Benchmarking resummed predictions for the p_t spectrum of Z and W bosons at the LHC

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M_W and p_t^W at hadron colliders

Tevatron and LHC W mass results use precise Z measurements to obtain a prediction for W p_t via

$$\frac{1}{\sigma^W}\frac{d\sigma^W}{p_\perp^W} \sim \frac{1}{\sigma^Z_{\text{data}}}\frac{d\sigma^Z_{\text{data}}}{p_\perp^Z}\frac{\frac{1}{\sigma^W_{\text{theory}}}\frac{d\sigma^W_{\text{theory}}}{p_\perp^W}}{\frac{1}{\sigma^Z_{\text{theory}}}\frac{d\sigma^Z_{\text{theory}}}{p_\perp^Z}}$$

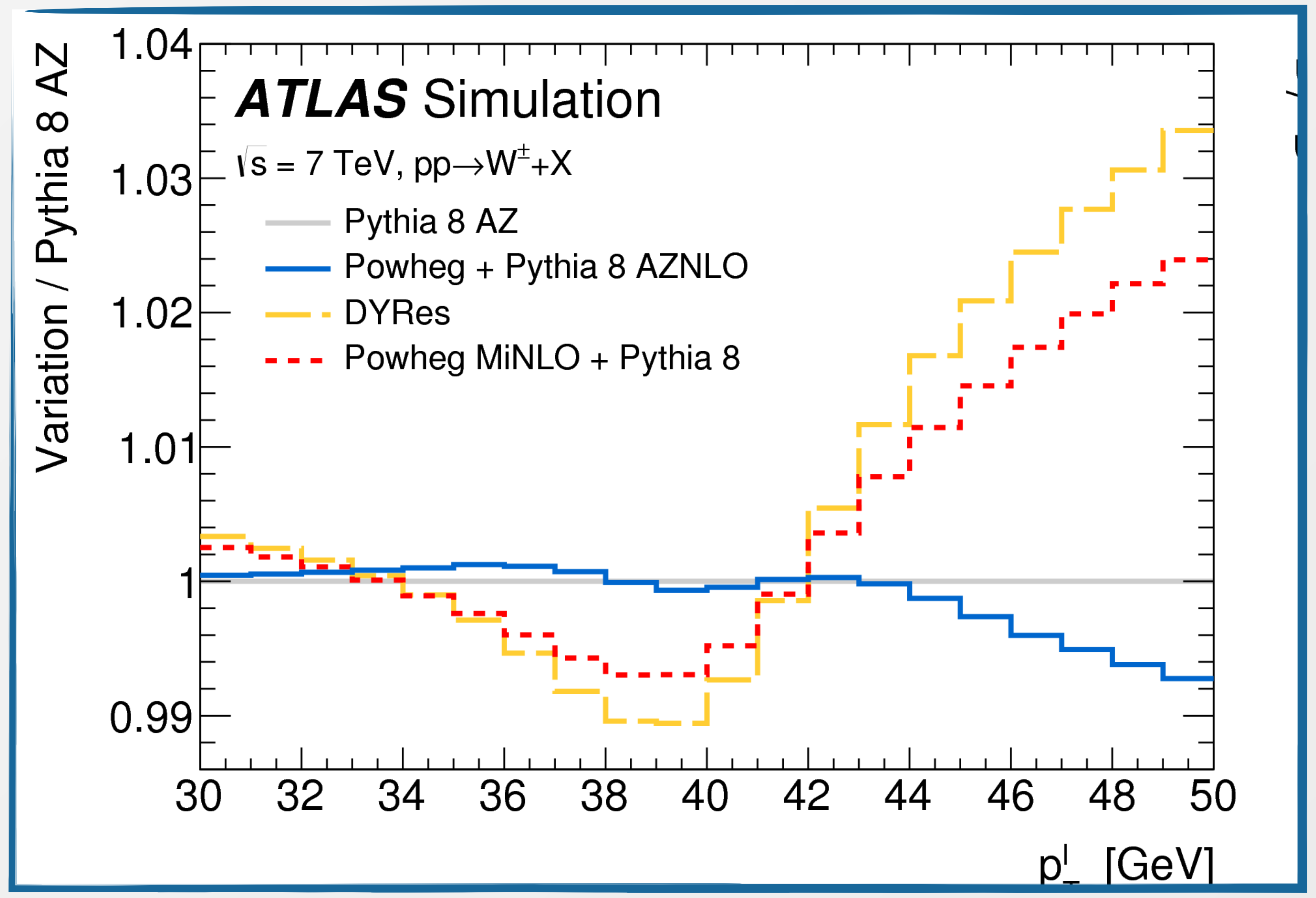
ATLAS: ~2% uncertainty in W p_t translates to ~10 MeV uncertainty on M_W

Recent ATLAS measurement used Pythia8 to 'fit' the Z p_t distribution and extrapolate to W p_t

Resulting tune (AZ) reproduces the Z pT at spectrum 1-2% level

Highly desirable to use calculations with **state-of-the-art accuracy** (**NNLO+N^{3}LL**) to describe the Z and W spectra

W p_t modeling and uncertainties is of great interest to experimentalists working on the W mass measurement

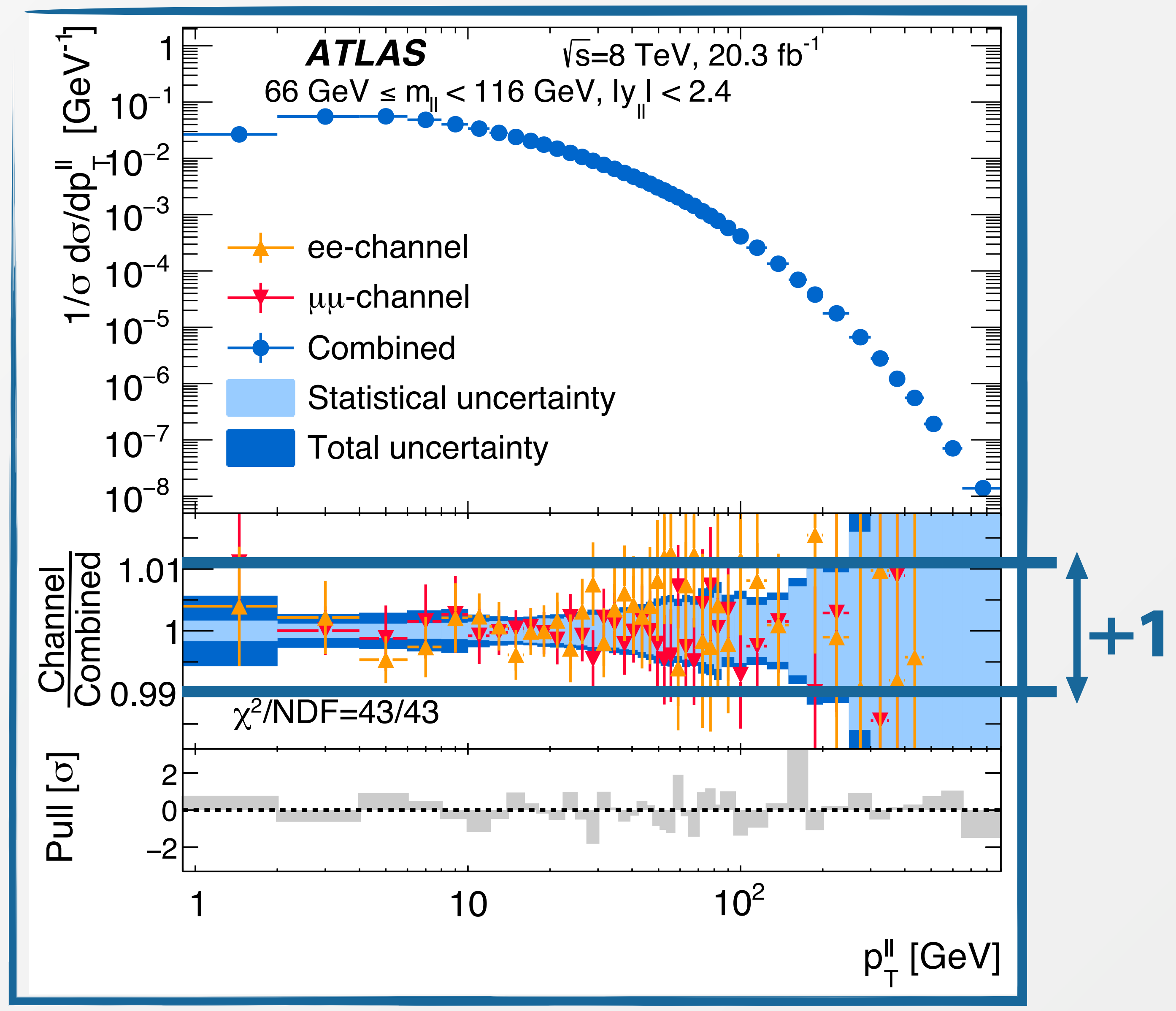


W/Z p_t spectra in QCD

Great experimental precision of the Z p_t spectrum (sub-% level) challenges current theory predictions

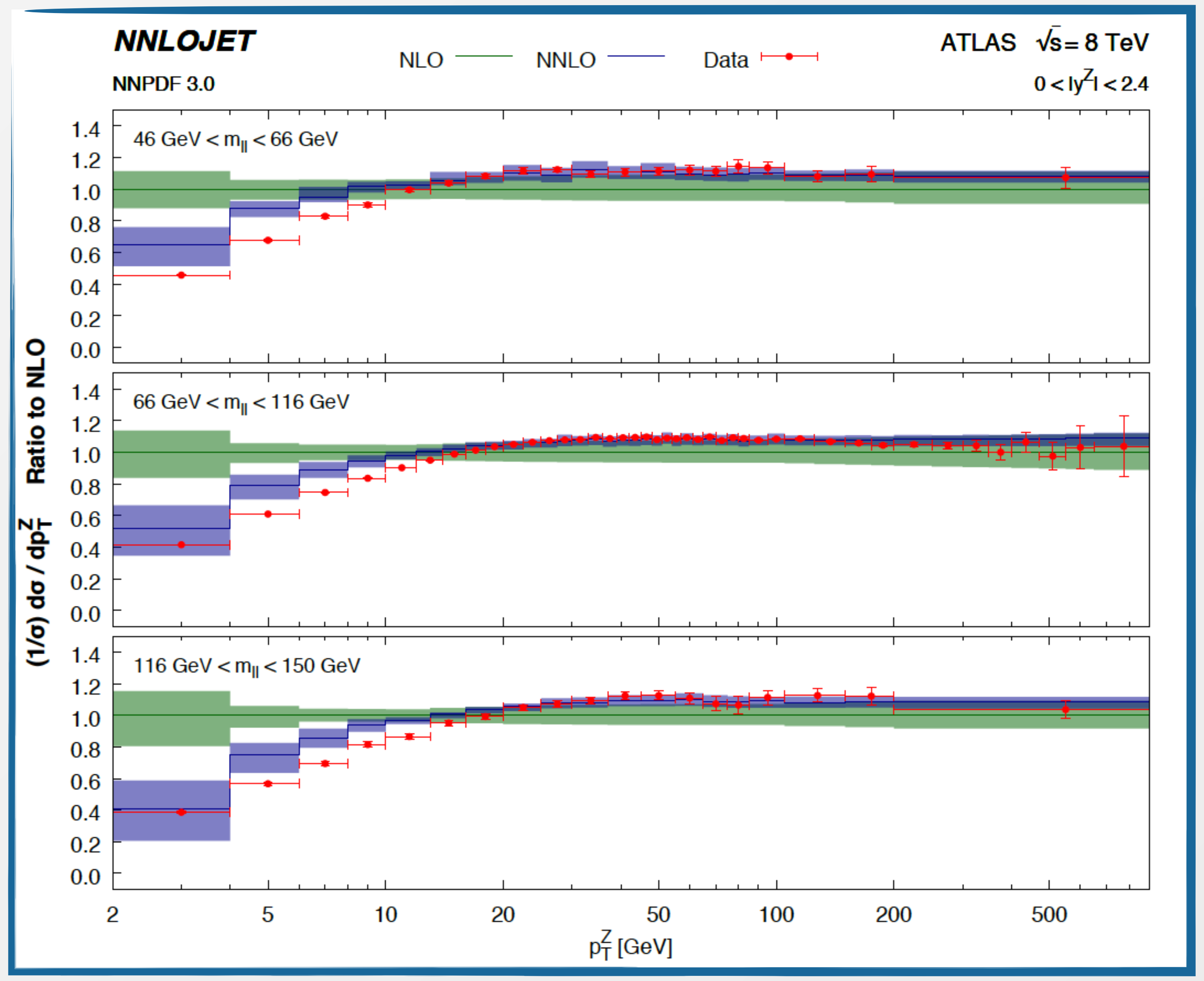
NNLO total cross section known for many years

[Hamberg, van Neerven, Matsuura '91]
[van Neerven, Zijlstra '92]
[Anastasiou, Dixon, Melnikov, Petriello '03]
[Melnikov, Petriello '06]
[Catani, Cieri, Ferrera, de Florian, Grazzini '09]
[Catani, Ferrera, Grazzini '10]
[Gavin, Li, Petriello, Quackenbush '10]

