Drell-Yan transverse observables with RadISH+NNLOJET

Luca Rottoli

Dipartimento di Fisica G. Occhialini, University of Milan-Bicocca

Based on ongoing work with

W. Bizon, P.F. Monni, A. Huss, D.M. Walker, E. Re, and P. Torrielli

Transverse observables in colour-singlet production

Clean experimental and theoretical environment for precision physics

Parameterized as

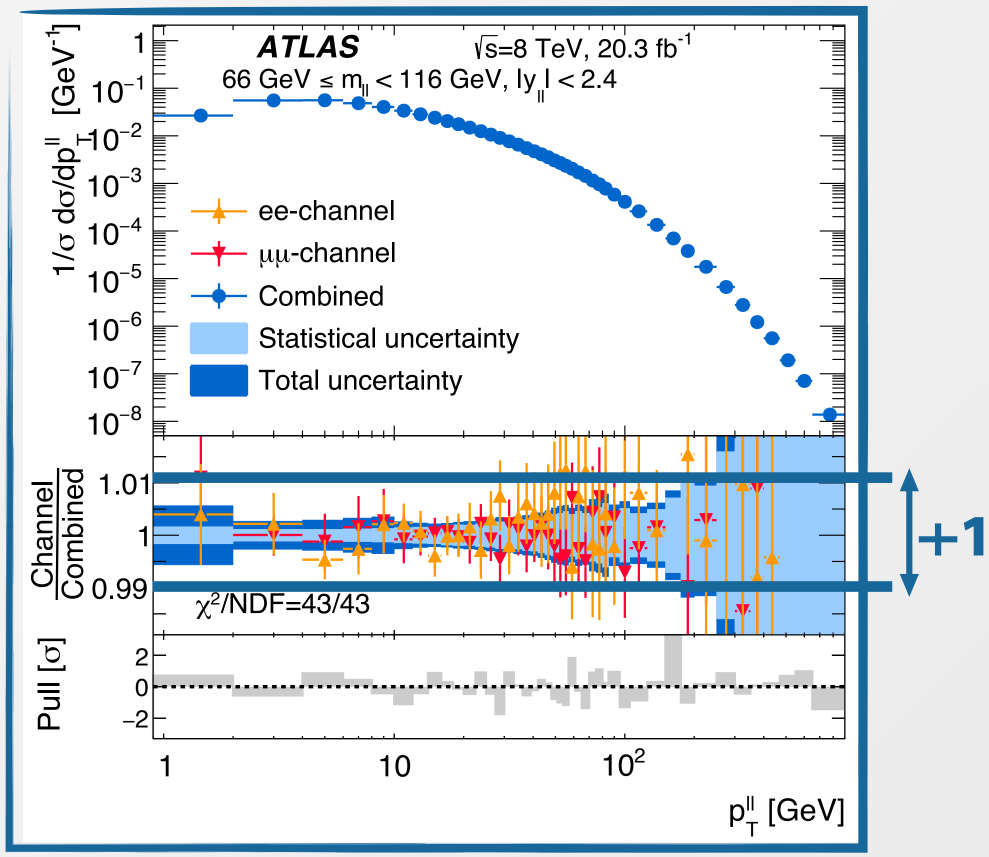
$$V(k) = \left(\frac{k_t}{M}\right)^a f(\phi)$$

for a **single soft** QCD emission *k* **collinear** to incoming leg. Independent of the rapidity of radiation. $V \to 0$ for soft/collinear radiation.

Inclusive observables (e.g. transverse momentum p_t) probe directly the kinematics of the colour singlet

$$V(k_1, \ldots k_n) = V(k_1 + \ldots + k_n)$$

- negligible or no sensitivity to multi-parton interactions
- reduced sensitivity to non-perturbative effects
- measured extremely precisely at experiments



Very accurate theoretical predictions needed

Precision physics at the LHC: theory

$$\sigma(s, Q^2) = \sum_{a,b} \int dx_1 dx_2\, f_{a/h_1}(x_1, Q^2) f_{b/h_2}(x_2, Q^2) \hat{\sigma}_{ab\to X}(Q^2, x_1 x_2 s) + \mathcal{O}(\Lambda^p_{\rm QCD}/Q^p)$$

Input parameters:

strong coupling α_s

PDFs f

few percent uncertainty; improvable

Non-perturbative effects

percent effect; not yet under control

Precision physics at the LHC: theory

$$\sigma(s,Q^2) = \sum_{a,b} \int dx_1 dx_2\, f_{a/h_1}(x_1,Q^2) f_{b/h_2}(x_2,Q^2) \hat{\sigma}_{ab\to X}(Q^2, x_1 x_2 s) + \mathcal{O}(\Lambda^p_{\rm QCD}/Q^p)$$

$$\hat{\sigma} = \hat{\sigma}_0(1 + \alpha_s C_1 + \alpha_s^2 C_2 + \alpha_s^3 C_3 + \ldots)$$

