The Drell-Yan fiducial cross section at N^{3}LO+N^{3}LL

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The Drell-Yan process: a standard for precision at the LHC

Lepton-pair production constitutes the most important **standard candle** at hadron colliders

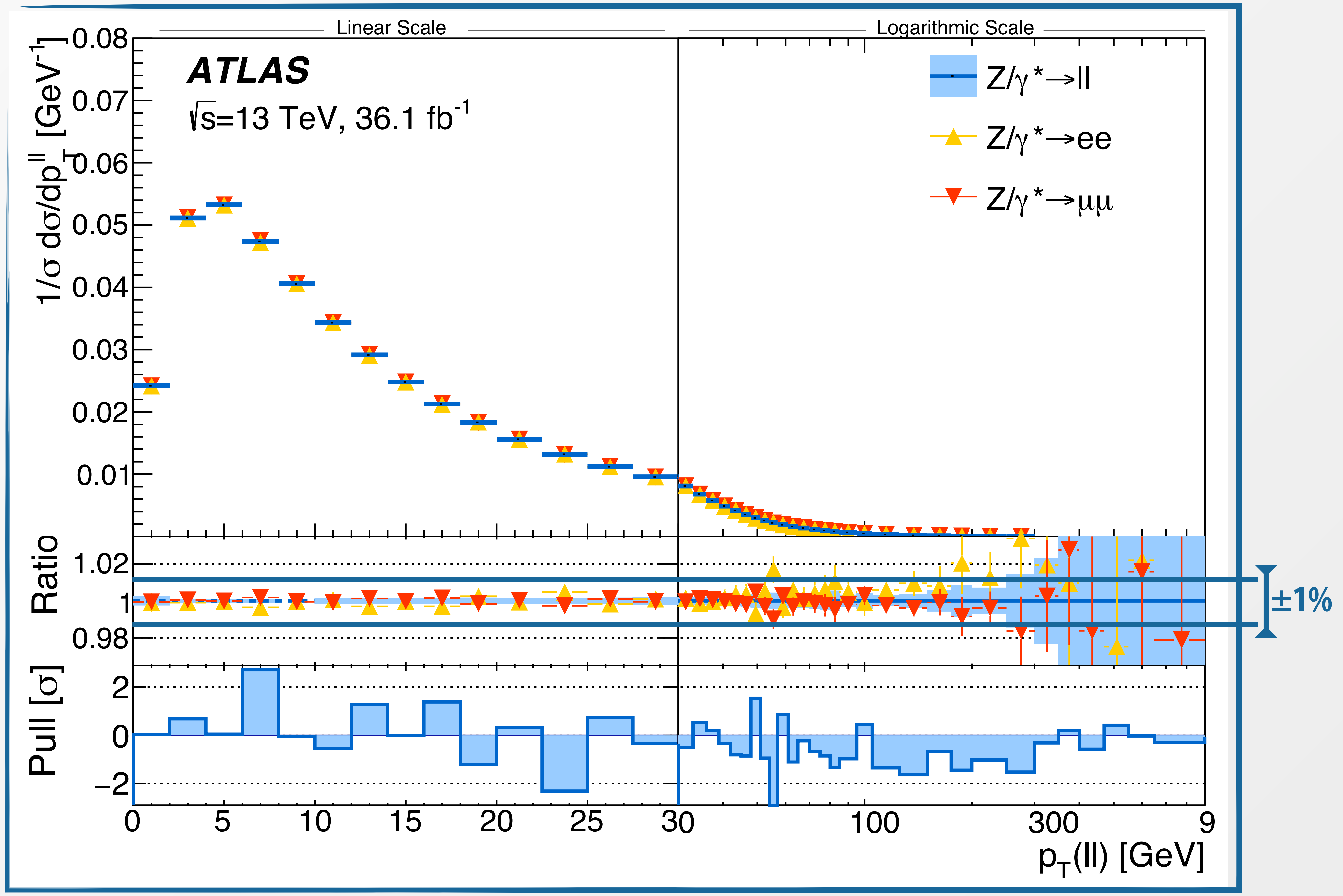
Talk by Chris Pollard

The wealth of data collected enables a **broad spectrum** of applications to different areas:

- determination of SM parameters such as the **W mass**
- extraction of parton densities of the proton
- exploration of BSM scenarios

Theoretical predictions now reach **highest level of accuracy**

- N^3LO for inclusive cross section and rapidity
  Talk by Tonghzi Yang
- N^3LO for fiducial cross section and distributions
  This talk
- NLO EW and mixed QCD-EW at NNLO
  Talk by Luca Buonocore



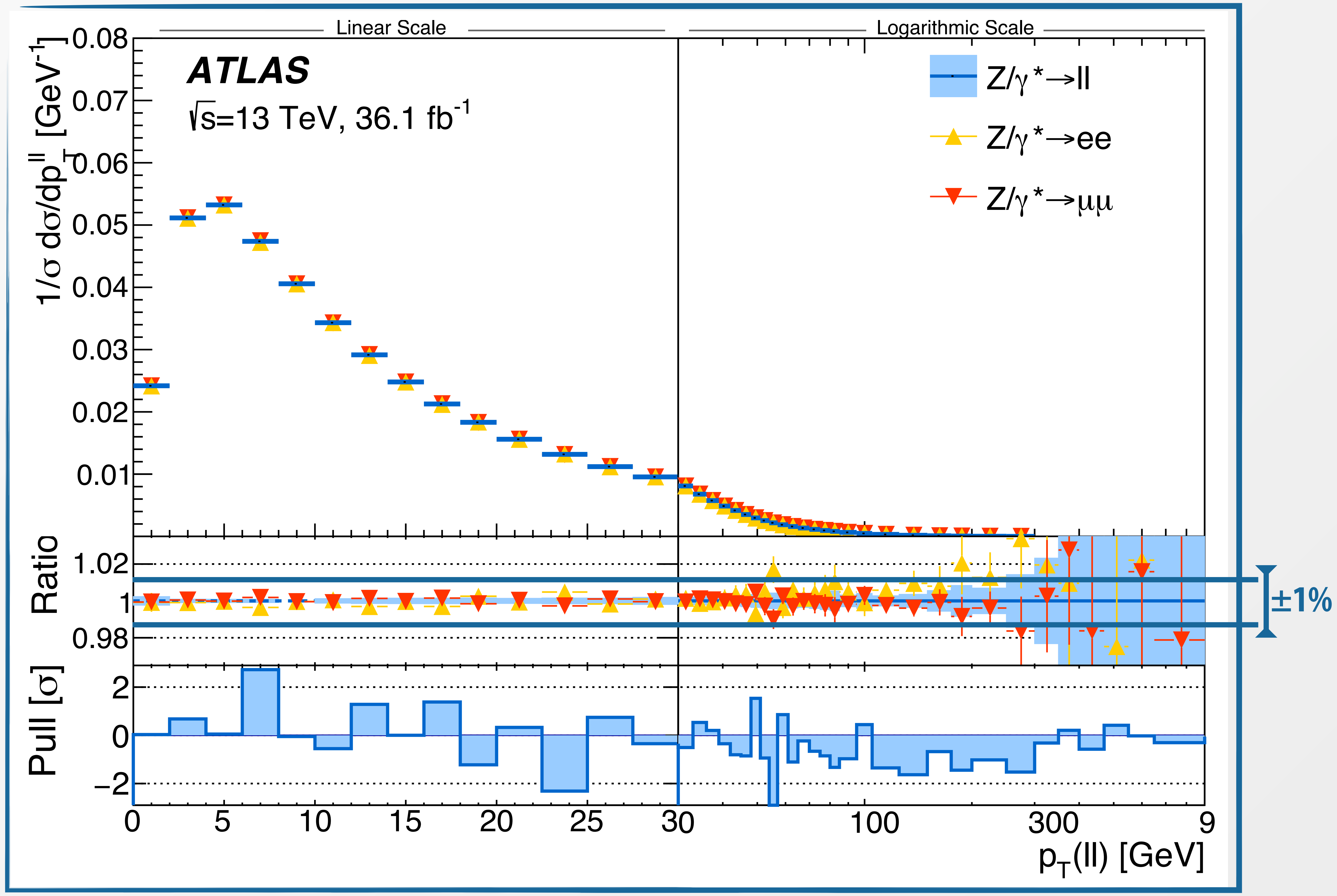
[ATLAS 2019]

The transverse momentum spectrum

Clean experimental and theoretical environment for precision physics

- little or no sensitivity to multi-parton interactions
- reduced sensitivity to non-perturbative effects
- measured extremely precisely at experiments

Very accurate theoretical predictions needed



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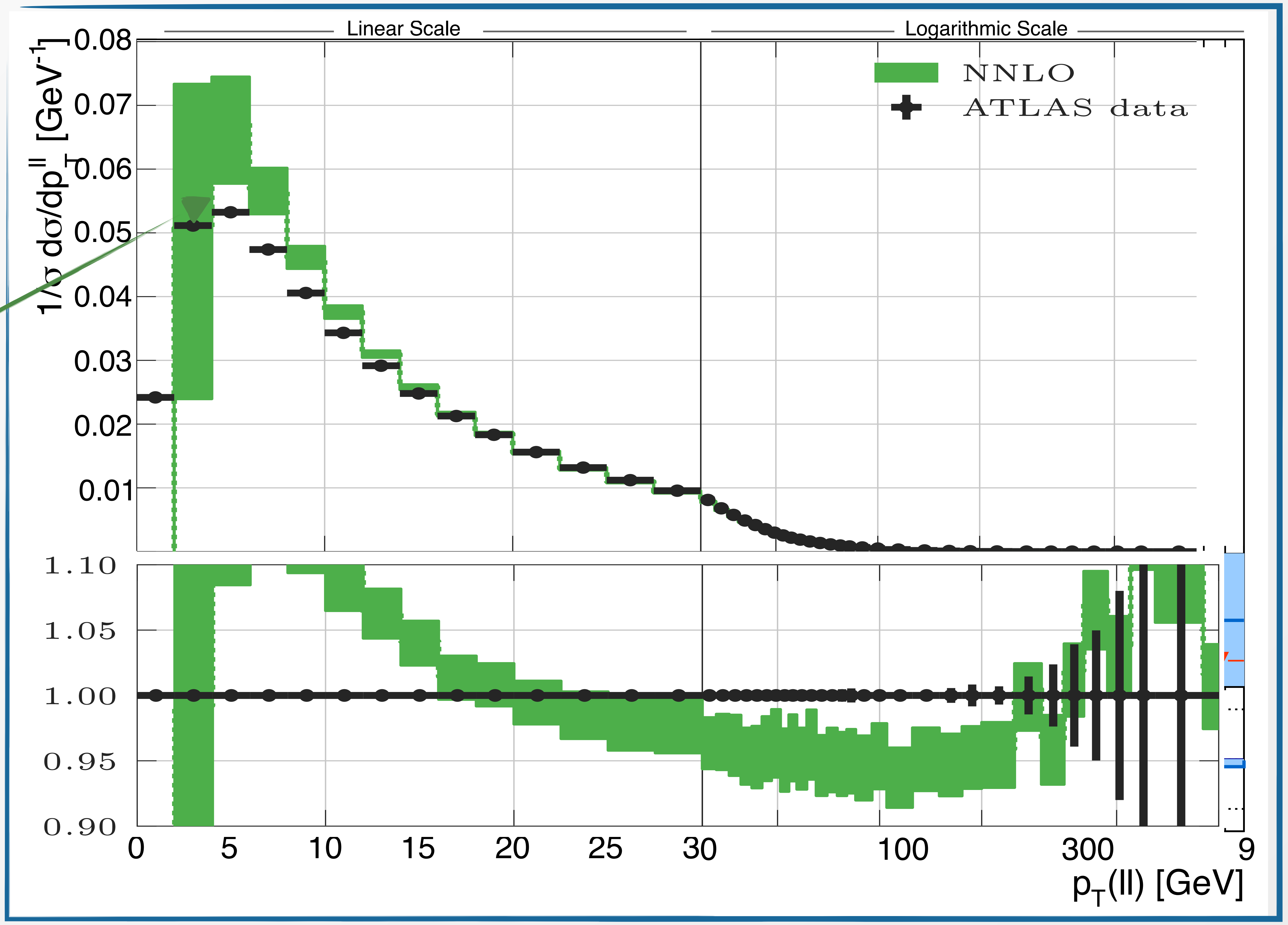
$$L = \ln(p_t^H/m_H) \qquad p_t^H \ll m_H$$



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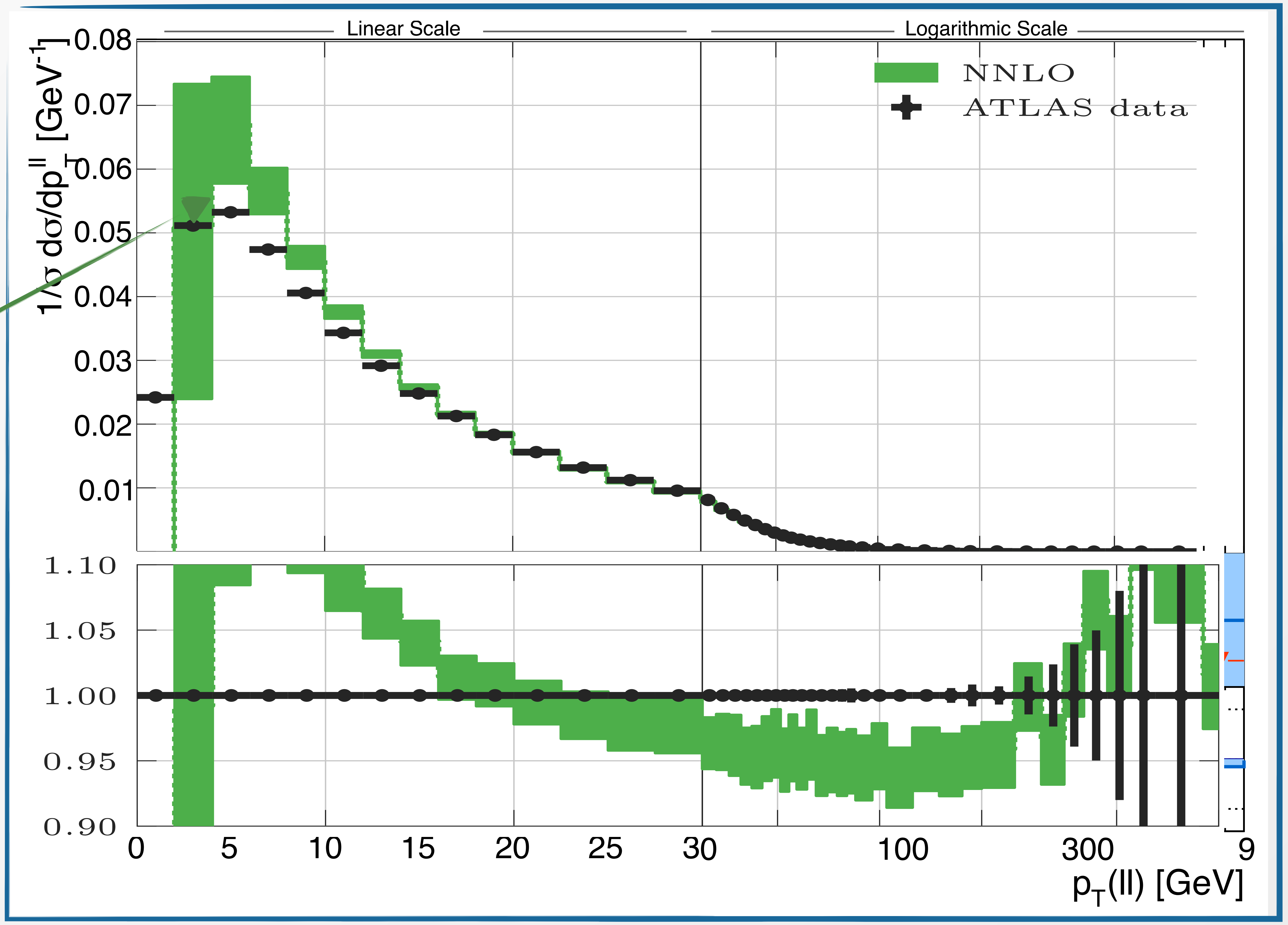


