Resummation in PDF fits

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related to work with Marco Bonvini, Simone Marzani, Juan Rojo, Valerio Bertone, Richard Ball, Stefano Forte, and the NNPDF Collaboration

Resummation of enhanced contributions

Single (double) logarithmic enhancement

$$\alpha_s^k \ln^j \qquad 0 \leq j \leq (2)k$$

Perturbative convergence is spoiled when

$$\alpha_s \ln^{(2)} \sim 1$$

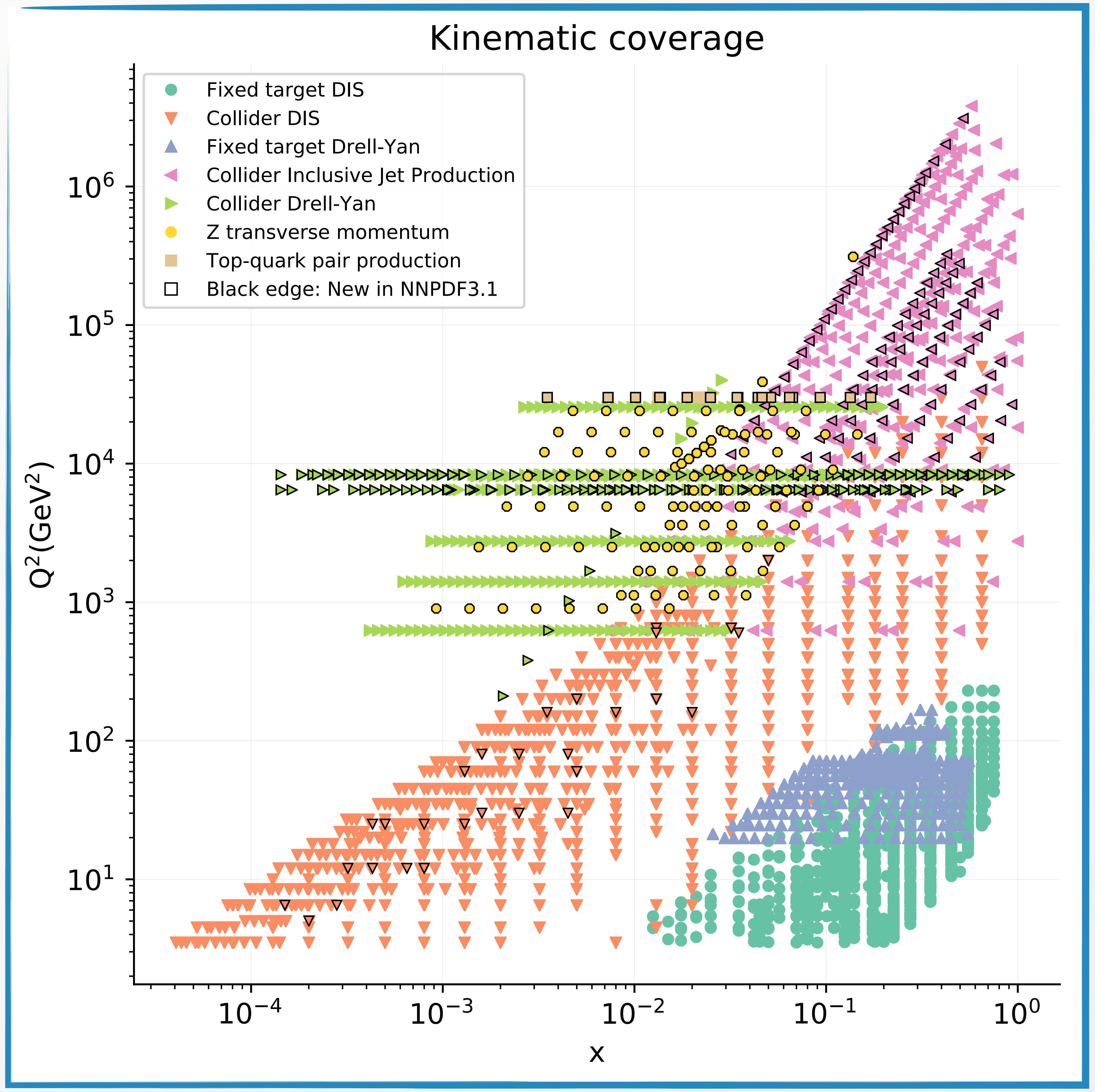
All-order **resummation** of the logarithmically enhanced terms

Including resummation in PDF fits:

- Provides **consistent predictions** when resummed computations are used
- Improves the **quality** of the PDF fits
- Helps in investigating the impact of **missing higher orders**

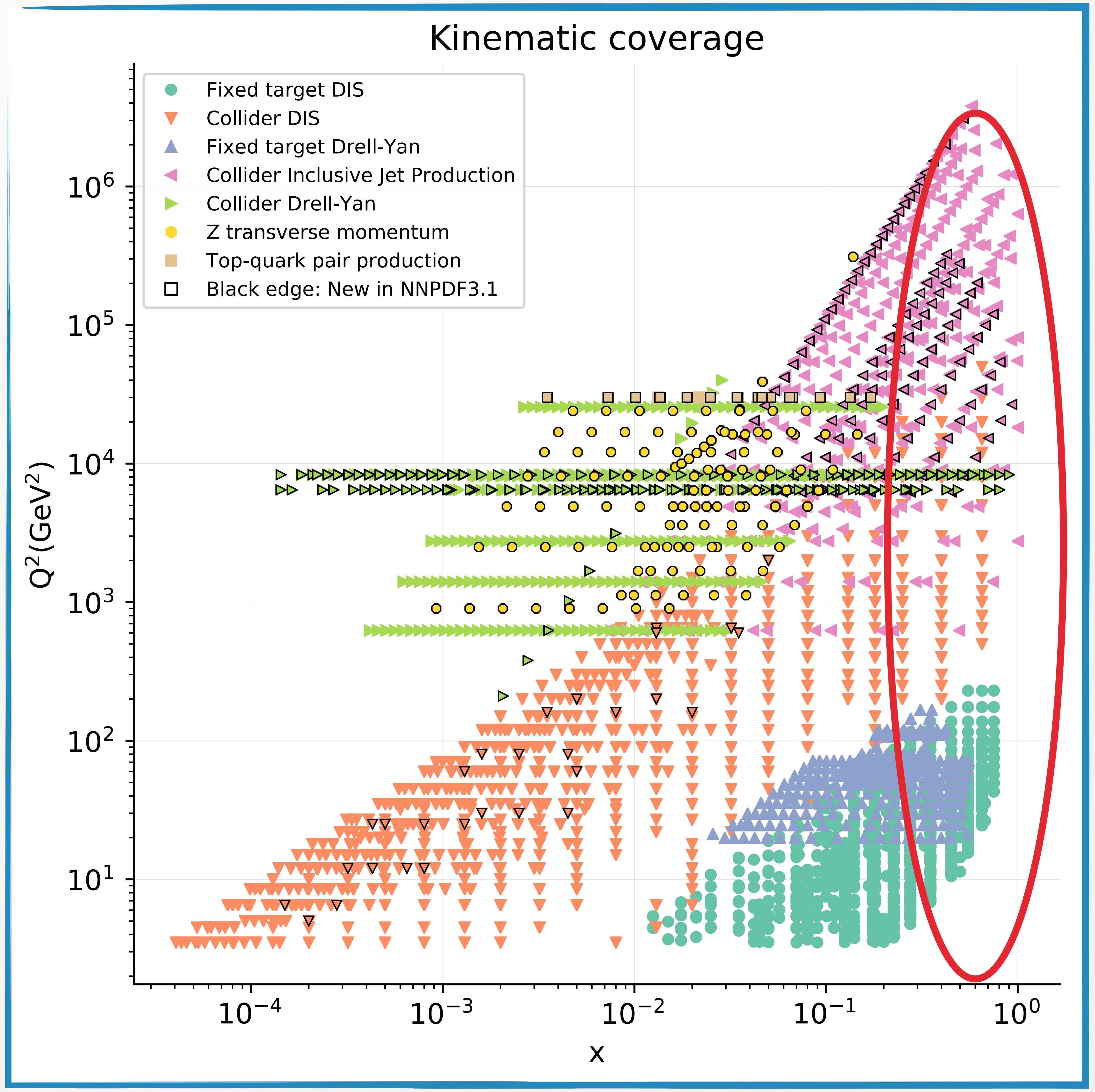
… it brings us closer to 'all-order' PDFs

Resummations



Oxford Physics

Resummations



Large x: **threshold** resummation

double logs due to **soft** gluon emission

$$\left(\frac{\ln^k(1-x)}{(1-x)}\right)_+$$

In Mellin space

$$\ln N \qquad N \to \infty$$

[Bonvini,Marzani,Rojo,LR,Ubiali,Ball,Bertone, Carrazza,Hartland 1507.01006]

Resummations

Small x: **high energy** resummation

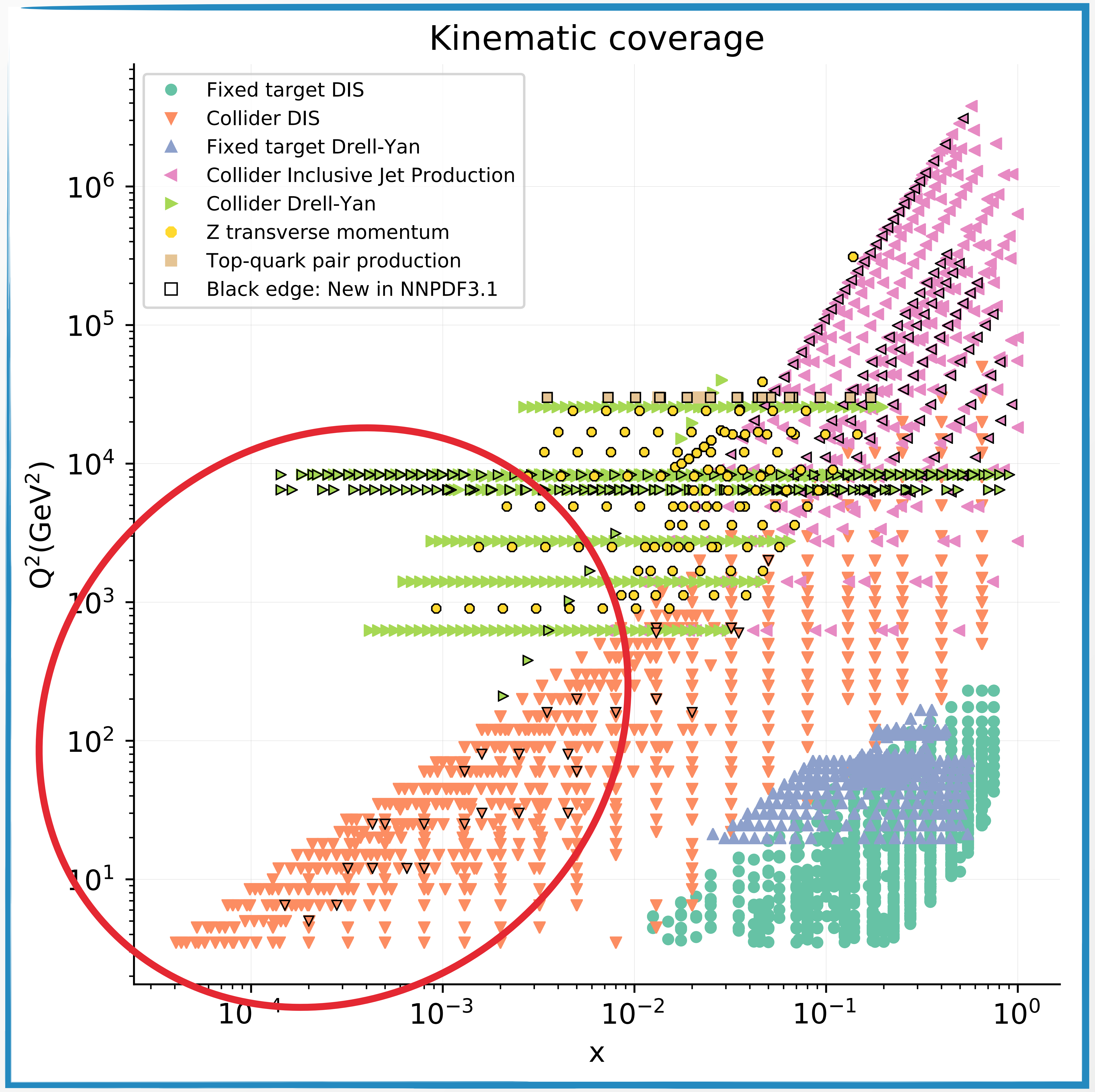
single logs due to high-energy gluon emission

$$\frac{1}{x} \ln^k x$$

In Mellin space, poles at

$$\frac{1}{N-1} \qquad N \to 1$$

[NNPDF, in progress]



