Missing higher order uncertainties

in the inclusive Higgs cross section

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Based on: arXiv 1603.08000, Marco Bonvini, Simone Marzani, Claudio Muselli, LR

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LHC, New Physics, and the pursuit of Precision

LHC as a **discovery machine**

- Higgs Boson ✓
- BSM particles ✗ (as of today)

Focus in LHC run II

- Measurement of the Standard Model parameters with **very high precision**
- Signals of New Physics **beyond the Standard Model**

A theorist's Quest:

- New BSM scenarios to be tested
- New techniques to enhance signal/background ratio and isolate tiny deviations from SM predictions
- Development of **accurate** and **precise** theoretical predictions

Higgs production in gluon fusion

Estimation of theoretical uncertainties for inclusive gluon fusion Higgs production involves **different sources of uncertainty**

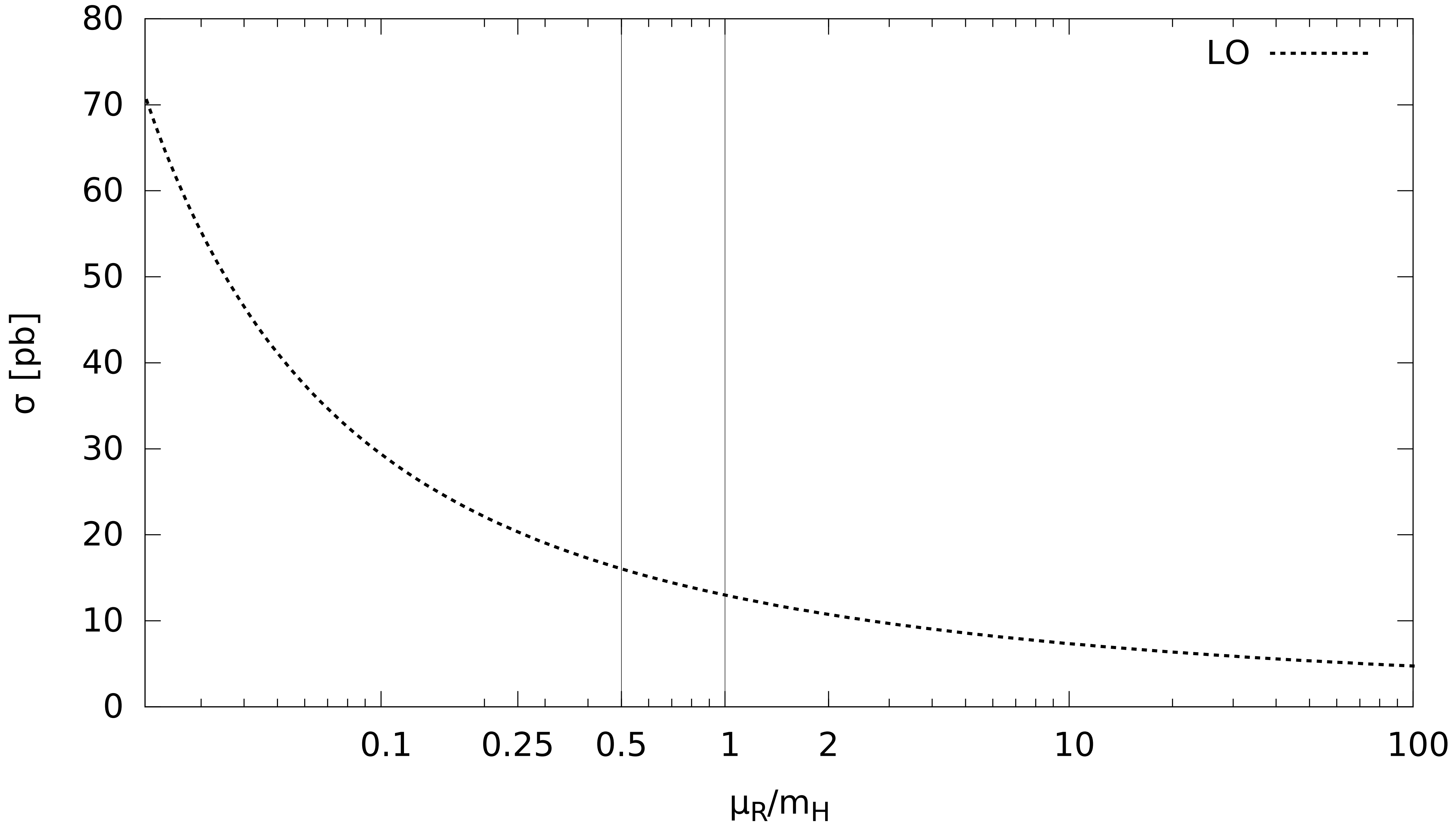
- Missing electroweak corrections
- Bottom and charm effects
- Finite top mass effects
- Truncation of the soft expansion
- Missing higher orders uncertainties — **This talk**
- …sociological uncertainty: uncertainty in the interpretation of the uncertainties

In this world nothing can be said to be certain, except death and taxes.
(Benjamin Franklin)

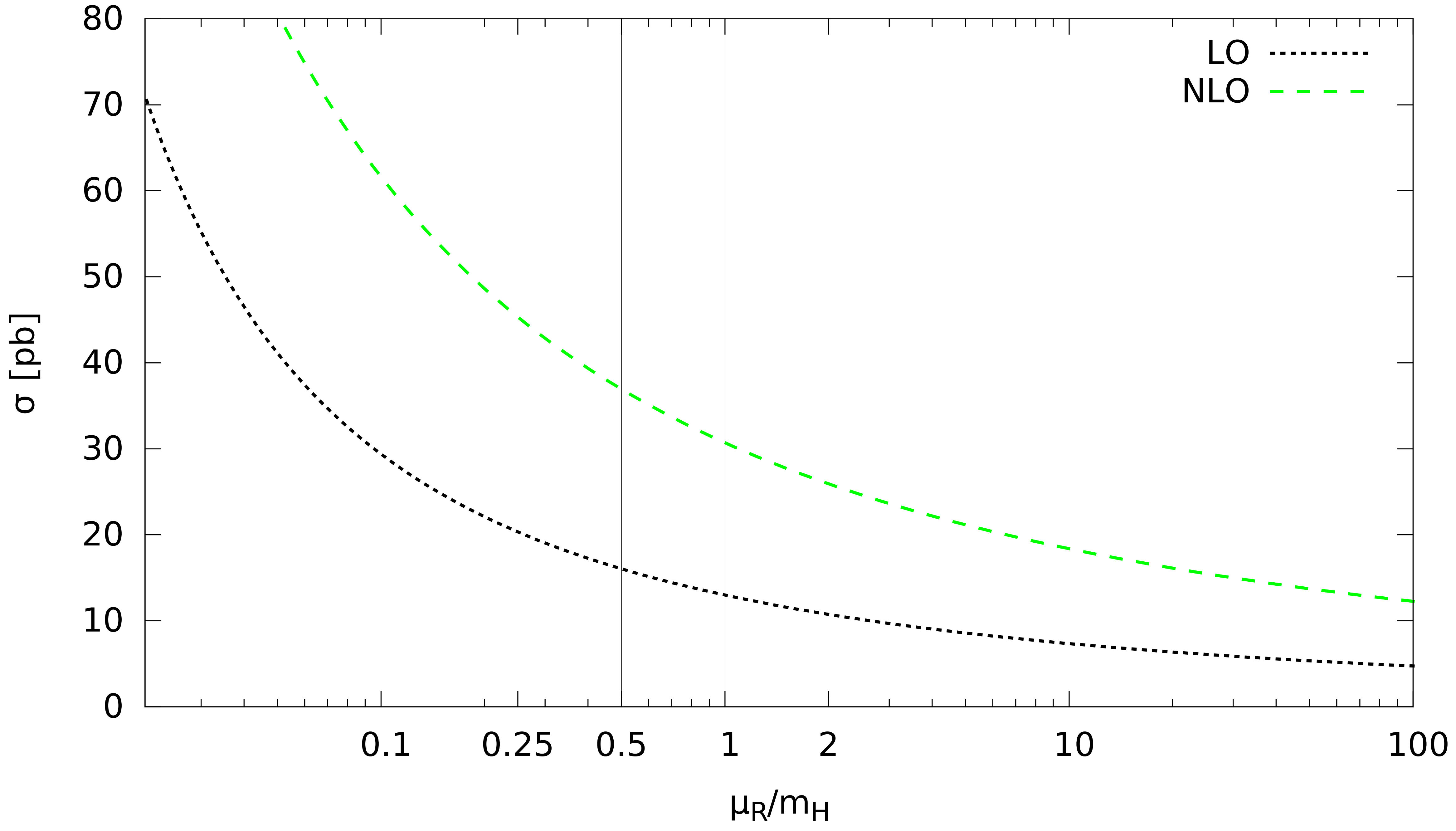
The curious case of the inclusive Higgs cross section

