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## The atmospheric prompt neutrino flux revisited

## Luca Rottoli

Rudolf Peierls Centre for Theoretical Physics, University of Oxford


Based on: arXiv 1506.08025, R. Gauld, J. Rojo, LR, J. Talbert
arXiv 1511.06346 , R. Gauld, J. Rojo, LR, S. Sarkar, J. Talbert

## Prompt vs. conventional flux

The energy spectrum from semi-leptonic decay products depends on a hadronic critical energy, below which the decay probability is > interaction probability


For pions \& kaons, this critical energy is low (decay length is long) hence the leptonic energy spectrum is soft. For charmed mesons, the critical energy is high: they decay promptly to highly energetic leptons

Courtesy: Anne Schukraft
The atmospheric neutrino flux from the decay of pions \& kaons is the conventional flux, whereas that from charm decay is called the prompt flux

## Where are the prompt neutrinos?

The flux of prompt neutrinos is harder than that of conventional neutrinos, and was predicted to dominate the total atmospheric flux at energies above $\sim 105-6 \mathrm{GeV}$


No prompt flux seen so far, but an astrophysical signal with similar spectrum has been discovered Astrophysical neutrinos


Recent data put an upper limit on the prompt flux above 1 TeV , which is less than
$\sim 1.5 \mathrm{x}$ the benchmark ERS 2008 calculation arXiv 0806.0418

Even stronger limit of 0.54×ERS @ 90\% C.L. from combined IC59 + IC79 + IC86 data

## Cascade Formalism

1. $\frac{d \phi_{p}}{d X}=-\frac{\phi_{p}}{\lambda_{p}}+Z_{p p} \frac{\phi_{p}}{\lambda_{p}}$
2. $\frac{d \phi_{h}}{d X}=-\frac{\phi_{h}}{\rho d_{h}(E)}-\frac{\phi_{h}}{\lambda_{h}}+Z_{h h} \frac{\phi_{h}}{\lambda_{h}}+Z_{p h} \frac{\phi_{p}}{\lambda_{p}}$
3. $\frac{d \phi_{l}}{d X}=\sum_{h} Z_{h \rightarrow l} \frac{\phi_{h}}{\rho d_{h}}$

Asymptotic solutions

$$
\pm 1
$$

$$
\left.\phi_{l}\right|_{\text {low }}=\phi_{p}(E) Z_{h \rightarrow l}^{\text {low }} \frac{Z_{p h}}{\left(1-Z_{p p}\right)}
$$

$$
\left.\phi_{l}\right|_{\text {high }}=\frac{Z_{h \rightarrow l} \epsilon_{h}}{E} \frac{Z_{p h} \phi_{p}(E)}{\left(1-Z_{p p}\right)\left(1-\frac{\Lambda_{p}}{\Lambda_{h}}\right)} \ln \frac{\Lambda_{h}}{\Lambda_{p}}
$$

Geometric Interpolation

$$
\phi_{l}=\sum_{h} \frac{\phi_{l}^{l o w} \phi_{l}^{h i g h}}{\phi_{l}^{l o w}+\phi_{l}^{h i g h}}
$$

Our final flux includes all (interpolated) contributions from charmed hadrons

Full series of cascade equations, from incoming cosmic ray nucleons to final state leptons

## Cascade Formalism: Z-moments

## For particle production:

$$
Z_{k h}=\int_{E}^{\infty} d E^{\prime} \frac{\phi_{k}\left(E^{\prime}, X, \theta\right)}{\phi_{k}(E, X, \theta)} \frac{\lambda_{k}(E)}{\lambda_{k}\left(E^{\prime}\right)} \frac{d n\left(k A \rightarrow h Y ; E^{\prime}, E\right)}{d E} \quad \frac{d n\left(p A \rightarrow h Y ; E^{\prime}, E\right)}{d E}=\frac{1}{\sigma_{p A}\left(E^{\prime}\right)} \frac{d \sigma\left(p A \rightarrow h Y ; E^{\prime}, E\right)}{d E}
$$

