Resummation of Transverse Observables

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All-order resummation

All-order resummation for an infrared-collinear safe (IRC) observable relies on

- **factorization of the QCD amplitude** in the IRC limit
- **factorization of the observable:** hard and singular IRC modes do not mix when one considers radiative corrections, separation of soft and collinear modes

Last requirement usually is interpreted as the existence of a factorized formula in a **conjugate space**

Very complicated to obtain such a factorization in general

Separation of the singular and the hard modes can be traslated into a scaling requirement for the observable in the presence of radiation to achieve a systematic solution in **direct space**

recursive IRC safety* allows to define a logarithmic hierarchy for the squared amplitudes at all orders

resummation can be formulated systematically

a) in the presence of multiple soft and/or collinear emissions, observable has the same scaling properties as with just one of them;

b) for sufficiently small values of the observable, emissions below ϵv do not significantly contribute to the observable

Transverse observables in colour-singlet production

$$V(\{\hat{p}\},k) \equiv V(k) = \left(\frac{k_t}{M}\right)^a f(\phi)$$

Clean experimental and theoretical environment for **precision physics**

- ~40 inverse fb collected in 2016
- Increase in statistics enables study of **differential distributions** in detail

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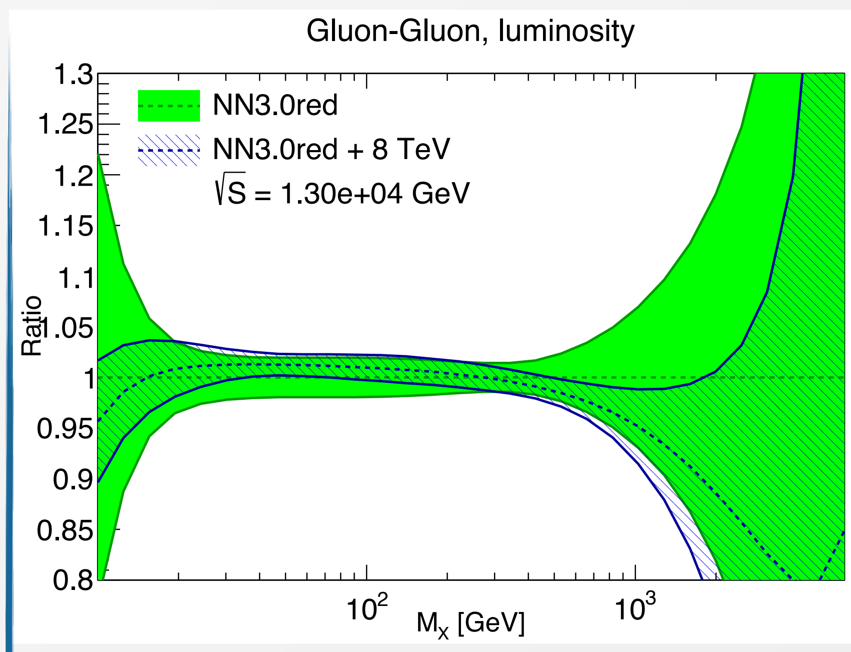
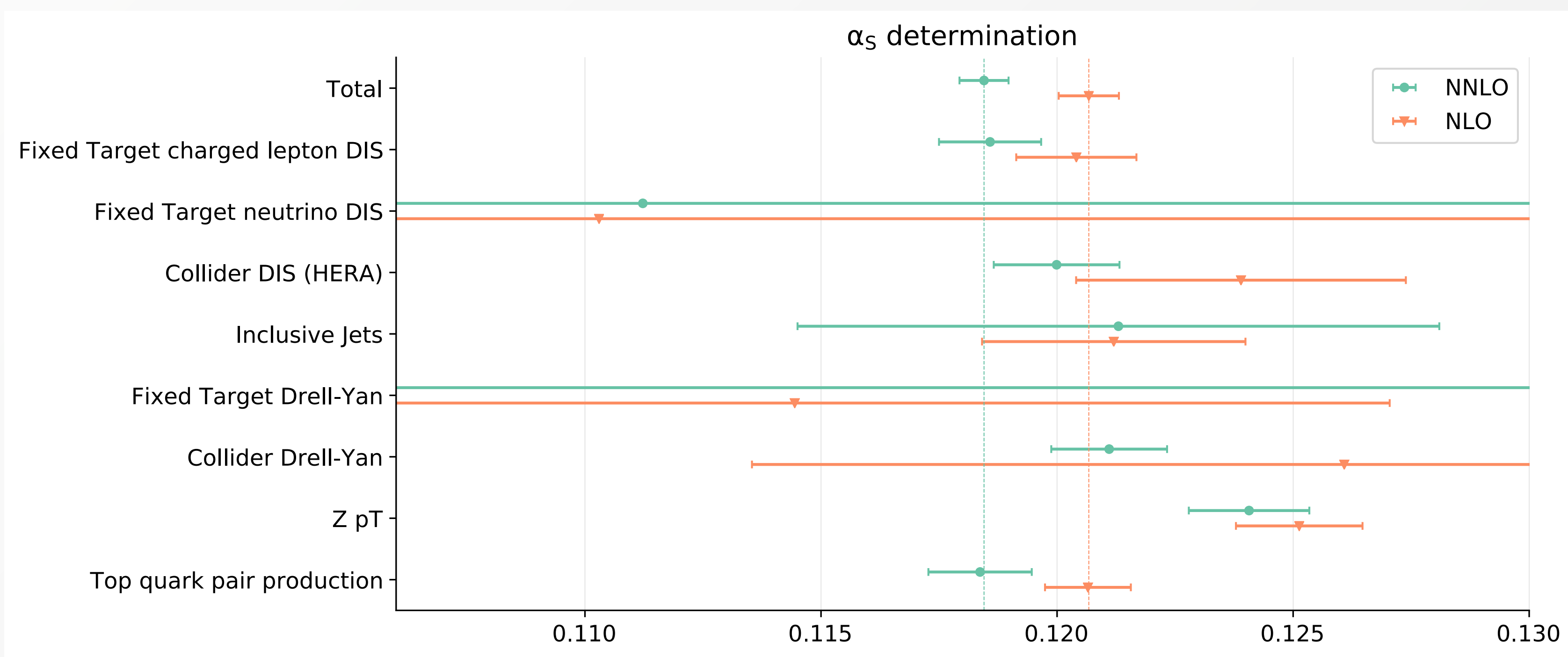
- Implications both for **SM measurements**. . .

Parton distribution functions



[Boughezal et al.,1705.00343]

Strong coupling



NNPDF, preliminary

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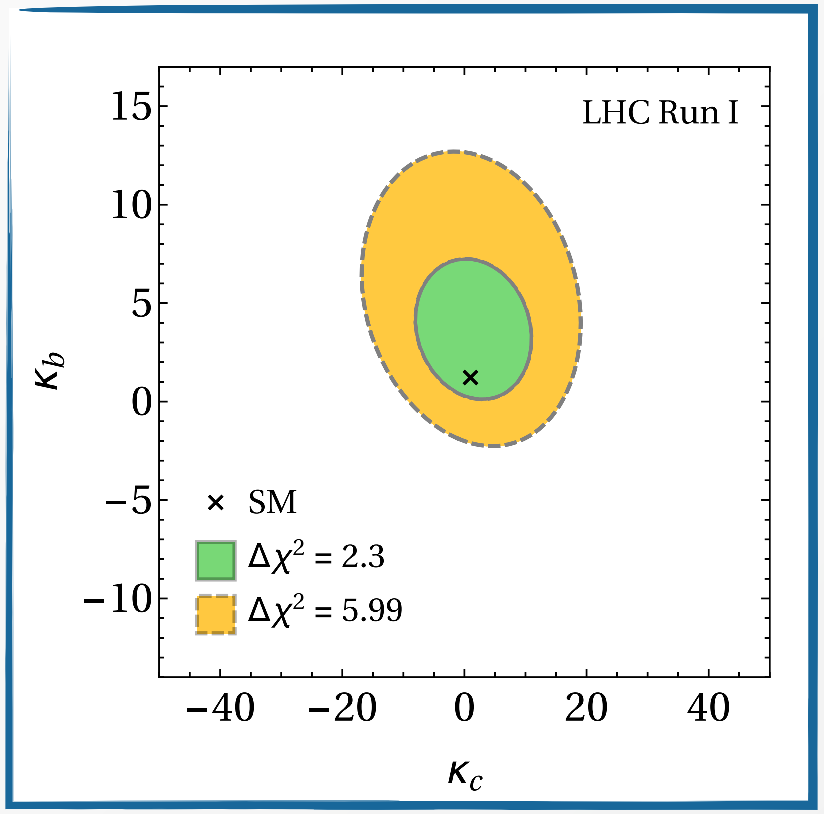
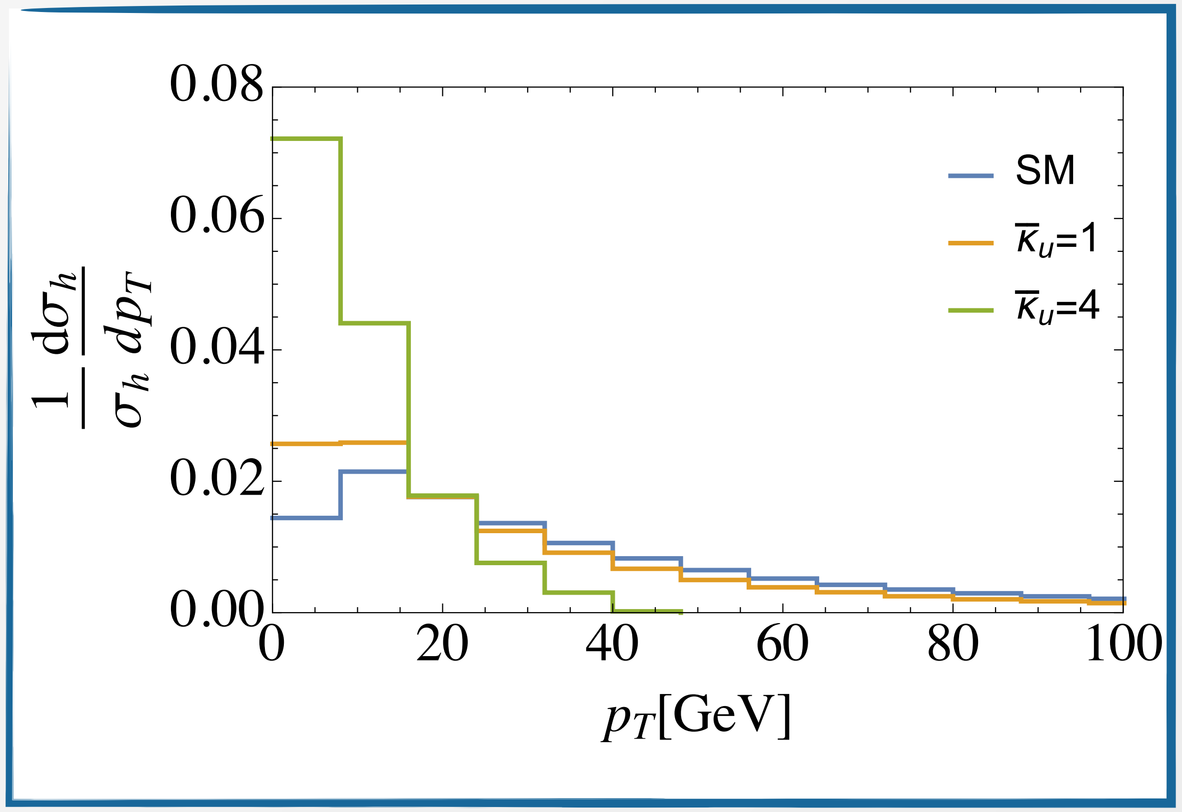
- Implications both for **SM measurements**...
- ...and **BSM measurements** (e.g. light Yukawa)



