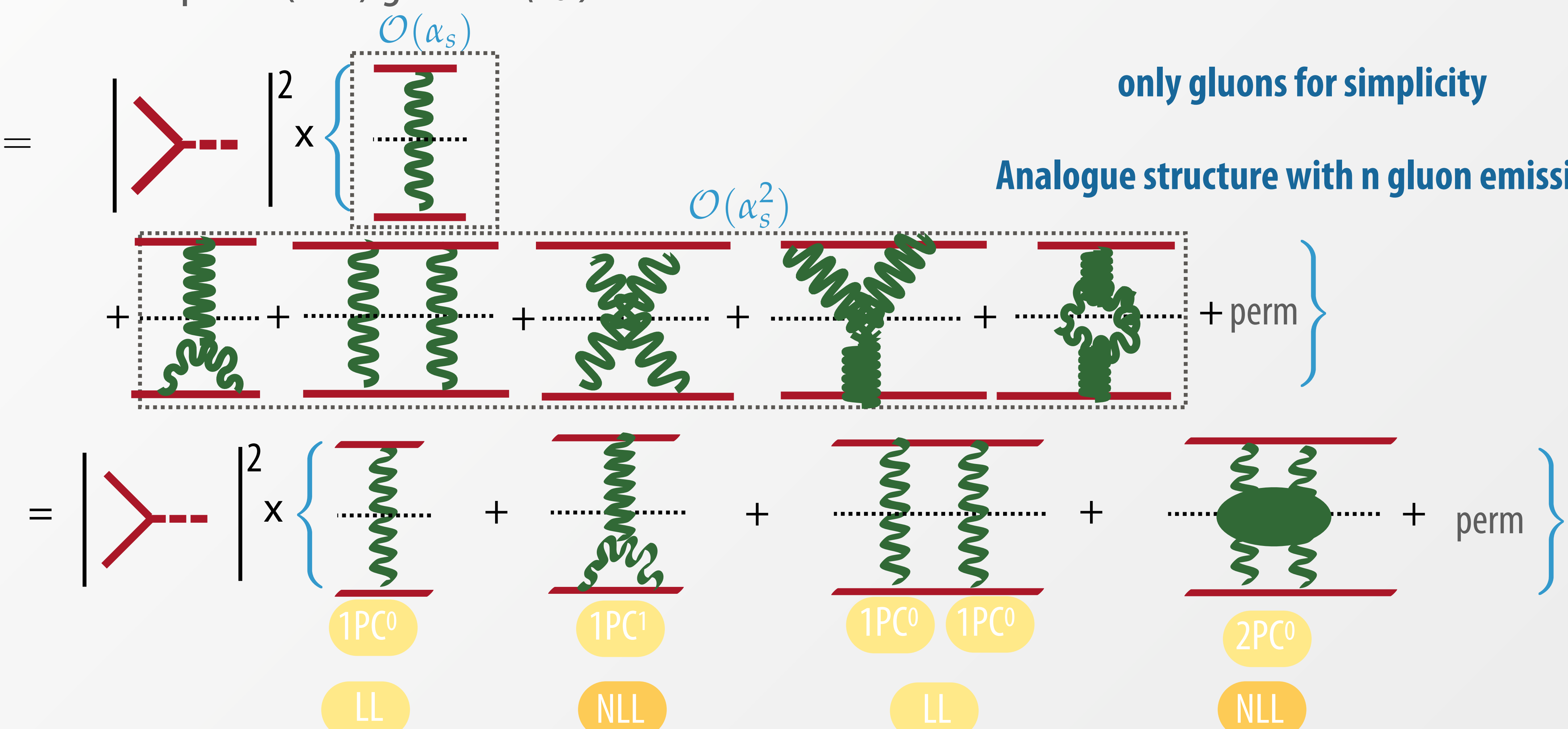
Logarithmic counting

Necessary to establish a **well defined logarithmic counting**: possibile to do that by decomposing the squared amplitude in terms of n-particle correlated blocks (**nPC**)

e.g. $pp \to H +$ emission of up to 2 (soft) gluons $O(\alpha_s^2)$

outgoing partons

$|M(p_1, p_2, k_1, k_2)|^2 =$



only gluons for simplicity

Analogue structure with n gluon emission

Logarithmic counting defined in terms of **nPC blocks** (owing to rIRC safety of the observable)

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Resolved and unresolved emissions

For inclusive observables (such as Higgs p_T) $V(\{\hat{p}\}, k_1, \ldots k_n) = V(\{\hat{p}\}, k_1 + \ldots + k_n)$

$$|M(p_1, p_2, k_1, \ldots, k_n)|^2 = |M_B(p_1, p_2)|^2 \times \frac{1}{n!}\left\{\prod_{i=1}^{n}\left(|M(k_i)|^2 + \int [dk_a][dk_b]|\tilde{M}(k_a, k_b)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} - \vec{k}_{ti})\delta(Y_{ab} - Y_i) + \int [dk_a][dk_b][dk_c]|\tilde{M}(k_a, k_b, k_c)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} + \vec{k}_{tc} - \vec{k}_{ti})\delta(Y_{abc} - Y_i) + \ldots \right)\right\}$$