[ATLAS 2019]

Large **transverse momentum** logarithms

$$L = \ln(p_t^H / m_H) \qquad p_t^H \ll m_H$$

Fixed order predictions no longer reliable:
all order resummation of the perturbative series mandatory

Resummation of the transverse momentum spectrum

Solution 1: move to **conjugate space** where phase space factorization is manifest

Talk by Ignazio Scimemi

b-space formulation

[Parisi, Petronzio '79; Collins, Soper, Sterman '85]

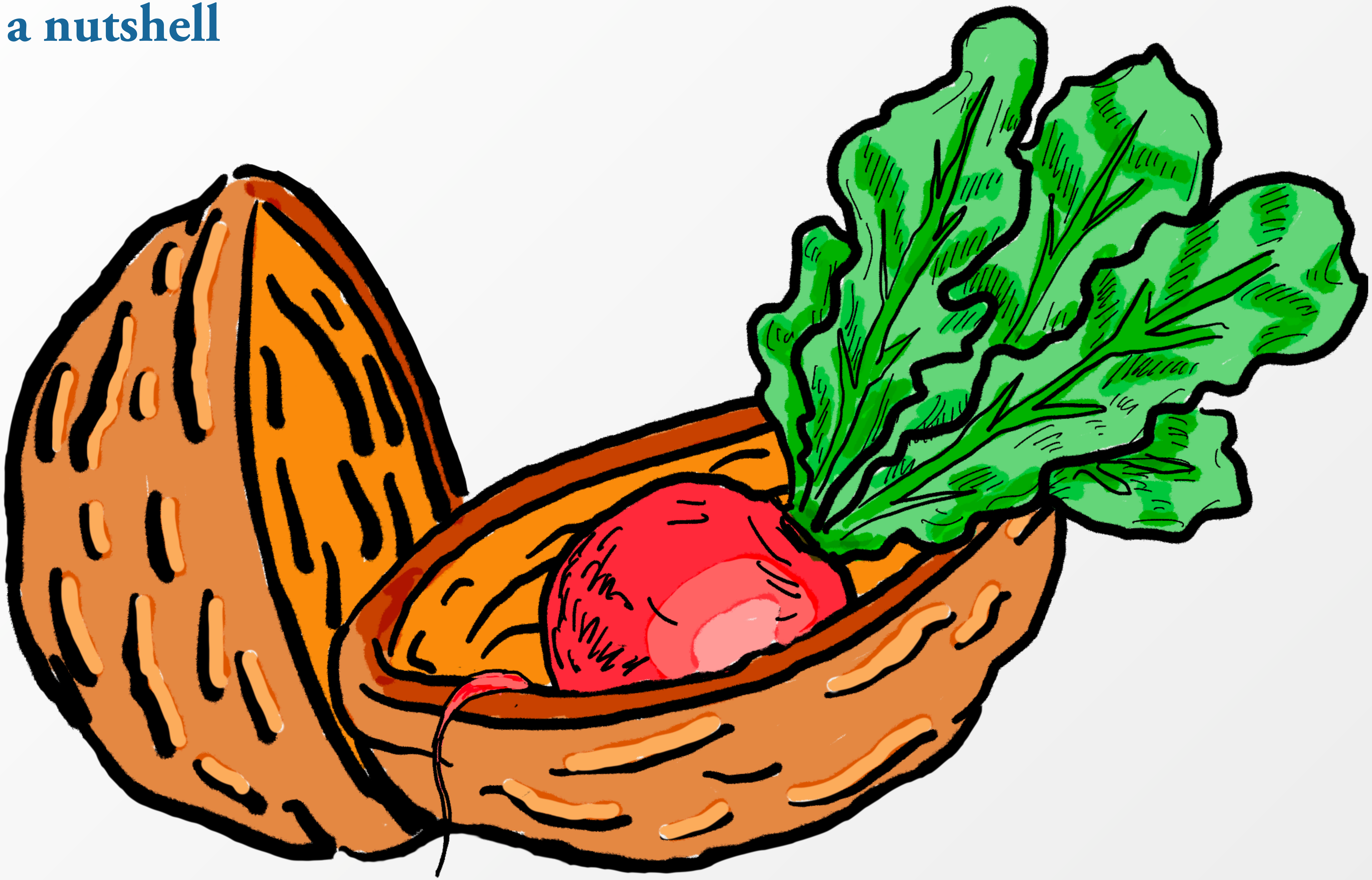
Resummation of the transverse momentum spectrum

Solution 2: resummation in **direct space** exploiting the properties of the observable in the presence of multiple radiation [Banfi, Salam, Zanderighi '03]

RadISH formulation

[Bizon, Monni, Re '16][Bizon, Monni, Re, LR, Torrielli '17]

RadISH in a nutshell



Resummation of the transverse momentum spectrum in direct space

Result at NLL accuracy with scale-independent PDFs

$$\sigma(p_\perp) = \sigma_0 \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(\epsilon v_1)}$$

$$\times R'\left(v_1\right) \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'\left(\zeta_i v_1\right) \Theta\left(p_\perp - |\vec{k}_{t,i} + \cdots \vec{k}_{t,n+1}|)\right)$$

$$v_i = k_{t,i}/m_H, \quad \zeta_i = v_i/v_1$$

Resummation of the transverse momentum spectrum in direct space

Result at NLL accuracy with scale-independent PDFs

Simple observable

$$\sigma(p_\perp) = \sigma_0 \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(\epsilon v_1)}$$

$$v_i = k_{t,i}/m_H, \quad \zeta_i = v_i/v_1$$

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Transfer function

Formula can be evaluated with Monte Carlo method; dependence on ϵ vanishes (as $\mathcal{O}(\epsilon)$) and result is finite in four dimensions

Logarithmic accuracy defined in terms of $\ln(m_H/k_{t1})$

Result formally equivalent to the b-space formulation [Bizon, Monni, Re, LR, Torrielli '17]

Resummation available at N^{3}LL accuracy [Bizon, Monni, Re, LR, Torrielli '17][Re, LR, Torrielli '21]

Matching with fixed order: N^3LO+N^3LL

To obtain predictions valid across the whole q_T spectrum the resummation result must be matched with the fixed order result

Fully differential formula in the **transverse momentum** q_T and in the Born kinematic variables for the production of a colour singlet V

Finite for $q_T \to 0$: integral over q_T allows one to obtain N^kLO+N^kLL predictions within fiducial cuts

$$d\sigma_V^{\mathrm{N^kLO+N^kLL}} \equiv d\sigma_V^{\mathrm{N^kLL}} + d\sigma_{\mathrm{V+jet}}^{\mathrm{N^{k-1}LO}} - \left[d\sigma_V^{\mathrm{N^kLL}}\right]_{\mathcal{O}(\alpha_s^k)}$$

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$\mathrm{N^kLL}$ resummed q_T distribution

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differential q_T distribution at N^{k-1}LO

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Expansion of the N^kLL resummed q_T distribution at order $\mathcal{O}(\alpha_s^k)$

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Both **diverge logarithmically** for $q_T \to 0$: high numerical precision required in the $d\sigma_{\mathrm{V+jet}}^{\mathrm{N^{k-1}LO}}$ down to very small values of q_T

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Setting $d\sigma_{\mathrm{V+jet}}^{\mathrm{N^{k-1}LO}} - \left[d\sigma_V^{\mathrm{N^kLL}} \right]_{\mathcal{O}(\alpha_s^k)} = 0$ for $q_T \leq q_T^{\mathrm{cut}}$ introduces a **slicing error** of order $\mathcal{O}((q_T^{\mathrm{cut}}/M)^n)$

q_T-subtraction and power corrections

The perturbative expansion of the N^{k}LL+N^{k}LO fiducial cross section to third order in α_s leads to the N^{k}LO prediction as obtained according to the q_T-subtraction formalism [Catani, Grazzini '07]

$$d\sigma_V^{\mathrm{N^kLO}} \equiv \mathscr{H}_{\mathrm{V}}^{\mathrm{N^kLO}} \otimes d\sigma_{\mathrm{V}}^{\mathrm{LO}} + \left(d\sigma_{\mathrm{V+jet}}^{\mathrm{N^{k-1}LO}} - \left[d\sigma_V^{\mathrm{N^kLL}} \right]_{\mathcal{O}(\alpha_s^k)} \right) \Theta(q_T > q_t^{\mathrm{cut}}) + \mathcal{O}((q_T^{\mathrm{cut}}/M)^n)$$

Virtual correction after subtraction of IR singularities and contribution of soft/collinear origin **(beam, soft, jet functions)**

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Missing **power corrections** below the slicing cut-off

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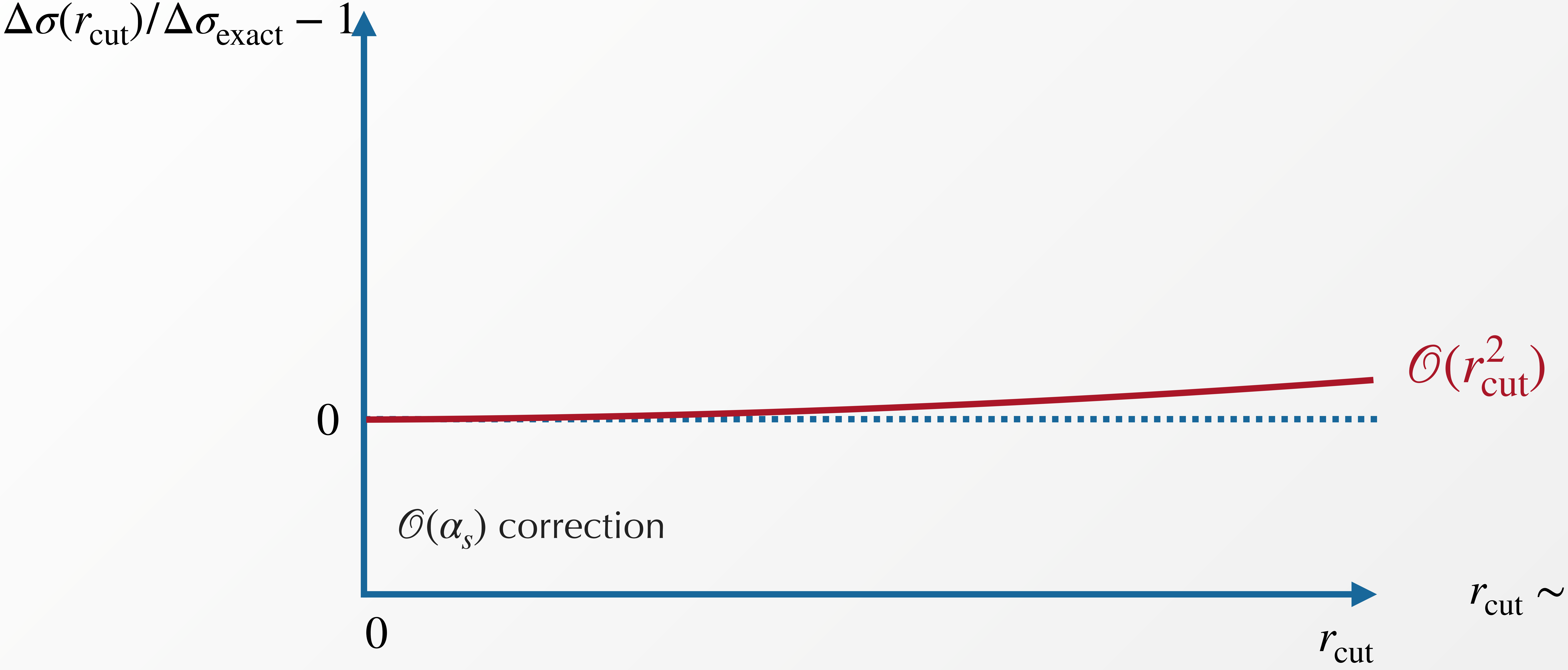
Missing **power corrections** below the slicing cut-off

Relative size of power corrections affects **stability and performance** of non-local subtraction methods

The larger the power corrections, the lower are the values of the slicing parameters needed for extrapolation of correct result (CPU consuming, numerically unstable)

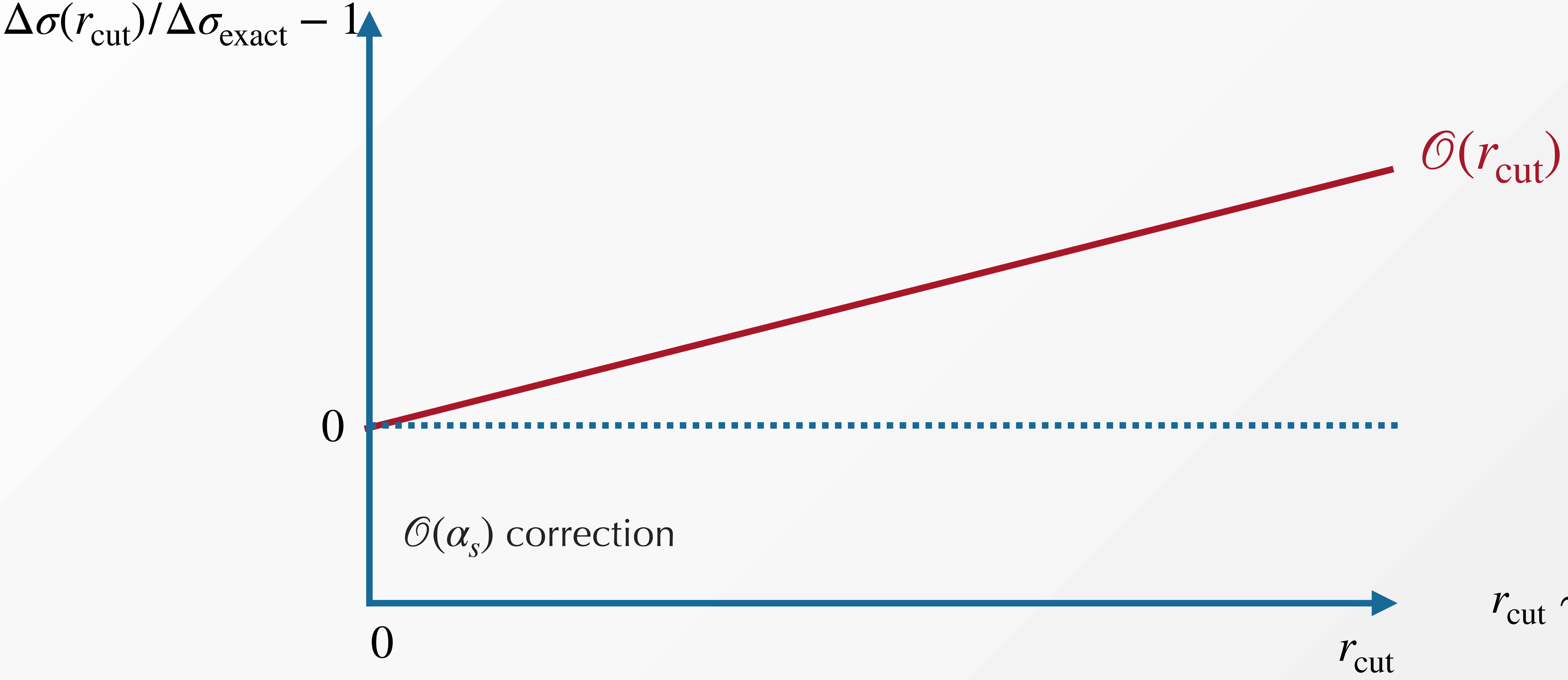
Non-local subtraction and power corrections