- Inclusive gluon fusion Higgs production cross section has a **very slowly convergent** perturbative expansion
- NLO and NNLO **large QCD corrections**
- **Missing higher order uncertainties** as large as PDF uncertainties

The curious case of the inclusive Higgs cross section



The curious case of the inclusive Higgs cross section



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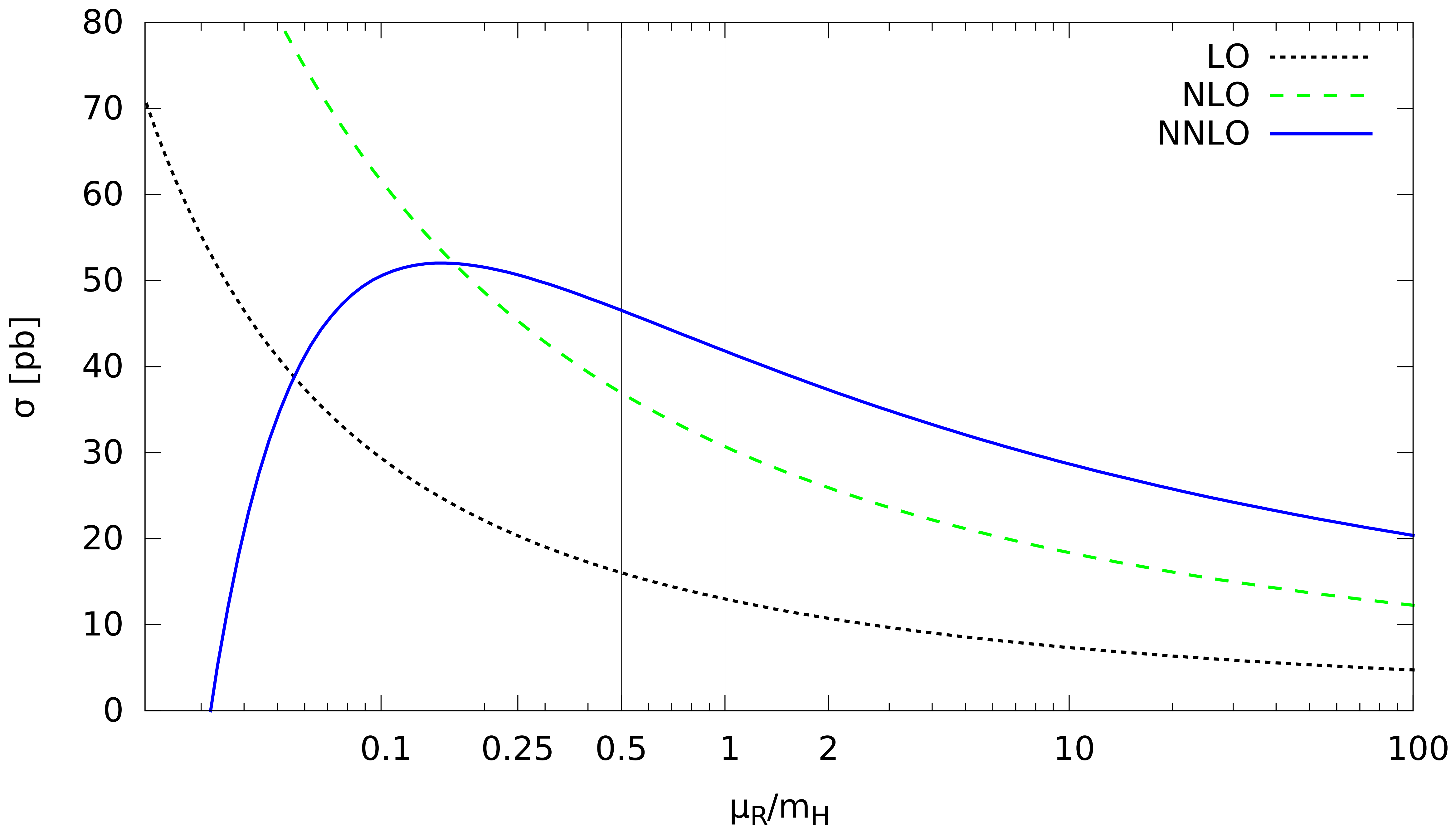
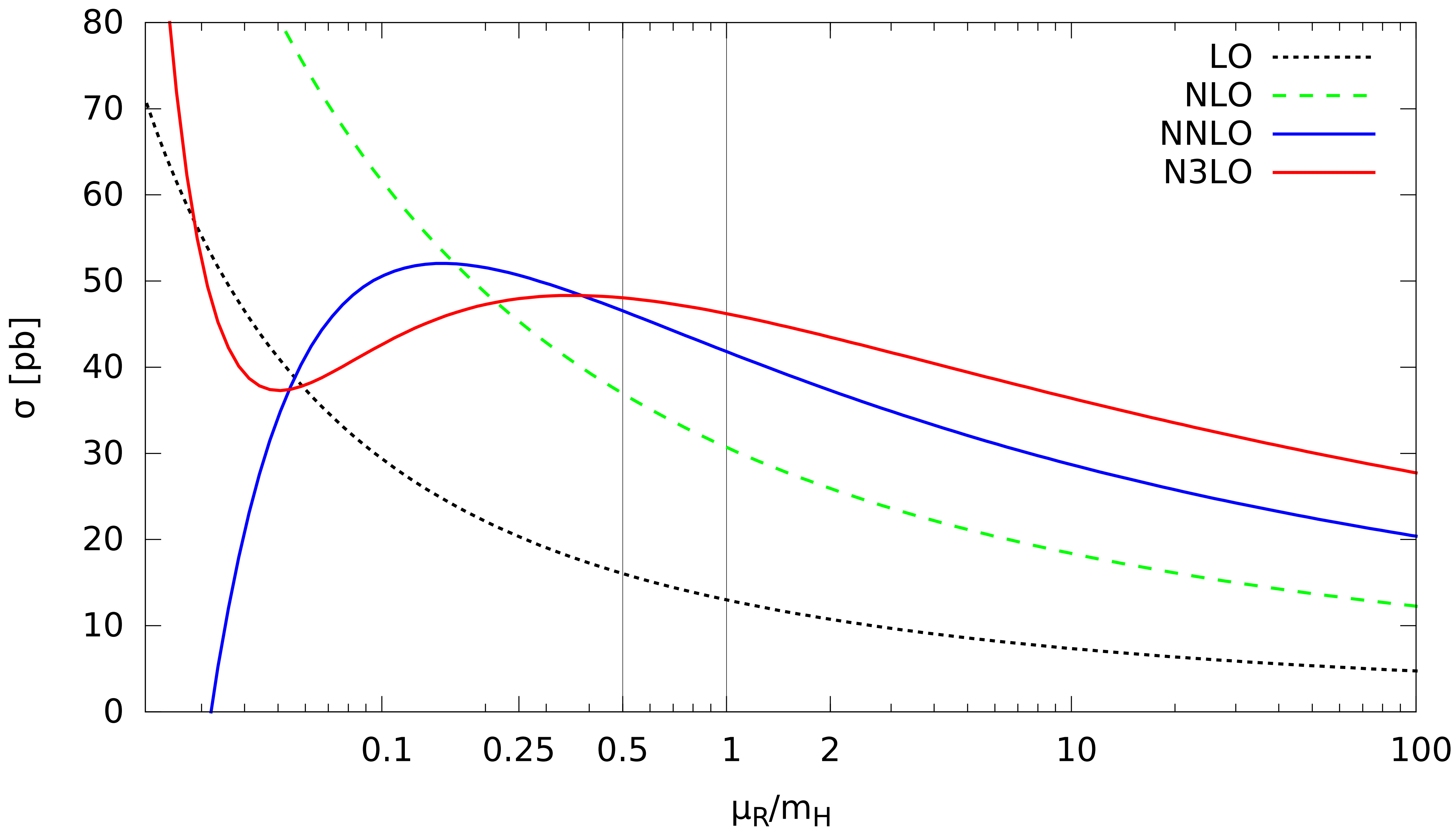