LO **NLO** **NNLO** **N³LO**

$\alpha_s \sim 0.1$

δ~10-20% **NLO**

δ~1-5% **NNLO** (or even **N³LO**)

All-order resummation

Cumulative cross section

$$\Sigma(v) = \int_0^v dV \frac{d\sigma}{dV} \sim \sigma_0[1 + \alpha_s \# + \alpha_s^2 \# + \ldots]$$

Fixed-order prediction: reliable for **inclusive enough** observables and in regions not marred by **soft/collinear radiation** ($v \to 0$)

Real and virtual contributions can become **highly unbalanced** in processes where the real radiation is strongly constrained by kinematics

Large logarithms appear at **all order** as a left-over of the real-virtual cancellation of IRC divergences

$$\ln \Sigma(v) = \sum_n \{ \underbrace{\mathcal{O}(\alpha_s^n L^{n+1})}_{\textbf{LL}} + \underbrace{\mathcal{O}(\alpha_s^n L^n)}_{\textbf{NLL}} + \underbrace{\mathcal{O}(\alpha_s^n L^{n-1})}_{\textbf{NNLL}} + \ldots \}$$

$$L = \ln 1/v$$

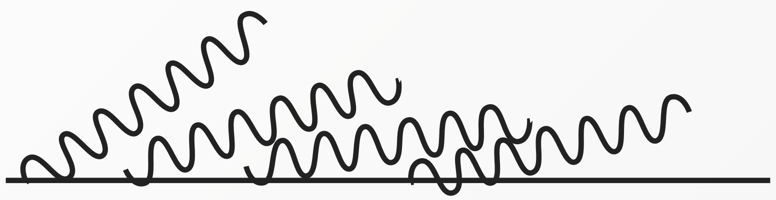
$v = p_t/M$ in the transverse momentum case

Fixed order predictions no longer reliable:
all-order resummation of the perturbative series

Case study: transverse momentum p_t

Resummation of transverse momentum is particularly delicate because p_t is a **vectorial quantity**

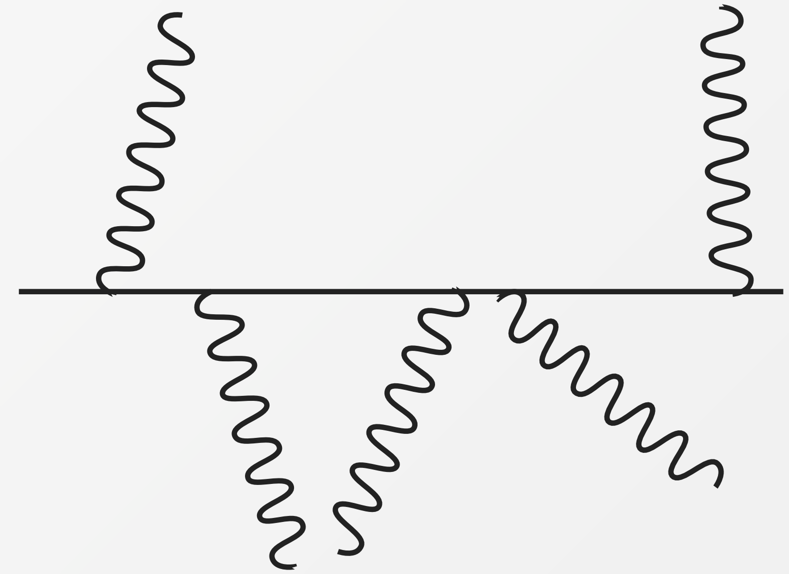
Two concurring mechanisms leading to a system with small p_t



$$p_t^2 \sim k_{t,i}^2 \ll M^2$$

cross section naturally suppressed as there is no phase space left for gluon emission (Sudakov limit)

Exponential suppression



$$\sum_{i=1}^{n} \vec{k}_{t,i} \simeq 0$$

Large kinematic cancellations

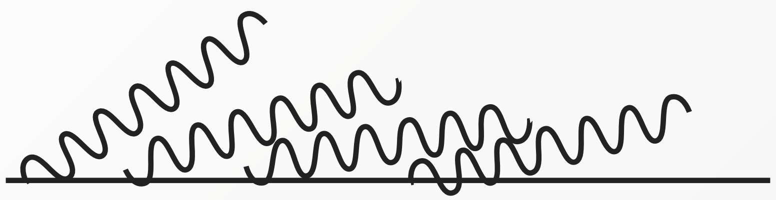
p_t ~0 far from the Sudakov limit

Power suppression

Case study: transverse momentum p_t

Resummation of transverse momentum is particularly delicate because it is a **vectorial quantity**

Two concurring mechanisms leading to a system with small p_t



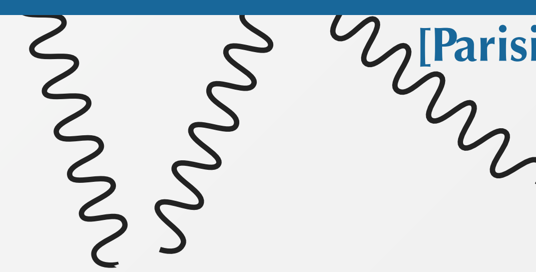
$$p_t^2 \sim k_{t,i}^2 \ll M^2$$

cross section naturally suppressed as there is no phase space left for gluon emission (Sudakov limit)