$r_{\rm cut} \sim q_T/Q$

q_T**-subtraction** with inclusive cuts and in various fiducial setups

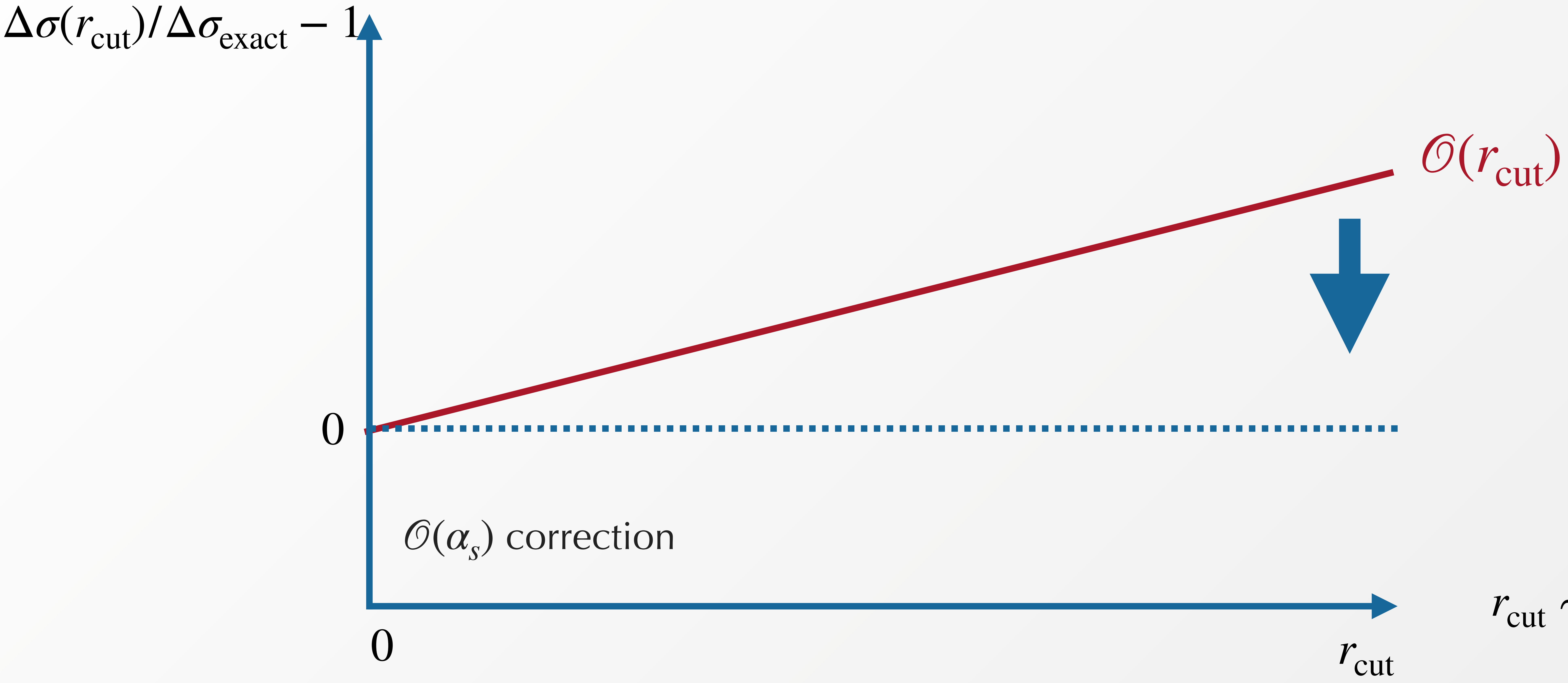
Non-local subtraction and power corrections



q_T**-subtraction** for $2 \to 2$ processes with (a)symmetric cuts

$r_{\text{cut}} \sim q_T/Q$

Non-local subtraction and power corrections



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For $2 \to 2$ processes with (a)symmetric cuts, fiducial linear power corrections can be resummed at all orders via a simple recoil prescription

[Catani, de Florian, Ferrera, Grazzini '15][Billis, Dehnadi, Ebert, Michel, Stewart, Tackmann '21]

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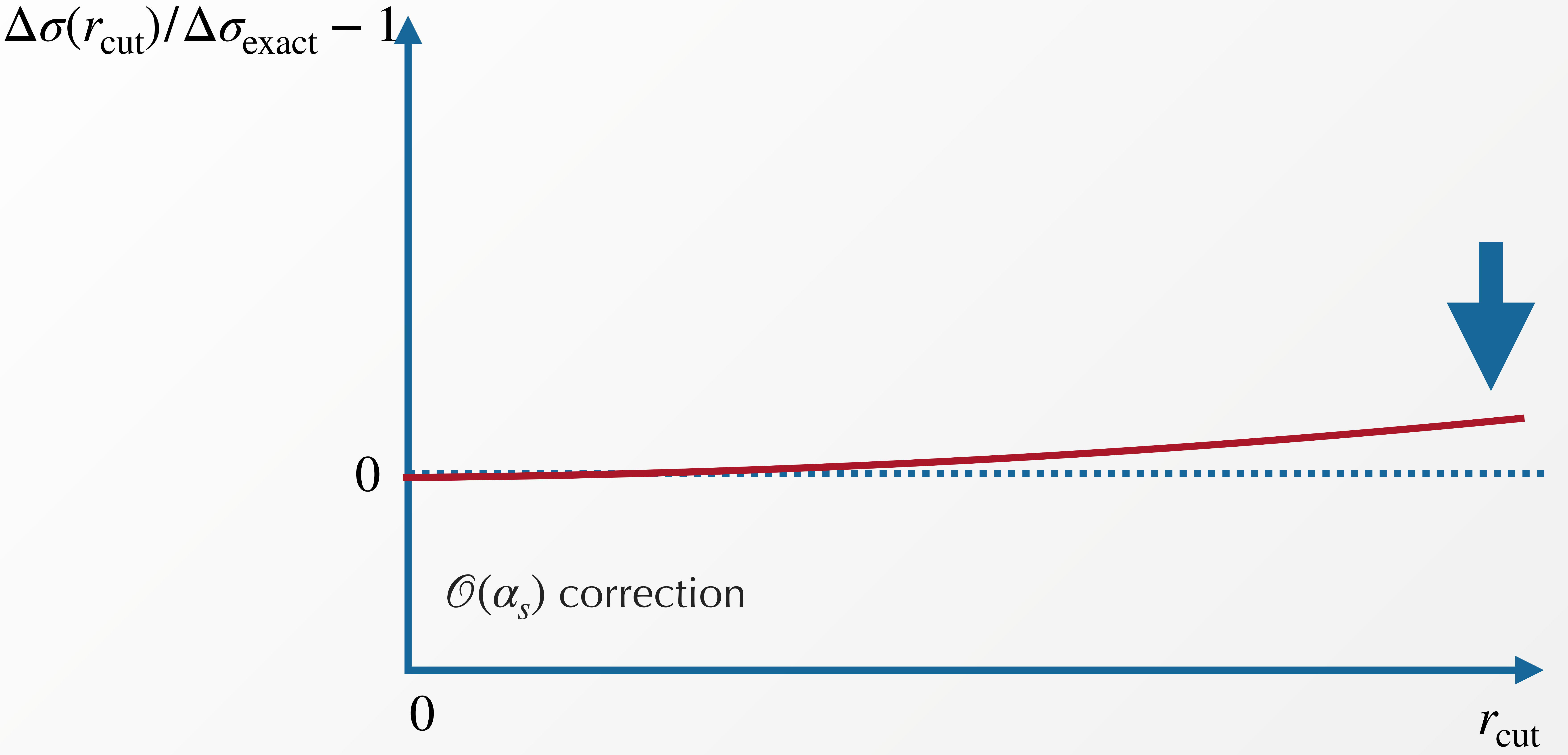

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Inclusion of transverse recoil effects

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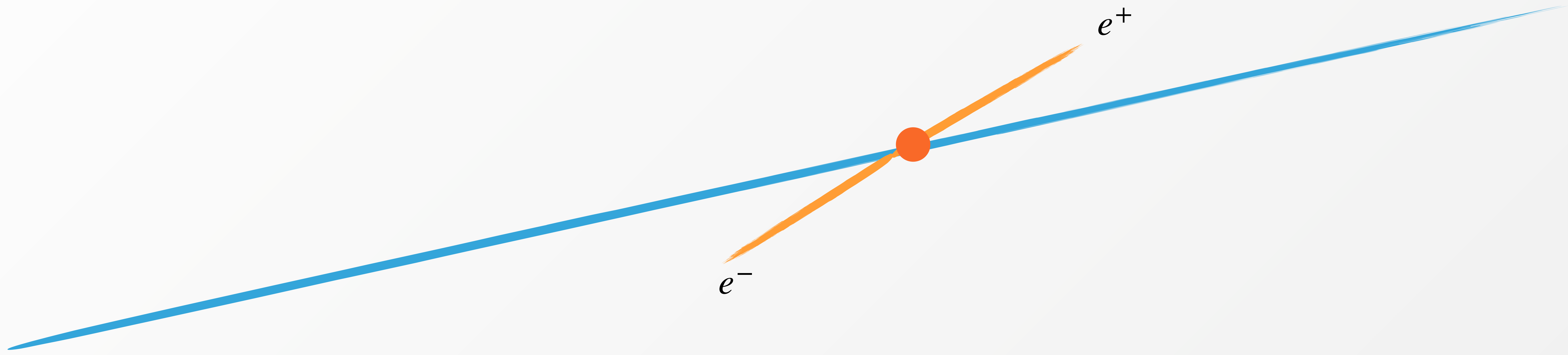
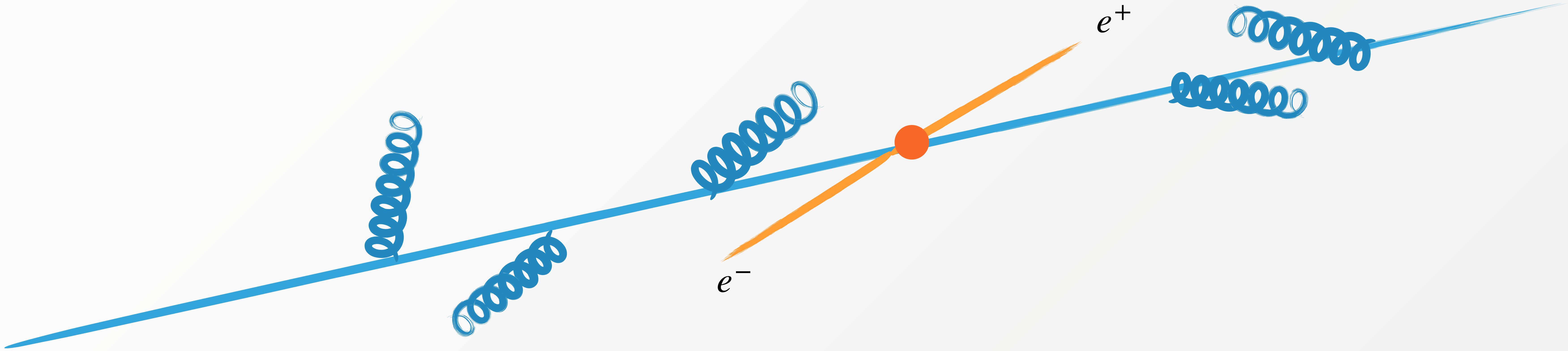


Born matrix element
evaluated at $q_T = 0$

Inclusion of transverse recoil effects

[Catani, de Florian, Ferrera, Grazzini '15]



Generate singlet q_T by QCD radiation

Inclusion of transverse recoil effects

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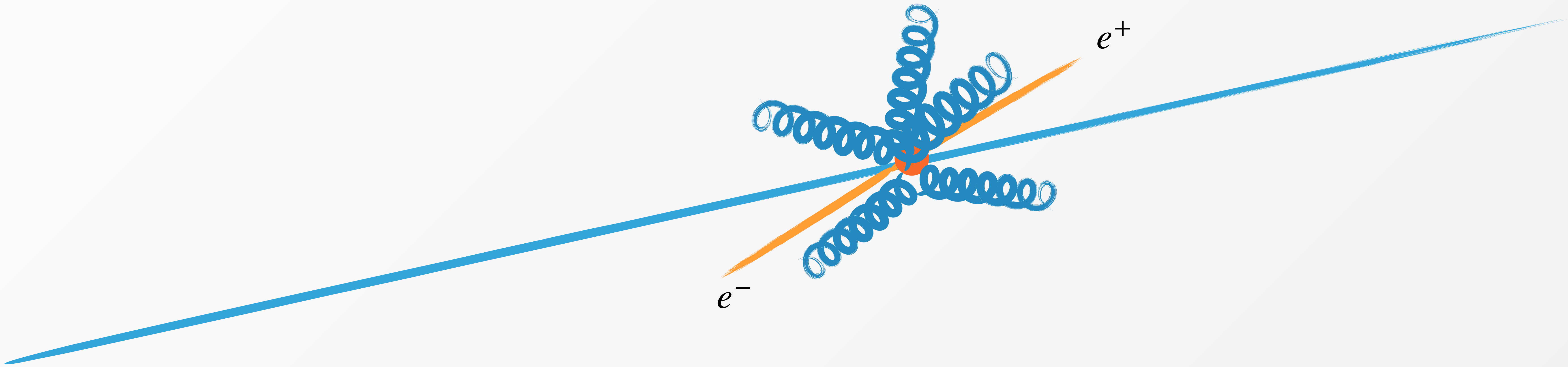

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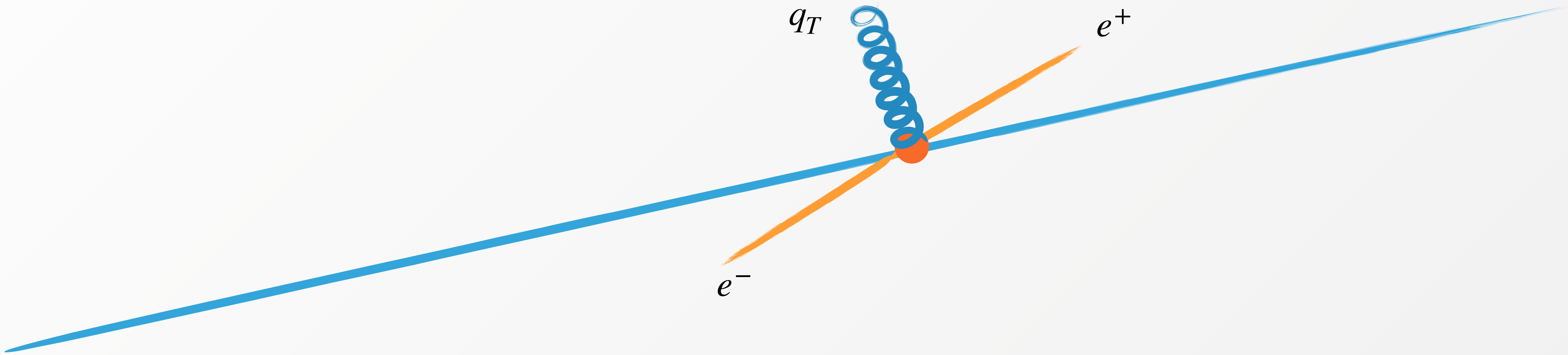

