

# Recent developments in resummation

Luca Rottoli

Rudolf Peierls Centre for Theoretical Physics, University of Oxford



# QCD beyond fixed order

Perturbative QCD at fixed order

$$\tilde{\sigma} = 1 + \alpha_s \tilde{\sigma}_1 + \alpha_s^2 \tilde{\sigma}_2 + \alpha_s^3 \tilde{\sigma}_3 + \dots$$

**LO    NLO    NNLO    N<sup>3</sup>LO**

**NLO** now standard and largely automated

**NNLO** available for an increasing number of processes

**N<sup>3</sup>LO** Higgs production in gluon fusion and VBF (hadron-collider processes)

# QCD beyond fixed order

Perturbative QCD at fixed order

$$\tilde{\sigma} = 1 + \alpha_s \tilde{\sigma}_1 + \alpha_s^2 \tilde{\sigma}_2 + \alpha_s^3 \tilde{\sigma}_3 + \dots$$

**LO    NLO    NNLO    N<sup>3</sup>LO**

**NLO** now standard and largely automated

**NNLO** available for an increasing number of processes

**N<sup>3</sup>LO** Higgs production in gluon fusion and VBF (hadron-collider processes)

**Assumption:** perturbative coefficients  $\tilde{\sigma}_n$  are well behaved (renormalon ambiguity)

Many observables studied at the LHC depend on more than one scale; **single** or **double** logs of the ratio of those scales at all orders in perturbation theory

$$(\alpha_s \ln R)^n$$

$$(\alpha_s \ln^2 R)^n$$

If the logarithms are large the convergence of the series is spoiled

# QCD beyond fixed order

Perturbative QCD at fixed order

$$\tilde{\sigma} = 1 + \alpha_s \tilde{\sigma}_1 + \alpha_s^2 \tilde{\sigma}_2 + \alpha_s^3 \tilde{\sigma}_3 + \dots$$

LO    NLO    NNLO    N<sup>3</sup>LO

NLO now standard and largely automated

NNLO available for an increasing number of processes

N<sup>3</sup>LO Higgs production in gluon fusion and VBF (hadron collider processes)

**Fixed order predictions no longer reliable:  
all order resummation of the perturbative series mandatory**

Many observables studied at the LHC depend on more than one scale; **single** or **double** logs of the ratio of those scales at all orders in perturbation theory

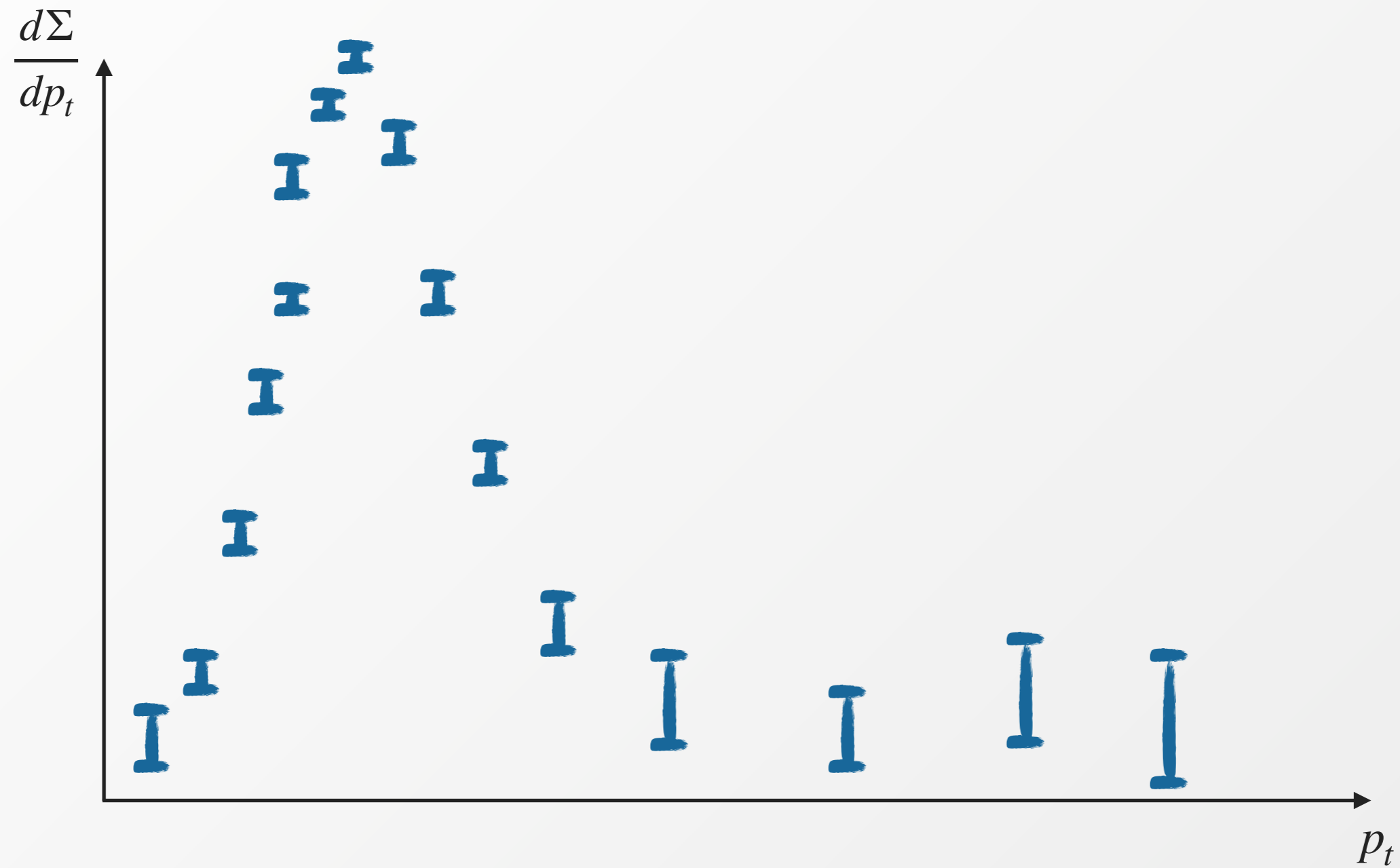
$$(\alpha_s \ln R)^n$$

$$(\alpha_s \ln^2 R)^n$$

If the logarithms are large the convergence of the series is spoiled

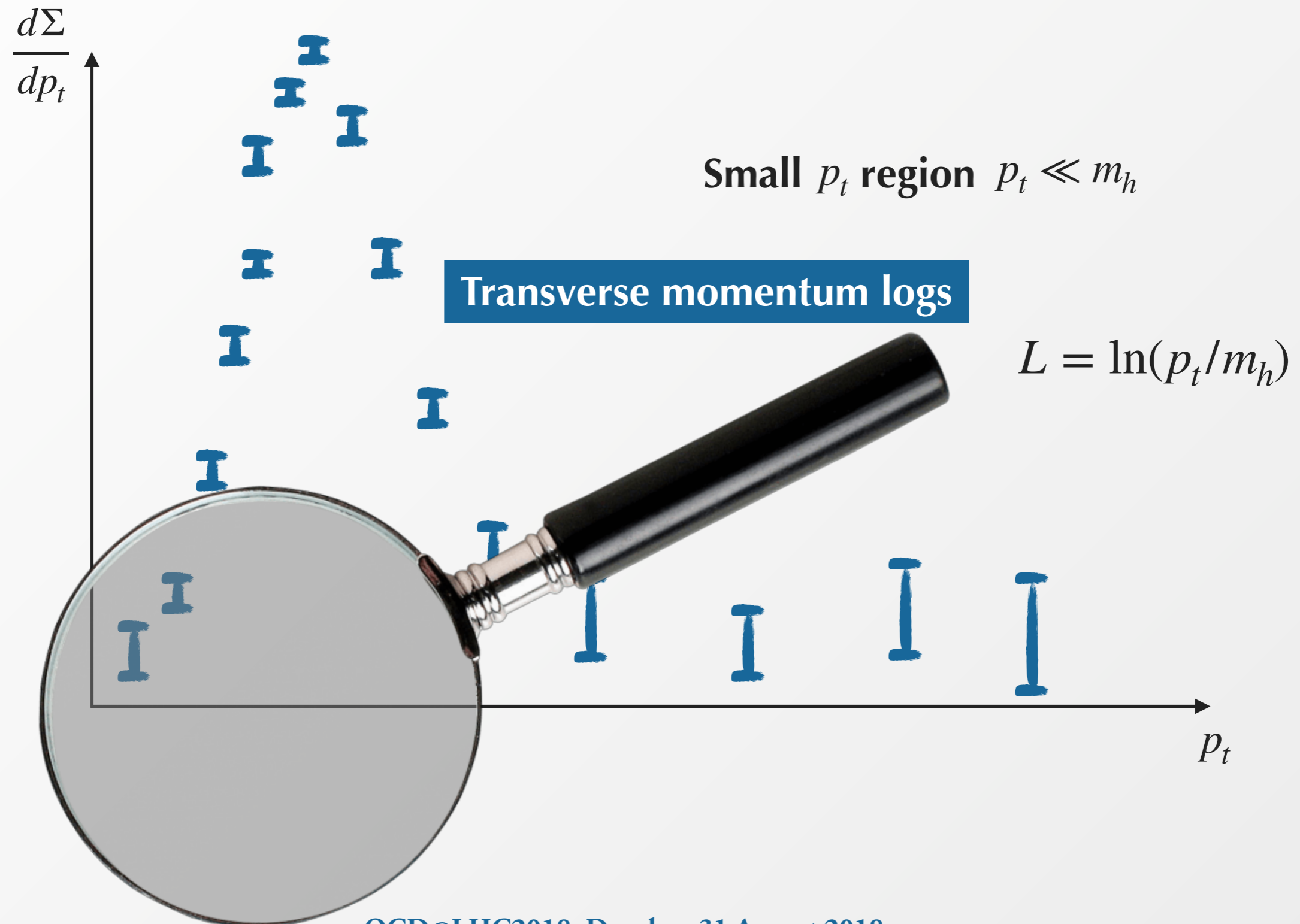
# Resum what?

Example: **transverse momentum distribution** in Higgs production



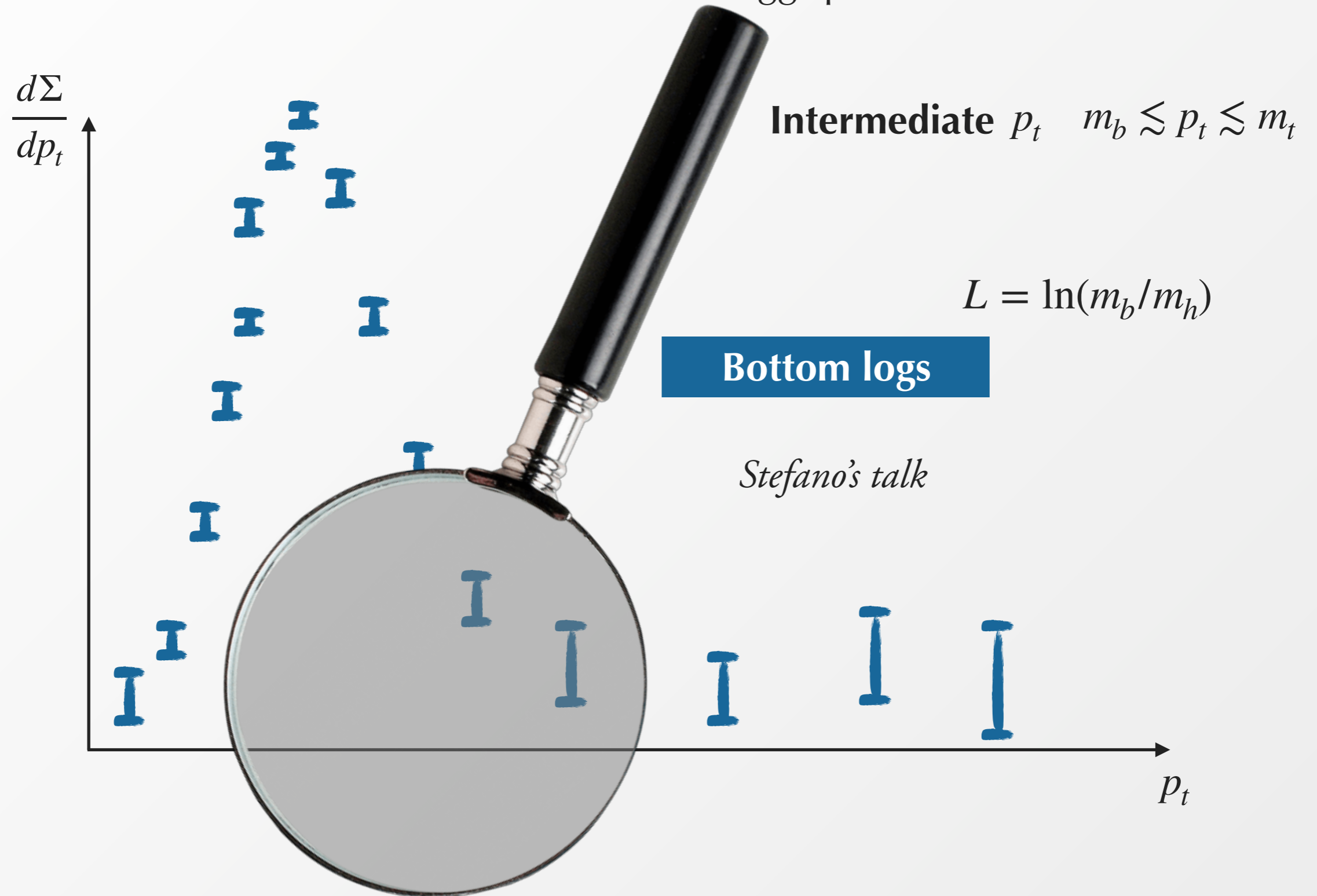
# Resum what?