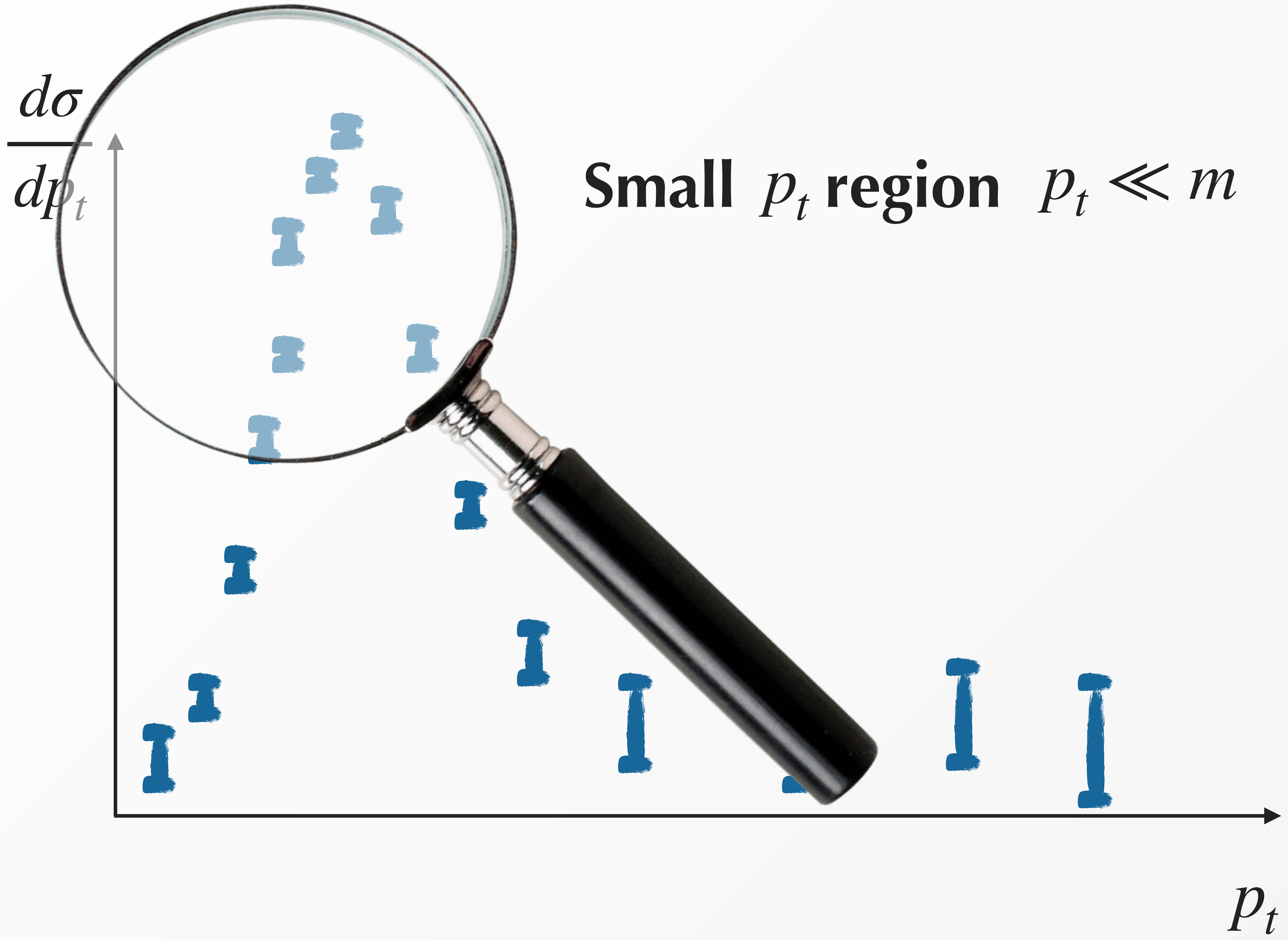


State of the art for fixed order p_t spectrum is NNLO: Z/W recoiling against at least one hard radiation

[Gehrmann - De Ridder, Gehrmann, Glover, Huss, Morgan '15-'16]
[Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello '15]
[Boughezal, Focke, Liu, Petriello '15]
[Gehrmann - De Ridder, Gehrmann, Glover, Huss, Walker '17]



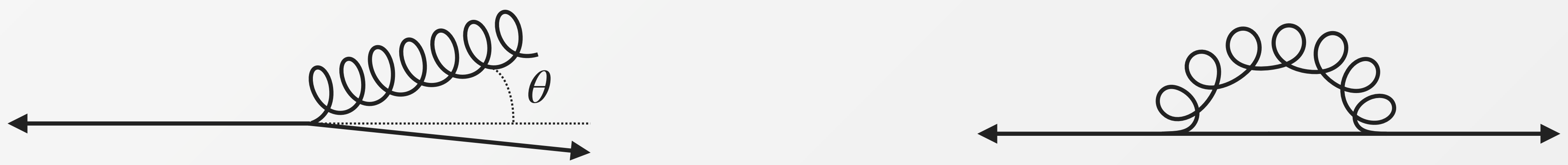
W/*Z* spectra at small transverse momentum: resummation



Small p_t region $p_t \ll m$

Double logarithms **leftovers** of the real-virtual cancellation of IRC divergences

$$\tilde{\sigma}_1(p_t) \sim \underbrace{\int \frac{d\theta}{\theta}\frac{dE}{E}\Theta\left(p_t - E\theta\right)} - \underbrace{\int \frac{d\theta}{\theta}\frac{dE}{E}}$$



$$\sim -\int \frac{dE}{E}\frac{d\theta}{\theta}\Theta(E\theta - p_t) \sim -\frac{1}{2}\ln^2\frac{p_t}{m}$$

Sudakov logarithms

Origin of the logs is simple. Resum them to all orders by **reorganizing** the series and make pQCD great again

$$\ln\tilde{\sigma}(p_t) = \sum_n \left(\underset{\textbf{LL}}{\mathcal{O}(\alpha_s^n L^{n+1})} + \underset{\textbf{NLL}}{\mathcal{O}(\alpha_s^n L^{n})} + \underset{\textbf{NNLL}}{\mathcal{O}(\alpha_s^n L^{n-1})} + \ldots\right) \qquad L = \ln(p_t/m)$$

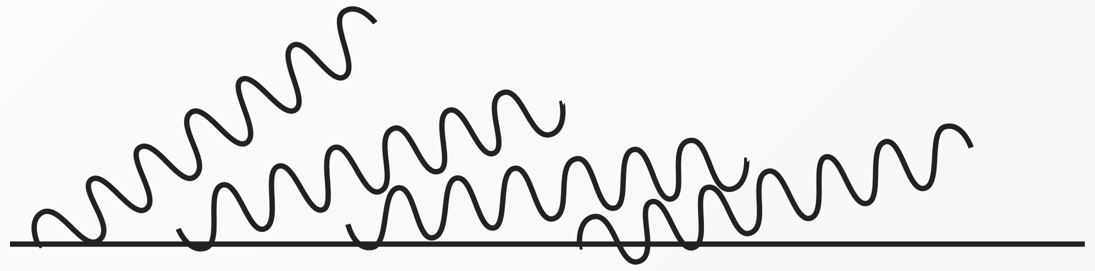
All ingredients to perform resummation at **N³LL accuracy** are now available

[Catani *et al.* ’11, ’12][Gehrmann *et al.* ’14][Li, Zhu ’16, Vladimirov ’16][Moch *et al.* ’18, Lee *et al.* ‘19]

Transverse momentum resummation

Resummation of transverse momentum is particularly delicate because p_t is a **vectorial quantity**

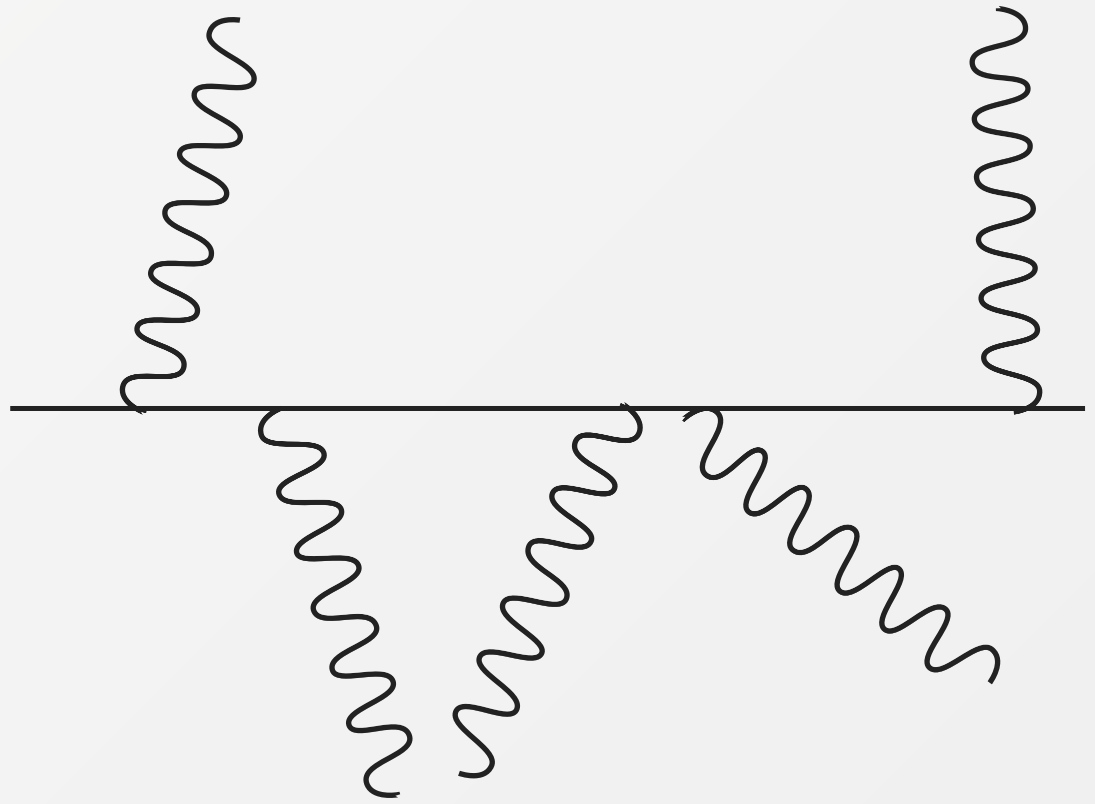
Two concurring mechanisms leading to a system with small p_t



$$p_t^2 \sim k_{t,i}^2 \ll M^2$$

cross section naturally suppressed as there is no phase space left for gluon emission (Sudakov limit)

Exponential suppression



$$\sum_{i=1}^{n} \vec{k}_{t,i} \simeq 0$$

Large kinematic cancellations

p_t ~0 far from the Sudakov limit

Power suppression

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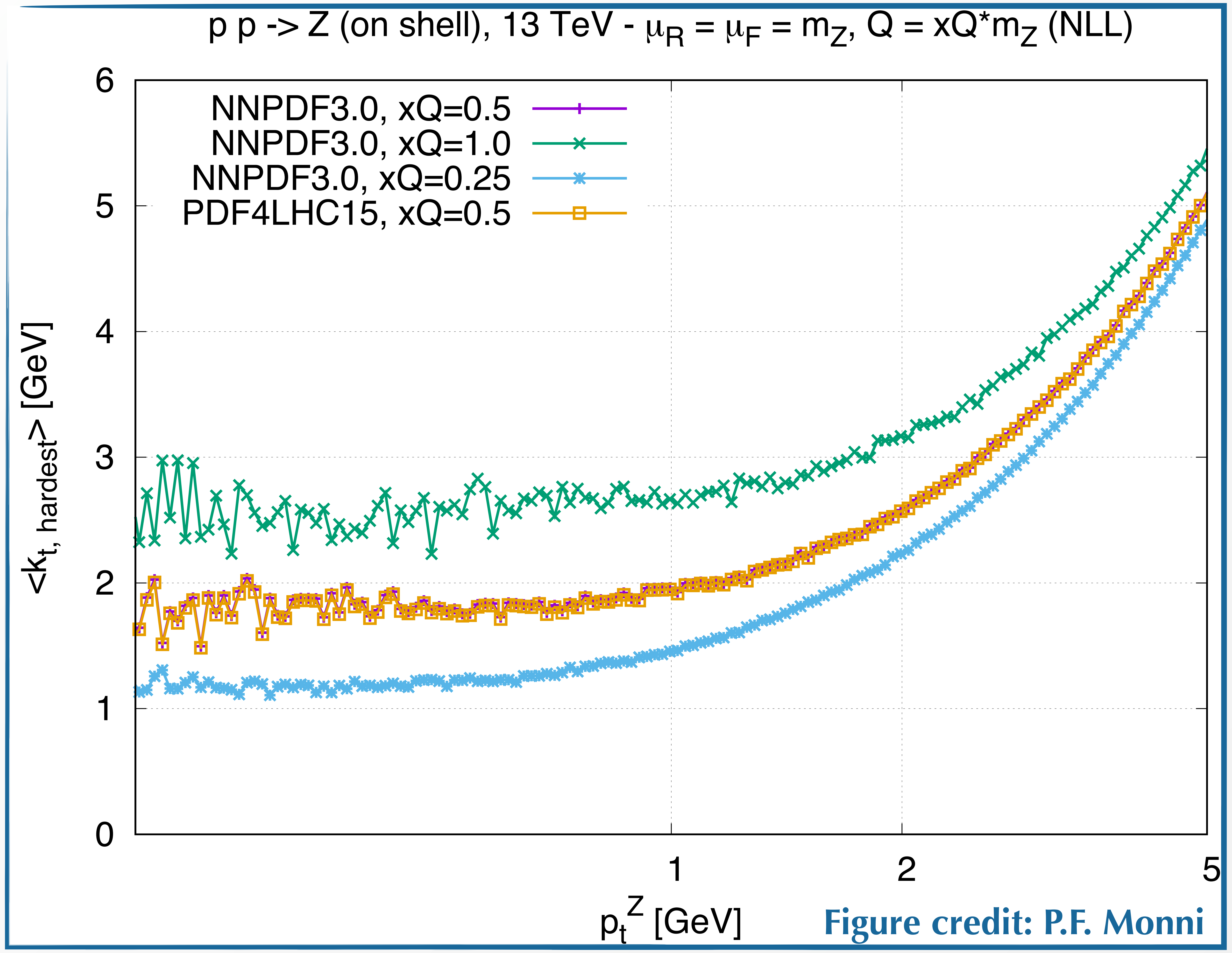
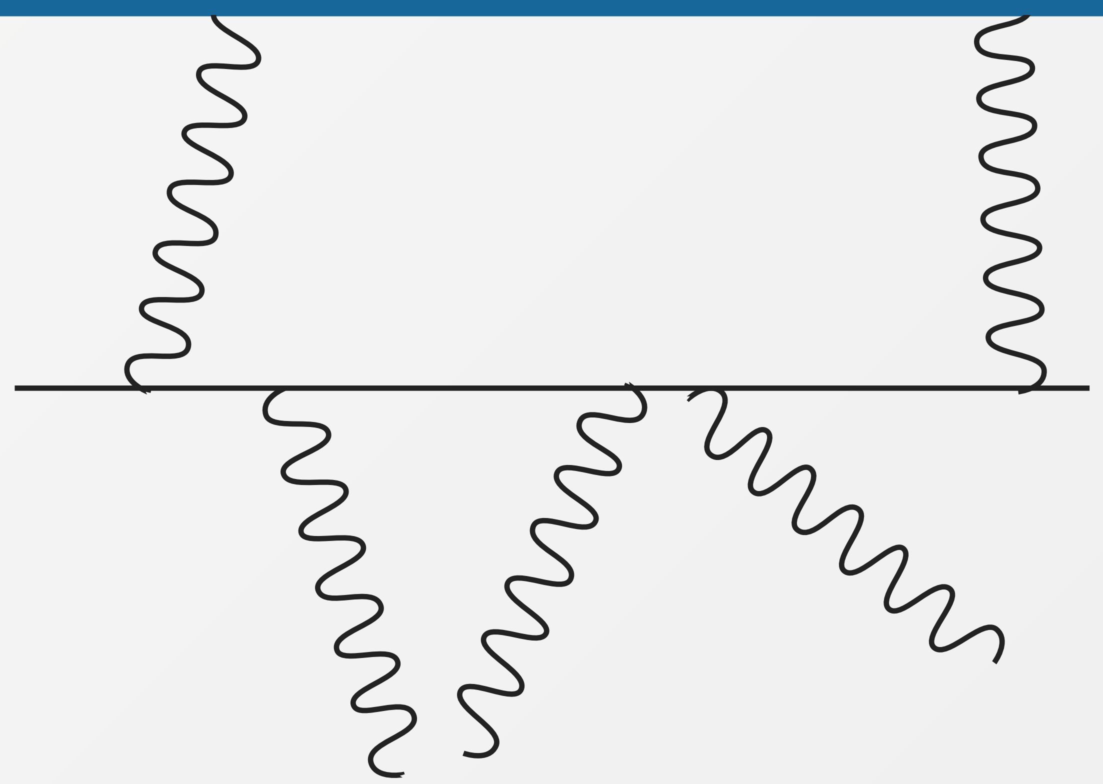