For particle decay:

$$
Z_{h \rightarrow l}=\int_{E}^{\infty} d E^{\prime} \frac{\phi_{h}\left(E^{\prime}, X\right)}{\phi_{h}(E, X)} \frac{d_{h}(E)}{d_{h}\left(E^{\prime}\right)} \frac{d n\left(h \rightarrow l Y ; E^{\prime}, E\right)}{d E} \quad \frac{d n\left(h \rightarrow l Y ; E^{\prime}, E\right)}{d E}=\frac{1}{\Gamma} \frac{d \Gamma}{d E}
$$

Calculating the prompt flux of atmospheric neutrinos requires a synthesis of QCD, atmospheric physics, and neutrino physics

## Incident Cosmic Ray Fluxes: $\phi_{N}^{0}(E)$

Cosmic ray spectrum constrained $\sim$ up to $10^{5} \mathrm{GeV}$ by balloon and space experiments, e.g. AMS and CREAM Higher energies rely on air shower arrays, e.g. Kascade, Auger \& TA. . . many uncertainties regarding CR composition

## Broken-Power-Law (BPL)

Gaisser et al. fluxes:
arXiv:astro-ph/1111.6675
arXiv:astro-ph/1303.3565
The effect of the new
parametrizations is significant above $\sim 10^{6} \mathrm{GeV}$, and we are interested in making predictions up to $\sim 10^{8} \mathrm{GeV}$...

$$
\phi_{i}(E)=\Sigma_{j=1}^{3} a_{i, j} E^{-\gamma_{i, j}} \times \exp \left[-\frac{E}{Z_{i} R_{c, j}}\right]
$$



## The QCD input: $Z_{p h}$

$$
Z_{p h}=\int_{E}^{\infty} d E^{\prime} \frac{\phi_{p}\left(E^{\prime}\right)}{\phi_{p}(E)} \frac{A}{\sigma_{p A}(E)} \frac{d \sigma\left(p p \rightarrow c \bar{c} Y ; E^{\prime}, E\right)}{d E}
$$

- The differential cross-section can be calculated in a variety of formalisms, e.g. the colour dipole model of ERS which is empirical (hard to estimate uncertainties)
- However, there is no evidence that perturbative QCD (with DGLAP evolution) cannot describe charm production data for the entire kinematic region of interest, hence our calculation is performed with NLO+PS Monte-Carlo event generators
- Boosting from CM to the rest frame of the (atmospheric) fixed target, one finds:

$$
\sqrt{s}=7[\mathrm{TeV}] \longleftrightarrow E_{b}=2.6 \times 10^{7}[\mathrm{GeV}]
$$

- Thus there is complementarity with LHC physics. We will predict the prompt neutrino flux at energies up to $10^{8} \mathbf{G e V} \ldots$ at these energies, the charm production cross section is dominated by gluon fusion, hence we are sensitive to the behaviour of the gluon PDF (parton distribution function) at small- $x$


## Gluon PDF Sensitivities



$\mathrm{xg}(\mathrm{x}, \mathrm{Q})$, comparison

$\mathrm{xg}(\mathrm{x}, \mathrm{Q})$, comparison


## Small-x Gluon NNPDF: LHCb constraints

- We utilize charm production data from LHCb to reduce the uncertainties in the small- $x$ gluon PDF
- Similar strategy as the one used by the PROSA collaboration in the HERAfitter framework arXiv: 1503.04581
- By using a Bayesian re-weighting technique, the impact of the new data is estimated. 75 data points added to NNPDF3.0 analysis
- The impact is negligible for $x>10^{-4}$, but substantive in the small- $x$ region where data was previously unavailable. At $x \sim 10^{-5}$, we achieve a $3 x$ reduction in uncertainty
- We utilize these improved PDFs to make predictions for 13 TeV physics