Resummation: what and how

Resummation affects:

Observable (coefficient functions) $$\sigma = \sigma_0 C(\alpha_s(\mu)) \otimes f(\mu)\,[\otimes f(\mu)]$$

Evolution (splitting functions) $$\mu^2 \frac{d}{d\mu^2} f(\mu) = P(\alpha_s(\mu)) \otimes f(\mu)$$

	observable (coefficient function)	evolution (splitting function)
small x	**NLLx***	**NLLx**
large x	**(N)NNLL**	—

*starts at NLLx

PDFs with Threshold Resummation: NNPDF3.0res

Datasets considered in NNPDF3.0res

[Bonvini,Marzani,Rojo,LR,Ubiali,Ball,Bertone, Carrazza,Hartland 1507.01006]

process	observable	included?
DIS	**$d\sigma/(dxdQ^2)$ (NC, CC, F2c…)**	✔
DY Z/γ	**$d\sigma/(dydM^2)$**	✔
DY W	**differential in lepton kinematics**	✘ no public code available yet
$t\bar{t}$	**total σ**	✔
jets	**inclusive $d\sigma/(dydp_T)$**	✘ NLL known to be poor

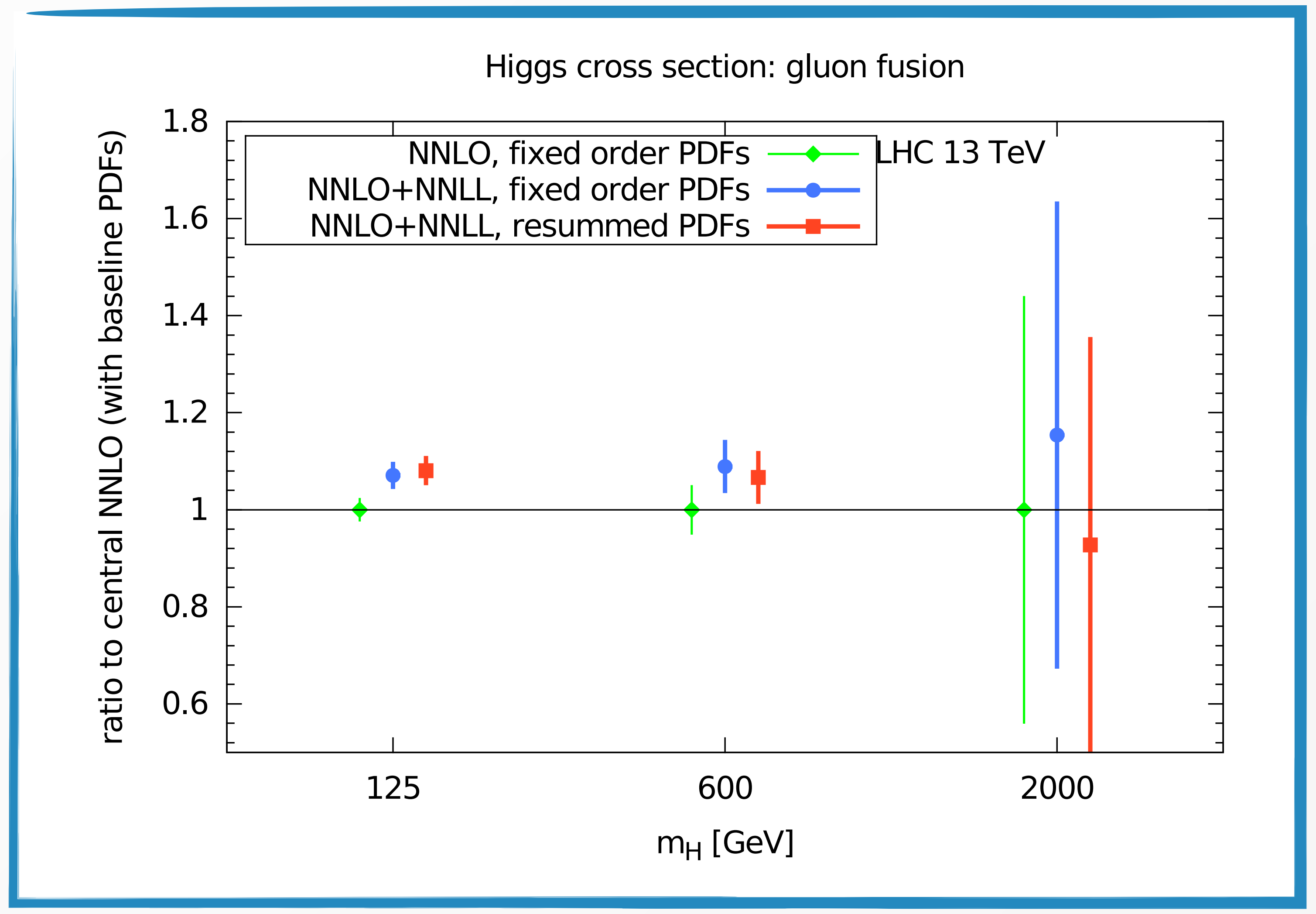
Accuracy is competitive with global fit, except for large-x gluon (jets not included)

Resummation is included supplementing fixed-order computations with **K-factors**

$$K^{\mathrm{N^kLO+N^kLL}} = \frac{\sigma^{\mathrm{N^kLO+N^kLL}}}{\sigma^{\mathrm{N^kLO}}}$$

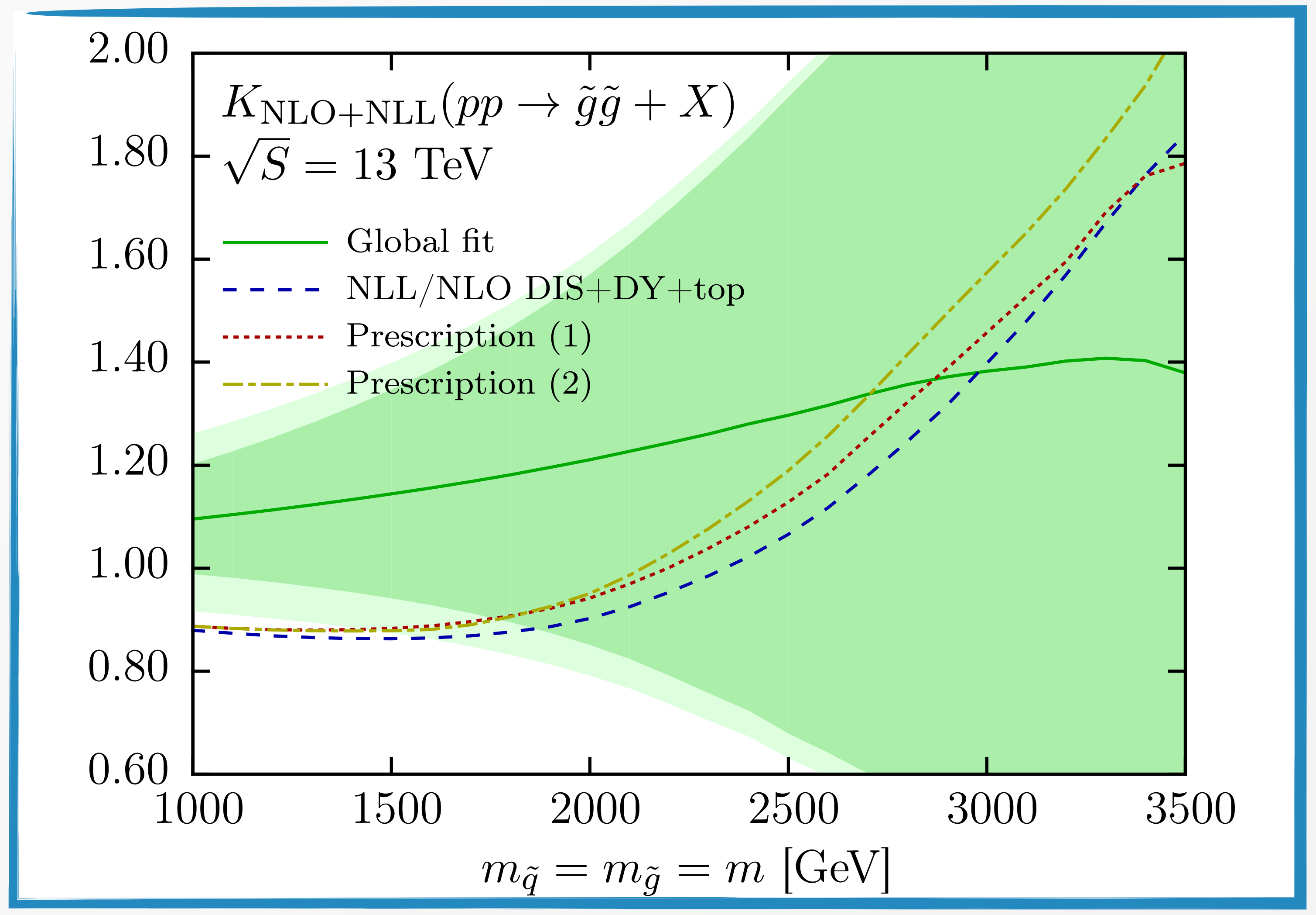
Impact on phenomenology

Higgs



- SM Higgs is not affected by resummation of PDFs
- m_H ~600 GeV cancellation of 1/2 of the enhancement
- m_H~2 TeV NNLO+NNLL with resummed PDFs is similar to FO PDFs (larger uncertainty)

Susy particles



- Predictions for MSSM particles are modified when using resummed PDFs
- However, PDF errors are very large

[Beenakker,Borschensky,Krämer,Kulesza,Laenen,Marzani,Rojo 1510.00375]