Figure credit: P.F. Monni

Dominant at small p_t



$$\sum_{i=1}^{n} \vec{k}_{t,i} \simeq 0$$

Large kinematic cancellations

p_t ~0 far from the Sudakov limit

Power suppression

Transverse momentum resummation: impact parameter space

The two competing effects are usually handled in **impact parameter** (*b*) space, where the phase-space constraints factorise

$$\frac{d\sigma}{d^2\vec{p}_t} \sim \sigma_0 \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_t} e^{-R_{\mathrm{NLL}}}$$

[Parisi, Petronzio '79; Collins, Soper, Sterman '85]

Exponentiation in conjugate space; **inverse transform** to move back to direct space

Logarithmic accuracy defined in terms of $L = \ln(b_0/b)$ $b_0 = 2e^{-\gamma_E}$

Extremely successful approach; resummation for DY production performed within a variety of formalisms to NNLL accuracy ('**direct QCD**', **SCET**, **TMD**) [Bozzi et al '10; Becher, Neubert '10; Banfi et al '12; Echevarria et at '11]

Transverse momentum resummation: direct space

Resummation in **direct space**: non-trivial problem. A naive logarithmic counting at small p_t is not sensible, as one loses the **correct power-suppressed scaling** if only logarithms are retained

New method that solves the problem in transverse-momentum space recently proposed: **RadISH**

at NLL

$$\sigma(p_t) \sim \sigma_0 \int \frac{dk_{t,1}}{k_{t,1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R_{\mathrm{NLL}}(k_{t,1})} \epsilon^{R'_{\mathrm{NLL}}(k_{t,1})} R'\left(k_{t,1}\right)$$
$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^{1} \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'_{\mathrm{NLL}}\left(\zeta_i k_{t,1}\right) \Theta\left(p_t - \left|\sum_{j=1}^{n+1} \vec{k}_{t,j}\right|\right)$$

[Monni, Re, Torrielli '16, Bizon, Monni, Re, LR, Torielli '17]

see also [Ebert, Tackmann '16] for an alternative approach within SCET formalism

Logarithmic accuracy defined in terms of $L = \ln(k_{t,1}/m)$

Access to multi-differential information. This is effectively similar to a **semi-inclusive parton shower**, but with higher-order logarithms, and control on **formal N^{3}LL accuracy**

Other parton-shower based formulations have been recently used in the context of TMD at NLL accuracy to compute predictions for the transverse momentum, rapidity and φ^* spectra of Z bosons

[Martinez et al '19]

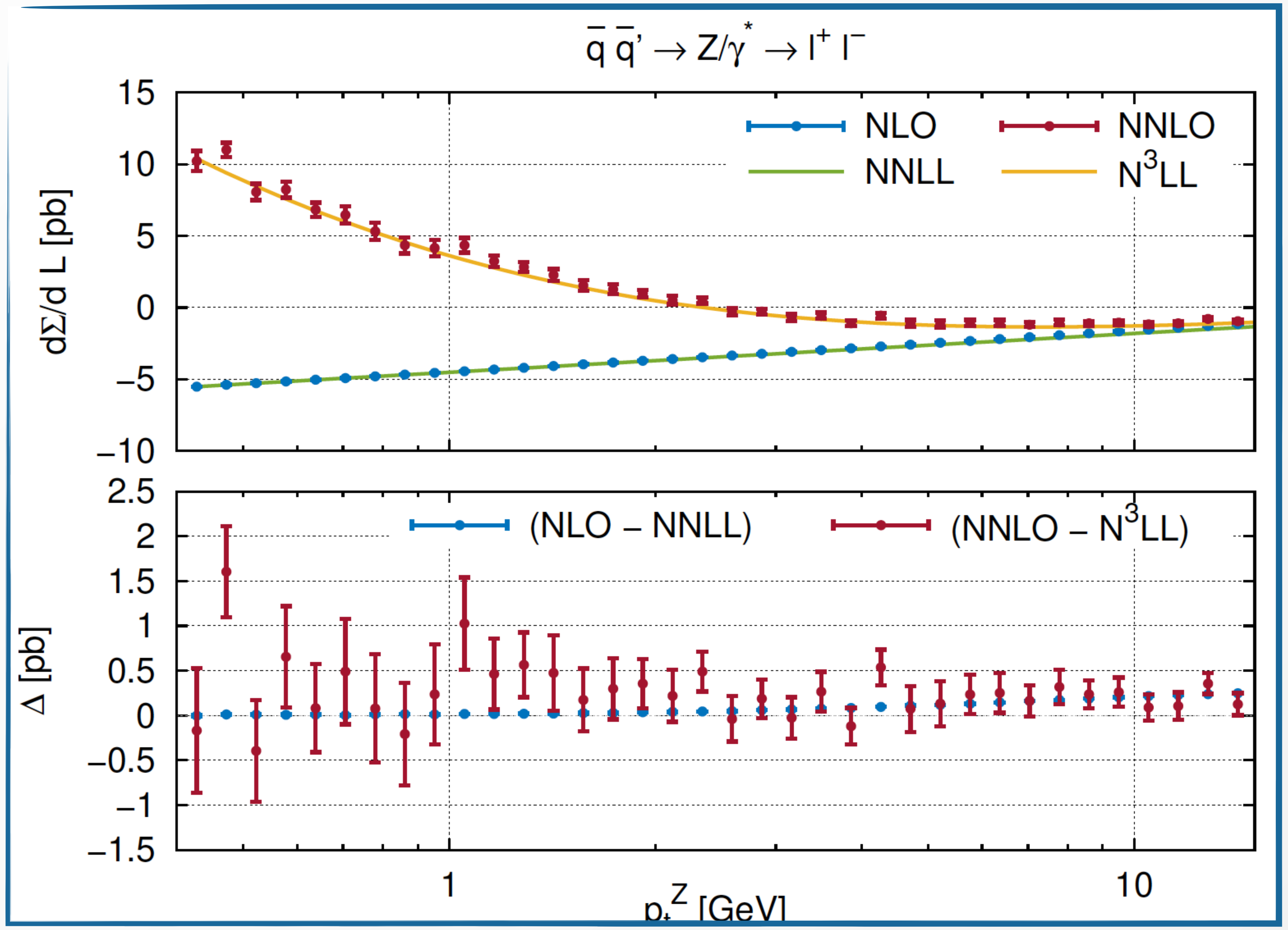
Matching fixed order and resummed calculations

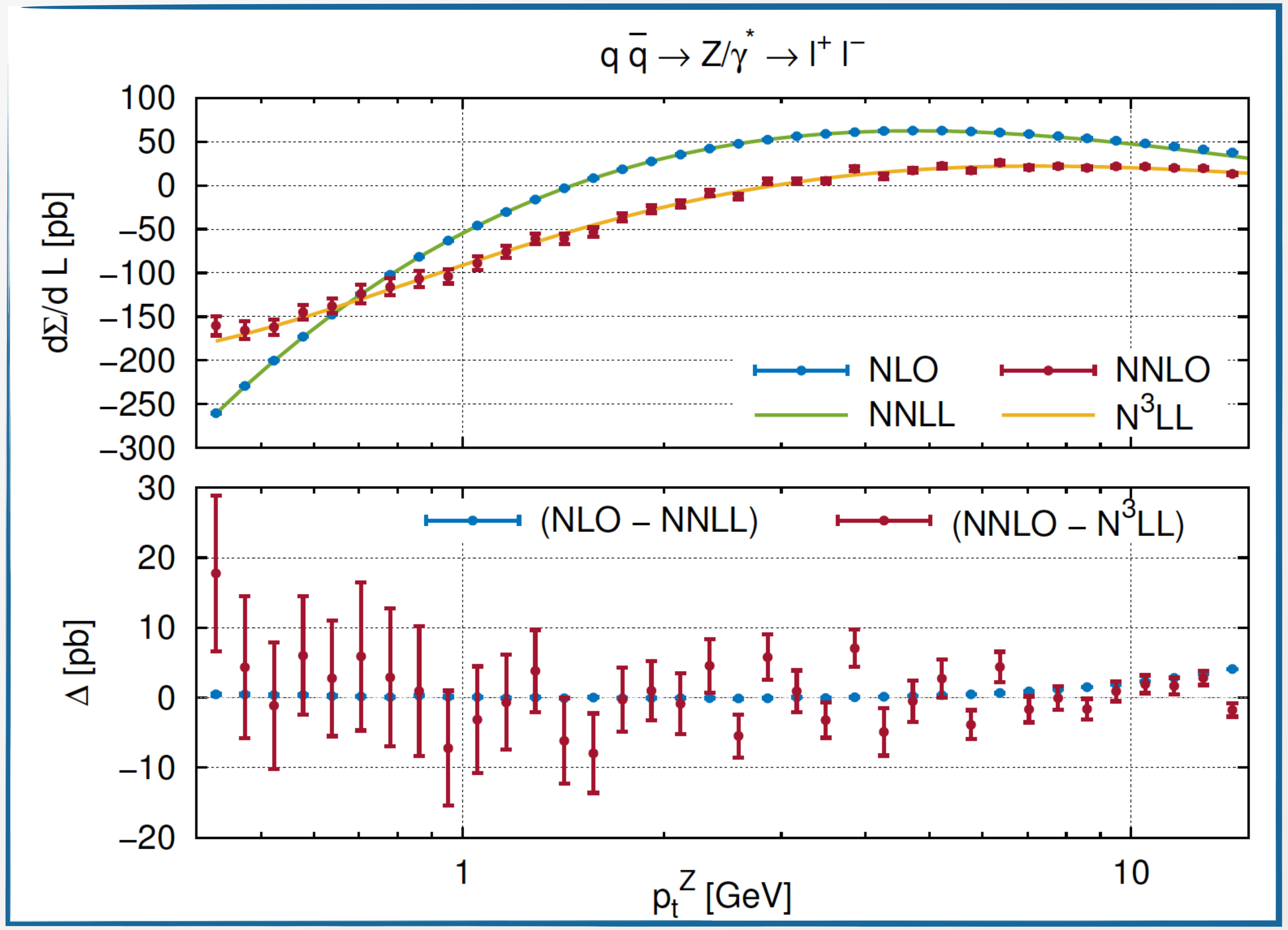
State-of-the-art **N³LL resummation is matched to NNLO calculations** for the differential spectrum

[Bizon, Chen, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Monni, Re, LR, Torrielli, Walker '18, '19]

Matching: **subtract** all logarithms from NNLO calculation and replace them with their **all-order summation**

few-% level cancellations: **numerically challenging**





Matching fixed order and resummed calculations

Combine the two predictions with a **matching scheme**

Additive $$\Sigma_{\text{add}}^{\text{N}^3\text{LL}+\text{N}^3\text{LO}}(p_t) = \int_0^{p_t} \frac{d\sigma}{dp_t} dp_t \sim \Sigma^{\text{N}^3\text{LL}}(p_t) + \Sigma^{\text{N}^3\text{LO}}(p_t) - \Sigma_{\text{exp}}^{\text{N}^3\text{LL}}(p_t),$$

RadISH+NNLOJET

Multiplicative $$\boxed{\Sigma_{\text{mult}}^{\text{N}^3\text{LL}+\text{N}^3\text{LO}}(p_t) \sim \Sigma^{\text{N}^3\text{LL}}(p_t) \left[\frac{\Sigma^{\text{N}^3\text{LO}}(p_t)}{\Sigma_{\text{exp}}^{\text{N}^3\text{LL}}(p_t)}\right]_{\text{exp}}}$$

Effect of N³LO total cross section subleading (N⁴LL) in the differential spectrum

$$\Sigma^{\text{N}^3\text{LO}}(p_t) = \overset{\sigma^{\text{NNLO}}}{\sigma^{\cancel{\text{N}^3\text{LO}}}} - \int_v^{\infty} \frac{d\sigma^{\text{NNLO}}}{dp_t} dp_t$$

Several strategies to ensure that resummation does not affect the hard region of the spectrum when matching is performed

RadISH+NNLOJET: **modified logarithms** (corresponds to restrict the rapidity phase space at large k_t)

$$\ln(Q/k_{t1}) \to \frac{1}{p} \ln\left(1 + \left(\frac{Q}{k_{t1}}\right)^p\right)$$

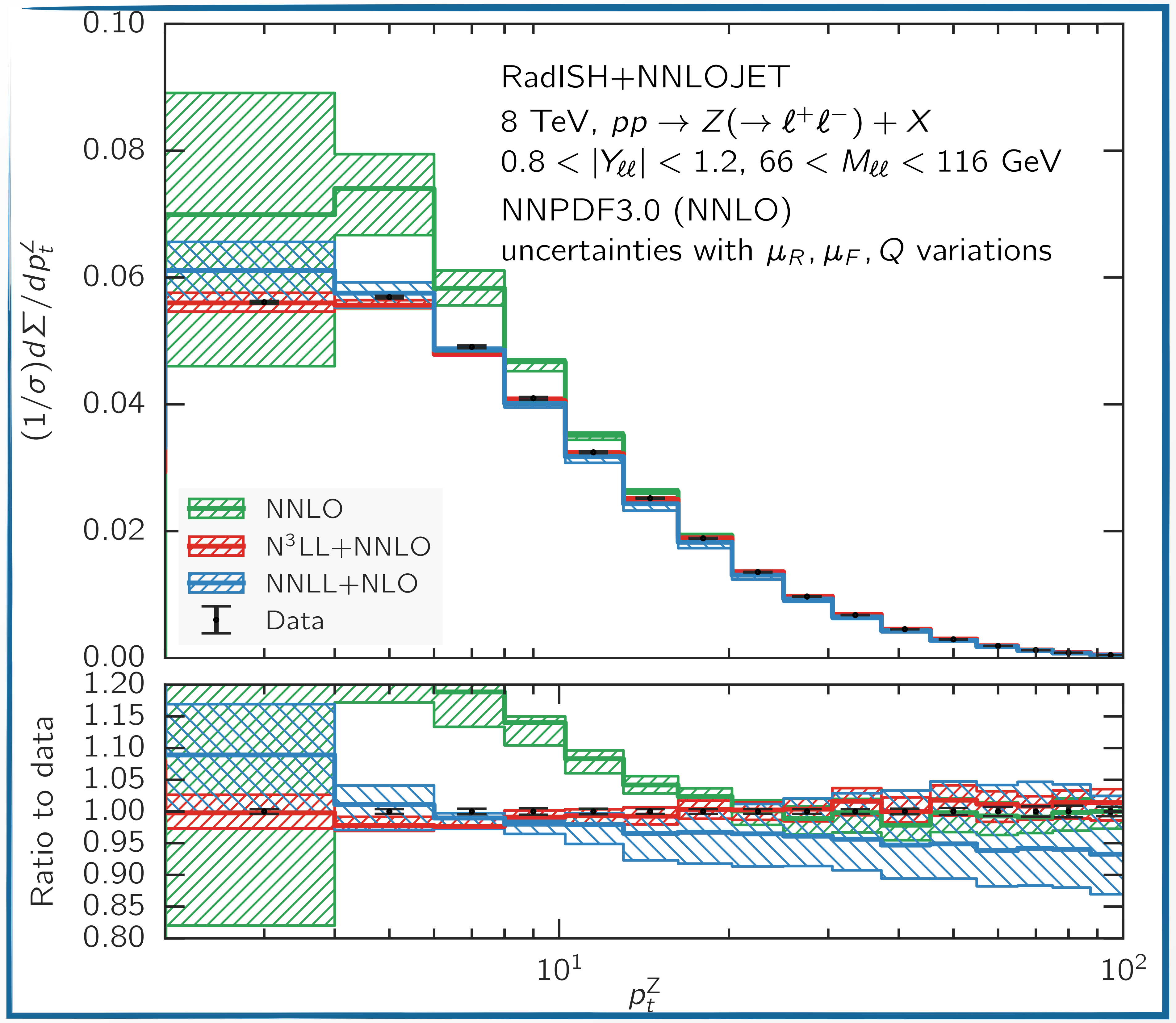
Q : **perturbative resummation scale** used to probe the size of **subleading** logarithmic corrections