Example: **transverse momentum distribution** in Higgs production



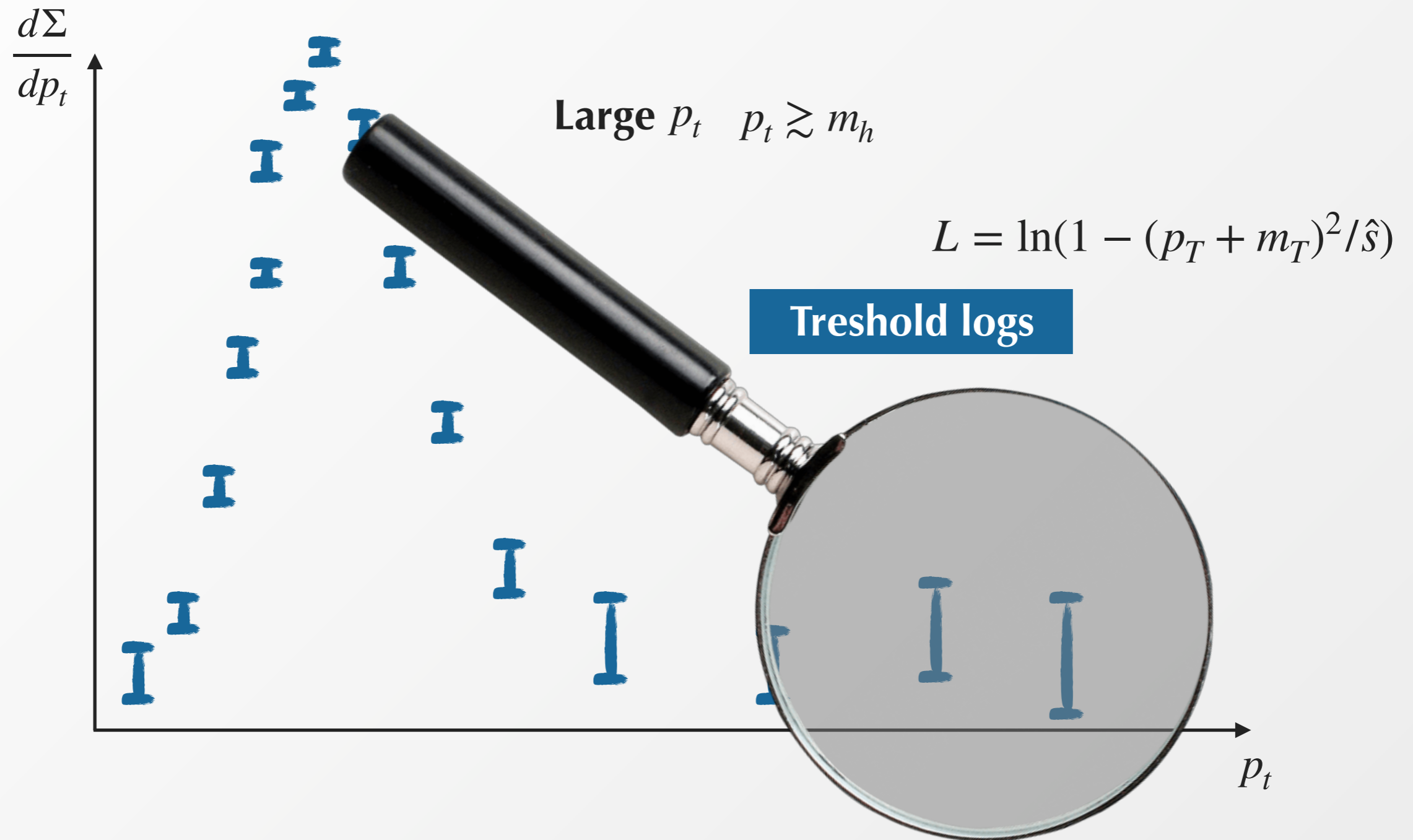
# Resum what?

Example: **transverse momentum distribution** in Higgs production



# Resum what?

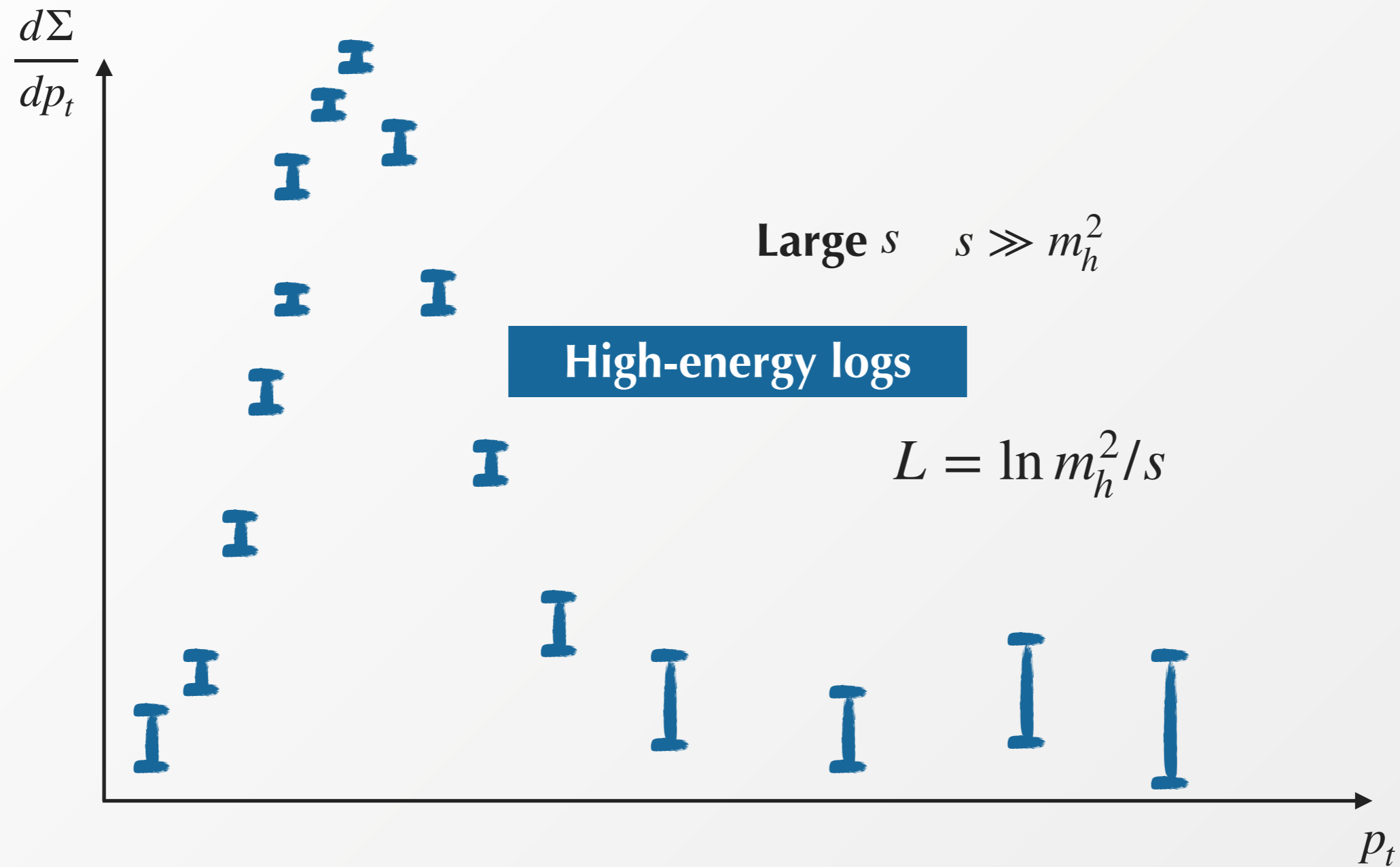
Example: **transverse momentum distribution** in Higgs production





# Resum what?

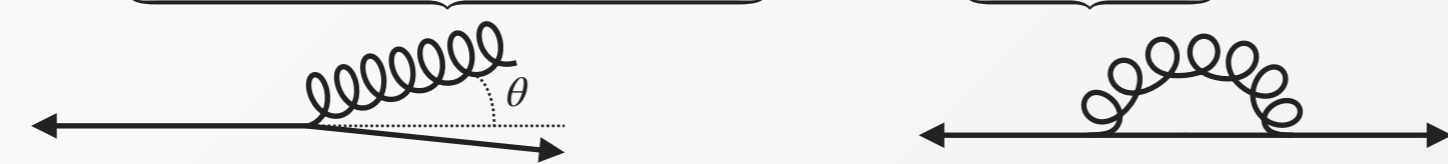
Example: **transverse momentum distribution** in Higgs production



# It's not a bug, it's a feature

Real emission diagrams singular for **soft/collinear emission**. Singularities are cancelled by virtual counterparts for IRC safe observables

Consider processes where real radiation is **constrained** in a corner of the phase space, (exclusive boundary of the phase space, **restrictive cuts**)

$$\tilde{\sigma}_1(\nu) \sim \underbrace{\int \frac{d\theta}{\theta} \frac{dE}{E} \Theta(\nu - E\theta/Q)}_{\text{Real emission diagram}} - \underbrace{\int \frac{d\theta}{\theta} \frac{dE}{E}}_{\text{Virtual counterpart}}$$


$$\sim - \int \frac{dE}{E} \frac{d\theta}{\theta} \Theta(E\theta/Q - \nu) \sim -\frac{1}{2} \ln^2 \nu \text{ Sudakov logarithms}$$

$\nu \rightarrow 0$  observable can become negative even in the perturbative regime

**Double** logarithms **leftovers** of the real-virtual cancellation of IRC divergences

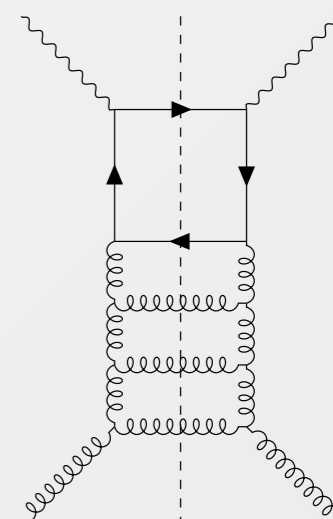
**Single** logarithms appear also when **exchanged gluon** is soft (**no collinear contribution**). **High-energy resummation** of  $\alpha_s \ln m^2/s$

Large phase space for emission of a cascade of partons with a very large fraction of the parent parton's **longitudinal** momentum

NB double logs in ggH in the heavy-top approximation

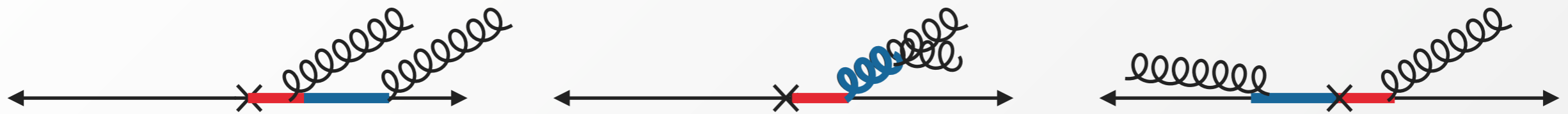
[Catani, Ciafaloni, Hautmann '90,'91,'04]

QCD@LHC2018, Dresden, 31 August 2018



# Making pQCD great again: all-order resummation

Soft-collinear emission of two gluons



Two propagators nearly on shell, 4 divergences. Diagrams can potentially give  $\alpha_s^2 \ln^4 v$

**All order** structure

$$\tilde{\sigma}(v) = \sum_{n=0}^{\infty} \alpha_s^n \sum_{m=1}^{2n} c_{nm} L^m + \dots \quad L = \ln(v)$$

Origin of the logs is simple. Resum them to all orders by **reorganizing** the series

$$\tilde{\sigma}(v) = \boxed{f_1(\alpha_s L^2)} + \frac{1}{L} f_2(\alpha_s L^2) + \dots$$

**Leading logarithmic (LL) resummation of the perturbative series**

Accurate for  $L \sim 1/\sqrt{\alpha_s}$

# All-order resummation

$$\tilde{\sigma}(v) = f_1(\alpha_s L^2) + \frac{1}{L} f_2(\alpha_s L^2) + \dots$$



*“It's the sum that makes the total”\**

*\*È la somma che fa il totale*

# All-order resummation: exponentiation

Independent emissions  $k_1, \dots, k_n$  (plus corresponding virtual contributions) in the soft and collinear limit (**eikonal approximation**)

$$d\Phi_n | \mathcal{M}(k_1, \dots, k_n) |^2 \rightarrow \frac{1}{n!} \alpha_s^n \prod_{i=1}^n \frac{dE_i}{E_i} \frac{d\theta_i}{\theta_i}$$

Calculate observable with arbitrary number of emissions: **exponentiation**

$$\tilde{\sigma} \simeq \sum_{n=0}^{\infty} \frac{1}{n!} \alpha_s^n \prod_{i=1}^n \int \frac{dE_i}{E_i} \frac{d\theta_i}{\theta_i} \Theta(E_i \theta_i / Q - \nu) \simeq e^{-\alpha_s L^2} \quad \text{[Sudakov '54]}$$

**Sudakov suppression**  
Price for constraining real radiation

Exponentiated form allows for a **more powerful reorganization**

$$\tilde{\sigma}(\nu) = \exp \left[ \sum_n \left( \underbrace{\mathcal{O}(\alpha_s^n L^{n+1})}_{\text{LL}} + \underbrace{\mathcal{O}(\alpha_s^n L^n)}_{\text{NLL}} + \underbrace{\mathcal{O}(\alpha_s^n L^{n-1})}_{\text{NNLL}} + \dots \right) \right]$$

Region of applicability now valid up to  $L \sim 1/\alpha_s$ , successive terms suppressed by  $\alpha_s$

Exponentiation not always possible, e.g. Jade Jet Resolution [Brown, Stirling '90] or jet mass pruning (convolution of two exponentials) [Dasgupta, Marzani, Salam '13]

# All-order resummation: (re)-factorization

Phase-space constraints do not usually factorize in **direct space**

$$\tilde{\sigma}(v) \sim \int \prod_i^n [dk_i] \mathcal{M}(k_1, \dots, k_n) \Theta_{\text{PS}}(v - V(k_1, \dots, k_n))$$

Solution: move to **conjugate space** where phase space factorization is manifest

e.g.  $p_t$  resummation  
 [Parisi, Petronzio '79; Collins, Soper, Sterman '85]

$$\delta^{(2)}\left(\vec{p}_t - \sum_{i=1}^n \vec{k}_{t,i}\right) = \int d^2b \frac{1}{4\pi^2} e^{i\vec{b} \cdot \vec{p}_t} \prod_{i=1}^n e^{-i\vec{b} \cdot \vec{k}_{t,i}}$$

two-dimensional momentum conservation

Exponentiation in conjugate space; **inverse transform** to move back to direct space

**Extremely successful** approach

**direct QCD**

- Catani, Trentadue, Mangano, Marchesini, Webber, Nason, Dokshitzer...

Emphasis on properties of QCD matrix elements and QCD radiation

- Collins, Soper, Sterman, Laenen, Magnea...

Factorization properties in the singular region and associated RGE (factorization → evolution → resummation)

**SCET**

- Manohar, Bauer, Stewart, Becher, Neubert....

+ many others!