Comments and outlook

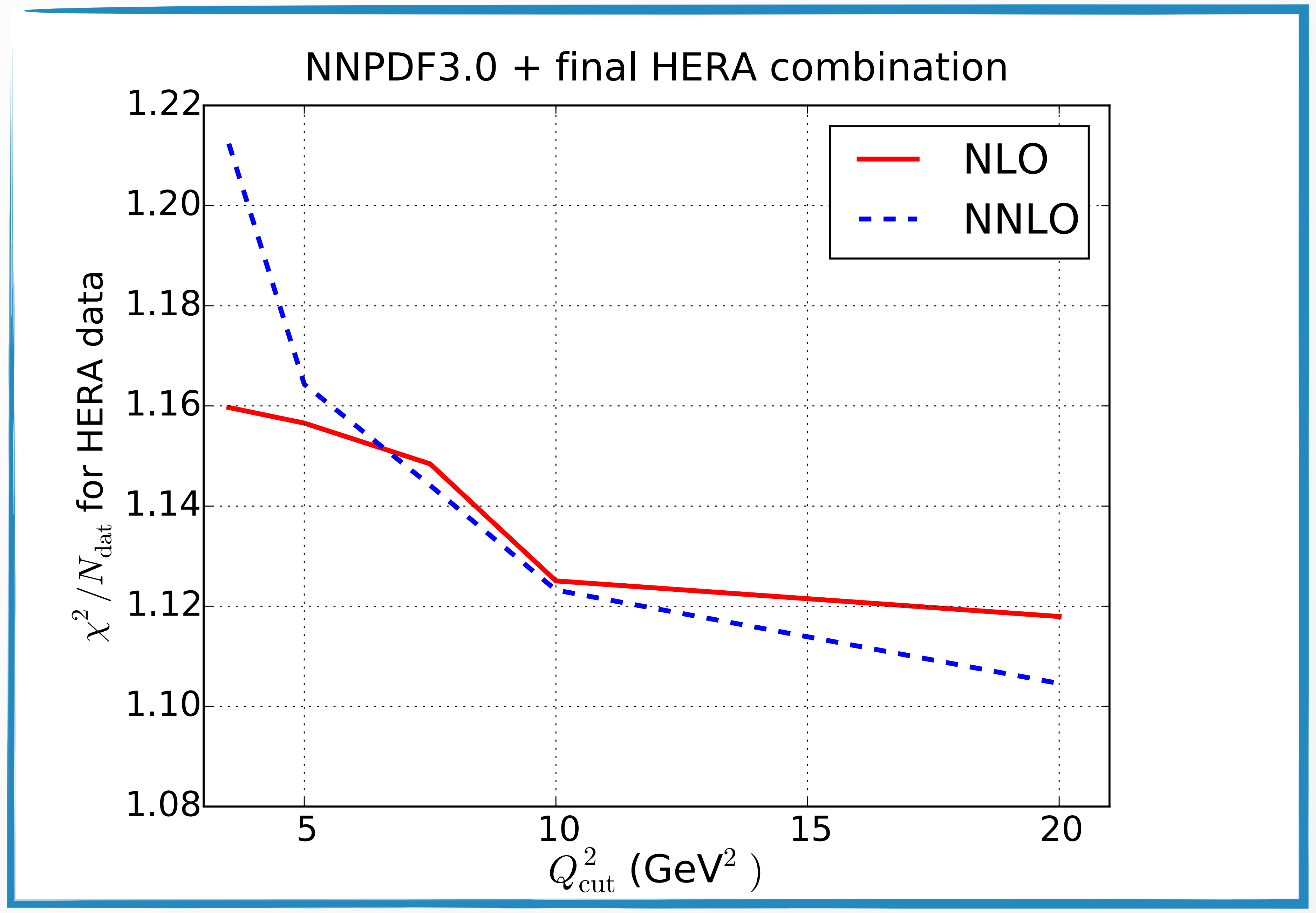
- First ever **global** fit of PDFs with **threshold resummation**
- PDFs are **suppressed in the large-x region**; at intermediate values of x quark PDFs are slightly enhanced (sum rule); negligible effects at $x<0.01$
- Inclusion of resummation **compensates the enhancement** from resummation in partonic cross sections
- Consistent resummed calculations might be closer to fixed order results

Limitations: larger uncertainties due to **reduced dataset**.

Methodology enables to have truly global resummed PDFs when calculations for missing processes will be available.

New processes to be included: DY Z/γ (Zp_T), $t\bar{t}$ (differential). . .

Need for small-x resummation?



Courtesy of Juan Rojo

Description of HERA data poorer when data points at smaller values of x are included and fixed-order theory is used

Fixed order theory could be not sufficient to describe data points at small-x and/or small Q^2

Effect is more pronounced if NNLO theory is used

This can indicate the need for **small-x resummation**

Overview of small-x resummation

Small-x resummation based on kt-factorization and BFKL. Developed mostly in the 90s-00s [Catani,Ciafaloni,Colferai,Hautmann,Salam,Stasto][Altarelli,Ball,Forte] [Thorne,White]

Affects both **evolution** (LLx, NLLx) and **coefficient functions** (NLLx, lowest logarithmic order) in the singlet sector

Splitting functions are resummed using **ABF** (Altarelli,Ball,Forte) procedure

New formalism for **coefficient function** [Bonvini,Marzani,Peraro 1607.02153] and further improvements on the ABF formalism [Bonvini,Marzani,Muselli,Peraro 170x.xxxx]

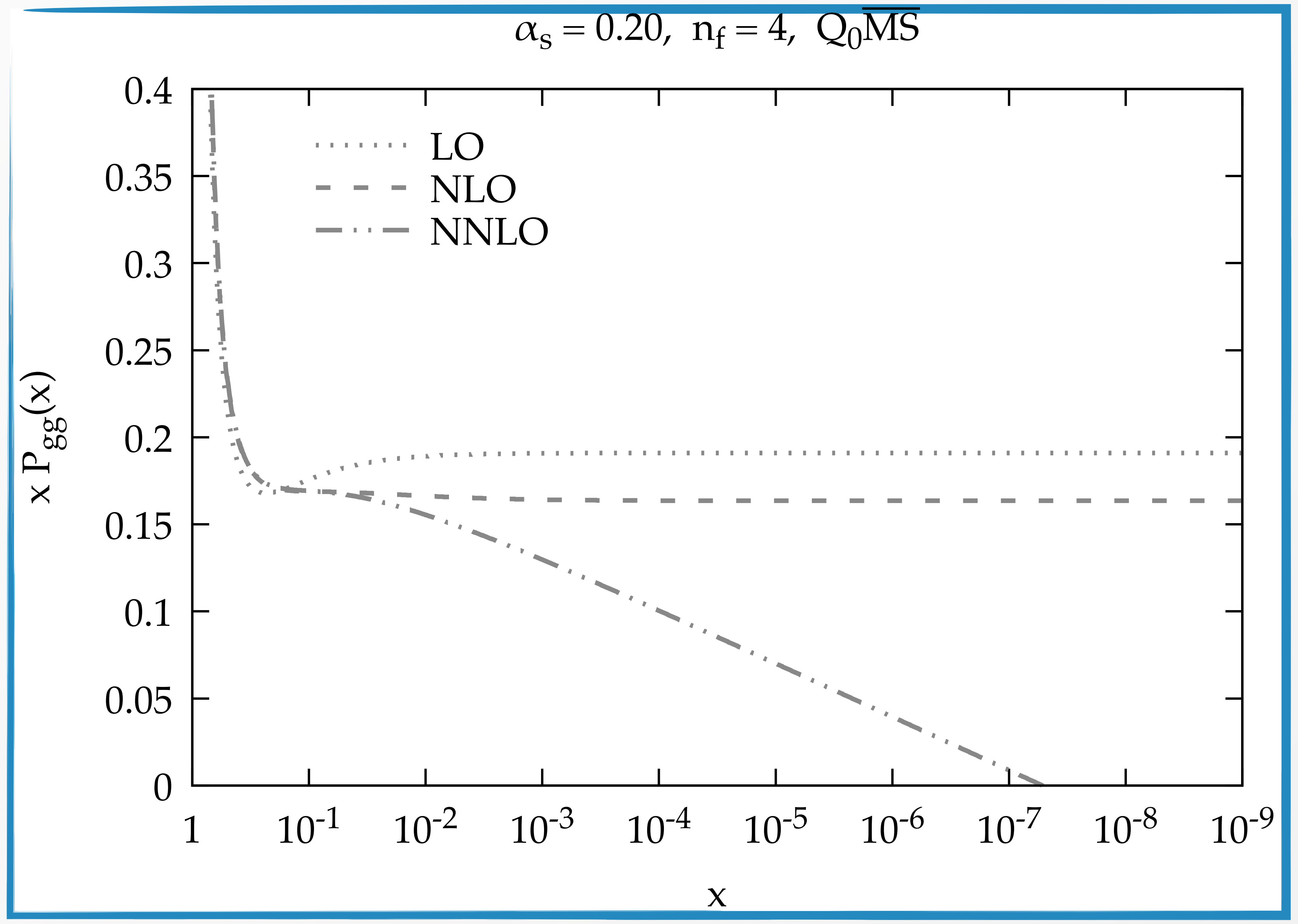
Resummed splitting functions and coefficient functions available through public code **HELL** www.ge.infn.it/~bonvini/hell

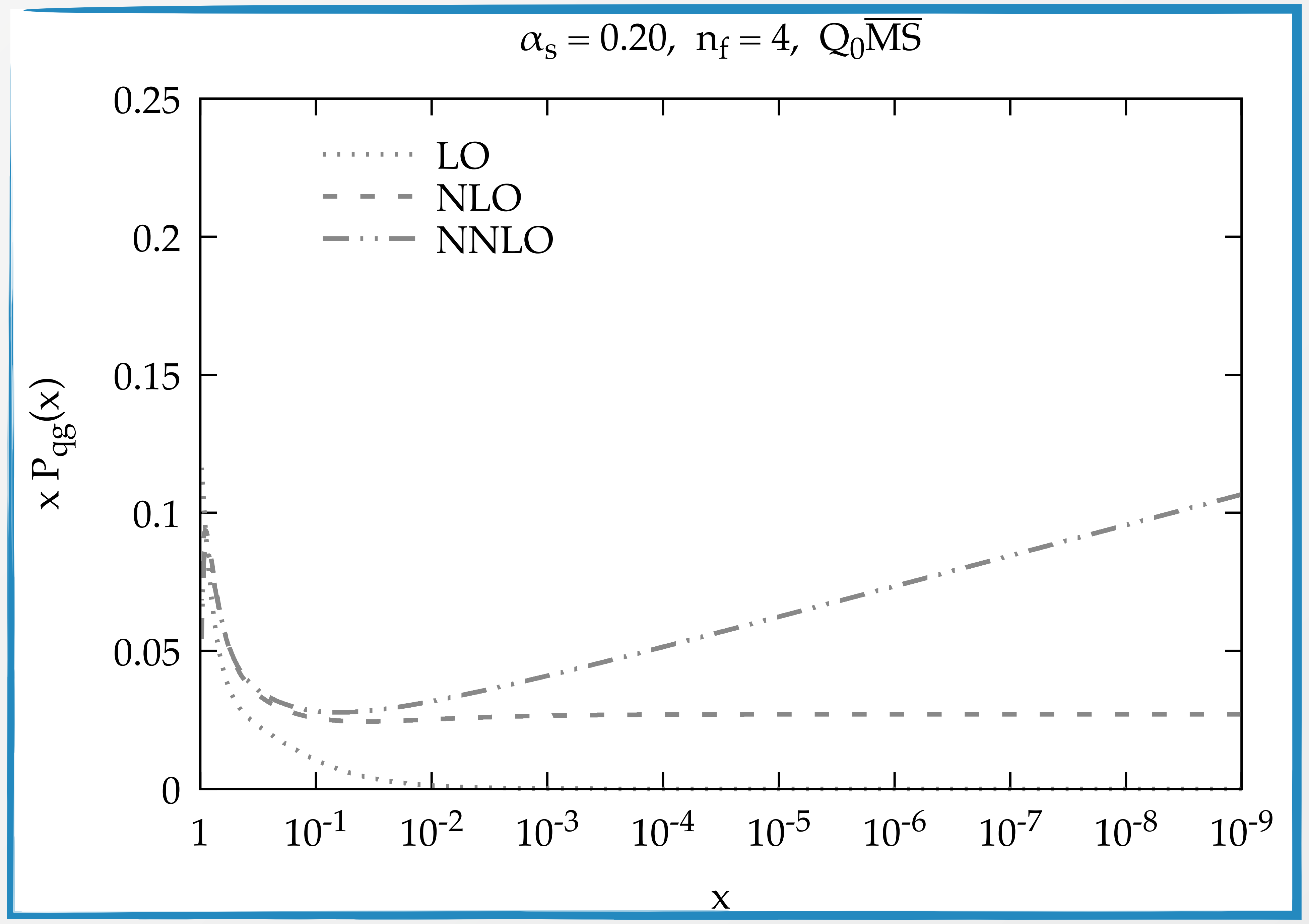
Use in PDF fits possible thanks to the interface with **APFEL** apfel.hepforge.org