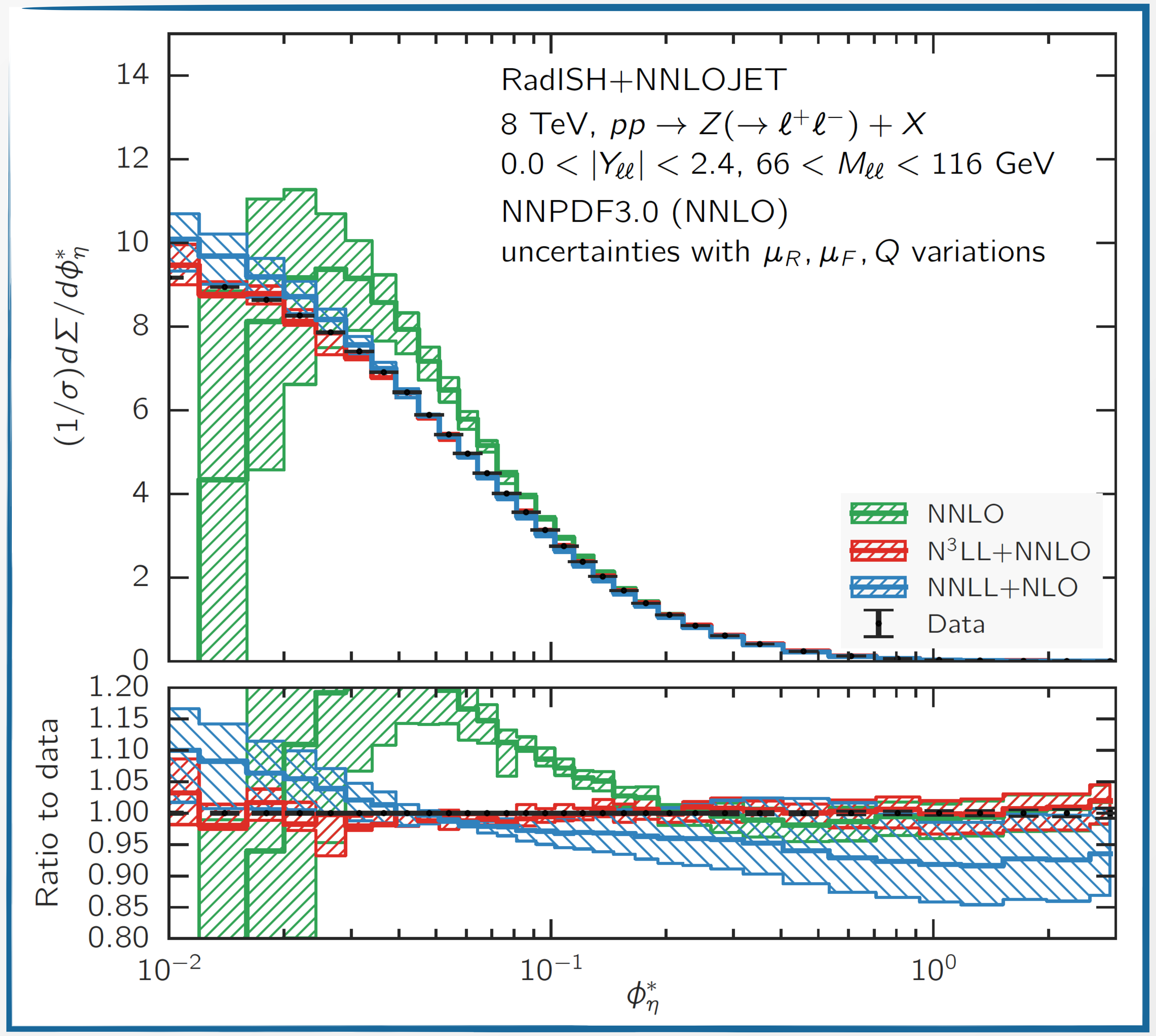
Alternative approaches use different prescriptions for turning off resummation (**profile functions**, **transition functions**…)

Results at N^3LL+NNLO: 8 TeV (Z, p_t and φ^*)

[Bizon, Chen, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Monni, Re, LR, Torrielli '18]

Data and fiducial cuts from [**ATLAS 1512.02192**] $p_t^{\ell^\pm} > 20\,\mathrm{GeV}, \qquad |\eta^{\ell^\pm}| < 2.4$





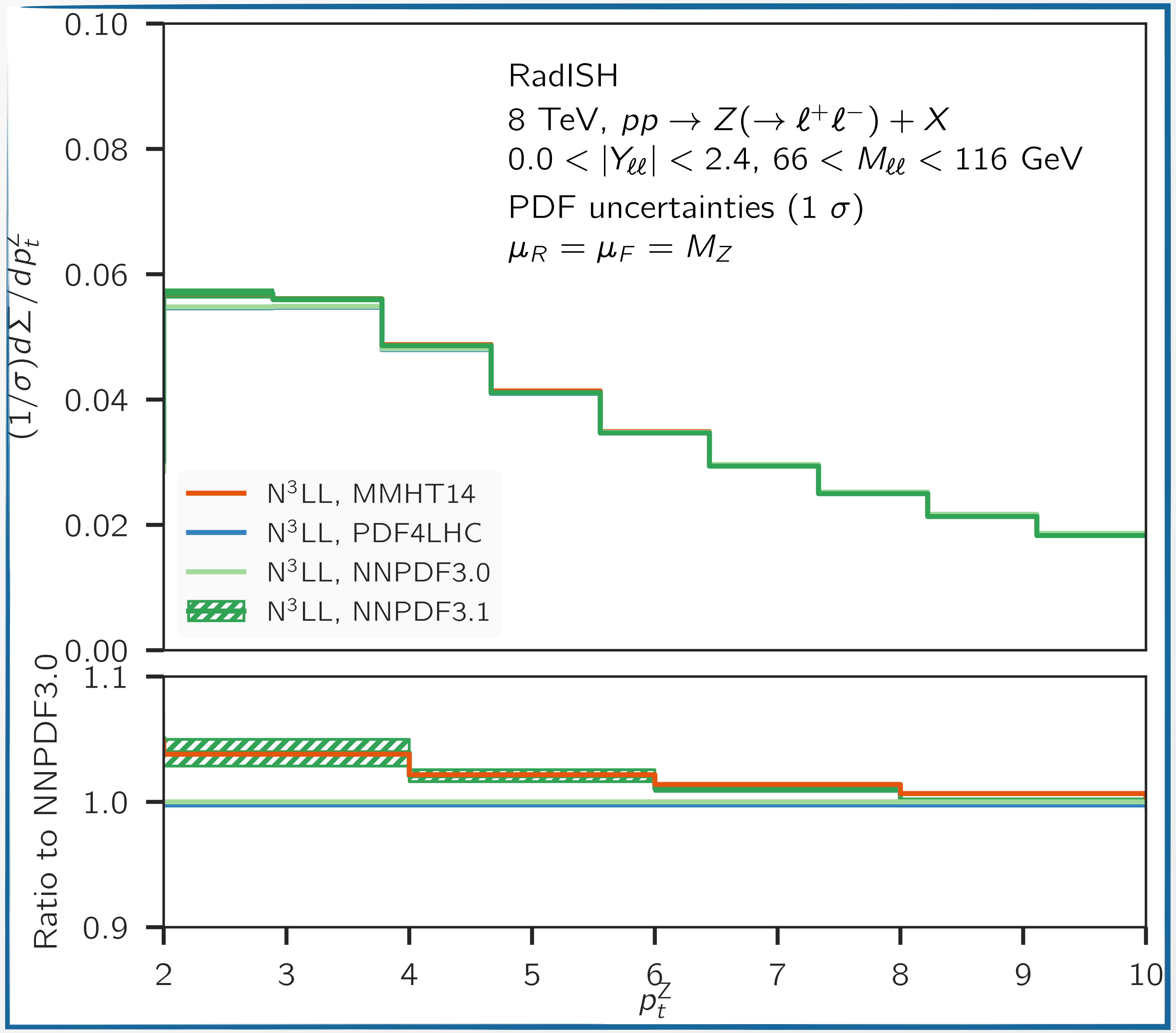
- **~7%-10% corrections** w.r.t. NNLL+NLO
- Scale uncertainties **below the 5% level**

Similar findings for the φ^* angular observable

Results at N³LL 8 TeV: PDF uncertainties

PDF errors at the 1% level, but difference between sets can be as large as 3.5%

Theory uncertainties in PDFs become relevant [NNPDF '19]



QED corrections and uncertainties

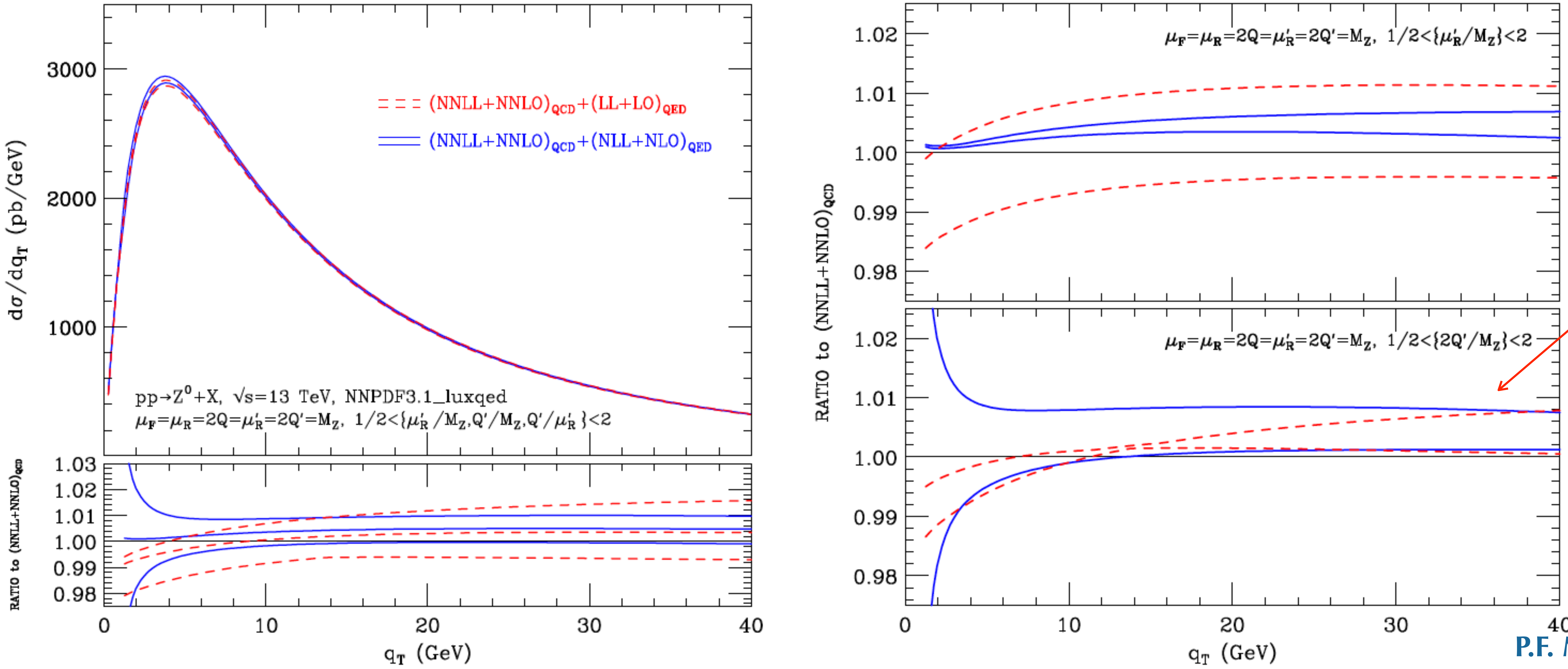
More on QED/EQ corrections in A. Vicini's talk later today

- QED $\mathcal{O}(\alpha^2)$ and mixed $\mathcal{O}(\alpha_s\alpha)$ QED/QCD corrections contribute at the permille level to the total cross section

[de Florian, Der, Fabre '18]

- QED effects lead to a 1-2% correction to the differential distribution

[Cieri, Ferrera, Sborlini '18]