The curious case of the inclusive Higgs cross section



The Canon

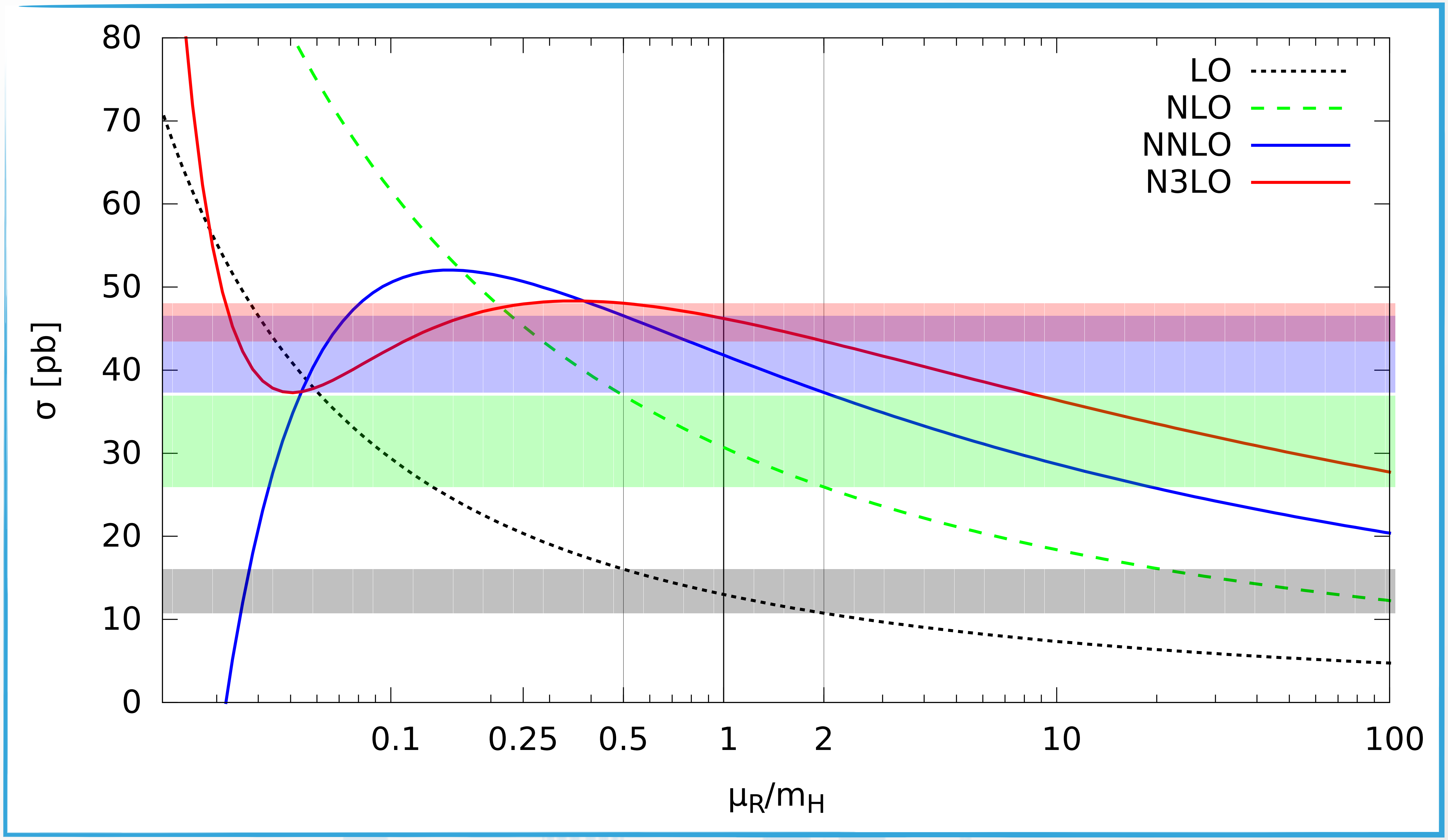
- Canonical way to probe higher order uncertainties: **scale variation**
- Theoretical computation of the inclusive Higgs cross section depends on scales (factorization and renormalization scales - more scales in a SCET approach)

$$\sigma(m_H) = \sum_{i,j} C_{ij}\left(\alpha_s(\mu_R), \frac{\mu_R}{m_H}, \frac{\mu_F}{m_H}\right) \otimes \mathcal{L}_{ij}(\mu_F)$$

- Scale dependence is higher order and vanishes to all orders
- **Arbitrary** choice of a central scale and variation of the scales

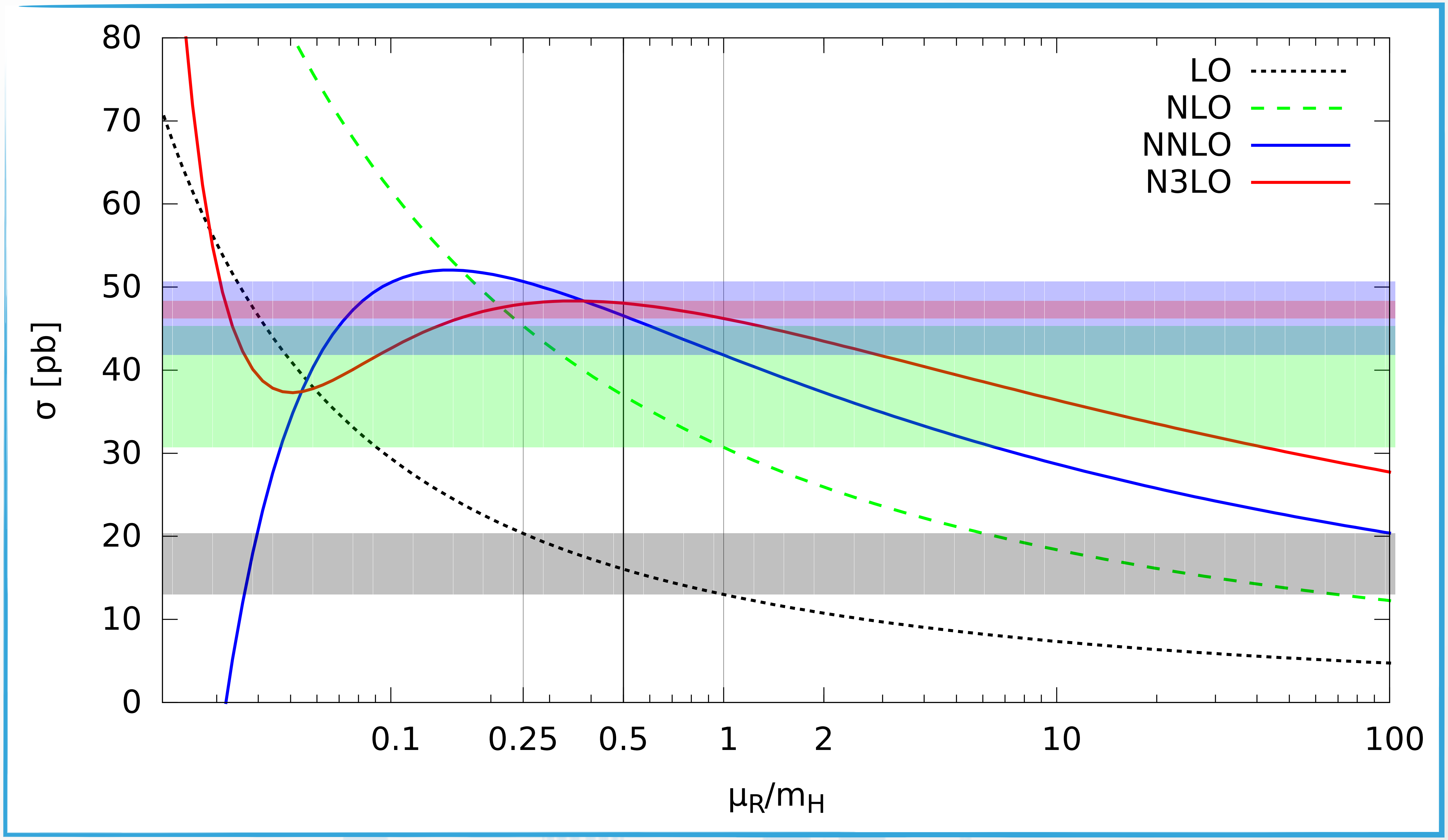
Advantages	Limitations
Simple procedure	Choice of central scale
Universal	Range of the variation
	Interpretation of the uncertainty
	Limited class of higher order terms being probed