[Bishara et al.,1606.09253]



[Soreq et al, 1606.09621]

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Transverse observables in colour-singlet production

$$V(\{\hat{p}\},k) \equiv V(k) = \left(\frac{k_t}{M}\right)^a f(\phi)$$

Inclusive observables probe directly the kinematics of the colour singlet

$$V(\{\hat{p}\},k_1,\dots k_n) = V(\{\hat{p}\},k_1+\dots+k_n)$$

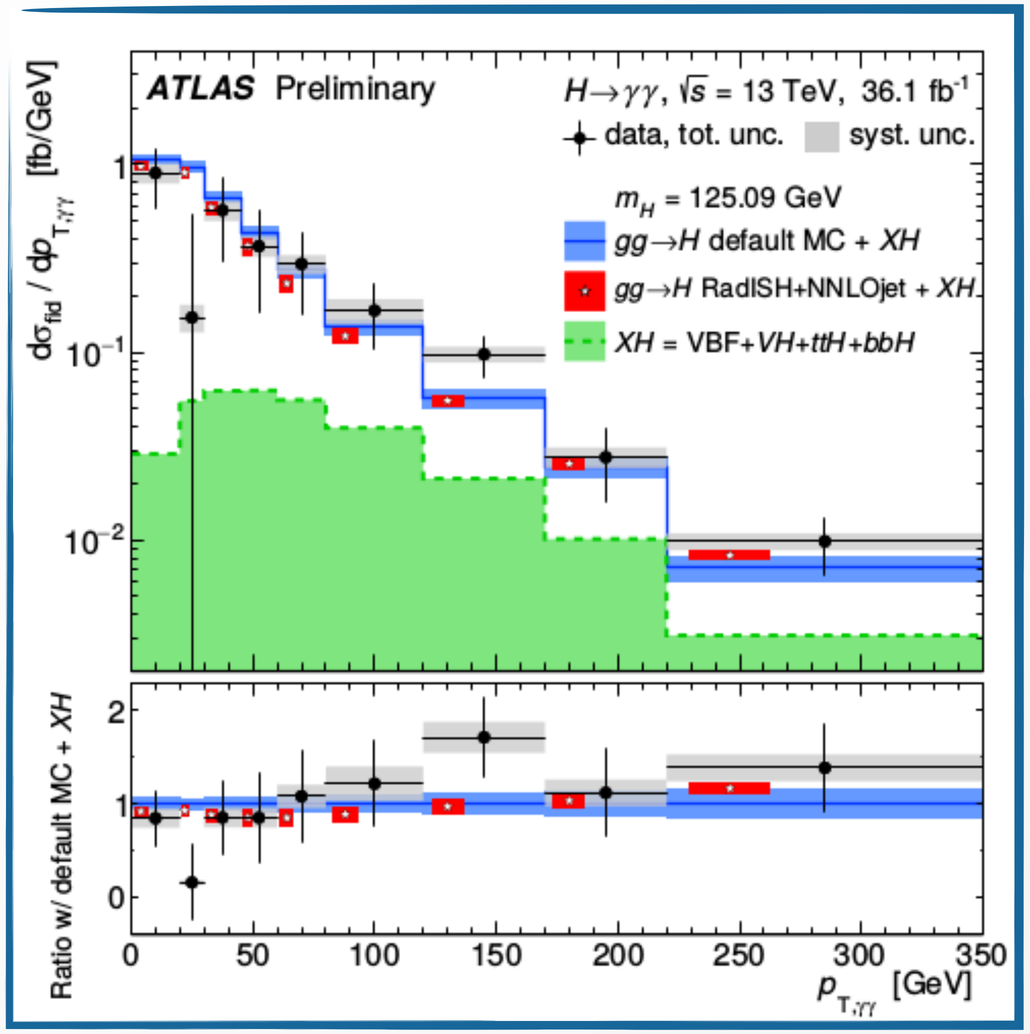
- negligible sensitivity to multi-parton interactions
- reduced sensitivity to non -perturbative effects (only through transverse recoil)
- measured extremely precisely at experiments (e.g. ~1% errors in Z p_T distributions)

Necessary to push perturbation theory to its limit

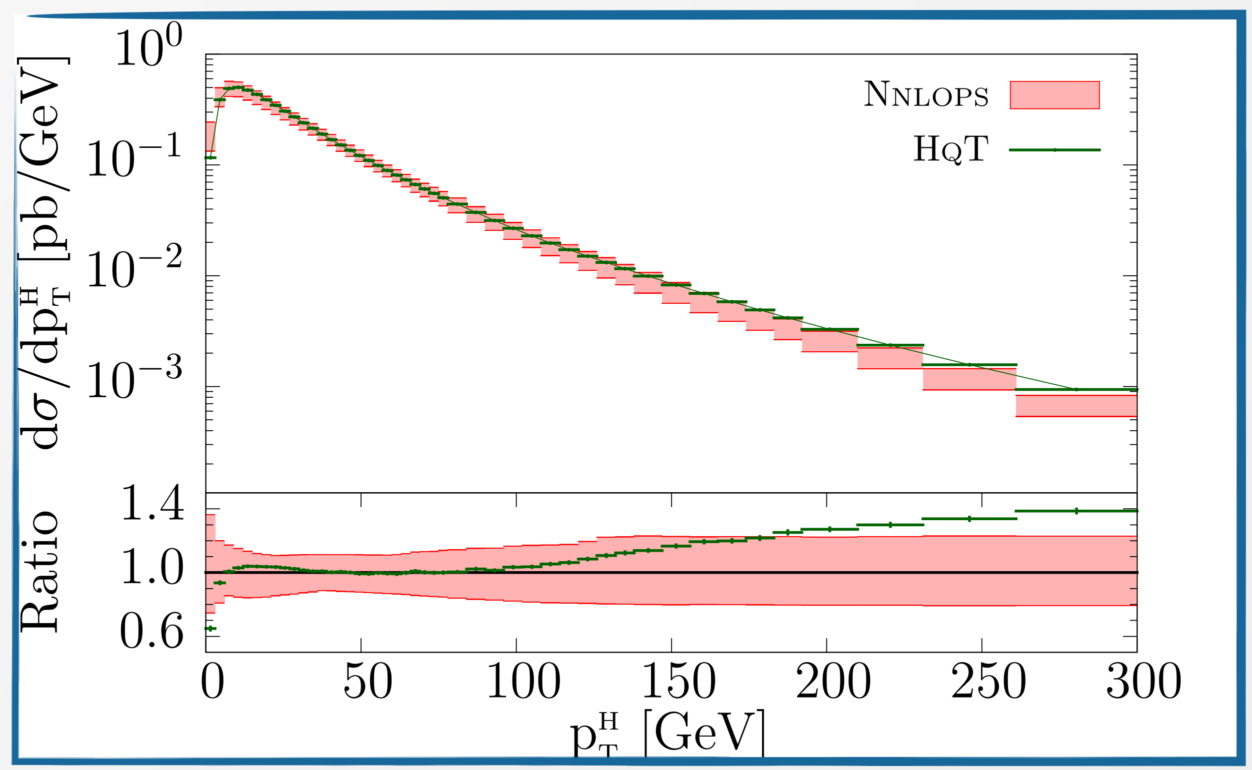
Higgs p_T

Higgs transverse momentum: an archetype

Currently one of the most important observables for current Higgs studies at the LHC



- High luminosity allows for precision studies
- Large p_T region: heavy degrees of freedom
- Medium and low p_T region: larger cross section, sensitive to Higgs couplings



[HqT, Bozzi et al.]

