Linear power corrections and small-q_T resummation

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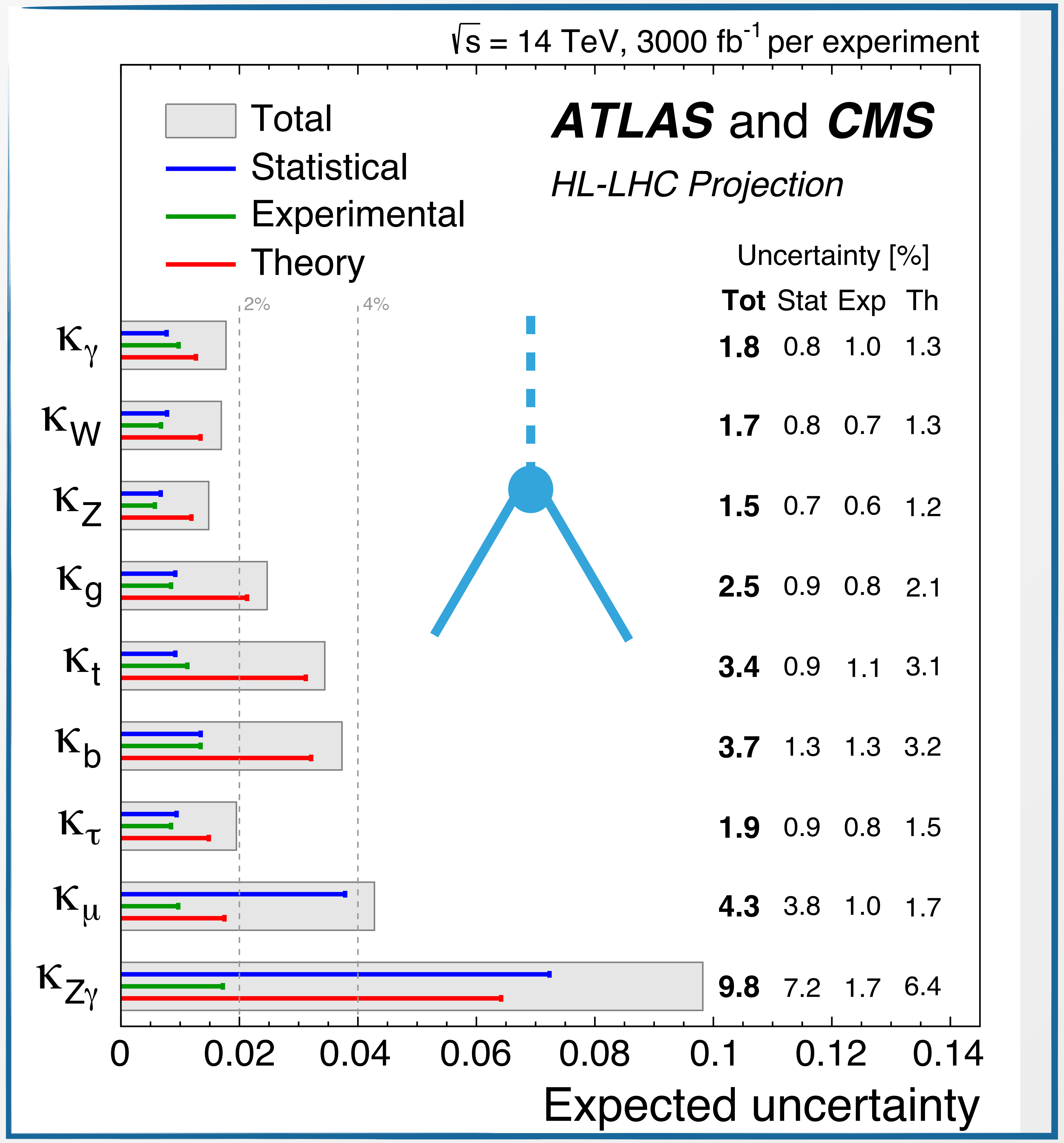
LHC as a precision machine

Luminosity expected to reach 3000 fb^{-1} at the end of the **HL-LHC** run

Substantial improvement in experimental precision with **increased statistics** and better understanding of **systematic uncertainties**

Precision target (Higgs couplings): 1-3%

Sensitivity to deviations of Higgs interactions from SM predictions



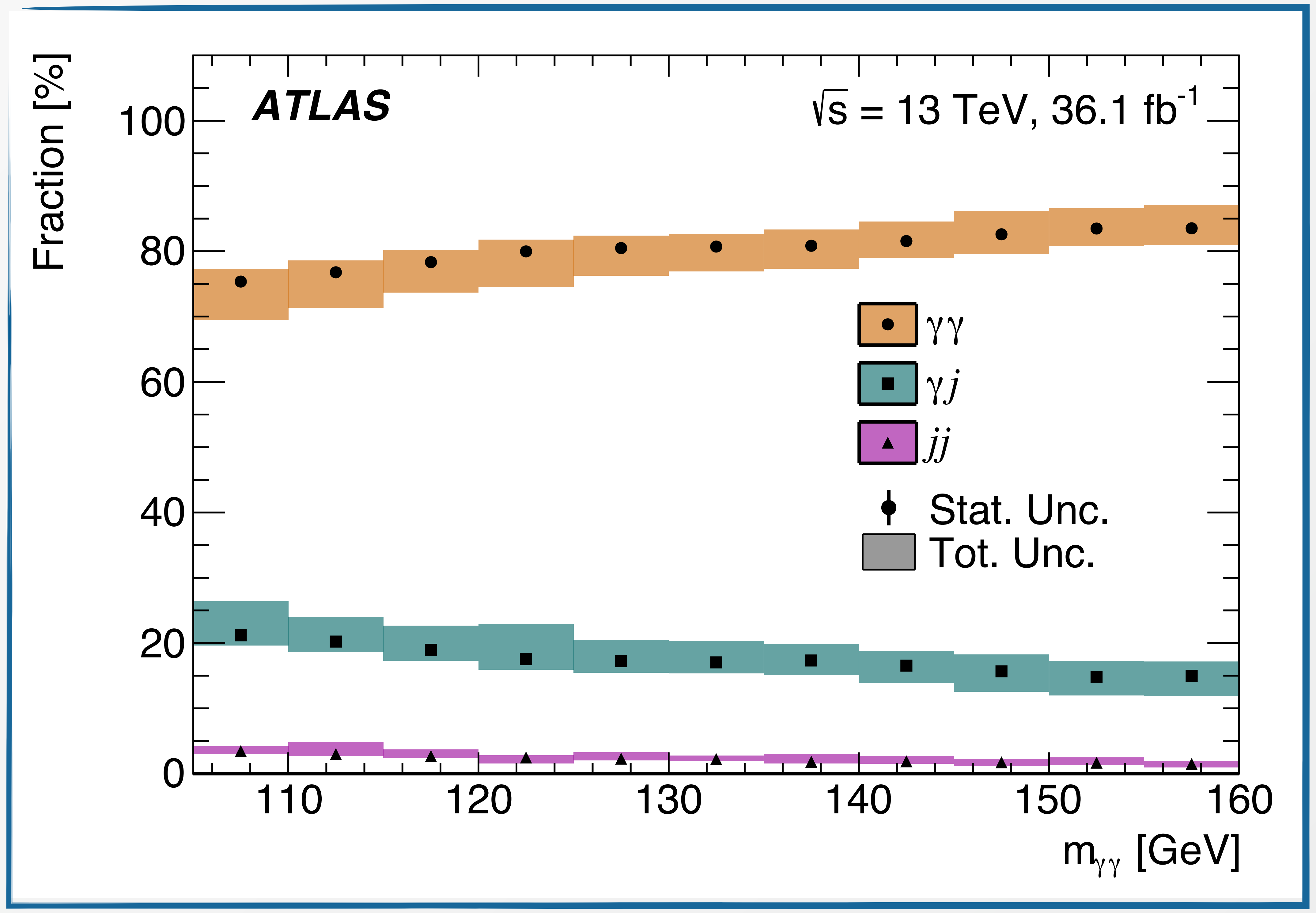
[Higgs Physics Report at HE/HL-LHC 2019]

Precision and fiducial acceptances

Experimental analyses performed within fiducial region corresponding to the phase space of experimental apparatuses

Additional selection cuts applied to enhance signal or eliminate/reduce experimental background (e.g. particles with low-p_T)

Data-theory comparison within fiducial region is a core principle in the LHC precision programme



[ATLAS 1802.04146]

$H \to \gamma\gamma$: cuts on final-state photons can improve the efficiency of the selection of pure $\gamma\gamma$ final state

Cut to the chase: fiducial acceptances and perturbative convergence

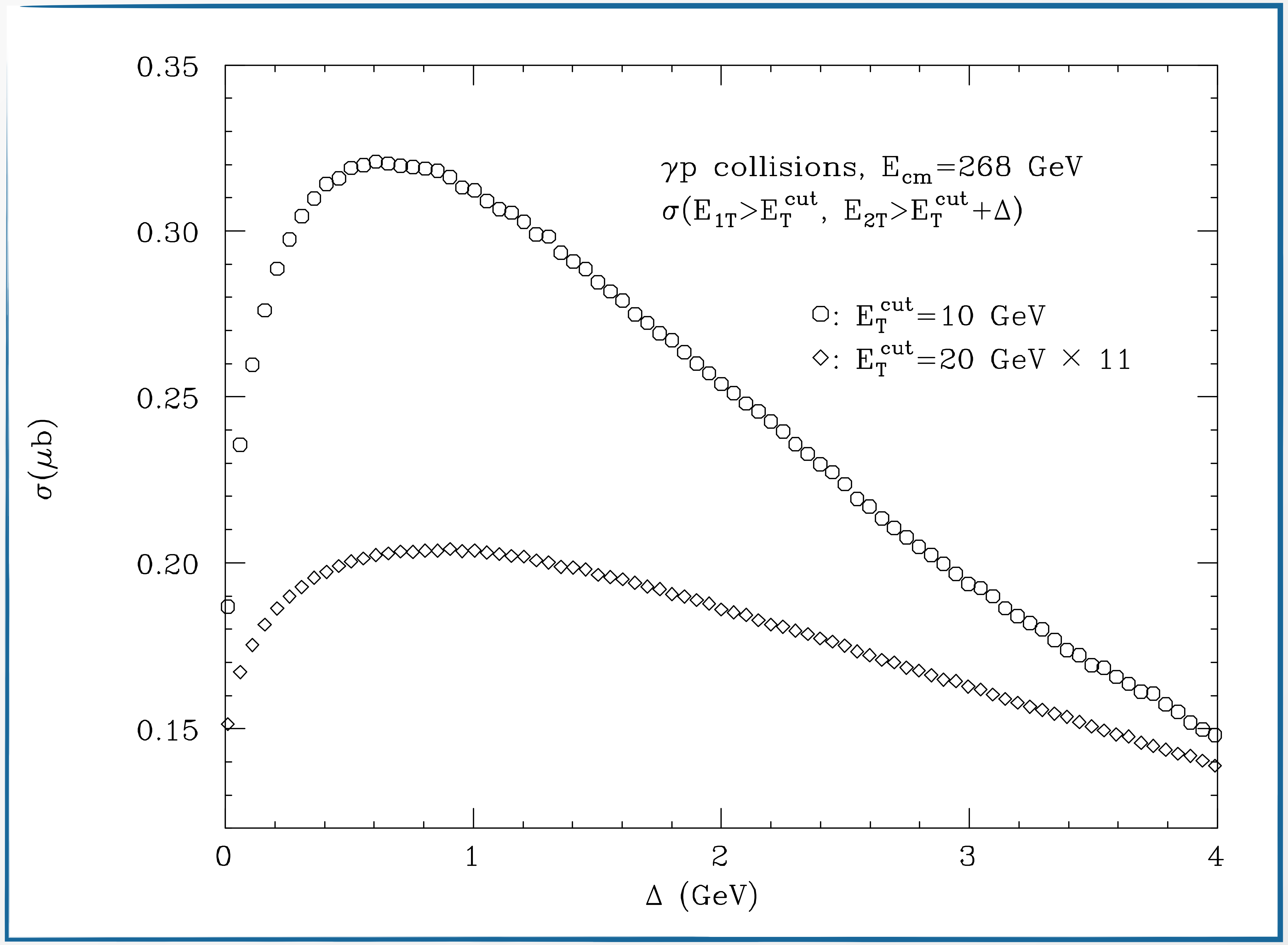
$$\sigma_2(\Delta) = \sigma(E_{1T} > E_T^{cut}, E_{2T} > E_T^{cut} + \Delta)$$

The definition of fiducial cuts can be delicate for configurations with final states with two objects in back-to-back configurations

Perturbative instability induced by sensitivity to soft radiation in configurations close to the back-to-back limit

Some key observations:
[Klasen, Kramen ’96][Harris, Owen ’97][Frixione, Ridolfi ’97]

- **Resummation** can be beneficial
- Choice of cuts has an impact on the perturbative convergence
- Subtraction methods based on **slicing techniques** might require **special care** with certain cuts

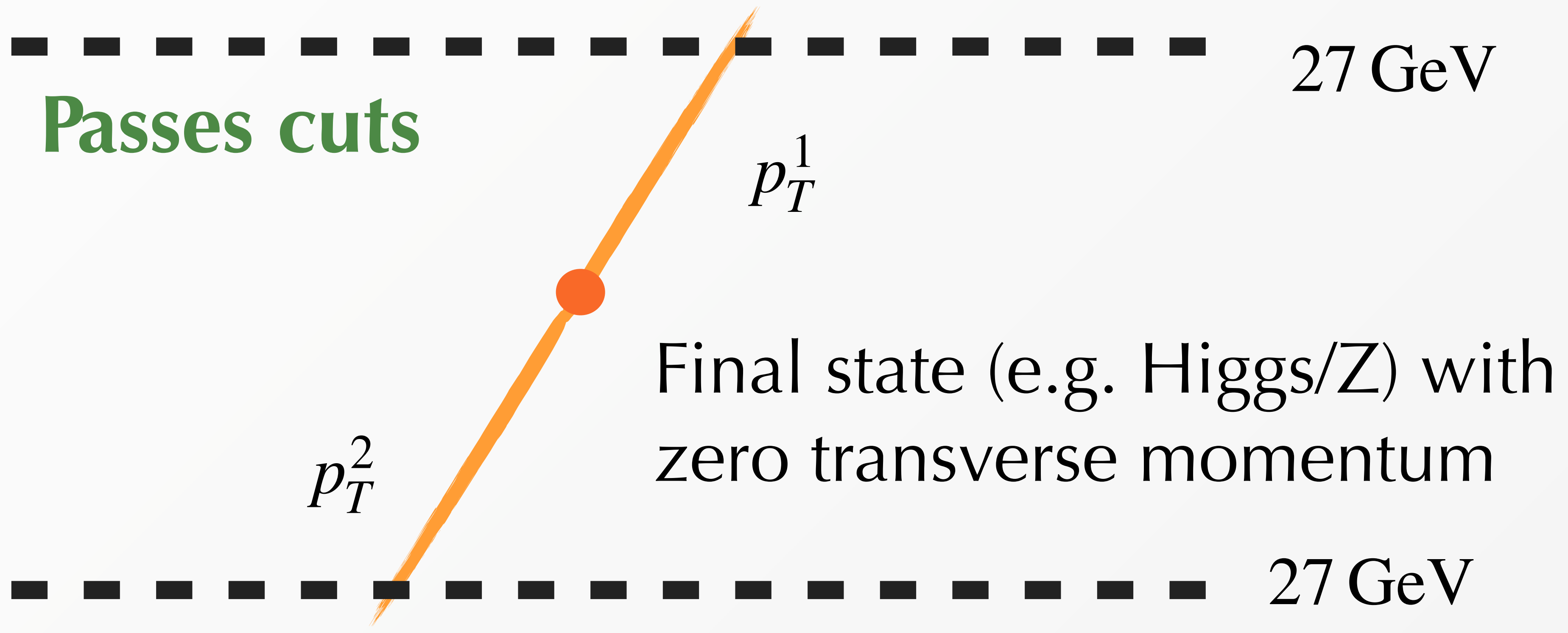


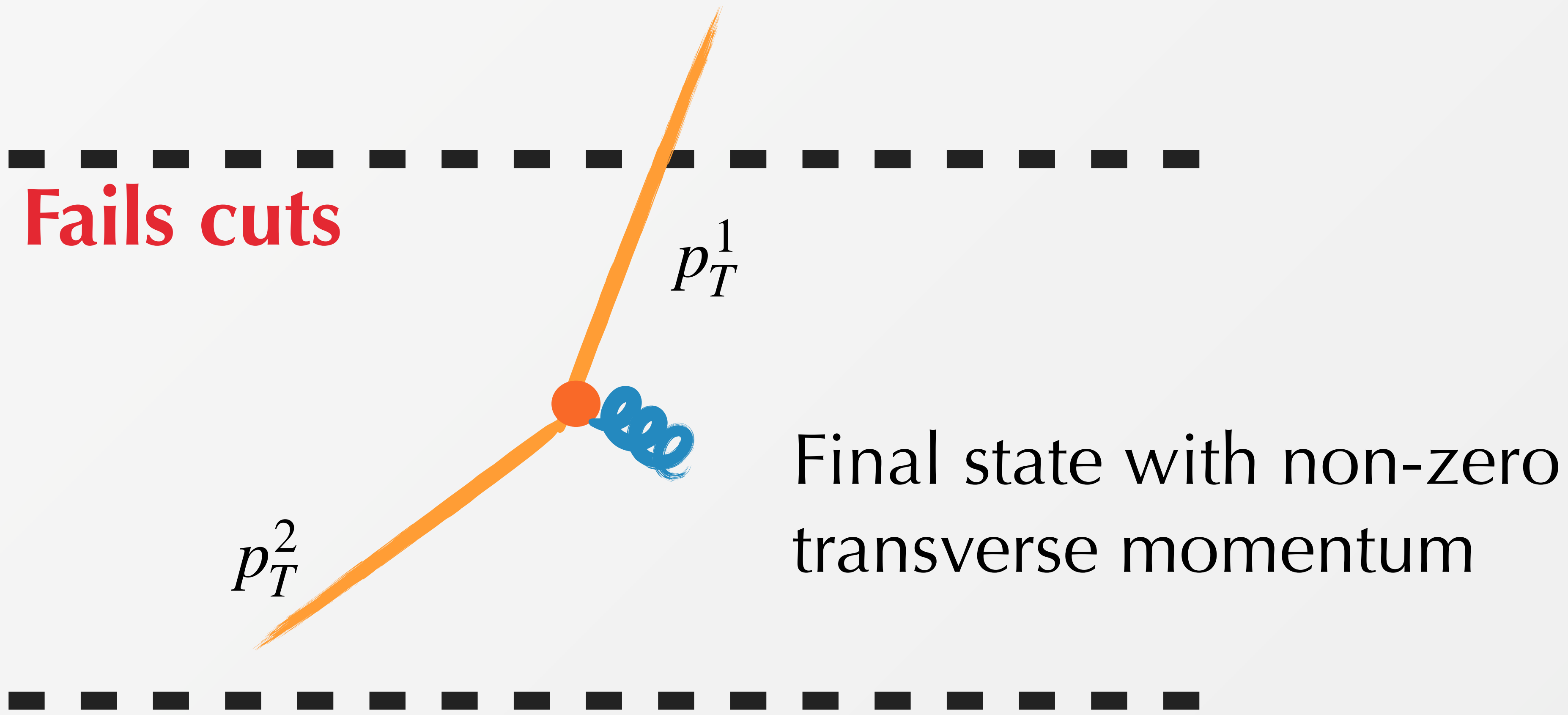
[Frixione, Ridolfi ’97]

Cuts and linear power corrections

Symmetric and asymmetric cuts induce a linear dependence on the acceptance

[Tackmann, Ebert '19][Alekhin, Kardos, Moch, Trócsányi '21][Salam, Slade '21]





Drell-Yan production cuts (ATLAS, CMS, LHCb…)

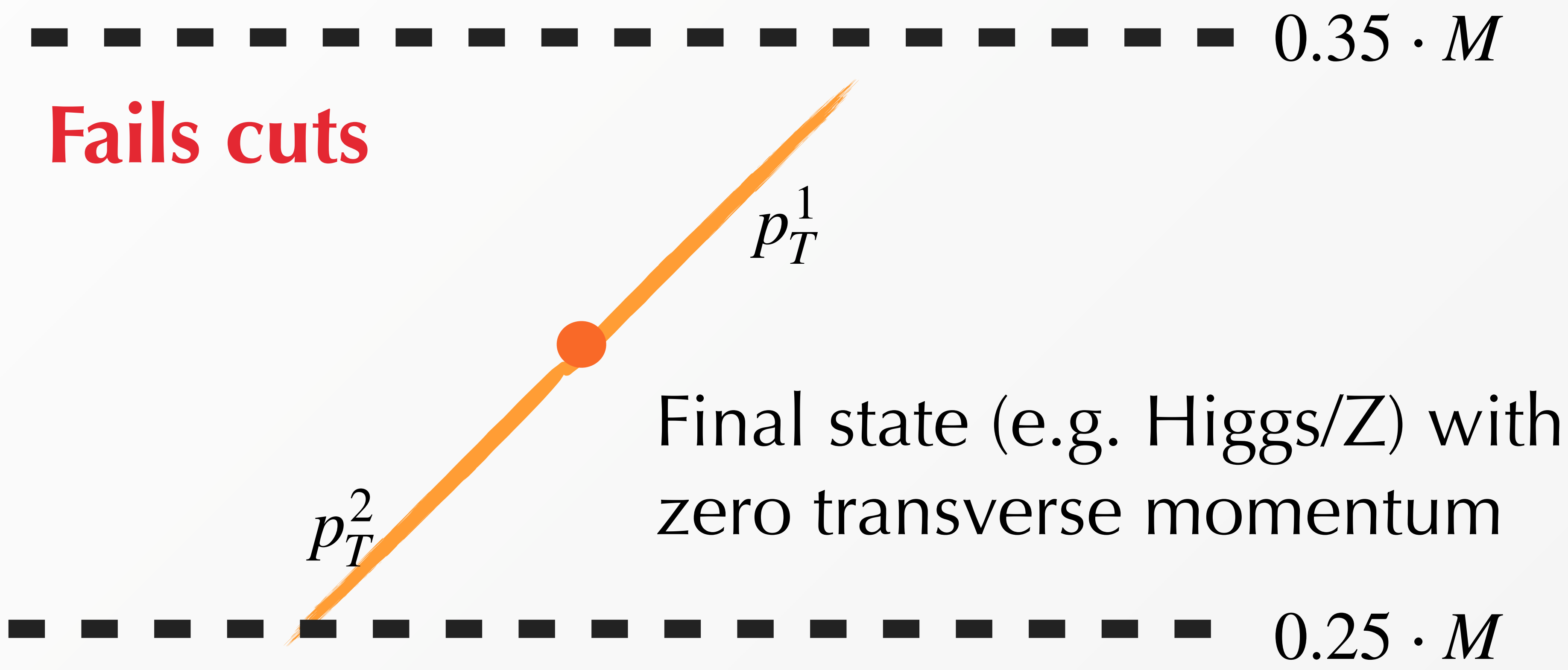
$$f^{\mathrm{sym}}(p_T) = f_0 + f_1^{\mathrm{sym}} \cdot \frac{p_T}{M} + \mathcal{O}_2$$