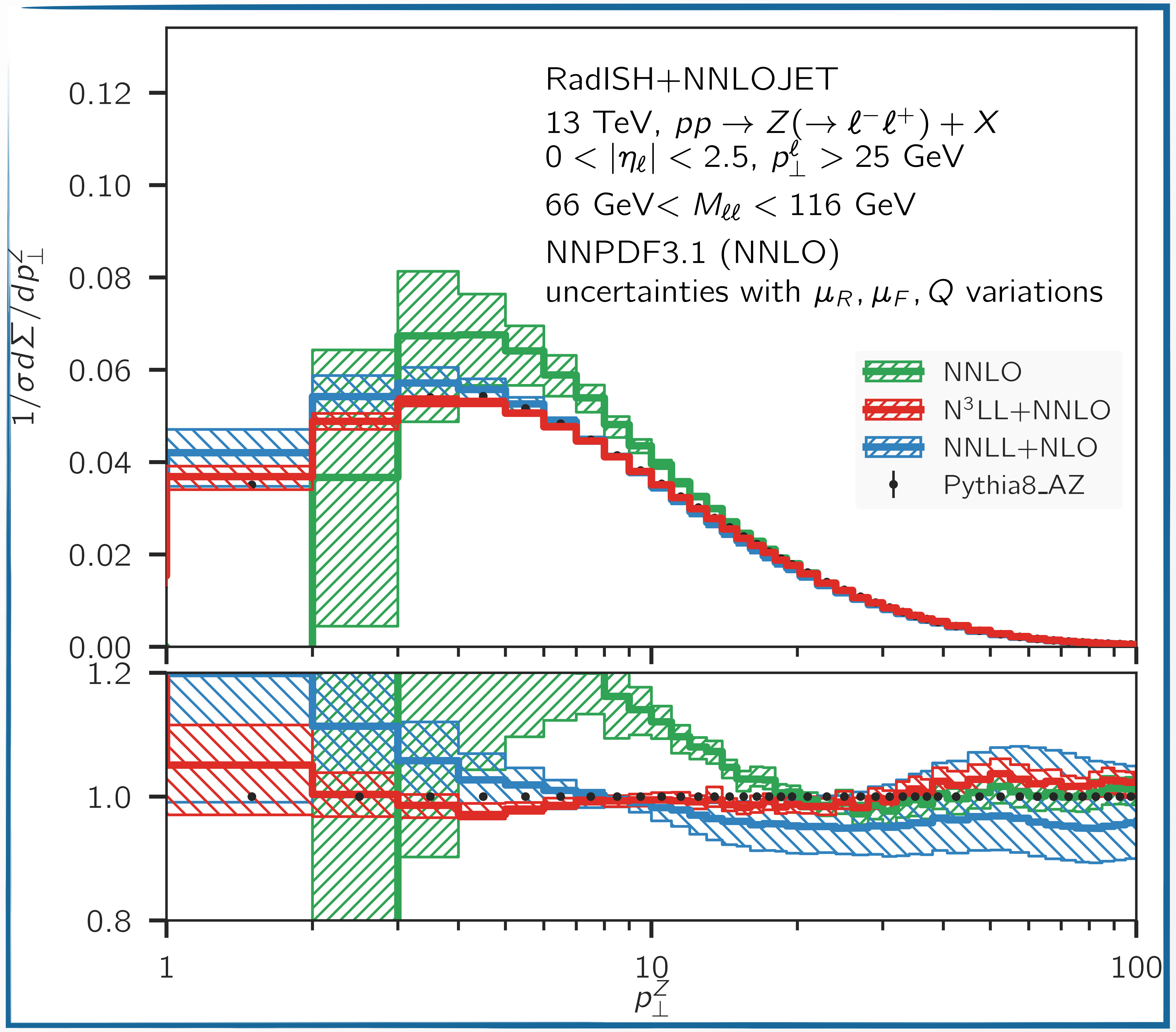
Error dominated by resummation uncertainties

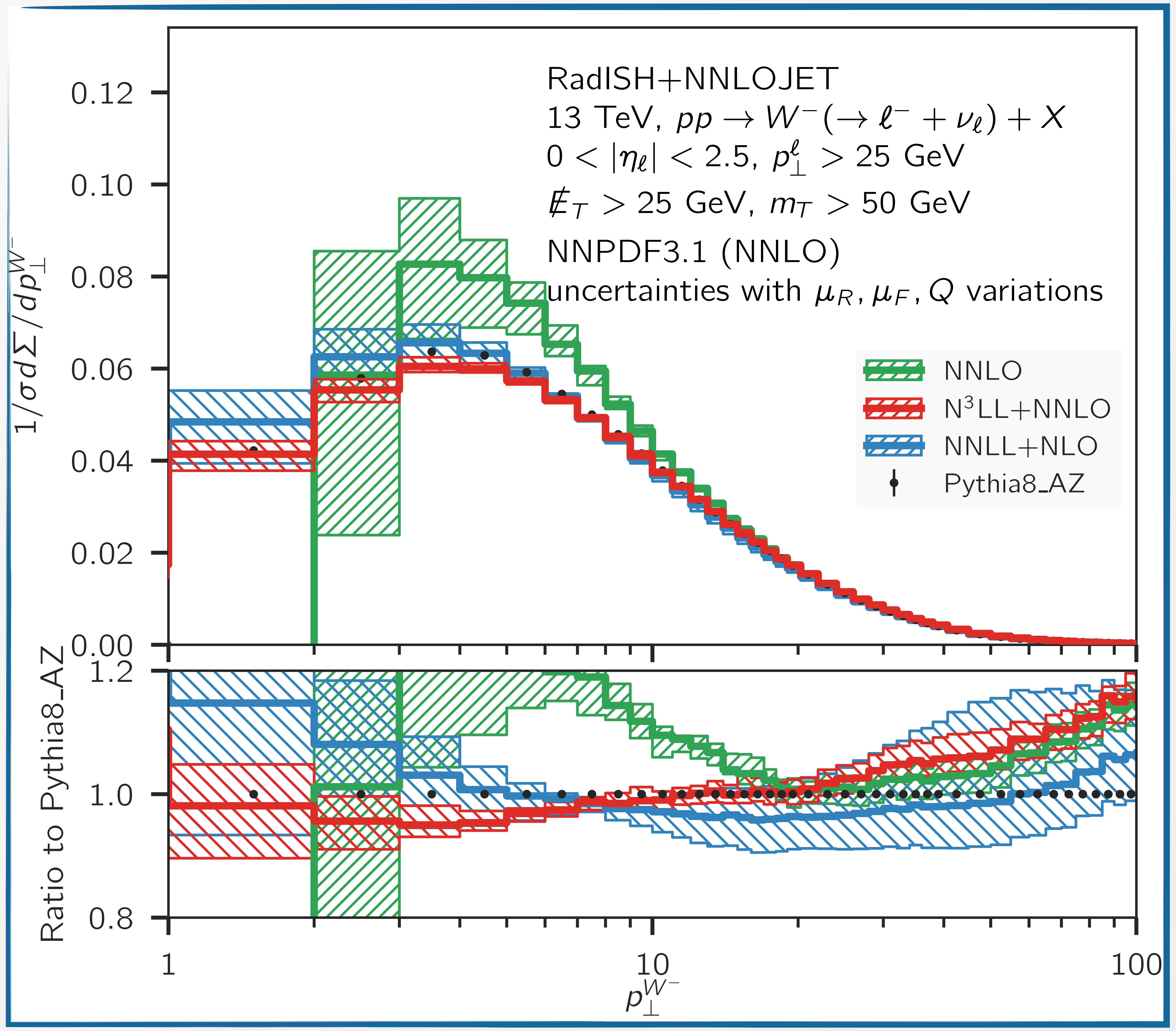
P.F. Monni, SM@LHC 2019

Results at N^{3}LL+NNLO: 13 TeV (Z & W p_t)

Thanks to Jan Kretzschmar for providing the PYTHIA8 AZ tune results

[Bizon, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Monni, Re, LR, Walker ’19]





Some discrepancies with Pythia8 [AZ tune, tuned to p_t^Z at 7 TeV]: is this tune reliable at 13 TeV ?

Resummation and matching ambiguities

Different approaches may have **same nominal (perturbative) accuracy,** but may differ by subleading logarithmic and/or higher orders terms.

Several sources of such differences:

For additional details, see G. Bozzi's slides

- subleading contributions
- ***b*-space** vs. **direct space**
- order of PDF evolution
- matching schemes: **additive** vs. **multiplicative**
- turning off resummation effects in the hard region of the spectrum: **modified logs** (and associated scaling parameter)**, profile scales, transition functions**…
- non-perturbative corrections

Not related to subleading effects, but relevant phenomenologically

- thresholds and treatment of heavy quarks

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Benchmark of resummed calculations

Not related to subleading effects, but relevant phenomenologically

- thresholds and treatment of heavy quarks

Logarithmic accuracy and counting

Ingredients needed to reach a given logarithmic accuracy

		Boundary conditions (FO hard, coll., soft)	Anomalous dimensions		FO matching (nonsingular)
			γ_i	$\Gamma_{\mathrm{cusp}}, \beta$	
g_0	LL	1	-	1-loop	-
g_1	NLL	1	1-loop	2-loop	-
g_2	NLL′+NLO$_0$	α_s	1-loop	2-loop	α_s
	NNLL+NLO$_0$	α_s	2-loop	3-loop	α_s
g_3	NNLL′+NNLO$_0$	α_s^2	2-loop	3-loop	α_s^2
	N^{3}LL+NNLO$_0$	α_s^2	3-loop	4-loop	α_s^2

Credits: F. Tackmann

E.g. in *b* space, in a **very** schematic way

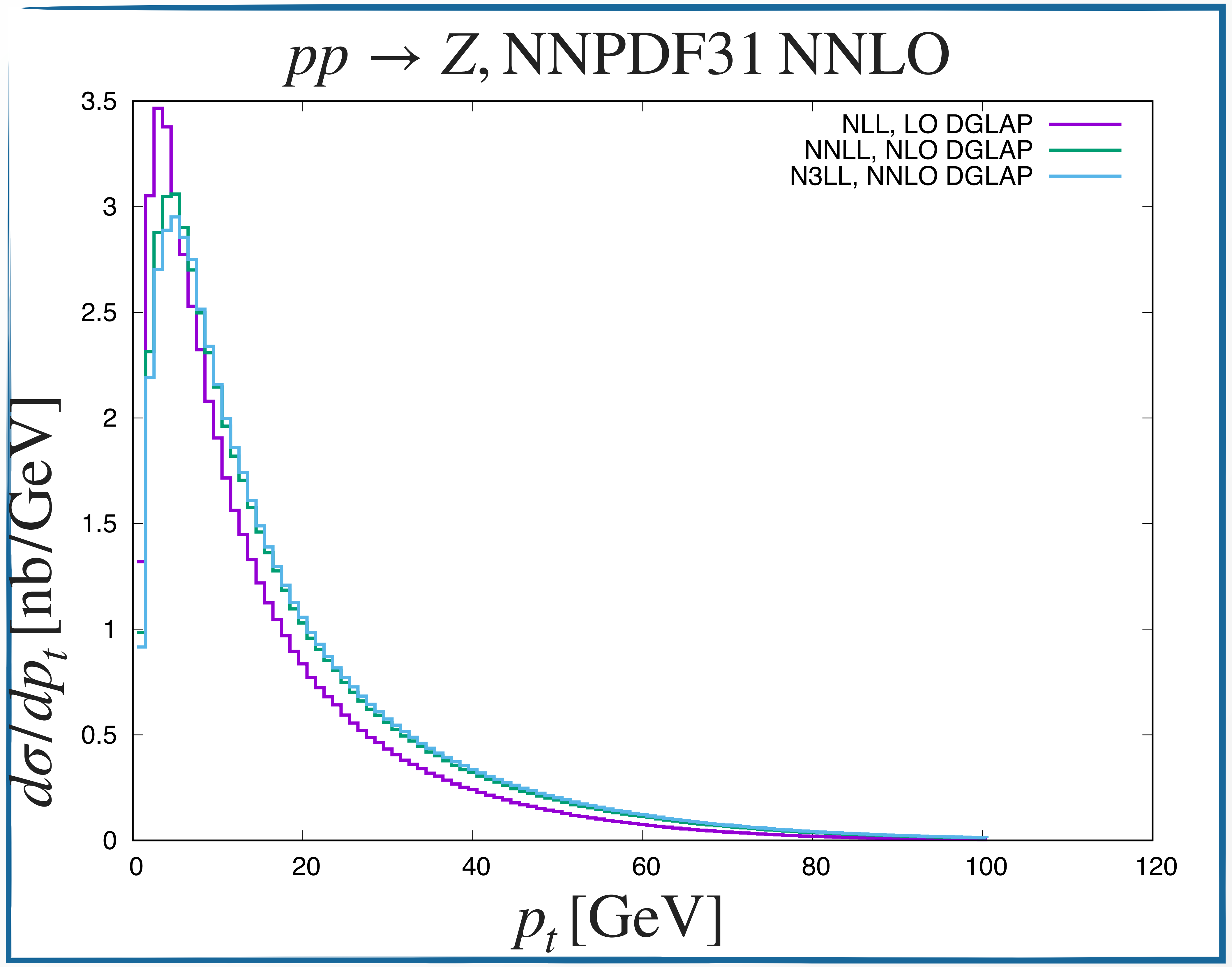
$$\Sigma_{\mathrm{NNLL}}(v) \sim \exp[Lg_0(\alpha_s L) + g_1(\alpha_s L) + \alpha_s g_2(\alpha_s L)]$$

$$\Sigma^{(1)}_{\mathrm{NNLL}}(v) \sim \exp[Lg_0(\alpha_s L) + g_1(\alpha_s L)](1 + \alpha_s g_2(\alpha_s L) + \ldots)$$

$$\Sigma^{(2)}_{\mathrm{NNLL}}(v) \sim \exp[Lg_0(\alpha_s L) + g_1(\alpha_s L) + \alpha_s \tilde{g}_2(\alpha_s L)]\{1 + \alpha_s[g_2(\alpha_s L) - \tilde{g}_2(\alpha_s L)] + \ldots\}, \qquad \tilde{g}_2(x) \neq g_2(x)$$