- Also, particularly clean environment for MC calibration

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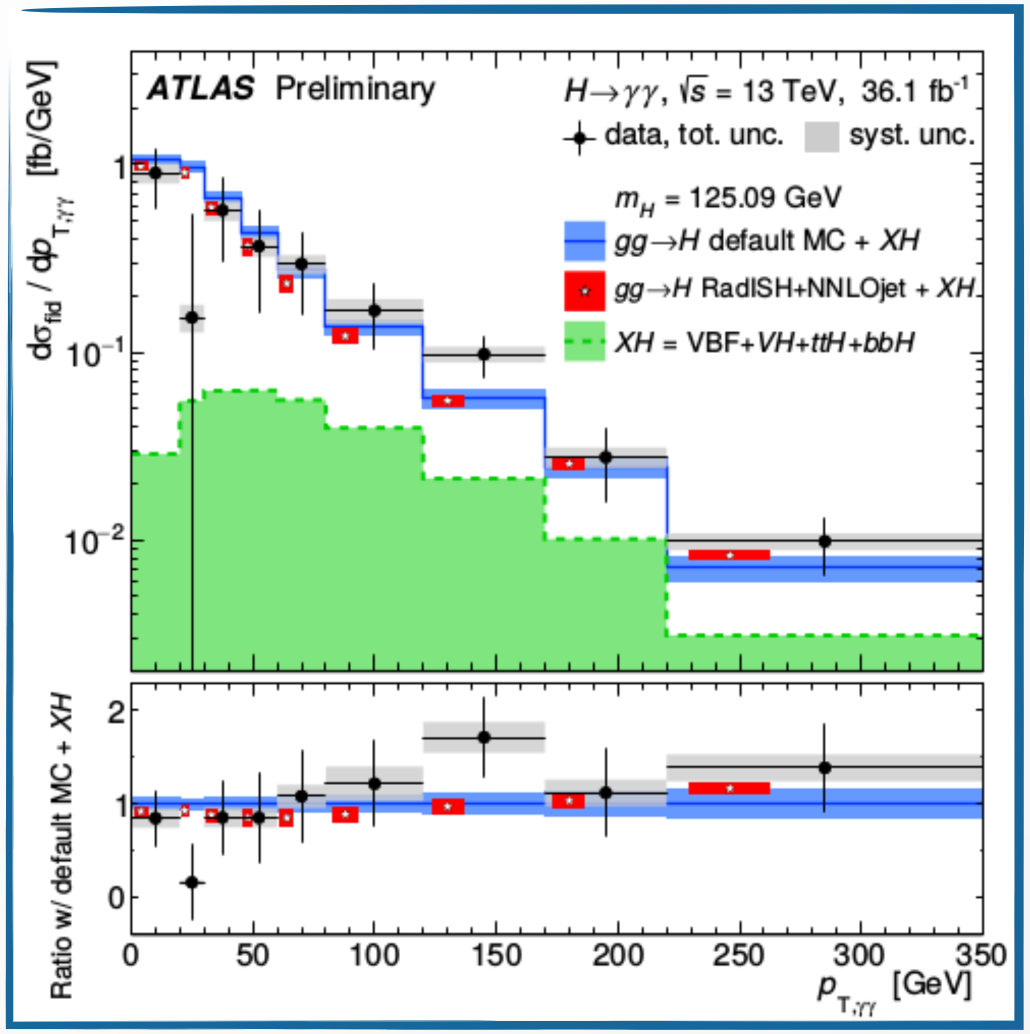

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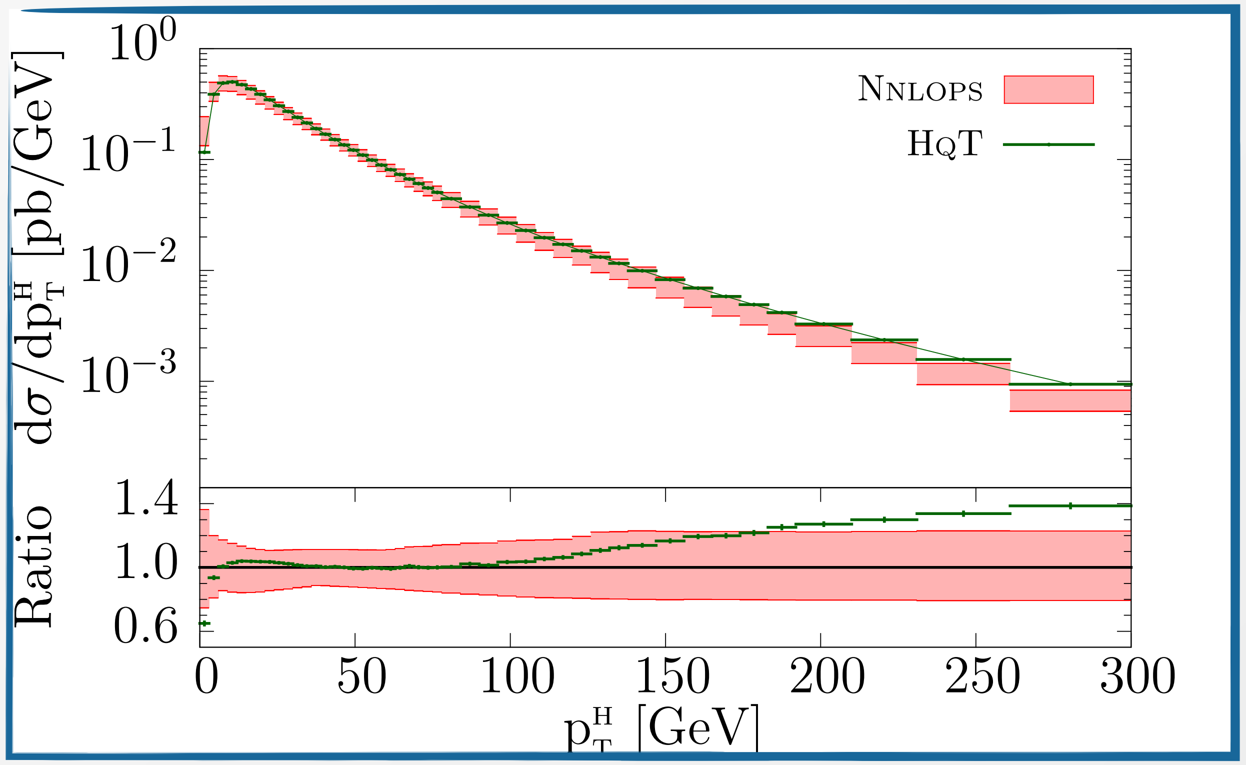

- Also, particularly clean environment for MC calibration

[HqT, Bozzi et al.]

high logarithmic accuracy is important for an accurate description of the low-p_T region

Zeros in the small-p_T region and b-space formulation

Two different mechanisms give a contribution in the small p_T region

- configurations where the transverse momenta of the radiated partons is small (**Sudakov limit**) — Exponential suppression — Sudakov peak region
- configurations where p_T tends to zero because of cancellations of non-zero transverse momenta of the emissions (**azimuthal cancellations**) — Power suppression — $p_T \to 0$ limit

Power-law scaling at very small p_T

For inclusive observables the vectorial nature of the cancellations can be handled via a **Fourier transform**

[Parisi, Petronzio '78; Collins, Soper, Sterman '85]

[Catani, Grazzini '11][Catani et al. '12,Gehrmann][Luebbert, Yang '14]

coefficient functions

$$\frac{d^2\Sigma(v)}{d\Phi_B dp_t} = \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \int b\, db\, p_t J_0(p_t b)\, \mathbf{f}^T(b_0/b) \mathbf{C}^{c_1;T}_{N_1}(\alpha_s(b_0/b)) H_{\mathrm{CSS}}(M) \mathbf{C}^{c_2}_{N_2}(\alpha_s(b_0/b)) \mathbf{f}(b_0/b)$$

hard-virtual corrections

$$\times \exp\left\{-\sum_{\ell=1}^{2}\int_0^M \frac{dk_t}{k_t} \mathbf{R}'_{\mathrm{CSS},\ell}(k_t)\, \Theta(k_t - \frac{b_0}{b})\right\}$$

$$R_{\mathrm{CSS}}(b) = \sum_{l=1}^{2}\int_{b0/b}^{M}\frac{dk_T}{k_T} R'_{\mathrm{CSS},l}(k_T) = \sum_{l=1}^{2}\int_{b_0/b}^{M}\frac{dk_T}{k_T}\left(A_{\mathrm{CSS},\ell}(\alpha_s(k_T))\ln\frac{M^2}{k_T^2} + B_{\mathrm{CSS},\ell}(\alpha_s(k_T))\right)$$

anomalous dimensions

[Davies, Stirling '84] [De Florian, Grazzini '01] [Becher, Neubert '10][Li, Zhu '16][Vladimirov '16]

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for the reasons stressed above, interesting to find a solution in momentum space

Fo ... m

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[Catani, Grazzini '11][Catani et al. '12,Gehrmann][Luebbert, Yang '14]

coefficient functions

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Momentum space formulation

[Monni, Re, Torrielli '16]
[Bizon, Monni, Re, LR, Torrielli '17]
(SCET: [Ebert, Tackmann '16])