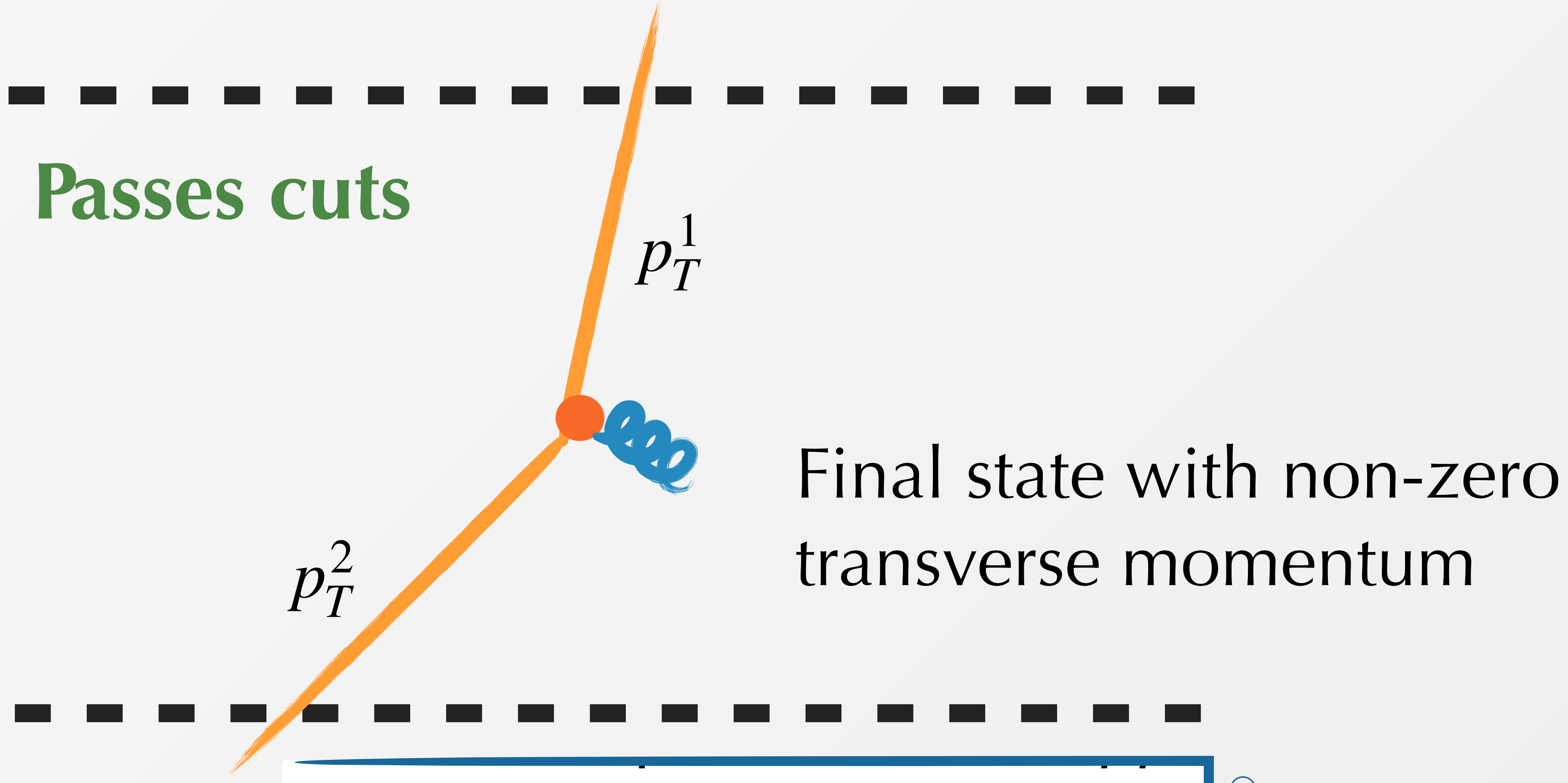
Coefficients depend on the specific choice of cuts

Cuts and linear power corrections

Symmetric and **asymmetric** cuts induce a linear dependence on the acceptance

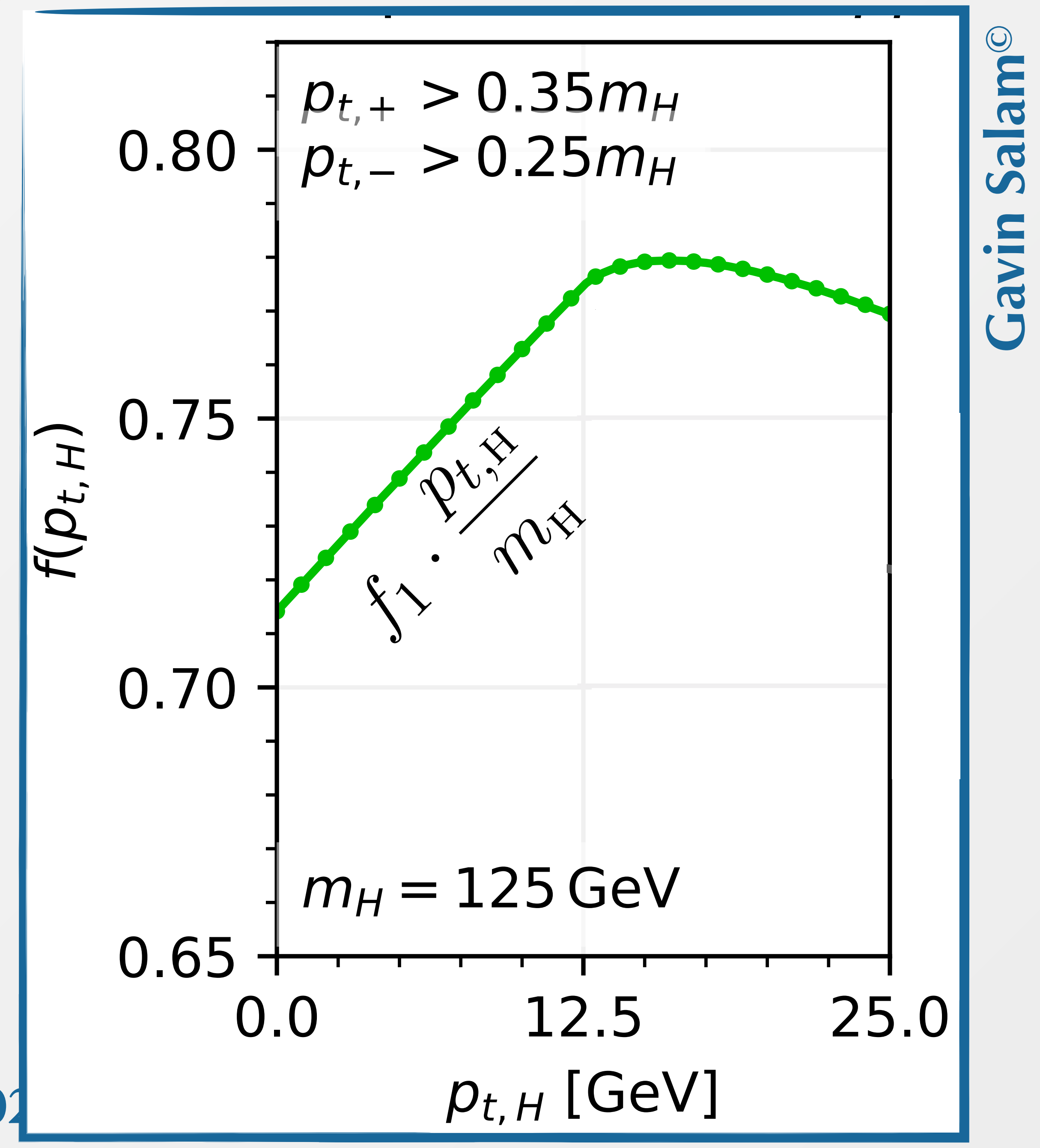
[Tackmann, Ebert '19][Alekhin, Kardos, Moch, Trócsányi '21][Salam, Slade '21]





$H \to \gamma\gamma$ selection cuts

$$f^{\text{asym}}(p_T) = f_0 + f_1^{\text{asym}} \cdot \frac{p_T}{M} + \mathcal{O}_2$$



Origin of linear power corrections

Symmetric and asymmetric cuts induce a linear dependence on the acceptance

[Tackmann, Ebert '19][Alekhin, Kardos, Moch, Trócsányi '21][Salam, Slade '21]

Kinematics of the two-body decay

$$q^\mu = (m_T \cosh Y, q_T, 0, m_T \sinh Y)$$

$$p_1^\mu = p_{T,1}\left(\cosh(Y+\Delta y), \cos\phi, \sin\phi, m_T \sinh(Y+\Delta y)\right)$$

$$p_1^\mu = q^\mu - p_1^\mu$$

$$p_{T,1} = \frac{Q}{2\cosh \Delta y}\left[1 + \frac{q_T}{Q}\frac{\cos\phi}{\cosh \Delta Y} + \mathcal{O}\left(q_T^2/Q^2\right)\right]$$

$$p_{T,2} = p_{T,1} - q_T \cos\phi + \mathcal{O}\left(q_T^2/Q^2\right)$$

$$\eta_1 = Y + \Delta y$$

$$\eta_2 = Y - \Delta y - 2\frac{q_T}{Q}\cos\phi \sinh \Delta y + \mathcal{O}\left(q_T^2/Q^2\right)$$

The two-body decay **phase space** with cuts is given by

$$\Phi_{q\to p_1+p_2}(q_T) = \frac{1}{8\pi^2}\int_0^{2\pi} d\phi \int d\Delta y \frac{p_{T,1}^2}{Q^2}\Theta_{\text{cuts}}(q_T, \phi, \Delta y; \text{cuts})$$

presence of **linear fiducial power corrections** ⟷ use of cuts **breaking the azimuthal symmetry**

Linear power corrections and candle processes

Z boson production

Symmetric cuts:

$p_{T,i} > p_T^{\rm cut}, \quad i = 1,2 \implies \min(p_{T,1}, p_{T,2}) > p_T^{\rm cut}$

$$\begin{cases} p_{T,1} > p_T^{\rm cut} & \cos\phi < 0 \\ p_{T,1} > p_T^{\rm cut} + q_T \cos\phi & \cos\phi > 0 \end{cases}$$

W boson production

Transverse mass cuts:

$p_{T,1} > p_T^{\rm cut}, \qquad m_T > m_T^{\rm cut}$ **with** $m_T \sim 2p_{T,1} - \cos(\phi) q_T$

If $m_T^{\rm cut} = 2p_T^{\rm cut}$ (as in 2022 CDF m_W analysis)

$$\begin{cases} p_{T,1} > p_T^{\rm cut} & \cos\phi > 0 \\ p_{T^1} > p_T^{\rm cut} + \frac{q_T}{2}\cos\phi & \cos\phi < 0 \end{cases}$$

two different integrands: **breaking of azimuthal symmetry**

$$\Phi(q_T) - \Phi(0) \sim \frac{1}{2\pi^2}\frac{q_T}{Q}\frac{p_T^{\rm cut}/Q}{\sqrt{1-(2p_T^{\rm cut})^2/Q^2}}\int_0^{\pi/2} d\phi \cos\phi$$

Linear power corrections and perturbative convergence

What happens at the level of the fiducial cross section?

$$\sigma_{\rm fid} = \int \frac{d\sigma}{dp_T} f(p_T) dp_T$$

Drastic impact on the behaviour of calculations in perturbative QCD

Simple **double-logarithmic** approximation for p_T distribution

$$\frac{d\sigma}{dp_T} \sim \frac{4C_A\alpha_s L}{\pi p_T} e^{-\frac{2C_A\alpha_s}{\pi}L^2} \sim \frac{\sigma_{\rm tot}}{p_T} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2\ln^{2n-1}\frac{M}{2p_T}}{(n-1)!} \left(\frac{2C_A\alpha_s}{\pi}\right)^n \qquad L = \ln\frac{p_T}{2M}$$

Upon integration, pathological perturbative behaviour

$$\sigma_{\rm fid} = \sigma_{\rm tot}\left[f_0 + f_1 \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(2n)!}{2(n!)} \left(\frac{2C_A\alpha_s}{\pi}\right)^n + \cdots\right]$$

(alternating sign) factorial growth

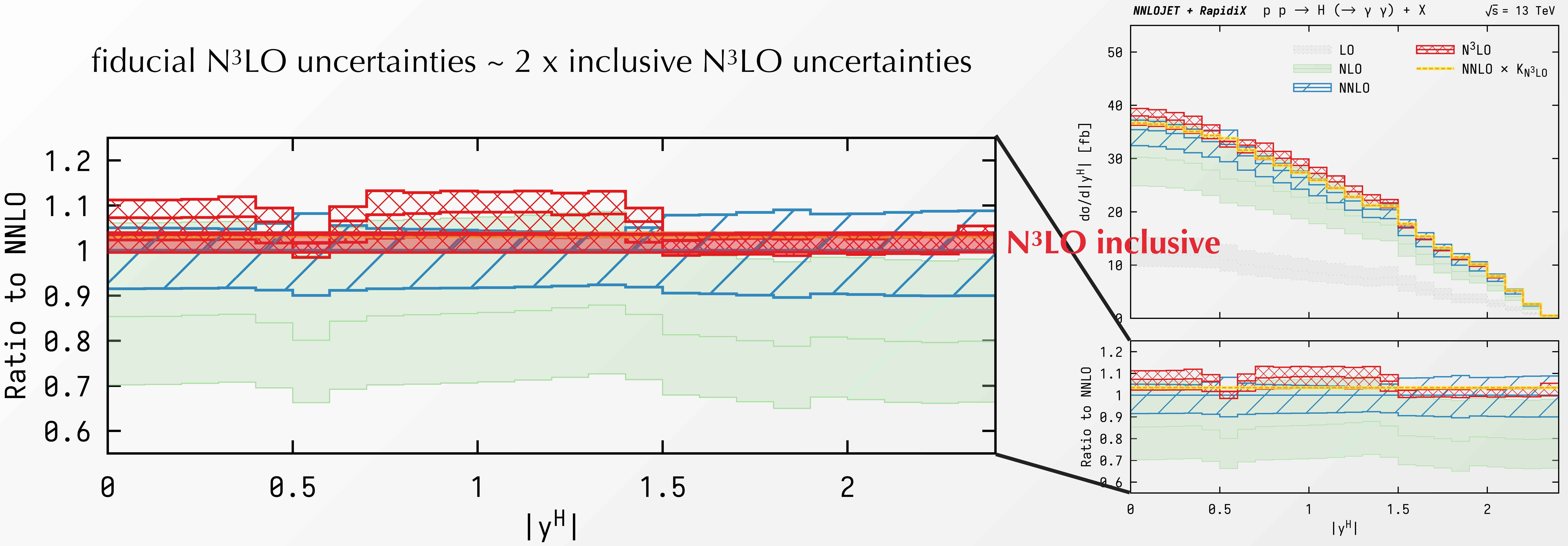
Linear power corrections and perturbative convergence

$$\frac{\sigma^{\rm DL}_{\rm asym}}{f_0\sigma_{\rm tot}} - 1 \simeq \frac{f_1^{\rm asym}}{f_0}\left(\underbrace{0.16}_{\alpha_s} - \underbrace{0.33}_{\alpha_s^2} + \underbrace{0.82}_{\alpha_s^3} - \underbrace{2.73}_{\alpha_s^4} + \underbrace{11.72}_{\alpha_s^5} + \ldots\right)$$

$pp \to H(\to \gamma\gamma)$

[Salam, Slade '21]

fiducial N³LO uncertainties ~ 2 x inclusive N³LO uncertainties



[Chen, Gehrmann, Glover, Huss, Mistlberger, Pelloni, '21]

Detailed explanation: Alex Huss' slides @ Higgs2021

Linear power corrections and perturbative convergence

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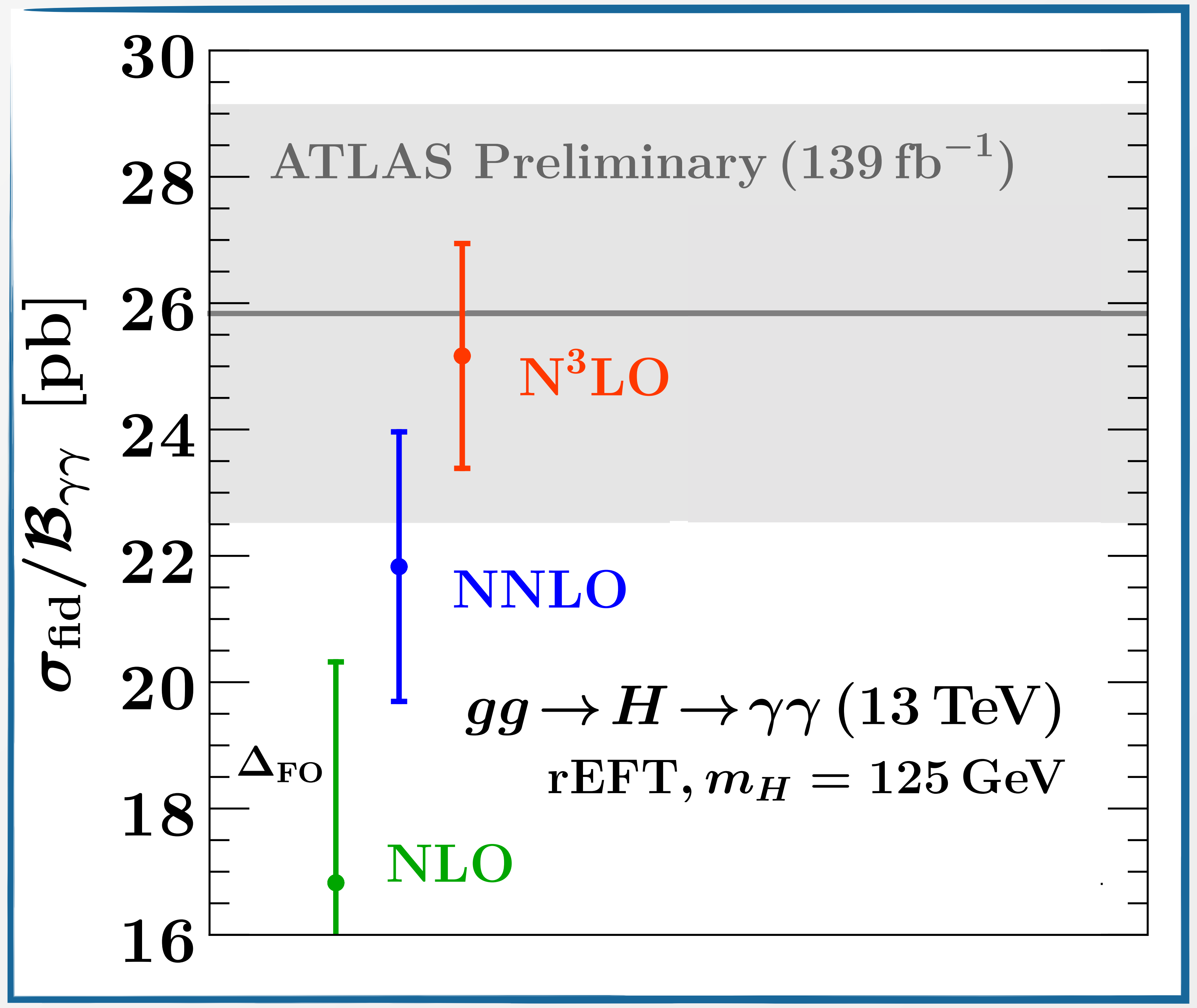
$pp \to H(\to \gamma\gamma)$

[Salam, Slade '21]

$$\sigma^{\rm FO}_{\rm incl} = 13.80\,[1 + 1.291 + 0.783 + 0.299]\,\text{pb}$$

$$\sigma^{\rm FO}_{\rm fid}/\mathcal{B}_{\gamma\gamma} = 6.928\,[1 + (1.300 + 0.129_{\rm fpc}) + (0.784 - 0.061_{\rm fpc}) + (0.331 + 0.150_{\rm fpc})]\,\text{pb}\,.$$

Effect of **linear fiducial corrections**



[Billis, Dehnadi, Ebert, Michel, Stewart, Tackmann '21]

It's the sum that makes up the total

[Totò, '60]

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[Salam, Slade '21]

Sum of divergent series with alternating sign factorial growth is **Borel-summable**

$$\frac{\sigma^{\rm DL}_{\rm asym}}{f_0\sigma_{\rm tot}} - 1 \simeq \frac{f_1^{\rm asym}}{f_0}\left(\underbrace{0.16}_{\alpha_s} - \underbrace{0.33}_{\alpha_s^2} + \underbrace{0.82}_{\alpha_s^3} - \underbrace{2.73}_{\alpha_s^4} + \underbrace{11.72}_{\alpha_s^5} + \dots\right) \simeq \frac{f_1^{\rm asym}}{f_0} \times \underbrace{0.05}_{\rm resummed}\,.$$

Divergent behaviour can be associated to a **theoretical ambiguity**, which can be estimated by looking at the size of the smallest term, $(\Lambda/M)^{0.2}$ (compared to $(\Lambda/M)^2$ expected for inclusive cross sections)

It's the sum that makes up the total

[Totò, '60]