Small-x resummation of DGLAP evolution

ABF procedure based on

- **duality** with BFKL evolution
- **symmetry** of the BFKL kernel
- **momentum conservation**
- resummation of (subleading, but fundamental) **running coupling effects**





Courtesy of Marco Bonvini

Small-x resummation of DGLAP evolution

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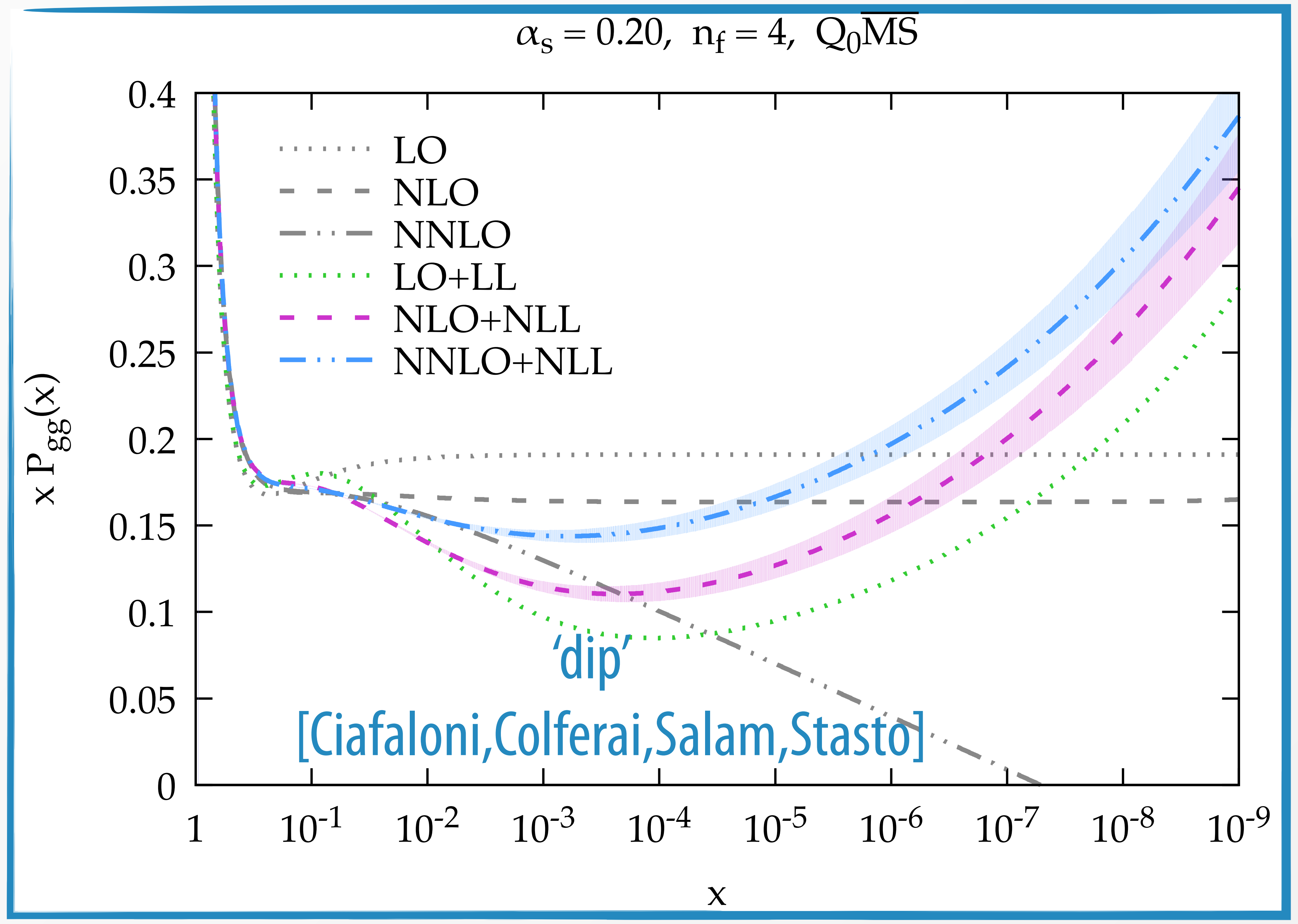
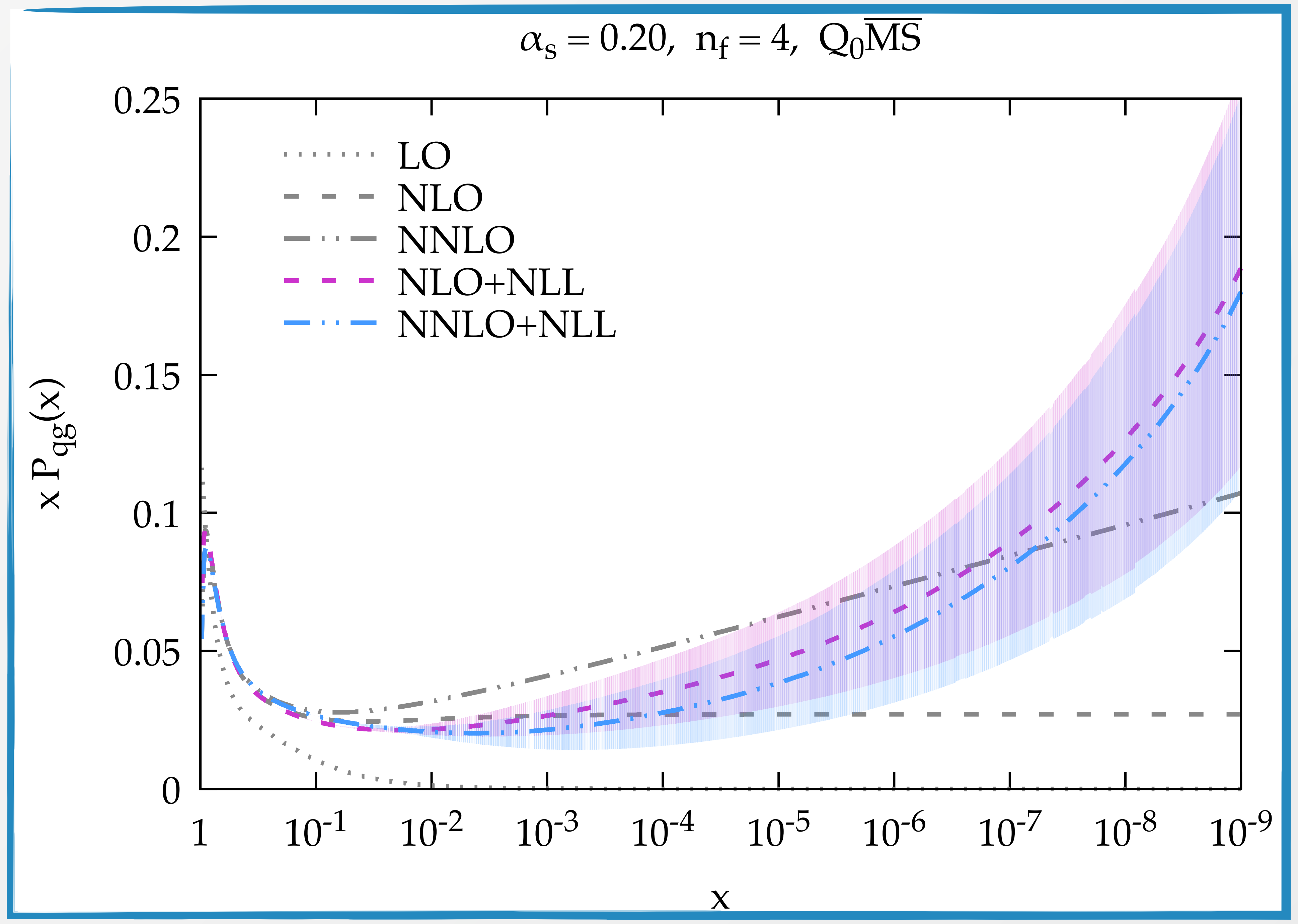
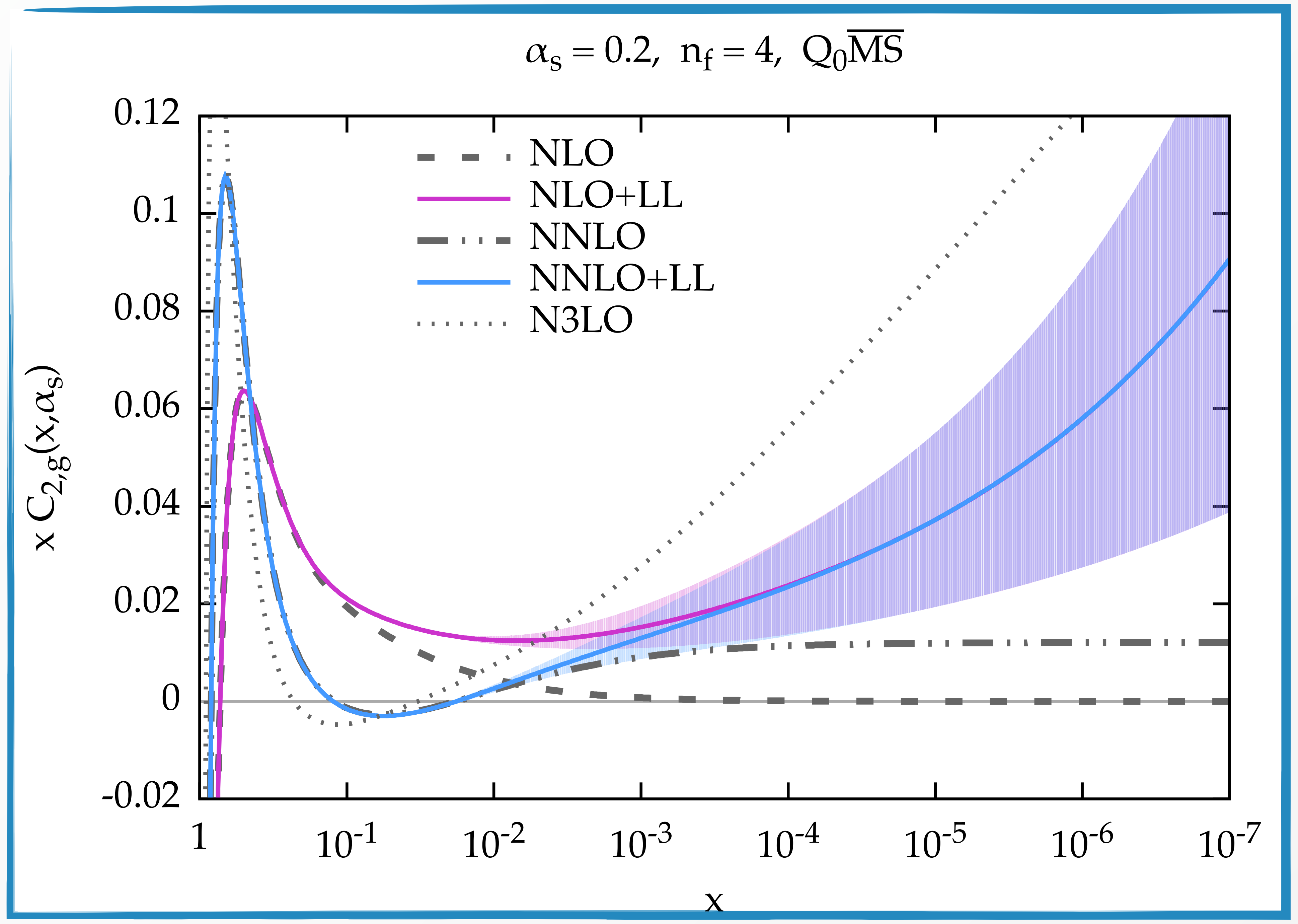
Now matching at NNLO available!