All-order cumulative cross section can be written as

$$\Sigma(v) = \int d\Phi_B \mathcal{V}(\Phi_B) \sum_{n=0}^{\infty} \int \prod_{i=1}^{n} [dk_i] |M(\tilde{p}_1, \tilde{p}_2, k_1, \ldots k_n)|^2 \Theta(v - V(\{\tilde{p}\}, k_1, \ldots k_n))$$

all-order form factor e.g. [Dixon, Magnea, Sterman '08] **real emissions**

rIRC safety of the observable allows to establish a well defined logarithmic counting

Possibile to do that by decomposing the squared amplitude in terms of *n*-particle correlated blocks: correlated blocks with *n* particles start contributing one logarithmic order higher than those with *n-1* particles

$$\sum_{n=0}^{\infty} |M(\tilde{p}_1, \tilde{p}_2, k_1, \ldots, k_n)|^2 = |M_B(\tilde{p}_1, \tilde{p}_2)|^2$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \left\{ \prod_{i=1}^{n} \left(\underbrace{|M(k_i)|^2}_{\text{LL}} + \int [dk_a][dk_b] |\underbrace{\tilde{M}(k_a, k_b)}_{\text{NLL}}|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} - \vec{k}_{ti}) \delta(Y_{ab} - Y_i) \right.\right.$$

$$\left.\left. + \int [dk_a][dk_b][dk_c] |\underbrace{\tilde{M}(k_a, k_b, k_c)}_{\text{NNLL}}|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} + \vec{k}_{tc} - \vec{k}_{ti}) \delta(Y_{abc} - Y_i) + \ldots \right) \right\}$$

$$\equiv |M_B(\tilde{p}_1, \tilde{p}_2)|^2 \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} |M(k_i)|^2_{\text{inc}}$$

for inclusive observables

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Momentum space formulation

Result can be expressed as

$$\frac{d\Sigma(v)}{d\Phi_B} = \int_{\mathcal{C}_1} \frac{dN_1}{2\pi i} \int_{\mathcal{C}_2} \frac{dN_2}{2\pi i} x_1^{-N_1} x_2^{-N_2} \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \mathbf{f}^T_{N_1}(\mu_0)\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v)\mathbf{f}_{N_2}(\mu_0)$$

DGLAP anomalous dimensions

RG evolution of coefficient functions

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0))H(\mu_R)\mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi}$$

$$\times e^{-\mathbf{R}(\epsilon k_{t1})} \exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t}\frac{\alpha_s(k_t)}{\pi}\mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t}\mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$

Result valid for all inclusive observables (e.g. p_T, φ^*)

unresolved emission + virtual corrections

$$\sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi}\mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$

$$\times \sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=2}^{n+1}\int_\epsilon^1 \frac{d\zeta_i}{\zeta_i}\int_0^{2\pi}\frac{d\phi_i}{2\pi}\sum_{\ell_i=1}^{2}\left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi}\mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right)$$

$$\times\,\Theta\left(v - V(\{\tilde{p}\},k_1,\dots,k_{n+1})\right)$$

resolved emission

Formulation **equivalent to *b*-space** result (up to a scheme change in the anomalous dimensions)

$$\frac{d^2\Sigma(v)}{d\Phi_B dp_t} = \sum_{c_1,c_2}\frac{d|M_B|^2_{c_1c_2}}{d\Phi_B}\int b\,db\,p_t J_0(p_t b)\,\mathbf{f}^T(b_0/b)\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(b_0/b))H(M)\mathbf{C}^{c_2}_{N_2}(\alpha_s(b_0/b))\mathbf{f}(b_0/b)$$

$$\times \exp\left\{-\sum_{\ell=1}^{2}\int_0^M\frac{dk_t}{k_t}\mathbf{R}'_\ell(k_t)\left(1-J_0(bk_t)\right)\right\}$$

$$(1-J_0(bk_t)) \simeq \Theta(k_t - \frac{b_0}{b}) + \frac{\zeta_3}{12}\frac{\partial^3}{\partial\ln(Mb/b_0)^3}\Theta(k_t-\frac{b_0}{b})$$

Result at NLL accuracy

Formulation in Mellin space already implementable. However, it is convenient to perform the evaluation entirely in momentum space

Integrands can be expanded about $k_{Ti} \sim k_{T1}$ to the desired accuracy: more efficient

we expanded around k_{T1}

The divergences cancel with the terms contained in the resolved real radiation

$$= e^{-R'(k_{T1}) \ln \frac{1}{\epsilon}} \qquad R' = \frac{d}{d\ln(M/k_{t1})} R$$

$$\frac{d\Sigma(v)}{d\Phi_B} = \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \partial_L \left(-e^{-R(k_{t1})} \mathcal{L}_{\rm NLL}(k_{t1}) \right) \epsilon^{R'(k_{t1})} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^{1} \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'(k_{t1}) \Theta(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}))$$

$$\zeta_i = k_{ti}/k_{t1}$$

resolved emission

parton luminosity at NLL reads

$$\mathcal{L}_{\rm NLL}(k_{t1}) = \sum_{c,c'} \frac{d|M_B|^2_{cc'}}{d\Phi_B} f_c(k_{t1}, x_1) f_{c'}(k_{t1}, x_2)$$

At higher logarithmic accuracy, it includes coefficient functions and hard-virtual corrections

This formula can be evaluated by means of fast Monte Carlo methods

Result at N3LL accuracy

$$
\begin{aligned}
\frac{d\Sigma(v)}{d\Phi_B} &= \int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}\partial_L\left(-e^{-R(k_{t1})}\mathcal{L}_{\mathrm{N^3LL}}(k_{t1})\right)\int d\mathcal{Z}[\{R',k_i\}]\Theta\left(v - V(\{\tilde{p}\},k_1,\dots,k_{n+1})\right)\\
&+ \int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}e^{-R(k_{t1})}\int d\mathcal{Z}[\{R',k_i\}]\int_0^1\frac{d\zeta_s}{\zeta_s}\frac{d\phi_s}{2\pi}\Bigg\{\left(R'(k_{t1})\mathcal{L}_{\mathrm{NNLL}}(k_{t1}) - \partial_L\mathcal{L}_{\mathrm{NNLL}}(k_{t1})\right)\\
&\times\left(R''(k_{t1})\ln\frac{1}{\zeta_s} + \frac{1}{2}R'''(k_{t1})\ln^2\frac{1}{\zeta_s}\right) - R'(k_{t1})\left(\partial_L\mathcal{L}_{\mathrm{NNLL}}(k_{t1}) - 2\frac{\beta_0}{\pi}\alpha_s^2(k_{t1})\hat{P}^{(0)}\otimes\mathcal{L}_{\mathrm{NLL}}(k_{t1})\ln\frac{1}{\zeta_s}\right)\\
&+\frac{\alpha_s^2(k_{t1})}{\pi^2}\hat{P}^{(0)}\otimes\hat{P}^{(0)}\otimes\mathcal{L}_{\mathrm{NLL}}(k_{t1})\Bigg\}\Big\{\Theta\left(v - V(\{\tilde{p}\},k_1,\dots,k_{n+1},k_s)\right) - \Theta\left(v - V(\{\tilde{p}\},k_1,\dots,k_{n+1})\right)\Big\}\\
&+\frac{1}{2}\int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}e^{-R(k_{t1})}\int d\mathcal{Z}[\{R',k_i\}]\int_0^1\frac{d\zeta_{s1}}{\zeta_{s1}}\frac{d\phi_{s1}}{2\pi}\int_0^1\frac{d\zeta_{s2}}{\zeta_{s2}}\frac{d\phi_{s2}}{2\pi}R'(k_{t1})\\
&\times\Bigg\{\mathcal{L}_{\mathrm{NLL}}(k_{t1})\left(R''(k_{t1})\right)^2\ln\frac{1}{\zeta_{s1}}\ln\frac{1}{\zeta_{s2}} - \partial_L\mathcal{L}_{\mathrm{NLL}}(k_{t1})R''(k_{t1})\left(\ln\frac{1}{\zeta_{s1}}+\ln\frac{1}{\zeta_{s2}}\right)\\
&+\frac{\alpha_s^2(k_{t1})}{\pi^2}\hat{P}^{(0)}\otimes\hat{P}^{(0)}\otimes\mathcal{L}_{\mathrm{NLL}}(k_{t1})\Bigg\}\\
&\times\Big\{\Theta\left(v - V(\{\tilde{p}\},k_1,\dots,k_{n+1},k_{s1},k_{s2})\right) - \Theta\left(v - V(\{\tilde{p}\},k_1,\dots,k_{n+1},k_{s1})\right) -\\
&\Theta\left(v - V(\{\tilde{p}\},k_1,\dots,k_{n+1},k_{s2})\right) + \Theta\left(v - V(\{\tilde{p}\},k_1,\dots,k_{n+1})\right)\Big\} + \mathcal{O}\left(\alpha_s^n\ln^{2n-6}\frac{1}{v}\right), \qquad (3.18)
\end{aligned}
$$

Implementation: matching to fixed order

$$\Sigma(p_T, \phi_B) = \int_0^{p_T} dp'_T \frac{d\sigma}{dp'_T d\phi_B}$$

Cumulative cross section should reduce to the fixed order at large p_T

$$\to \Sigma_{\text{res}} \qquad p_t \ll M_B$$

$$\to \Sigma_{\text{f.o.}} \qquad p_t \gtrsim M_B$$

Additive matching

$$\Sigma^{\text{add}}_{\text{matched}}(p_T) = \Sigma_{\text{res}}(p_T) + \Sigma_{\text{f.o.}}(p_T) - \Sigma_{\text{res,exp}}(p_T)$$

- perhaps more natural, simpler
- numerically delicate in the very small p_T limit as f.o. can be unstable