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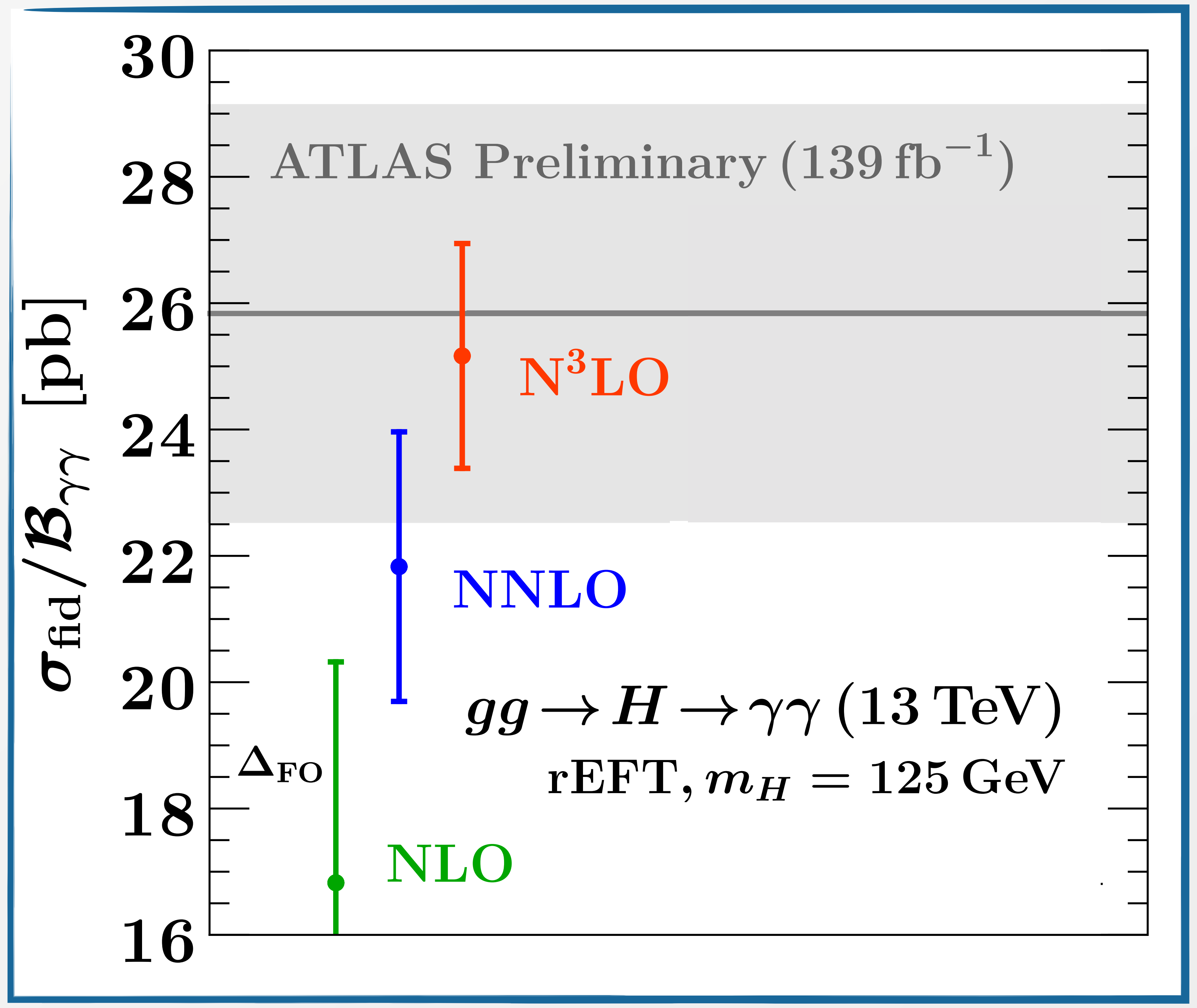
Linear **fiducial power corrections** can be resummed at all orders in perturbation theory

[Ebert, Michel, Stewart, Tackmann '20]



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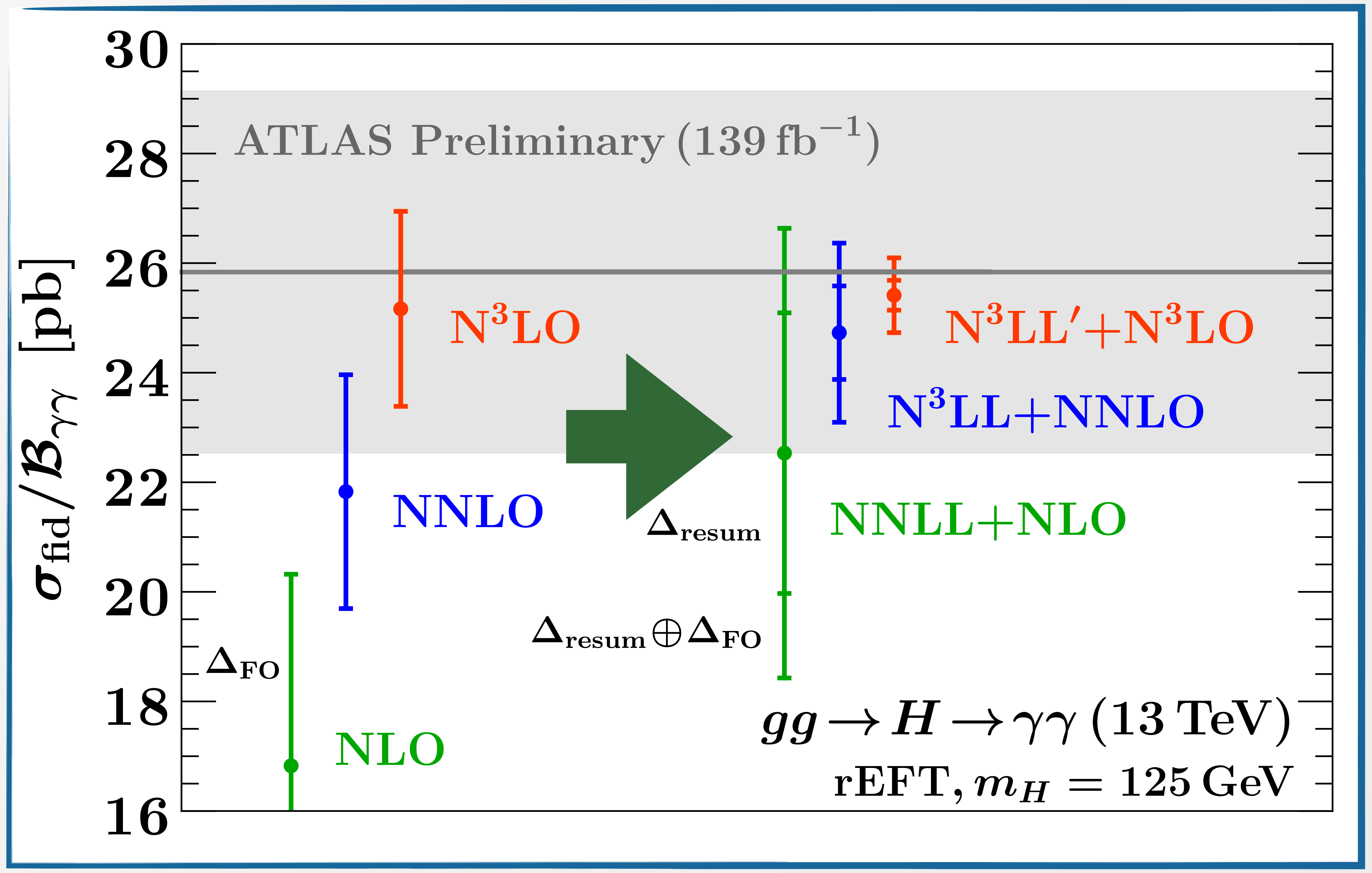


[Billis, Dehnadi, Ebert, Michel, Stewart, Tackmann '21]

All cuts are equal but some cuts are more equal than others

Do we need to abandon the idea of **fixed-order accuracy** for fiducial cross sections?

For **legacy measurements**, resummation appears the only viable solution

Resorting to alternative definition of cuts for future analyses can resolve the issue of linear fiducial power corrections altogether

[Salam, Slade '21]

Simplest options:

Product cuts:

Replace the symmetric/asymmetric cuts on $p_T^{(1)}, p_T^{(2)}$ with a cut on $p_T^{(1)} \cdot p_T^{(2)}$, **keeping a cut on the softer final state particle** $\min(p_T^{(1)}, p_T^{(2)}) > p_{\mathrm{T}}^{\mathrm{min}}$

Staggered cuts: [Grazzini, Kallweit, Wiesemann '17][Alekhin, Kardos, Moch, Trócsányi '21]

Rather than imposing an asymmetric cut on leading/subleading $p_T^{(1)}, p_T^{(2)}$, apply an asymmetric cut on **identified final state particles** (e.g. lepton/antilepton in NC DY production, lepton/neutrino in CC DY, photon with higher/lower rapidity in $pp \to H(\to \gamma\gamma)$)

More performing (and refined) choice of cuts possible [Salam, Slade '21]

Linear power corrections and q_T-subtraction

For candle processes like Drell-Yan production it would still be desirable to have predictions at **fixed order** (relevant for e.g. parton densities extraction)

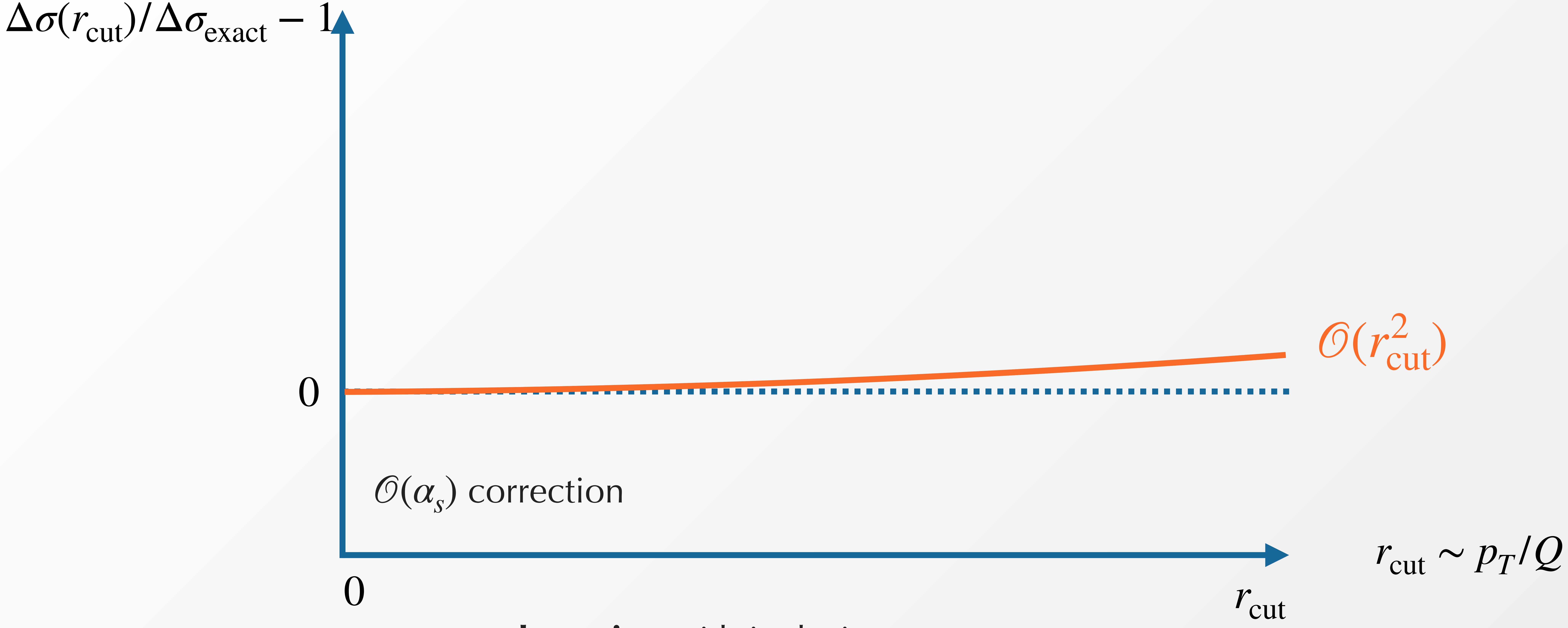
Linear dependence on p_T affects efficiency *and* precision of non-local subtraction techniques such as q_T-subtraction [Catani, Grazzini '07]

$$d\sigma_V^{\mathrm{N^3LO}} \equiv \mathcal{H}_V^{\mathrm{N^3LO}} \otimes d\sigma_V^{\mathrm{LO}} + \left(d\sigma_{V+\mathrm{jet}}^{\mathrm{NNLO}} - \left[d\sigma_V^{\mathrm{N^3LL}} \right]_{\mathcal{O}(\alpha_s^3)} \right) \Theta(p_T > p_T^{\mathrm{cut}}) + \mathcal{O}((p_T^{\mathrm{cut}}/M)^n)$$

Missing **power corrections** below the slicing cut-off

Relative size of **power corrections** affects **stability and performance** of non-local subtraction methods

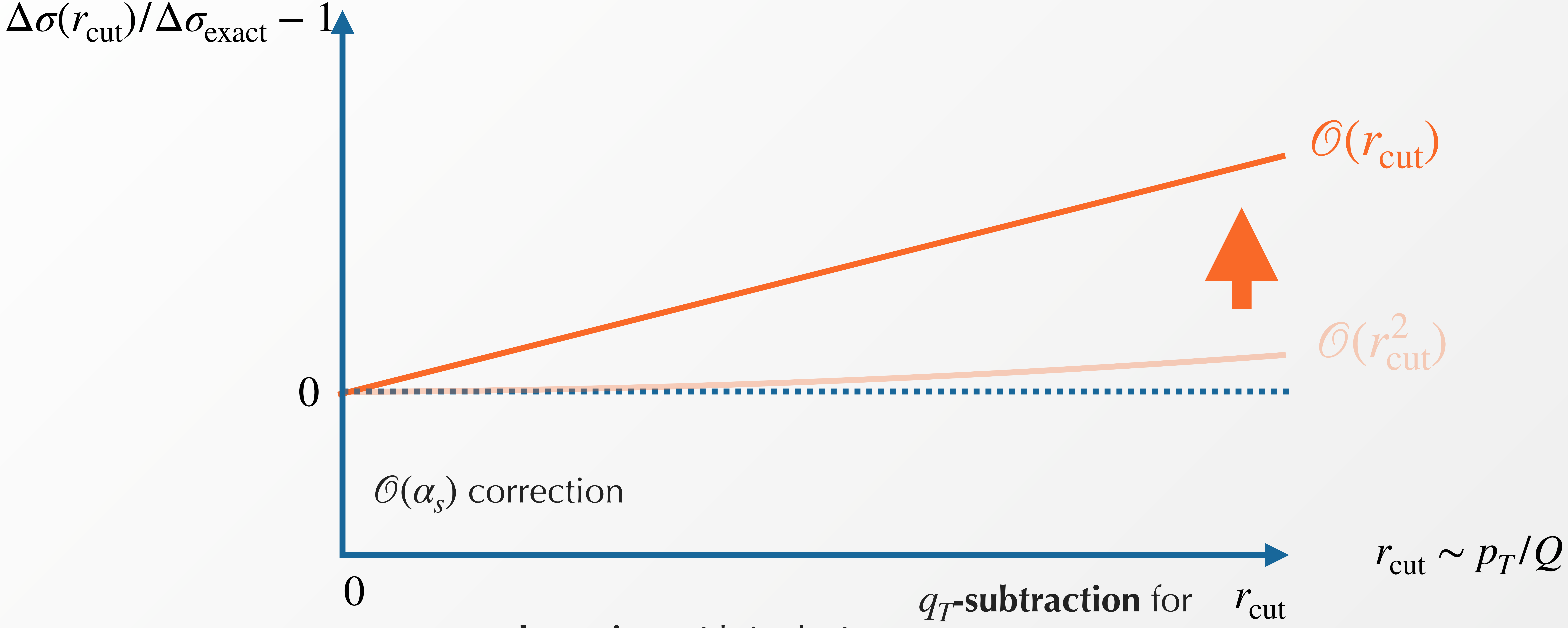
Linear power corrections and q_T-subtraction



q_T**-subtraction** with inclusive cuts (see LB's slides)

Quadratic dependence

Linear power corrections and q_T-subtraction



q_T**-subtraction** with inclusive cuts (see LB's slides)

Quadratic dependence

q_T**-subtraction** for $2 \to 2$ processes with (a)symmetric cuts

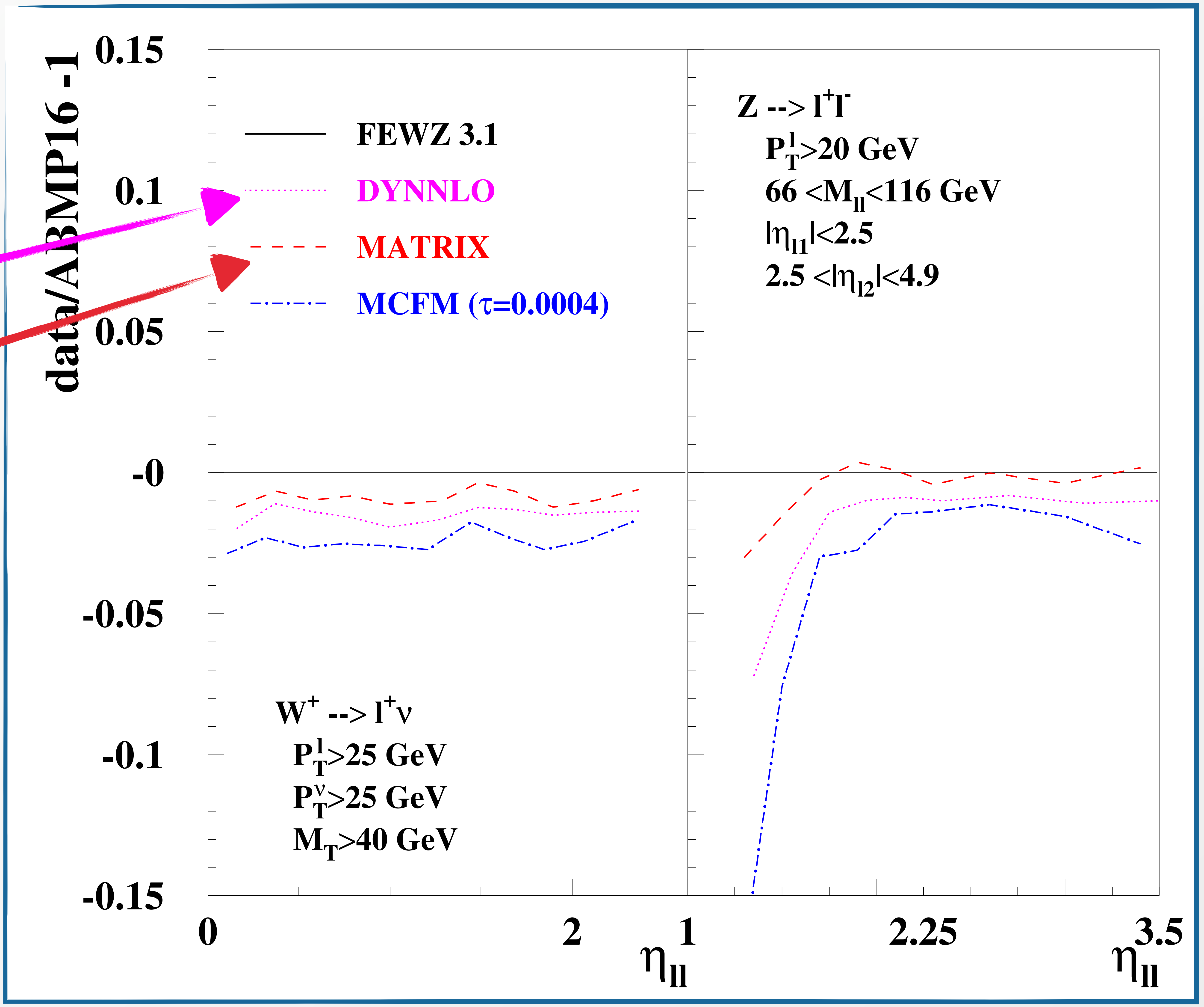
Linear dependence

Perturbative convergence within fiducial cuts

Values of $r_{\mathrm{cut}} \sim p_T/Q$ too large, or lack of extrapolation to $r_{\mathrm{cut}} \to 0$, can lead to **percent-level** effects when compared to results obtained with **local subtractions**

$r_{\mathrm{cut}} \sim 0.01$

$r_{\mathrm{cut}} \sim 0.0005 - 0.001$



[Alekhin, Kardos, Moch, Trócsányi '21]

Can this situation be improved?

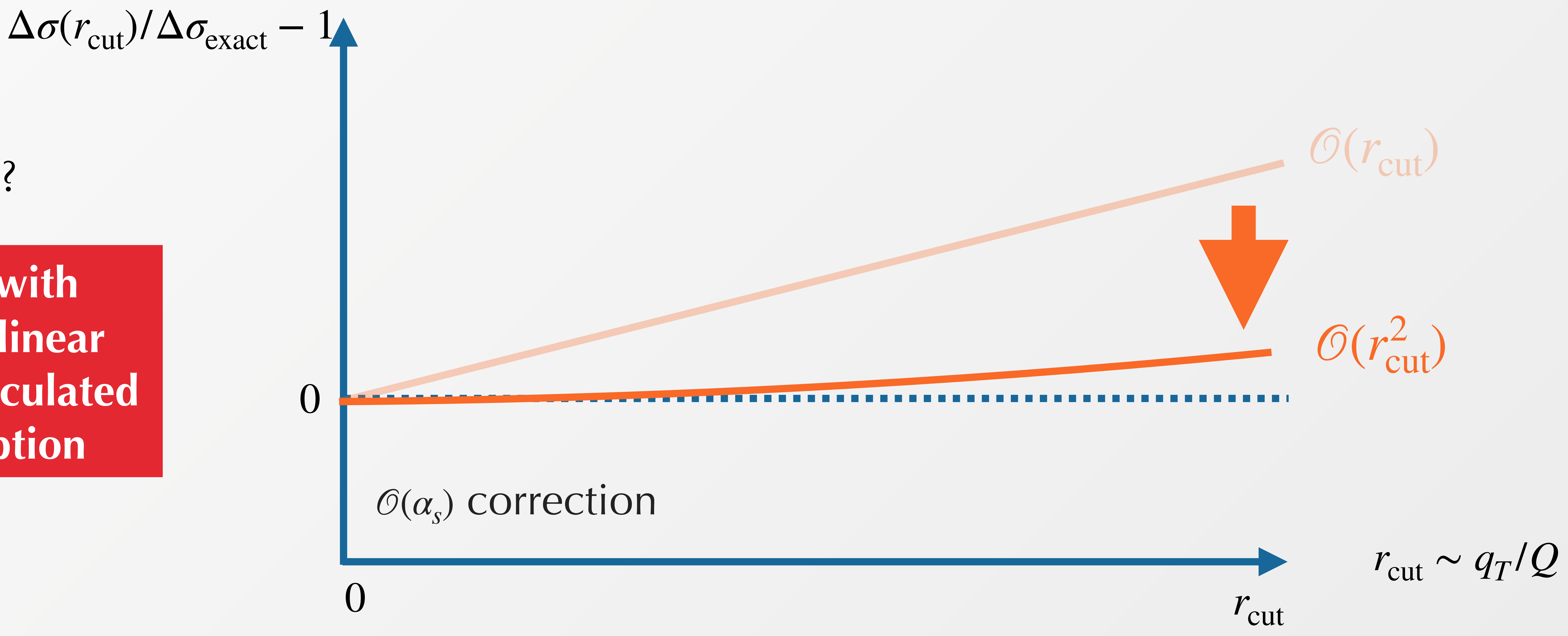
Perturbative convergence within fiducial cuts

Values of $r_{\text{cut}} \sim p_T/Q$ too large, or lack of extrapolation to $r_{\text{cut}} \to 0$, can lead to **percent-level** effects when compared to results obtained with **local subtractions**

Can this situation be improved?