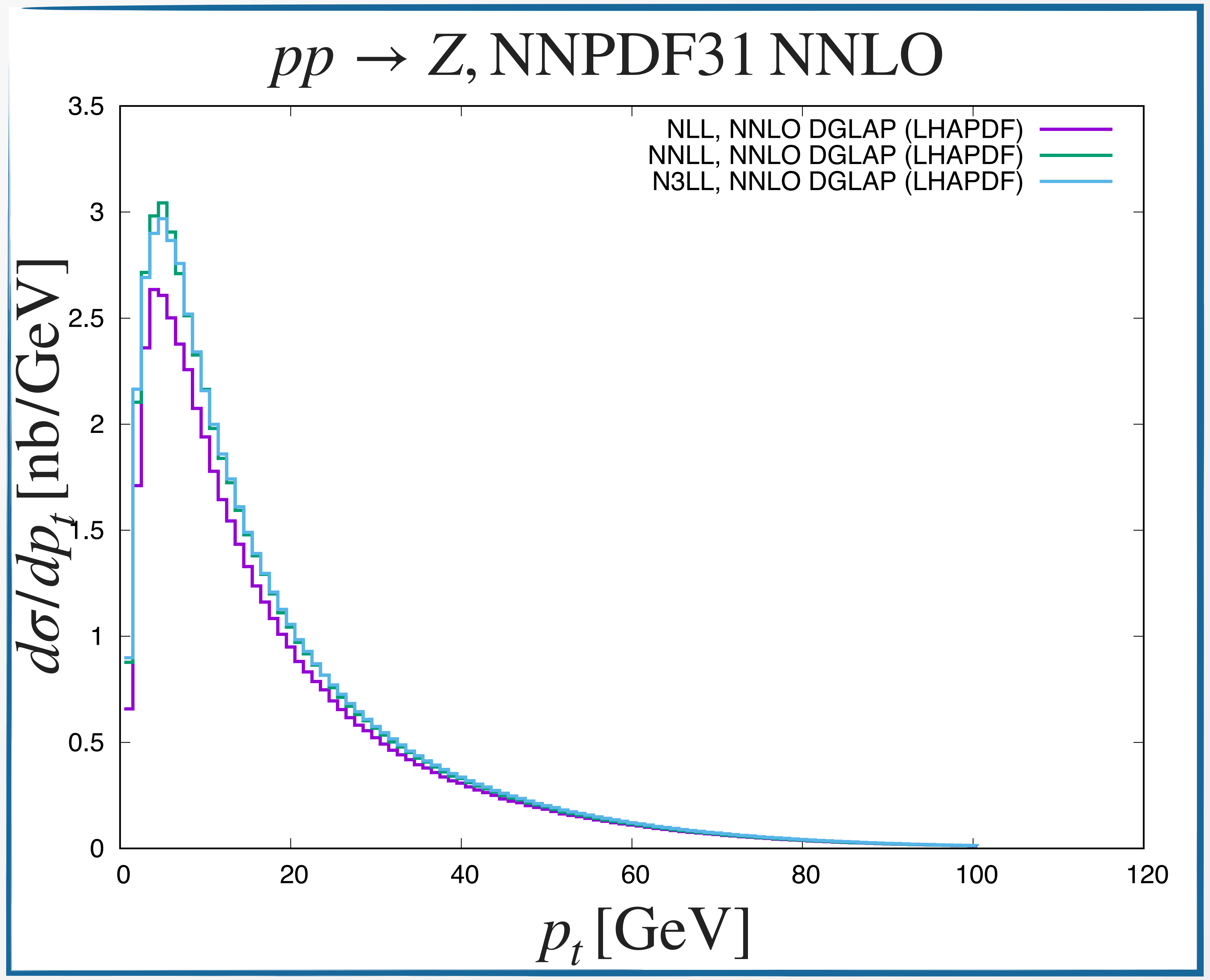
Results all **formally equivalent** at NNLL accuracy

Logarithmic accuracy and counting: the role of DGLAP evolution



PDF evolution at LO, NLO, NNLO at NLL, NNLL, N^{3}LL

Default in e.g. DYRes/DYTURBO, ReSolve



PDF evolution at NNLO at NLL, NNLL, N^{3}LL through LHAPDF

Default in e.g. RadISH, ResBos2, SCETLib

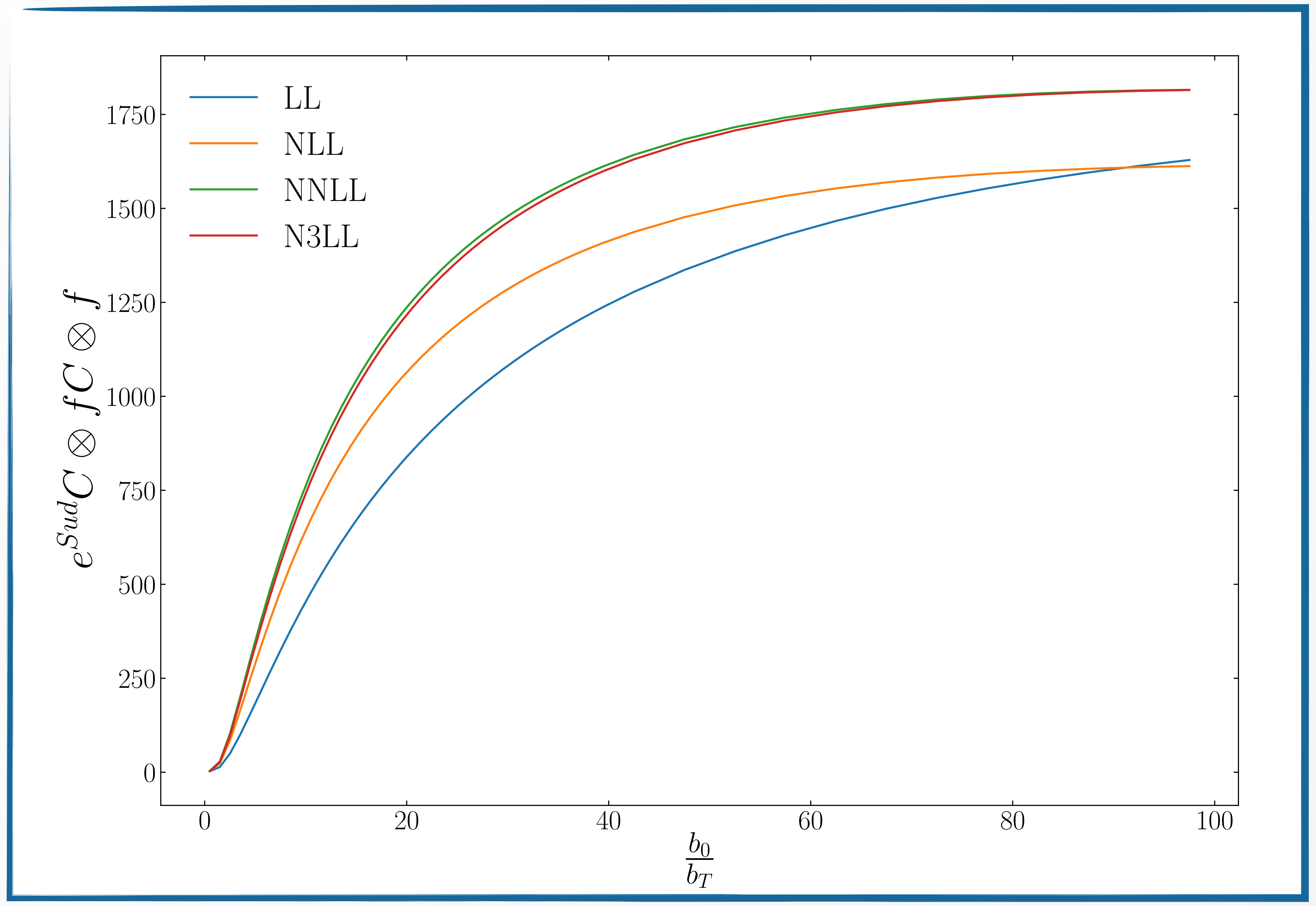
Advantage in using LHAPDF: (partial) information on quark thresholds

Differences **can be important** at NLL and NNLL and are an indication of the size of subleading corrections

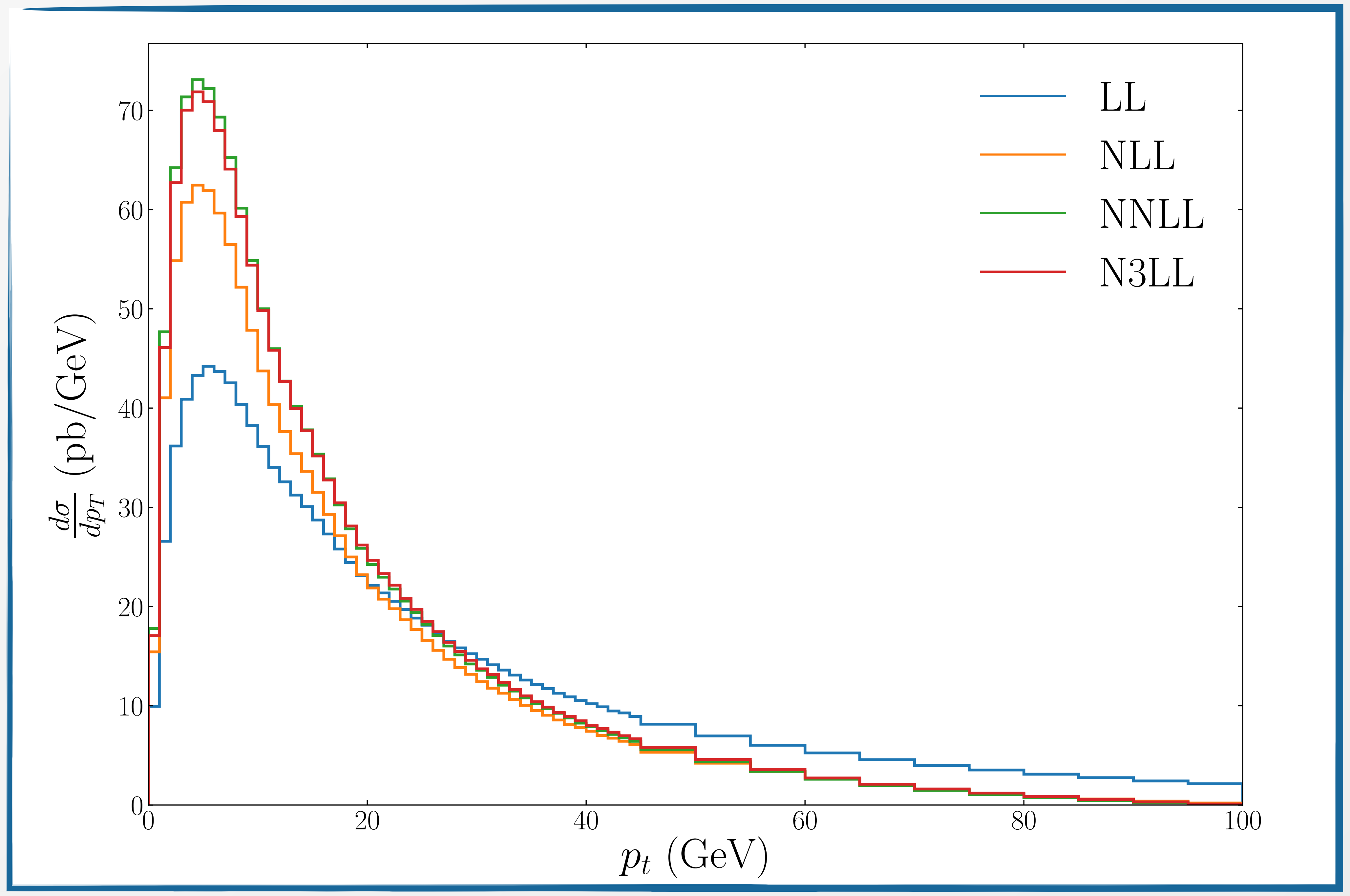
b-space results vs. p_t space results

For codes whose formal accuracy is defined in *b*-space, it may be of some interest to compare the results both in impact-parameter space and in p_t-space after the inverse Fourier transform