Multiplicative matching

$$\Sigma^{\text{mult}}_{\text{matched}}(p_T) = \Sigma_{\text{res}}(p_T) \frac{\Sigma_{\text{f.o.}}(p_T)}{\Sigma_{\text{res,exp}}(p_T)}$$

- constant terms can be included from fixed order
- numerically more stable as the physical suppression at small p_T cures potential instabilities

No rigorous theory argument to favour matching prescriptions

Implementation: matching to fixed order

Multiplicative matching at N3LL+NNLO allows to recover the constant terms at order α_s^3 which are currently not known analytically

$$\Sigma_{\rm f.o.} = \sigma^{\rm N^3LO}_{pp\to H} - \int_{p_T} p'_T \frac{d\Sigma^{\rm NNLO}_{\rm f.o.}}{dp'_T}$$

Additive matching would require knowledge of $C^{(3)}$ and $H^{(3)}$ in $\mathcal{L}_{\rm N3LL}$

Multiplicative matching however gives rise to higher-order terms in the p_T tail whose effect can be large

Damping factor Z to turn off smoothly the resummation at large p_T

$$Z = \left(1 - \left(\frac{p_T/M}{v_0}\right)^u\right)^h \Theta(v_0 - p_T/M)$$

$$\Sigma^{\rm mult}_{\rm matched}(p_T) = (\Sigma_{\rm res}(p_T))^Z \frac{\Sigma_{\rm f.o.}(p_T)}{(\Sigma_{\rm res,exp}(p_T))^Z}$$

$u=1$, $v_0=1/2$, $h=3$ in the following. Results stable with variations

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Implementation: resummation scale

Introduction of a resummation scale Q to estimate higher-order corrections

$$\ln \frac{M}{k_{T,1}} = \ln \frac{Q}{k_{T,1}} + \ln \frac{M}{Q} \qquad \ln \frac{Q}{k_{T,1}} \gg \ln \frac{M}{Q}$$

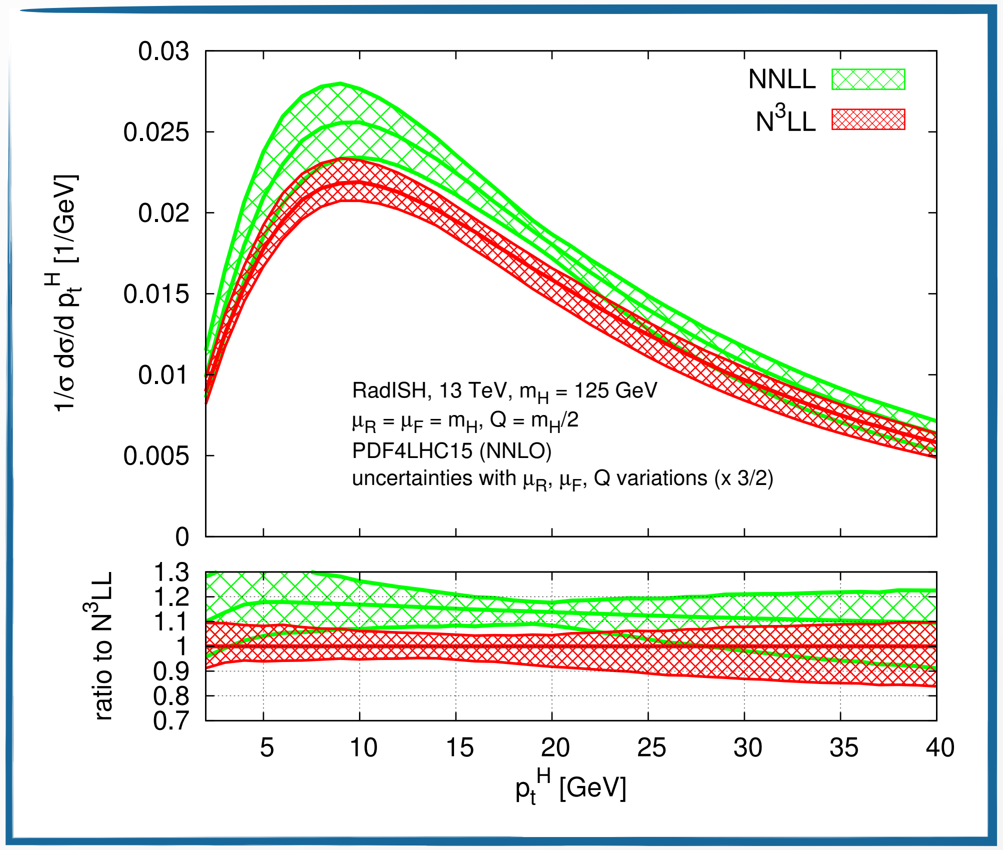
Logarithms of M/Q are then reabsorbed in H and C functions

In the following, $Q=m_H/2$, $Q \in [m_H/3, 3m_H/4]$ (no visible difference wrt to usual 2 variation in the resummed region)

Ensures that resummation is reliable in the peak region and avoids artifacts in the matched spectrum