The Canon for ggH



$$1/2 < \mu_R/m_H < 2$$

The Canon for ggH



$$1/4 < \mu_{\rm R}/m_H < 1$$

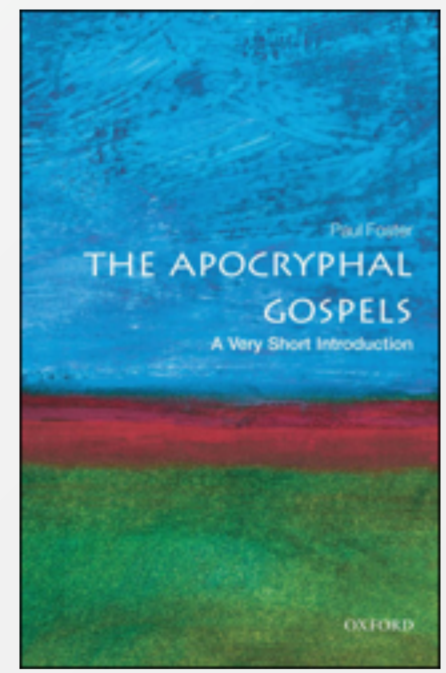
Apocrypha

Are there any **alternatives** to canonical scale variation?

1. 'Stretching' the canon

- **Symmetrization** of the uncertainties to avoid stationary points
- Modification of the range

2. Thinking outside the canon
   - **Rearranging** the perturbative expansion
     - **Resummation**
     - **Series acceleration**
   - Resort to a completely **different approach**



Threshold resummation

Reorganization of the perturbative expansion by performing an **all order** summation of classes of logs

Inclusive cross section: large logarithms of $1 - z$ $\qquad z = \frac{m_H^2}{\hat{s}}$

Tower of logarithms $\qquad \alpha_s^n \left(\frac{\ln^k(1-z)}{1-z} \right)_+ , \qquad 0 \leq k \leq 2n - 1$

Enhancement in the partonic soft limit $z \to 1$

Double logarithmic enhancement due to soft gluon emission

$$C(N) = 1 + \sum_{n=1}^{\infty} \alpha_s^n \sum_{k=0}^{2n} c_{nk} \ln^k N + \mathcal{O}(1/N)$$ Mellin space expression

Theme and Variations

$$C(N) = g_0(\alpha_s) \exp\left[\frac{1}{\alpha_s} g_1(\alpha_s \ln N) + g_2(\alpha_s \ln N) + \alpha_s g_3(\alpha_s \ln N) + \ldots\right]$$

Standard dQCD resummation

- **Subdominant** contribution not fixed by resummation → Freedom over $1/N$ terms
- Exponentiation of constants provides a handle on **subleading** terms

Improvements:

- Single gluon emission kinematics
- Collinear contributions from the full AP splitting function

Bonvini, Marzani 2014

Resummation is beneficial

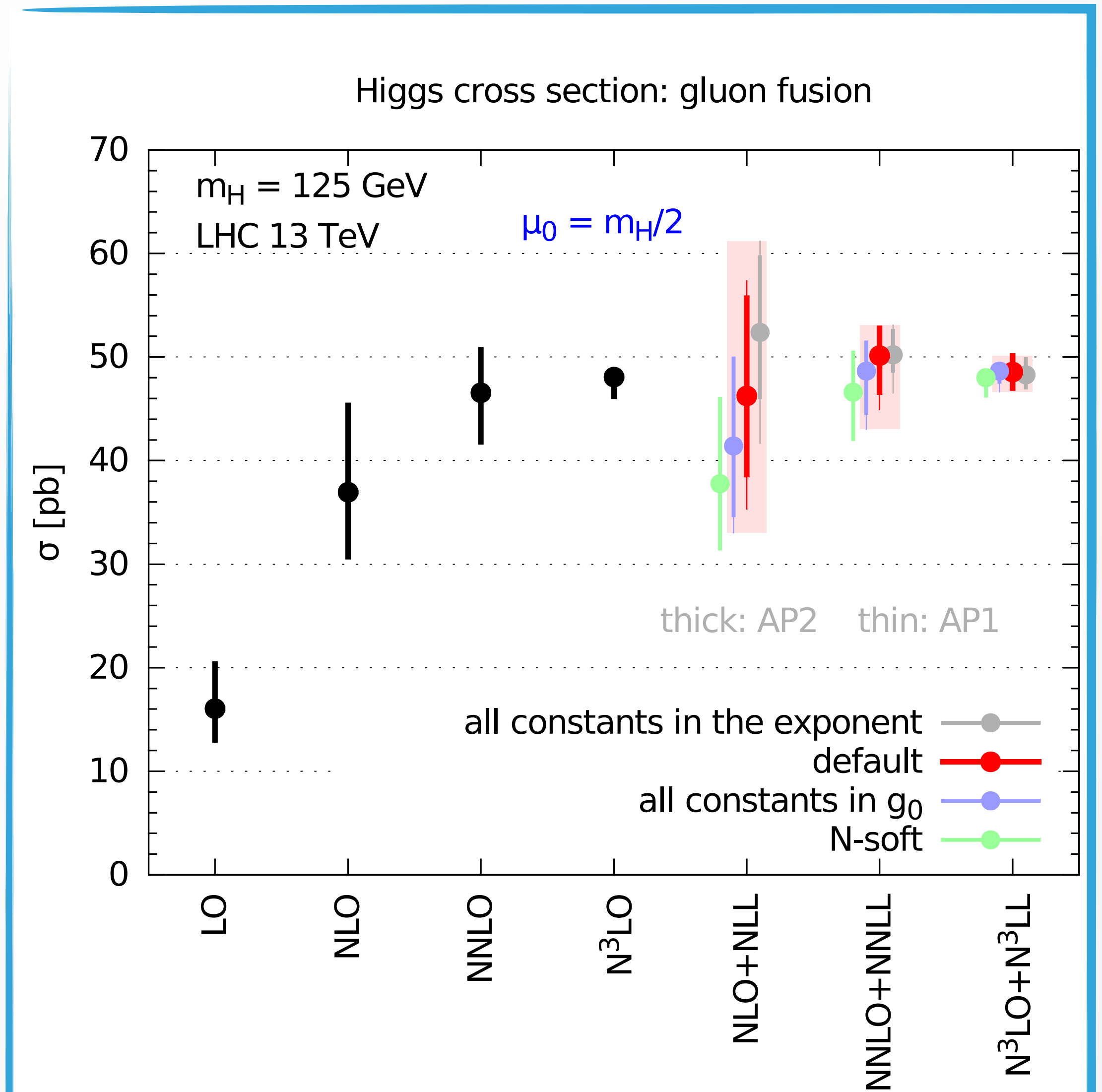
- Perturbative convergence improved
- Theory uncertainty from scale variations
- Theory uncertainty from **subleading** (all order) and **subdominant** (all order) contributions