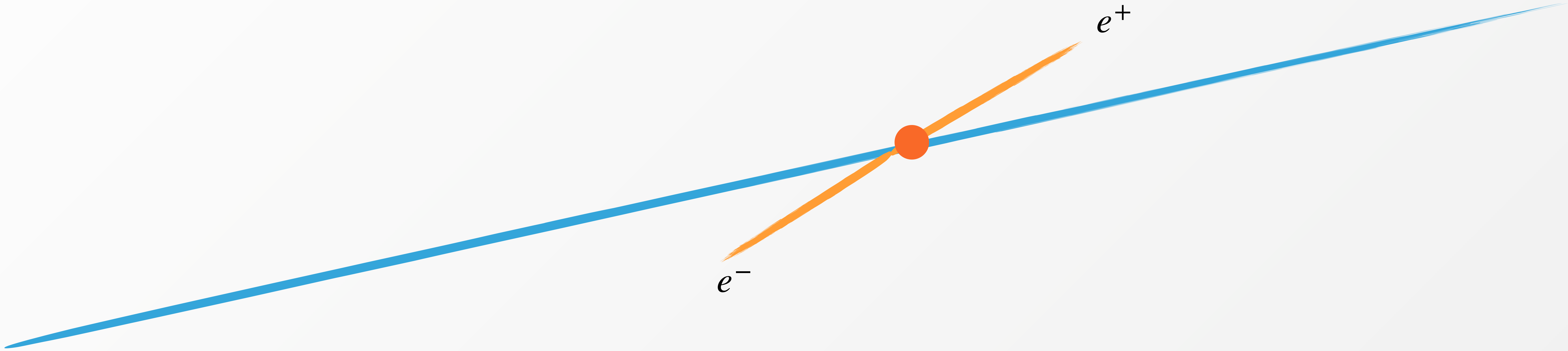
Yes! For $2 \to 2$ processes with (a)symmetric cuts, fiducial linear power corrections can be calculated via a simple recoil prescription

[Catani, de Florian, Ferrera, Grazzini '15]
[Ebert, Michel, Stewart, Tackmann '20]



Inclusion of transverse recoil effects

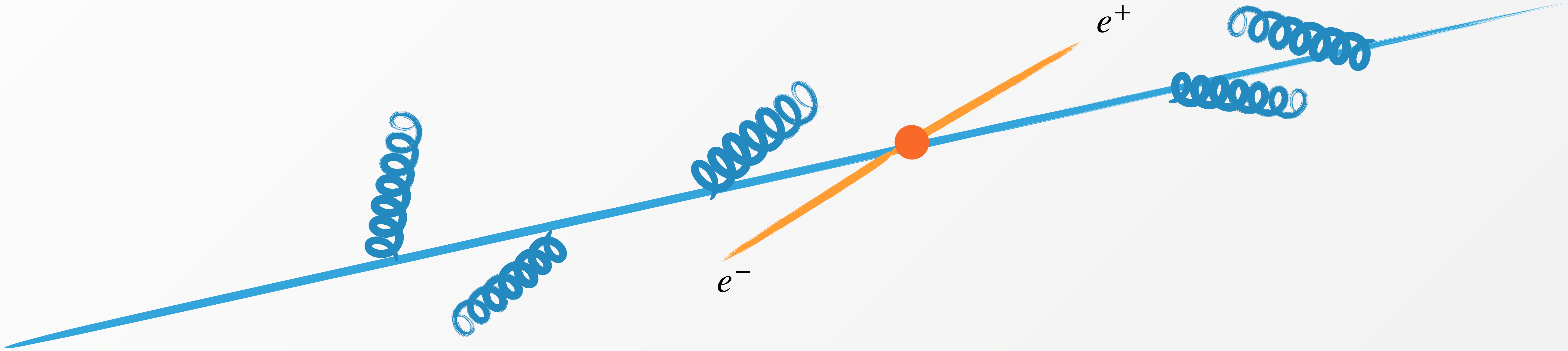
[Catani, de Florian, Ferrera, Grazzini '15]



Born matrix element
evaluated at $q_T = 0$

Inclusion of transverse recoil effects

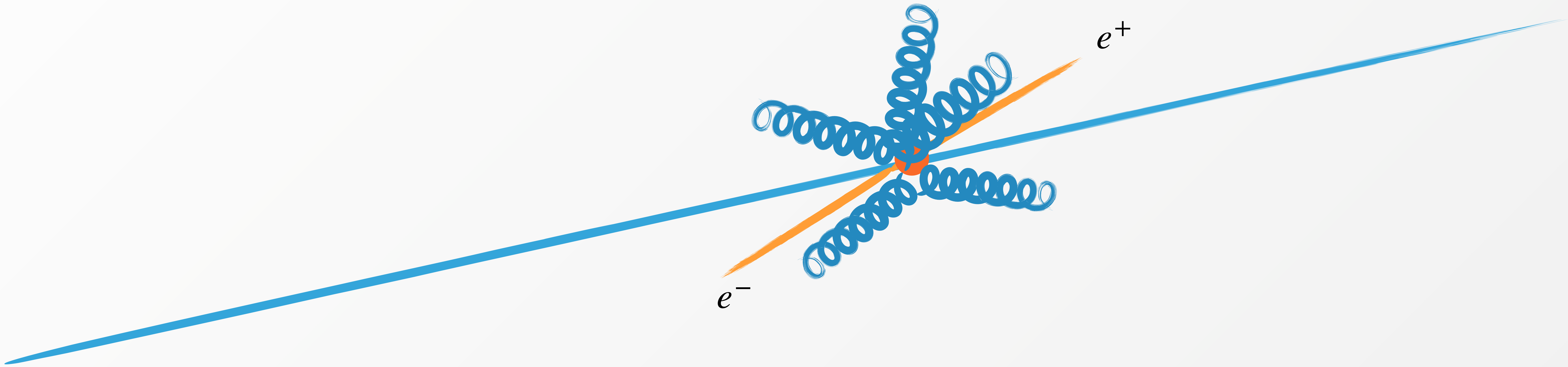
[Catani, de Florian, Ferrera, Grazzini '15]



Generate singlet q_T by
QCD radiation

Inclusion of transverse recoil effects

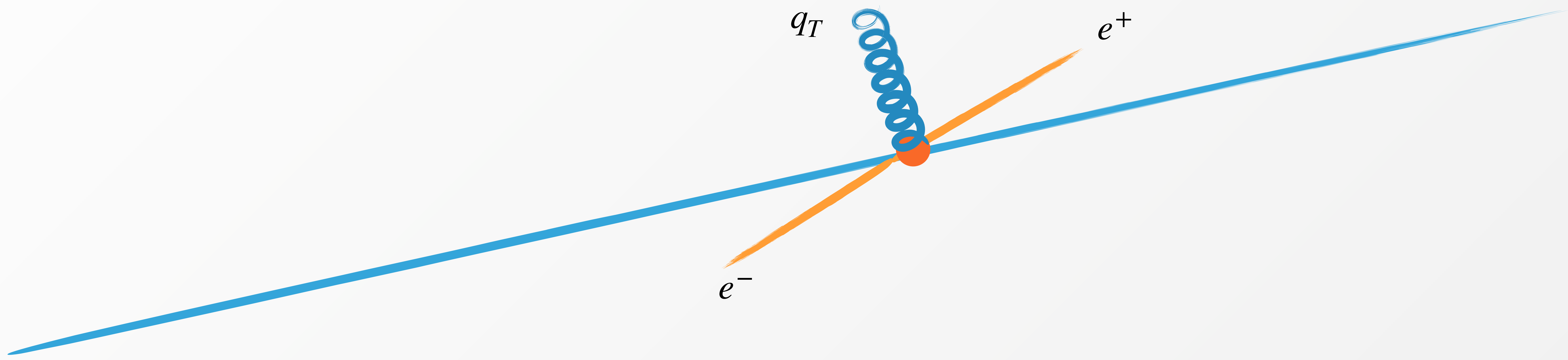
[Catani, de Florian, Ferrera, Grazzini '15]



Generate singlet q_T by QCD radiation

Inclusion of transverse recoil effects

[Catani, de Florian, Ferrera, Grazzini '15]



Generate singlet q_T by QCD radiation

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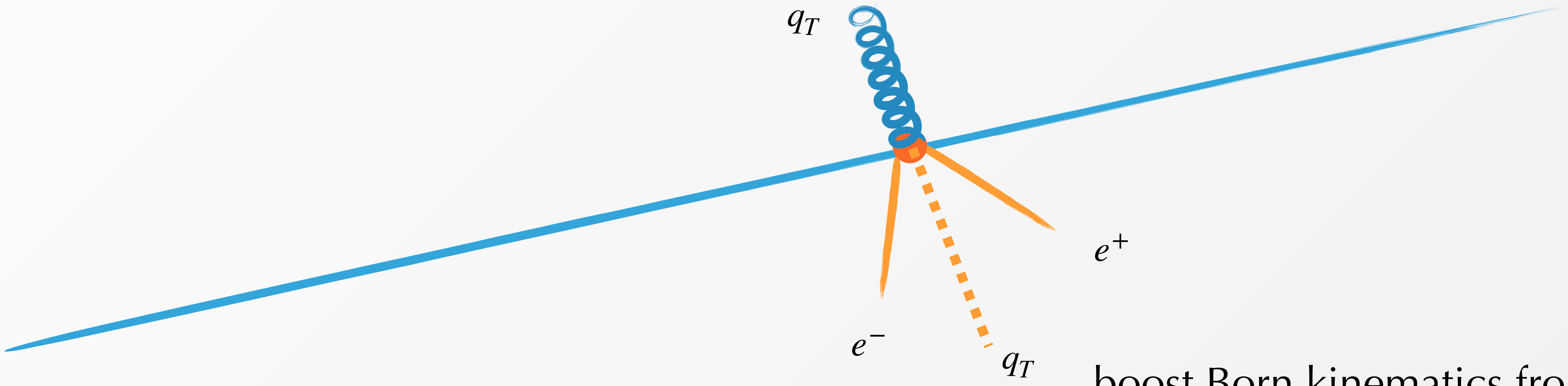
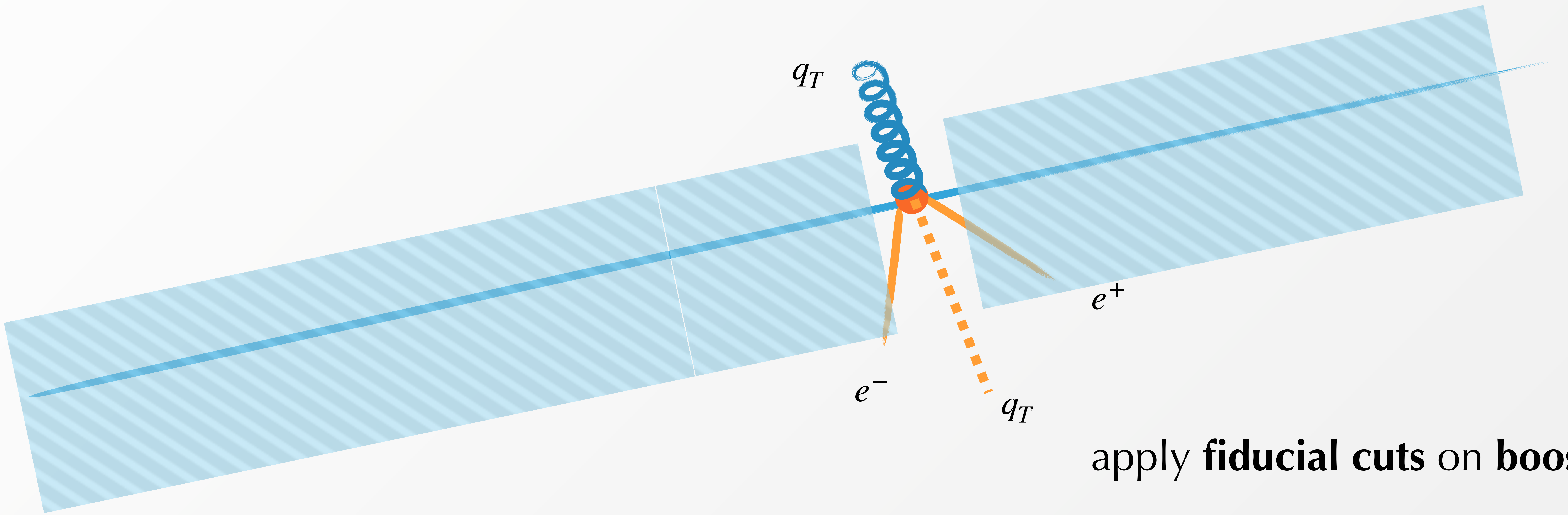


boost Born kinematics from boson rest frame (e.g. CS) to lab frame with that q_T

Inclusion of transverse recoil effects

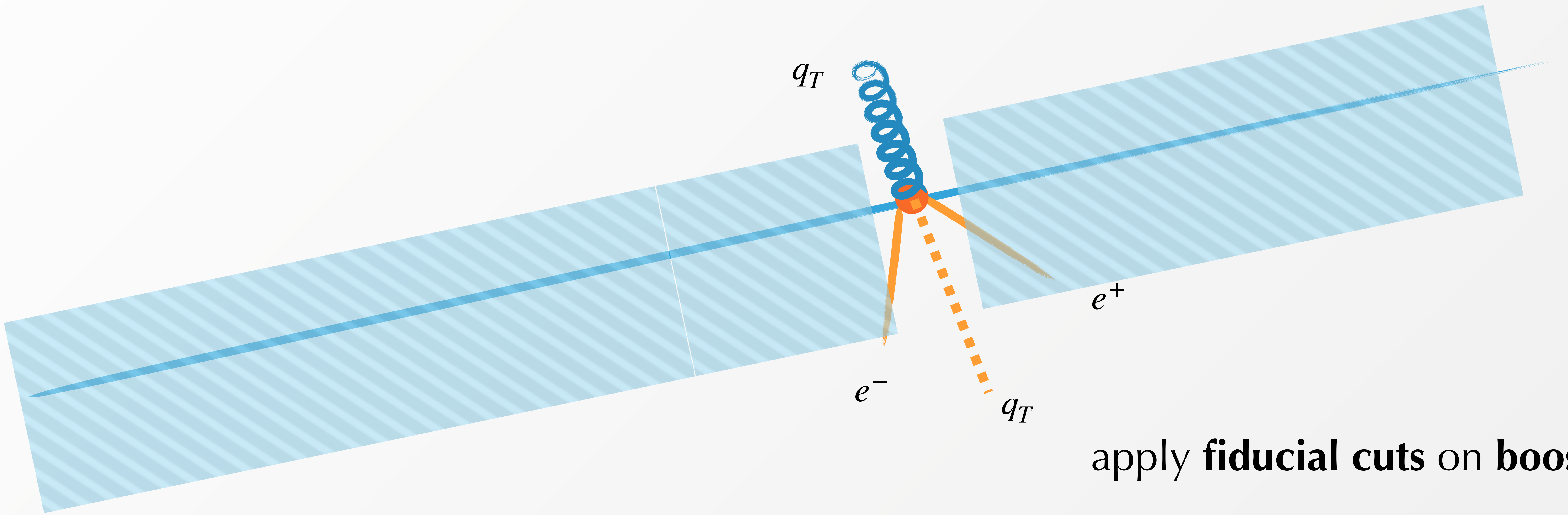
[Catani, de Florian, Ferrera, Grazzini '15]



apply **fiducial cuts** on **boosted Born kinematics**

Inclusion of transverse recoil effects

[Catani, de Florian, Ferrera, Grazzini '15]



apply **fiducial cuts** on **boosted Born kinematics**

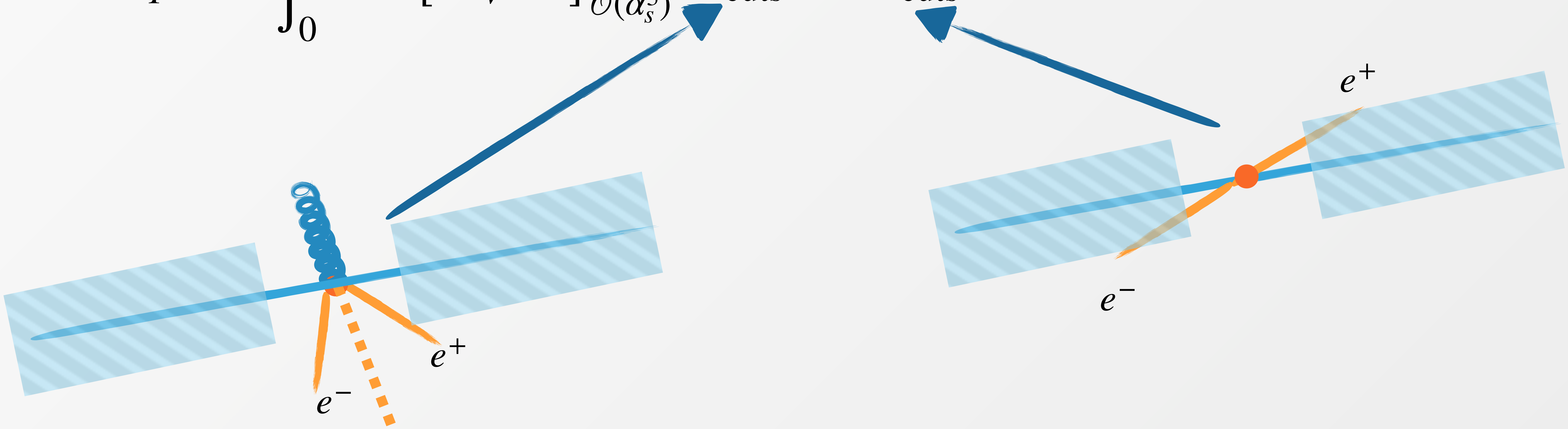
Sufficient to capture the **full linear fiducial power correction** for q_T [Ebert, Michel, Stewart, Tackmann '20]

Linear power corrections and q_T-subtraction

Resorting to the recoil prescription allows for the inclusion of all missing fiducial linear power corrections below $p_T^{\rm cut}$, improving dramatically the efficiency of the non-local subtraction [Buonocore, Kallweit, LR, Wiesemann'21] [Camarda, Cieri, Ferrera '21]

$$d\sigma_V^{\rm N^3LO} \equiv \mathcal{H}_{\rm V}^{\rm N^3LO} \otimes d\sigma_{\rm V}^{\rm LO} + \left(d\sigma_{\rm V+jet}^{\rm NNLO} - \left[d\sigma_V^{\rm N^3LL} \right]_{\mathcal{O}(\alpha_s^3)} \right) \Theta(p_T > p_T^{\rm cut}) + \Delta\sigma^{\rm linPCs}(p_T^{\rm cut}) + \mathcal{O}((p_T^{\rm cut}/M)^2)$$

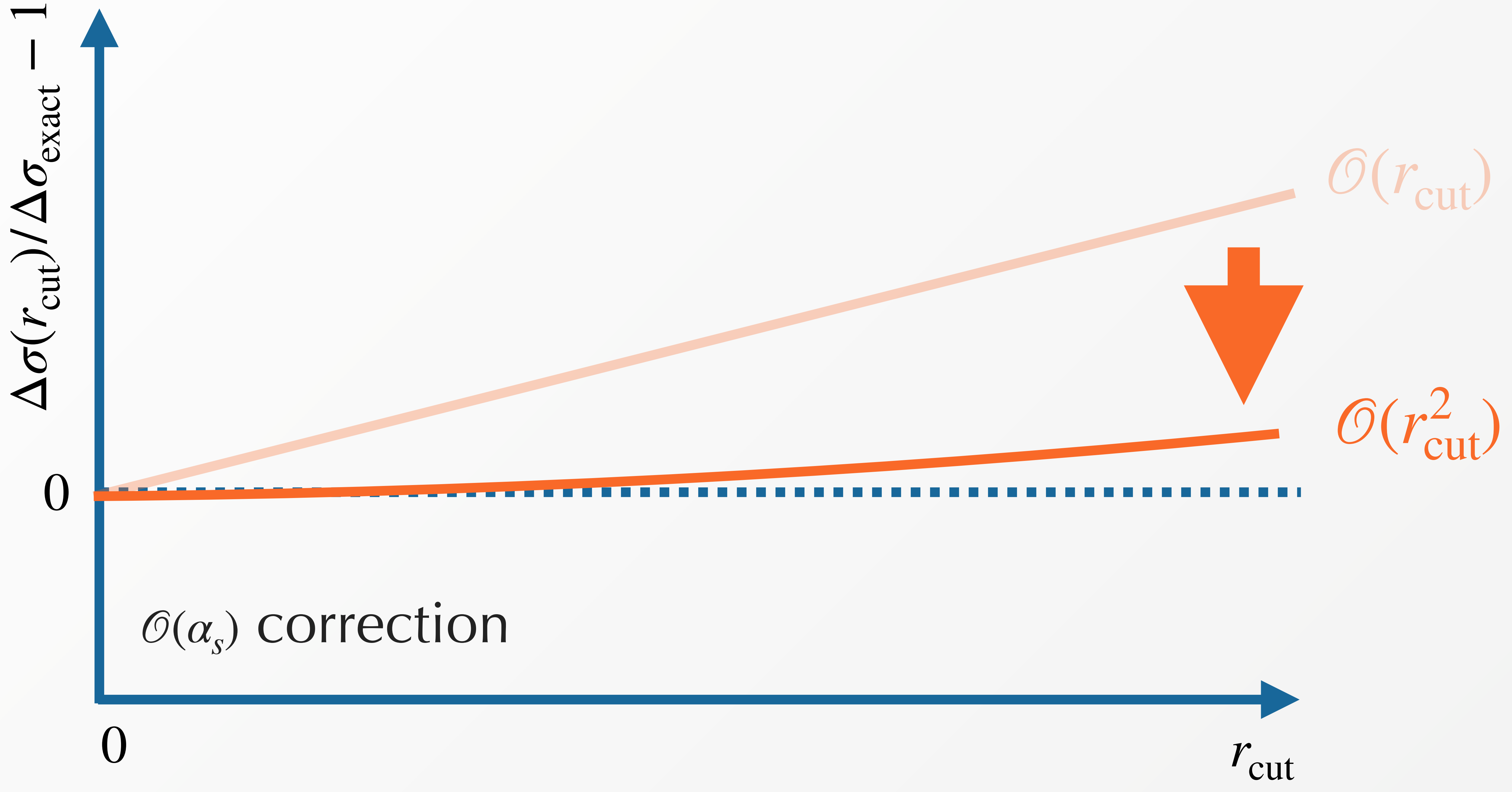
$$\Delta\sigma^{\rm linPCs}(p_T^{\rm cut}) = \int_0^{r_{\rm cut}} dr' \left[d\sigma_V^{\rm N^3LL} \right]_{\mathcal{O}(\alpha_s^3)} \left(\Theta_{\rm cuts}^{\rm recoil} - \Theta_{\rm cuts}^{\rm Born} \right)$$