Exponential suppression

Dominant at small p_t

[Parisi, Petronzio '78]



$$\sum_{i=1}^{n} \vec{k}_{t,i} \simeq 0$$

Large kinematic cancellations

p_t ~0 far from the Sudakov limit

Power suppression

Resummation in direct and in conjugate space

Phase-space constraints do not usually factorize in **direct space**

Resummation usually performed in impact-parameter (b) space where the two competing mechanisms are handled trough a **Fourier transform. Transverse-momentum conservation** is respected

$$\delta\left(\vec{p}_t - \sum_{i=1}^{n} \vec{k}_{t,i}\right) = \int d^2b \frac{1}{4\pi^2} e^{i\vec{b}\cdot\vec{p}_t} \prod_{i=1}^{n} e^{-i\vec{b}\cdot\vec{k}_{t,i}}$$

[Parisi, Petronzio '78; Collins, Soper, Sterman '85]

Resummation in direct space: not possible to find a closed analytic expression in direct space which is both

[Frixione, Nason, Ridolfi '98]

a) free of logarithmically subleading corrections

b) free of singularities at finite p_t values

A naive logarithmic counting at small p_t is not sensible, as one loses the **correct power-suppressed scaling** if only logarithms are retained

Resummation in direct space now possible

[Monni, Re, Torrielli '16, Bizon, Monni, Re, LR, Torielli '17]
[Ebert, Tackmann '16] see also [Kang,Lee,Vaidya '17]

Transverse observable resummation with RadISH

[Monni, Re, Torrielli '16, Bizon, Monni, Re, LR, Torielli '17]

Translate the resummability of the observable into properties of the observable in the presence of multiple radiation: **recursive infrared and collinear (rIRC) safety**

[Banfi, Salam, Zanderighi '01, '03, '04]

Existence of a **resolution scale** q_0, **independent of the observable**, such that emissions below q_0 (**unresolved**) do not contribute significantly to the observable's value.

Starting point: all-order cumulative cross section

single-particle phase space

matrix element for *n* real emissions

$$\Sigma(v) = \int d\Phi_B \mathcal{V}(\Phi_B) \sum_{n=0}^{\infty} \int \prod_{i=1}^{n} [dk_i] \, |\mathcal{M}(\Phi_B, k_1, \ldots k_n)|^2 \, \Theta(v - V(\{\Phi_B\}, k_1, \ldots k_n))$$

all-order form factor (virtuals)

$v = p_t/M$

Transverse observable resummation with RadISH

[Monni, Re, Torrielli '16, Bizon, Monni, Re, LR, Toriellli '17]

1. Establish a **logarithmic counting** for the squared matrix element $|\mathcal{M}(\Phi_B, k_1, \ldots k_n)|^2$

Decompose the squared amplitude in terms of ***n*-particle correlated blocks,** denoted by $|\tilde{\mathcal{M}}(k_1, \ldots, k_n)|^2 \quad (|\tilde{\mathcal{M}}(k_1)|^2 = |\mathcal{M}(k_1)|^2)$

*expression valid for inclusive observables

LL | NLL | NNLL

$$\sum_{n=0}^{\infty} |\mathcal{M}(\Phi_B, k_1, \ldots, k_n)|^2 = |\mathcal{M}_B(\Phi_B)|^2$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \left\{ \prod_{i=1}^{n} \left(|\mathcal{M}(k_i)|^2 + \int [dk_a][dk_b] |\tilde{\mathcal{M}}(k_a, k_b)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} - \vec{k}_{ti}) \delta(Y_{ab} - Y_i) \right. \right.$$

$$\left. \left. + \int [dk_a][dk_b][dk_c] |\tilde{\mathcal{M}}(k_a, k_b, k_c)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} + \vec{k}_{tc} - \vec{k}_{ti}) \delta(Y_{abc} - Y_i) + \ldots \right) \right\}$$

$$\equiv |\mathcal{M}_B(\Phi_B)|^2 \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} |\mathcal{M}(k_i)|^2_{\rm inc}$$

Upon integration over the phase space, the expansion can be put in a **one to one correspondence** with the logarithmic structure

Systematic recipe to include terms up to the desired logarithmic accuracy

Transverse observable resummation with RadISH

[Monni, Re, Torrielli '16, Bizon, Monni, Re, LR, Toriellli '17]

2. Exploit rIRC safety to single out the IRC singularities of the real matrix element and achieve the cancellation of the exponentiated divergences of virtual origin