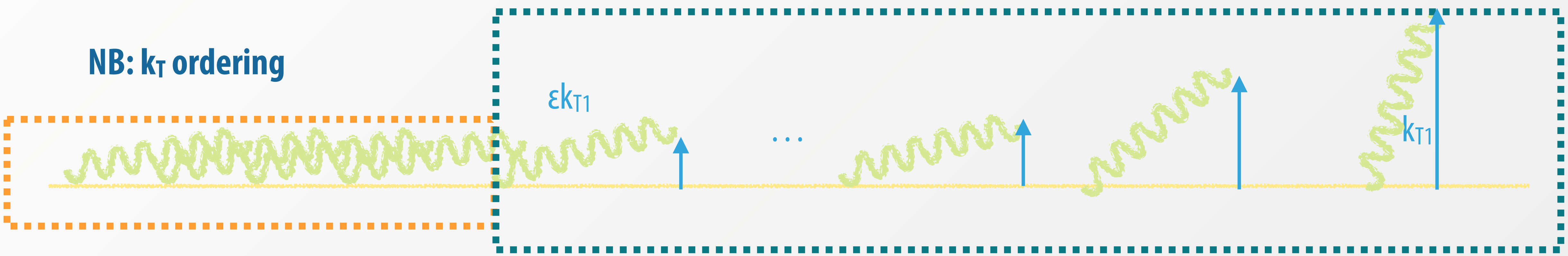
1PC

2PC

3PC

Introduction of a **resolution scale** ϵk_{T1}

NB: k_T ordering



unresolved emission

resolved emission

can be integrated inclusively to cancel the divergences of the virtuals (rIRC): exponential factor

$$e^{-R(\epsilon k_{t1})}$$

Sudakov form factor

ε dependence cancels against the resolved real corrections

treated exclusively: for inclusive observables can be parametrised exactly as a Sudakov **unintegrated** in k_t and azimuthal angle

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Momentum space formulation

Result can be expressed as

$$\frac{d\Sigma(v)}{d\Phi_B} = \int_{\mathcal{C}_1} \frac{dN_1}{2\pi i} \int_{\mathcal{C}_2} \frac{dN_2}{2\pi i} x_1^{-N_1} x_2^{-N_2} \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \mathbf{f}^T_{N_1}(\mu_0) \hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) \mathbf{f}_{N_2}(\mu_0)$$

DGLAP anomalous dimensions

RG evolution of coefficient functions

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi}$$
$$\times e^{-\mathbf{R}(\epsilon k_{t1})} \exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$
$$\sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi} \mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$
$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \sum_{\ell_i=1}^{2}\left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi} \mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right)$$
$$\times \Theta\left(v - V(\{\tilde{p}\}, k_1, \dots, k_{n+1})\right)$$

Result valid for all inclusive observables (e.g. p_T, φ^*)

unresolved emission + virtual corrections

resolved emission

Formulation **equivalent to *b*-space** result (up to a scheme change in the anomalous dimensions)

$$\frac{d^2\Sigma(v)}{d\Phi_B dp_t} = \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \int b\, db\, p_t J_0(p_t b)\, \mathbf{f}^T(b_0/b) \mathbf{C}^{c_1;T}_{N_1}(\alpha_s(b_0/b)) H(M) \mathbf{C}^{c_2}_{N_2}(\alpha_s(b_0/b)) \mathbf{f}(b_0/b)$$
$$\times \exp\left\{-\sum_{\ell=1}^{2} \int_0^M \frac{dk_t}{k_t} \mathbf{R}'_\ell(k_t)\left(1 - J_0(bk_t)\right)\right\}$$

$$(1 - J_0(bk_t)) \simeq \Theta(k_t - \frac{b_0}{b}) + \frac{\zeta_3}{12} \frac{\partial^3}{\partial \ln(Mb/b_0)^3} \Theta(k_t - \frac{b_0}{b})$$

Resummation in momentum space

Formulation in Mellin space already implementable. However, it is convenient to perform the evaluation entirely in momentum space

In previous formula, resummation of logarithms of $k_{T,i}/M$

$k_{Ti}/k_{T1} \sim O(1)$

(everywhere in the resolved phase space, due to rIRC safety)

subleading logarithms in p_T

free of singularity at low p_T values

(power-law scaling)

Integrands can be expanded about $k_{Ti} \sim k_{T1}$ to the desired accuracy: more efficient

Sudakov region: $k_{T1} \sim p_T$

$ln(M/p_T)$ resummed at the desired accuracy

+ additional subleading terms that **cannot be neglected**

azimuthal region: $k_{Ti} \sim k_{T1}$

correct description of the kinematics after expansion $k_{Ti} \sim k_{T1}$

correct scaling of the cumulant $\boldsymbol{O(p_T^2)}$

Checks and remarks

- **b-space** formulation **reproduced analytically** at the resummed level
- **correct scaling** at small p_T computed analytically
- **numerical checks** down to very low p_T against b-space codes (`HqT`, `CuTe`) [Grazzini et al.][Becher et al.]
- check that the FO expansion of the final expression in momentum space up to $O(\alpha^5)$ yields the corresponding expansion in b-space (CSS)
- expansion checked against `MCFM` up to $O(\alpha^4)$ [Campbell et al.]