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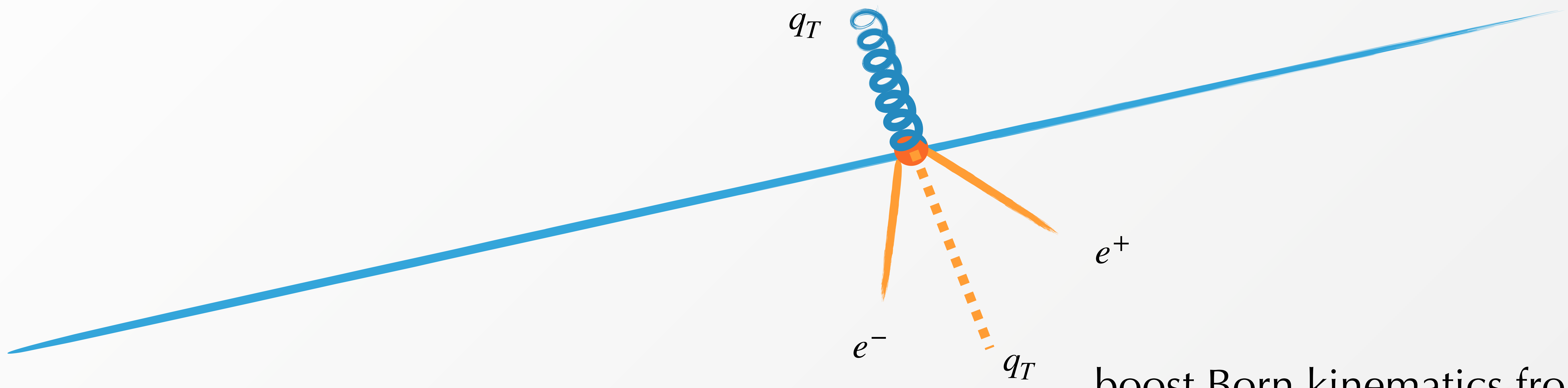
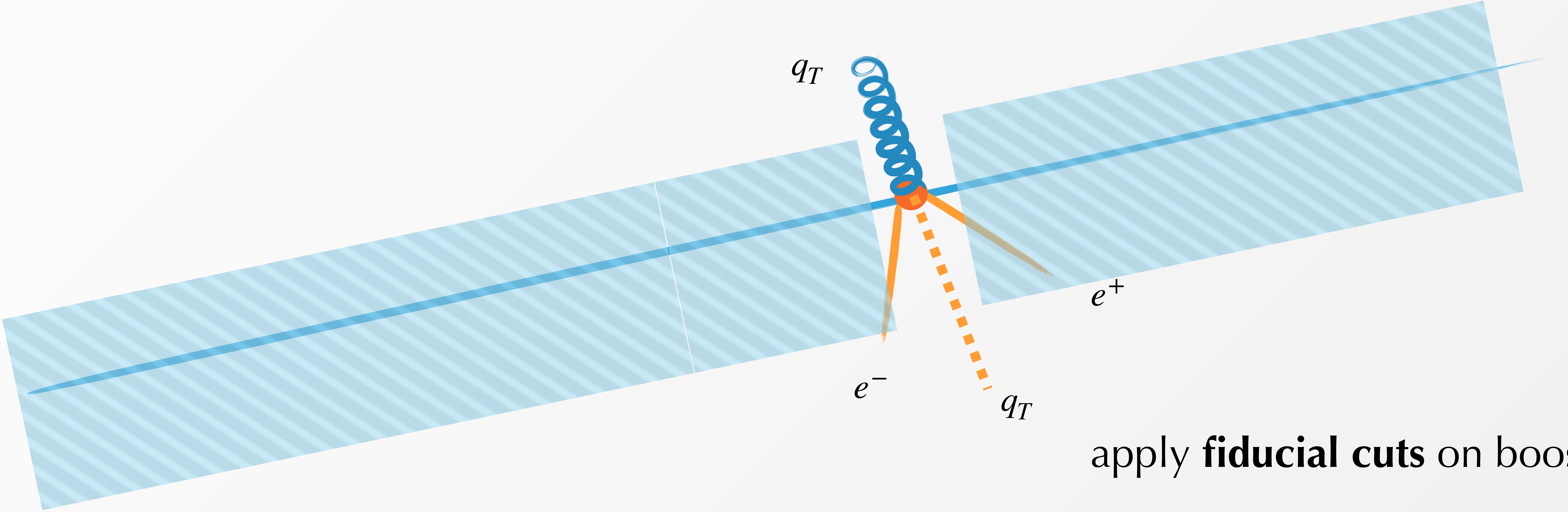


boost Born kinematics from boson rest frame (e.g. CS) to lab frame with that q_T

Inclusion of transverse recoil effects

[Catani, de Florian, Ferrera, Grazzini '15]



apply **fiducial cuts** on boosted Born kinematics

Inclusion of transverse recoil effects

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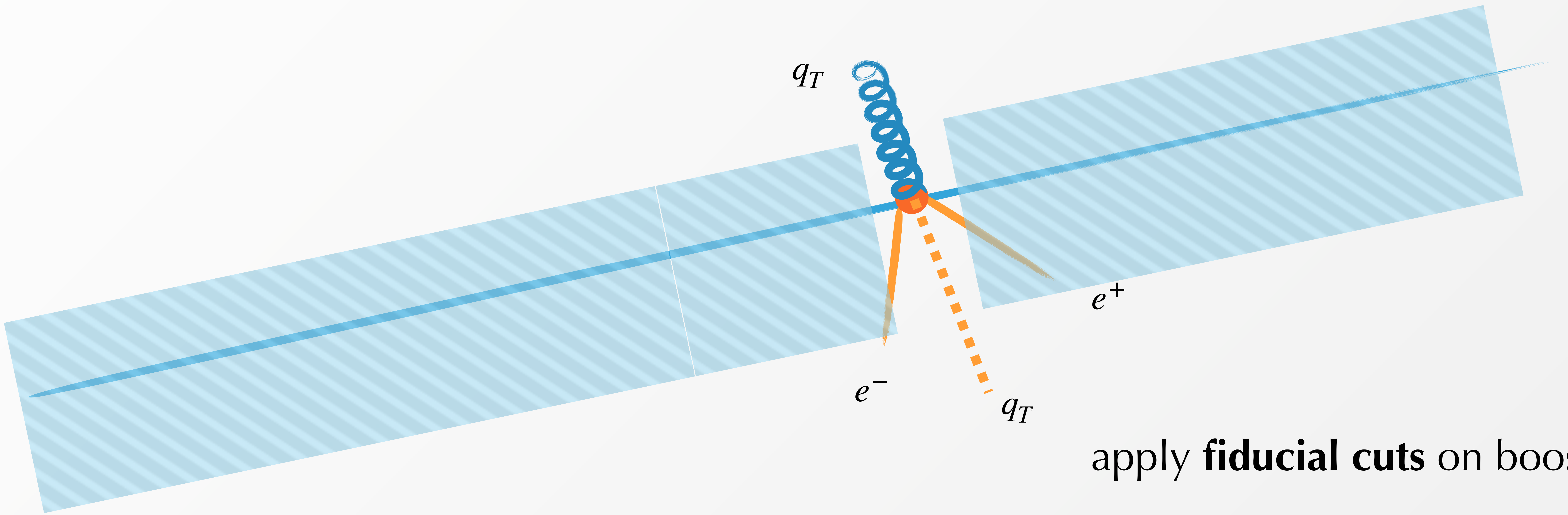
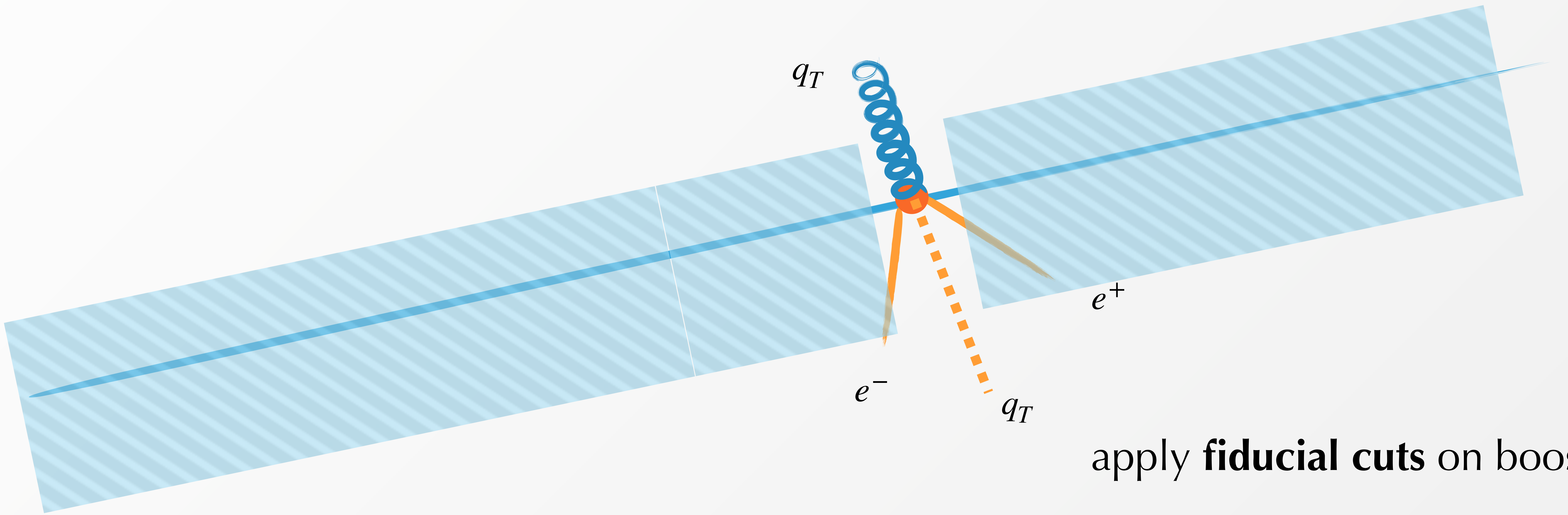


apply **fiducial cuts** on boosted Born kinematics

Sufficient to capture the **full linear fiducial power correction** for q_T [Ebert et al. '20]

Inclusion of transverse recoil effects

[Catani, de Florian, Ferrera, Grazzini '15]



apply **fiducial cuts** on boosted Born kinematics

Implementation in RadISH: [Re, LR, Torrielli '21]

- Each contribution in the resummation formula **boosted in the corresponding frame**
- Derivative of the expansion computed on-the-fly, boost computed according to the value of q_T

The Drell-Yan fiducial cross section at N³LO and N³LO+N³LL

The above considerations are particularly relevant for the case of **Drell-Yan productions within fiducial cuts**

ATLAS (and CMS) experiments define their fiducial region using ***symmetric*** cuts on the lepton transverse momenta

ATLAS fiducial region $p_T^{\ell^\pm} > 27\,\text{GeV} \qquad |\eta^{\ell^\pm}| < 2.5$

Remark: linear power corrections in the **symmetric/asymmetric** case are related to **ambiguities** in the perturbative expansion and **can be avoided with different sets of cuts**

[Salam, Slade '21]

All necessary ingredients available to calculate N^3LO cross section using q_T-subtraction

[Gehrmann et al '10][Catani, Cieri, de Florian,Ferrera,Grazzini '12][Gehrmann, Luebbert, Yang '14][Li, Zhu '17][Luo,Yang,Zhu,Zhu '19, '21][Ebert,Mistlberger,Vita '20]

N^3LO cross section for on-shell Drell-Yan production calculated using q_T-subtraction and compared to analytic calculation [Chen, Gehrmann, Glover, Huss, Yang, Zhu '21]

Talk by Tonghzi Yang

First estimates of the N^3LO correction in the fiducial region obtained using these ingredients [Camarda, Cieri, Ferrera '21]

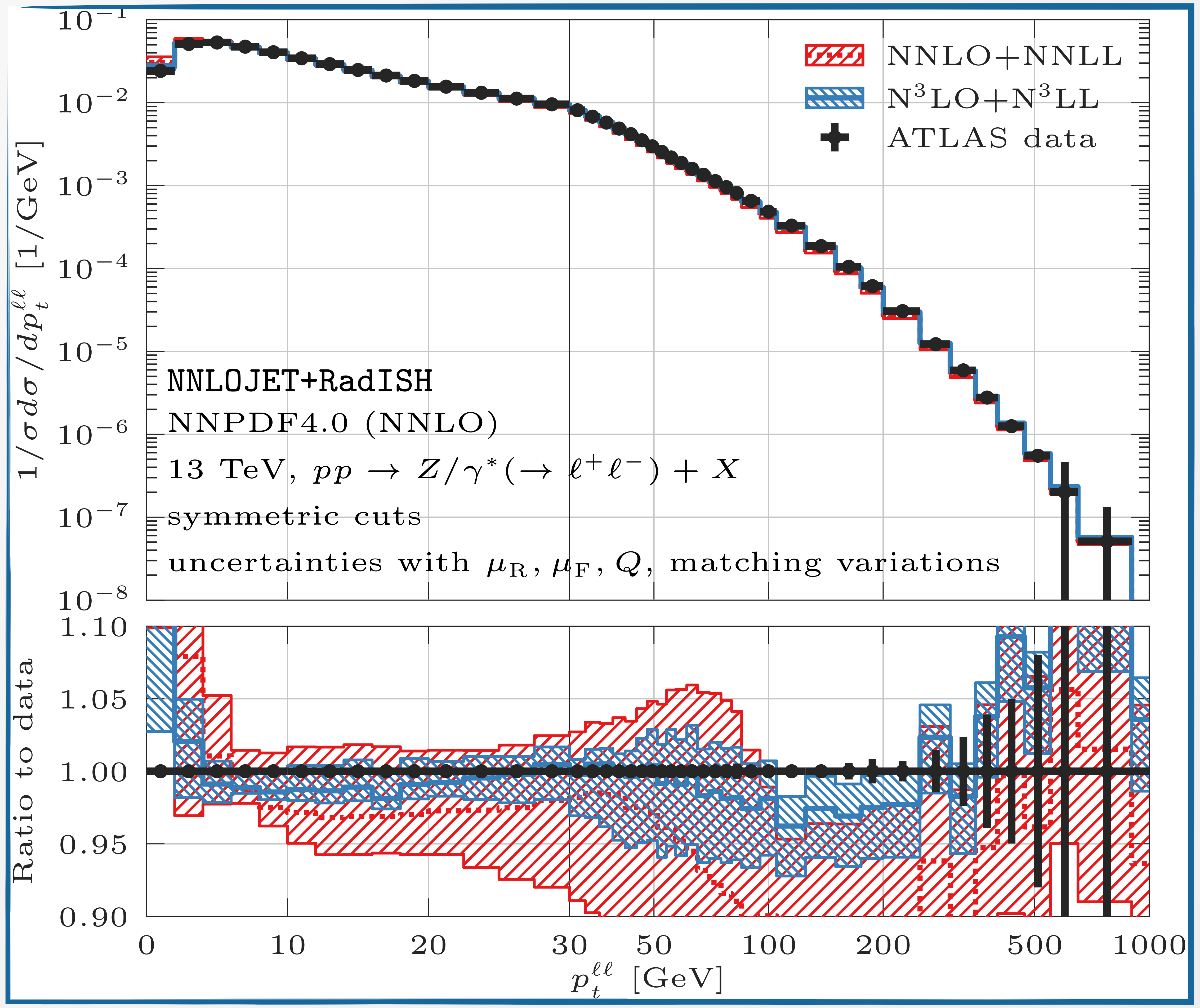
Full control on the **theory systematics** is paramount due to the astonishing precision of the experimental data (permille-level!)