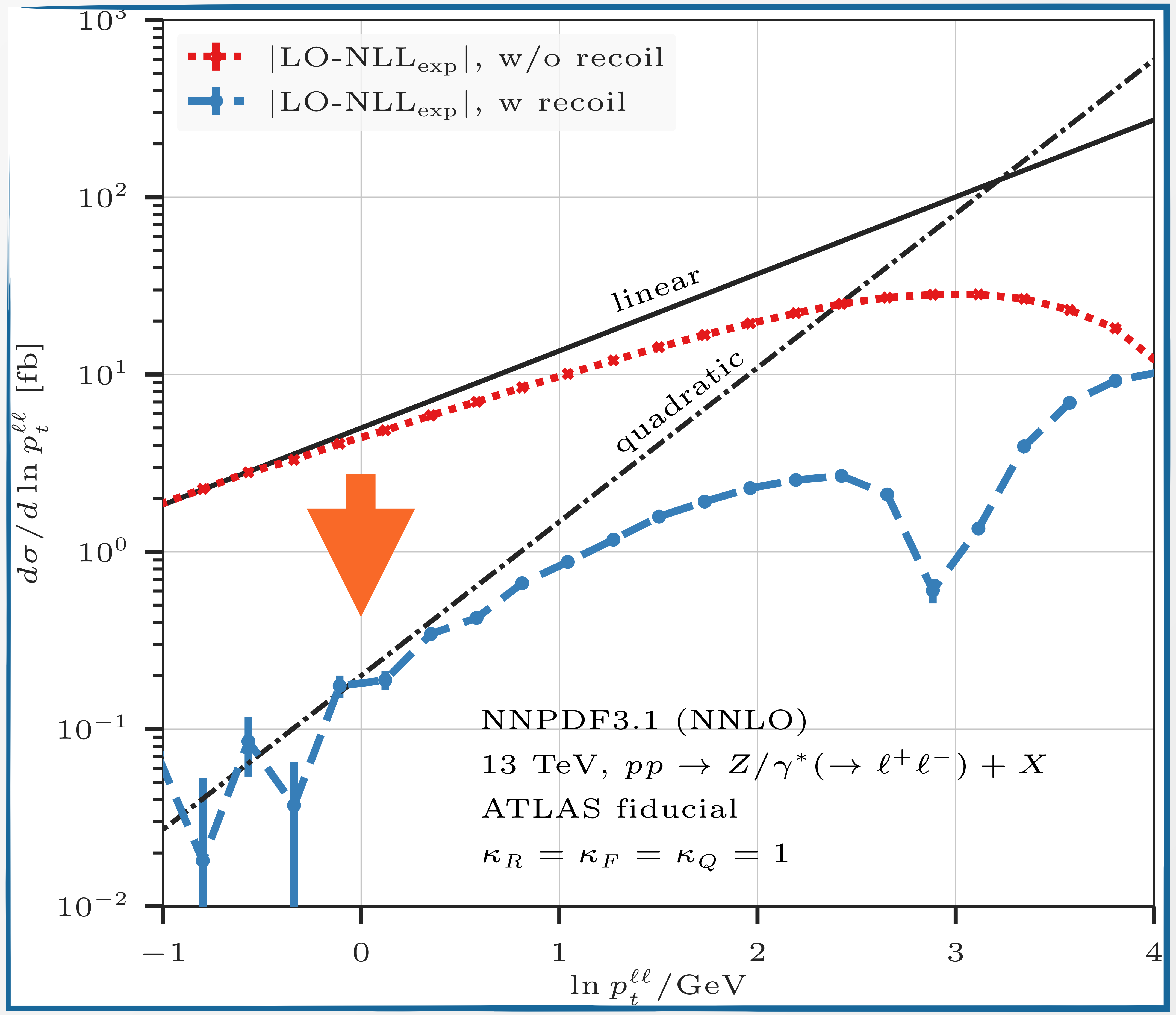


Linear power corrections have a **purely kinematical origin** and can **be predicted by factorisation**

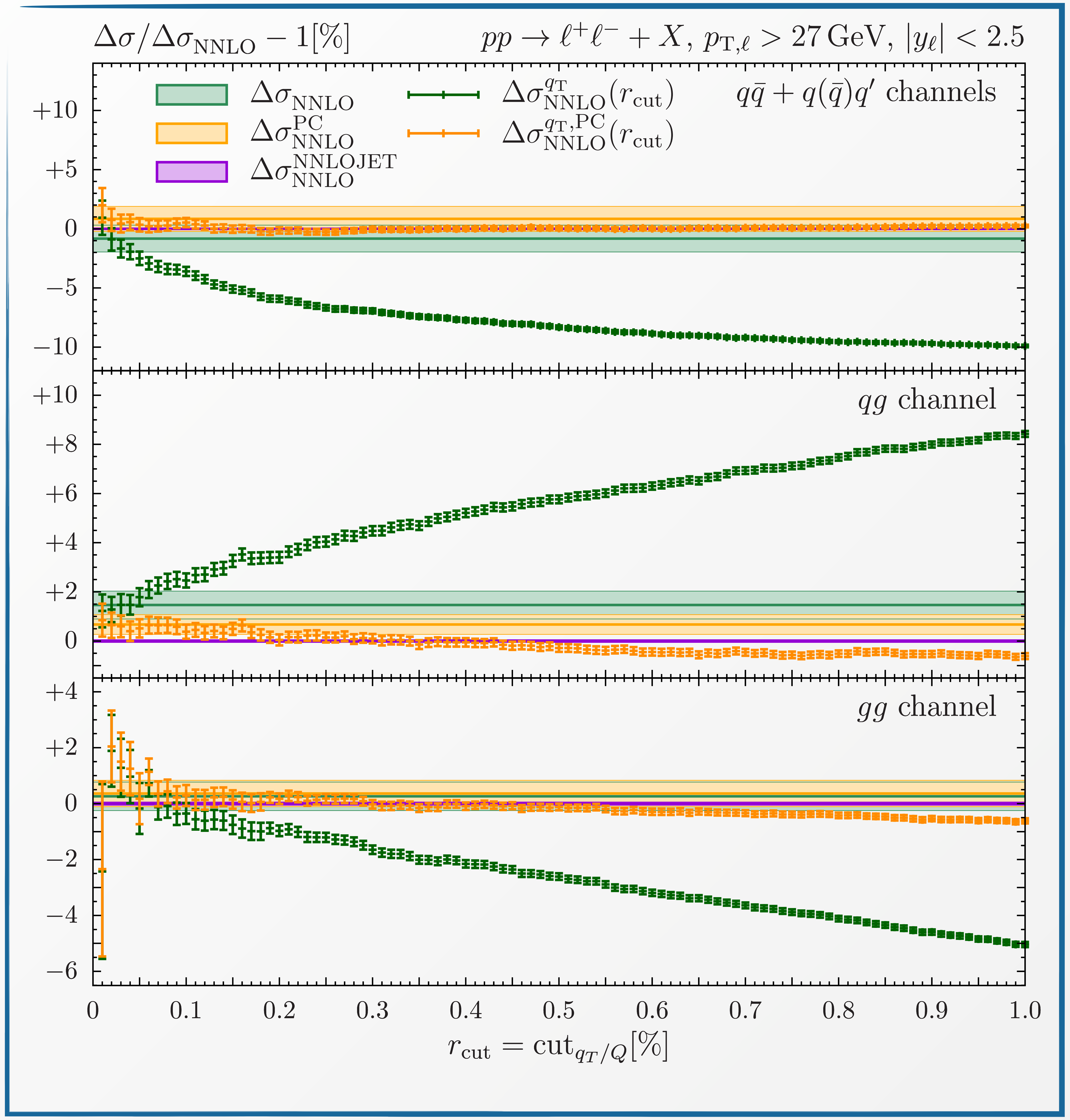
Linear power corrections and q_T-subtraction

Recoil effectively captures the **full linear fiducial power correction** for p_t





Linear power corrections for q_T-subtraction



[Buonocore, Kallweit, LR, Wiesemann '21]

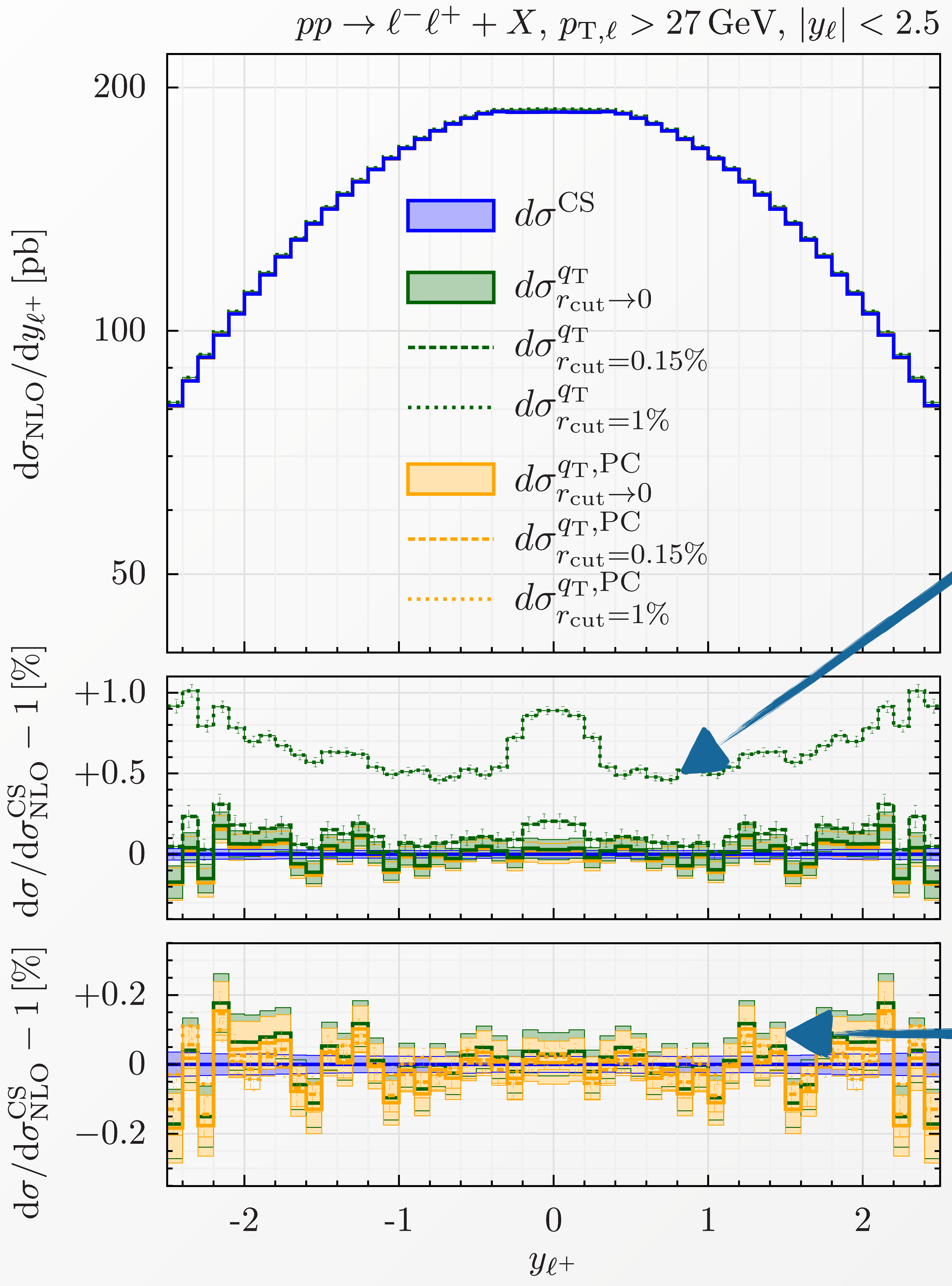
Much improved convergence over linear power correction case

Accurate computation of the NNLO correction without the need to push $r_{\rm cut}$ to very low values

Nice agreement up to NNLO with NNLOJET, which uses a local subtraction method

Now available in the new public version of **MATRIX 2.1**

Linear power corrections for q_T-subtraction: differential distributions



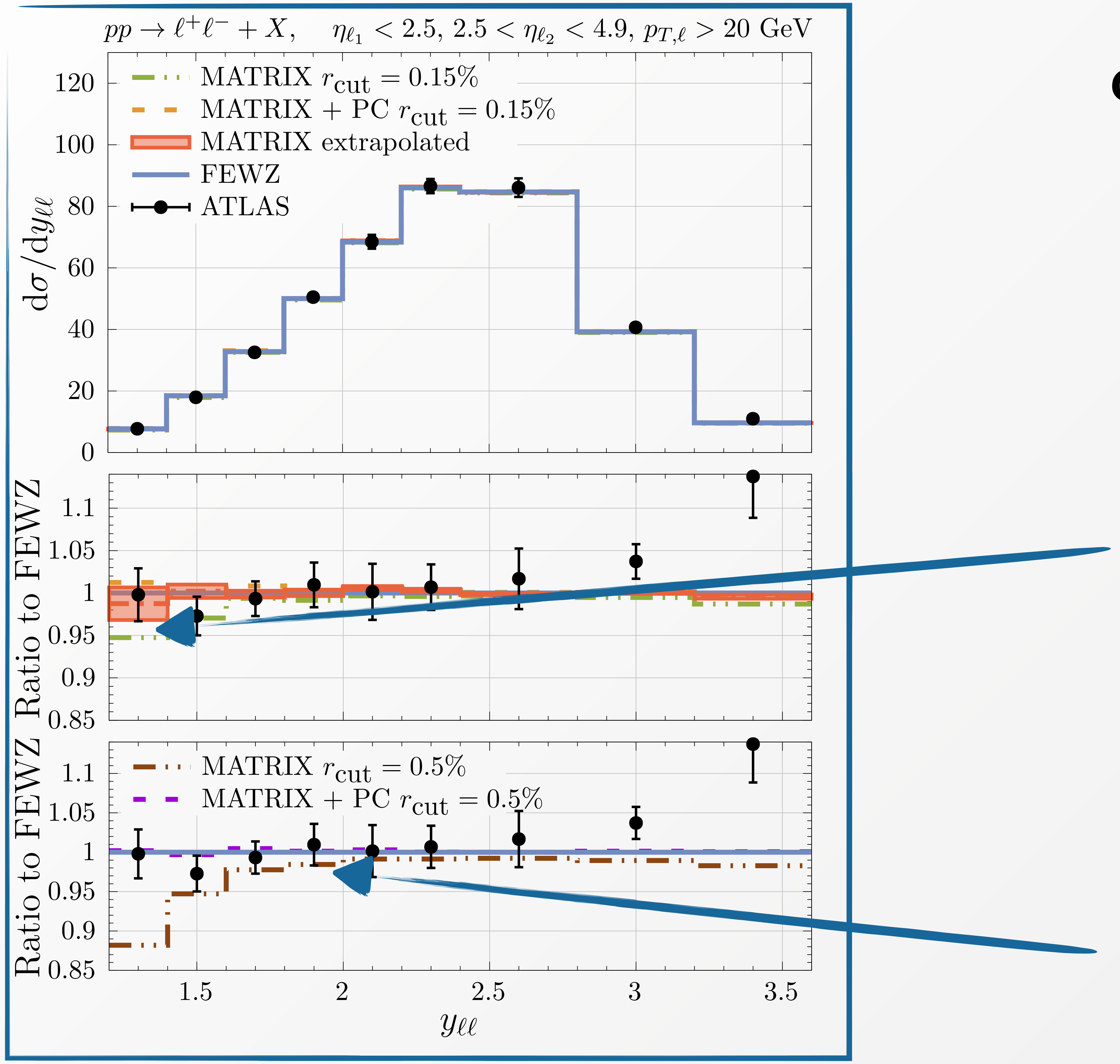
[Buonocore, Kallweit, LR, Wiesemann '21]

Without power corrections percent-level discrepancies are visible for too larger values of the slicing parameters

Smaller values of the slicing parameters are required to reliably compute extrapolated result

The inclusion of linear power corrections substantially improves the convergence towards result obtained with local subtractions, even at large-ish values of r_{cut}

Linear power corrections for q_T-subtraction: differential distributions



Comparison with ATLAS 8 TeV dataset

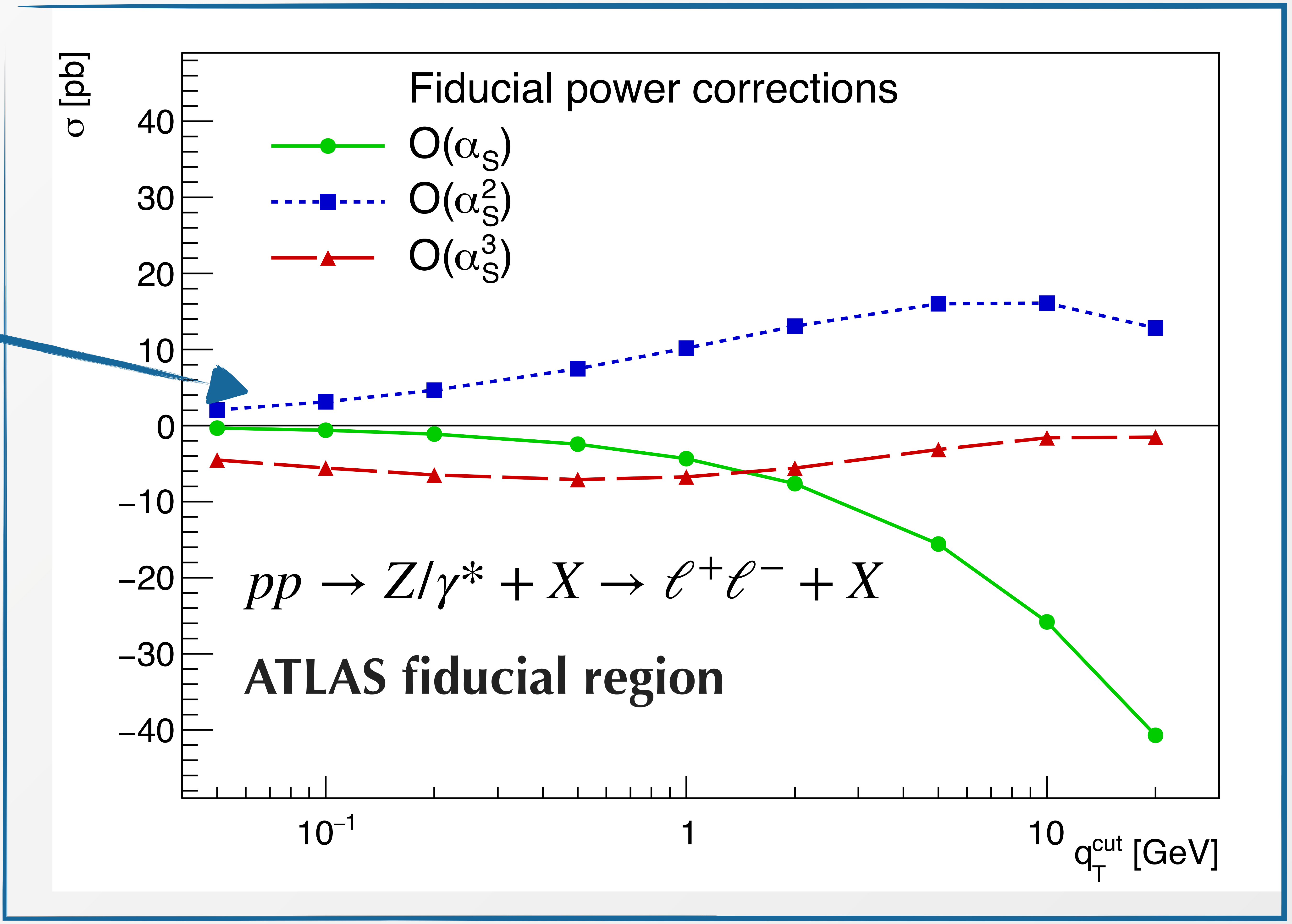
- setup from [Alekhin, Kardos, Moch, Trócsányi '21]
- At NNLO, discrepancies between local and non local subtraction can reach the few-percent level in certain configurations
- large corrections at small $y_{\ell\ell}$: 5%(10%) at r_{cut}=0.15%(0.5%)
- for the standard q_T subtraction computation, **bin-wise extrapolation required** to correctly model the first two bins
- excellent agreement when including $\Delta\sigma^{\text{linPCs}}$ already at r_{cut}=0.5%

Linear power corrections and N³LO calculations

$$\Delta\sigma^{\text{linPCs}}(p_T^{\text{cut}}) = \int_0^{r_{\text{cut}}} dr' \left[d\sigma_V^{\text{N}^3\text{LL}}\right]_{\mathcal{O}(\alpha_s^3)} (\Theta_{\text{cuts}}^{\text{recoil}} - \Theta_{\text{cuts}}^{\text{Born}})$$

No sign of perturbative convergence in the size of linear power corrections

Inclusion of power corrections becomes mandatory to obtain predictions at N³LO with q_T slicing if symmetric cuts are used