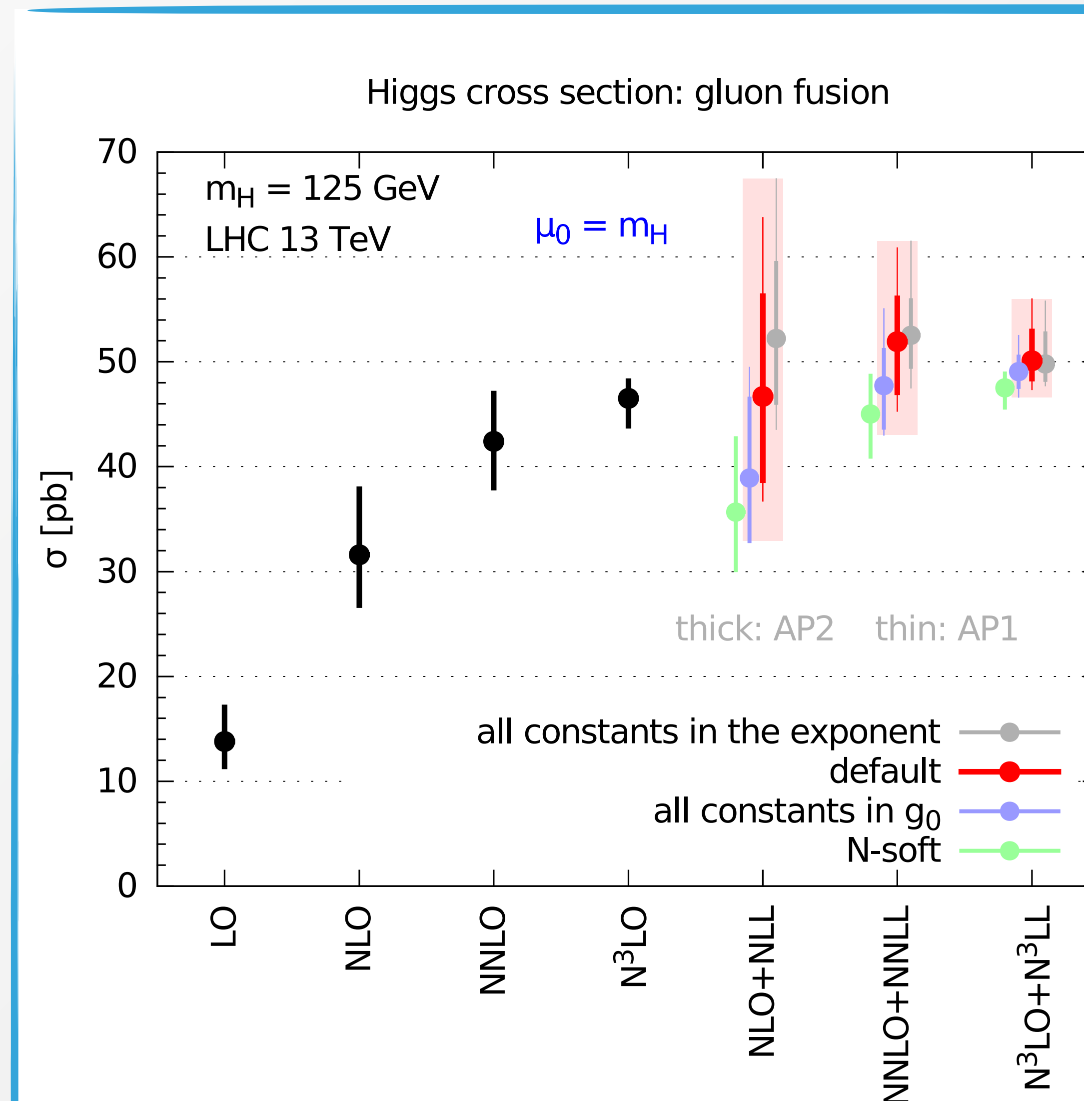
Perform all these variations together…

Higgs cross section results

Perform **42** variations and take the envelope

The answer to the ultimate question of life, the universe and everything!!!!!!!!!





Sequence transformations according to Weniger

Idea (Stirling, Euler): speed up convergence by applying a **transformation** to the sequence s_n

$$\lim_{n\to\infty} \frac{s'_n - s}{s_n - s} = 0$$

Very wide class of sequence transformation

$$\mathcal{G}_k^{(n)}(q_m, s_n, \omega_n) = \frac{\displaystyle\sum_{j=0}^{k} (-1)^j \binom{k}{j} \prod_{m=1}^{k-1} \frac{n+j+q_m}{n+k+q_m} \frac{s_{n+j}}{\omega_{n+j}}}{\displaystyle\sum_{j=0}^{k} (-1)^j \binom{k}{j} \prod_{m=1}^{k-1} \frac{n+j+q_m}{n+k+q_m} \frac{1}{\omega_{n+j}}}$$

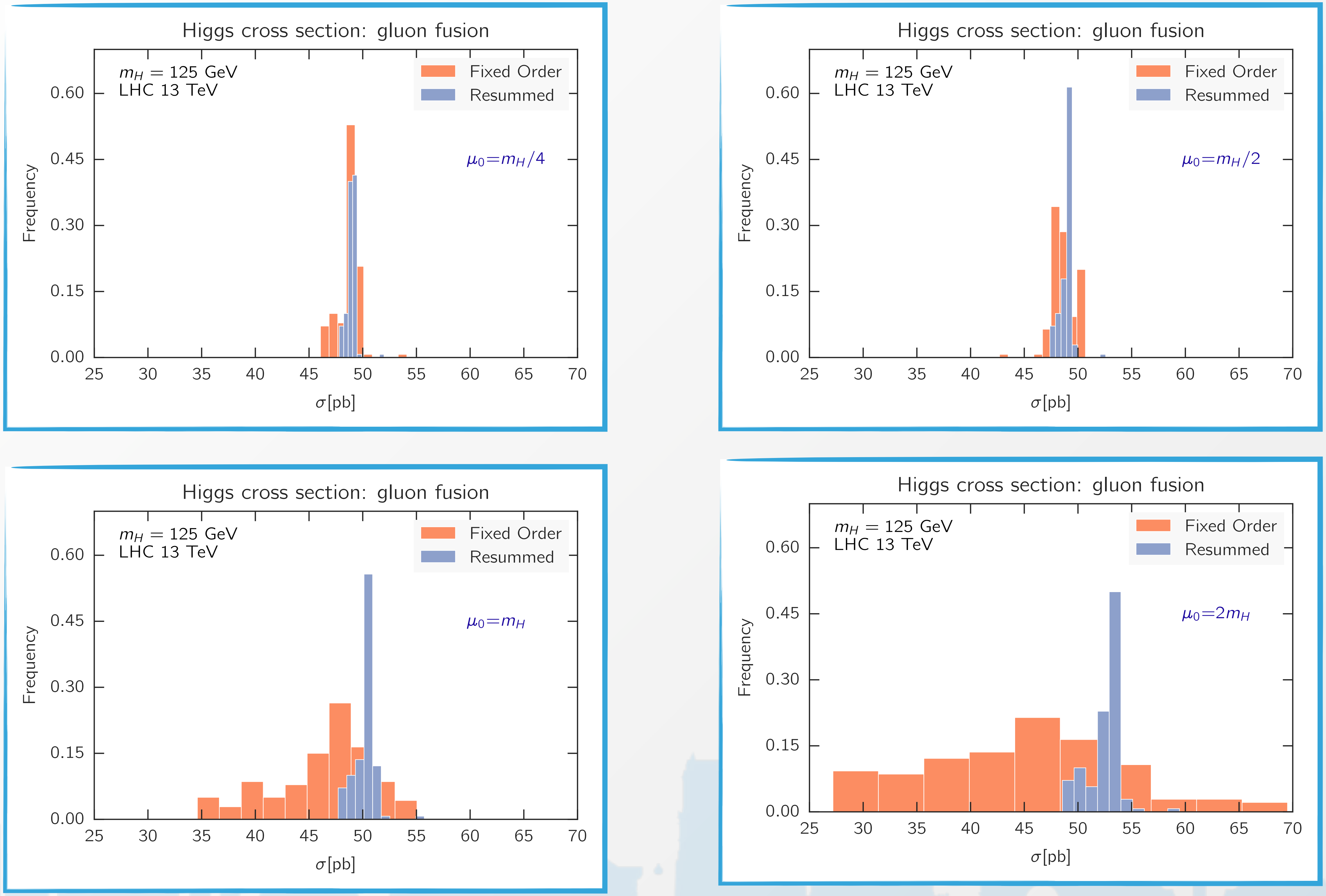
Application to the inclusive Higgs cross section

Choose some good algorithms and compute some guesses David, Passarino 2013

Choose **many** O(100) algorithms and compute **many** guesses Bonvini, Marzani, Muselli, LR 2016

- No information on the asymptotic behaviour of the series, so it is not clear how to prefer an algorithm rather than another
- Result **should not depend** on the scale

Higgs cross section results



The Cacciari-Houdeau approach

I believe that we do not know anything for certain, but everything probably (Christiaan Huygens)

Statistical model for the interpretation of theory errors, from which one can compute the uncertainty on the truncated perturbative series for a given degree of belief (DoB) given the first terms in the expansion. Cacciari, Houdeau (2011)

Probability density for σ

CH $$\sigma = \sigma_{\mathrm{LO}} \sum_{k=0}^{\infty} c_k(\lambda) \left(\frac{\alpha_s}{\lambda}\right)^k$$ **Possible power growth**

$\overline{\mathrm{CH}}$ $$\sigma = \sigma_{\mathrm{LO}} \sum_{k=0}^{\infty} b_k(\lambda, k_0)(k + k_0)! \left(\frac{\alpha_s}{\lambda}\right)^k$$ **Possible factorial growth**