Joshua Isaacson, ResBos2



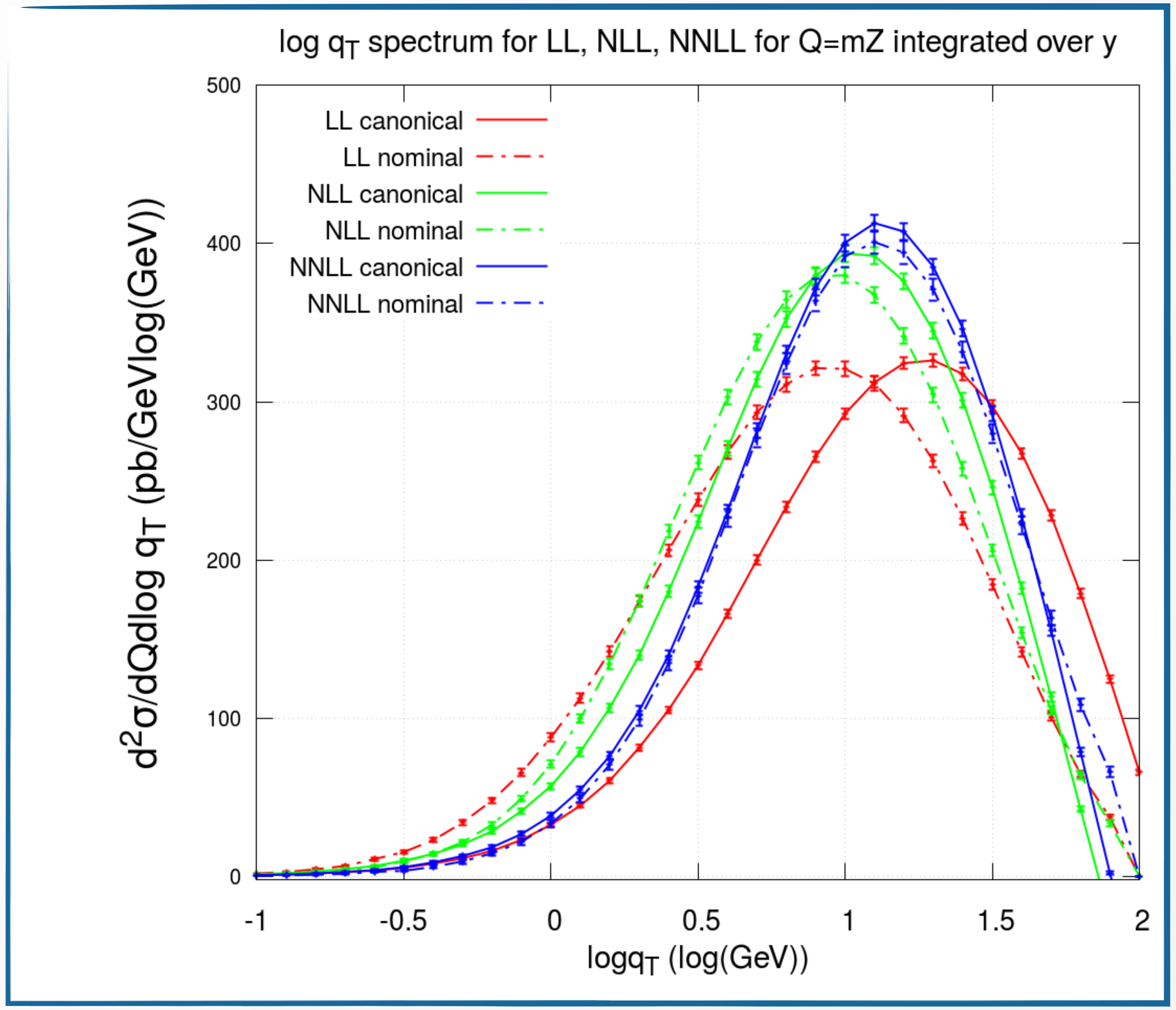
Inverse Fourier Transform



Matching ambiguities

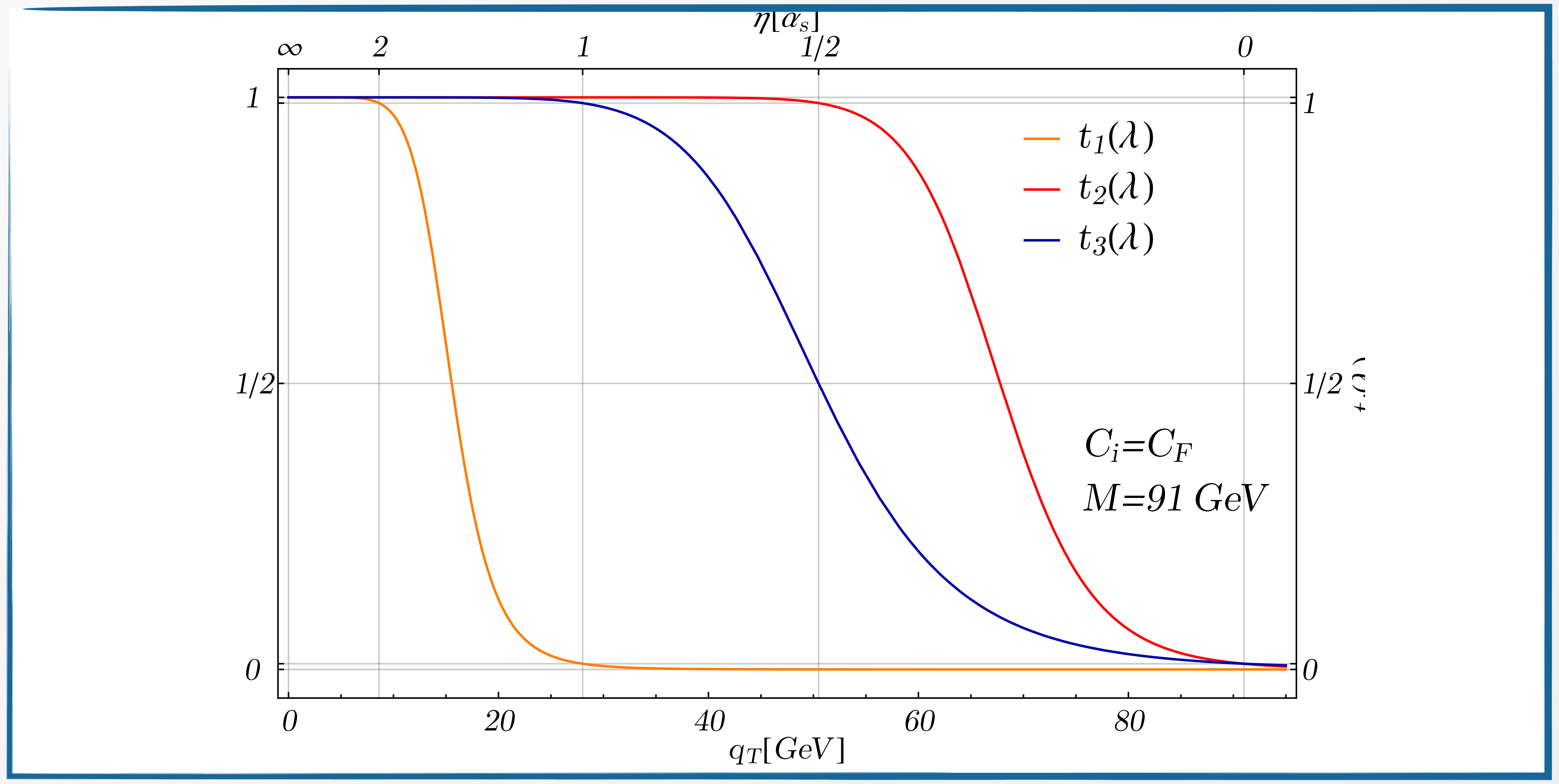
F. Coradeschi/T. Cridge, ReSolve

T. Becher, CuTe



Nominal (**un-modified**) vs. canonical (**modified**) logs

most of the differences due to the different resummation scales used in the two cases



Transition functions and matching functions used to turn off resummation at large q_t

$$\frac{d\sigma_{\rm ms}}{dq_T} = t\left(\lambda\right)\frac{d\sigma_{\rm res}}{dq_T} + \left[R_{\rm sud}\left(\mu_{\rm ms}\right)\right]^{t(\lambda)}\left[\frac{d\sigma_{\rm fo}}{dq_T} - t\left(\lambda\right)\frac{d\sigma_{\rm sqt}}{dq_T}\right]$$

Matching details play an important role in the transition region, but at lower accuracy might induce differences also in the small-p_t limit

Non-perturbative corrections

1. All formalisms have to deal with the **Landau pole**
   - direct space: Sudakov radiator hit Landau pole at $\alpha_s(\mu_R^2)\beta_0 \ln Q/k_{t1} = \frac{1}{2}$

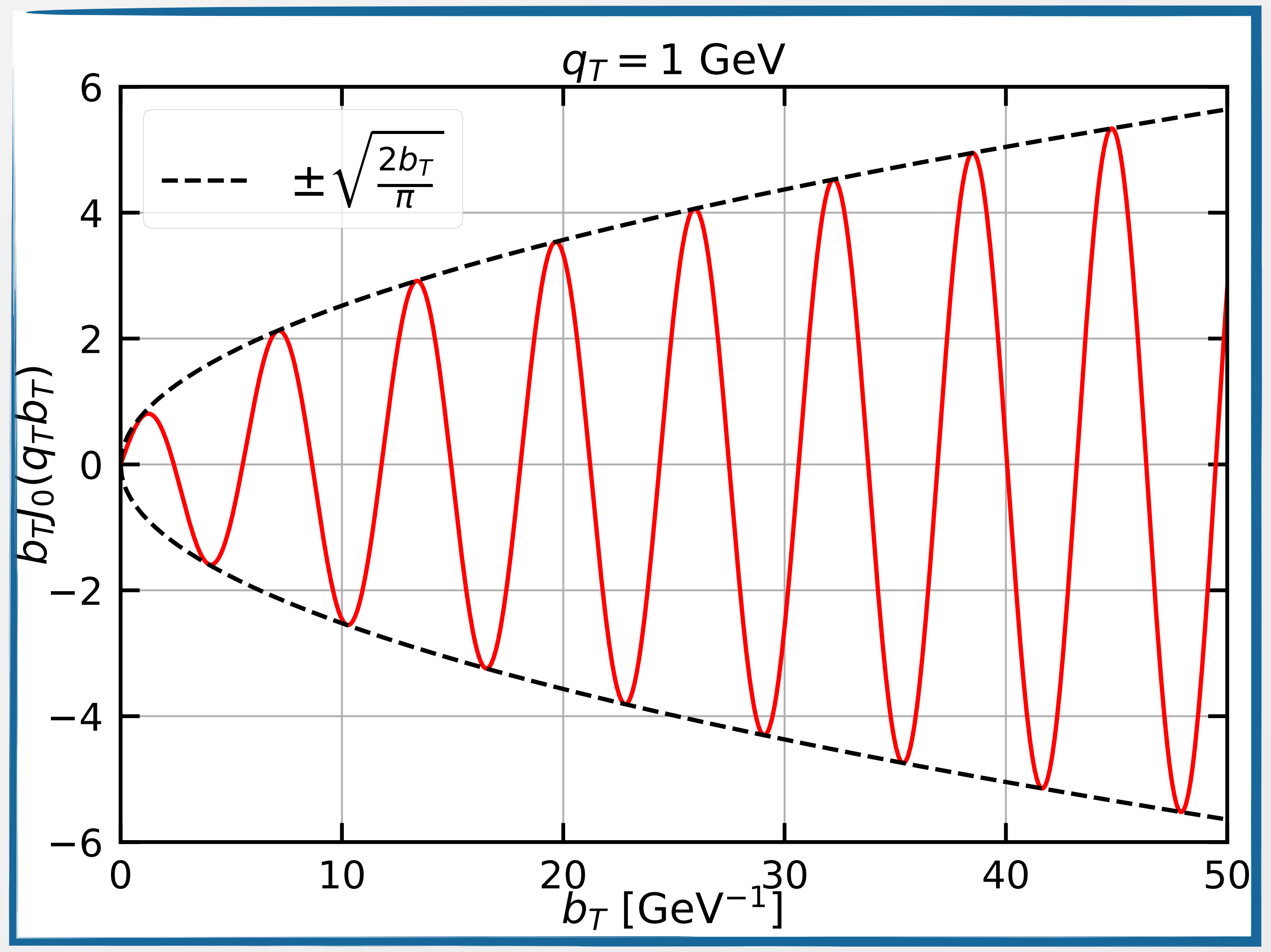
     n.b. since at small p_t the large azimuthal cancellations dominate, this cutoff is never an issue in practice
   - b space, when integrating over b, the integral hits the Landau pole at large values of b

Several solutions available

E.g. b* prescription: impact parameter frozen at a value

$$b_* = \frac{b}{\sqrt{1 + (b/b_{\lim})}}, \quad b_* < b_{\lim}$$

V. Bertone/G. Bozzi, NangaParbat



2. intrinsic quark transverse momentum (initial condition for TMDs)
   - **non-perturbative, fitted factor to model the non-perturbative region**, in principle kinematics- and flavour- dependent
   - **Fitted factor** may help to stabilize the numerical integral when computing b-integral

Heavy-quark effects

Bottom quarks in the initial state yield ~4% of the total Z cross section (CKM suppressed for W)

Collinear logarithmic contributions encoded in DGLAP evolution in the 5FS; accounting for bottom mass can be important at scales $p_t \sim m_b \sim$ peak region

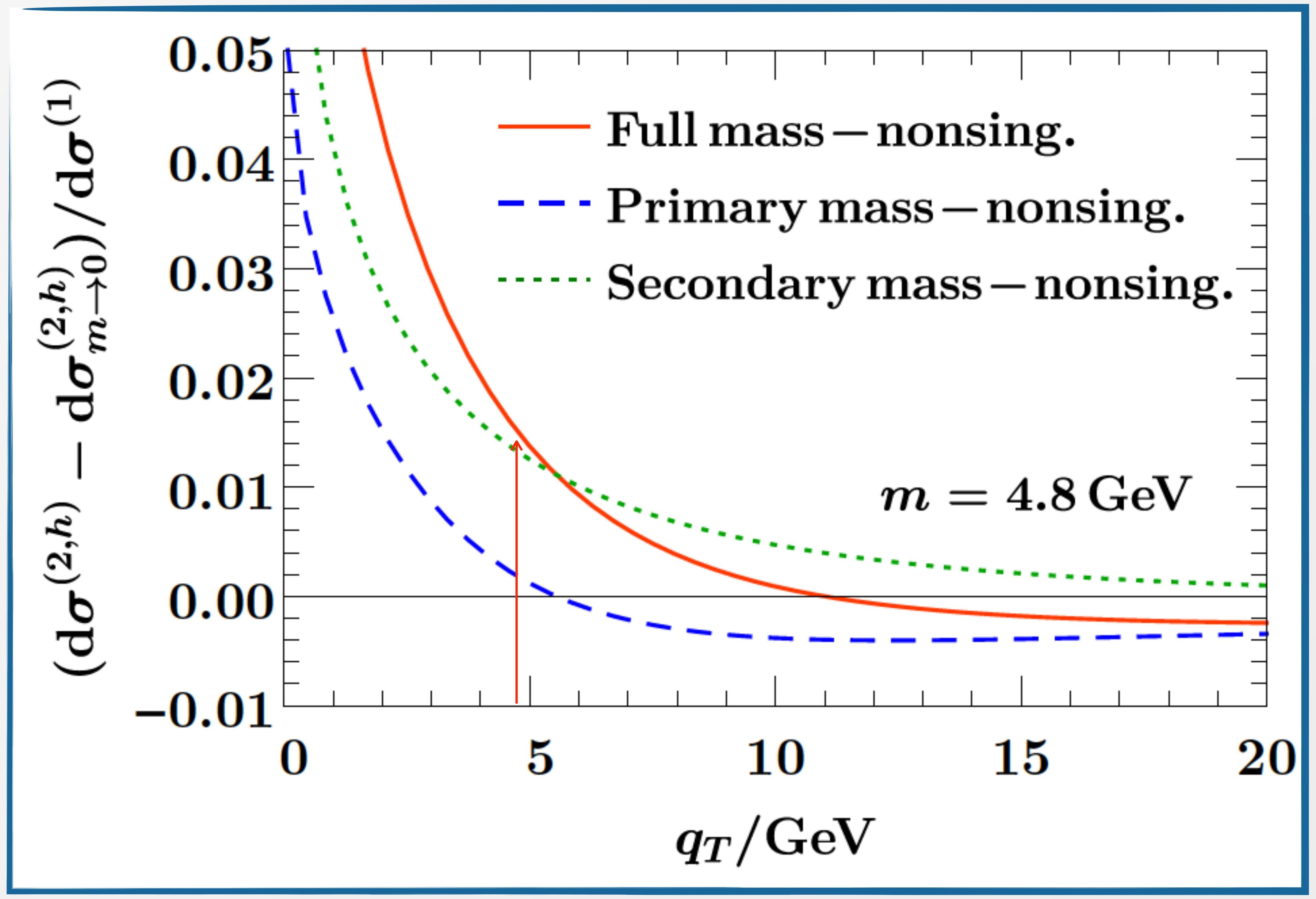
Existing studies indicate very small corrections ~ 1%

[Bagnaschi, Maltoni, Vicini, Zaro ’18]

Exact shape details remain an open question: fully consistent treatment in resummations useful for %-level precision

[Aivazis, Collins, Olness, Tung ’93]
[Nadolsky, Kidonakis, Olness, Yuan ’02]
[Berge, Nadolsky, Olness ‘05]
[Pietrulewicz, Samitz, Spiering, Tackmann ’17][

Full calculation still unavailable, but partial results indicate a percent effect at $p_t \sim m_b$



[Pietrulewicz, Samitz, Spiering, Tackmann ’17]

Benchmark

Benchmark: address most (all?) of the issues by comparing different resummed predictions

Various groups involved, different default choices and formalisms

	b-space	k_t-space	add.	mult.	m. logs	profile	trans. fun	NP corr
PB-TMD		✓						✓
CuTe		✓	✓				✓	✓
DYres/DYTURBO	✓		✓		✓			(✓)
NangaParbat	✓		✓		✓			✓
RadISH		✓	(✓)	✓	✓			
ResBos2	✓		✓		✓			✓
Resolve	✓		✓		✓			✓
SCETLib	✓		✓			✓		

Non-trivial effort, need to decide what needs to be prioritised. Work in progress in the subgroup

Benchmark

Benchmark: address

Various groups invol

	NP corr
PB-TMD	✓
CuTe	✓
DYres/DYTURBO	(✓)
NangaParbat	✓
RadISH	
ResBos2	✓
Resolve	✓
SCETLib	