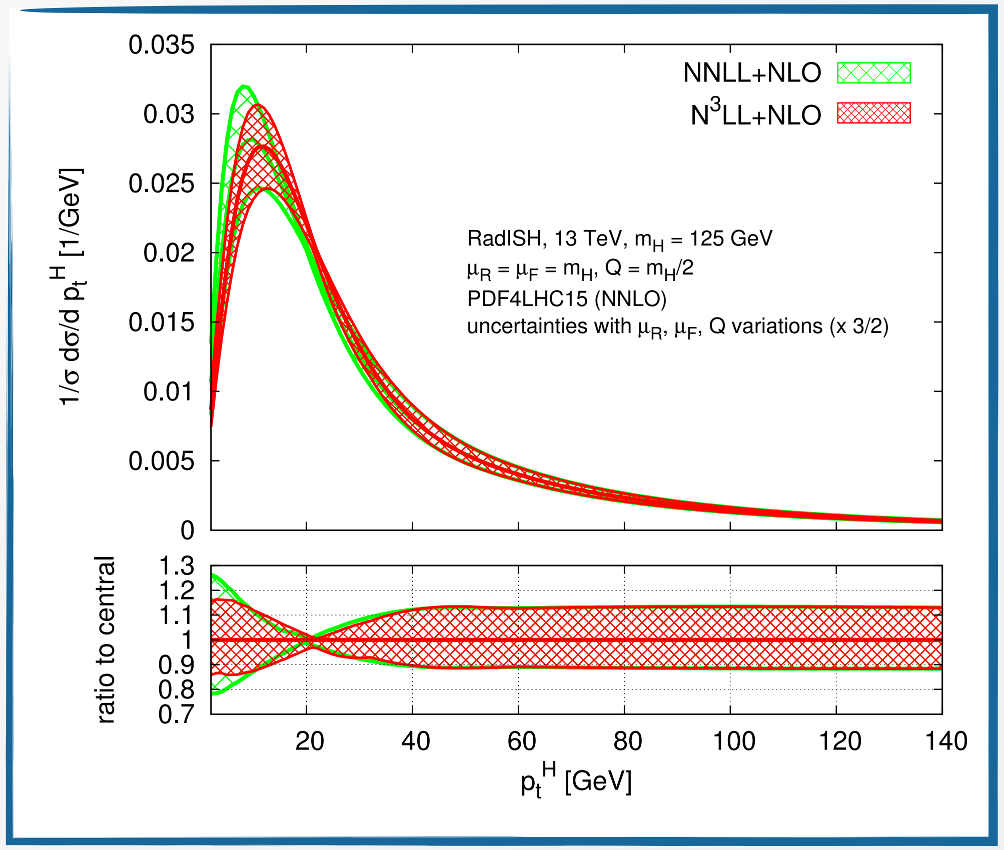
Impact of resummation

Pure resummation



- reduction of scale uncertainties

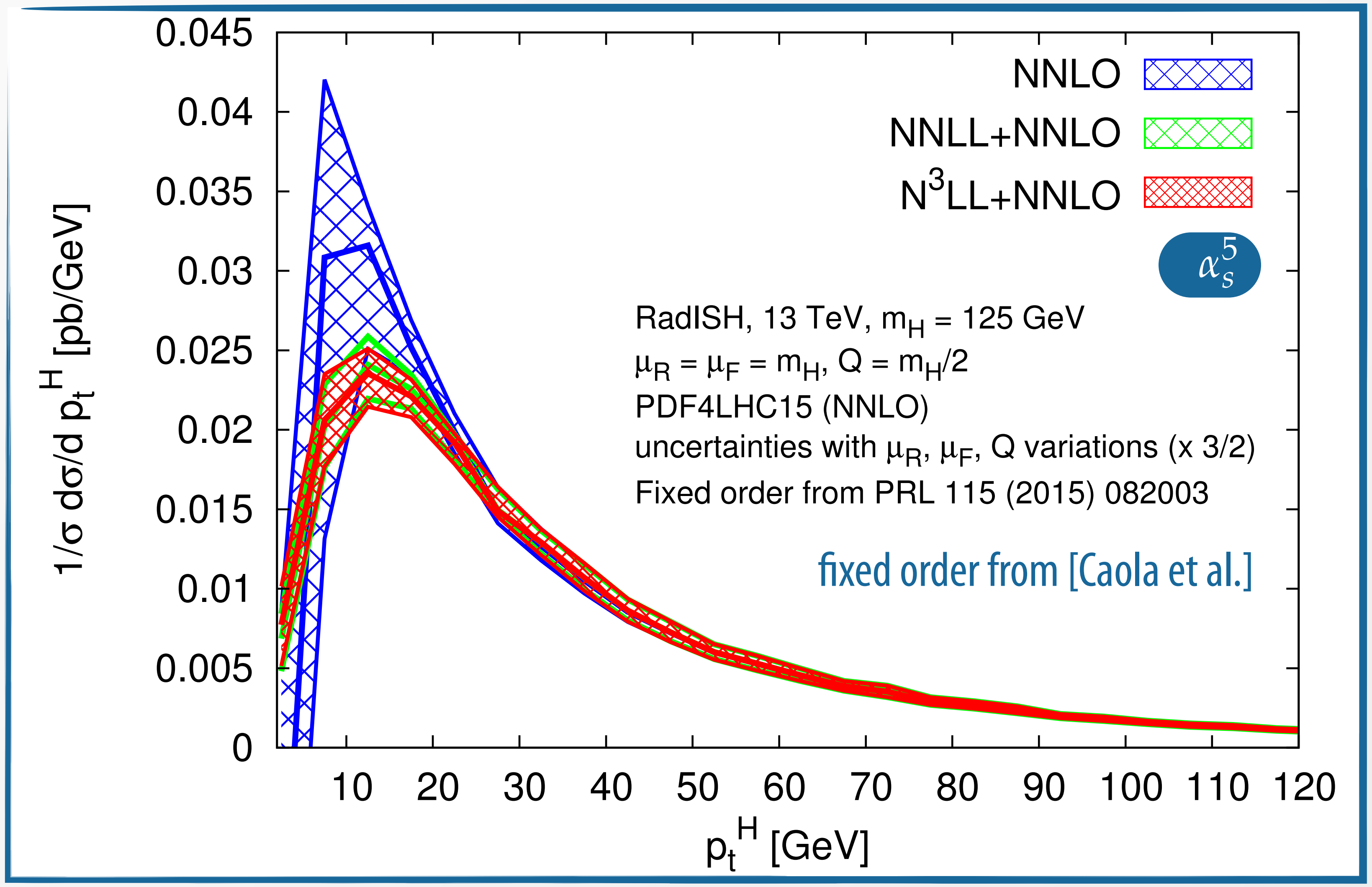
Matched results



- 10% correction at the peak
- scale uncertainties halved below 10 GeV

Results at N3LL+NNLO

- When matched to NNLO, the N³LL correction is a few % at the peak, and *O*(10%) at smaller values of p_T
- Rather moderate reduction of scale dependence at N³LL+NNLO. Need for very stable NNLO distributions below 15 GeV to appreciate reduction
- Mass effects corrections necessary to improve further
- Details of the matching also play a role: lots of studies ongoing, will be part of future work



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Drell-Yan $p_{T,ll}$ and φ^*

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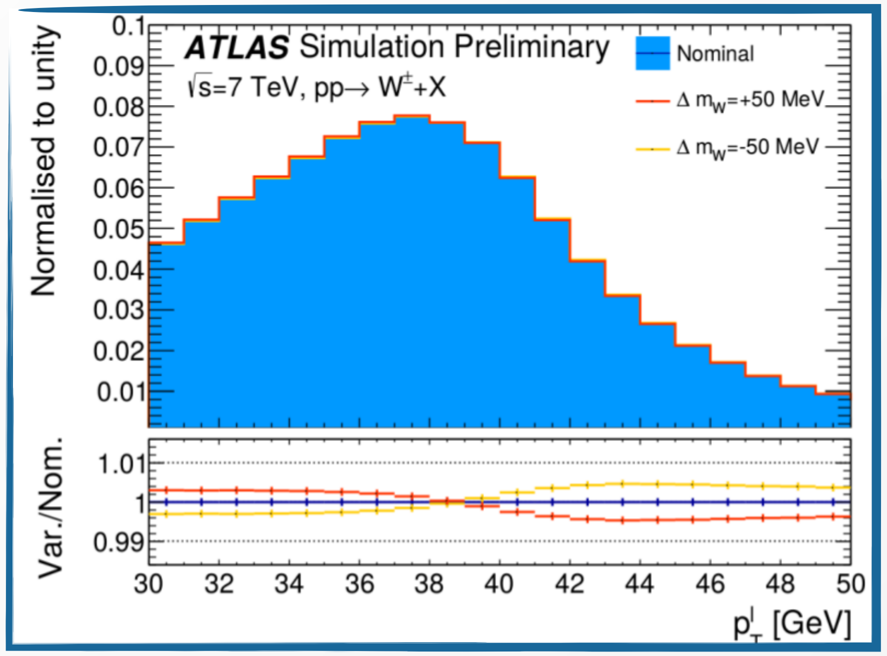
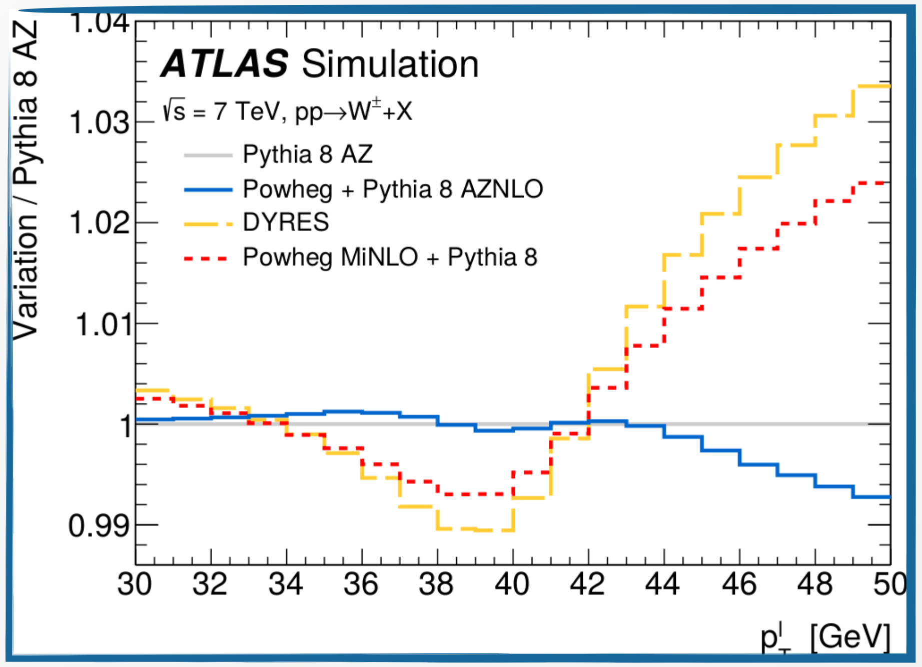
Drell-Yan

- Astonishing precision reached at the LHC. Many applications (PDF, strong coupling. . .)
- Extreme precision is needed e.g. for *W* mass extraction



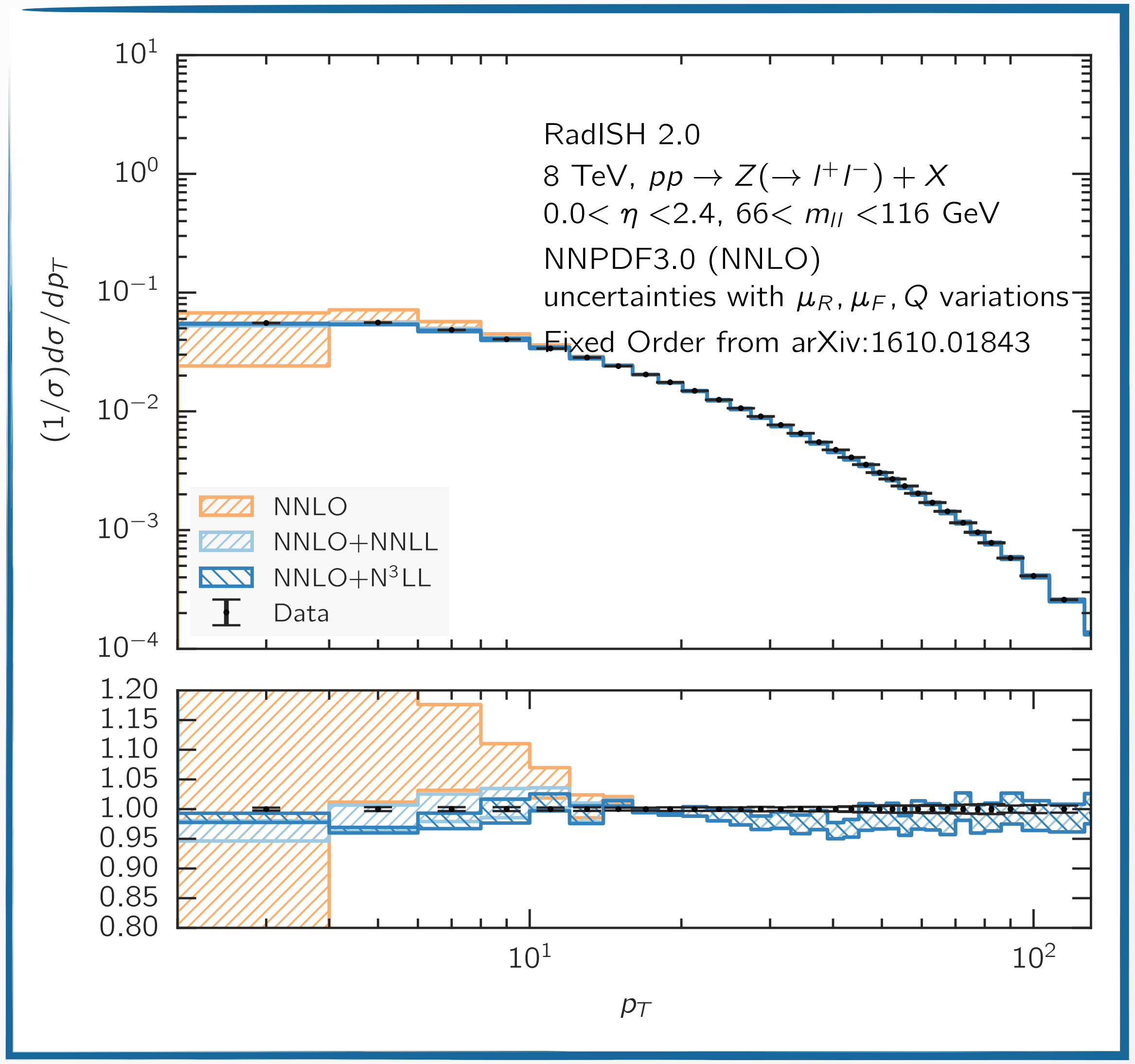
- Template fits to lepton observables
- Modelling of $p_{T,W}$ is crucial. Fit predictions to *Z* data, apply to *W*