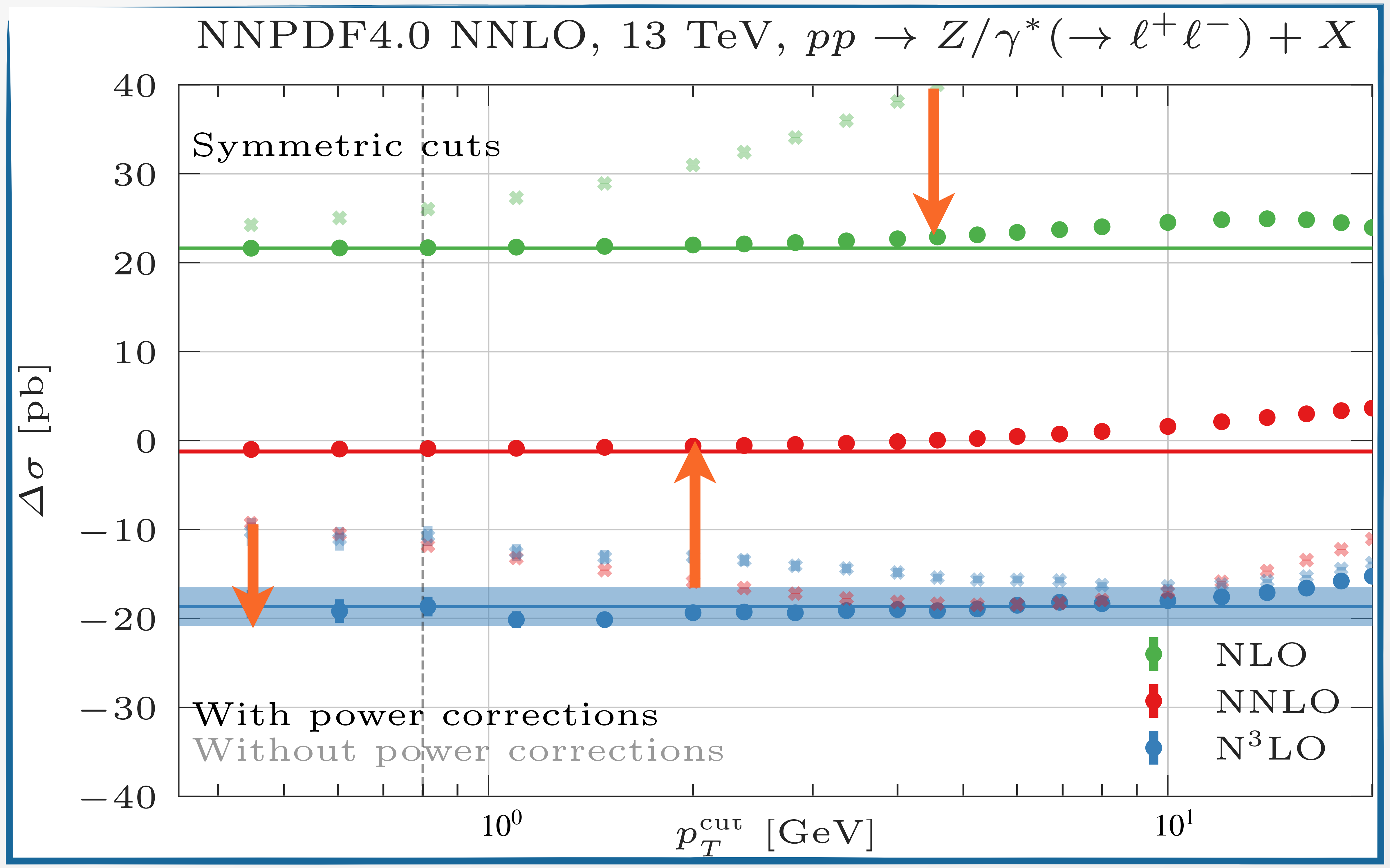
[Camarda, Cieri, Ferrera '21]

The Drell-Yan fiducial cross section at N³LO

ATLAS fiducial region $p_T^{\ell^\pm} > 27\,\text{GeV} \qquad |\eta^{\ell^\pm}| < 2.5$

$$d\sigma_V^{N^3LO} \equiv \mathcal{H}_V^{N^3LO} \otimes d\sigma_V^{LO} + \left(d\sigma_{V+jet}^{NNLO} - \left[d\sigma_V^{N^3LL} \right]_{\mathcal{O}(\alpha_s^k)} \right) \Theta(p_T > p_T^{cut}) + \mathcal{O}((p_T^{cut}/M)^n)$$

- Exquisite control on the **fixed order component** (from **NNLOJET**) allows to push to low values of the slicing parameter p_T^{cut}
- Mandatory to include missing linear **power corrections** to reach a **precise control of the N^kLO correction** down to small values of p_T^{cut}
- Plateau at small p_T^{cut} indicates the desired independence of the slicing parameter
- Result without power correction does not converge yet to the correct value at N^kLO



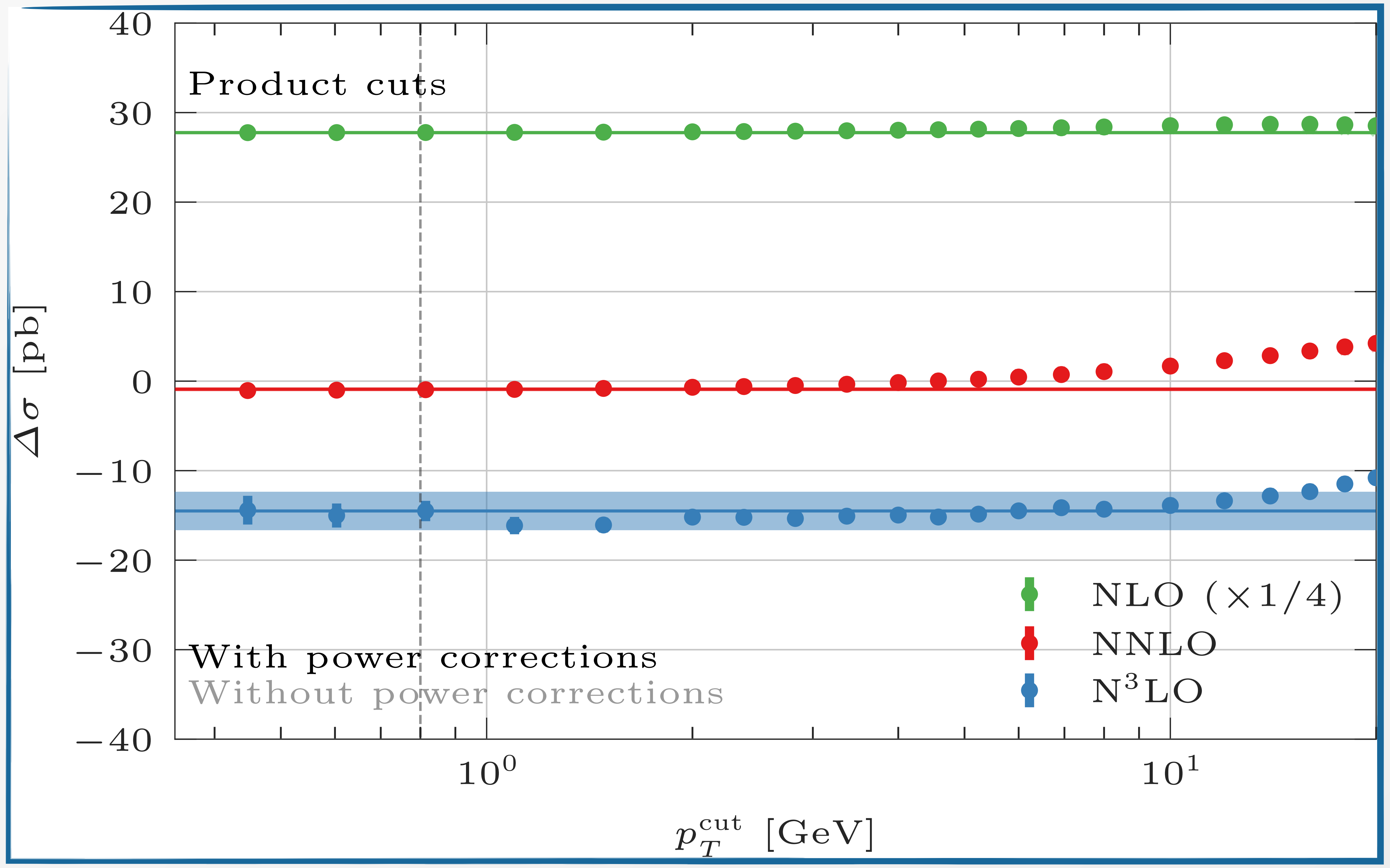
[Chen, Gehrmann, Glover, Huss, Monni, Re, LR, Torrielli '22]

The Drell-Yan fiducial cross section at N³LO

Product cuts
[Salam, Slade '21]

$$\sqrt{|\vec{p}_T^{\,\ell^+}||\vec{p}_T^{\,\ell^-}|} > 27\,\text{GeV} \qquad \min\{|\vec{p}_T^{\,\ell^\pm}|\} > 20\,\text{GeV} \qquad |\eta^{\ell^\pm}| < 2.5$$

- **Alternative set of cuts** which does not suffer from linear power corrections
- Improved convergence, result independent of the recoil procedure



[Chen, Gehrmann, Glover, Huss, Monni, Re, LR, Torrielli '22]

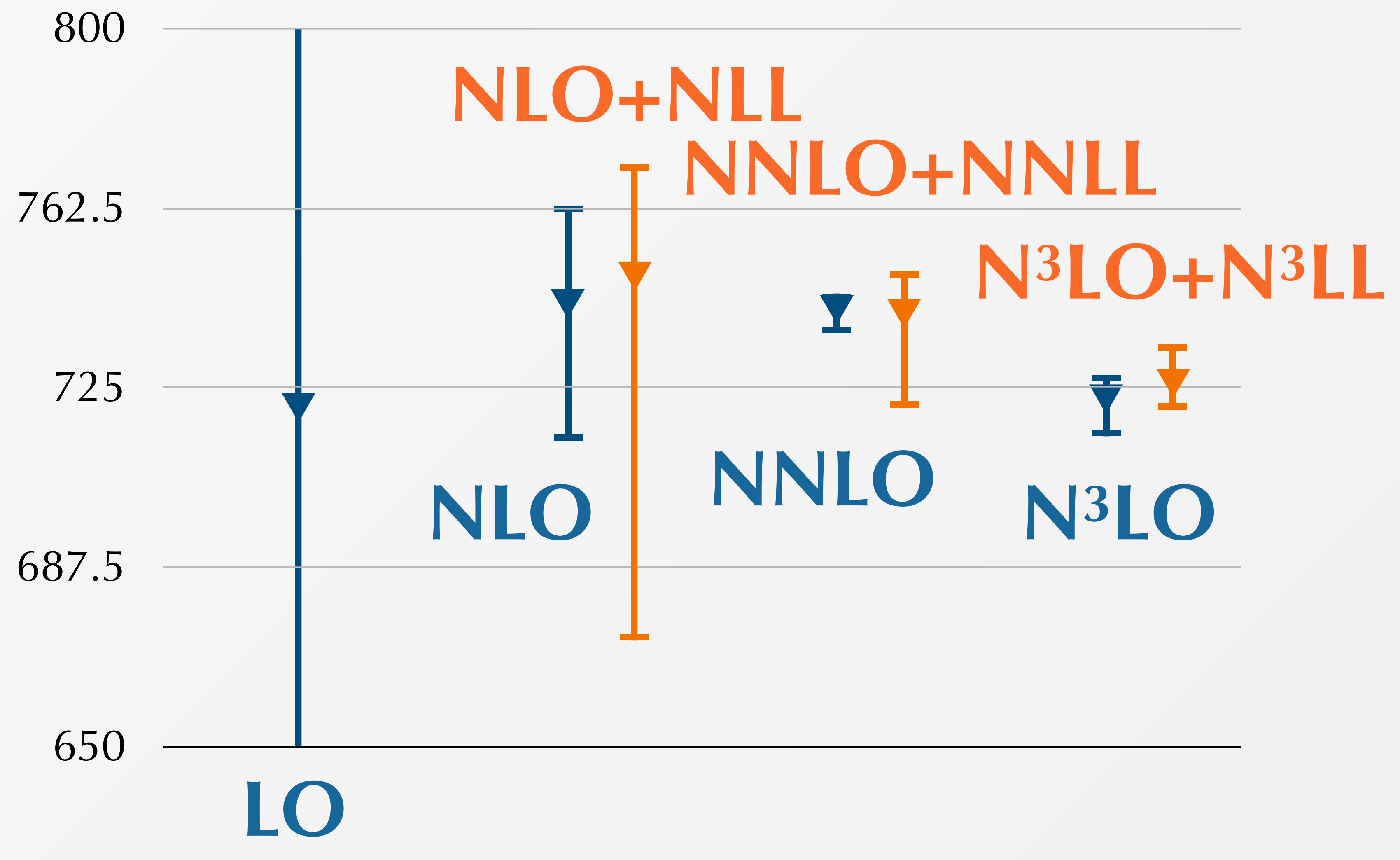
The Drell-Yan fiducial cross section at N3LO and N3LO+N3LL

[Chen, Gehrmann, Glover, Huss, Monni, Re, LR, Torrielli '22]

Includes resummation of linear power corrections

Order k	σ [pb] Symmetric cuts: $\mathrm{N}^k\mathrm{LO}$	$\mathrm{N}^k\mathrm{LO}+\mathrm{N}^k\mathrm{LL}$
0	$721.16^{+12.2\%}_{-13.2\%}$	—
1	$742.80(1)^{+2.7\%}_{-3.9\%}$	$748.58(3)^{+3.1\%}_{-10.2\%}$
2	$741.59(8)^{+0.42\%}_{-0.71\%}$	$740.75(5)^{+1.15\%}_{-2.66\%}$
3	$722.9(1.1)^{+0.68\%}_{-1.09\%} \pm 0.9$	$726.2(1.1)^{+1.07\%}_{-0.77\%}$

$q_T^{\mathrm{cut}} = 0.8\,\mathrm{GeV}$



- 2.5% negative correction at N3LO in the ATLAS fiducial region. N3LO larger than the NNLO correction and outside its error band
- More robust estimate of the theory uncertainty when **resummation effects are included**
- Slicing error computed conservatively by considering the cutoff within the [0.45-1.5] GeV interval

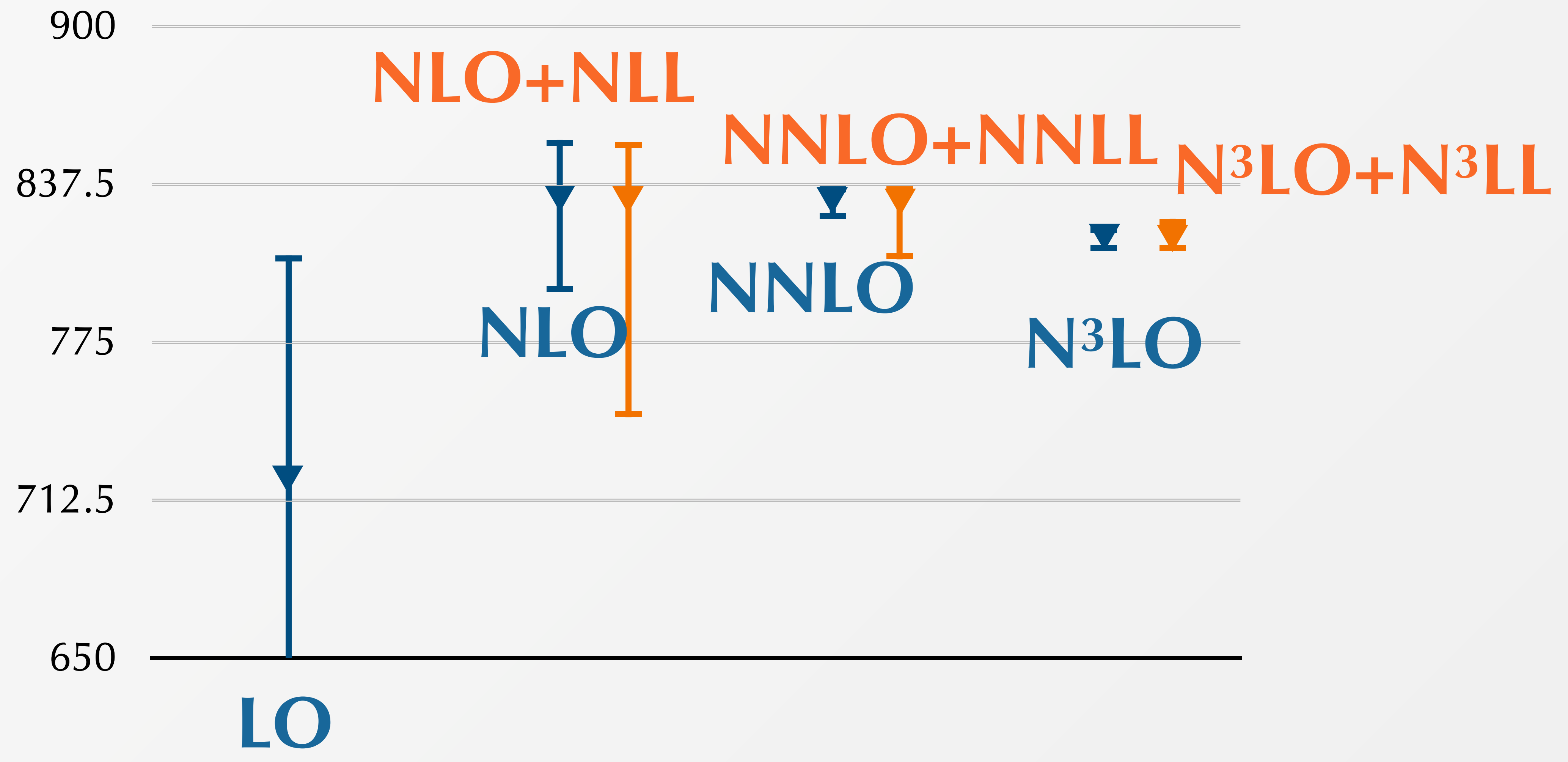
The Drell-Yan fiducial cross section at N³LO and N³LO+N³LL

[Chen, Gehrmann, Glover, Huss, Monni, Re, LR, Torrielli '22]

Includes resummation of linear power corrections

Order	σ [pb] Product cuts	
k	$\mathrm{N}^k\mathrm{LO}$	$\mathrm{N}^k\mathrm{LO}+\mathrm{N}^k\mathrm{LL}$
0	$721.16^{+12.2\%}_{-13.2\%}$	—
1	$832.22(1)^{+2.7\%}_{-4.5\%}$	$831.91(2)^{+2.7\%}_{-10.4\%}$
2	$831.32(3)^{+0.59\%}_{-0.96\%}$	$830.98(4)^{+0.74\%}_{-2.73\%}$
3	$816.8(1.1)^{+0.45\%}_{-0.73\%} \pm 0.8$	$816.6(1.1)^{+0.87\%}_{-0.69\%}$

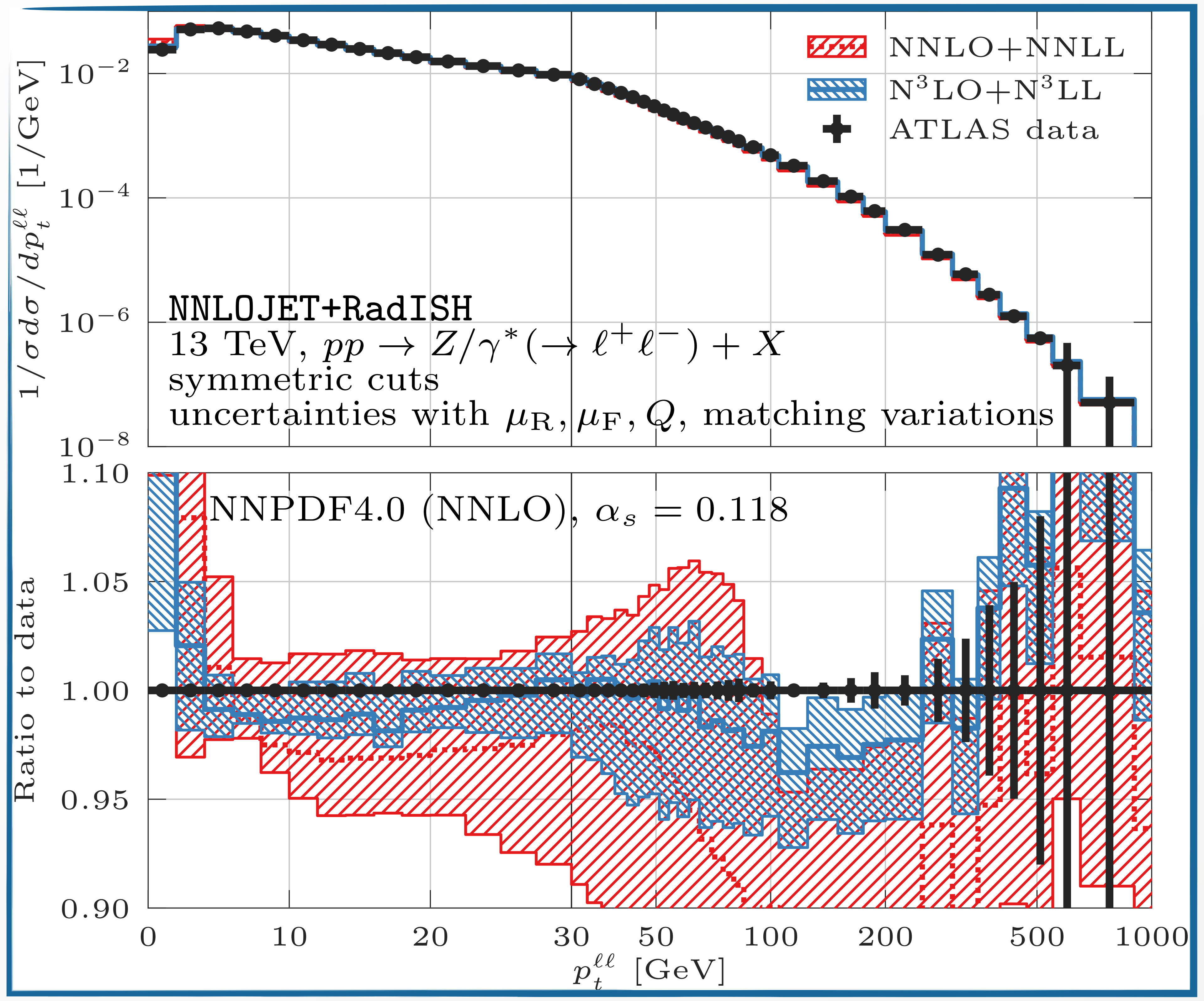
$q_T^{\mathrm{cut}} = 0.8\,\mathrm{GeV}$