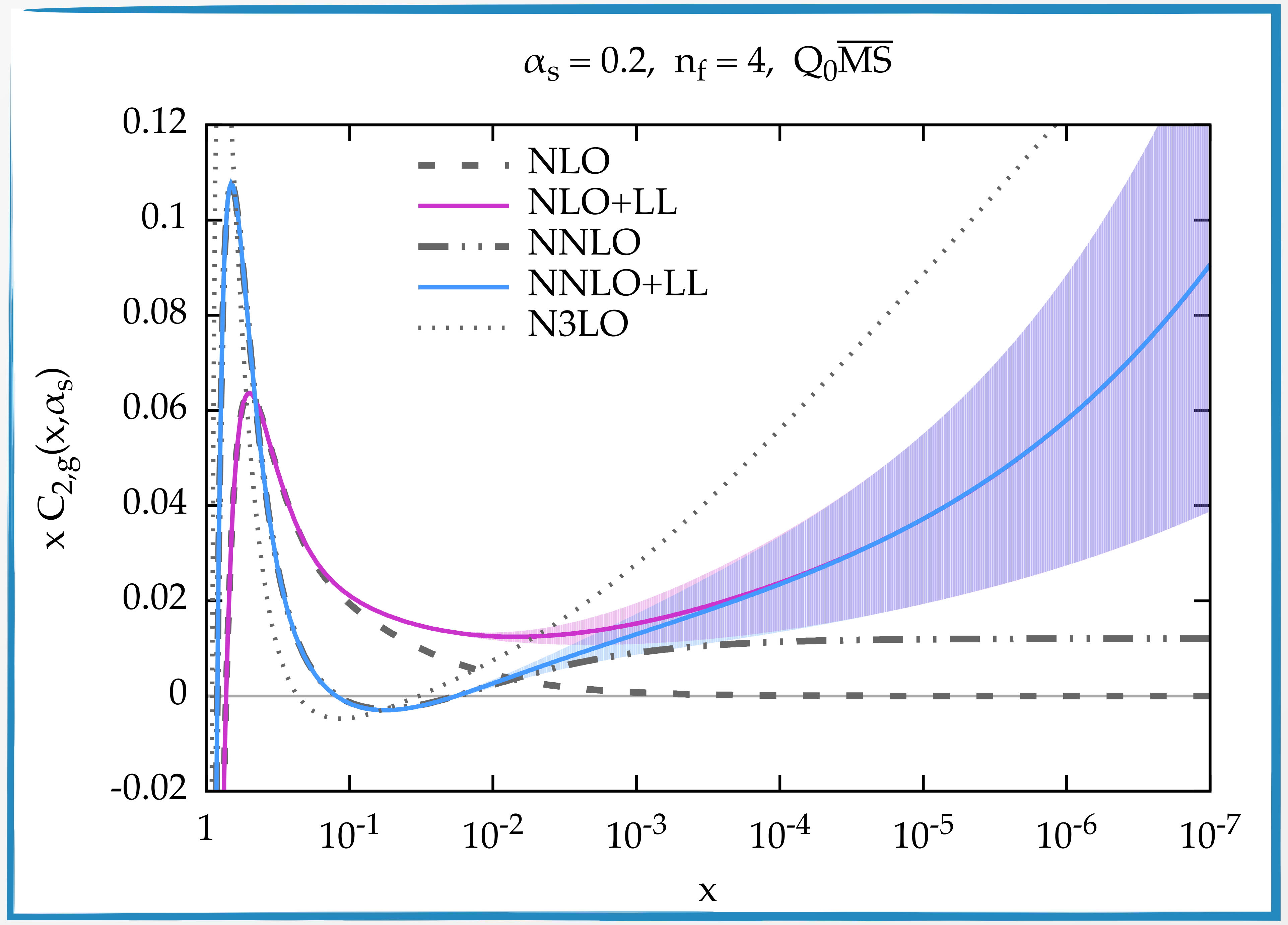




Courtesy of Marco Bonvini

Small-x resummation of coefficient functions





Courtesy of Marco Bonvini

- **massive** DIS coefficient functions available and implemented in **HELL**
- **VFNS** (FONLL = S-ACOT) implementation $\quad C_{L,g}^{[n_f+1]}(m) = C_{L,g}^{[n_f]}(m), \qquad C_{2,g}^{[n_f+1]}(m) = C_{2,g}^{[n_f]}(m) - K_{hg}(m)$
- resummed **matching conditions** in **HELL** $\quad f_i^{[n_f+1]}(m) = \sum_{j=g,q_i\ldots q_{n_f}} K_{ij}(m) f_j^{[n_f]}, \qquad i = g, q, \ldots q_{n_f+1}$

See also [Thorne,White]

Towards a global small-x resummed fit

All ingredients for a PDF fit to DIS data are now available

In principle, one should add additional processes:

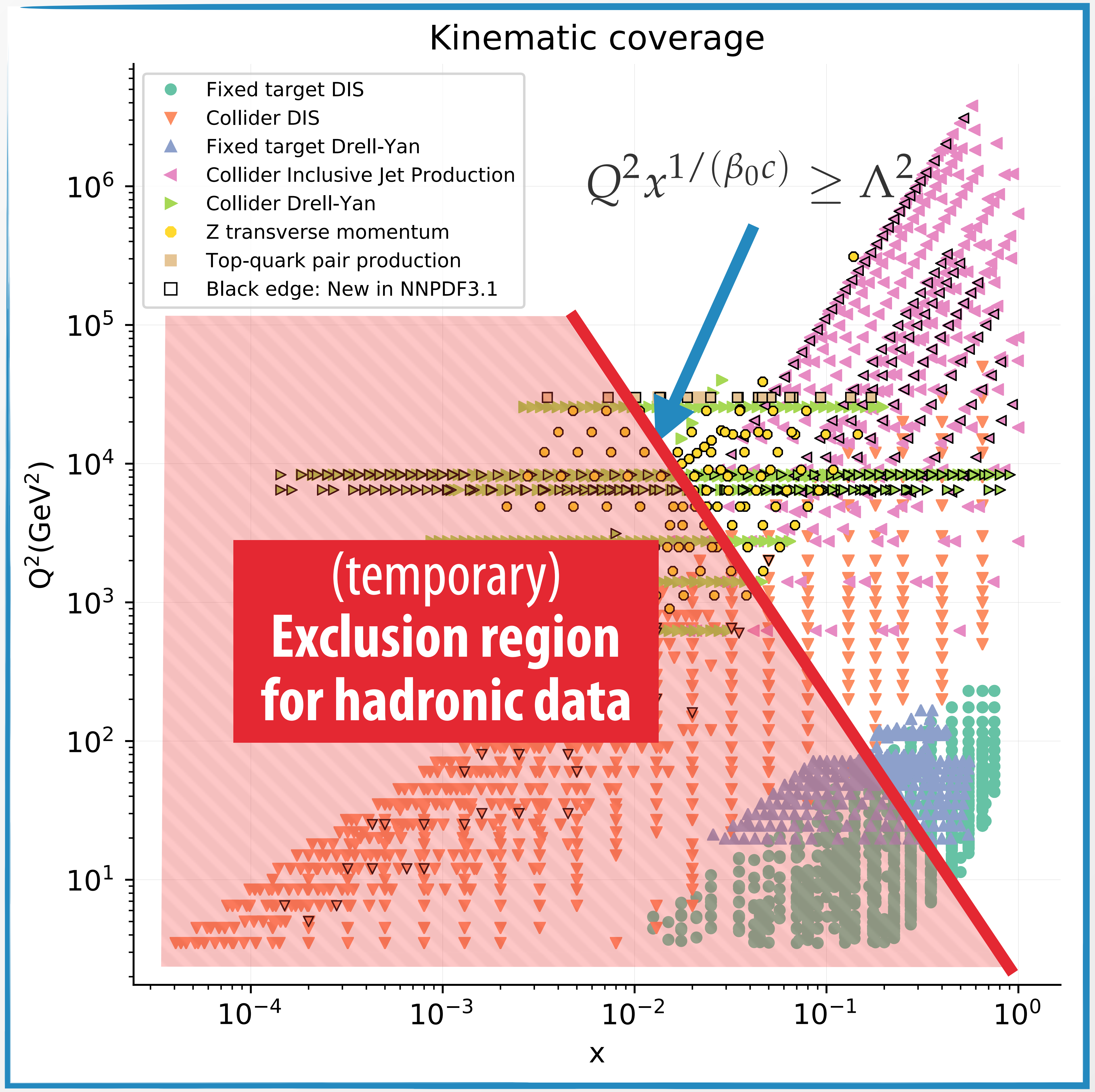
- DY
- Jets
- top
- …

Ongoing work in this direction

However, a global fit is possible if **conservatives cuts** on hadronic data are applied and points which may feature small-x enhancement are excluded

$$\alpha_s(Q^2)\log\frac{1}{x} \geq c \sim 1$$

Value of *c* (slope of the line) selects the exclusion region



DIS-only fit results

Experiment	Dataset	DOF	Current ^{2}	Reference ^{2}	Current ^{2}	Reference ^{2}
NMC		325	1.28627	1.28758	1.27370	1.28742
	NMCPD	121	0.92561	0.91453	0.89964	0.91669
	NMC	204	1.50019	1.50885	1.49558	1.50731
SLAC		67	1.00929	1.01376	0.87304	0.83805
	SLACP	33	1.02654	1.02160	0.86848	0.84938
	SLACD	34	0.94462	0.96066	0.83496	0.78457
BCDMS		581	1.18540	1.19748	1.20455	1.21112
	BCDMSP	333	1.23390	1.25020	1.26626	1.27350
	BCDMSD	248	1.11995	1.12657	1.12252	1.12718
CHORUS		832	0.97194	0.97820	0.98387	0.97908
	CHORUSNU	416	0.93686	0.93564	0.94292	0.94093
	CHORUSNB	416	0.97409	0.98321	0.99881	0.99085
NTVDMN		76	0.64439	0.67227	0.69993	0.69213
	NTVNUDMN	39	0.62988	0.55987	0.63087	0.70683
	NTVNBDMN	37	0.64793	0.78956	0.76609	0.67187
HERACOMB		1145	1.12111	1.13084	1.12411	1.17376
	HERACOMBNCEM	159	1.45607	1.44595	1.44561	1.44855
	HERACOMBNCEP460	204	1.07735	1.09569	1.07618	1.09723
	HERACOMBNCEP575	254	0.87031	0.87236	0.86894	0.91757
	HERACOMBNCEP820	70	1.00489	1.04616	1.04623	1.18655
	HERACOMBNCEP920	377	1.17811	1.18217	1.18983	1.27363
	HERACOMBCCEM	42	0.94844	0.96002	0.96945	1.00185
	HERACOMBCCEP	39	1.30369	1.29350	1.23654	1.21963
HERAF2CHARM		47	2.15652	1.75245	1.75765	1.62864
F2BOTTOM		29	1.00797	1.01885	1.05043	1.10405
	H1HERAF2B	12	0.77889	0.76393	0.75769	0.81308
	ZEUSHERAF2B	17	1.16968	1.19879	1.25708	1.30944
Total (exps)		**3102**	**1.11098**	**1.11341**	**1.10824**	**1.12602**

NLO+NLL NLO NNLO+NLL NNLO

Hierarchy as expected

$\chi^2_{\text{NNLO+NLL}}$ smallest

$\chi^2_{\text{(N)NLO+NLL}} < \chi^2_{\text{(N)NLO}}$

$\chi^2_{\text{NLO}} < \chi^2_{\text{NNLO}}$

Global fit results

Partial results with very tight cut (c=0.5)

Improvement of the χ^2 at NNLO+NLL

$\chi^2_{\mathrm{NNLO}} = 1.108$ $\chi^2_{\mathrm{NNLO+NLL}} = 1.087$

Fit particularly conservative: several datasets are excluded compared with NNPDF3.1.