- State-of-the-art QCD prediction do not match the precision of the data
- LO MC are used, tuned on *Z* data
- Would be preferable to use more accurate theoretical predictions

Drell-Yan: $p_{T,ll}$

[Bizon et al, ongoing]

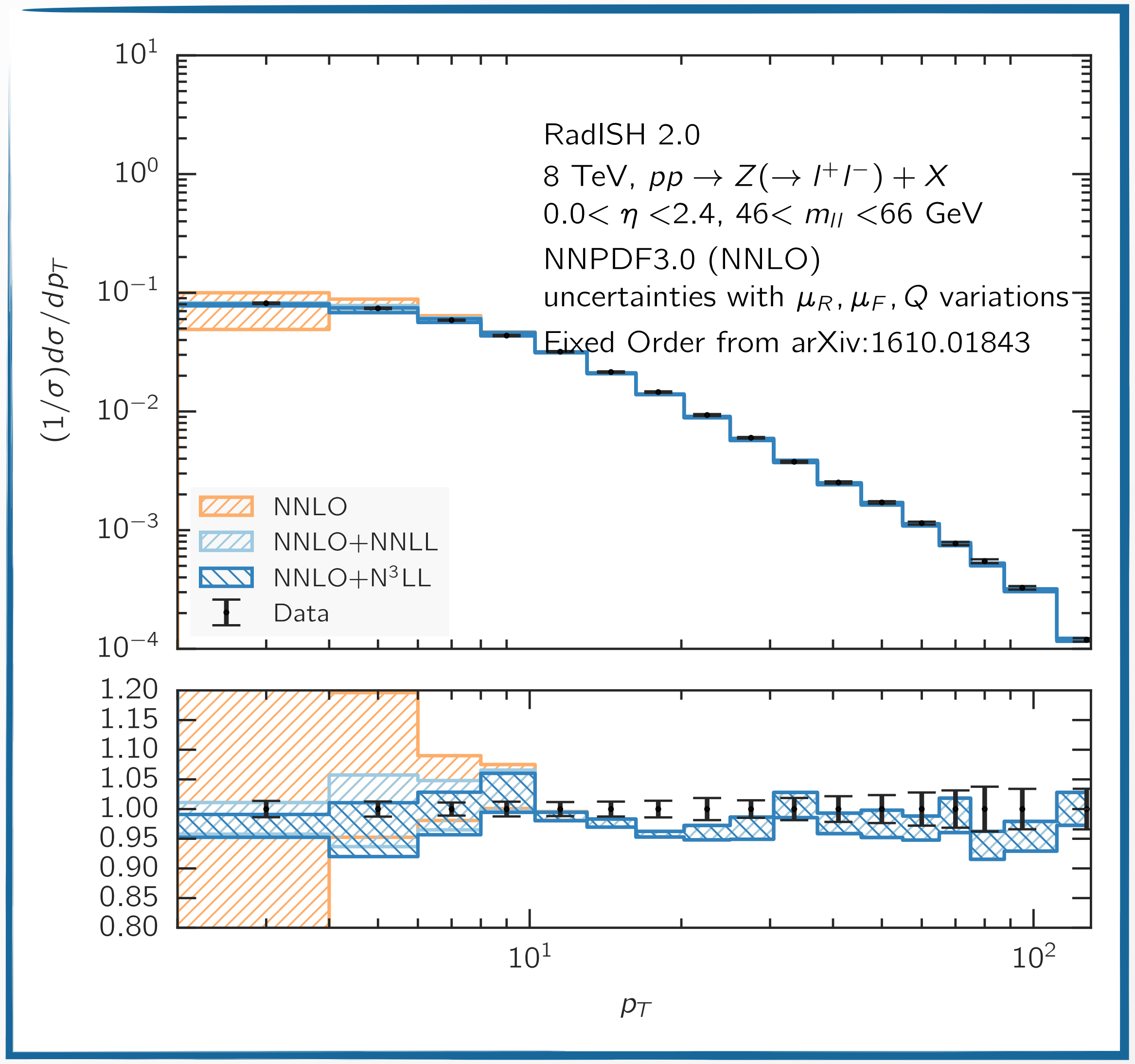


fixed order from [NNLOJET, Gehrmann-De Ridder et al., '16]

- Preliminary, data-driven choices: $Q=m_Z/4$; $h=5$
- Focus on a fiducial region and on an interval where we are confident that we are dealing only with matching ambiguities ($p_T \gtrsim 10$ GeV, far from non-perturbative region)

Drell-Yan: $p_{T,ll}$

[Bizon et al, ongoing]



fixed order from [NNLOJET, Gehrmann-De Ridder et al., '16]

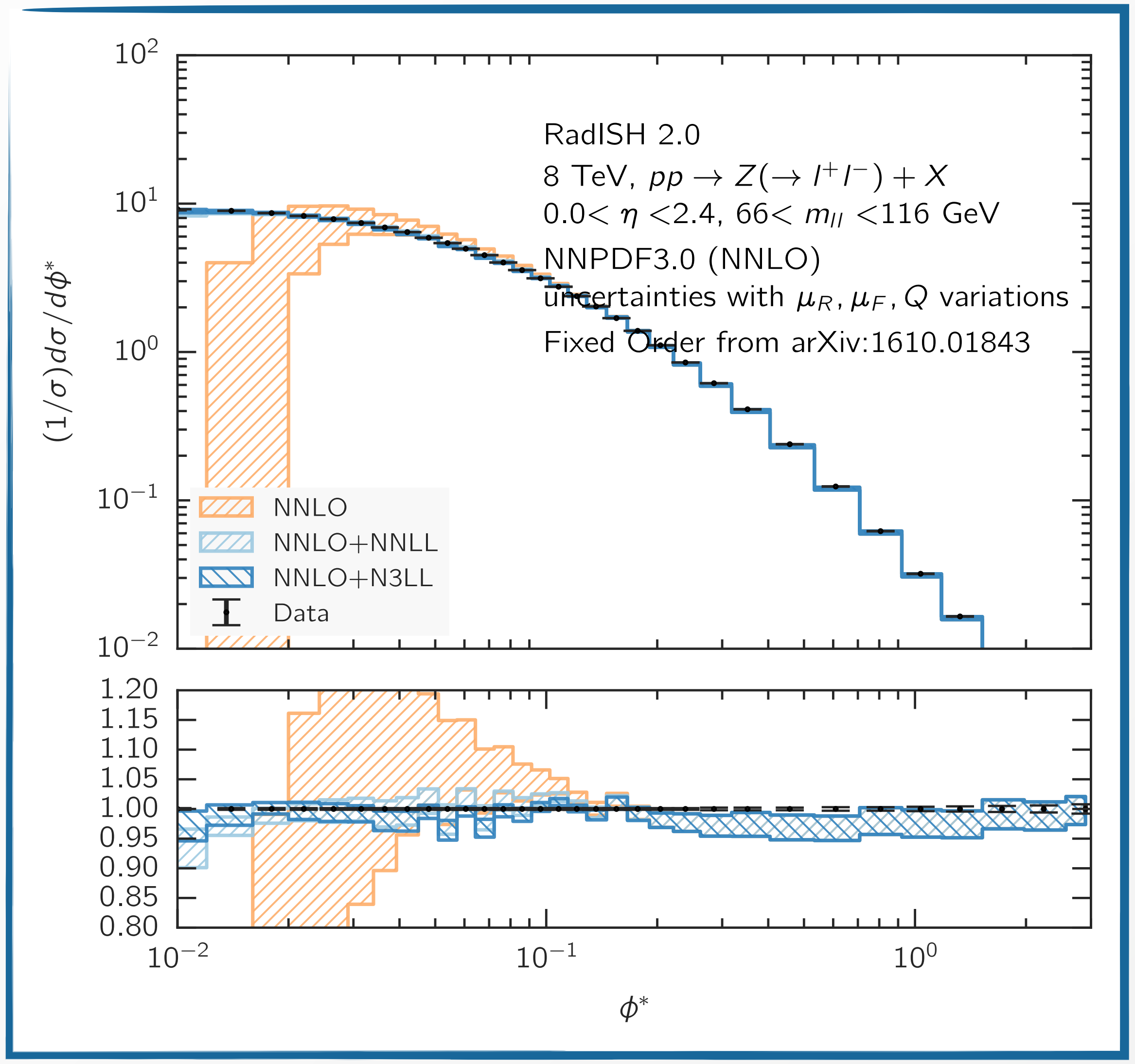
- Preliminary, data-driven choices: $Q=m_Z/4$; $h=5$
- Focus on a fiducial region and on an interval where we are confident that we are dealing only with matching ambiguities ($p_T \gtrsim 10$ GeV, far from non-perturbative region)
- Then look at other fiducial regions

progress in the computation of next-to-eikonal/power corrections may help to solve matching ambiguities

Drell-Yan: φ^*

[Bizon et al, ongoing]

Our approach can be used for resumming other transverse observables; e.g φ^*



fixed order from [NNLOJET, Gehrmann-De Ridder et al., '16]

$$\phi^* = \tan\left(\frac{\pi - \Delta\phi}{2}\right) \sin\theta^*$$

angle between electron and beam axis, in Z boson rest frame

- Preliminary, data-driven choices: $Q=m_Z/4$; $h=5$ (same as p_T)

Conclusions

- New formalism for all-order resummation up to **N^{3}LL accuracy** for inclusive, transverse observables.
- Method formulated in **momentum space**, formally equivalent to the standard *b*-space formalism
- Method allows for an **efficient implementation in a computer code**. Code `RadISH` can process any colour singlet with arbitrary cuts in the Born phase space. Public release soon.
- Extension to more general transverse observables possible thanks to the universality of the Sudakov radiator
- Encouraging phenomenological results. At this level of accuracy, several subleading effects start to play a role (details of the matching scheme, resummation scale choices, etc)

Advantages

- Method can be extended to other observables for which *b*-space formulation is not yet available
- Important to understand the dynamics of the radiation to improve generators
- Broader application range, possible generalization beyond the simple inclusive-observable case
- Possibility to perform **joint resummation** of observables

Backup

Formulation in momentum space

Is it possible to obtain a formulation in momentum space?