NNPDF3.0 NLO $\alpha_{s}=0.118$



Due to the improved NNPDF3.0+LHCb, the PDF errors are moderate even @ 13 TeV

arXiv.org 1510.01707

## $\mathrm{Z}_{\mathrm{ph}}$ with NNPDF3.0+LHCb

$$
Z_{p h}=\int_{E}^{\infty} d E^{\prime} \frac{\phi_{p}\left(E^{\prime}\right)}{\phi_{p}(E)} \frac{A}{\sigma_{p A}(E)} \frac{d \sigma\left(p p \rightarrow c \bar{c} Y ; E^{\prime}, E\right)}{d E}
$$

The differential cross-section is generated at various E' between $10^{3}$ and $10^{10} \mathrm{GeV}$ with POWHEG+PYTHIA8, and incorporates our updated NNPDF3.0+LHCb ... Cross-checks made with aMC@NLO
We perform an interpolation over $E_{\text {inc }}$ and $E_{h}$.


## Benchmark NNPDF3.0+LHCb flux

We present the following predictions for prompt atmospheric neutrino flux adopting the broken power-law (BPL) as well as H3A and H3P cosmic-ray spectra



Scale, PDF, and charm mass uncertainty
Different cosmic ray spectrum parameterisations
$\Rightarrow$ significant differences in the expected flux above $\sim 10^{6} \mathrm{GeV}$

## Consistency with IceCube bounds



## Consistency with previous calculations

Prompt Neutrino Flux (BPL)


## Input PDF dependency

Prompt Neutrino Flux (BPL)


## Response from the astrophysics community



KM3nET Letter of Intent
arxiv.org/1601.07459

## Conclusions

We have presented updated predictions for the flux of prompt atmospheric neutrinos at ground-based detectors.

Our approach is grounded in perturbative QCD, and incorporates:

1. State-of-the-art calculation of charmed hadron production in the forward region, validated against recent LHCb measurements
2. A small-x gluon PDF which is also constrained by LHCb data

Our estimates are consistent with previous studies but provide a more reliable estimate of uncertainties and alleviate the tension between the previous benchmark (ERS) calculation and IceCube data

The prompt flux should be seen soon (and provide a probe of low-x QCD)

## Back-up

## Previous calculations

- Volkova, Sov. J. Nucl. Physics 12 (1980) 784
- Bugaev, Naumov, Sinegovksy, Zaslavskaya, II Nuovo Cimento C 12 (1989) 41
- Lipari, Astroparticle Physics 1 (1993) 195
- Thunman, Ingelman, Gondolo (TIG), Astroparticle Physics 5 (1993) 309
- Pasquali, Reno, Sarcevic (PRS), Physical Review D59 (1999) 034020
- Gelmini, Gondolo, Varieschi (GGV1), Physical Review D61 (2000) 036005
- Gelmini, Gondolo, Varieschi (GGV2), Physical Review D61 (2000) 056011
- Martin, Ryskin, Stasto (MRS), Acta Physica Polonica B34 (2003) 3273
- Enberg, Reno, Sarcevic (ERS), Physical Review D78 (2008) 043005
- Bhattacharya, Enberg, Reno, Sarcevic, Stasto (BERSS), JHEP 1506 (2015) 110
- Garzelli, Moch, Sigl (GMS), JHEP 1510 (2015) 115

Calculating the prompt flux of atmospheric neutrinos requires a synthesis of QCD, atmospheric physics, and neutrino physics

## Prompt vs. conventional flux

The energy spectrum from semi-leptonic decay products depends on a hadronic 'critical energy', below which the decay probability is > interaction probability:

$$
\begin{array}{rlrl}
\epsilon_{h} & =\frac{m_{h} c^{2} h_{0}}{c \tau_{h} \cos \theta} & \epsilon_{\pi^{ \pm}} & =115[\mathrm{GeV}] \\
\epsilon_{K^{ \pm}} & =850[\mathrm{GeV}]
\end{array}
$$