Transverse momentum spectrum at N³LO+N³LL

[Chen, Gehrmann, Glover, Huss, Monni, Re, LR, Torrielli '22]

$$d\sigma_V^{\mathrm{N^kLO+N^kLL}} \equiv d\sigma_V^{\mathrm{N^kLL}} + d\sigma_{\mathrm{V+jet}}^{\mathrm{N^{k-1}LO}} - \left[d\sigma_V^{\mathrm{N^kLL}}\right]_{\mathcal{O}(\alpha_s^k)}$$

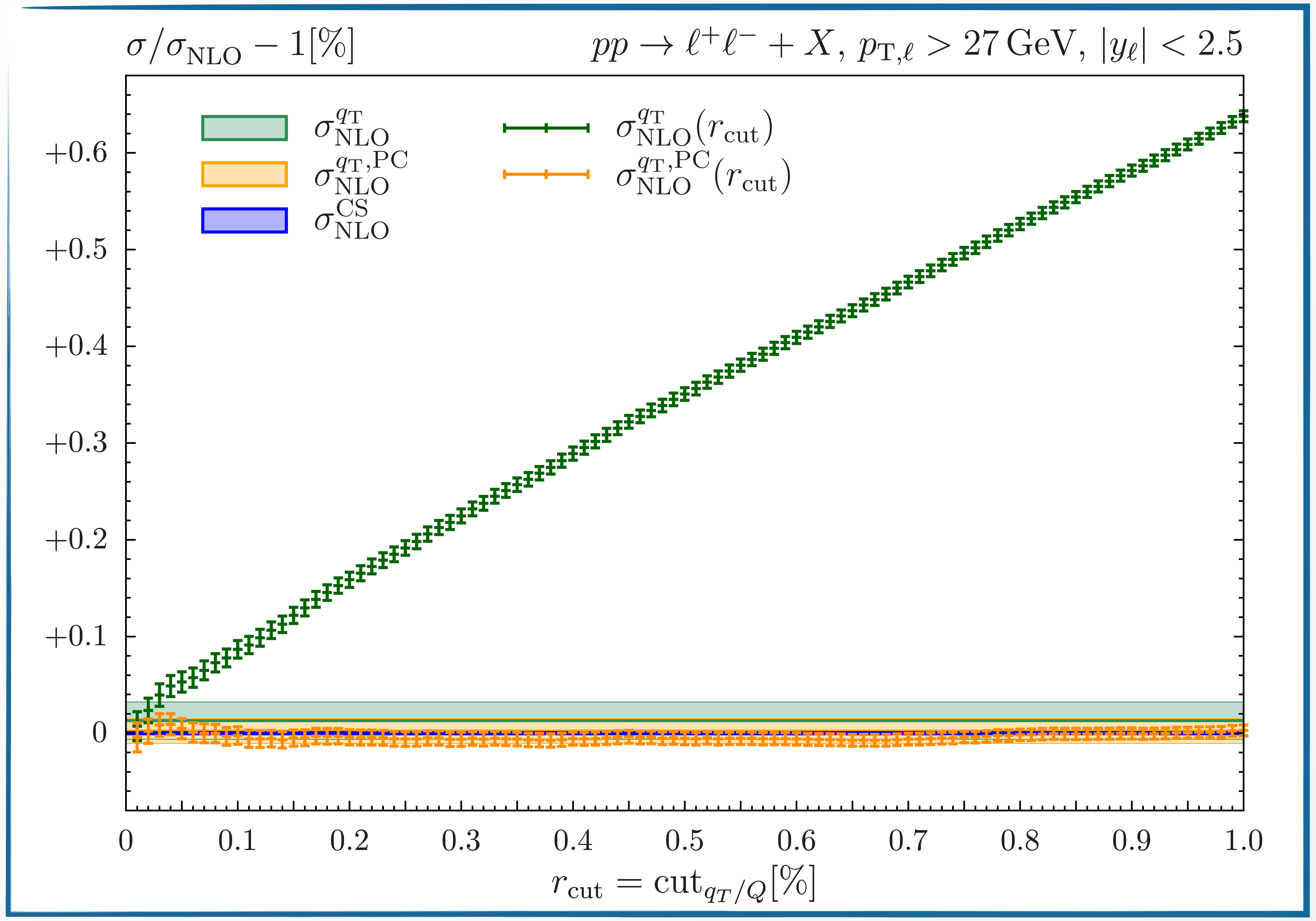
- **Excellent description** of the data across the whole q_T spectrum,
- First bin which is susceptible to non-perturbative corrections
- Non-singular (matching) correction **non-negligible** even below $q_T \lesssim 15$ GeV
- Residual theoretical uncertainty in the intermediate q_T region is at the **few-percent level**, about 5% for $q_T \gtrsim 50$ GeV



Linear power corrections for q_T-subtraction

Resorting to the recoil prescription allows the inclusion of all missing fiducial linear power corrections below $r_{\rm cut}$, improving dramatically the efficiency of the non-local subtraction

[Buonocore, Kallweit, LR, Wiesemann'21] [Camarda, Cieri, Ferrera '21]



[Buonocore, Kallweit, LR, Wiesemann '21]

Much improved convergence over linear power correction case

Accurate computation of the N^{k}LO correction without the need to push $r_{\rm cut}$ to very low values

Now available in **MATRIX 2.1**

Talk by Simone Devoto

Resorting to this prescription allows one to obtain precise and reliable predictions at N^{3}LO

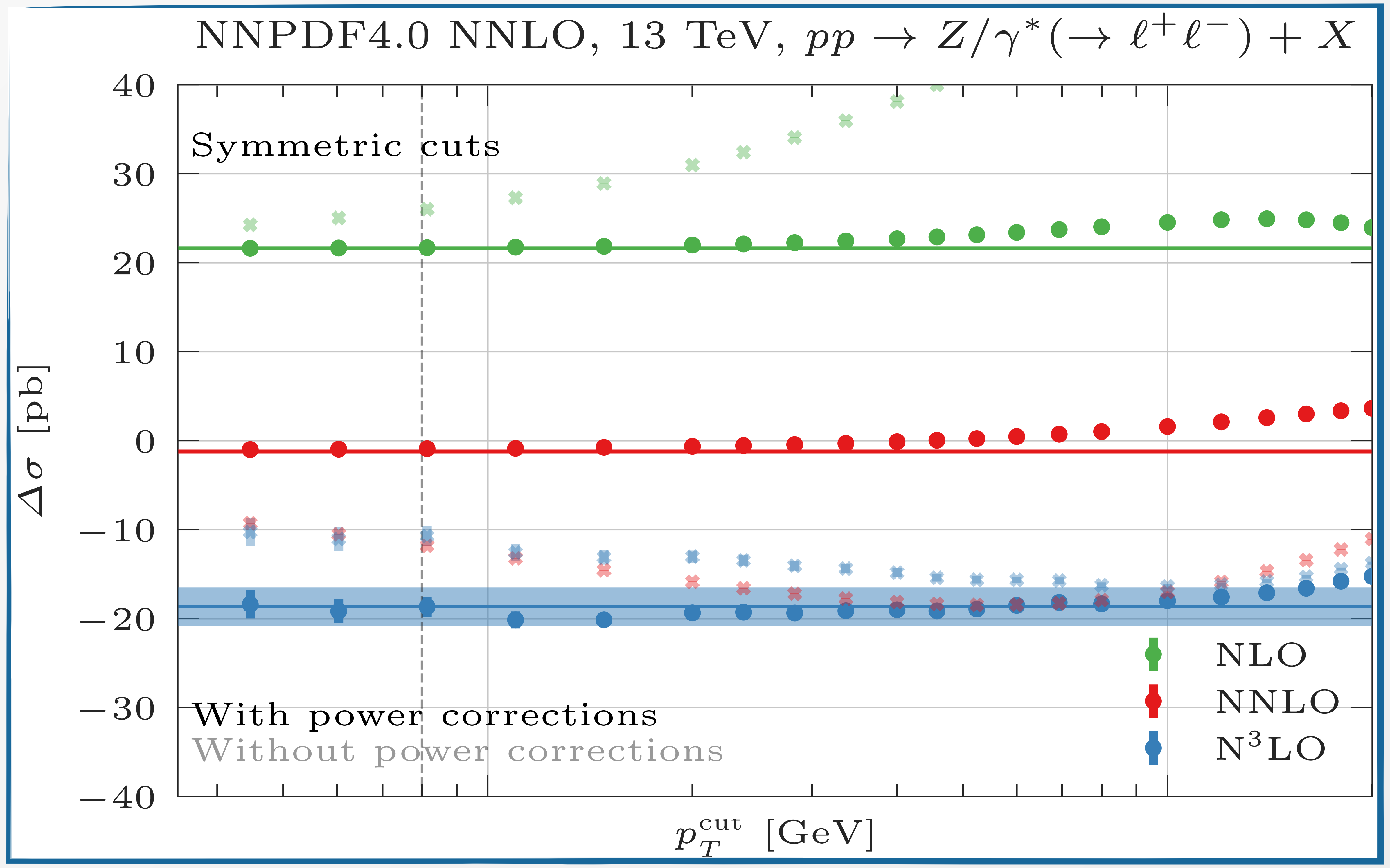
The Drell-Yan fiducial cross section at N³LO

ATLAS fiducial region $p_T^{\ell^\pm} > 27\,\mathrm{GeV}$ $|\eta^{\ell^\pm}| < 2.5$

- Mandatory to include missing linear power corrections to reach a **precise control of the N^k^LO correction** down to small values of q_T^{cut}

- Plateau at small q_T^{cut} indicates the desired independence of the slicing parameter

- Result without power correction does not converge yet to the correct value at N^k^LO



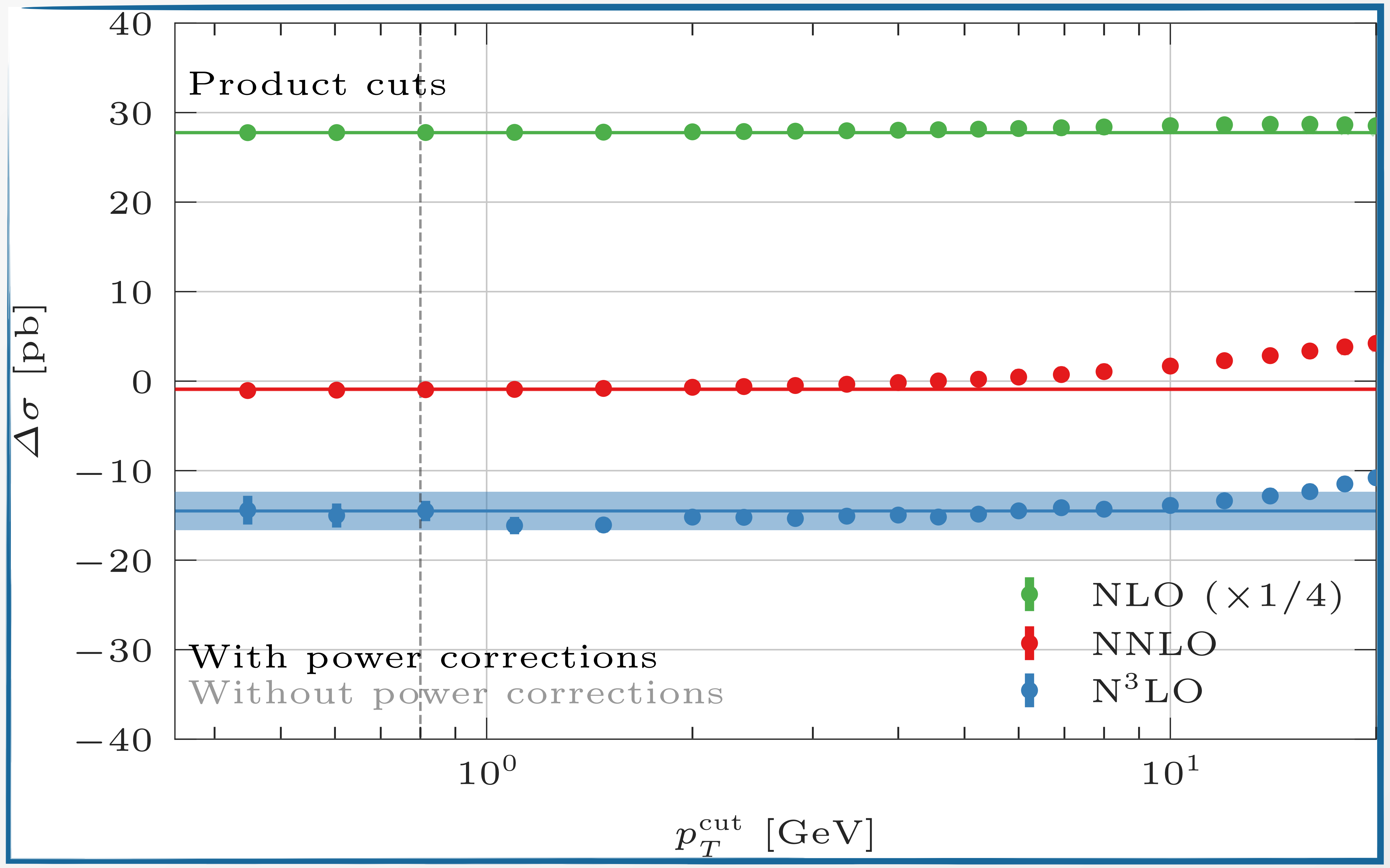
[Chen, Gehrmann, Glover, Huss, Monni, Re, LR, Torrielli '22]

The Drell-Yan fiducial cross section at N3LO

Product cuts
[Salam, Slade '21]

$$\sqrt{|\vec{p}_T^{\,\ell^+}||\vec{p}_T^{\,\ell^-}|} > 27\,\mathrm{GeV} \qquad \min\{|\vec{p}_T^{\,\ell^\pm}|\} > 20\,\mathrm{GeV} \qquad |\eta^{\ell^\pm}| < 2.5$$

- **Alternative set of cuts** which does not suffer from linear power corrections
- Improved convergence, result independent of the recoil procedure



[Chen, Gehrmann, Glover, Huss, Monni, Re, LR, Torrielli '22]

The Drell-Yan fiducial cross section at N^{3}LO and N^{3}LO+N^{3}LL

Includes resummation of linear power corrections

$q_T^{\text{cut}} = 0.8\,\text{GeV}$

Order	σ [pb] Symmetric cuts		σ [pb] Product cuts	
k	N^{k}LO	N^{k}LO+N^{k}LL	N^{k}LO	N^{k}LO+N^{k}LL
0	$721.16^{+12.2\%}_{-13.2\%}$	—	$721.16^{+12.2\%}_{-13.2\%}$	—
1	$742.80(1)^{+2.7\%}_{-3.9\%}$	$748.58(3)^{+3.1\%}_{-10.2\%}$	$832.22(1)^{+2.7\%}_{-4.5\%}$	$831.91(2)^{+2.7\%}_{-10.4\%}$
2	$741.59(8)^{+0.42\%}_{-0.71\%}$	$740.75(5)^{+1.15\%}_{-2.66\%}$	$831.32(3)^{+0.59\%}_{-0.96\%}$	$830.98(4)^{+0.74\%}_{-2.73\%}$
3	$722.9(1.1)^{+0.68\%}_{-1.09\%} \pm 0.9$	$726.2(1.1)^{+1.07\%}_{-0.77\%}$	$816.8(1.1)^{+0.45\%}_{-0.73\%} \pm 0.8$	$816.6(1.1)^{+0.87\%}_{-0.69\%}$