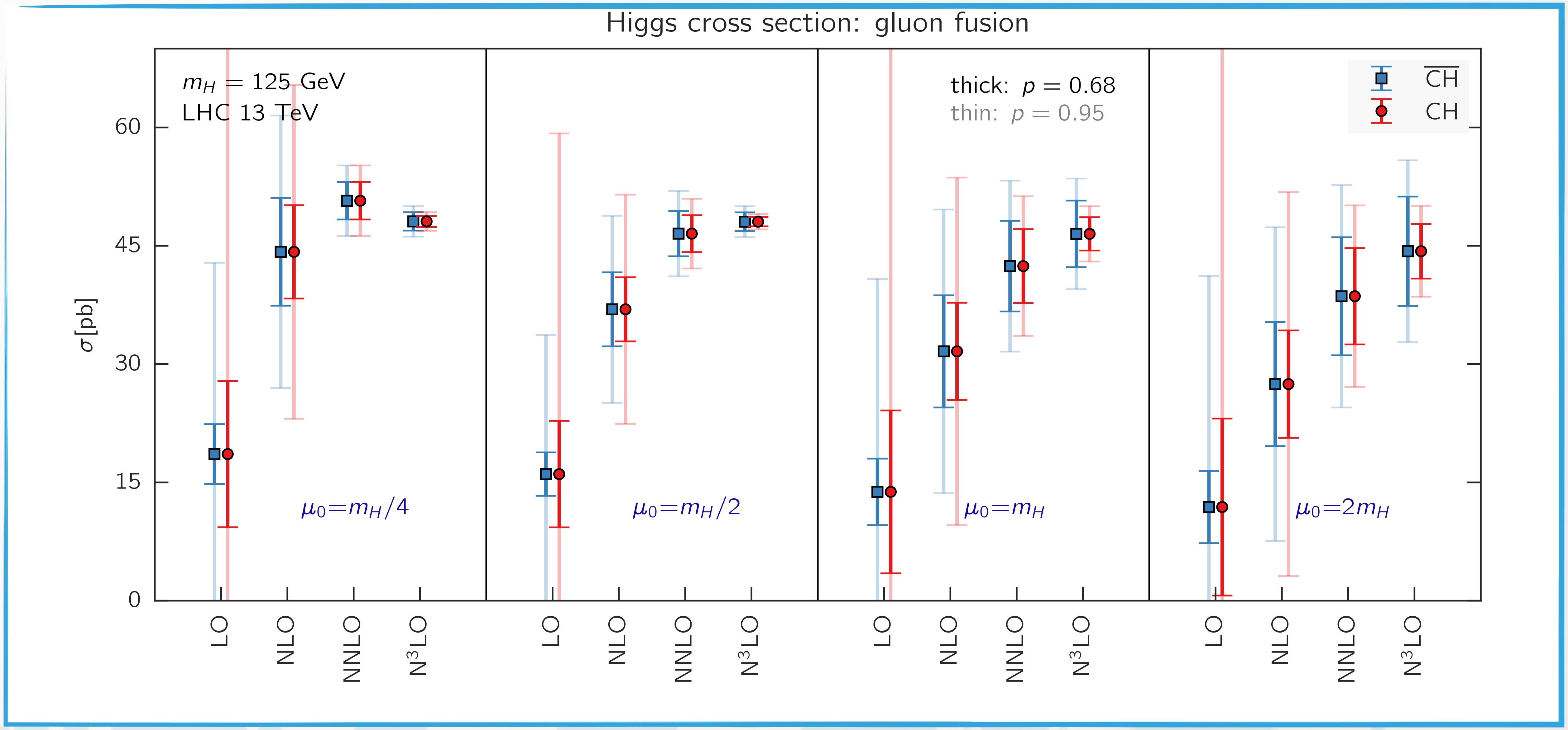
Bagnaschi, Cacciari, Guffanti, Jenniches (2014)

Determination of λ

- Survey over several observables (assumes λ is process-independent)
  Bagnaschi, Cacciari, Guffanti, Jenniches (2014)
- fit λ requiring the first known coefficients are of the same size
  Forte, Isgrò, Vita (2013)

Higgs cross section results

Pascal bet on the existence of God basing on calculation of probabilities, we use calculation of probabilities to bet on the value of the cross section of the ~~God's particle~~ Higgs



All Things Considered

Scale Variations

	$\mu_0 = m_H/4$	$\mu_0 = m_H/2$	$\mu_0 = m_H$	$\mu_0 = 2m_H$
LO	$18.6^{+5.8}_{-3.9}$	$16.0^{+4.3}_{-3.1}$	$13.8^{+3.2}_{-2.4}$	$11.9^{+2.5}_{-1.9}$
NLO	$44.2^{+12.0}_{-8.5}$	$36.9^{+8.4}_{-6.2}$	$31.6^{+6.3}_{-4.8}$	$27.5^{+4.9}_{-3.9}$
NNLO	$50.7^{+3.4}_{-4.6}$	$46.5^{+4.2}_{-4.7}$	$42.4^{+4.6}_{-4.4}$	$38.6^{+4.4}_{-4.0}$
N^3LO	$48.1^{+0.0}_{-7.5}$	$48.1^{+0.1}_{-1.8}$	$46.5^{+1.6}_{-2.6}$	$44.3^{+2.5}_{-2.9}$

The answer to the ultimate question of life, the universe and everything!!!

	$\mu_0 = m_H/4$	$\mu_0 = m_H/2$	$\mu_0 = m_H$	$\mu_0 = 2m_H$
LO+LL	$24.0^{+8.9}_{-6.8}$	$20.1^{+6.2}_{-5.0}$	$16.9^{+4.5}_{-3.7}$	$14.3^{+3.3}_{-2.8}$
NLO+NLL	$46.9^{+15.1}_{-12.6}$	$46.2^{+15.0}_{-13.2}$	$46.7^{+20.8}_{-13.8}$	$47.3^{+26.1}_{-15.8}$
NNLO+NNLL	$50.2^{+5.5}_{-5.3}$	$50.1^{+3.0}_{-7.1}$	$51.9^{+9.6}_{-8.9}$	$54.9^{+17.6}_{-11.5}$
N^3LO+N^3LL	$47.7^{+1.0}_{-6.8}$	$48.5^{+1.5}_{-1.9}$	$50.1^{+5.9}_{-3.5}$	$52.9^{+13.1}_{-5.3}$

Acceleration

	$\mu_0 = m_H/4$	$\mu_0 = m_H/2$	$\mu_0 = m_H$	$\mu_0 = 2m_H$
Fixed-order expansion	48.7 ± 1.0	48.7 ± 1.2	46.3 ± 4.6	44.6 ± 9.3
Resummed expansion	48.9 ± 0.5	48.9 ± 0.6	50.2 ± 1.0	52.6 ± 1.6

Cacciari-Hodeau

	$\mu_0 = m_H/4$	$\mu_0 = m_H/2$	$\mu_0 = m_H$	$\mu_0 = 2m_H$
CH	$48.1 \pm 0.7(1.2)$	$48.1 \pm 0.6(1.0)$	$46.5 \pm 2.1(3.5)$	$44.3 \pm 3.5(5.8)$
$\overline{CH}$	$48.1 \pm 1.2(1.9)$	$48.1 \pm 1.2(2.0)$	$46.5 \pm 4.2(7.0)$	$44.3 \pm 6.9(11.5)$

The Quest for the Holy Scale

The series has better convergence at $m_H/2$

Do we understand **why**?

Is there a way to choose the **optimal** scale?