~700 proton-(anti)proton collider data now included

Final fits will likely have a larger value of c: studies ongoing

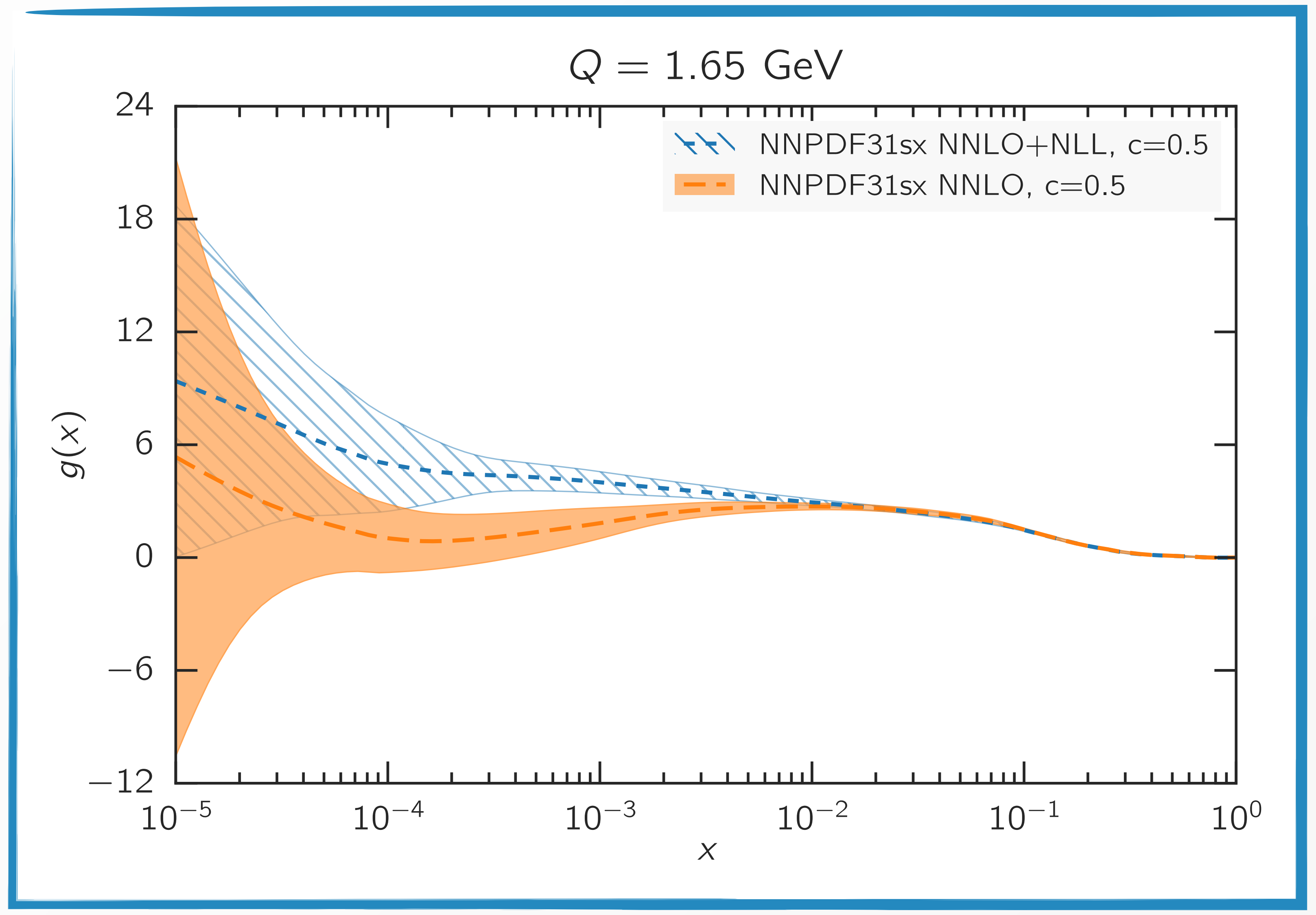
NMC		325	1.30933	1.30658
	NMCPD	121	0.90340	0.90975
	NMC	204	1.55010	1.54195
SLAC		67	0.75438	0.71791
	SLACP	33	0.77612	0.76136
	SLACD	34	0.69468	0.64219
BCDMS		581	1.23231	1.23749
	BCDMSP	333	1.29846	1.29965
	BCDMSD	248	1.14657	1.15807
CHORUS		832	0.98529	0.99100
	CHORUSNU	416	0.95963	0.96959
	CHORUSNB	416	0.96967	0.97604
NTVDMN		76	0.69705	0.66020
	NTVNUDMN	39	0.64168	0.61377
	NTVNBDMN	37	0.75480	0.70862
HERACOMB		1145	1.13637	1.20053
	HERACOMBNCEM	159	1.41686	1.42561
	HERACOMBNCEP460	204	1.07268	1.09846
	HERACOMBNCEP575	254	0.87425	0.92498
	HERACOMBNCEP820	70	1.00094	1.14444
	HERACOMBNCEP920	377	1.21586	1.33538
	HERACOMBCCEM	42	1.18772	1.21120
	HERACOMBCCEP	39	1.29966	1.25821
HERAF2CHARM		37	1.55062	1.43096
F2BOTTOM		29	1.08301	1.14559
	H1HERAF2B	12	0.77492	0.83800
	ZEUSHERAF2B	17	1.30048	1.36271
DYE886		66	0.77774	0.82618
	DYE886R	11	0.35450	0.35305
	DYE886P	55	0.86239	0.92081
DYE605		85	1.03631	1.04339
CDF		88	0.91494	0.96510
	CDFZRAP	12	1.43559	1.50530
	CDFR2KT	76	0.80545	0.83996
D0		20	[illegible]	[illegible]
	D0ZRAP	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]
ATLAS		[illegible]	[illegible]	1.00621
	[illegible]	5	1.56893	1.50833
	ATLASR04JETS36PB	81	0.87952	0.88701
	ATLASR04JETS2P76TEV	56	[illegible]	0.98552
	[illegible]	[illegible]	[illegible]	1.09379
	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	1.01556
	[illegible]	10	1.57159	1.54724
CMS		234	0.88203	0.89675
	CMSDY2D11	8	0.86674	0.61011
	CMSJETS11	133	0.79988	0.83660
	CMS1JET276TEV	81	0.99556	1.02734
	CMSZDIFF12	3	1.62160	1.21982
	CMSTTBARTOT	3	0.47858	0.23342
	CMSTOPDIFF8TEVTTRAPNORM	6	0.83594	0.82121
Total (exps)		**3816**	**1.08710**	**1.10849**