Introduce a slicing parameter $\epsilon \ll 1$ such that all inclusive blocks with $k_{t,i} < \epsilon k_{t,1}$, with $k_{t,1}$ hardest emission, can be neglected in the computation of the observable

$$\Sigma(v) = \int d\Phi_B |\mathcal{M}_B(\Phi_B)|^2 \mathcal{V}(\Phi_B)$$

unresolved emissions

$$\times \int [dk_1] |\mathcal{M}(k_1)|^2_{\rm inc} \left(\sum_{l=0}^{\infty} \frac{1}{l!} \int \prod_{j=2}^{l+1} [dk_j] |\mathcal{M}(k_j)|^2_{\rm inc}\, \Theta(\epsilon V(k_1) - V(k_j)) \right)$$

$$\times \left(\sum_{m=0}^{\infty} \frac{1}{m!} \int \prod_{i=2}^{m+1} [dk_i] |\mathcal{M}(k_i)|^2_{\rm inc}\, \Theta(V(k_i) - \epsilon V(k_1)) \Theta\left(v - V(\Phi_B, k_1, \ldots, k_{m+1})\right) \right)$$

resolved emissions

Unresolved emission doesn't contribute to the evaluation of the observable: it can be exponentiated directly and employed to cancel the virtual divergences, giving rise to a Sudakov radiator

$$\mathcal{V}(\Phi_B) \exp\left\{ \int [dk] |\mathcal{M}(k)|^2_{\rm inc}\, \Theta(\epsilon V(k_1) - V(k)) \right\} \simeq e^{-R(\epsilon V(k_1))}$$

Transverse observable resummation with RadISH

[Monni, Re, Torrielli '16, Bizon, Monni, Re, LR, Torielli '17]

Final result at NLL

$$\frac{d\Sigma(v)}{d\Phi_B} = \int \frac{dk_{t,1}}{k_{t,1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(k_{t,1})} \epsilon^{R'(k_{t,1})} \mathscr{L}_{\mathrm{NLL}}(k_{t,1}) R'\left(k_{t,1}\right)$$
$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^{1} \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'\left(\zeta_i k_{t,1}\right) \Theta\left(v - V(\Phi_B, k_1, \ldots, k_{n+1})\right)$$

Parton luminosity at NLL reads

$$\mathscr{L}_{\mathrm{NLL}}(k_{t,1}) = \sum_c \frac{d|M_B|^2_{c\bar{c}}}{d\Phi_B} f_c\left(x_1, k_{t,1}^2\right) f_{\bar{c}}\left(x_2, k_{t,1}^2\right)$$

At higher logarithmic accuracy, it includes **coefficient functions** and **hard-virtual** corrections

All ingredients to perform resummation at **N^{3}LL accuracy** are now available

[Catani *et al.* '11, '12][Gehrmann *et al.* '14][Li, Zhu '16][Moch *et al.* '18]

Fixed-order predictions now available at **NNLO**

[A. Gehrmann-De Ridder *et al.* '15, 16, '17][Boughezal *et al.* '15, 16]

Matching to fixed order: multiplicative matching

Cumulative cross section should reduce to the fixed order at large v

$$\Sigma^{\text{mult}}_{\text{matched}}(v) \sim \Sigma_{\text{res}}(v)\left[\frac{\Sigma_{\text{f.o.}}(v)}{\Sigma_{\text{res}}(v)}\right]_{\text{expanded}} \, *$$

- allows to include constant terms from NNLO (if N^{3}LO total xs available)
- physical suppression at small v cures potential instabilities

*actual scheme slightly more involved

To ensure that resummation does not affect the hard region of the spectrum when the matching is performed we introduce **modified logarithms**

This corresponds to restrict the rapidity phase space at large k_t

$$\ln(Q/k_{t1}) \to \frac{1}{p}\ln\left(1+\left(\frac{Q}{k_{t1}}\right)^p\right)$$

Q : **perturbative resummation scale** used to probe the size of subleading logarithmic corrections

p : arbitrary matching parameter

Theoretical predictions for Z and W observables at 13 TeV

Results obtained using the fiducial cuts of the 13 TeV ATLAS data measurement

$$Z \qquad p_t^{\ell^\pm} > 25\,\text{GeV}, \quad |\eta^{\ell^\pm}| < 2.5, \quad 66\,\text{GeV} < M_{\ell\ell} < 116\,\text{GeV}$$

$$W \qquad p_t^{\ell} > 25\,\text{GeV}, \quad |\eta^{\ell}| < 2.5, \quad E_T^{\nu_\ell} > 25\,\text{GeV}, \quad m_T > 50\,\text{GeV}$$

using NNPDF3.1 with $\alpha_s(M_Z)$=0.118 and setting the central scales to

$$\mu_R = \mu_F = M_T = \sqrt{M_{\ell\ell} + p_T^2}, \quad Q = \frac{M_{\ell\ell}}{2}$$