For pions \& kaons, this critical energy is low (decay length is long) hence the leptonic energy spectrum is soft. For charmed mesons, the critical energy is high . . . they decay promptly to highly energetic leptons

$$
\begin{aligned}
\epsilon_{D^{0}} & =9.71 \times 10^{7}[\mathrm{GeV}] \\
\epsilon_{D^{ \pm}} & =3.84 \times 10^{7}[\mathrm{GeV}] \\
\epsilon_{D_{s}^{ \pm}} & =8.40 \times 10^{7}[\mathrm{GeV}] \\
\epsilon_{\Lambda_{c}} & =24.4 \times 10^{7}[\mathrm{GeV}]
\end{aligned}
$$

The atmospheric neutrino flux from the decay of pions \& kaons is the conventional flux, whereas that from charm decay is called the prompt flux

## Tracing a particle through the atmosphere

The flux of particle $j$ can be generically written as:

$$
\frac{d \phi_{j}}{d X}=-\frac{\phi_{j}}{\lambda_{j}}-\frac{\phi_{j}}{\lambda_{j}^{d e c}}+\sum S(k \rightarrow j)
$$

This depends on the slant depth $X$ measuring the atmosphere traversed:

$$
X(l, \theta)=\int_{l}^{\infty} \rho\left(H\left(l^{\prime}, \theta\right) d l^{\prime} \quad H(l, \theta) \simeq l \cos \theta+\frac{l^{2}}{2 R_{0}} \sin ^{2} \theta\right.
$$

We adopt a simple isothermal model of the atmosphere:

$$
\begin{array}{ll}
\rho(H)=\rho_{0} e^{-\frac{H}{H_{0}}} & \rho_{0}=2.03 \times 10^{-3}\left[\frac{g}{c m^{3}}\right] \\
H_{0}=6.4[\mathrm{~km}]
\end{array}
$$

Such that sample values of $X$ are:

$$
\begin{array}{ll}
X=0\left[\frac{g}{c m^{2}}\right](\text { space }) & X=1300\left[\frac{g}{{c m^{2}}^{2}}\right](\theta=0) \\
X=\infty\left[\frac{g}{c m^{2}}\right](\text { ground }) & X=36000\left[\frac{g}{{c m^{2}}^{2}}\right]\left(\theta=\frac{\pi}{2}\right)
\end{array}
$$

## Atmospheric hadron flux

$$
\frac{d \phi_{h}}{d X}=-\frac{\phi_{h}}{\rho d_{h}(E)}-\frac{\phi_{h}}{\lambda_{h}}+Z_{h h} \frac{\phi_{h}}{\lambda_{h}}+Z_{p h} \frac{\phi_{p}}{\lambda_{p}}
$$

In the low energy limit, the probability for hadron interaction is minimal, and thus we neglect the interaction and regeneration terms:

$$
\left.\phi_{h}\right|_{\text {low }}=\frac{Z_{p h}}{\Lambda_{p}\left(1-Z_{p p}\right)} \rho d_{h} \phi_{p}(E) e^{-\frac{X}{\Lambda_{p}}}
$$

At high energies the decay length becomes large, hence we neglect the decay term:

$$
\left.\phi_{h}\right|_{h i g h}=\frac{Z_{p h} \phi_{p}(E)}{\left(1-Z_{p p}\right)} \frac{\left(e^{-\frac{X}{\Lambda_{h}}}-e^{-\frac{X}{\Lambda_{p}}}\right)}{\left(1-\frac{\Lambda_{p}}{\Lambda_{h}}\right)}
$$

These solutions then feed into asymptotic solutions for the final leptonic flux (note that the low-energy solution scales with an additional power of $E$ ):

$$
\begin{aligned}
\text { high } & \phi_{h} \propto \phi_{p} \\
\text { low } & \phi_{h} \propto E \phi_{p}
\end{aligned}
$$

## Cascade Formalism: Sources \& Z-moments

$$
S(k \rightarrow j)=\int_{E}^{\infty} \frac{\phi_{k}\left(E_{k}^{\prime}\right)}{\lambda_{k}\left(E_{k}^{\prime}\right)} \frac{d n\left(k \rightarrow j ; E^{\prime}, E\right)}{d E} d E^{\prime}
$$