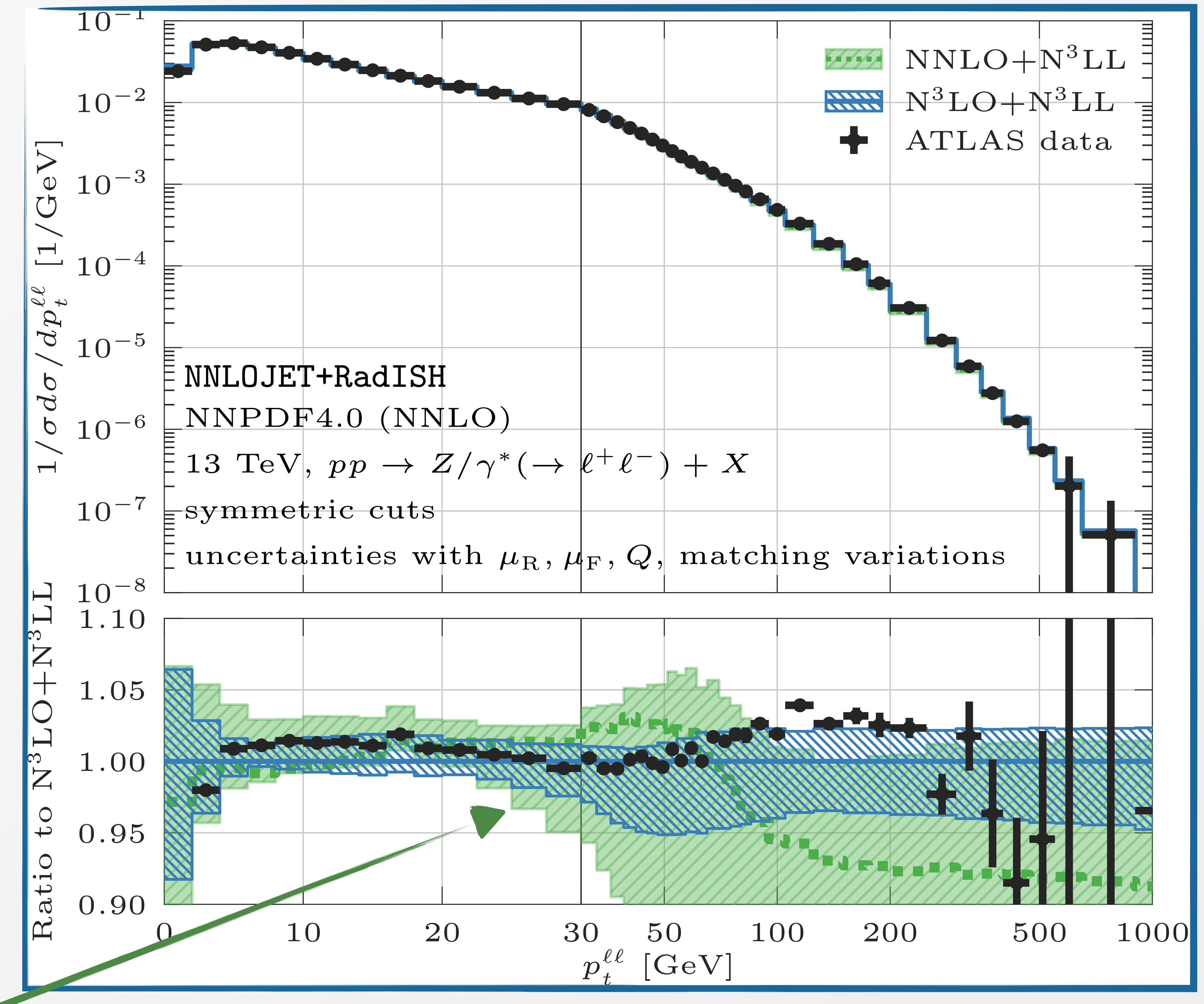


- 2.5% negative correction at N³LO in the ATLAS fiducial region. N³LO larger than the NNLO correction and outside its error band
- More robust estimate of the theory uncertainty when **resummation effects are included**
- Slicing error computed conservatively by considering the cutoff within the [0.45-1.5] GeV interval
- Central value very similar at $\mathrm{N^kLO}$ and $\mathrm{N^kLO+N^kLL}$ for product cuts, compatible with the absence of linear power corrections

Transverse momentum resummation and power corrections in DY

Resummation of **linear power corrections** captures the bulk of the non-singular component at low values of p_T





However, effect of higher-order power correction from fixed-order corrections has 1-3% effect even at low values of the transverse momentum

Fixed order matching at small values of transverse momentum **essential** for applications in **Drell-Yan precision physics** (PDF, α_s extraction, …)

Summary

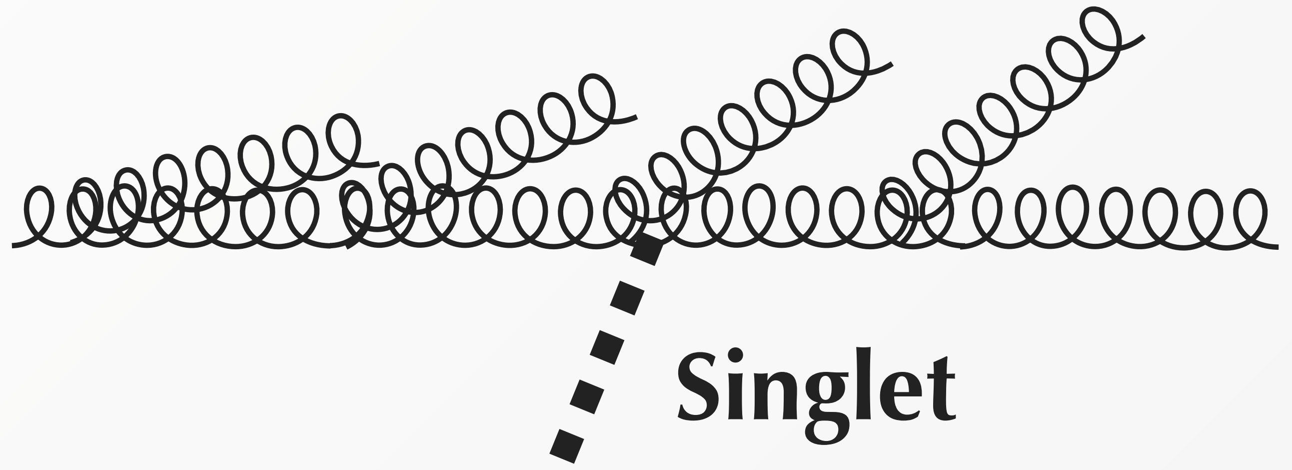
- Fiducial cuts currently applied in experimental analyses in **two-body final states** (Drell-Yan, Higgs to diphotons) cause **undesired instabilities** in fixed-order perturbation theory
- **Resummation** provides a viable solution for legacy measurements. Robust physical results **also require fixed order predictions** for practical applications (e.g. PDF extraction)
- Sensitivity to unresolved region challenges **non-local subtraction methods**, which are widely used in data-theory comparison at NNLO (e.g. MATRIX, MCFM) or are currently the only viable method to get to N^3LO accuracy for key processes (e.g. **fiducial DY production**)
- Reliable results up to N^3LO can be obtained using q_T-subtraction methods by computing **fiducial linear power corrections**
- **Resummation of linear fiducial power corrections** can be performed alongside q_T-resummation to provide reliable all-order results for legacy measurements
- **Robustness of data-theory comparison within fiducial regions require rethinking of fiducial acceptances for future LHC measurements in run 3**

Backup

Resummation of the transverse momentum spectrum

Resummation of transverse momentum is delicate because p_t is a **vectorial quantity**

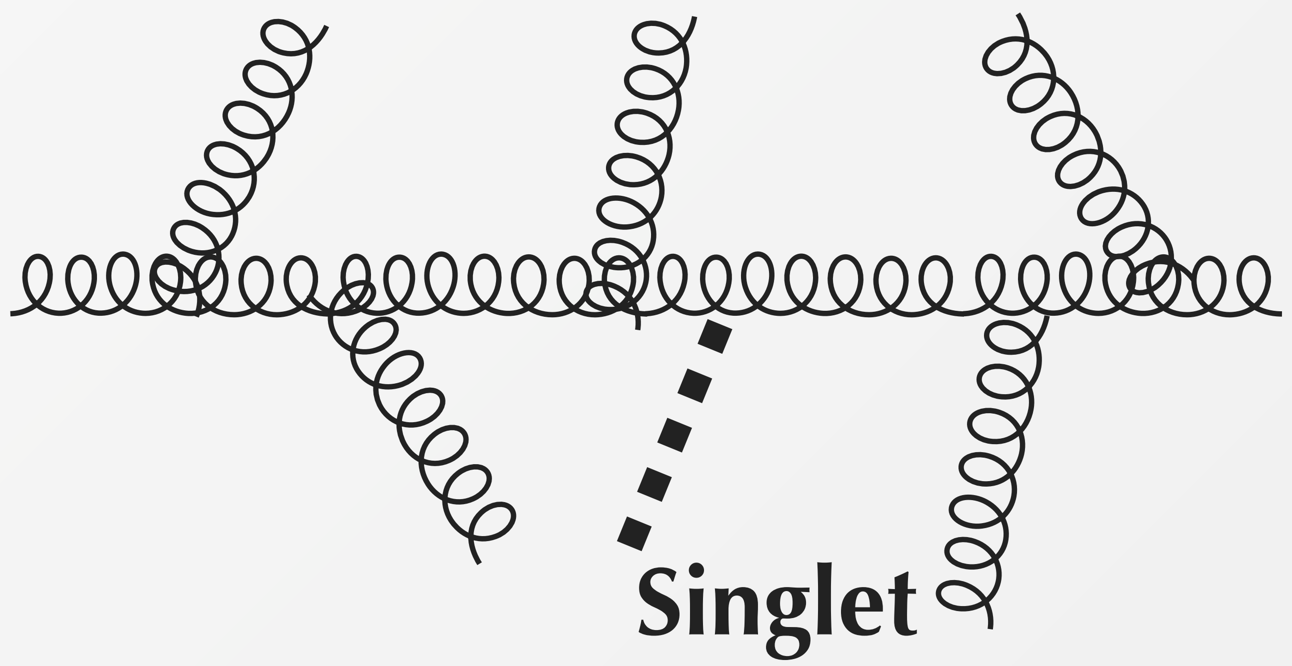
Two concurring mechanisms leading to a system with small p_t



$$p_t^2 \sim k_{t,i}^2 \ll m_H^2$$

cross section naturally suppressed as there is no phase space left for gluon emission (Sudakov limit)

Exponential suppression



$$\sum_{i=1}^{n} \vec{k}_{t,i} \simeq 0$$

Large kinematic cancellations

p_t ~0 far from the Sudakov limit

Power suppression

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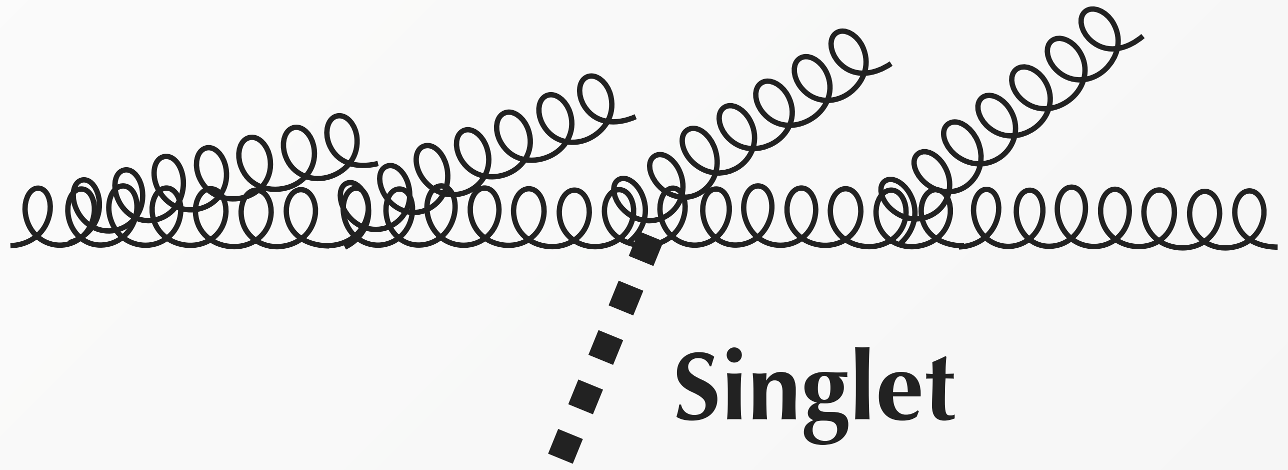

$$p_t^2 \sim k_{t,i}^2 \ll m_H^2$$

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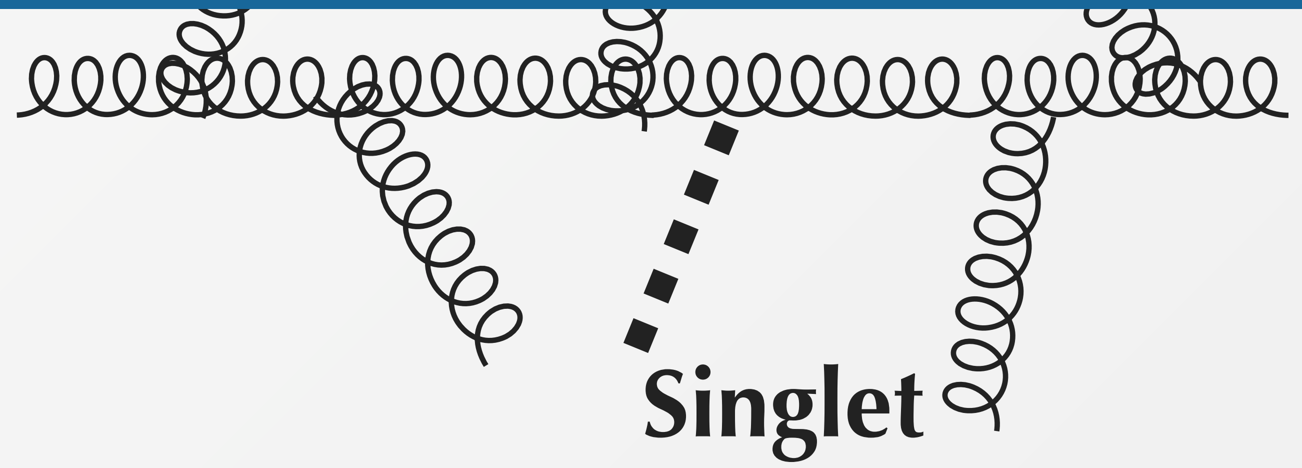
Exponential suppression

Dominant at small p_t

[Parisi, Petronzio, '79]



$$\sum_{i=1}^{n} \vec{k}_{t,i} \simeq 0$$

Large kinematic cancellations

p_t ~0 far from the Sudakov limit

Power suppression

Resummation of the transverse momentum spectrum in b space

two-dimensional momentum conservation $\delta^{(2)}\left(\vec{p}_t - \sum_{i=1}^{n} \vec{k}_{t,i}\right) = \int d^2b \frac{1}{4\pi^2} e^{i\vec{b}\cdot\vec{p}_t} \prod_{i=1}^{n} e^{-i\vec{b}\cdot\vec{k}_{t,i}}$

Exponentiation in conjugate space

virtual corrections

$$\sigma = \sigma_0 \int d^2\vec{p}_\perp^H \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_\perp^H} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \int [dk_i] |M(k_i)|^2 \left(e^{i\vec{b}\cdot\vec{k}_{t,i}} - 1\right) = \sigma_0 \int d^2\vec{p}_\perp^H \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_\perp^H} e^{-R_{\rm NLL}(L)}$$

NLL formula with scale-independent PDFs

$$R_{\rm NLL}(L) = -Lg_1(\alpha_s L) - g_2(\alpha_s L) \qquad L = \ln(m_H b/b_0)$$

Logarithmic accuracy defined in terms of $\ln(m_H b/b_0)$

Talk by Ignazio Scimemi

All-order formula in Mellin space at N^{3}LL

[Bizon, Monni, Re, LR, Torrielli '17] [Re, LR, Torrielli '21]

Now include effect of **collinear radiation** and terms beyond NLL accuracy

$$\frac{d\Sigma(v)}{d\Phi_B} = \int_{\mathscr{C}_1} \frac{dN_1}{2\pi i} \int_{\mathscr{C}_2} \frac{dN_2}{2\pi i} x_1^{-N_1} x_2^{-N_2} \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \mathbf{f}^T_{N_1}(\mu_0) \hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) \mathbf{f}_{N_2}(\mu_0),$$

Unresolved

$v = p_t/M$

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-\mathbf{R}(\epsilon k_{t1})}$$

$$\times \exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$

Resolved

$$\times \sum_{\ell_1=1}^{2} \left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi} \mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right),$$

$$\times \sum_{\ell_i=1}^{2} \left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi} \mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right)$$

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Sudakov radiator

$$R(k_{t1}) = -\log\frac{M}{k_{t1}} g_1 - g_2 - \left(\frac{\alpha_s}{\pi}\right) g_3 - \left(\frac{\alpha_s}{\pi}\right)^2 g_4 - \left(\frac{\alpha_s}{\pi}\right)^3 g_5$$

Resummation scale $Q \sim M$

$$\ln\frac{M}{k_{t1}} \to \ln\frac{Q}{k_{t1}} + \ln\frac{M}{Q}$$

Constant terms expanded in α_s and included in H

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-\mathbf{R}(\epsilon k_{t1})}$$

$$\times \exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$

$$\times \sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi} \mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^{1} \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right),$$

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All-order formula in Mellin space at N³LL

[Bizon, Monni, Re, LR, Torrielli '17] [Re, LR, Torrielli '21]

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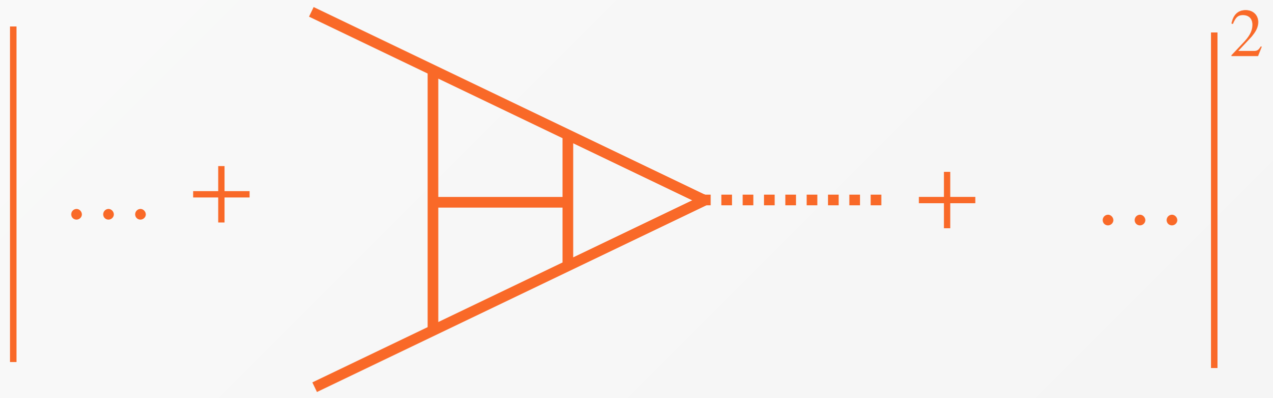
Three-loop hard-virtual coefficient

$$H(\alpha_s) = 1 + \left(\frac{\alpha_s}{2\pi}\right) H_1 + \left(\frac{\alpha_s}{2\pi}\right)^2 H_2 + \left(\frac{\alpha_s}{2\pi}\right)^3 H_3$$



[Gehrmann et al. '10]

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$$\times \exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$

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Three-loop coefficient functions and their evolution

$$C(\alpha_s, z) = \delta(1-z) + \left(\frac{\alpha_s}{2\pi}\right) C_1(z) + \left(\frac{\alpha_s}{2\pi}\right)^2 C_2(z) + \left(\frac{\alpha_s}{2\pi}\right)^3 C_3(z)$$

[Li, Zhu '16][Vladimirov '16][Luo et al. '19][Ebert et al. '20]

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DGLAP evolution

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-\mathbf{R}(\epsilon k_{t1})}$$

$$\times \exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$

$$\times \sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi}\mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^{1} \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right),$$

$$\times \sum_{\ell_i=1}^{2}\left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi}\mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right)$$

All-order formula in Mellin space at N³LL

[Bizon, Monni, Re, LR, Torrielli '17] [Re, LR, Torrielli '21]

$$
\begin{aligned}
\frac{d\Sigma(v)}{d\Phi_B} =& \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \partial_L \left(-e^{-R(k_{t1})} \mathcal{L}_{\mathrm{N^3LL}'}(k_{t1})\right) \int d\mathcal{Z} \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right) \\
&+ \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(k_{t1})} \int d\mathcal{Z} \int_0^1 \frac{d\zeta_s}{\zeta_s} \frac{d\phi_s}{2\pi} \Bigg\{ \left(R'(k_{t1}) \mathcal{L}_{\mathrm{NNLL}}(k_{t1}) - \partial_L \mathcal{L}_{\mathrm{NNLL}}(k_{t1}) \right) \\
&\times \left(R''(k_{t1}) \ln \frac{1}{\zeta_s} + \frac{1}{2} R'''(k_{t1}) \ln^2 \frac{1}{\zeta_s} \right) - R'(k_{t1}) \left(\partial_L \mathcal{L}_{\mathrm{NNLL}}(k_{t1}) - 2 \frac{\beta_0}{\pi} \alpha_s^2(k_{t1}) \hat{P}^{(0)} \otimes \mathcal{L}_{\mathrm{NLL}}(k_{t1}) \ln \frac{1}{\zeta_s} \right) \\
&+ \frac{\alpha_s^2(k_{t1})}{\pi^2} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathcal{L}_{\mathrm{NLL}'}(k_{t1}) - \beta_0 \frac{\alpha_s^3(k_{t1})}{\pi^2} \left(\hat{P}^{(0)} \otimes \hat{C}^{(1)} + \hat{C}^{(1)} \otimes \hat{P}^{(0)} \right) \otimes \mathcal{L}_{\mathrm{NLL}}(k_{(t1)}) + \frac{\alpha_s^3(k_{t1})}{\pi^2} 2\beta_0 \ln \frac{1}{\zeta_s} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathcal{L}_{\mathrm{NLL}}(k_{(t1)}) \\
&+ \frac{\alpha_s^3(k_{t1})}{2\pi^2} \left(\hat{P}^{(0)} \otimes \hat{P}^{(1)} + \hat{P}^{(1)} \otimes \hat{P}^{(0)} \right) \otimes \mathcal{L}_{\mathrm{NLL}}(k_{(t1)}) \Bigg\} \times \left\{ \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_s)\right) - \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right) \right\} \\
&+ \frac{1}{2} \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(k_{t1})} \int d\mathcal{Z} \int_0^1 \frac{d\zeta_{s1}}{\zeta_{s1}} \frac{d\phi_{s1}}{2\pi} \int_0^1 \frac{d\zeta_{s2}}{\zeta_{s2}} \frac{d\phi_{s2}}{2\pi} R'(k_{t1}) \Bigg\{ \mathcal{L}_{\mathrm{NLL}}(k_{t1}) \left(R''(k_{t1})\right)^2 \ln \frac{1}{\zeta_{s1}} \ln \frac{1}{\zeta_{s2}} - \partial_L \mathcal{L}_{\mathrm{NLL}}(k_{t1}) R''(k_{t1}) \left(\ln \frac{1}{\zeta_{s1}} + \ln \frac{1}{\zeta_{s2}} \right) \\
&+ \frac{\alpha_s^2(k_{t1})}{\pi^2} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathcal{L}_{\mathrm{NLL}}(k_{t1}) + \frac{\alpha_s^2(k_{t1})}{\pi^2} \left(\ln \frac{1}{\zeta_{s1}} + \ln \frac{1}{\zeta_{s2}} \right) R''(k_{t1}) \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathcal{L}_{\mathrm{NLL}}(k_{(t1)}) - \ln \frac{1}{\zeta_{s1}} \ln \frac{1}{\zeta_{s2}} (R''(k_{t1})^2 \partial_L \mathcal{L}_{\mathrm{NLL}}(k_{(t1)}) \\
&+ \frac{\alpha_s^2(k_{t1})}{\pi^3} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathcal{L}_{\mathrm{NLL}}(k_{t1}) \Bigg\} \times \Big\{ \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_{s1}, k_{s2})\right) - \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_{s1})\right) - \\
&\Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}, k_{s2})\right) + \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right) \Big\} + \mathcal{O}\left(\alpha_s^n \ln^{2n-7} \frac{1}{v}\right)
\end{aligned}
$$