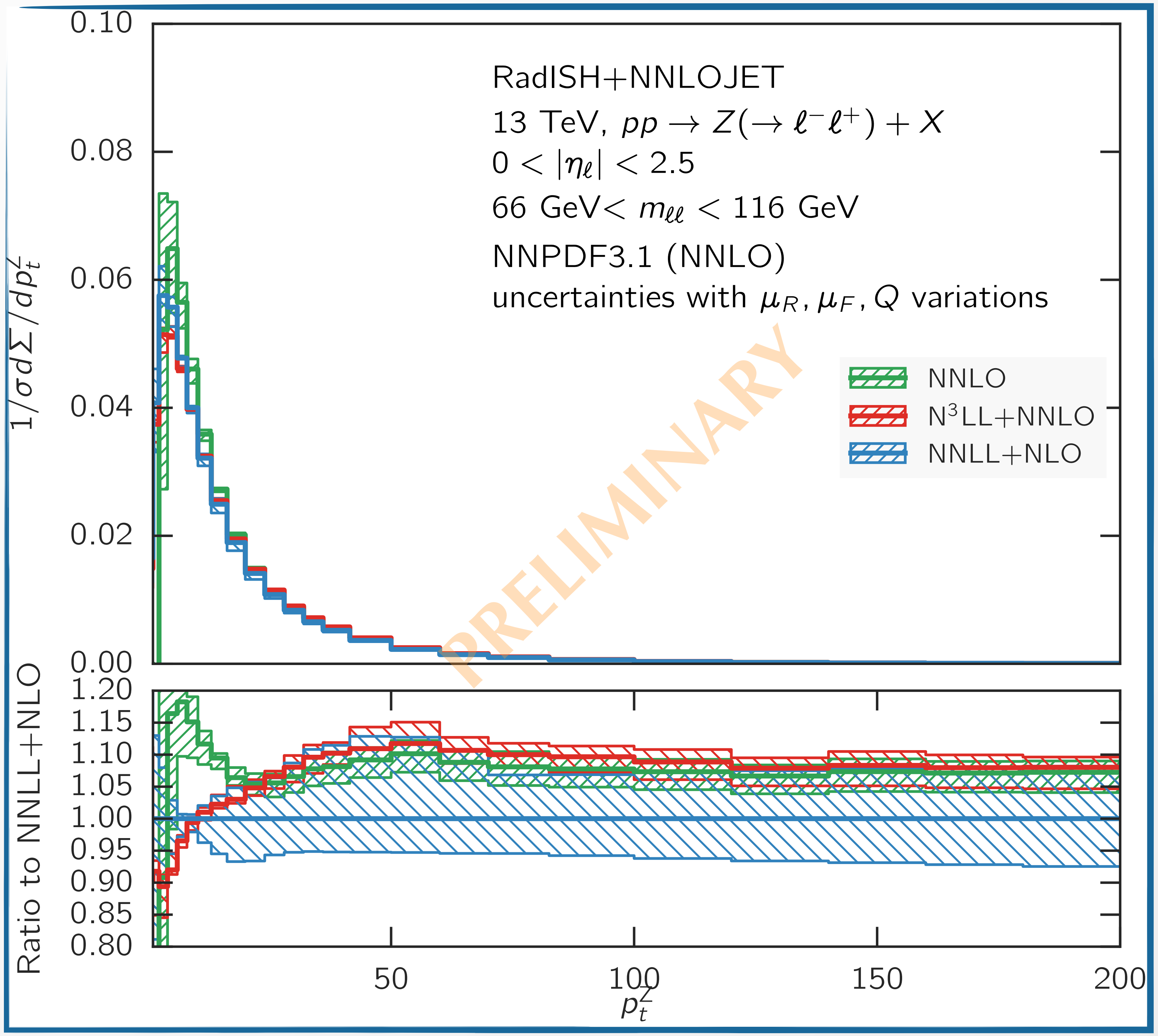
5 flavour (massless) scheme: no HQ effects and no PDF thresholds

Scale uncertainties estimated by varying **renormalization** and **factorization** scale by a factor of two around their central value (**7 point variation**) and varying the **resummation** scale by a factor of 2 around its central value for factorization and renormalization scales set to their central value: **9 point envelope**