Under reasonable assumptions, the $S$-moments simplify:

$$
S(k \rightarrow j)=\frac{\phi_{k}}{\lambda_{k}} Z_{k j}
$$

## For particle production:

$$
Z_{k h}=\int_{E}^{\infty} d E^{\prime} \frac{\phi_{k}\left(E^{\prime}, X, \theta\right)}{\phi_{k}(E, X, \theta)} \frac{\lambda_{k}(E)}{\lambda_{k}\left(E^{\prime}\right)} \frac{d n\left(k A \rightarrow h Y ; E^{\prime}, E\right)}{d E} \quad \frac{d n\left(p A \rightarrow h Y ; E^{\prime}, E\right)}{d E}=\frac{1}{\sigma_{p A}\left(E^{\prime}\right)} \frac{d \sigma\left(p A \rightarrow h Y ; E^{\prime}, E\right)}{d E}
$$

For particle decay:

$$
Z_{h \rightarrow l}=\int_{E}^{\infty} d E^{\prime} \frac{\phi_{h}\left(E^{\prime}, X\right)}{\phi_{h}(E, X)} \frac{d_{h}(E)}{d_{h}\left(E^{\prime}\right)} \frac{d n\left(h \rightarrow l Y ; E^{\prime}, E\right)}{d E} \quad \frac{d n\left(h \rightarrow l Y ; E^{\prime}, E\right)}{d E}=\frac{1}{\Gamma} \frac{d \Gamma}{d E}
$$

## Atmospheric Nucleon Flux

$$
\frac{d \phi_{N}}{d X}=-\frac{\phi_{N}}{\lambda_{N}}+S(N A \rightarrow N Y)=-\frac{\phi_{N}}{\lambda_{N}}+Z_{N N} \frac{\phi_{N}}{\lambda_{N}}
$$

Assume a factorisation of fluxes $\longrightarrow \phi_{k}(E, X)=\phi_{k}(E) \phi_{k}(X)$

Define the interaction length
$\longrightarrow \quad \lambda_{N}(E)=\frac{A}{N_{0} \sigma_{p A}(E)}$
Define the attenuation length $\qquad$

$$
\Lambda_{N}=\frac{\lambda_{N}}{\left(1-Z_{N N}\right)}
$$

$$
\begin{gathered}
\frac{d \phi_{N}}{d X}=\frac{\phi_{N}}{\lambda_{N}}\left(Z_{N N}-1\right) \rightarrow \frac{d \phi_{N}}{d X}+\frac{\phi_{N}}{\lambda_{N}}\left(1-Z_{N N}\right)=0 \\
\downarrow \downarrow \downarrow
\end{gathered}
$$

$$
\phi_{N}=\phi_{N}^{0}(E) e^{-\frac{X}{\Lambda_{N}}}
$$

What constitutes this primary nucleon flux?

## Gaisser et all. fluxes: $\quad \phi_{N}^{0}(E)$

arXiv:astro-ph/1111.6675 arXiv:astro-ph/1303.3565


|  | p | He | CNO | $\mathrm{Mg}-\mathrm{Si}$ | Fe |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Pop. 1: | 7860 | 3550 | 2200 | 1430 | 2120 |
| $R_{c}=4 \mathrm{PV}$ | 1.661 | 1.58 | 1.63 | 1.67 | 1.63 |
| Pop. 2: | 20 | 20 | 13.4 | 13.4 | 13.4 |
| $R_{c}=30 \mathrm{PV}$ | 1.4 | 1.4 | 1.4 | 1.4 | 1.4 |
| Pop. 3: | 1.7 | 1.7 | 1.14 | 1.14 | 1.14 |
| $R_{c}=2 \mathrm{EV}$ | 1.4 | 1.4 | 1.4 | 1.4 | 1.4 |
| Pop. 3(*): | 200 | 0.0 | 0.0 | 0.0 | 0.0 |
| $R_{c}=60 \mathrm{EV}$ | 1.6 |  |  |  |  |

## Input PDF dependency



Evaluations of charm production utilising multiple input PDFs, including our updated NNPDF3.0+LHCb, indicate substantive differences in the small-x region. This will trace through our calculation of the prompt atmospheric neutrino flux and lead to qualitative differences in the high-energy tail.
We are thus evaluating final uncertainties utilising multiple input PDFs.