SCET vs. dQCD **not an issue** [Sterman *et al.* '13, '14][Bonvini, Forte, Ghezzi, Ridolfi, LR '12, '13, '14]  
 [Becher, Neubert *et al.* '08, '11, 14]

# All-order resummation: (re)-factorization

Phase-space constraints do not usually factorize in **direct space**

$$\tilde{\sigma}(v) \sim \int \prod_i^n [dk_i] \mathcal{M}(k_1, \dots, k_n) \Theta_{\text{PS}}(v - V(k_1, \dots, k_n))$$

Solution: move to **conjugate space** where phase space factorization is manifest

e.g.  $p_t$  resummation  
[Parisi, Petronzio '79; Collins, Soper, Sterman '85]

$$\delta^{(2)}\left(\vec{p}_t - \sum_{i=1}^n \vec{k}_{t,i}\right) = \int d^2b \frac{1}{4\pi^2} e^{i\vec{b} \cdot \vec{p}_t} \prod_{i=1}^n e^{-i\vec{b} \cdot \vec{k}_{t,i}}$$

two-dimensional momentum conservation

Exponentiation in conjugate space; **inverse transform** to move back to direct space

**Extremely successful** approach

**Limitation: it is process-dependent, and must be performed manually and analytically for each observable (error prone)**

## All-order resummation: (re)-factorization

Phase-space constraints do not usually factorize in direct space

*Is it possible to achieve resummation without the need to establish factorization properties on a case-by-case basis?*

Limitation: it is process-dependent, and must be performed manually and analytically for each observable (error prone)



## All-order resummation: (re)-factorization

Phase-space constraints do not usually factorize in direct space

*Is it possible to achieve resummation without the need to establish factorization properties on a case-by-case basis?*

Limitation: it is process-dependent, and must be performed manually and analytically for each observable (error prone)

*Yes*

# CAESAR/ARES approach: towards automated resummation

Translate the resummability of the observable into properties of the observable in the presence of multiple radiation: **recursive infrared and collinear (rIRC) safety**

[Banfi, Salam, Zanderighi '01, '03, '04]

Existence of a **resolution scale**  $q_0$ , **independent of the observable**, such that emissions below  $q_0$  (**unresolved**) do not contribute significantly to the observable's value.

$\tilde{\sigma}(v) \sim \int d[k_1] \boxed{e^{-R(q_0 V(k_1))}}$  **Unresolved emission** can be treated as **totally uncorrelated**  
 $\rightarrow$  **exponentiation**

$$\times \left( \sum_{m=0}^{\infty} \frac{1}{m!} \int \prod_{i=2}^{m+1} [dk_i] |\mathcal{M}(k_i)|^2 \Theta(V(k_i) - q_0 V(k_1)) \Theta(v - V(k_1, \dots, k_{m+1})) \right)$$

**Resolved emission** treated exclusively with **Monte Carlo methods**

**Method entirely formulated in direct space**

- Generic structure of rIRC safe observables known at NNLL [Banfi, Monni, McAslan, Zanderighi '14, 16]
- Event shapes at hadron colliders [Banfi, Salam, Zanderighi '10]
- Observables with azimuthal cancellation (e.g.  $p_t$ ) [(Bizon), Monni, Re, (LR), Torrielli '16, '17]
- rIRC safe jet observables at NNLL [Banfi, El-Menoufi, Monni, '18]

# Some recent results

# Resummation ca. 2018



Welcome to INSPIRE, the High Energy Physics information system. Please direct questions, comments or concerns to [feedback@inspirehep.net](mailto:feedback@inspirehep.net).

HEP :: HEPNAMES :: INSTITUTIONS :: CONFERENCES :: JOBS :: EXPERIMENTS :: JOURNALS :: HELP

find t resummation and date > 2017

Brief format

Search

[Easy Search](#)  
[Advanced Search](#)

[find | "Phys.Rev.Lett.,105"](#) :: [more](#)

Sort by:

Display results:

earliest date desc. - or rank by - 25 results single list

HEP

35 records found 1 - 25 ▶ jump to record: 1

Search took 0.97 seconds.

## 1. Threshold and jet radius joint resummation for single-inclusive jet production

Sven-Olaf Moch, Engin Eren, Katerina Lipka, Xiaohui Liu, Felix Ringer. Aug 14, 2018. 8 pp.

DESY 18-135

Conference: [C18-04-29.1](#)

e-Print: [arXiv:1808.04574](#) [hep-ph] | [PDF](#)

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

[ADS Abstract Service](#)

[Detailed record](#)

## 2. Gradient resummation for nonlinear chiral transport: an insight from holography

Yanyan Bu (Harbin Inst. Tech.), Tuna Demircik, Michael Lublinsky (Ben Gurion U. of Negev). Jul 30, 2018. 44 pp.

e-Print: [arXiv:1807.11908](#) [hep-th] | [PDF](#)

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

[ADS Abstract Service](#)

[Detailed record](#)

## 3. Long distance behavior of

### $O(N)$ -model correlators in de Sitter space and the resummation of secular terms

Diana López Nacir (Buenos Aires U.), Francisco D. Mazzitelli (Balseiro Inst., San Carlos de Bariloche & Centro Atomico Bariloche), Leonardo G. Trombetta (Pisa, Scuola Normale Superiore & INFN, Pisa). Jul 16, 2018. 29 pp.

e-Print: [arXiv:1807.05964](#) [hep-th] | [PDF](#)

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

[ADS Abstract Service](#)

[Detailed record](#)

## 4. Resummation in QFT with Meijer G-functions

Oleg Antipin, Alessio Maiezza (Boskovic Inst., Zagreb), Juan Carlos Vasquez (CCTVal, Valparaiso). Jul 13, 2018. 22 pp.

e-Print: [arXiv:1807.05060](#) [hep-th] | [PDF](#)

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

[ADS Abstract Service](#)

[Detailed record](#) - [Cited by 1 record](#)

## 5. Threshold resummation in rapidity for colorless particle production at LHC

Pulak Banerjee (IMSc, Chennai), Goutam Das (DESY), Prasanna K. Dhani, V. Ravindran (HBNI, Mumbai). Jul 12, 2018. 8 pp.

DESY 18-115, DESY-18-115

Conference: [C18-04-29.1](#)

e-Print: [arXiv:1807.04583](#) [hep-ph] | [PDF](#)

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

[CERN Document Server](#); [ADS Abstract Service](#)

[Detailed record](#)

## 6. Soft gluon resummation for Higgs boson pair production including finite

### $M_t$ effects

Daniel De Florian (ICAS, UNSAM, Buenos Aires), Javier Mazzitelli (Zurich U.). Jul 10, 2018. 16 pp.

ZU-TH 26/18, ICAS 35/18, ZU-TH-26-18, ICAS-35-18

e-Print: [arXiv:1807.03704](#) [hep-ph] | [PDF](#)

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

[ADS Abstract Service](#)

[Detailed record](#) - [Cited by 1 record](#)

# Resummation ca. 2018



Welcome to INSPIRE, the High Energy Physics information system. Please direct questions, comments or concerns to [feedback@inspirehep.net](mailto:feedback@inspirehep.net).

HEP :: HEPNAMES :: INSTITUTIONS :: CONFERENCES :: JOBS :: EXPERIMENTS :: JOURNALS :: HELP

## Disclaimer & apology

find t resummation and date > 2017

find | "Phys.Rev.Lett.,105" :: more

Sort by:

Display results:

earliest date desc - or rank by - 25 results single list

HEP

35 records found 1 - 25 jump to record: 1

Search took 0.97 seconds.

### 1. Threshold and jet radius joint resummation for single-inclusive jet production

Sven-Olaf Moch, Engin Eren, Katerina Lipka, Xiaohui Liu, Felix Ringer. Aug 14, 2018. 8 pp.

DESY 18-135

Conference: C18-04-29.1

e-Print: [arXiv:1808.04574](https://arxiv.org/abs/1808.04574) [hep-ph] | PDF

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

[ADS Abstract Service](#)

[Detailed record](#)

### 2. Gradient resummation for nonlinear chiral transport: an insight from *personal*

Yanyan Bu (Harbin Inst. Tech.), Tuna Demircik, Michael Lublinsky (Ben Gurion U. of Neg.)

e-Print: [arXiv:1807.11908](https://arxiv.org/abs/1807.11908) [hep-th] | PDF

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

[ADS Abstract Service](#)

[Detailed record](#)

### 3. Long distance behavior of

$O(N)$ -model correlators in de Sitter space and the resummation of secular terms

Diana López Nacir (Buenos Aires U.), Francisco D. Mazzitelli (Balseiro Inst., San Carlos de Bariloche & Centro Atomico Bariloche), Leonardo G. Trombetta (Pisa, Scuola Normale Superiore & INFN, Pisa). Jul 16, 2018. 29 pp.

e-Print: [arXiv:1807.05964](https://arxiv.org/abs/1807.05964) [hep-th] | PDF

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

[ADS Abstract Service](#)

[Detailed record](#)

### 4. Resummation in QFT with Meijer G-functions

Oleg Antipin, Alessio Maiezza (Boskovic Inst., Zagreb), Juan Carlos Vasquez (CCTVal, Valparaiso). Jul 13, 2018. 22 pp.

e-Print: [arXiv:1807.05060](https://arxiv.org/abs/1807.05060) [hep-th] | PDF

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

[ADS Abstract Service](#)

[Detailed record](#) - [Cited by 1 record](#)

### 5. Threshold resummation in rapidity for colorless particle production at LHC

Pulak Banerjee (IMSc, Chennai), Goutam Das (DESY), Prasanna K. Dhani, V. Ravindran (HBNI, Mumbai). Jul 12, 2018. 8 pp.

DESY 18-115, DESY-18-115

Conference: C18-04-29.1

e-Print: [arXiv:1807.04583](https://arxiv.org/abs/1807.04583) [hep-ph] | PDF

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

[CERN Document Server](#); [ADS Abstract Service](#)

[Detailed record](#)

### 6. Soft gluon resummation for Higgs boson pair production including finite

$M_t$  effects

Daniel De Florian (ICAS, UNSAM, Buenos Aires), Javier Mazzitelli (Zurich U.). Jul 10, 2018. 16 pp.

ZU-TH 26/18, ICAS 35/18, ZU-TH-26-18, ICAS-35-18

e-Print: [arXiv:1807.03704](https://arxiv.org/abs/1807.03704) [hep-ph] | PDF

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

[ADS Abstract Service](#)

[Detailed record](#) - [Cited by 1 record](#)

*personal*  
~~a biased~~ selection of topics

not a comprehensive review or set of references

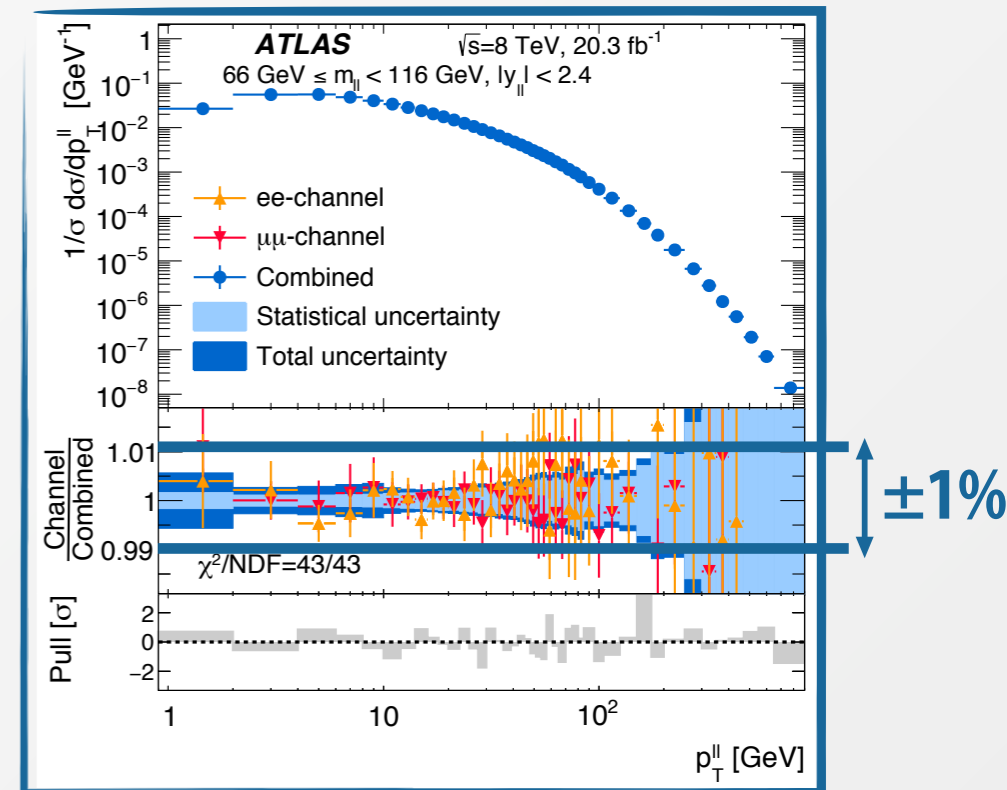
# Resummation for transverse observables

# Resummation for transverse observables at the LHC

If the scale for NP  $\Lambda_{\text{NP}}$  is a few TeV, rough estimate of **deviations from the SM** behaviour gives  $\delta \sim Q^2/\Lambda_{\text{NP}}^2 \rightarrow$  Bulk:  $Q^2 \sim 0.1 \text{ TeV}$   $\delta \sim 1\text{-}5\%$

This level of precision is within reach at the (HL)-LHC (e.g. astonishing precision in Z transverse observables)

**Very accurate theoretical predictions needed for transverse distributions**



Besides implications for indirect constraints on BSM physics, important implications for **extraction of SM parameters** (strong coupling and PDF determination,  $W$  mass...)