Caveat: factorization and renormalization scales are not physical scales

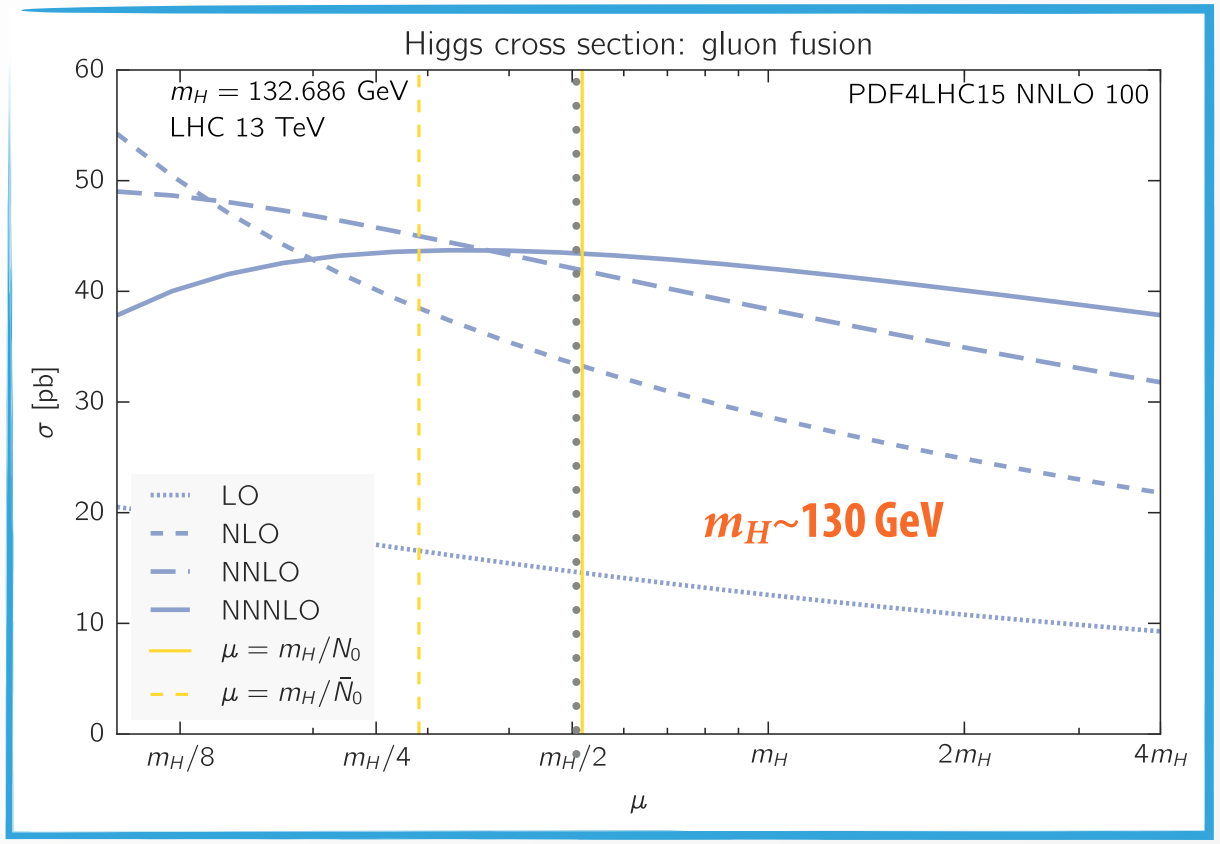
Central scale chosen such that there are not large logs in the coefficient function

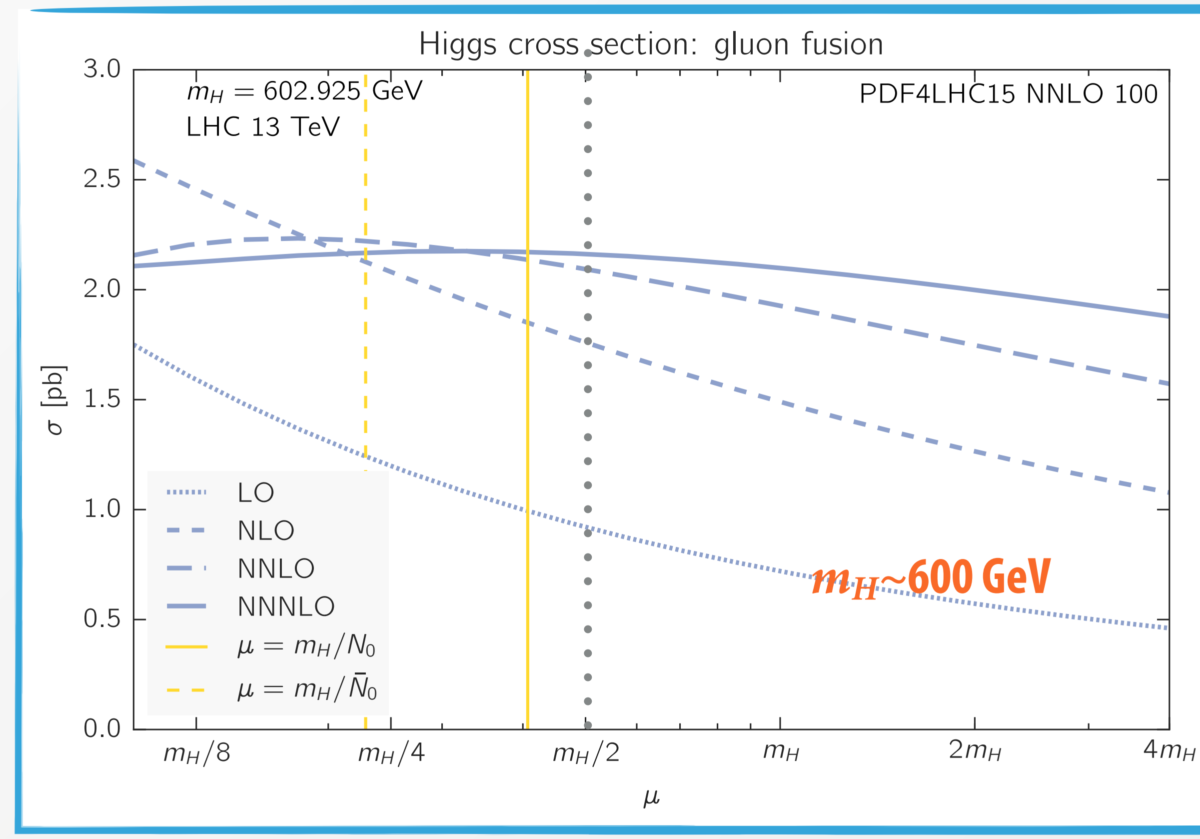
$$\ln \frac{\mu_\mathrm{F}}{m_H} \quad \ln \frac{\mu_\mathrm{R}}{m_H} \quad \text{but also} \quad \ln \frac{\mu_\mathrm{F} N}{m_H} \quad \ln \frac{\mu_\mathrm{R} N}{m_H}$$

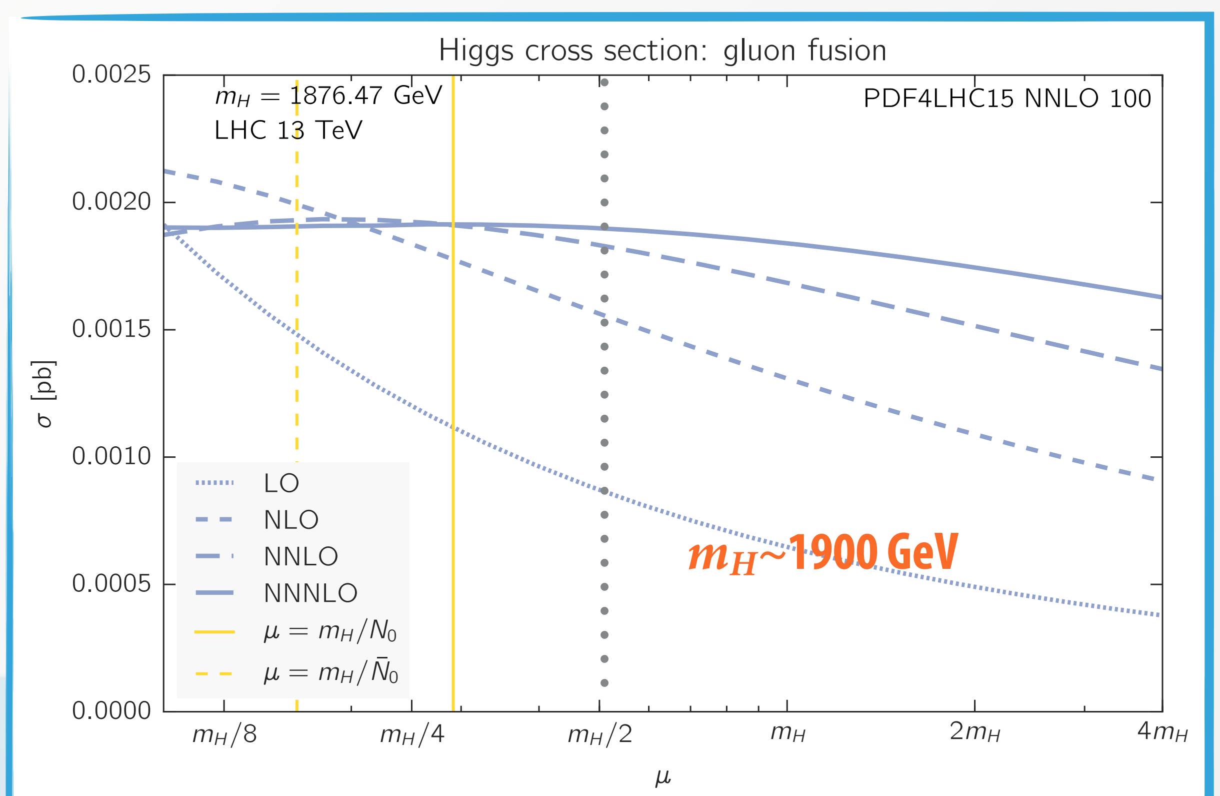
Could the best scale be the **soft scale**?