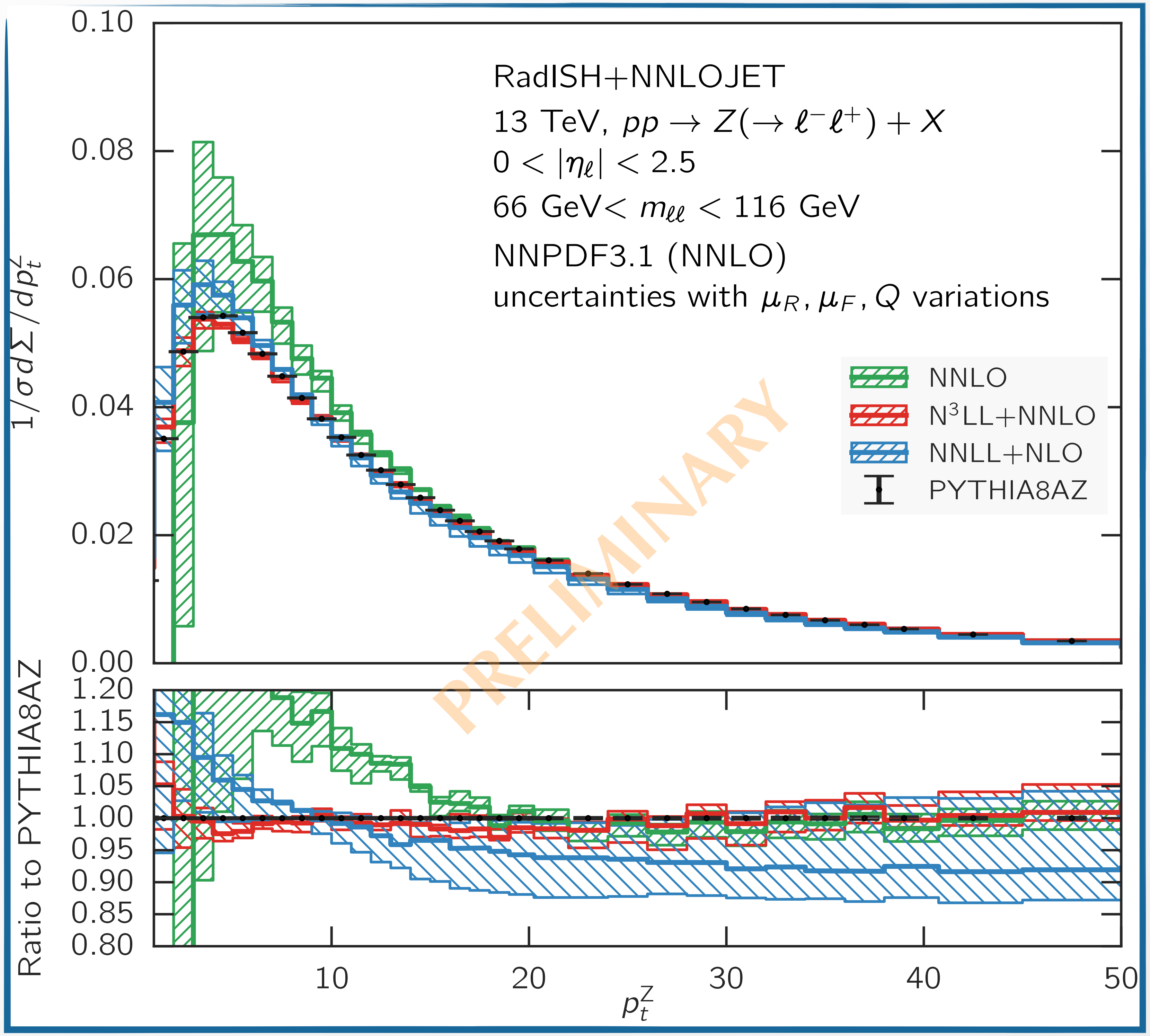
Matching parameter p set to 4 as a default