Fixed-order predictions now available at **NNLO** (in the EFT for Higgs production)

[Boughezal *et al.* '15][Caola *et al.* '15][Chen *et al.* '16]

All ingredients to perform resummation at **N<sup>3</sup>LL accuracy** are now available

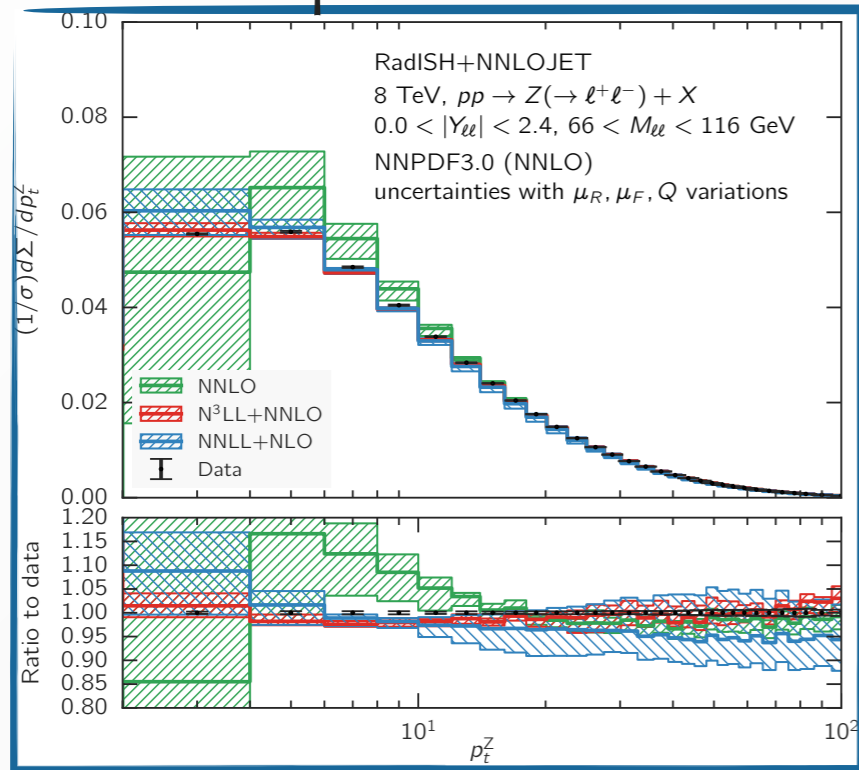
[Catani *et al.* '11, '12][Gehrmann *et al.* '14][Li, Zhu '16][Moch *et al.* '18]

# Drell-Yan transverse observables at N<sup>3</sup>LL+NNLO

Wojtek's talk

[Bizon, Monni, Re, LR, Torrielli + NNLOJET '18]

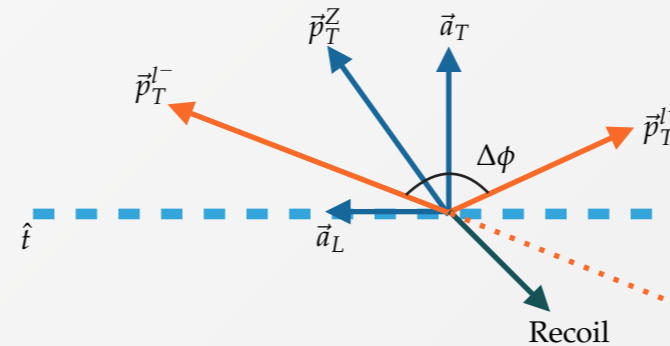
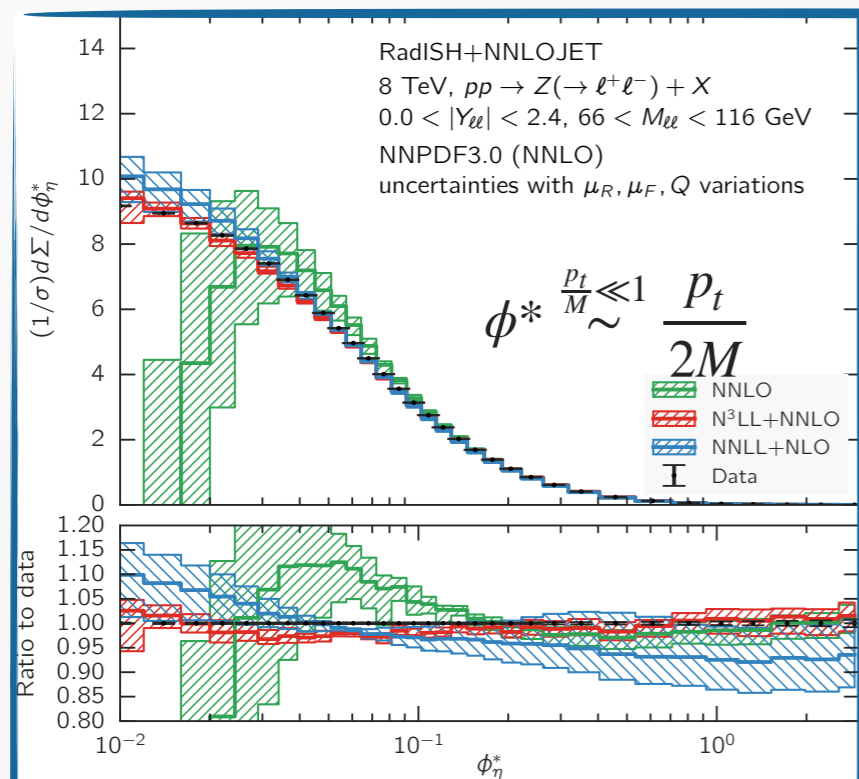
Momentum-space resummation approach



Comparison with ATLAS data @ 8 TeV [1512.02192]

- Good description of the data in all fiducial regions
- Perturbative uncertainty at the few percent level, still does not match the precision of the data

Approach can be used for resumming other transverse obs. e.g



$$\phi^* = \tan\left(\frac{\pi - \Delta\phi}{2}\right) \sin\theta^*$$

deviations from acoplanarity

$$\cos(\theta^*) \equiv \tanh\left(\frac{\eta^{l-} - \eta^{l+}}{2}\right)$$

- Similar situation as  $p_t$ , with perturbative uncertainty at the few percent level but with experimental errors at the sub-percent level
- Estimate of non-perturbative effects and of quark mass effects may start to be relevant

Goutam's talk

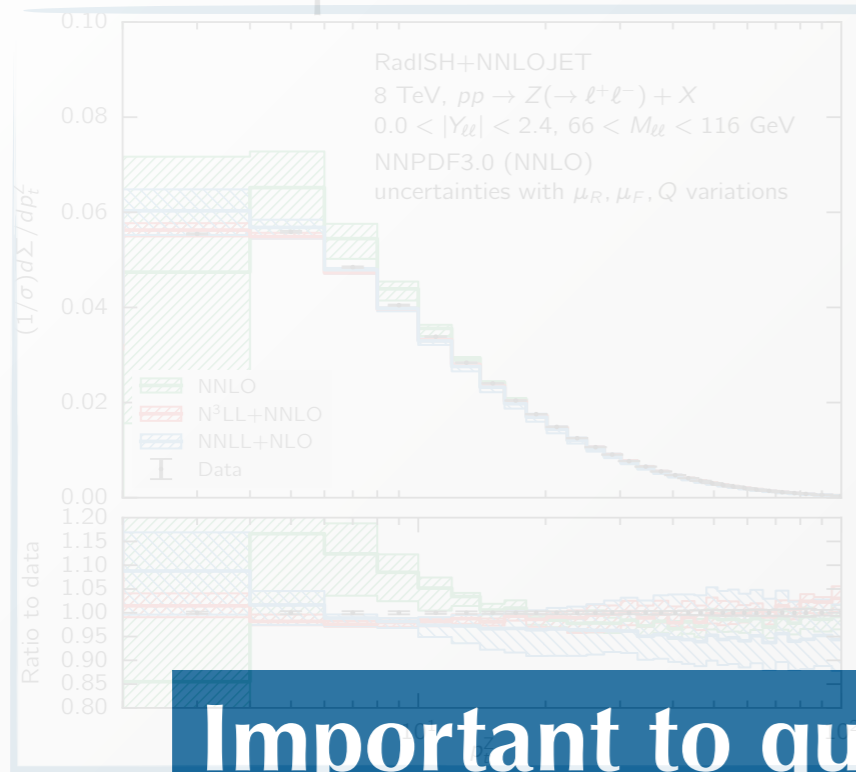


# Drell-Yan transverse observables at N<sup>3</sup>LL+NNLO

Wojtek's talk

[Bizon, Monni, Re, LR, Torrielli + NNLOJET '18]

Momentum-space resummation approach

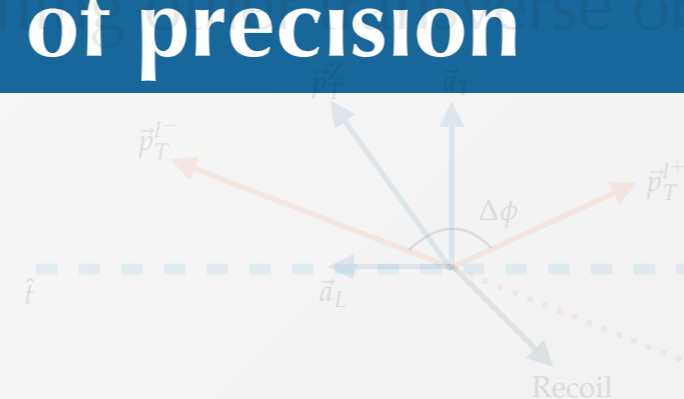
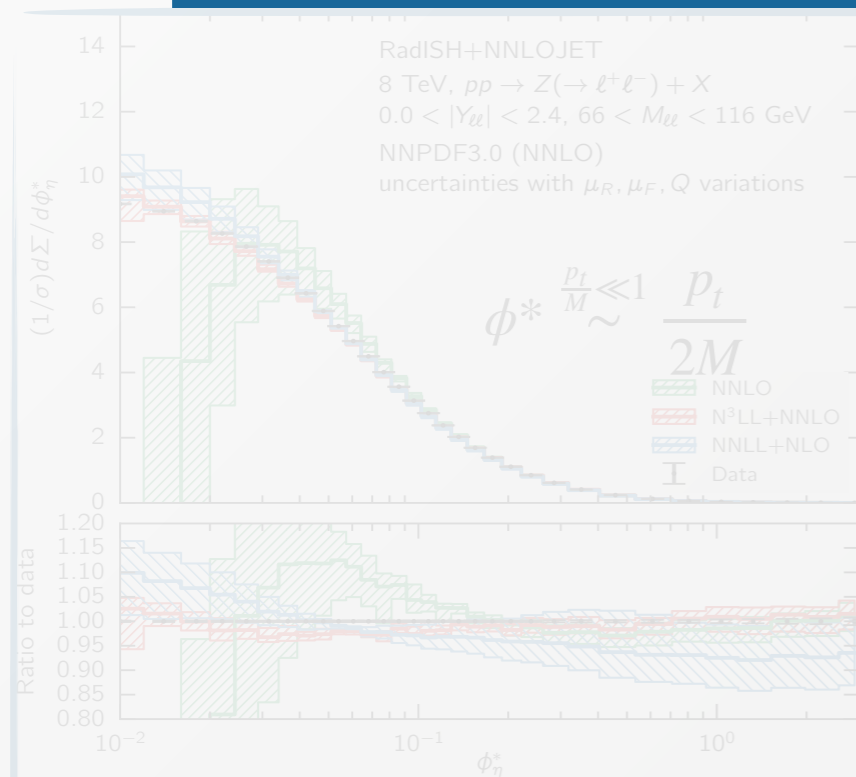


Comparison with ATLAS data @ 8 TeV [1512.02192]

- Good description of the data in all fiducial regions
- Perturbative uncertainty at the few percent level, still does not match the precision of the data

**Important to quantify QED effects at this level of precision**

Approach can be used for resummation of other transverse obs. e.g.



deviations from acoplanarity

$$\phi^* = \tan\left(\frac{\pi - \Delta\phi}{2}\right) \sin\theta^*$$

$$\cos(\theta^*) \equiv \tanh\left(\frac{\eta^{l-} - \eta^{l+}}{2}\right)$$

- Similar situation as  $p_T$ , with perturbative uncertainty at the few percent level but with experimental errors at the sub-percent level
- Estimate of non-perturbative effects and of quark mass effects may start to be relevant

Goutam's talk

# Combined QCD+QED transverse momentum resummation

[Cieri, Ferrera, Sborlini '18]

QED contribution corresponds to an  $\mathcal{O}(\alpha/\alpha_s)$  correction to the QCD result

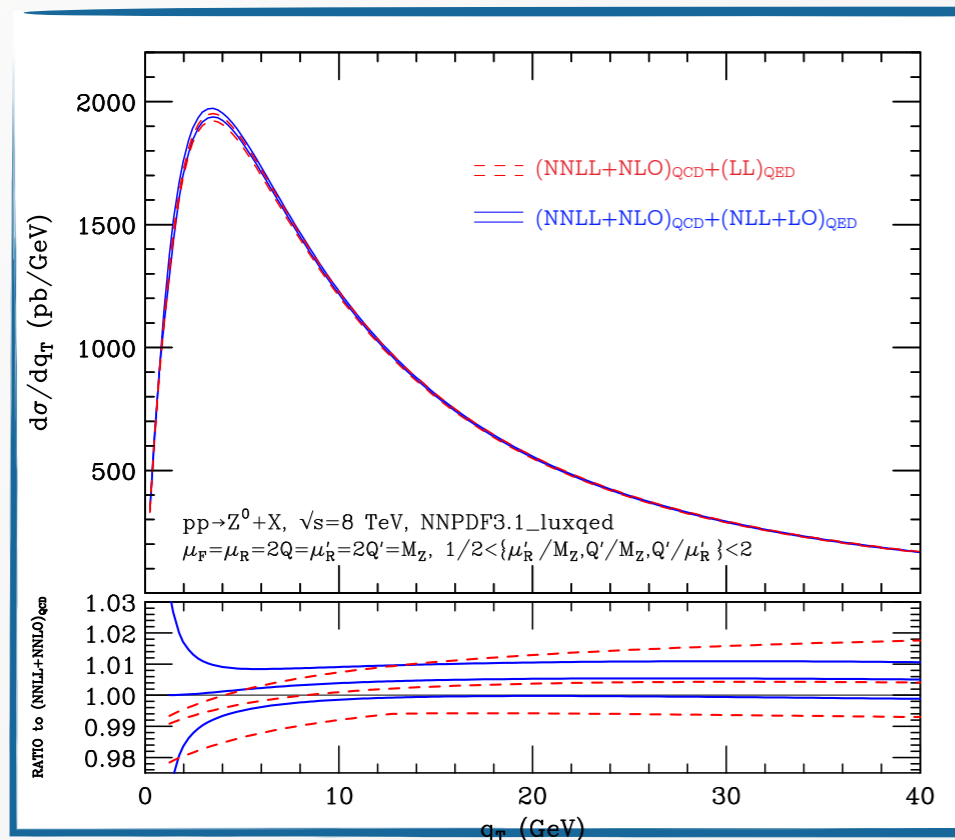
**All order resummation of QED emissions** can have non-negligible impact on pure QCD resummed results

Formulation of combined resummation obtained as an extension of the  $b$ -space formalism [Catani *et al.* '00]

$$\frac{d\hat{\sigma}}{dp_t^2} \simeq \frac{M^2}{\hat{s}} \int_0^\infty db \frac{b}{2} J(bp_t) \exp[\mathcal{G}(\alpha_s, L)] \longrightarrow \frac{d\hat{\sigma}}{dp_t^2} \simeq \frac{M^2}{\hat{s}} \int_0^\infty db \frac{b}{2} J(bp_t) \exp[\mathcal{G}(\alpha_s, \alpha, L)]$$