Luminosity factor: contains the three loop collinear coefficient functions C_3 and the three loop hard function H_3

[Li, Zhu '16][Vladimirov '16][Luo et al. '19][Ebert et al. '20] [Gehrmann et al. '10]

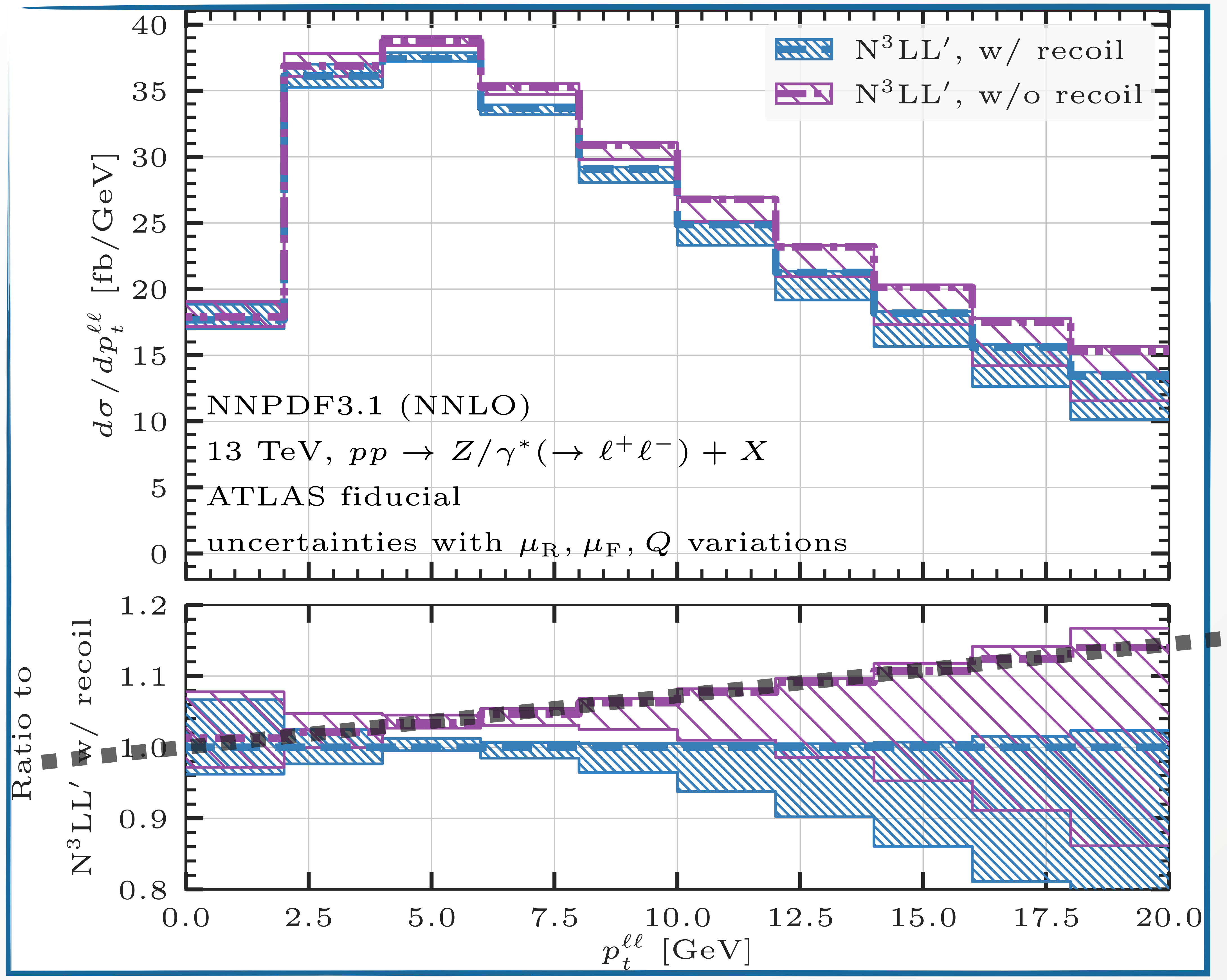
NNLL corrections

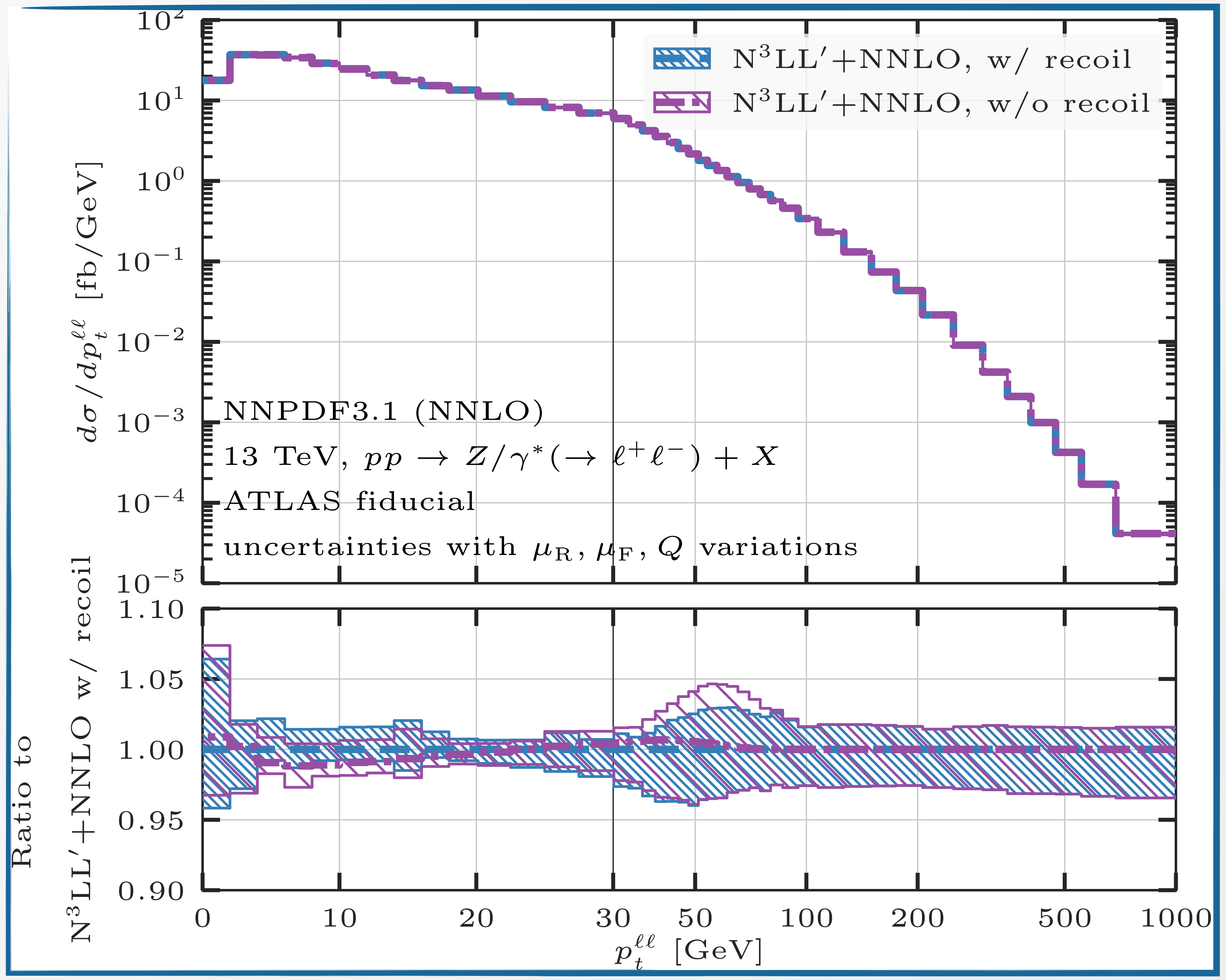
N³LL corrections

Subleading terms

Transverse recoil effects in fiducial DY setup

[Re, LR, Torrielli '21]





At the pure resummed level recoil prescription captures whole linear power corrections from fiducial cuts

Effect reduce at 1-2% level after matching to fixed order (effect becomes $\mathcal{O}(\alpha_s^4)$)

Pure resummed: band widening due to power corrections due to modified logs

$$\ln(Q/k_{t1}) \to 1/p \ln(1 + (Q/k_{t1})^p)$$

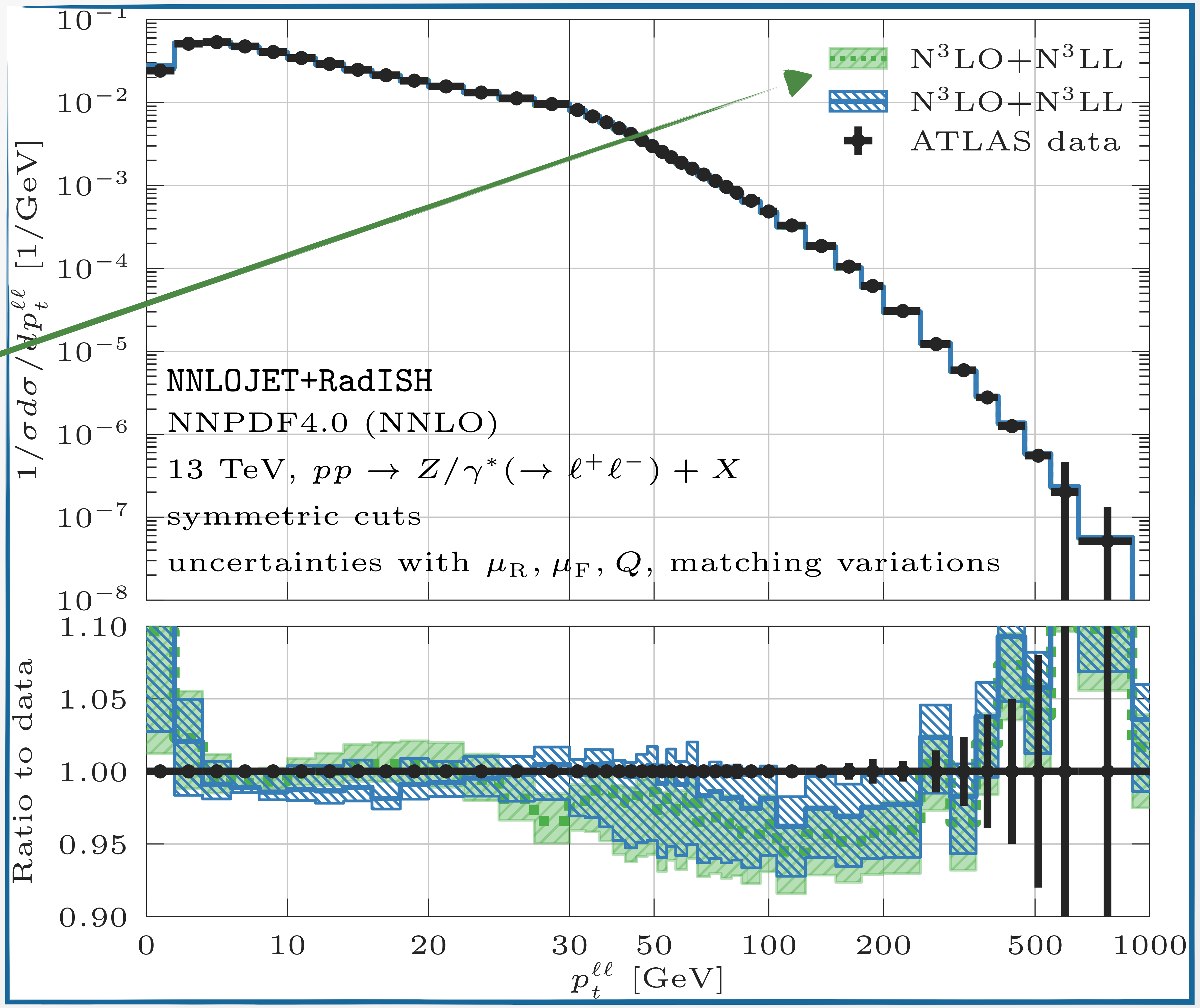
$$\int_0^M \frac{dk_{t1}}{k_{t1}} \to \int_0^\infty \frac{dk_{t1}}{k_{t1}} \frac{(Q/k_{t1})^p}{1 + (Q/k_{t1})^p}$$

Transverse momentum spectrum at N³LO+N³LL

[Chen, Gehrmann, Glover, Huss, Monni, Re, LR, Torrielli '22]

$$d\sigma_V^{\mathrm{N^kLO+N^kLL}} \equiv d\sigma_V^{\mathrm{N^kLL}} + d\sigma_{\mathrm{V+jet}}^{\mathrm{N^{k-1}LO}} - \left[d\sigma_V^{\mathrm{N^kLL}}\right]_{\mathcal{O}(\alpha_s^k)}$$

No fixed order component **below 30 GeV**



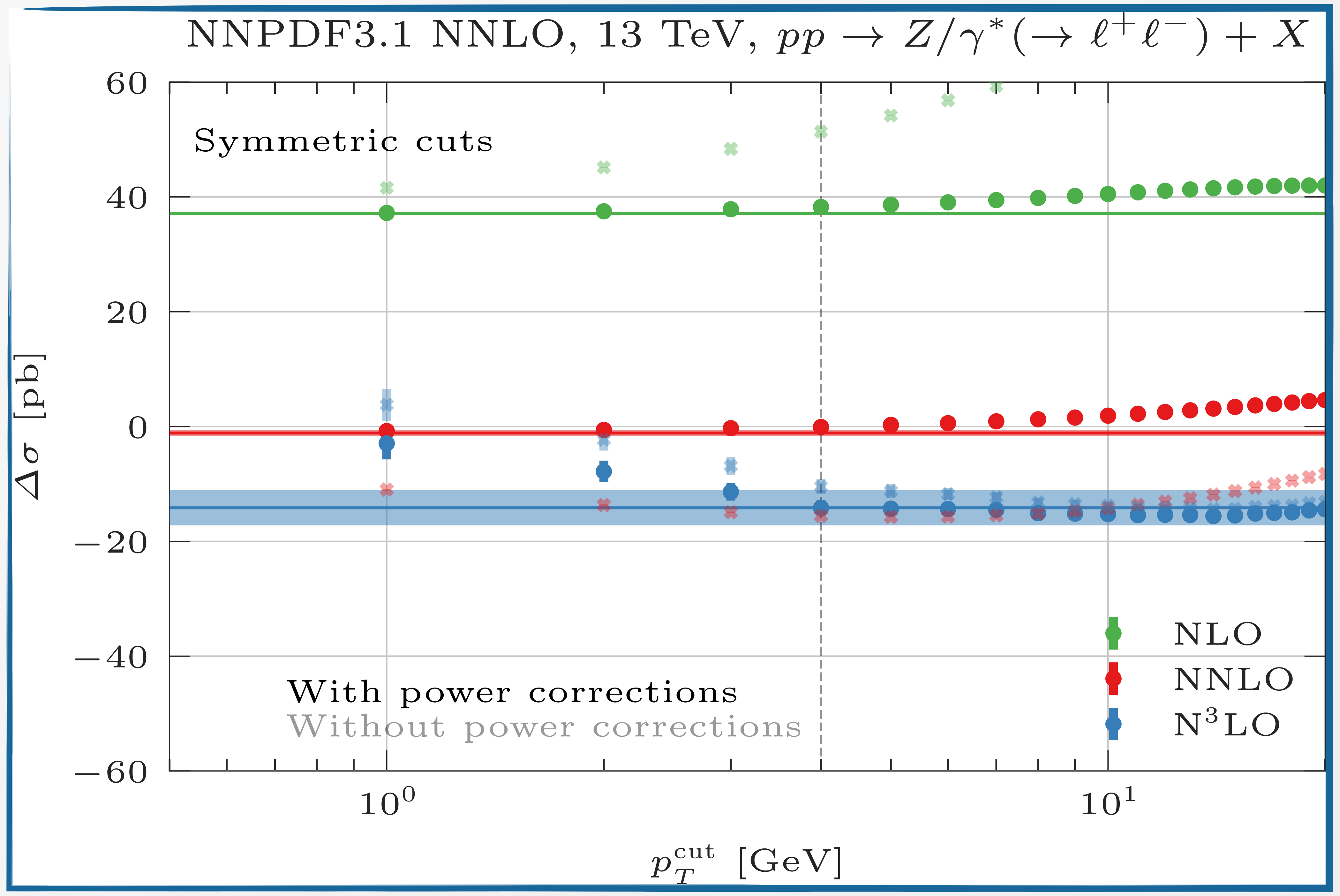
- Non-singular (matching) correction **non-negligible** even below $q_T \lesssim 15$ GeV
- Fixed order matching **crucial** to get correct shape

Comparison with previous N^3LO estimates

Symmetric cuts $p_T^{\ell^\pm} > 25\,\text{GeV}$ $|\eta^{\ell^\pm}| < 2.5$

- Omission of linear power corrections leads to incorrect estimate of N^kLO corrections
  [Camarda, Cieri, Ferrera ’21]

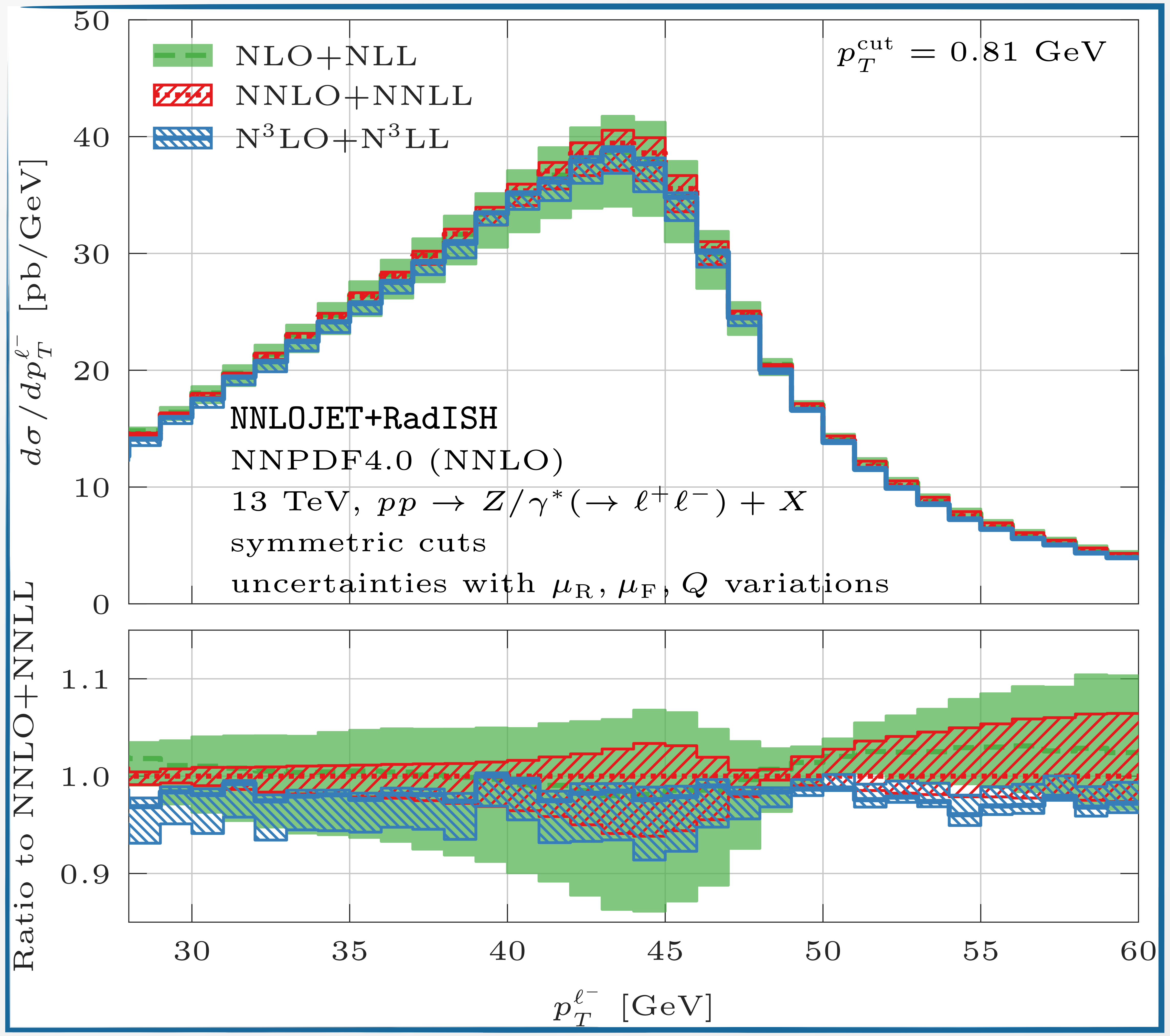
- Data at N^3LO not of sufficient quality to observe a stable plateau, inducing larger systematic uncertainties



Fiducial distributions and transverse momentum resummation

- Transverse momentum resummation affects observables sensitive to **soft gluon emission** as the lepton transverse momentum in Drell-Yan
  [Balázs, Yuan '97] [Catani, de Florian, Ferrera, Grazzini '15]
- Leptonic transverse momentum is a particularly relevant observable due to its importance in the **extraction of the *W* mass**
- Inclusion of resummation effects necessary to cure (integrable) divergences due to the presence of a **Sudakov shoulder** at $m_{\ell\ell}/2$ [Catani, Webber '97]

NB: **EW corrections** also relevant for correct shape



[Chen, Gehrmann, Glover, Huss, Monni, Re, LR, Torrielli '22]