Non-trivial effort, n

A theorist herder's reverie

The W/Z transverse momentum ratio: understanding correlations

Z and *W* production share a similar pattern of QCD radiative corrections

Crucial to understand correlation between *Z* and *W* spectra to exploit data-driven predictions

$$\frac{1}{\sigma^W}\frac{d\sigma^W}{p_\perp^W} \sim \frac{1}{\sigma^Z_{\rm data}}\frac{d\sigma^Z_{\rm data}}{p_\perp^Z}\,\frac{\frac{1}{\sigma^W_{\rm theory}}\frac{d\sigma^W_{\rm theory}}{p_\perp^W}}{\frac{1}{\sigma^Z_{\rm theory}}\frac{d\sigma^Z_{\rm theory}}{p_\perp^Z}}$$

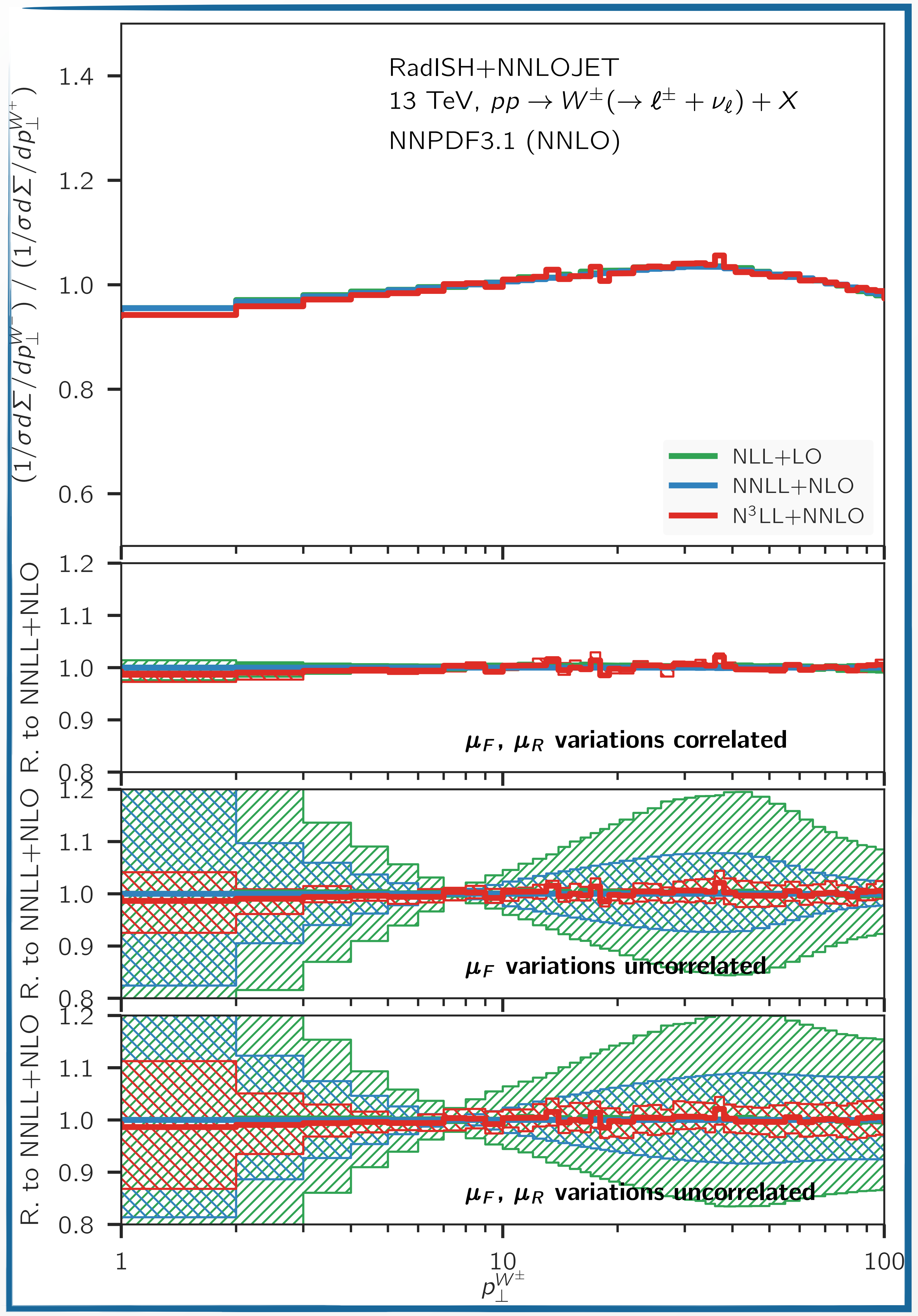
Several choices are possible. Within **canonical scale variations**:

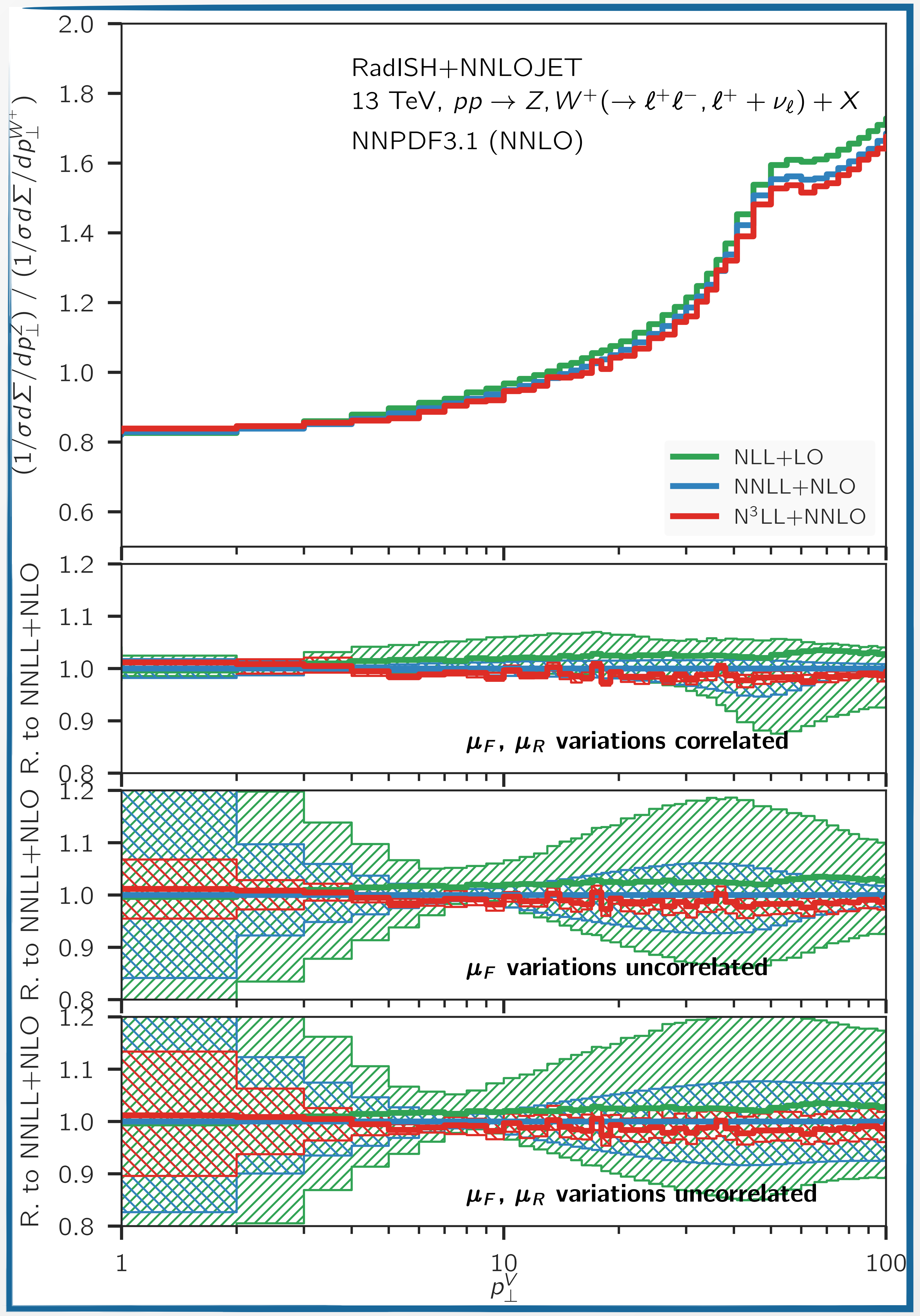
- **Correlate renormalisation and factorisation scales**
- **Correlate resummation and renormalisation** scale variations, keep **factorisation scale uncorrelated**, while keeping
$$\frac{1}{2} \leq \frac{\mu_{\rm F}^{\rm num}}{\mu_{\rm F}^{\rm den}} \leq 2$$
- More **conservative** estimate: vary both **renormalisation and factorisation scales in an uncorrelated** way with
$$\frac{1}{2} \leq \frac{\mu^{\rm num}}{\mu^{\rm den}} \leq 2$$

The W/Z transverse momentum ratio: understanding correlations

[Bizon, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Monni, Re, LR, Walker ’19]

Validate by studying the convergence of the perturbative predictions





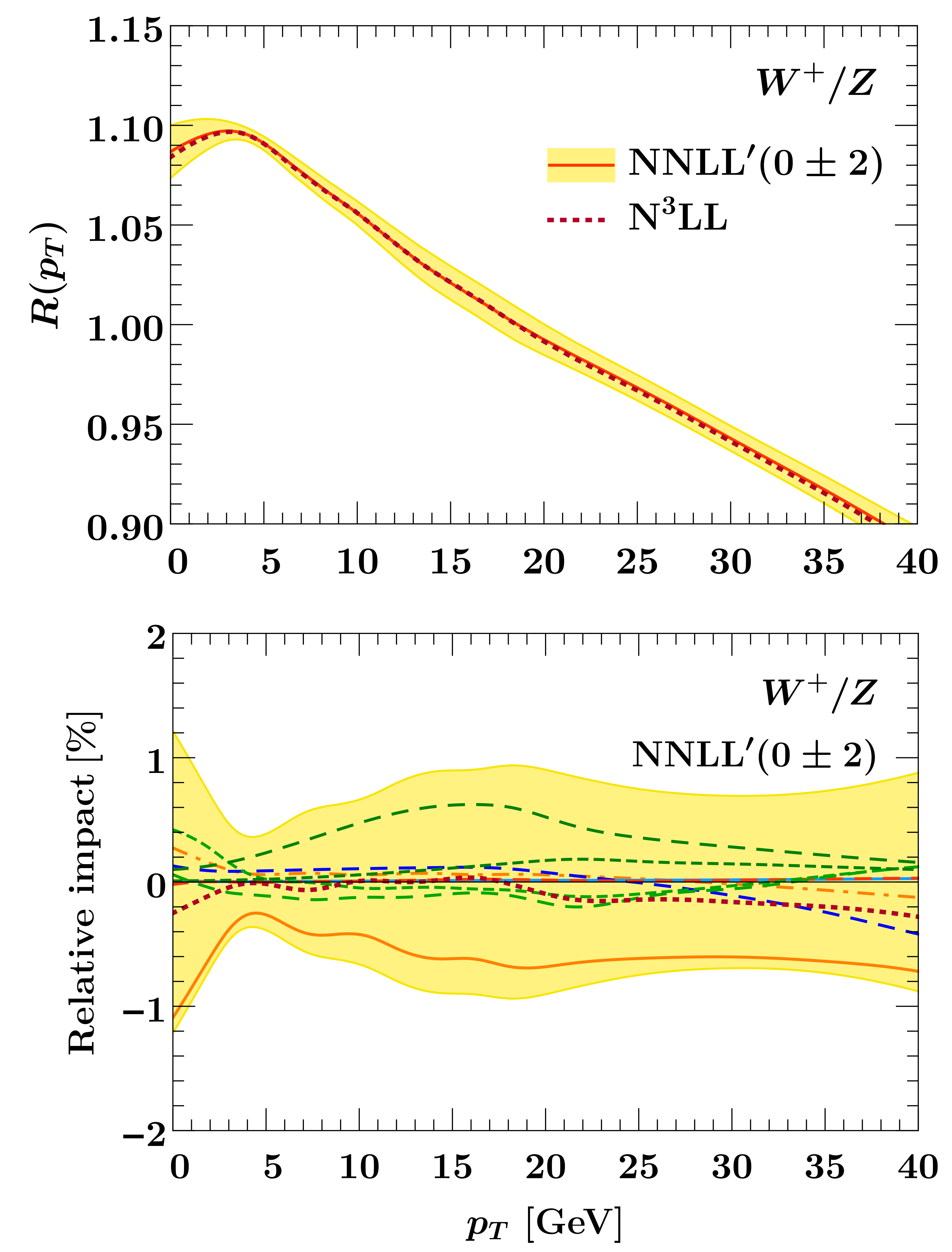
Less conservative prescription seems justified

The W/Z transverse momentum ratio: understanding correlations

Alternative uncertainty estimate: each resummation order only depends on a few semi-universal parameters

F. Tackmann, SCETlib

	boundary conditions			anomalous dimensions			
order	h_n	s_n	b_n	γ^h_n	γ^s_n	Γ_n	β_n
LL	h_0	s_0	b_0	–	–	Γ_0	β_0
NLL′	h_1	s_1	b_1	γ^h_0	γ^s_0	Γ_1	β_1
NNLL′	h_2	s_2	b_2	γ^h_1	γ^s_1	Γ_2	β_2
N^3LL′	h_3	s_3	b_3	γ^h_2	γ^s_2	Γ_3	β_3
N^4LL′	h_4	s_4	b_4	γ^h_3	γ^s_3	Γ_4	β_4



- Basic Idea: Treat them as theory nuisance parameters
  - ✓ Vary them independently to estimate the theory uncertainties
  - ✓ Impact of each independent nuisance parameter is fully correlated across all kinematic regions and processes
  - ✓ Impact of different nuisance parameters is fully uncorrelated
- Price to Pay: Calculation becomes quite a bit more complex

Advantages

- ✓ Encode correct correlations
- ✓ Can be propagated straightforwardly
  - ► Including Monte Carlo, BDTs, neural networks, ...
- ✓ Can be consistently included in a fit and constrained by data
  - ► Even okay to use control measurements to reduce theory uncertainties
  - ► Due to central-limit theorem, total theory uncertainty becomes Gaussian

Conclusion

- Modelling of theoretical uncertainties crucial for experimentalists working on the W mass measurement
- Resummation needed in the small p_t region. Different resummation approaches may have the same perturbative accuracy, but may differ by subleading logarithmic and/or higher orders terms, whose relevance should be assessed
- Work in progress in the subgroup. Eight different theory groups providing their best predictions and benchmarking their results
- Degree of correlation between various corrections to be understood at this level of precision. Insight on how we should estimate missing higher order uncertainties (e.g. PDFs) with this level of accuracy needed
- Monte Carlo tunes for sub-percent precision must be handled with care. Very careful study of what parameters are actually being tuned is necessary to avoid unphysical correlations