Contains pure QCD resummation

Contains pure QCD and QED resummation and mixed QCD+QED corrections



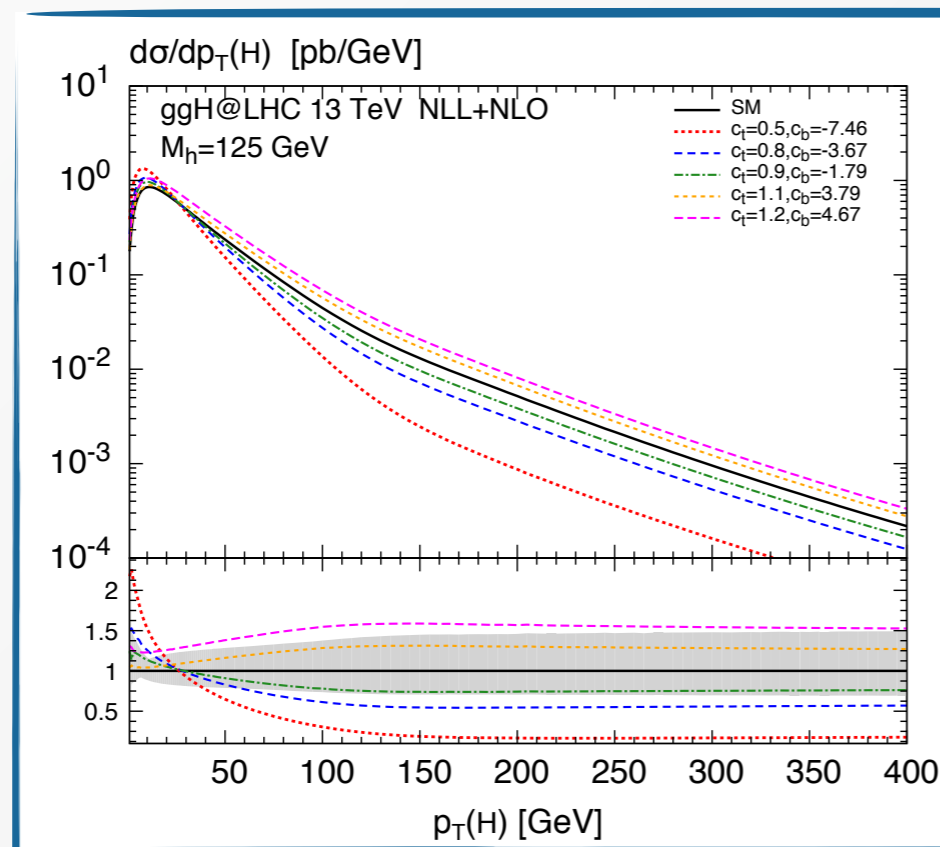
- Effects at  $(\text{LO}+\text{NLL})_{\text{QED}}$  on the  $(\text{NLO}+\text{NNLL})_{\text{QCD}}$  distribution at the 0.5-1% level
- Perturbative uncertainties dominated by resummation scale dependence at  $(\text{LO}+\text{NLL})_{\text{QED}}$

# Higgs transverse momentum at $N^3LL+NNLO$

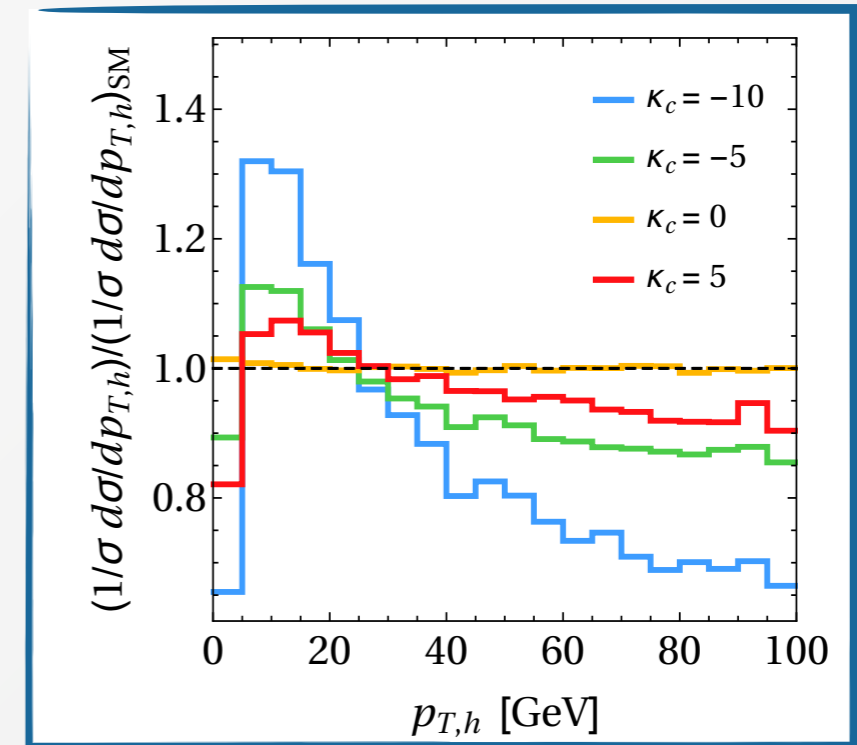
Probably the most studied distribution, known until recently up to NNLL+NLO

Accurate description of the spectrum at small  $p_t$  requires transverse momentum resummation.

- bound on light Yukawa coupling
- sensitivity to high-dimensional operators

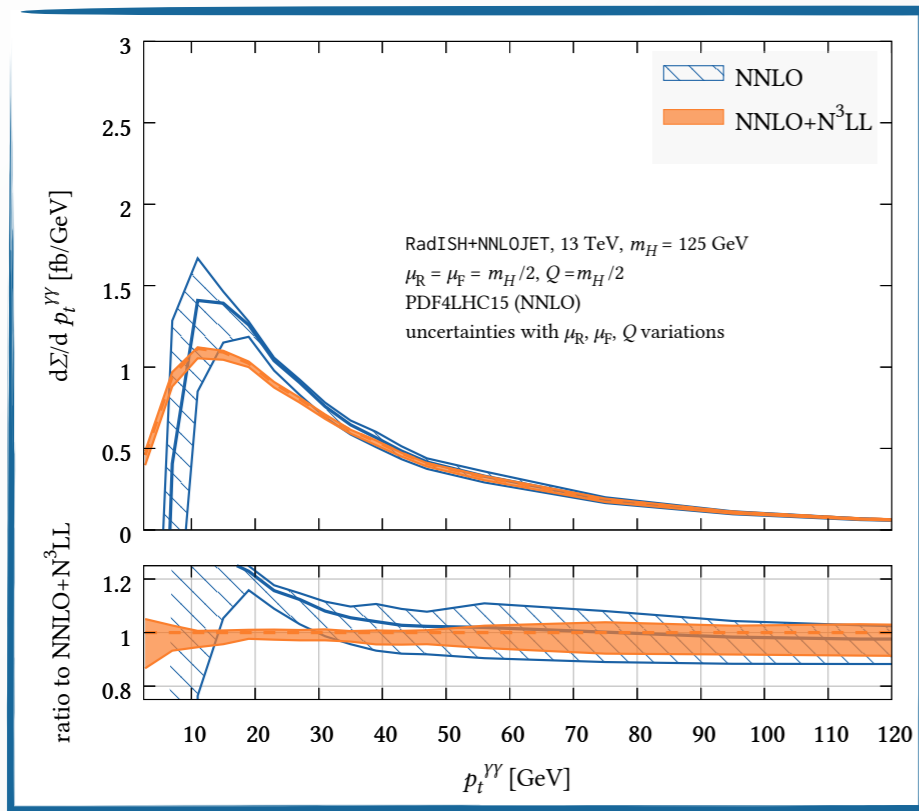


[Grazzini *et al.* '16]



[Bishara *et al.* '16]

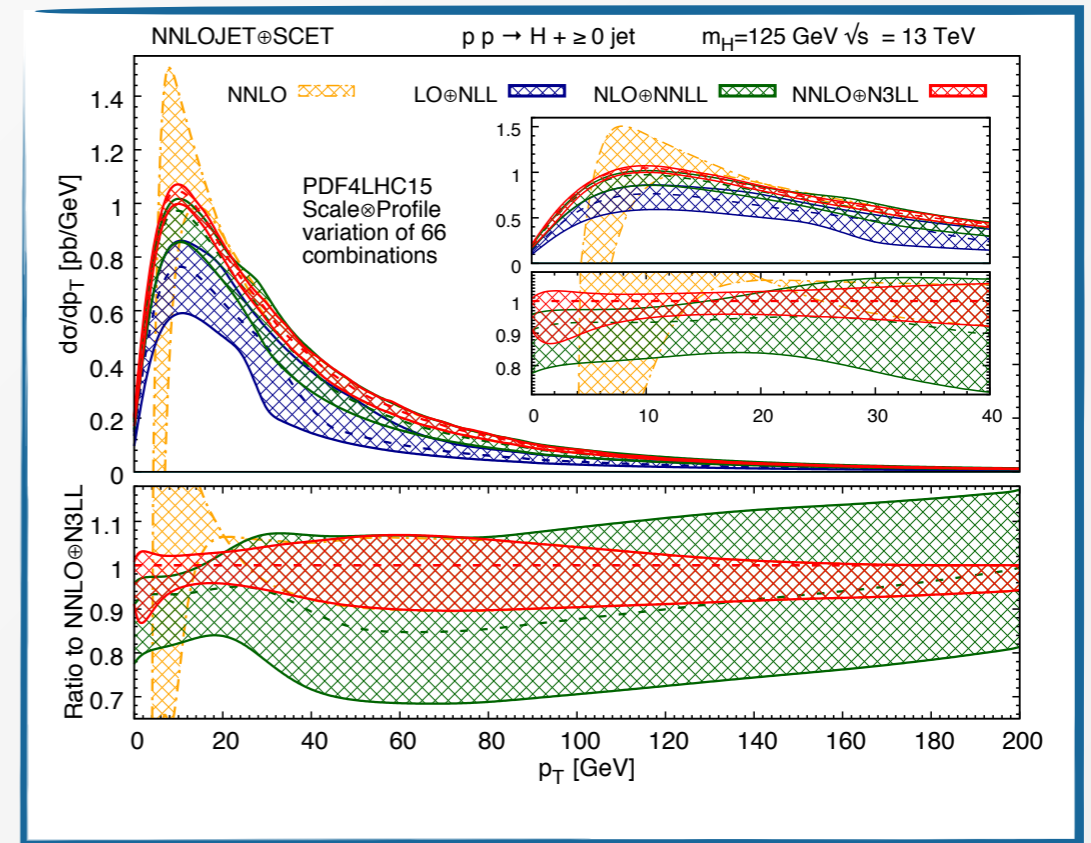
# Higgs transverse momentum at N<sup>3</sup>LL+NNLO



[Bizon, Monni, Re, LR, Torrielli + NNLOJET '17, '18]

- Resummation performed directly in momentum space
- Multiplicative matching allows to recover N<sup>3</sup>LO constant terms from the fixed-order (for inclusive distr.)
- Results for fiducial region within experimental cuts

*Wojtek's talk*



[Chen et al. '18]

- Resummation performed in  $b$ -space within a SCET approach
- Additive matching
- Effects relevant for  $p_t^h \lesssim 40$  GeV