Not possible to find a closed analytic expression in direct space which is both a) free of logarithmically subleading corrections and b) free of singularities at finite p_T values [Frixione, Nason, Ridolfi '98]

Why? A naive logarithmic counting at small p_T is not sensible, as one loses the correct power-suppressed scaling if only logarithms are retained: it's not possible to reproduce a power behaviour with logs of p_T/M

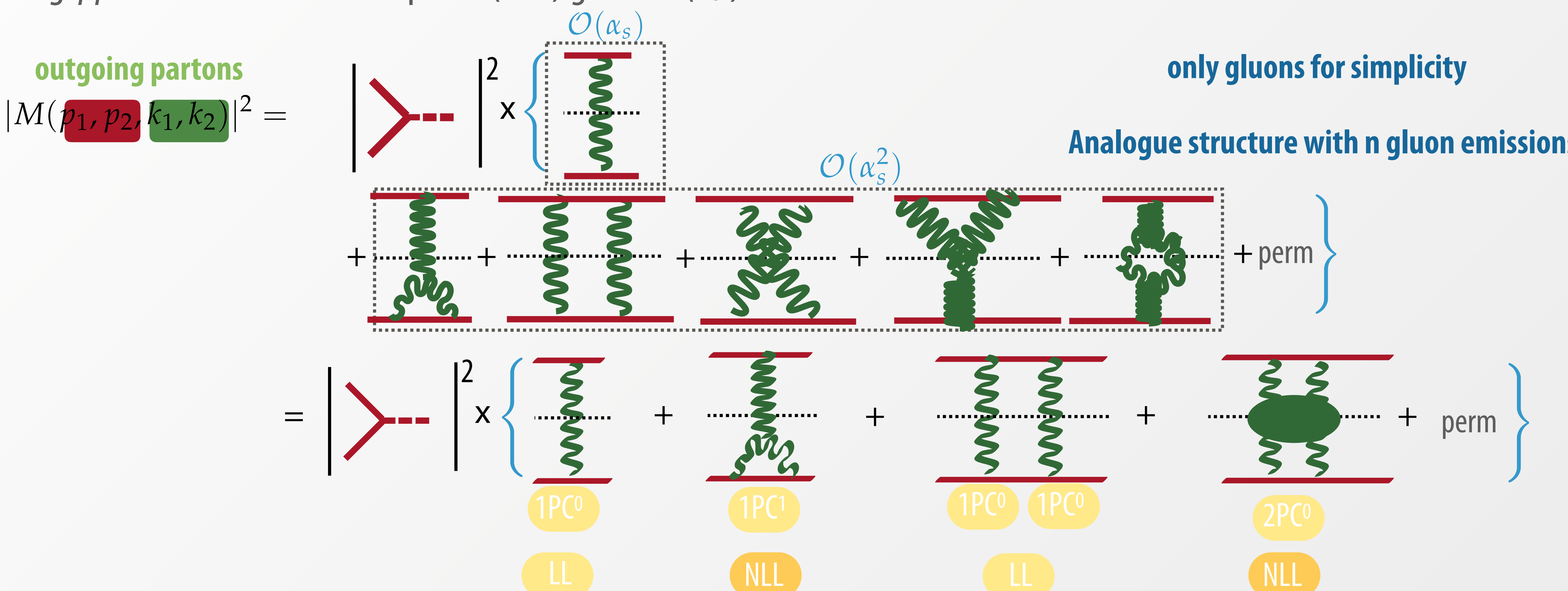
(logarithms of b do not correspond to logarithms of p_T)

Necessary to establish a well defined logarithmic counting in momentum space in order to reproduce the correct behaviour of the observable at small p_T

Logarithmic counting

Necessary to establish a **well defined logarithmic counting**: possibile to do that by decomposing the squared amplitude in terms of n-particle correlated blocks (**nPC**)

e.g. $pp \rightarrow H +$ emission of up to 2 (soft) gluons $O(\alpha_s^2)$



Logarithmic counting defined in terms of **nPC blocks** (owing to rIRC safety of the observable)

Resolved and unresolved emissions

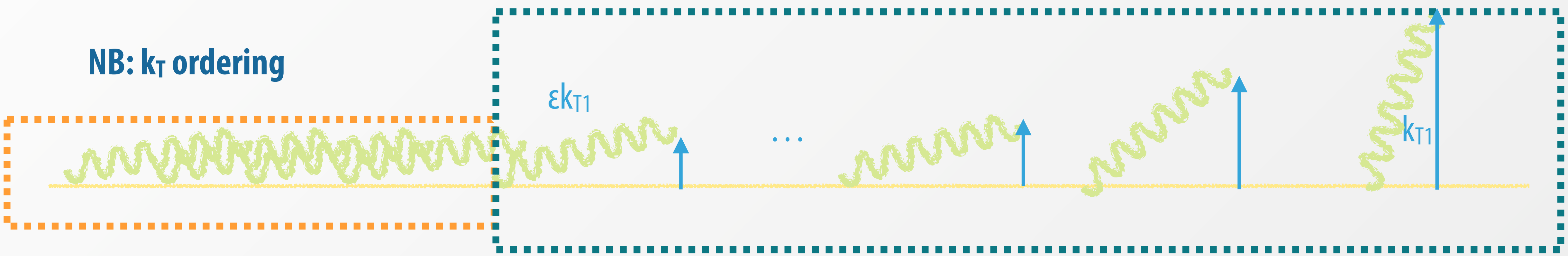
For inclusive observables (such as Higgs p_T) $\quad V(\{\hat{p}\},k_1,\ldots k_n) = V(\{\hat{p}\},k_1+\ldots+k_n)$

$$|M(p_1,p_2,k_1,\ldots,k_n)|^2 = |M_B(p_1,p_2)|^2$$
$$\times \frac{1}{n!}\left\{\prod_{i=1}^{n}\left(|M(k_i)|^2 + \int [dk_a][dk_b]|\tilde{M}(k_a,k_b)|^2\delta^{(2)}(\vec{k}_{ta}+\vec{k}_{tb}-\vec{k}_{ti})\delta(Y_{ab}-Y_i)\right.\right.$$
$$\left.\left. + \int [dk_a][dk_b][dk_c]|\tilde{M}(k_a,k_b,k_c)|^2\delta^{(2)}(\vec{k}_{ta}+\vec{k}_{tb}+\vec{k}_{tc}-\vec{k}_{ti})\delta(Y_{abc}-Y_i)+\ldots\right)\right\}$$

1PC 2PC 3PC

Introduction of a **resolution scale** εk_{T1}

NB: k_T ordering



unresolved emission

can be integrated inclusively to cancel the divergences of the virtuals (rIRC): exponential factor

$$e^{-R(\varepsilon k_{t1})}$$

Sudakov form factor

ε dependence cancels against the resolved real corrections

resolved emission

treated exclusively: for inclusive observables can be parametrised exactly as a Sudakov **unintegrated** in k_t and azimuthal angle

Resummation in momentum space

Formulation in Mellin space already implementable. However, it is convenient to perform the evaluation entirely in momentum space

In previous formula, resummation of logarithms of $k_{T,i}/M$

subleading logarithms in p_T

free of singularity at low p_T values

(power-law scaling)

$k_{Ti}/k_{T1} \sim O(1)$

(everywhere in the resolved phase space, due to rIRC safety)

Integrands can be expanded about $k_{Ti} \sim k_{T1}$ to the desired accuracy: more efficient

Sudakov region: $k_{T1} \sim p_T$

$ln(M/p_T)$ resummed at the desired accuracy

+ additional subleading terms that **cannot be neglected**

azimuthal region: $k_{Ti} \sim k_{T1}$

correct description of the kinematics after expansion $k_{Ti} \sim k_{T1}$

correct scaling of the cumulant $\boldsymbol{O(p_T^2)}$

Checks and remarks

- **b-space** formulation **reproduced analytically** at the resummed level
- **correct scaling** at small p_T computed analytically
- **numerical checks** down to very low p_T against b-space codes (`HqT`, `CuTe`) [Grazzini et al.][Becher et al.]
- check that the FO expansion of the final expression in momentum space up to $O(\alpha^5)$ yields the corresponding expansion in b-space (CSS)
- expansion checked against `MCFM` up to $O(\alpha^4)$ [Campbell et al.]

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