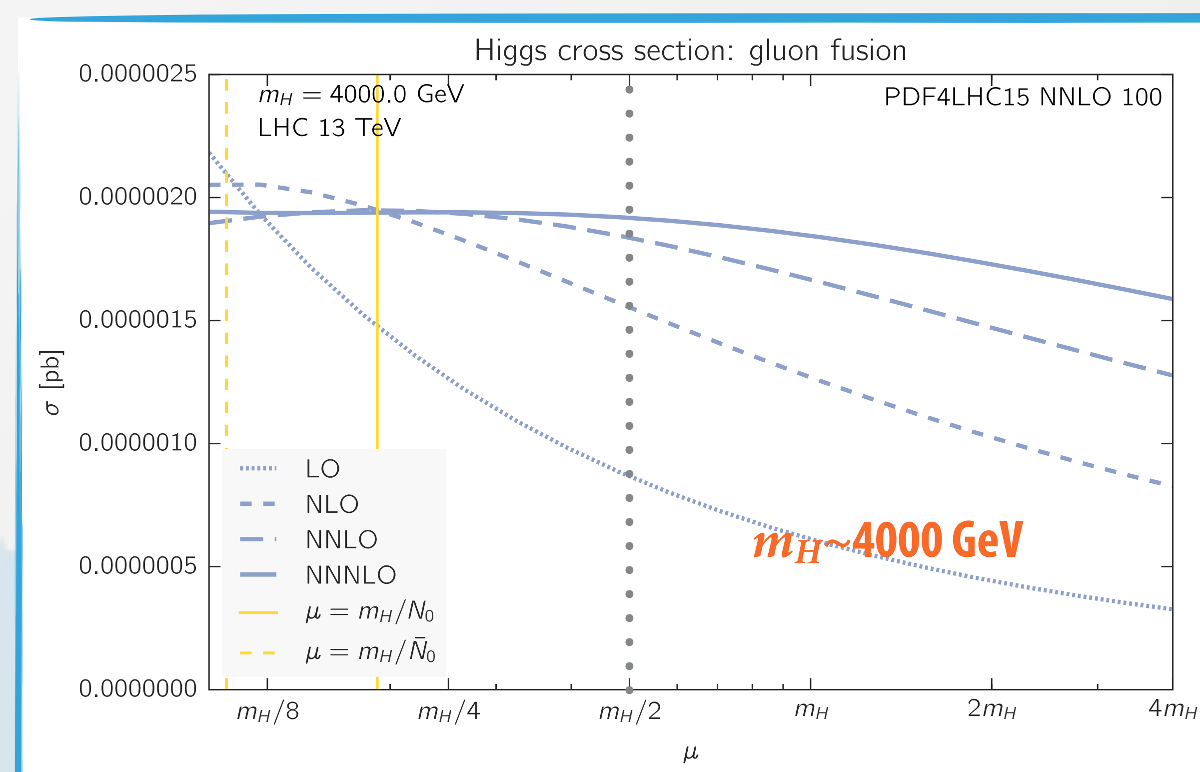
Saddle-point analysis: determination of $N_{\mathrm{saddle}} \sim 2$ for $m_H \sim 125$ GeV   Bonvini, Forte, Ridolfi (2012)

The soft spot









Summary

- Thorough understanding of theory errors mandatory at the LHC
- Canonical scale variation often does not guarantee a reliable estimate of the uncertainty from missing higher orders
- Reverting to a different expansion is promising since it goes beyond scale dependent terms
- CH Bayesian approach gives a statistical interpretation of the theory errors so far missing in other approaches
- All scales are equal, but some scales are more equal than others

The aim of science is not to open a door to infinite wisdom,
but to set a limit to infinite error
(Brecht)

Back-up

Theme and variations

$$C(N,\alpha_s) = \bar{g}_0(\alpha_s)\exp\overline{\mathcal{S}}(\alpha_s,N)$$

$$\overline{\mathcal{S}}(\alpha_s,N) = \int_0^1 dz \frac{z^{N-1}-1}{1-z}\left(\int_{m_H^2}^{m_H^2\frac{(1-z)^2}{z}} \frac{d\mu^2}{\mu^2} 2A\left(\alpha_s(\mu^2)\right) + D\left(\alpha_s([1-z]^2 m_H^2)\right)\right)$$

Computation of integrals in the large-N limit: N-soft

ψ-soft: variant of the N-soft resummation $\ln N \to \psi_0(N)$

Reproduces $\overline{\mathcal{S}}$ up to $O(1/N^2)$

Collinear improvement: class of next-to-soft terms, through inclusion of more terms in the soft expansion of P_{gg}

AP1 $\quad \overline{\mathcal{S}}(\alpha_s,N) \to \overline{\mathcal{S}}(\alpha_s,N+1)$

AP2 $\quad \overline{\mathcal{S}}(\alpha_s,N) \to 2\overline{\mathcal{S}}(\alpha_s,N) - 3\overline{\mathcal{S}}(\alpha_s,N+1) + 2\overline{\mathcal{S}}(\alpha_s,N+2)$

Bonvini, Marzani 2014

Theme and variations

μ_F/m_H	μ_R/m_H	ψ-soft default AP2		ψ-soft constants in exp AP2		ψ-soft constants in g_0 AP2		N-soft
		AP2	AP1	AP2	AP1	AP2	AP1	
4	4	56.8	66.0	56.8	66.0	51.2	58.7	49.4
4	2	55.1	62.3	54.9	62.0	52.2	58.6	50.5
2	4	53.2	57.2	53.7	57.9	48.2	51.4	46.0
2	2	52.9	56.0	52.7	55.8	49.9	52.5	47.9
2	1	51.2	53.0	50.9	52.6	50.5	52.1	48.9
1	2	50.2	50.4	50.6	50.9	47.6	47.7	45.6
1	1	50.1	50.1	49.8	49.8	49.1	49.0	47.5
1	1/2	48.5	48.3	48.3	48.0	49.1	48.8	48.3
1/2	1	48.4	47.4	48.8	47.7	47.6	46.6	46.3
1/2	1/2	48.5	48.0	48.3	47.8	48.6	48.1	48.0
1/2	1/4	47.0	47.1	47.1	47.2	47.7	47.7	47.9
1/4	1/2	47.8	47.4	48.2	47.7	48.0	47.6	47.6
1/4	1/4	47.7	48.0	47.6	47.9	48.0	48.2	48.2
1/4	1/8	44.7	45.1	45.4	45.7	44.6	45.0	44.9
1/8	1/4	45.5	46.1	46.1	46.6	46.2	46.6	46.5
1/8	1/8	41.0	40.9	41.4	41.2	40.9	40.8	40.9

The Cacciari-Houdeau approach

$$\sigma = \sum_n c_n \alpha_s^n$$

"We make the assumption that all the coefficients c_n in a perturbative series share some sort of upper bound $\bar{c} > 0$ to their absolute values, specific to the physical process studied. The calculated coefficients will give an estimate of this $\bar{c}$, restricting the possible values for the unknown c_n. "

Cacciari, Houdeau (2011)

λ: ensures that $\bar{c}$ exists

Assumption that all c_n are bounded broken by presence of factorial growths (**renormalons**)

$$\sigma = \sigma_{\mathrm{LO}} \sum_{k=0}^{\infty} b_k(\lambda, k_0)(k + k_0)! \left(\frac{\alpha_s}{\lambda}\right)^k$$

Weak processes starting at order $\alpha_s{}^{0}$: renormalon factorial growth behaves as $\alpha_s{}^k(k-1)!$

Drell-Yan: gluon appears first at NLO, correction to the gluon propagator at NNLO, $k_0 = -1$

Higgs: gluon appears at LO, correction to the gluon propagator at NLO, $k_0 = 0$