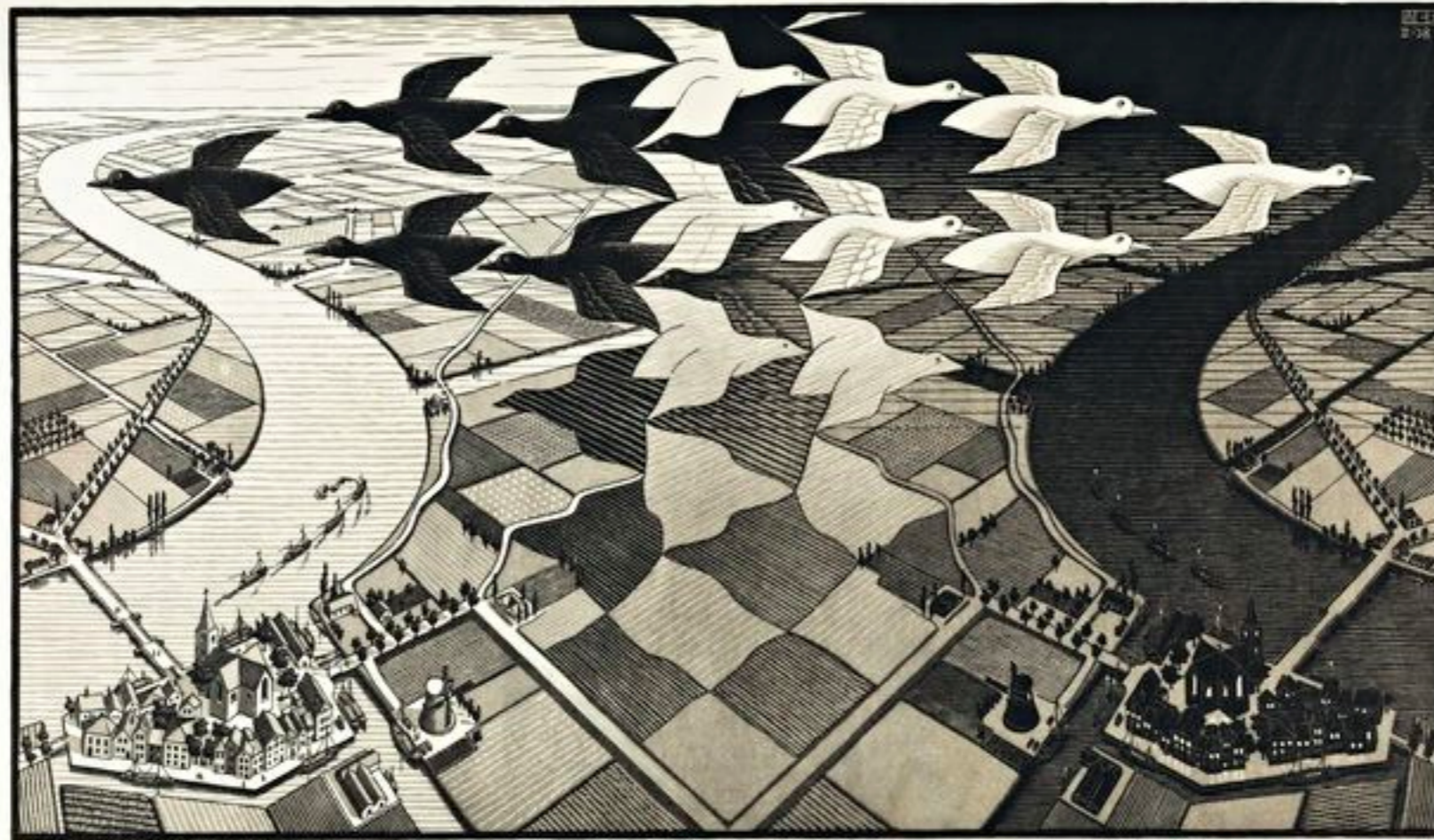
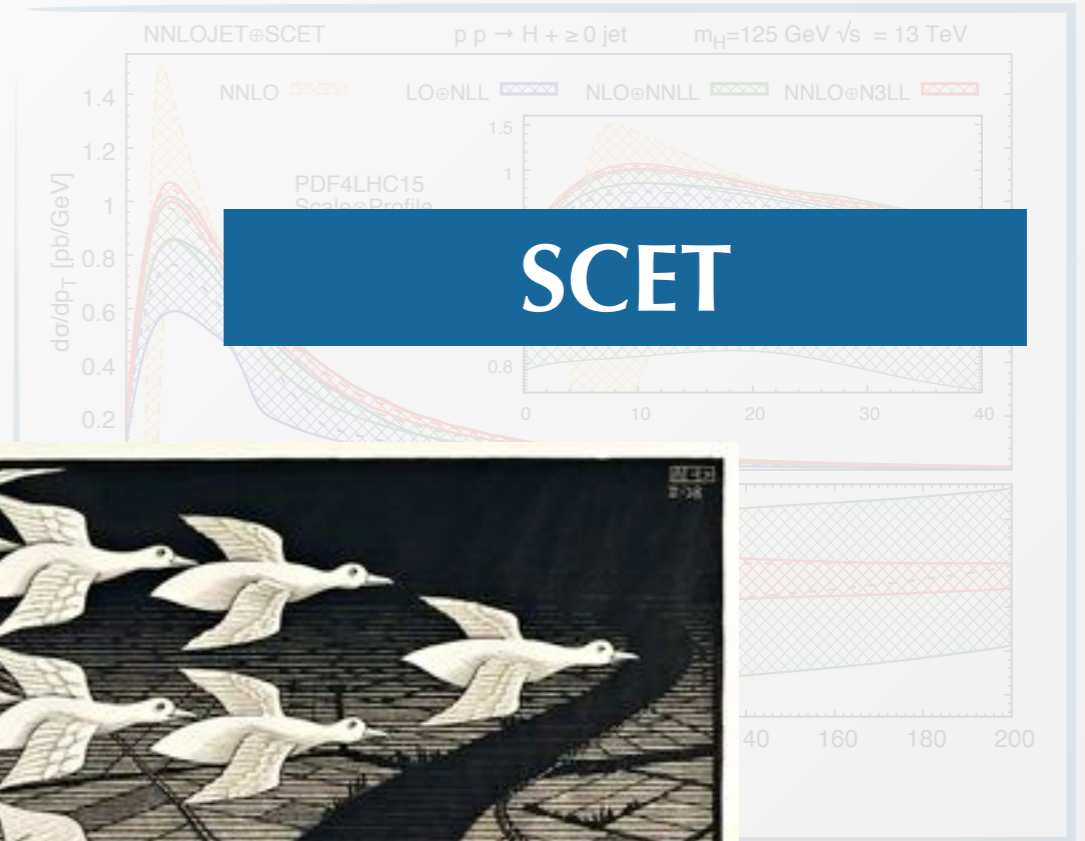
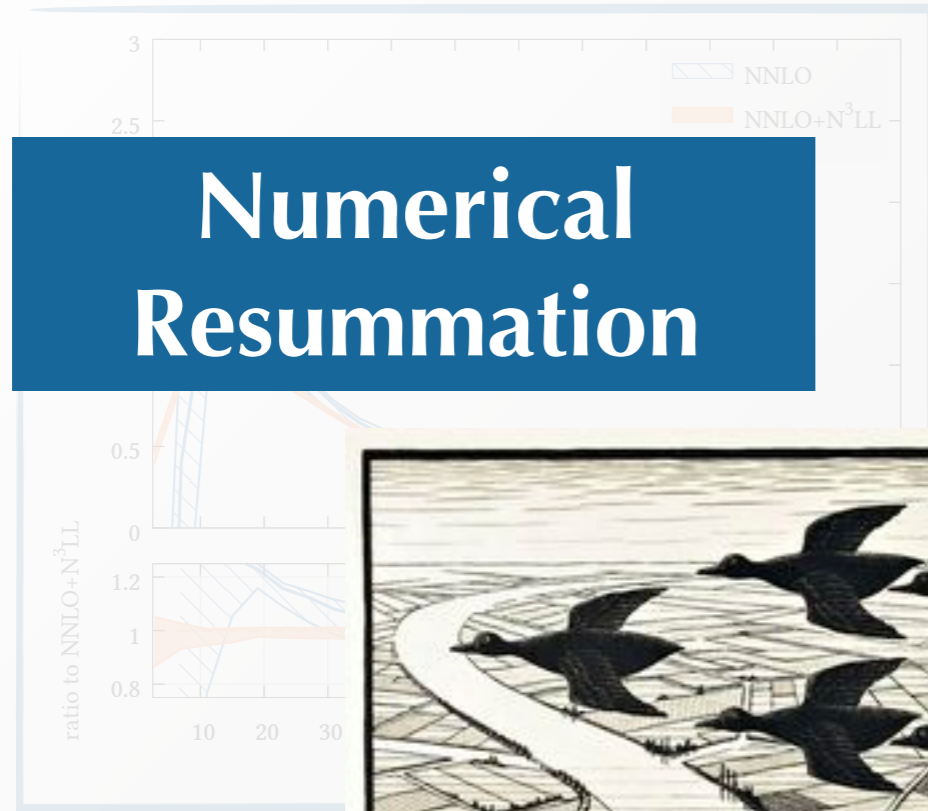
Heavy-quark mass effects start to be relevant at this level of precision

[Lindert et al., '17][Caola et al., '18]

*Stefano's talk*

# Higgs transverse momentum at $N^3LL+NNLO$

## Numerical Resummation



[Bizon, Monni, Re, ...]

[Chen et al '18]

- Resummation in moment
- Multiplicative recover  $N^3$  the fixed-order
- Results for fiducial region within experimental cuts

ed in  $b$ -approach

\$ 40 GeV

heavy-quark mass effects start to be relevant at this level of precision

[Lindert et al, '17, Caola et al, '18]

*Two separate worlds?*

Wojtek's talk

Stefano's talk

# Numerical resummation in SCET

# Resummation in SCET

Factorization of relevant modes is assumed at the level of the Lagrangian

$$\mathcal{L}_{\text{SCET}} = \mathcal{L}_s + \sum_i \mathcal{L}_{n_i}$$

General observables **mix soft and collinear modes** in their definition. If (re)-**factorization theorem** for the observables exist then

$$\Sigma(v, Q) = H(\mu, \mu_H) J(\mu, \mu_J) \otimes J(\mu, \mu_J) \otimes S(\mu, \mu_S)$$

Each contribution depends on a **single scale**: log dependence tied to the dependence on the renormalization scale. Logarithms resummed solving associated RGEs

$$\mu \frac{d}{d\mu} F(\mu, \mu_F) = \gamma_F(\mu, \mu_F) \otimes F(\mu, \mu_F) \quad \longrightarrow \quad F(\mu, \mu_F) = U(\mu, \mu_F) \otimes F(\mu_F, \mu_F)$$

## Pros

1. Systematic way to go to higher orders (more loops in anomalous dimensions)

## Cons:

1. Does not work if factorization theorem does not exist
2. Purely analytical calculation (numerical techniques for parts exist) *Rudi's talk*

# Numerical resummation

Based on rIRC properties of the observable. For rIRC safe observables

$$\Sigma(v) \sim \int d[k_1] e^{-R(q_0 V(k_1))} \times \left( \sum_{m=0}^{\infty} \frac{1}{m!} \int \prod_{i=2}^{m+1} [dk_i] |\mathcal{M}(k_i)|^2 \Theta(V(k_i) - q_0 V(k_1)) \Theta(v - V(k_1, \dots, k_{m+1})) \right)$$

$$\sim \boxed{\Sigma_s(v_s)} \boxed{\mathcal{F}(v, v_s)}. \quad \text{'Transfer function' is calculated numerically}$$

**'Simple' observable: Can be computed analytically and shares the same LL structure of  $\Sigma(v)$**

## Pros

1. Works for any rIRC safe observable
2. Everything done numerically except for analytical resummation for simple observable.

## Cons

1. Gets rapidly non trivial at higher orders



# Pros and Cons for both: numerical resummation being more generic and SCET more systematic

SCET	Numerical resummation (coherent branching formalism)
Only works for observables for which factorization theorem exists	Works for any observable (that is rIRC safe)
Purely analytical calculations (although numerical techniques for parts exist)	Only need analytical resummation for simple observable. Everything else done numerically
Very systematic way to go to higher orders (more loops in anomalous dimensions)	Somewhat of an art to go to higher orders (need to know exactly what was and was not included before)

# Pros and Cons for both: numerical resummation being more generic and SCET more systematic

SCET

Numerical resummation  
(coherent branching formalism)

**Combine the best of the two worlds by overcoming the need of factorization theorem and systematically perform numerical resummation within SCET**

[Bauer, Monni '18]

Purely analytical calculations  
(although numerical techniques for parts exist)

Only need analytical resummation for simple observable. Everything else done numerically

Very systematic way to go to higher orders (more loops in anomalous dimensions)

Somewhat of an art to go to higher orders (need to know exactly what was and was not included before)

# Numerical resummation in SCET

[Bauer, Monni '18]

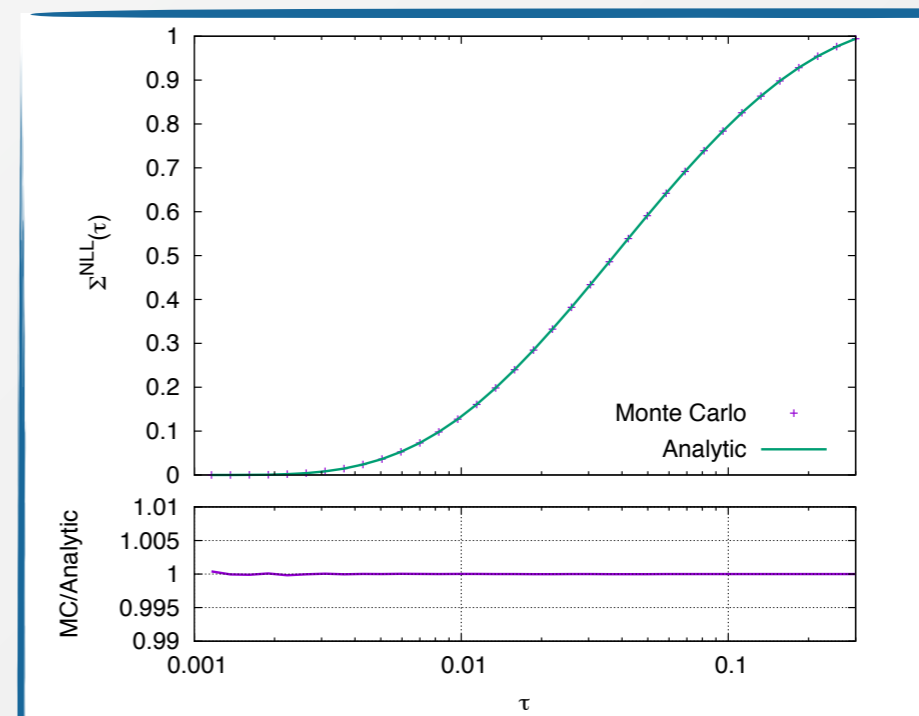
Steps:

1. Write most generic expression for observable to be resummed using separation of modes in the SCET Lagrangian [Bauer *et al.* '08]
2. Identify simple observables for which factorization can be derived in SCET in a very simple manner

$$\Sigma_s = H_s J_s J_s S_s$$

3. Perform resummation of the simple observable solving RGEs
4. Compute numerically relation between simple observable and full observable

Result: numerical vs. analytical computations for thrust



# PDFs with small- $x$ resummation

# Small- $x$ (high energy) resummation

See also Andreas' talk

Relevant at **high energy**  $x = Q^2/s$ ,  $Q^2 \ll s$

Small- $x$  logs in general affect both **DGLAP evolution** and **coefficient functions**

Predictions computed using the  $k_t$ -factorization formalism [Catani, Ciafaloni, Hautmann '90, '91]

Resummation achieved by combining **DGLAP** and **BFKL** evolution

$$\frac{d}{d(1/x)}G(x, M) = \chi(\alpha_s, M)G(x, M),$$

BFKL: evolution for Mellin  $M$  moments of parton densities

$$\frac{d}{dQ^2}G(N, Q^2) = \gamma(\alpha_s, N)G(N, Q^2),$$

DGLAP: evolution for Mellin  $N$  moments of parton densities

Mellin transform maps logs into poles  $\ln^k(Q^2/\mu^2) \rightarrow 1/M^{k+1}$

$$\ln^k(1/x) \rightarrow 1/N^{k+1}$$

A subject with a long history...



# Small- $x$ (high energy) resummation

*See also Andreas' talk*

...but number of phenomenological applications very limited

**Small- $x$  resummation is a HELL of a challenge!**

*The new wave*

ABF approach recently revived and improved to allow for phenomenological applications

- New formalism for **coefficient functions**  
[Bonvini, Marzani, Peraro '16]
- **Matching to NNLO** and further improvement aimed at producing PDFs with resummation (heavy quarks, VFNS, ...)
- **Public code** (aptly named HELL)

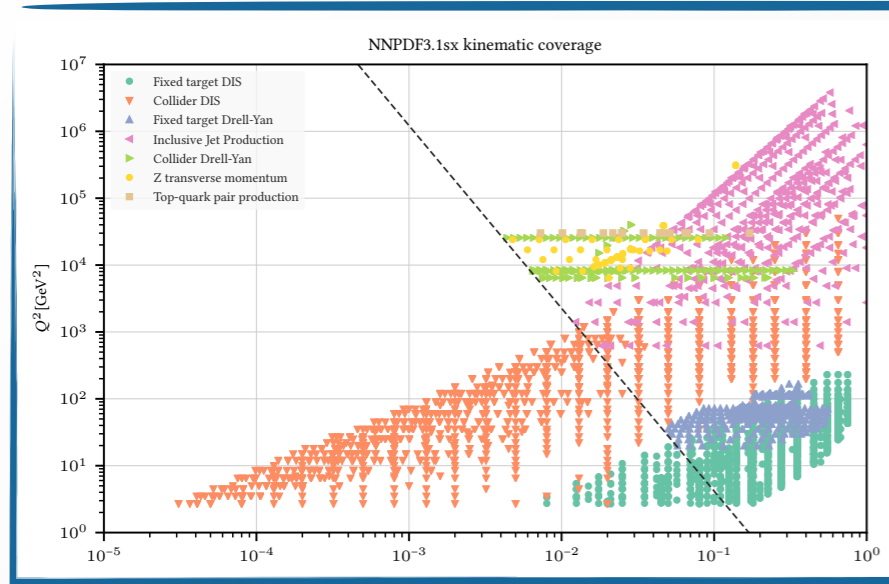
[Bonvini, Marzani, Muselli '17]

+also recent work using EFT approach [Rothstein and Stewart '16]