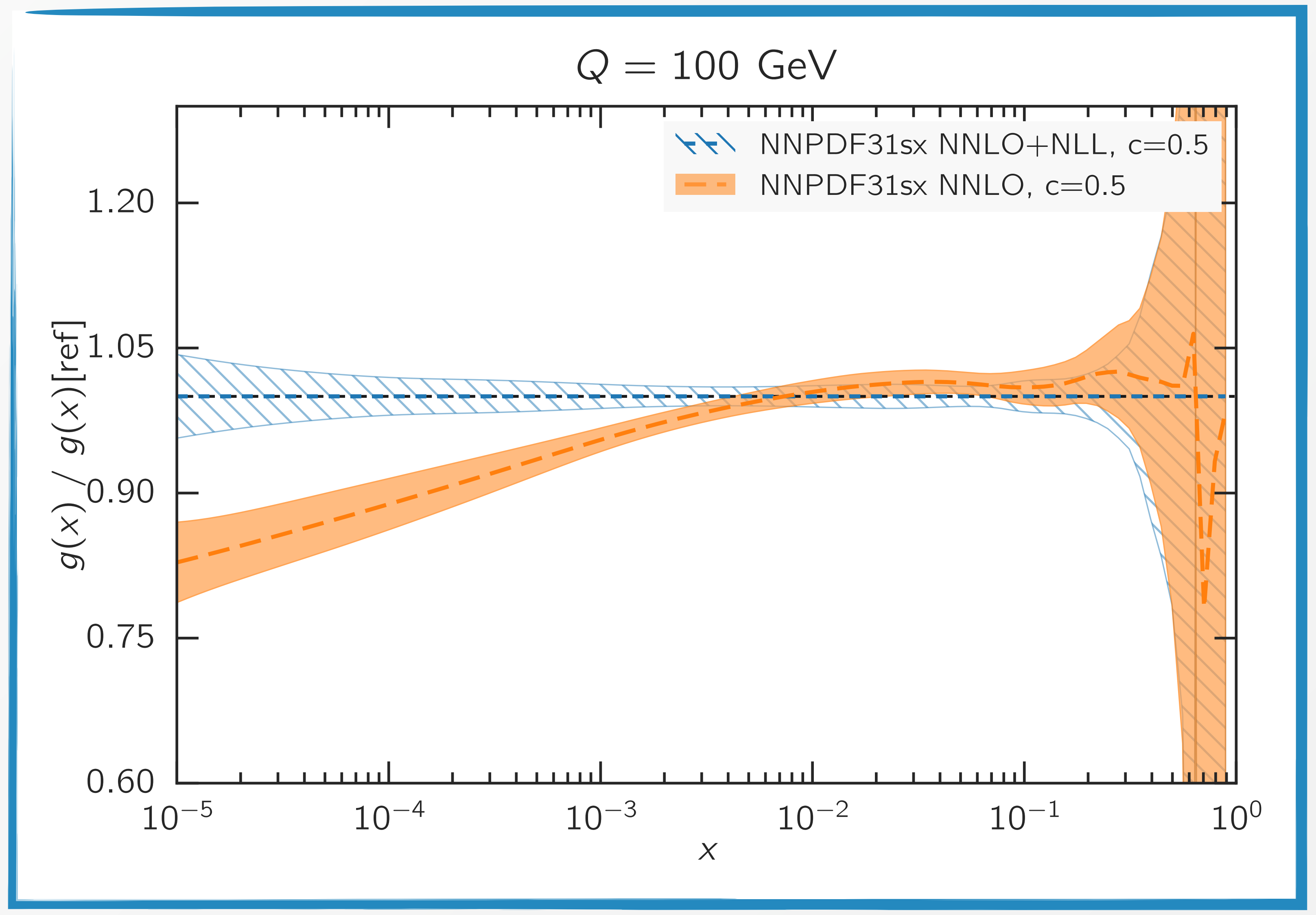
NNLO+NLL NNLO

Preliminary Results

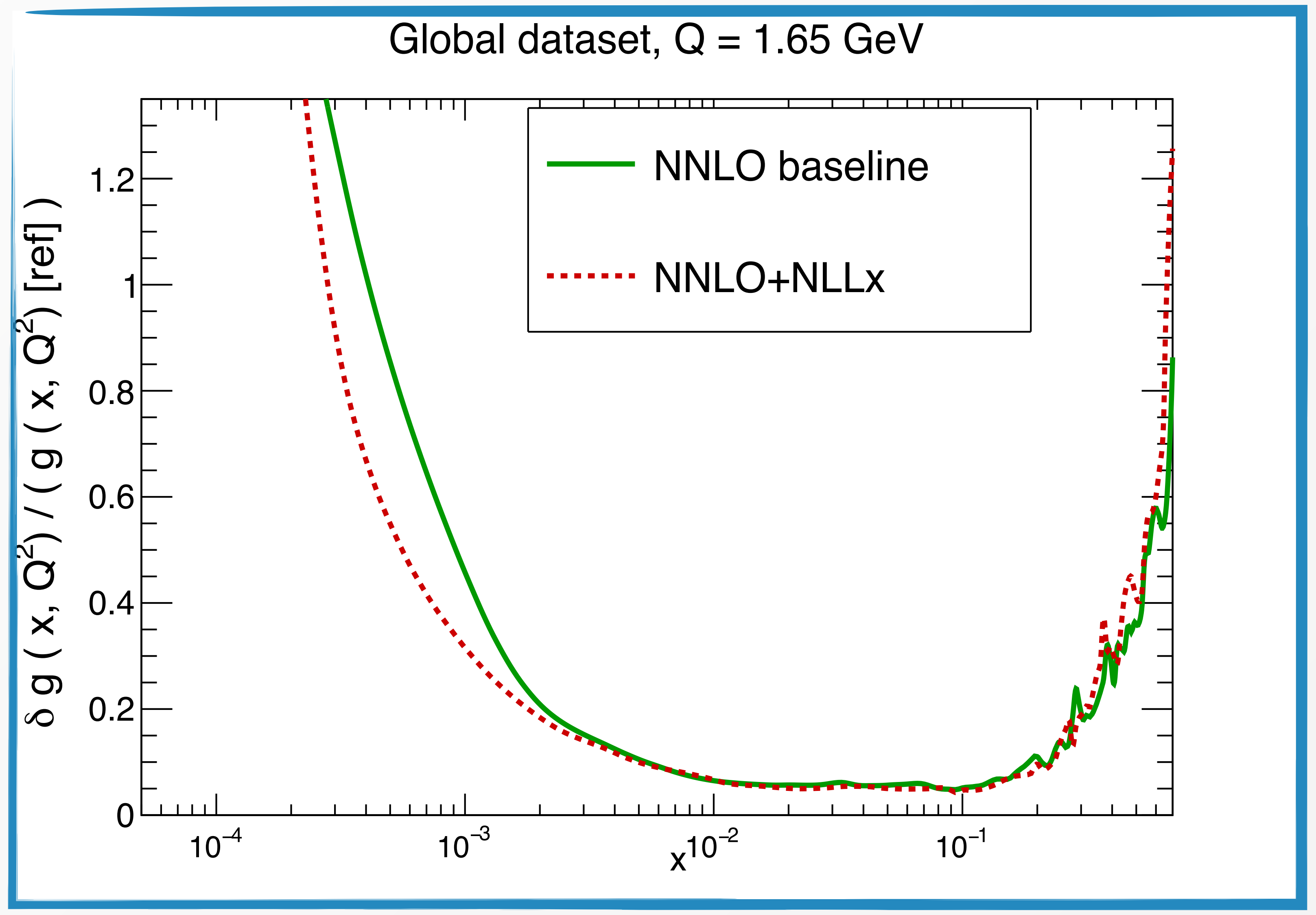
Oxford Physics

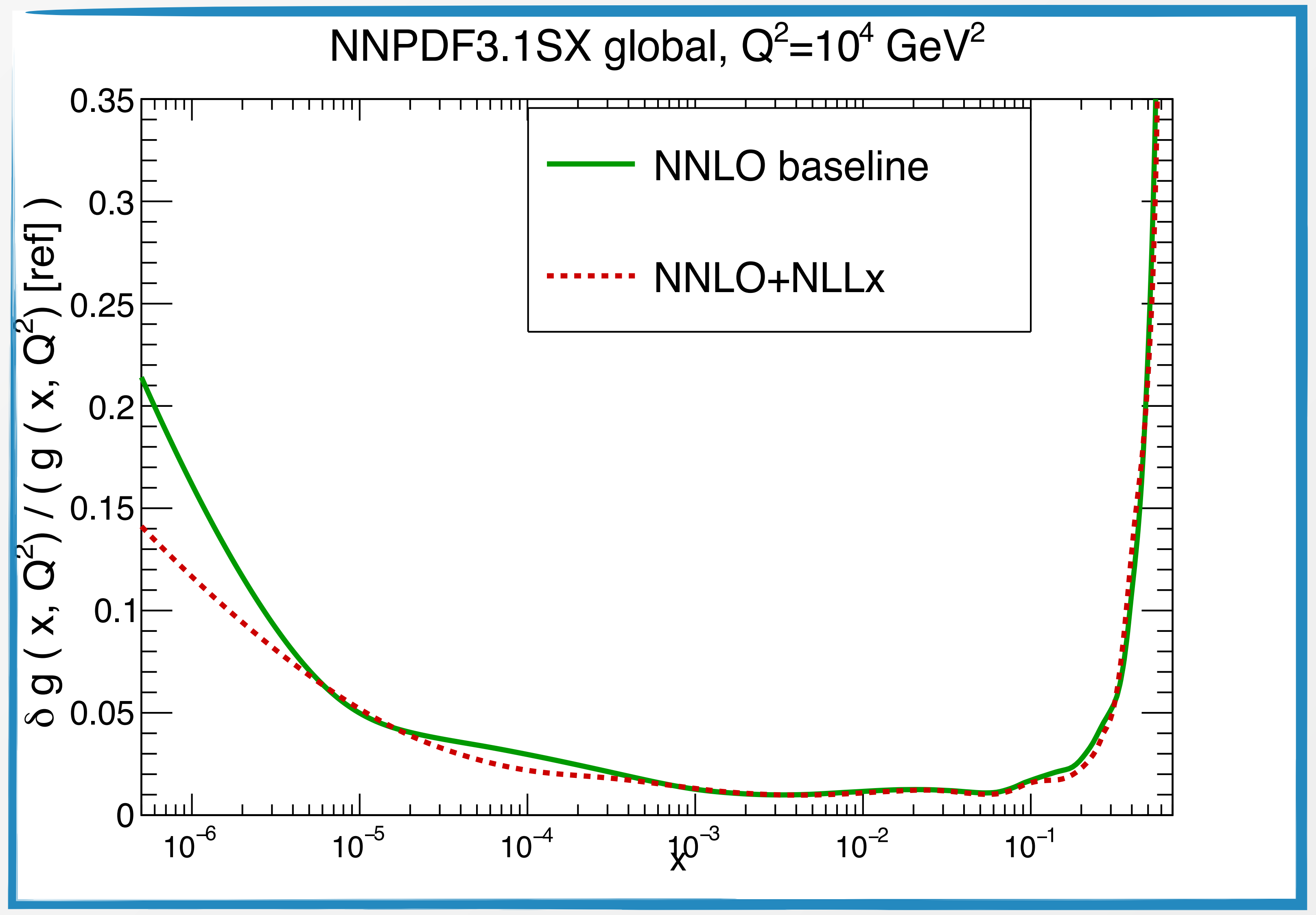
PDFs





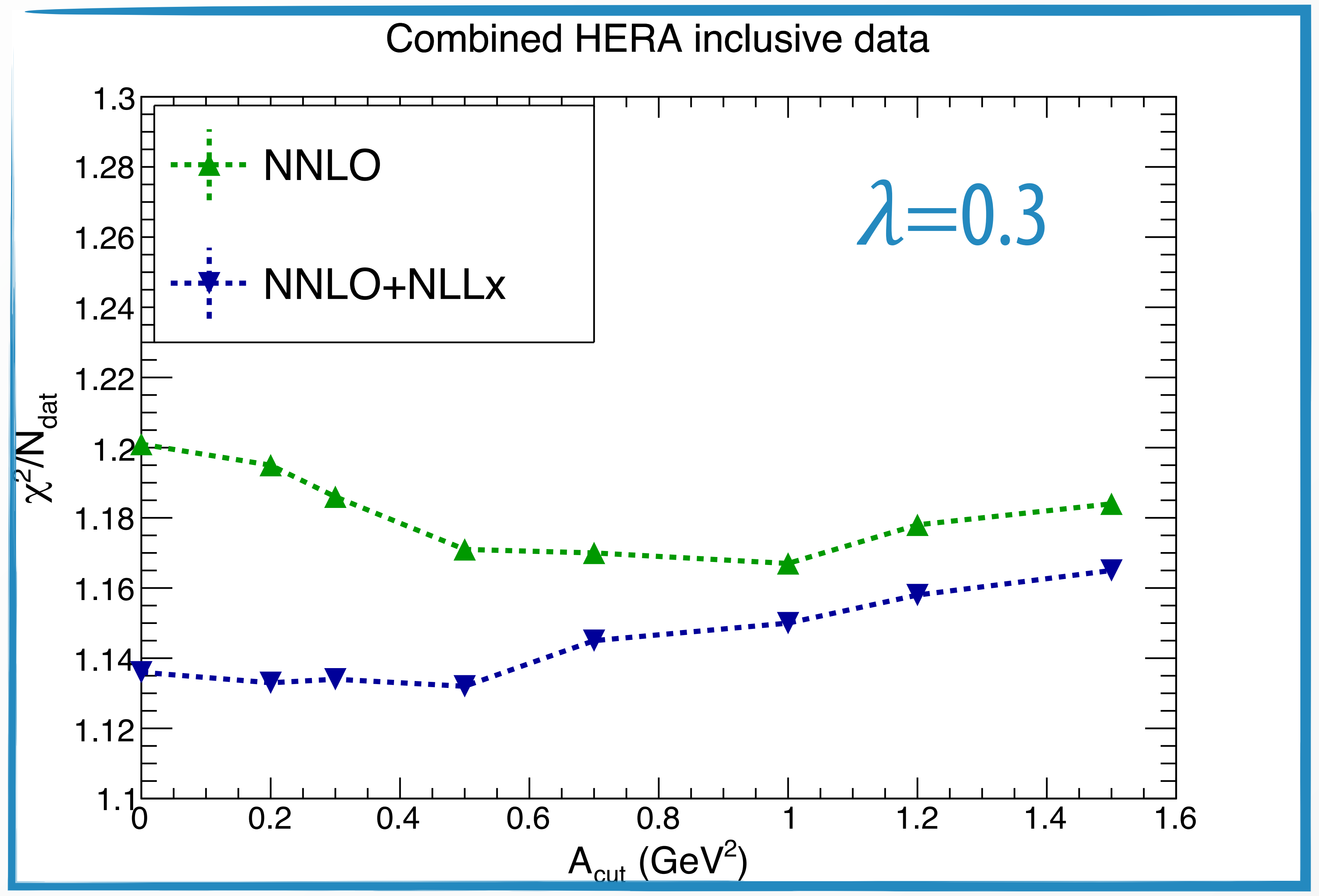
Resummed PDF enhanced in the small-x region