No NP parameters included in the following

Results for the $p^Z{}_t$ distribution



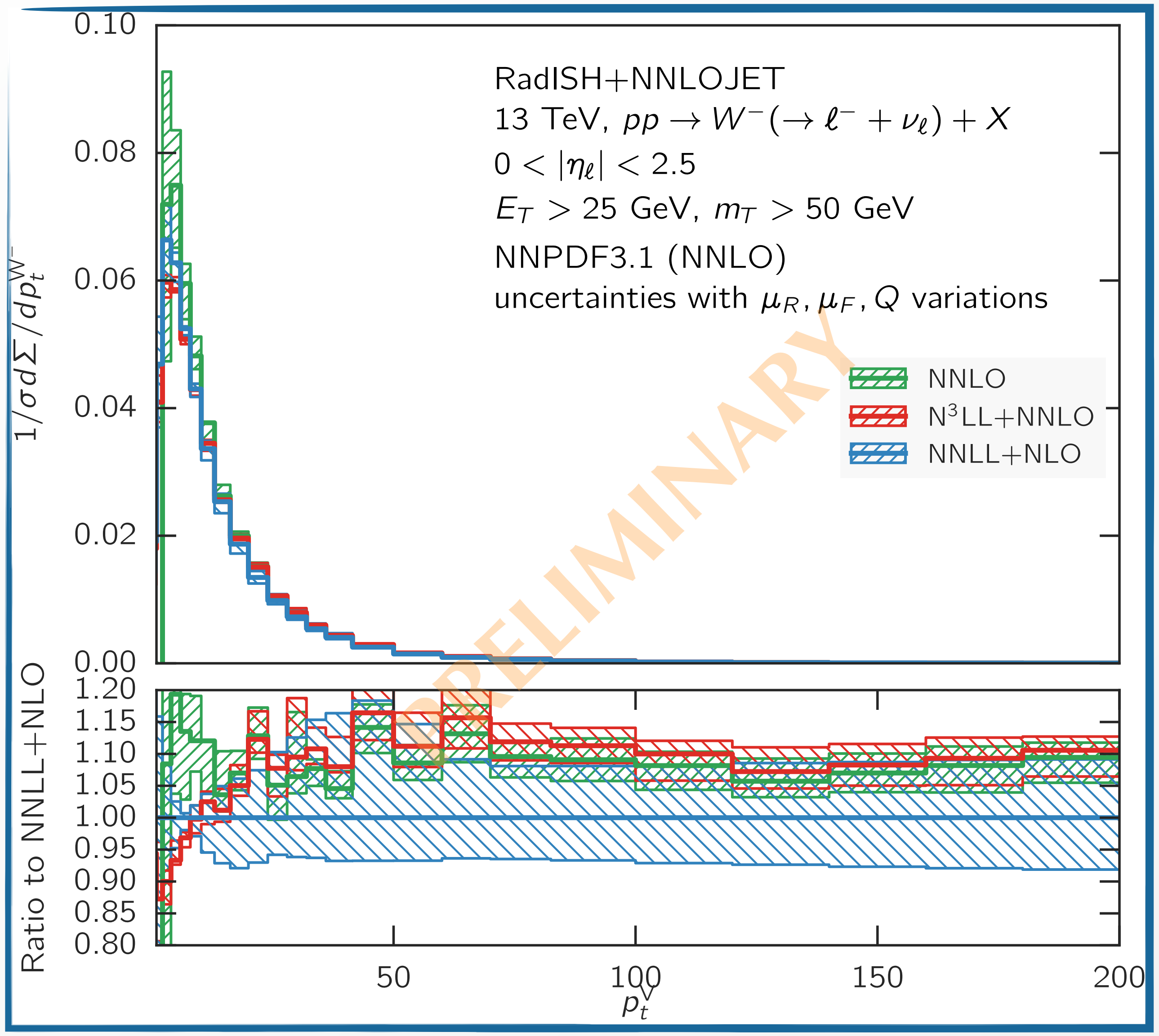
Results for the p^Z_t distribution

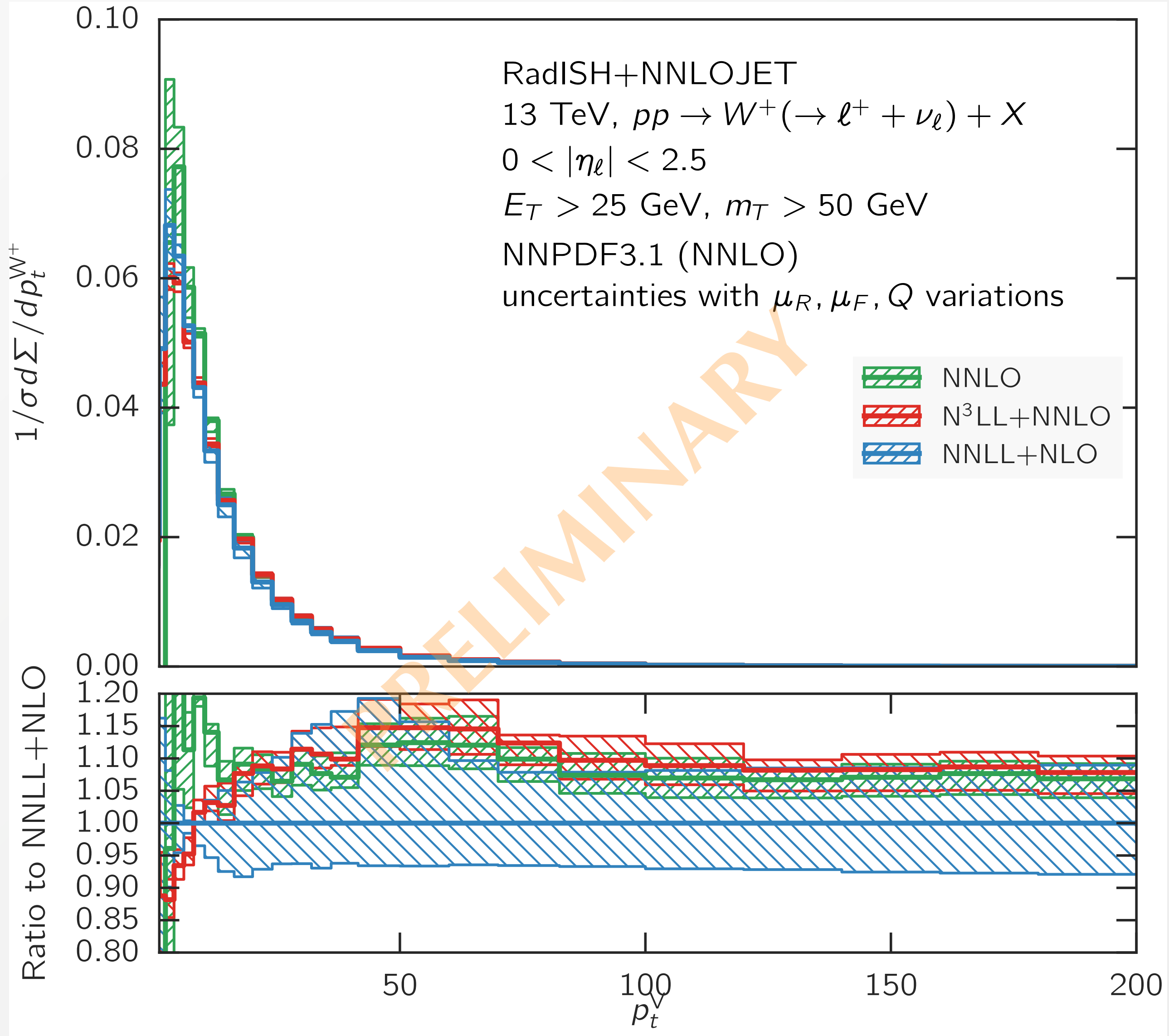