# Parton distribution functions with small- $x$ resummation

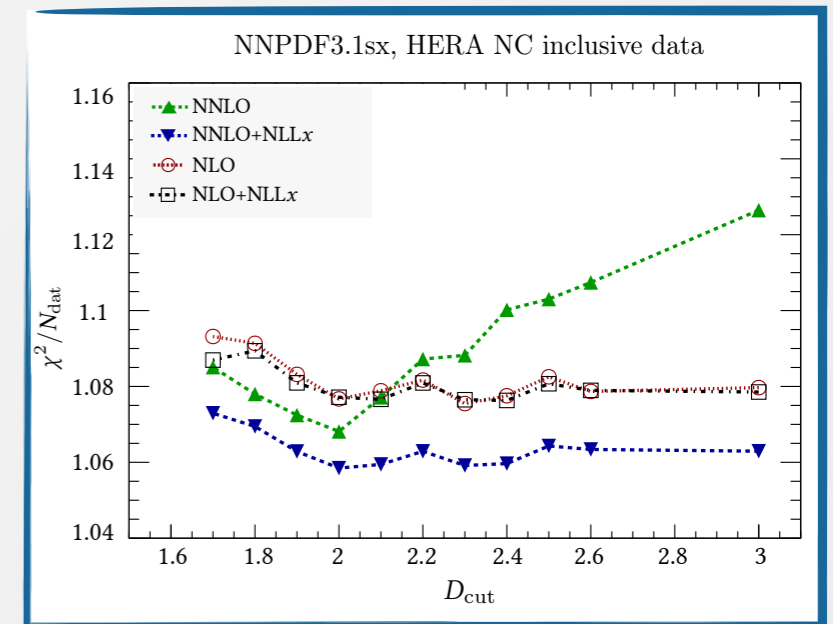
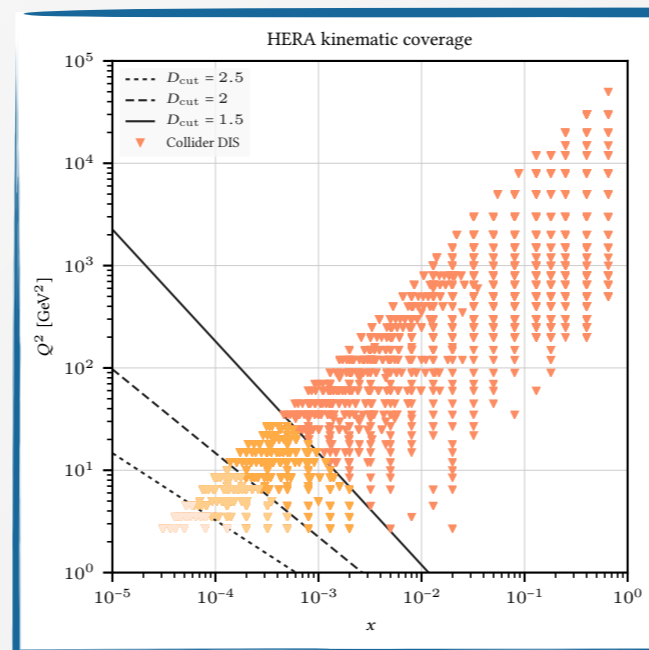
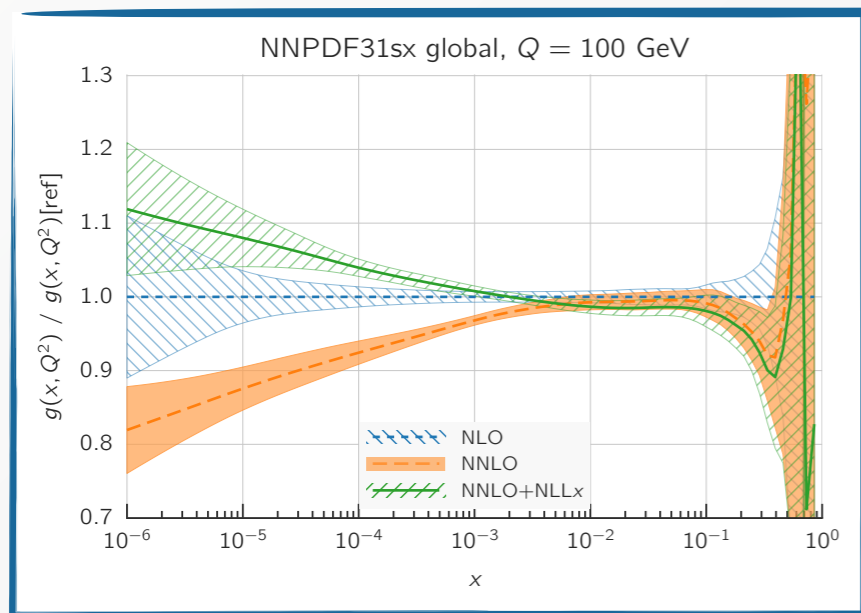
[Ball, Bertone, Bonvini, Marzani, Rojo, LR '17]



All ingredients available to produce a consistent DIS-only fit with small- $x$  resummation

To make the fit competitive with global fits, **cuts applied** to hadronic data

- Improvement on the description of the PDFs at small  $x$  (better perturbative behaviour)
- Improved description of HERA data at small- $x$  (emergence of **BFKL dynamics**)
- Effects at the LHC should be visible, either at low **invariant masses** or at **high rapidities**



Similar analysis (DIS only dataset) performed by xFitter collaboration

*Sasha's talk*

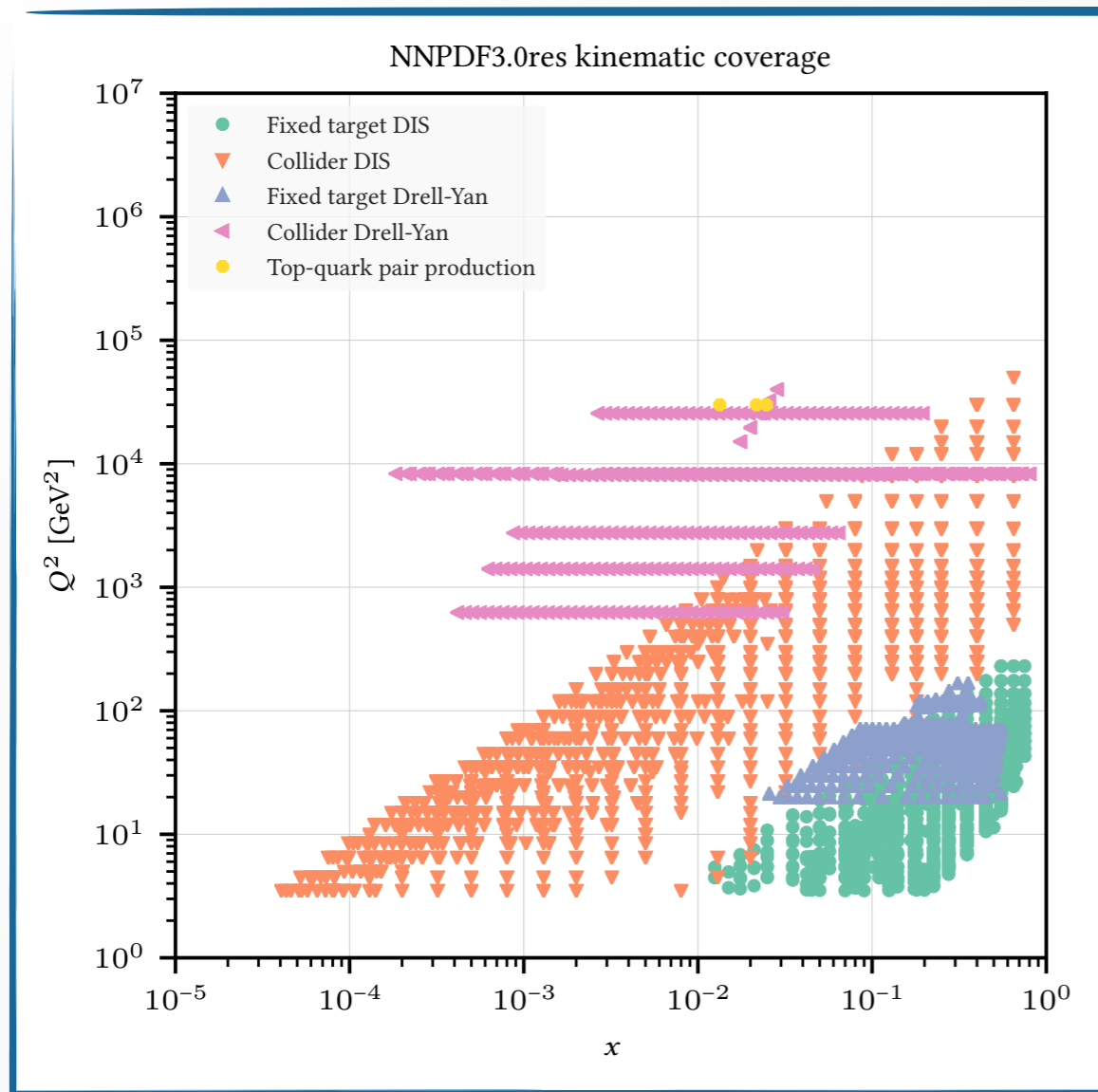
[Abdolmaleki et al'17]

QCD@LHC2018, Dresden, 31 August 2018

# What about large- $x$ ?



# The large- $x$ region



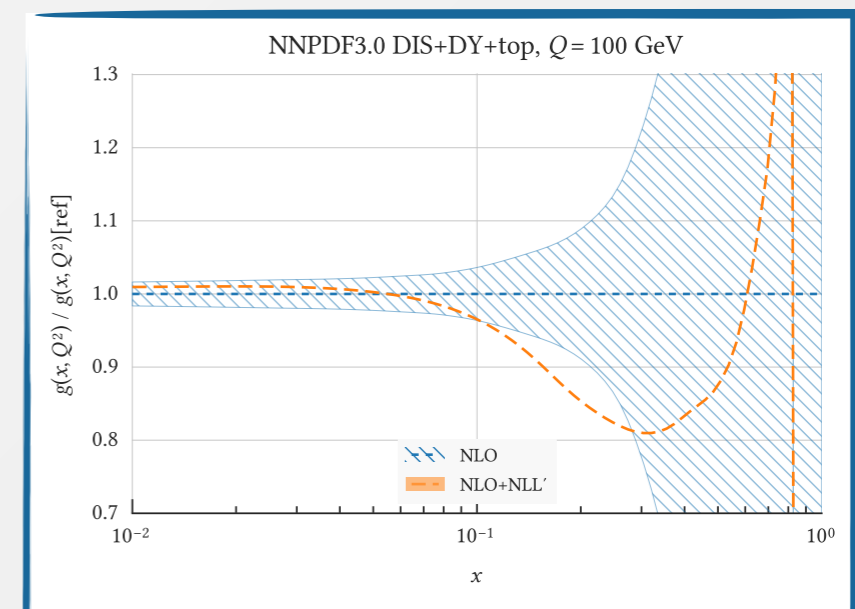
PDF datasets span a large region in the  $(x, Q^2)$  plane and require accurate description both at low and at large  $x \sim Q^2/s$

Description in the large- $x$  region might require **consistent resummation** of large- $x$  (threshold) logs  $L = \ln(1 - x)$  which **enhance** the **coefficient functions** ( $\overline{MS}$  scheme)

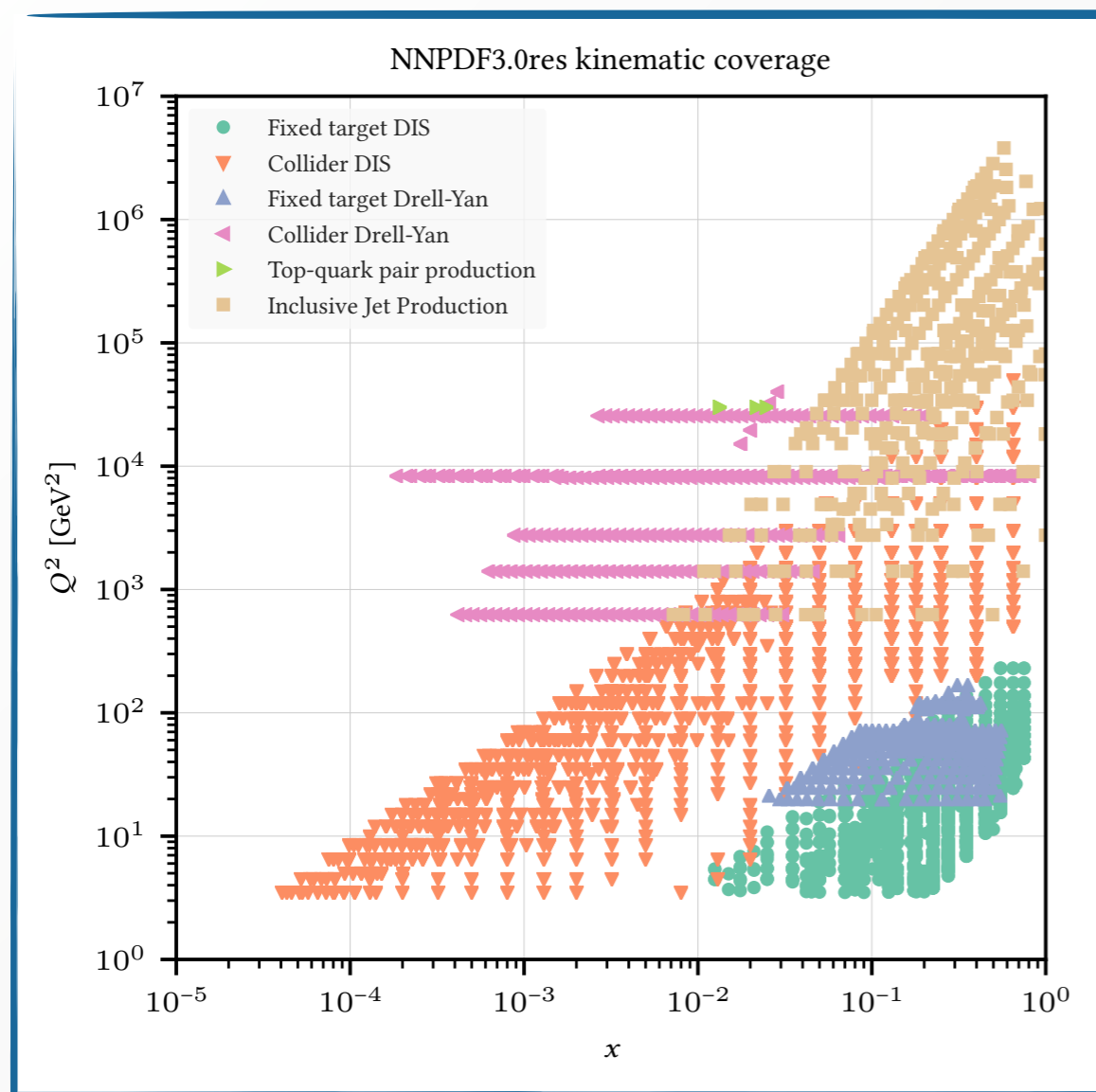
First (almost) global fit performed using a DIS+DY+(inclusive) top dataset [Bonvini et al. '14]