## Forward Charm Production \& LHCb






$$
\sqrt{s}=7[T e V]
$$

arXiv:1506.08025
arXiv:1302.2864 (LHCb)


We first validate our NLO predictions for forward charm production against recent LHCb data . . . finding good agreement between the 3 calculation schemes

## Small-x Gluon NNPDF: LHCb constraints

- We utilize charm production data from LHCb to reduce the uncertainties in the small- $x$ gluon PDF
- Similar strategy as the one used by the PROSA collaboration in the HERAfitter framework
- By using a Bayesian re-weighting technique, the impact of the new data is estimated. 75 data points added to NNPDF3.0 analysis
- The impact is negligible for $x>10^{-4}$, but substantive in the smaller-x region where data was previously unavailable. At $x \sim 10^{-5}$, we achieve a 3 x reduction in uncertainty
- We utilize these improved PDFs to make predictions for 13 TeV physics


Courtesy: Katerina Lipka

## Our principal new result: $\mathrm{Z}_{\mathrm{ph}}$

$$
Z_{p h}=\int_{E}^{\infty} d E^{\prime} \frac{\phi_{p}\left(E^{\prime}\right)}{\phi_{p}(E)} \frac{A}{\sigma_{p A}(E)} \frac{d \sigma\left(p p \rightarrow c \bar{c} Y ; E^{\prime}, E\right)}{d E}
$$

## The differential cross-section is generated at various $\mathrm{E}^{\prime}$ between $10^{3}$ and $10^{10} \mathrm{GeV}$ with POWHEG+PYTHIA8, and incorporates our updated NNPDF3.0+LHCb ... Cross-checks made with aMC@NLO



arXiv: 1506.08025

## Decay moments: $\mathbb{Z}_{h \rightarrow l}$

$$
Z_{h \rightarrow l}=\int_{E}^{\infty} d E^{\prime} \frac{\phi_{h}\left(E^{\prime}, X\right)}{\phi_{h}(E, X)} \frac{d_{h}(E)}{d_{h}\left(E^{\prime}\right)} \frac{d n\left(h \rightarrow l Y ; E^{\prime}, E\right)}{d E}
$$




The relative contributions of different species in the BPL cosmic ray scenario.

The relative contributions of the $\mathrm{D}^{+}$species in varying cosmic ray scenarios.

## Stitching things together...






## Decay moments: $Z_{h \rightarrow l}$

$$
Z_{h \rightarrow l}=\int_{E}^{\infty} d E^{\prime} \frac{\phi_{h}\left(E^{\prime}, X\right)}{\phi_{h}(E, X)} \frac{d_{h}(E)}{d_{h}\left(E^{\prime}\right)} \frac{d n\left(h \rightarrow l Y ; E^{\prime}, E\right)}{d E}
$$

The distribution for leptonic decay is known to obey the simple scaling law:

$$
d n\left(h \rightarrow l Y ; E^{\prime}, E\right)=F_{h \rightarrow l}\left(\frac{E}{E^{\prime}}\right) \frac{d E}{E^{\prime}}
$$

The moment then simplifies, and we generate $F$ with POWHEG:

$$
Z_{h \rightarrow l}=\int_{0}^{1} d x_{E} \frac{\phi_{h}\left(E / x_{E}\right)}{\phi_{h}(E)} F_{h \rightarrow l}\left(x_{E}\right)
$$

The following branching fractions are built into our decay moments:

$$
\begin{aligned}
\mathcal{B}\left(D^{ \pm} \rightarrow \nu_{l} X\right) & =.153 \\
\mathcal{B}\left(D^{0} \rightarrow \nu_{l} X\right) & =.101 \\
\mathcal{B}\left(D_{s}^{ \pm} \rightarrow \nu_{l} X\right) & =.06 \\
\mathcal{B}\left(\Lambda_{c} \rightarrow \nu_{l} X\right) & =.02
\end{aligned}
$$