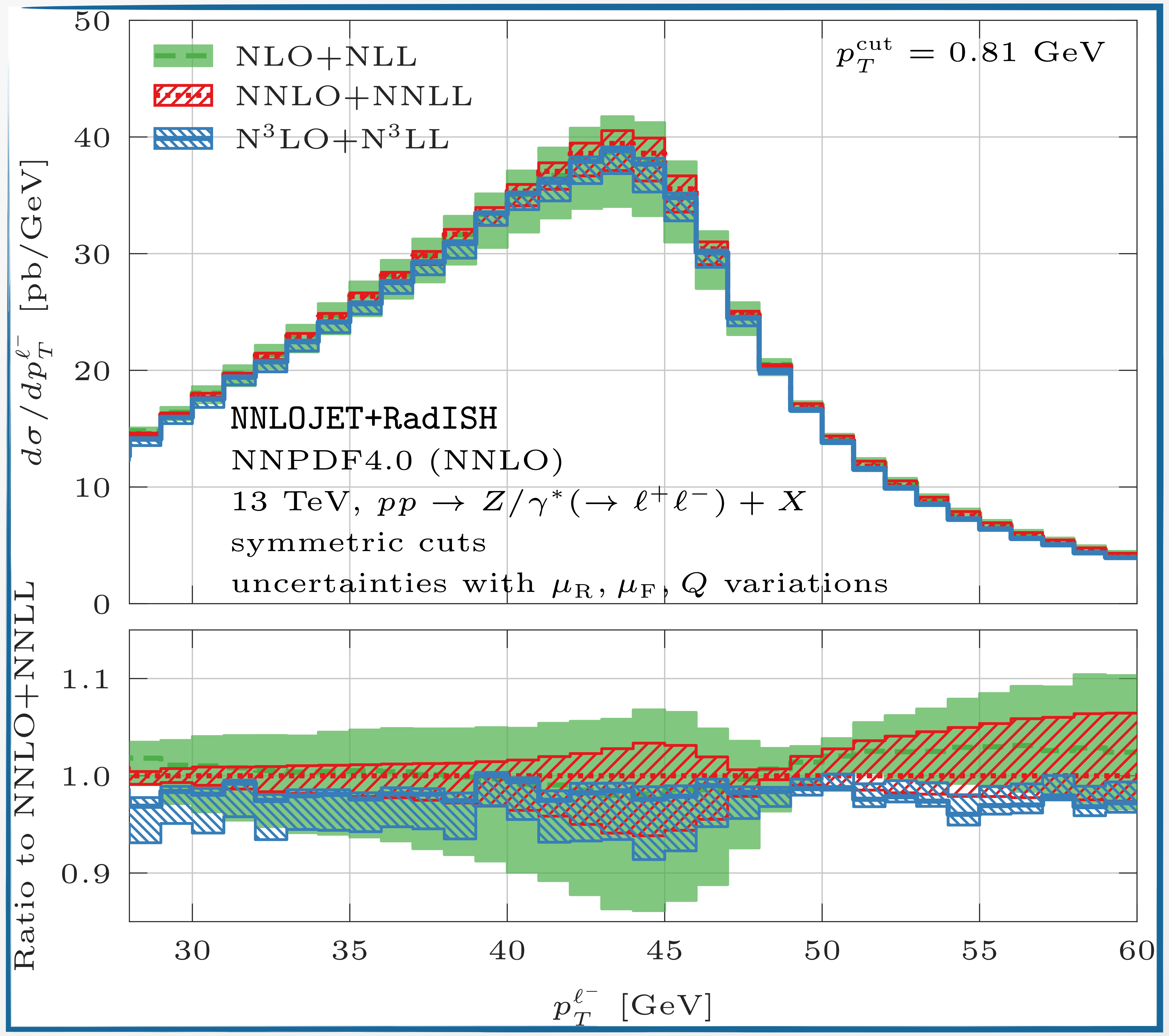
[Chen, Gehrmann, Glover, Huss, Monni, Re, LR, Torrielli '22]

- 2.5% negative correction at N^{3}LO in the ATLAS fiducial region. N^{3}LO larger than the NNLO correction and outside its error band
- More robust estimate of the theory uncertainty when **resummation effects are included**
- Central value very similar at N^{k}LO and N^{k}LO+N^{k}LL for product cuts, compatible with the absence of linear power corrections
- Slicing error computed conservatively by considering the cutoff within the [0.45-1.5] GeV interval

Fiducial distributions at N³LO+N³LL

$$d\sigma_V^{\mathrm{N^kLO+N^kLL}} \equiv d\sigma_V^{\mathrm{N^kLL}} + d\sigma_{\mathrm{V+jet}}^{\mathrm{N^{k-1}LO}} - \left[d\sigma_V^{\mathrm{N^kLL}}\right]_{\mathcal{O}(\alpha_s^k)}$$

- Fully differential calculation allows one to obtain N³LO+N³LL predictions for **fiducial observables**
- Leptonic transverse momentum is a particularly relevant observable due to its importance in the **extraction of the *W* mass**
- Inclusion of resummation effects necessary to cure (integrable) divergences due to the presence of a **Sudakov shoulder** at $m_{\ell\ell}/2$
[Catani, Webber '97]



[Chen, Gehrmann, Glover, Huss, Monni, Re, LR, Torrielli '22]

Summary

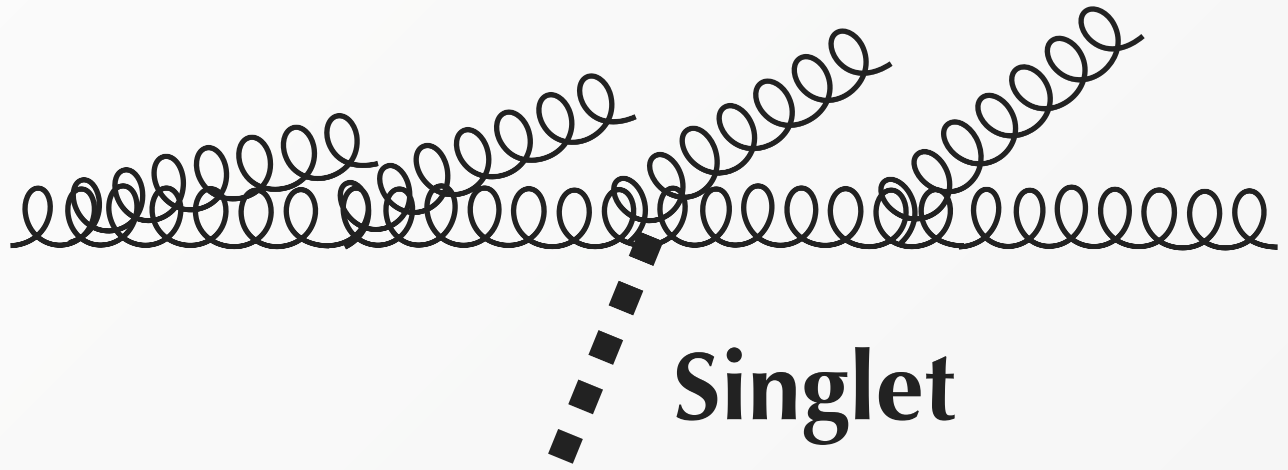
- State-of-the-art predictions for the **fiducial cross section and differential distributions** in the DY process at the LHC, through N^3LO and N^3LO+N^3LL in QCD.
- Thorough study of the performance of the computational method adopted, reaching an **excellent control over all systematic uncertainties** involved.
- Residual theoretical uncertainties at the $\mathcal{O}(1\%)$ **level** in the fiducial cross section, and at the **few-percent level** in differential distributions.

Backup

Resummation of the transverse momentum spectrum

Resummation of transverse momentum is delicate because p_t is a **vectorial quantity**

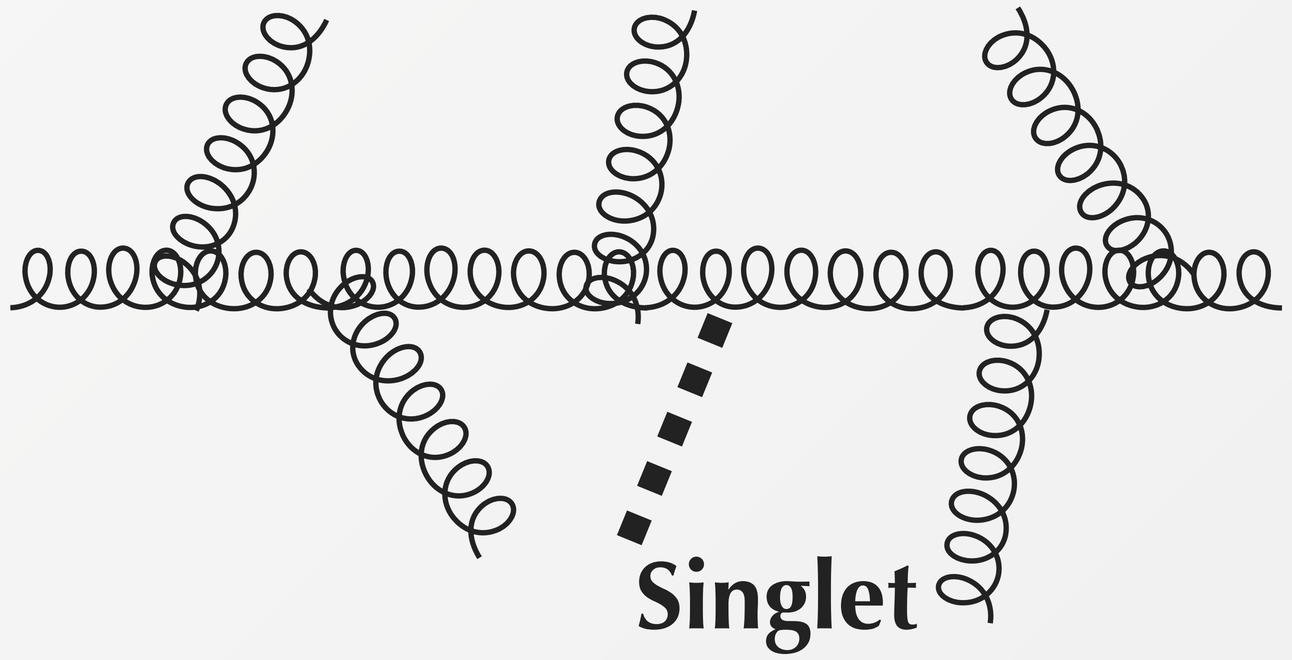
Two concurring mechanisms leading to a system with small p_t



$$p_t^2 \sim k_{t,i}^2 \ll m_H^2$$

cross section naturally suppressed as there is no phase space left for gluon emission (Sudakov limit)

Exponential suppression



$$\sum_{i=1}^{n} \vec{k}_{t,i} \simeq 0$$

Large kinematic cancellations

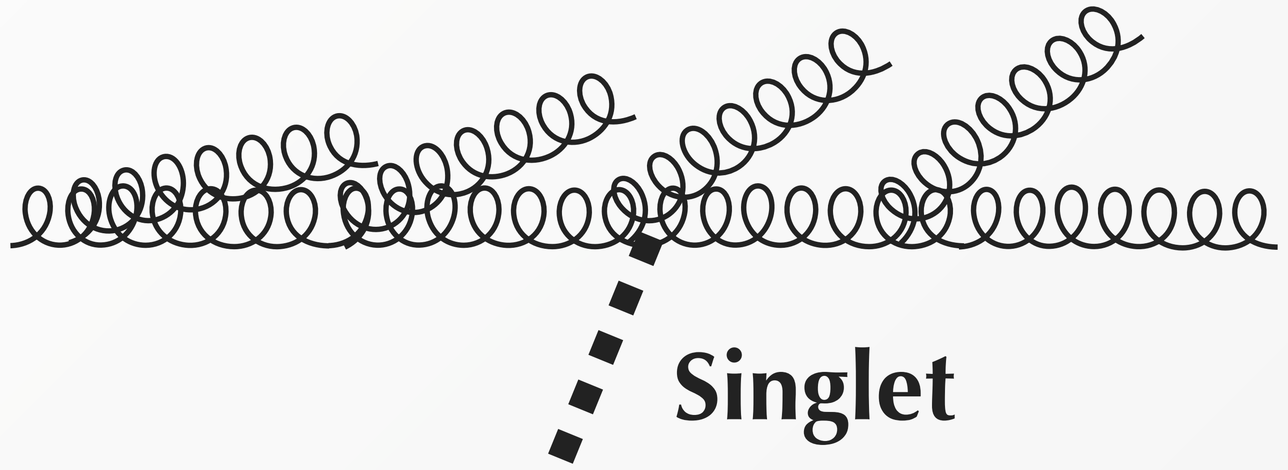
$p_t \sim 0$ far from the Sudakov limit

Power suppression

Resummation of the transverse momentum spectrum

Resummation of transverse momentum is delicate because p_t is a **vectorial quantity**

Two concurring mechanisms leading to a system with small p_t



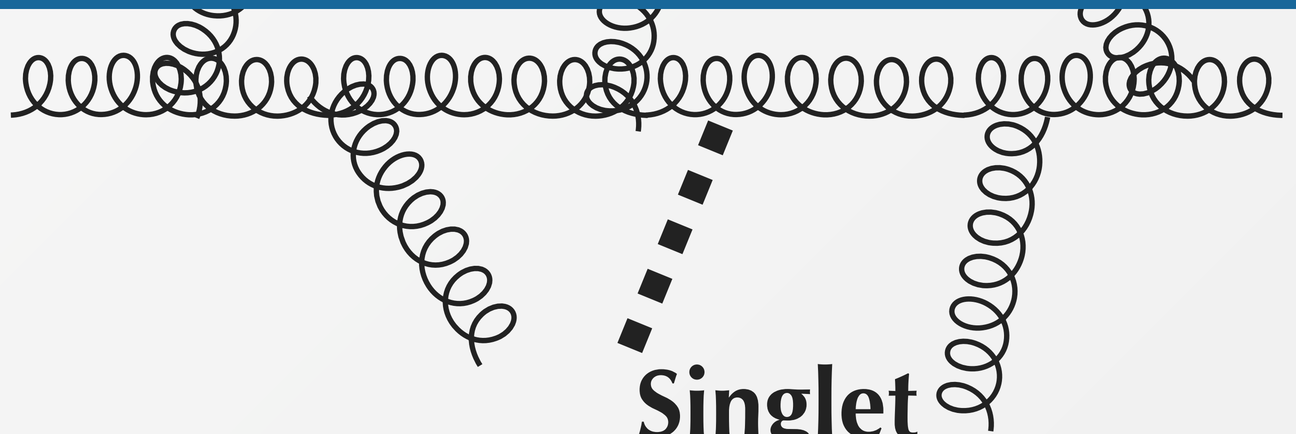
$$p_t^2 \sim k_{t,i}^2 \ll m_H^2$$

cross section naturally suppressed as there is no phase space left for gluon emission (Sudakov limit)

Exponential suppression

Dominant at small p_t

[Parisi, Petronzio, '79]



$$\sum_{i=1}^{n} \vec{k}_{t,i} \simeq 0$$

Large kinematic cancellations

p_t ~0 far from the Sudakov limit

Power suppression

Resummation of the transverse momentum spectrum in b space

two-dimensional momentum conservation $\delta^{(2)}\left(\vec{p}_t - \sum_{i=1}^{n} \vec{k}_{t,i}\right) = \int d^2b \frac{1}{4\pi^2} e^{i\vec{b}\cdot\vec{p}_t} \prod_{i=1}^{n} e^{-i\vec{b}\cdot\vec{k}_{t,i}}$

Exponentiation in conjugate space

virtual corrections

$$\sigma = \sigma_0 \int d^2\vec{p}_\perp^{\,H} \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_\perp^{\,H}} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \int [dk_i] |M(k_i)|^2 \left(e^{i\vec{b}\cdot\vec{k}_{t,i}} - 1\right) = \sigma_0 \int d^2\vec{p}_\perp^{\,H} \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_\perp^{\,H}} e^{-R_{\mathrm{NLL}}(L)}$$

NLL formula with scale-independent PDFs

$$R_{\mathrm{NLL}}(L) = -Lg_1(\alpha_s L) - g_2(\alpha_s L) \qquad L = \ln(m_H b/b_0)$$

Logarithmic accuracy defined in terms of $\ln(m_H b/b_0)$

Talk by Ignazio Scimemi

All-order formula in Mellin space at N^{3}LL

[Bizon, Monni, Re, LR, Torrielli '17] [Re, LR, Torrielli '21]