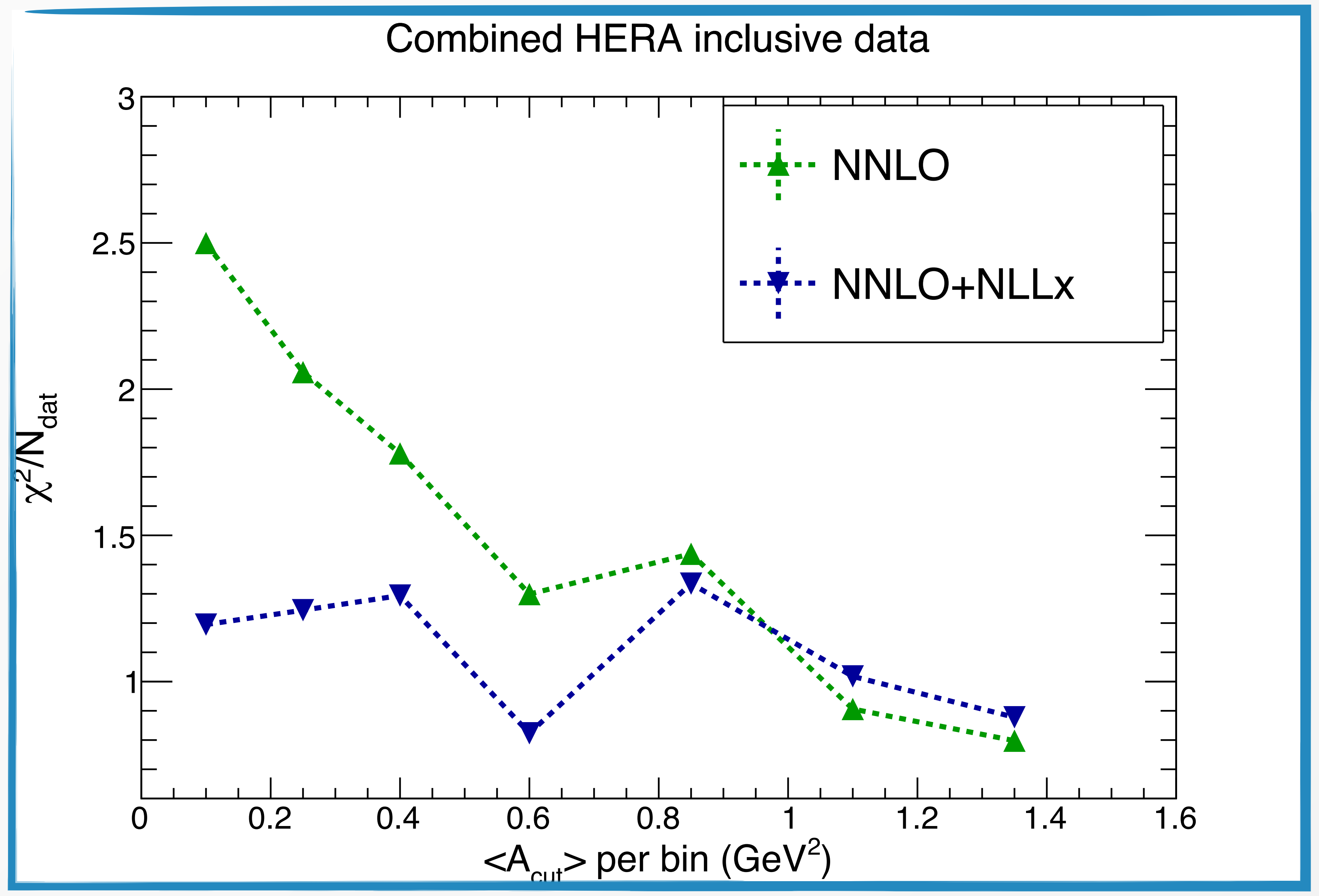




Reduction of the uncertainties at small x

The beginning of a new (H)ERA?





Impact of resummation is reduced if data at small x and small Q are removed

$$Q^2 \geq Q^2_{\rm cut} = A_{\rm cut} x^{-\lambda}$$

NNLO+NLL χ^2 flattens at smaller values of $A_{\rm cut}$

Effect is more pronounced if regions where small-x resummation should have bigger effect are isolated

$$A^{\rm min}_{\rm cut} \leq Q^2 x^{\lambda} \leq A^{\rm max}_{\rm cut}$$

The beginning of a new (H)ERA?

- Towards a **first global fit** with small-x resummation in the NNPDF framework
- Evidence that **NNLO+NLLx improves** with respect to NNLO
- Description of the data at small x/small Q^2 significantly improves when resummation effects are included
- Potential for reducing uncertainties for processes not necessarily related to small-x physics

- Non-negligible **impact on phenomenology***

	σ^{N3LO} (ggH) @ LHC 13 TeV
(preliminary) NNLO+NLLx	47.8
(preliminary) NNLO	47.2

Outlook

- Computation of small-x resummation for other processes
- Motivation to explore further probes of small-x dynamics at the LHC, such as low-mass DY at LHCb
- PDF sets with **joint** (large-x & small-x) resummation?

*For consistency, small-x resummation should be included in Higgs production

backup

Threshold resummation in a nutshell

$$\sigma(x,Q^2) = x\int_x^1 \frac{dz}{z}\mathcal{L}\left(\frac{x}{z},Q^2\right)\frac{\hat{\sigma}(z,Q^2)}{z}$$

Convolution integral diagonalise in **Mellin space**

$$\sigma(N,Q^2) = \mathcal{L}(N,Q^2)\sigma_0(N,Q^2)C(N)$$

Double logarithmic enhancement due to soft gluon emission

$$C(N) = 1 + \sum_{n=1}^{\infty}\alpha_s\sum_{k=0}^{2n} c_{nk}\ln^k N + \mathcal{O}(1/N)$$

N-soft

Exponentiation

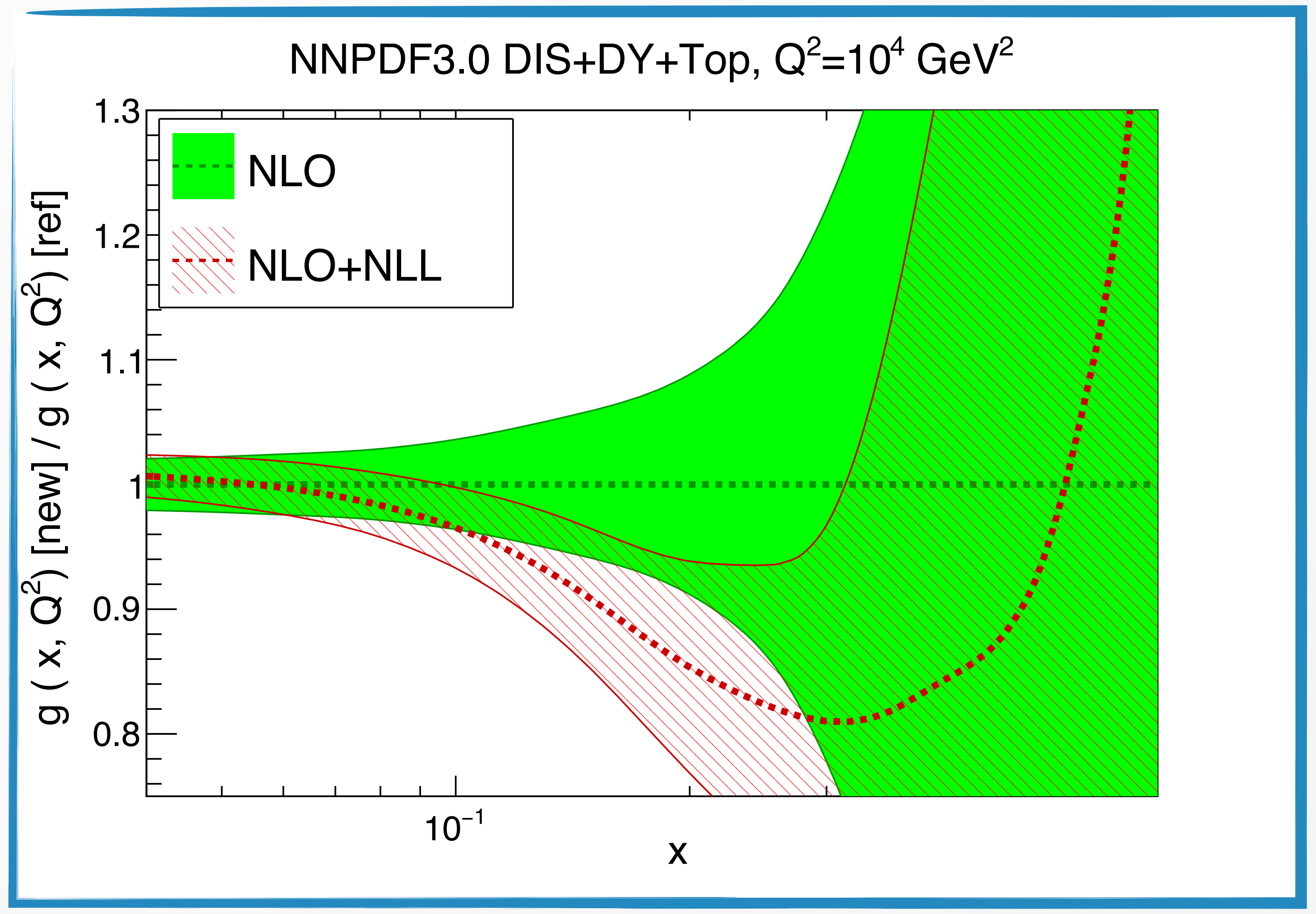
$$C(N) = g_0(\alpha_s)\exp\left[\frac{1}{\alpha_s}g_1(\alpha_s\ln N) + g_2(\alpha_s\ln N) + \alpha_s g_3(\alpha_s\ln N) + \ldots\right]$$

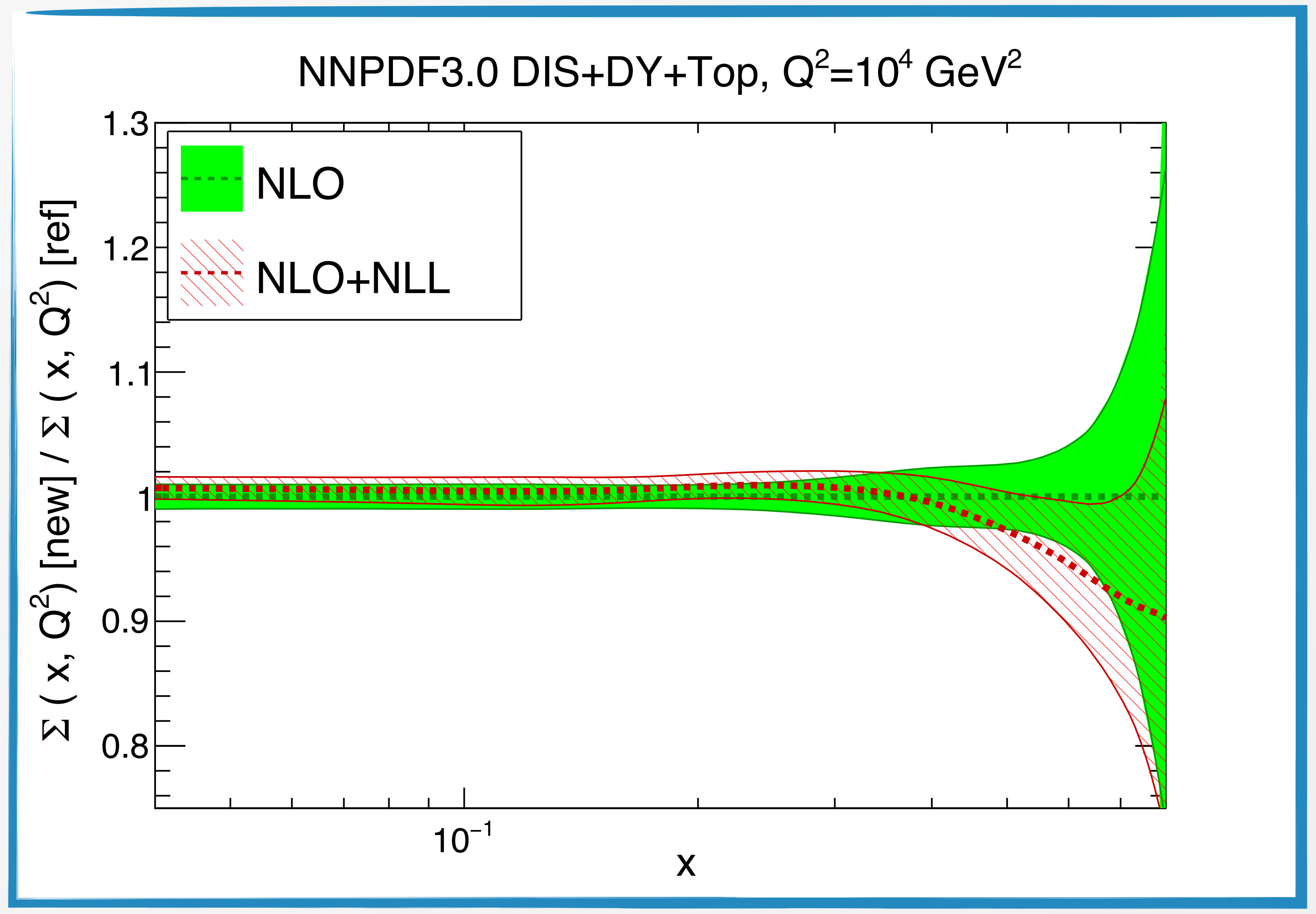
LL **NLL** **NNLL**

The functions g_i resum $\alpha_s^k \ln^k N$ to all orders

Impact on PDFs

NLO+NLL





NNLO+NNLL