May be relevant for **high-mass resonance searches**

Effect on PDFs small (especially at NNLO+NNLL) and often **below the level of PDF uncertainty**



# The large- $x$ region



PDF datasets span a large region in the  $(x, Q^2)$  plane and require accurate description both at low and at large  $x \sim Q^2/s$

Description in the large- $x$  region might require **consistent resummation** of large- $x$  (threshold) logs  $L = \ln(1 - x)$  which **enhance** the **coefficient functions** ( $\overline{MS}$  scheme)

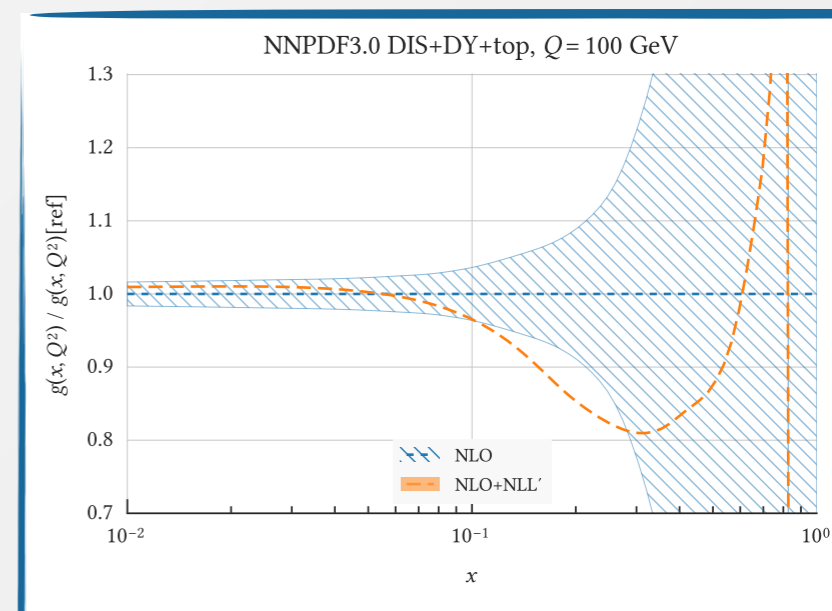
First (almost) global fit performed using a DIS+DY+(inclusive) top dataset [Bonvini et al. '14]

May be relevant for **high-mass resonance searches**

Effect on PDFs small (especially at NNLO+NNLL) and often **below the level of PDF uncertainty**

Main limitation: lack of constraint due to limited dataset (no resummed calculation available)

**No jet data, gluon poorly constrained**



# Threshold resummation for single-inclusive jet production

[Liu, Moch, Ringer, Eren, Lipka '17, '18]

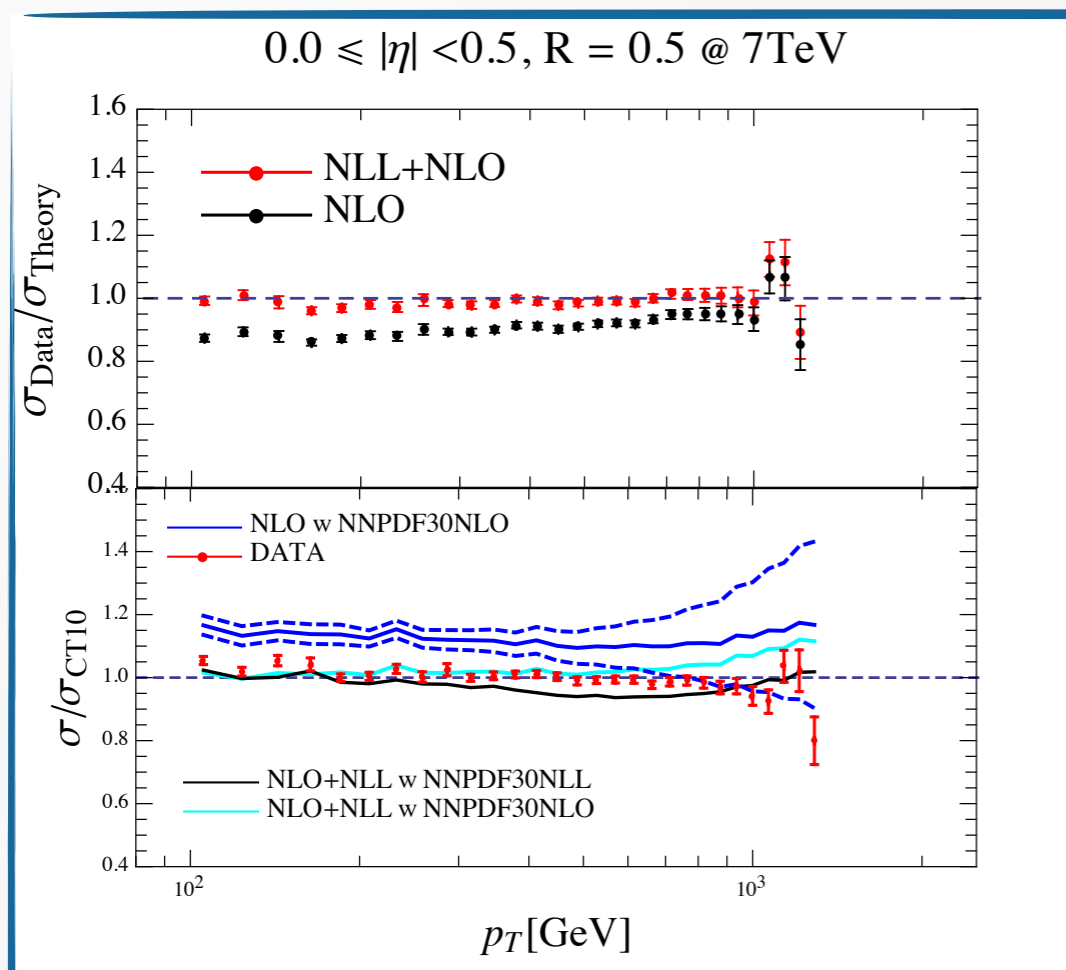
Calculations were available at NLL, but **no numerical implementation**

[Florian, Vogelsang '07]

**Complete jet kinematics** must be taken into account for meaningful predictions

Resummation is performed in a **SCET approach** at NLL accuracy

- **combined resummation** of threshold and small- $R$  (single) logarithms, **differential** in jet  $p_t$  and rapidity
- resummation in direct space avoid intricacies related to use of Mellin transform
- extension of the formalism to NNLL possible



- Improved agreement with data with respect to NLO (resummed predictions tend to somewhat overshoot the data at larger value of  $R$ )
- Reduction of scale uncertainty
- Overall better agreement wrt NLO if NLO+NLL predictions are computed using resummed PDF
- Interesting to use these predictions for future extractions of PDFs with threshold resummation

# Combining resummations

# Combining resummations

Previous result is an example of a **combined** (or **joint**) resummation: simultaneous resummation of threshold logarithms and small- $R$  logarithms

n.b. sometimes joint resummation is used to refer to simultaneous resummation of **two different** observables, e.g. two angularities [Larkoski, Mout, Neill '14] [Procura et al '18]

Results for combined resummation existed, e.g. combined transverse momentum and threshold resummation [Li '98][Laenen et al. '00][Kulesza et al. '02, '03][Fuks et al. '07, '11][Laenen, Banfi '05]

# Combining resummations

Previous result is an example of a **combined** (or **joint**) resummation: simultaneous resummation of threshold logarithms and small- $R$  logarithms

n.b. sometimes joint resummation is used to refer to simultaneous resummation of **two different** observables, e.g. two angularities [Larkoski, Moult, Neill '14] [Procura et al '18]

Results for combined resummation existed, e.g. combined transverse momentum and threshold resummation [Li '98][Laenen et al. '00][Kulesza et al. '02, '03][Fuks et al. '07, '11][Laenen, Banfi '05]

Recently, revived interest in combined resummation, various new results (list not exhaustive) see also [Procura, Waalewijn, Zeune '15][Li, Neill, Zhu '16]

## transverse momentum resummation

[Lustermans et al. '16]

[Muselli et al '17][Marzani, Theuves '17]

high-energy resummation

## threshold resummation

# Combining resummations

Previous result is an example of a **combined** (or **joint**) resummation: simultaneous resummation of threshold logarithms and small- $R$  logarithms

n.b. sometimes joint resummation is used to refer to simultaneous resummation of **two different** observables, e.g. two angularities [Larkoski, Moult, Neill '14] [Procura et al '18]

Results for combined resummation existed, e.g. combined transverse momentum and threshold resummation [Li '98][Laenen et al. '00][Kulesza et al. '02, '03][Fuks et al. '07, '11][Laenen, Banfi '05]

Recently, revived interest in combined resummation, various new results (list not exhaustive) see also [Procura, Waalewijn, Zeune '15][Li, Neill, Zhu '16]

**transverse momentum resummation**

[Marzani '15][Muselli, Forte '15]

**high-energy resummation**

threshold resummation

# Combining resummations

Previous result is an example of a **combined** (or **joint**) resummation: simultaneous resummation of threshold logarithms and small- $R$  logarithms

n.b. sometimes joint resummation is used to refer to simultaneous resummation of **two different** observables, e.g. two angularities [Larkoski, Mout, Neill '14] [Procura et al '18]

Results for combined resummation existed, e.g. combined transverse momentum and threshold resummation [Li '98][Laenen et al. '00][Kulesza et al. '02, '03][Fuks et al. '07, '11][Laenen, Banfi '05]

Recently, revived interest in combined resummation, various new results (list not exhaustive) see also [Procura, Waalewijn, Zeune '15][Li, Neill, Zhu '16]

transverse momentum resummation

high-energy resummation

[Bonvini, Marzani '18]

threshold resummation



# Double resummation for Higgs production

Simone's talk

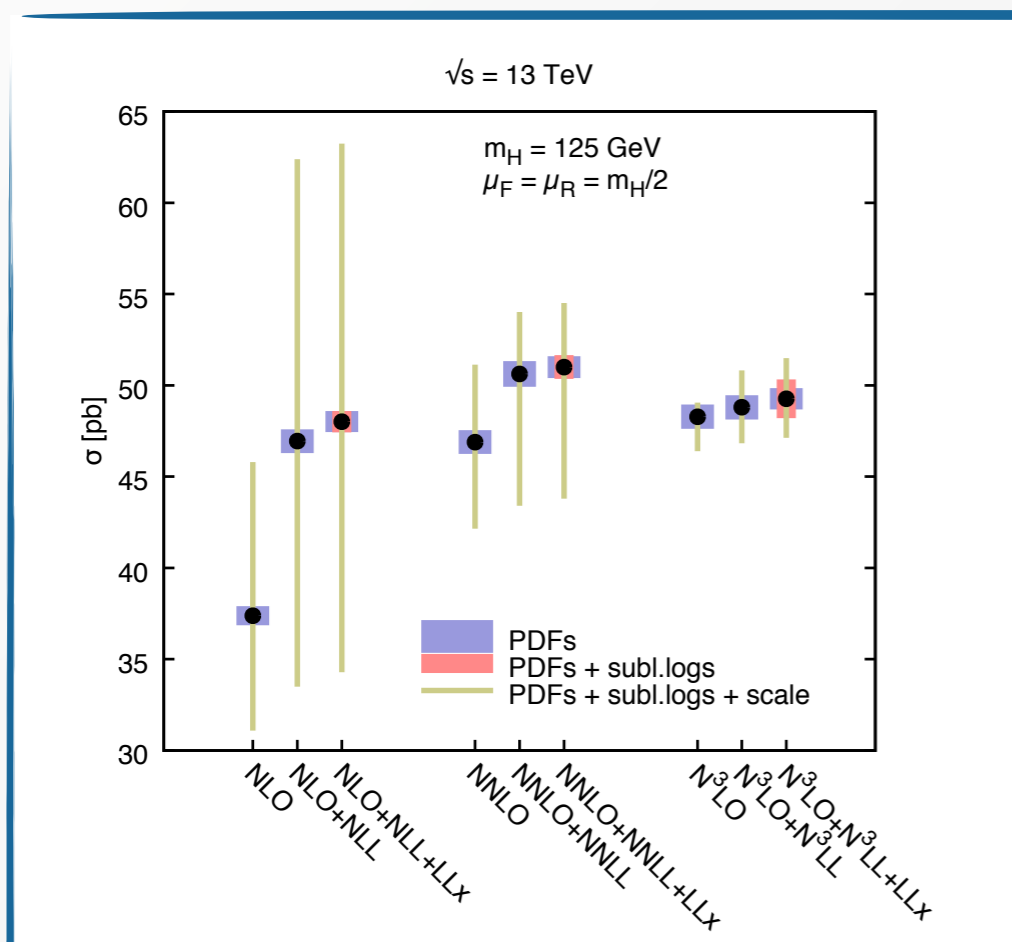
[Bonvini, Marzani '18]

To match the two resummation, look at the **analytical structure** of the coefficient functions in Mellin space [Ball *et al.* '14]

Double resummed coefficient functions should respect singularity structure order-by-order

$$C(N, \alpha_s) = C^{\text{fo}}(N, \alpha_s) + \Delta C^{\text{large-}x}(N, \alpha_s) + \Delta C^{\text{small-}x}(N, \alpha_s)$$

In principle, should use **double-resummed PDFs**. Effect of threshold resummation on gluon PDFs is small  $\rightarrow$  use PDFs with small- $x$  resummation



Results for Higgs production: small ( $\sim 2\%$ ) correction to N<sup>3</sup>LO at current LHC energies

Due to small- $x$  effects impact grows at larger collider energies ( $\sim 5\%$  at 27 TeV,  $\sim 10\%$  at 100 TeV)

Method presented rather general: can be applied to a variety of processes currently studied at the LHC (DY, heavy quarks, **differential in rapidity**)

# Summary & outlook

I presented a personal overview of some recent results in resummation

- Large logarithmic corrections are a **feature** of QCD
- Resummed calculations needed for **accurate predictions**
- Compelling connections between different approaches open up the possibility of achieving **systematically** resummation for a wider class of observables
- Progress towards **fully consistent**, exclusive, **combined** resummation
- Advances in analytical and automated resummation can be used to assess and improve the formal accuracy of parton showers [\[Hoche, Reichelt, Siegert '17\]](#)  
*Daniel's and Frédéric's talks* [\[Dasgupta, Dreyer, Hamilton, Monni, Salam '18\]](#)

Many topics I could not cover

- Resummation in jet substructures *Felix's talk*
- Resummation in  $e^+e^-$  and heavy-ions collisions
- Non-global logs, factorization-breaking effects *Dingyu's and Matthew's talk*
- Next-to-leading power *Leonardo's talk*
- $t\bar{t}$ ,  $t\bar{t}H$  & BSM processes *Stefano's, Jan's and Anna's talks*
- EW Sudakovs *Jonas' talk*
- ...

*Resummation just scratches the surface of QCD. But it makes a mark.*

George Sterman, CTEQ school 2006