Now include effect of **collinear radiation** and terms beyond NLL accuracy

$$\frac{d\Sigma(v)}{d\Phi_B} = \int_{\mathscr{C}_1} \frac{dN_1}{2\pi i} \int_{\mathscr{C}_2} \frac{dN_2}{2\pi i} x_1^{-N_1} x_2^{-N_2} \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \mathbf{f}^T_{N_1}(\mu_0) \hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) \mathbf{f}_{N_2}(\mu_0),$$

Unresolved

$v = p_t/M$

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-\mathbf{R}(\epsilon k_{t1})}$$

$$\times \exp\left\{ -\sum_{\ell=1}^{2} \left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t)) \right) \right\}$$

Resolved

$$\times \sum_{\ell_1=1}^{2} \left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi} \mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1})) \right)$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^{1} \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right),$$

$$\times \sum_{\ell_i=1}^{2} \left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi} \mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti})) \right)$$

All-order formula in Mellin space at N³LL

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Sudakov radiator

$$R(k_{t1}) = -\log\frac{M}{k_{t1}} g_1 - g_2 - \left(\frac{\alpha_s}{\pi}\right) g_3 - \left(\frac{\alpha_s}{\pi}\right)^2 g_4 - \left(\frac{\alpha_s}{\pi}\right)^3 g_5$$

Resummation scale $Q \sim M$

$$\ln\frac{M}{k_{t1}} \to \ln\frac{Q}{k_{t1}} + \ln\frac{M}{Q}$$

Constant terms expanded in α_s and included in H

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-\mathbf{R}(\epsilon k_{t1})}$$

$$\times \exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$

$$\times \sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi} \mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right),$$

$$\times \sum_{\ell_i=1}^{2}\left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi} \mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right)$$

All-order formula in Mellin space at N³LL

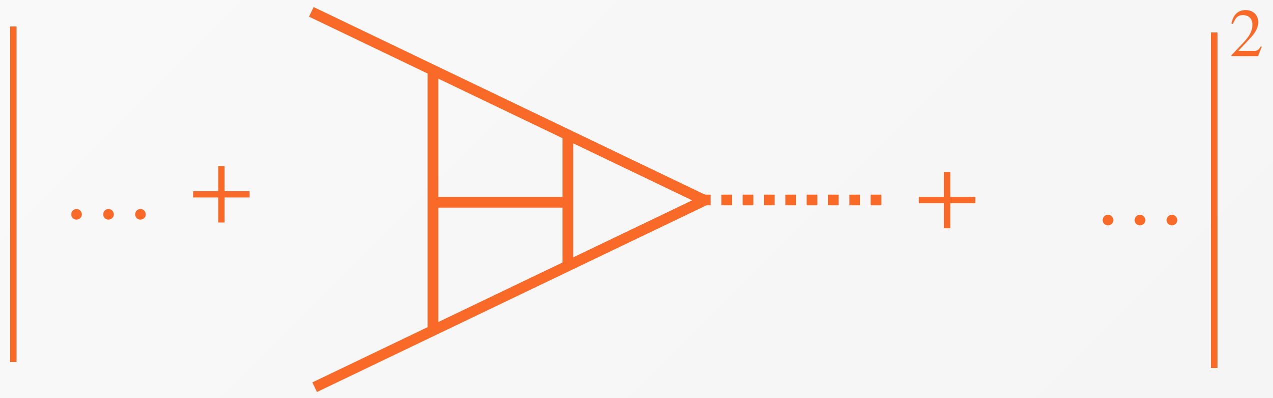
[Bizon, Monni, Re, LR, Torrielli '17] [Re, LR, Torrielli '21]

Now include effect of **collinear radiation** and terms beyond NLL accuracy

$$\frac{d\Sigma(v)}{d\Phi_B} = \int_{\mathcal{C}_1} \frac{dN_1}{2\pi i} \int_{\mathcal{C}_2} \frac{dN_2}{2\pi i} x_1^{-N_1} x_2^{-N_2} \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \mathbf{f}^T_{N_1}(\mu_0) \hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) \mathbf{f}_{N_2}(\mu_0),$$

Three-loop hard-virtual coefficient

$$H(\alpha_s) = 1 + \left(\frac{\alpha_s}{2\pi}\right) H_1 + \left(\frac{\alpha_s}{2\pi}\right)^2 H_2 + \left(\frac{\alpha_s}{2\pi}\right)^3 H_3$$



[Gehrmann et al. '10]

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-\mathbf{R}(\epsilon k_{t1})}$$

$$\times \exp\left\{-\sum_{\ell=1}^{2} \left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t)) \right)\right\}$$

$$\times \sum_{\ell_1=1}^{2} \left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi} \mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1})) \right)$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right),$$

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All-order formula in Mellin space at N³LL

[Bizon, Monni, Re, LR, Torrielli '17] [Re, LR, Torrielli '21]

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Three-loop coefficient functions and their evolution

$$C(\alpha_s, z) = \delta(1-z) + \left(\frac{\alpha_s}{2\pi}\right) C_1(z) + \left(\frac{\alpha_s}{2\pi}\right)^2 C_2(z) + \left(\frac{\alpha_s}{2\pi}\right)^3 C_3(z)$$

[Li, Zhu '16][Vladimirov '16][Luo et al. '19][Ebert et al. '20]

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-\mathbf{R}(\epsilon k_{t1})}$$

$$\times \exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$

$$\times \sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi} \mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right),$$

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All-order formula in Mellin space at N^3LL

[Bizon, Monni, Re, LR, Torrielli '17] [Re, LR, Torrielli '21]

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DGLAP evolution

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-\mathbf{R}(\epsilon k_{t1})}$$

$$\times \exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$

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All-order formula in Mellin space at N³LL [Bizon, Monni, Re, LR, Torrielli '17] [Re, LR, Torrielli '21]

$$\frac{d\Sigma(v)}{d\Phi_B} = \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \partial_L \left(-e^{-R(k_{t1})} \mathcal{L}_{\mathrm{N^3LL'}}(k_{t1})\right) \int d\mathcal{Z}\, \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right)$$

Luminosity factor: contains the three loop collinear coefficient functions C_3 and the three loop hard function H_3 [Li, Zhu '16][Vladimirov '16][Luo et al. '19][Ebert et al. '20] [Gehrmann et al. '10]

NNLL corrections

$$+ \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(k_{t1})} \int d\mathcal{Z} \int_0^1 \frac{d\zeta_s}{\zeta_s} \frac{d\phi_s}{2\pi} \Bigg\{ \left(R'(k_{t1}) \mathcal{L}_{\mathrm{NNLL}}(k_{t1}) - \partial_L \mathcal{L}_{\mathrm{NNLL}}(k_{t1})\right)$$
$$\times \left(R''(k_{t1}) \ln\frac{1}{\zeta_s} + \frac{1}{2} R'''(k_{t1}) \ln^2\frac{1}{\zeta_s}\right) - R'(k_{t1}) \left(\partial_L \mathcal{L}_{\mathrm{NNLL}}(k_{t1}) - 2\frac{\beta_0}{\pi} \alpha_s^2(k_{t1}) \hat{P}^{(0)} \otimes \mathcal{L}_{\mathrm{NLL}}(k_{t1}) \ln\frac{1}{\zeta_s}\right)$$
$$+ \frac{\alpha_s^2(k_{t1})}{\pi^2} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathcal{L}_{\mathrm{NLL'}}(k_{t1}) - \beta_0 \frac{\alpha_s^3(k_{t1})}{\pi^2} \left(\hat{P}^{(0)} \otimes \hat{C}^{(1)} + \hat{C}^{(1)} \otimes \hat{P}^{(0)}\right) \otimes \mathcal{L}_{\mathrm{NLL}}(k_{(t1}) + \frac{\alpha_s^3(k_{t1})}{\pi^2} 2\beta_0 \ln\frac{1}{\zeta_s} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathcal{L}_{\mathrm{NLL}}(k_{(t1})$$
$$+ \frac{\alpha_s^3(k_{t1})}{2\pi^2} \left(\hat{P}^{(0)} \otimes \hat{P}^{(1)} + \hat{P}^{(1)} \otimes \hat{P}^{(0)}\right) \otimes \mathcal{L}_{\mathrm{NLL}}(k_{(t1}) \Bigg\} \times \left\{\Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_s)\right) - \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right)\right\}$$

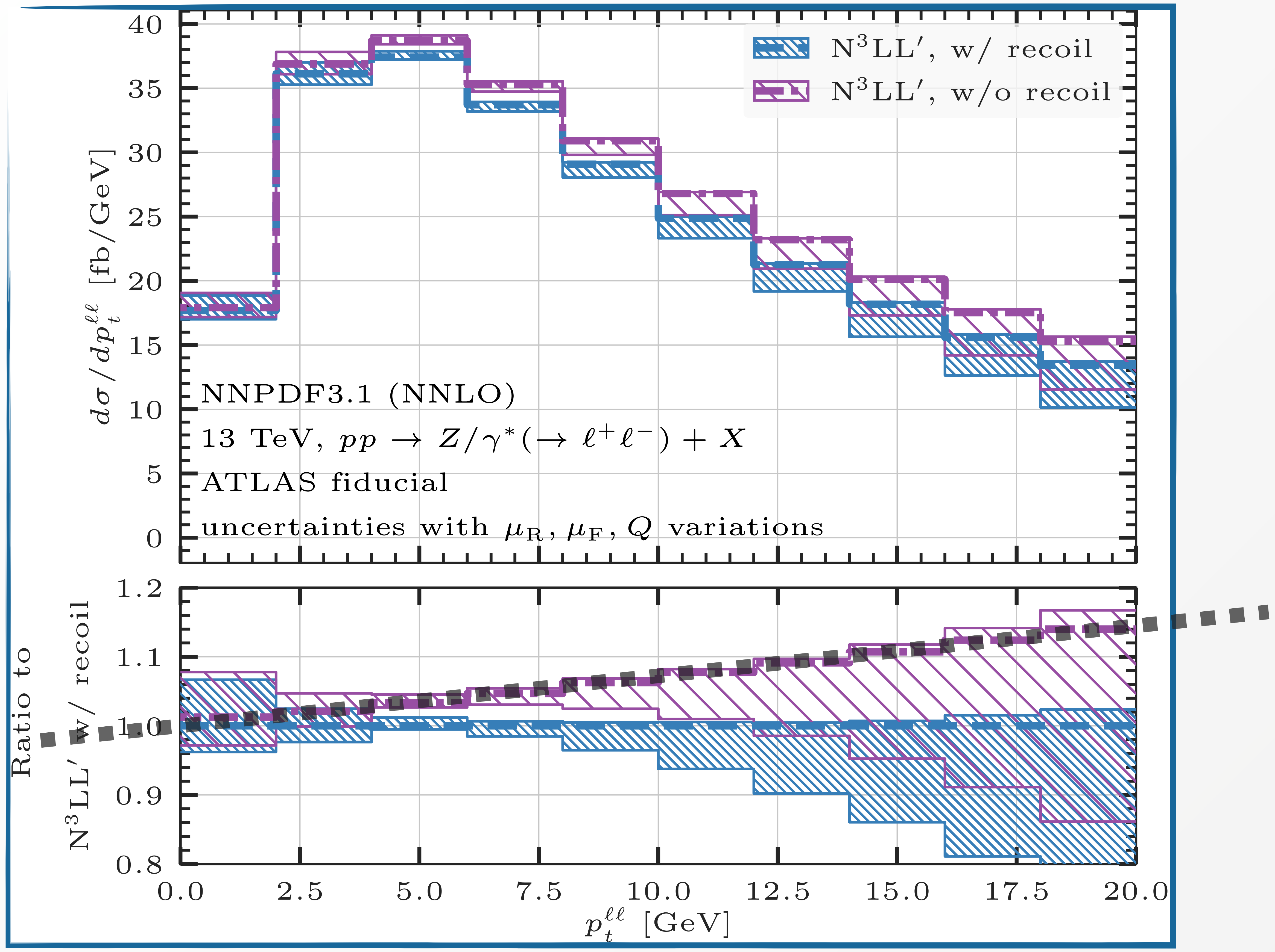
N³LL corrections

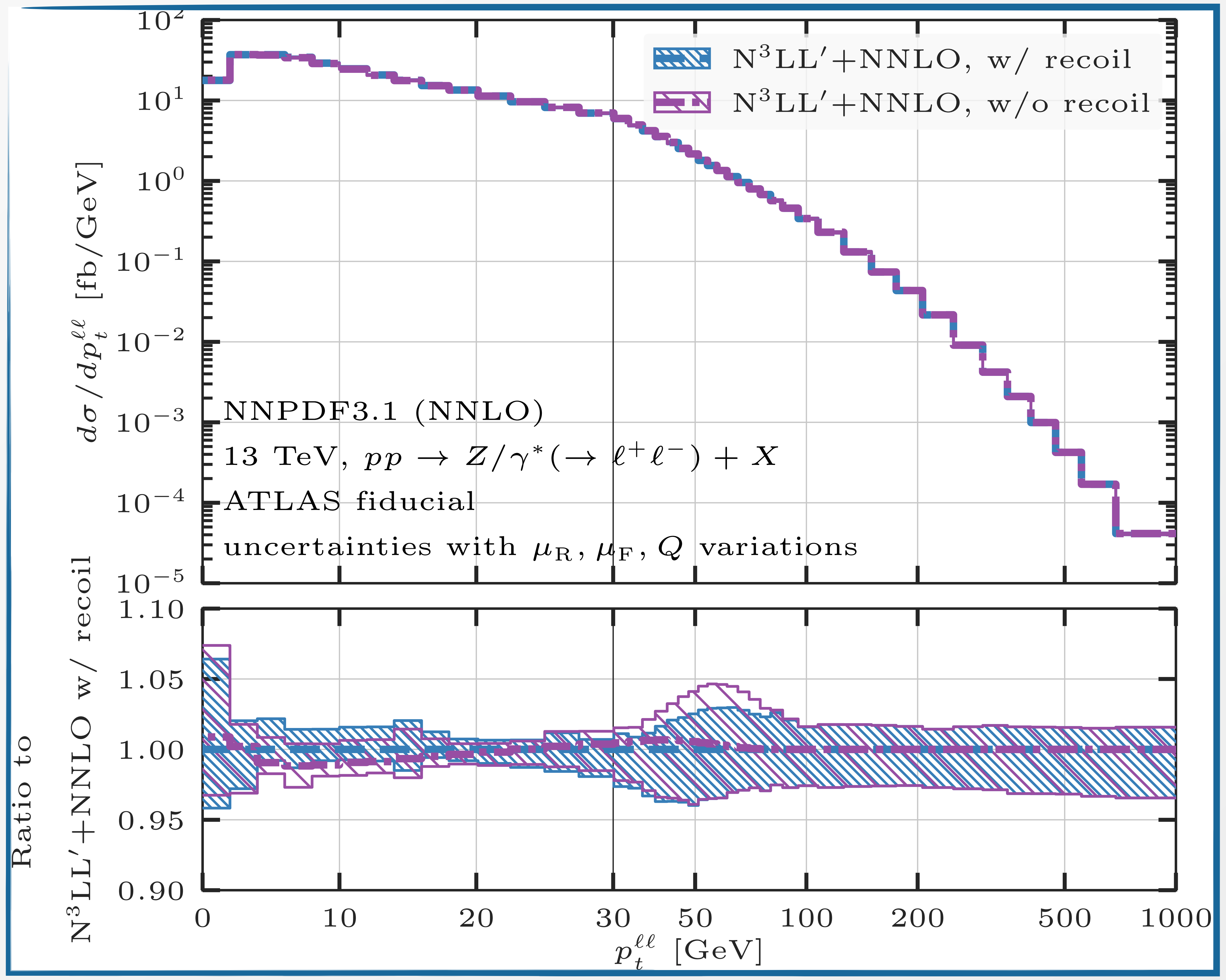
$$+ \frac{1}{2} \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(k_{t1})} \int d\mathcal{Z} \int_0^1 \frac{d\zeta_{s1}}{\zeta_{s1}} \frac{d\phi_{s1}}{2\pi} \int_0^1 \frac{d\zeta_{s2}}{\zeta_{s2}} \frac{d\phi_{s2}}{2\pi} R'(k_{t1}) \Bigg\{ \mathcal{L}_{\mathrm{NLL}}(k_{t1}) \left(R''(k_{t1})\right)^2 \ln\frac{1}{\zeta_{s1}} \ln\frac{1}{\zeta_{s2}} - \partial_L \mathcal{L}_{\mathrm{NLL}}(k_{t1}) R''(k_{t1}) \left(\ln\frac{1}{\zeta_{s1}} + \ln\frac{1}{\zeta_{s2}}\right)$$
$$+ \frac{\alpha_s^2(k_{t1})}{\pi^2} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathcal{L}_{\mathrm{NLL}}(k_{t1}) + \frac{\alpha_s^2(k_{t1})}{\pi^2} \left(\ln\frac{1}{\zeta_{s1}} + \ln\frac{1}{\zeta_{s2}}\right) R''(k_{t1}) \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathcal{L}_{\mathrm{NLL}}(k_{(t1}) - \ln\frac{1}{\zeta_{s1}} \ln\frac{1}{\zeta_{s2}} (R''(k_{t1})^2 \partial_L \mathcal{L}_{\mathrm{NLL}}(k_{(t1})$$
$$+ \frac{\alpha_s^2(k_{t1})}{\pi^3} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathcal{L}_{\mathrm{NLL}}(k_{t1}) \Bigg\} \times \Big\{ \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_{s1}, k_{s2})\right) - \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_{s1})\right) -$$
$$\Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_{s2})\right) + \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right) \Big\} + \mathcal{O}\left(\alpha_s^n \ln^{2n-7}\frac{1}{v}\right)$$

Subleading terms

Transverse recoil effects in fiducial DY setup

[Re, LR, Torrielli '21]





At the pure resummed level recoil prescription captures whole linear power corrections from fiducial cuts

Effect reduce at 1-2% level after matching to fixed order (effect becomes $\mathcal{O}(\alpha_s^4)$)

Pure resummed: band widening due to power corrections due to modified logs

$$\ln(Q/k_{t1}) \to 1/p \ln(1 + (Q/k_{t1})^p)$$

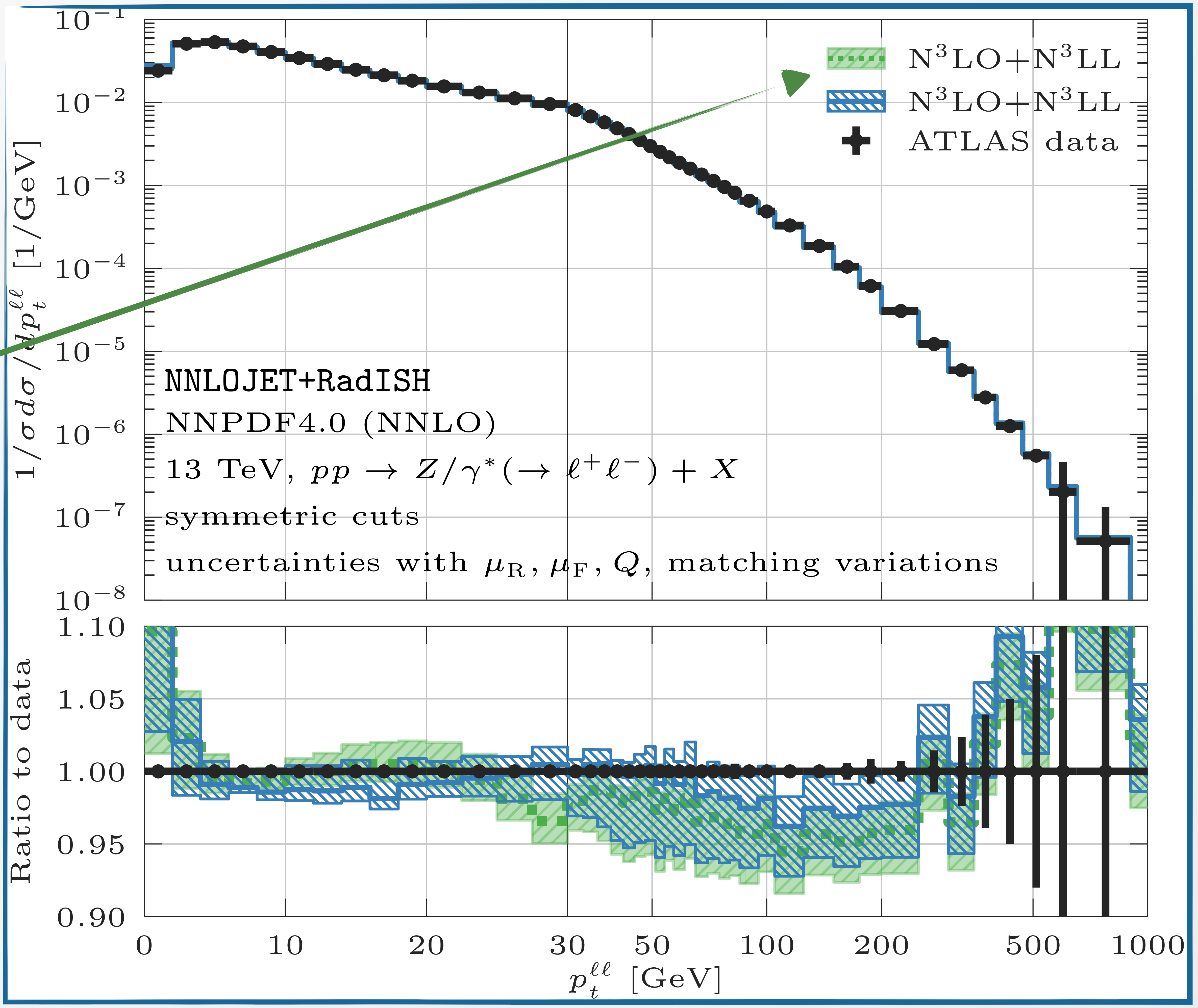
$$\int_0^M \frac{dk_{t1}}{k_{t1}} \to \int_0^\infty \frac{dk_{t1}}{k_{t1}} \frac{(Q/k_{t1})^p}{1 + (Q/k_{t1})^p}$$

Transverse momentum spectrum at N3LO+N3LL

[Chen, Gehrmann, Glover, Huss, Monni, Re, LR, Torrielli '22]

$$d\sigma_V^{\mathrm{N^kLO+N^kLL}} \equiv d\sigma_V^{\mathrm{N^kLL}} + d\sigma_{\mathrm{V+jet}}^{\mathrm{N^{k-1}LO}} - \left[d\sigma_V^{\mathrm{N^kLL}}\right]_{\mathcal{O}(\alpha_s^k)}$$

No fixed order component **below 30 GeV**



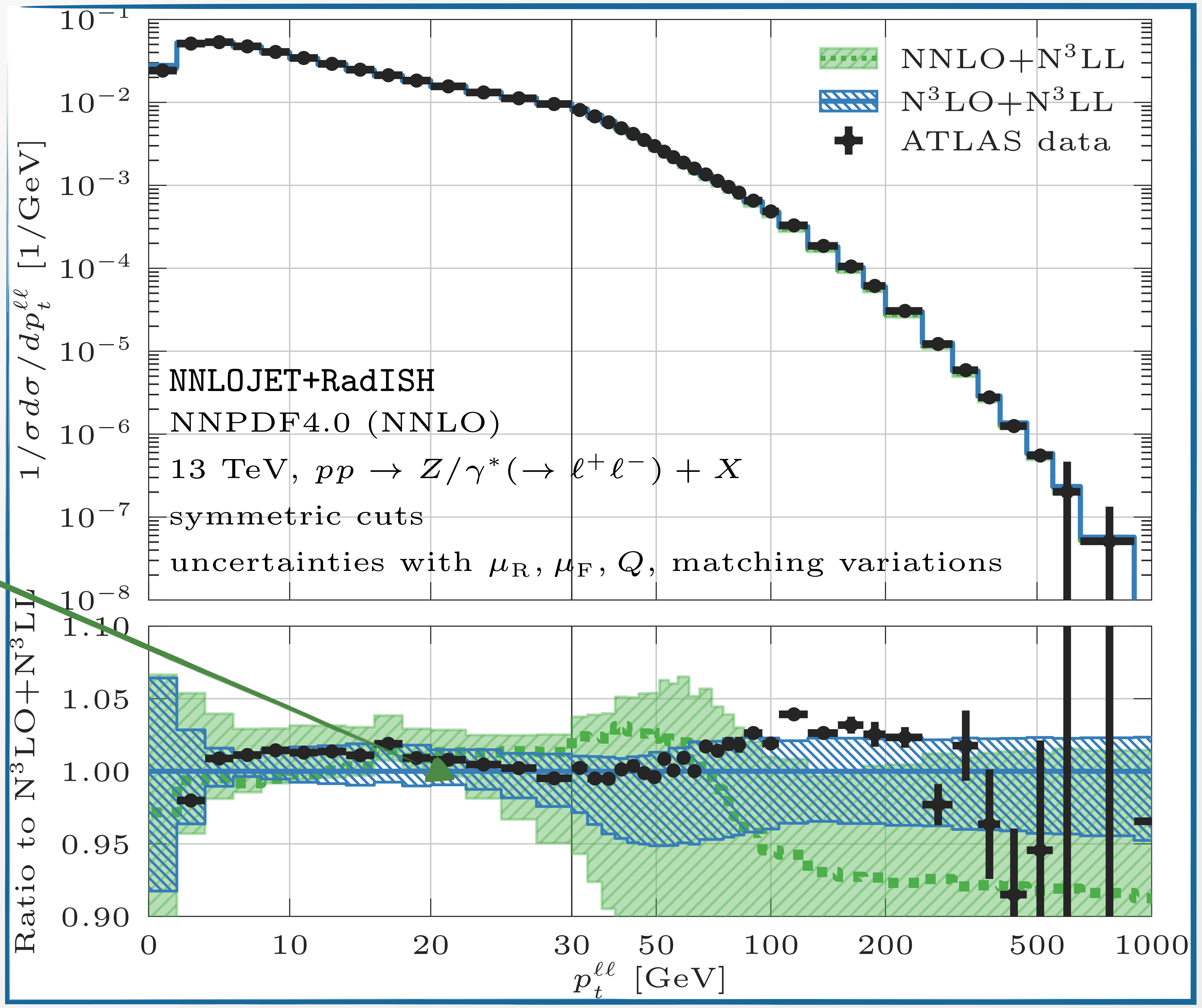
- Non-singular (matching) correction **non-negligible** even below $q_T \lesssim 15$ GeV
- Fixed order matching **crucial** to get correct shape

Transverse momentum spectrum at N³LO+N³LL

[Chen, Gehrmann, Glover, Huss, Monni, Re, LR, Torrielli '22]

$$d\sigma_V^{\mathrm{N^kLO+N^kLL}} \equiv d\sigma_V^{\mathrm{N^kLL}} + d\sigma_{\mathrm{V+jet}}^{\mathrm{N^{k-1}LO}} - \left[d\sigma_V^{\mathrm{N^kLL}}\right]_{\mathcal{O}(\alpha_s^k)}$$

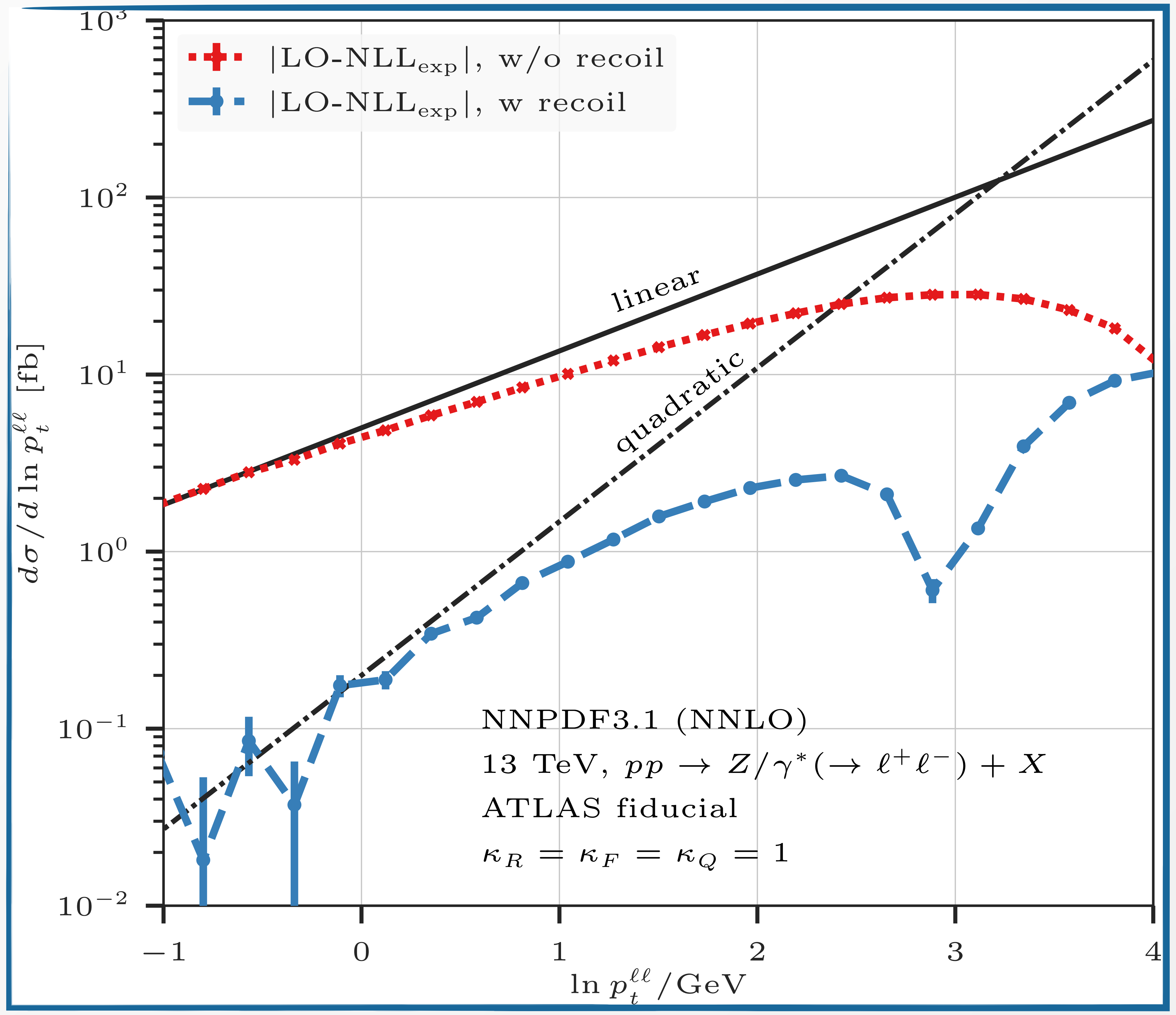
Effect of NNLO corrections in the V+jet calculation has 1-3% effect between 10 and 30 GeV



Transverse recoil effects in fiducial DY setup

Symmetric cuts on the dileptons induce linear power corrections in the fiducial spectrum

Can be avoided by suitable choice of cuts [Salam, Slade '21]



Recoil effectively captures the **full linear fiducial power correction** for p_t

Comparison with previous N^3LO estimates

Symmetric cuts $p_T^{\ell^\pm} > 25\,\text{GeV} \qquad |\eta^{\ell^\pm}| < 2.5$

- Omission of linear power corrections leads to incorrect estimate of N^kLO corrections [Camarda, Cieri, Ferrera '21]
- Data at N^3LO not of sufficient quality to observe a stable plateau, inducing larger systematic uncertainties