Thanks to Jan Kretzschmar for providing the PYTHIA8 AZ tune results

Results for the p^W_t distribution

Preliminary results - low statistics

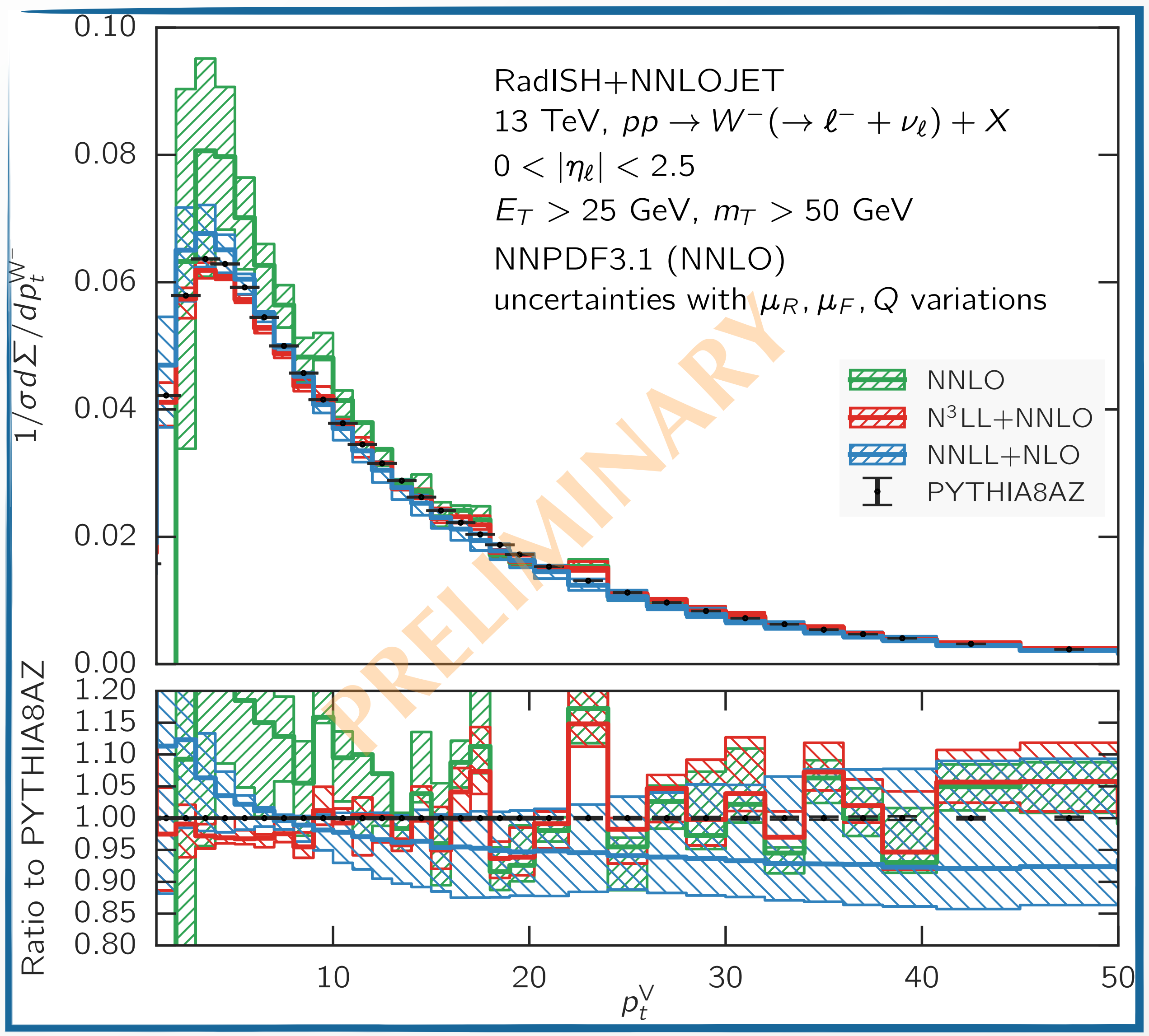


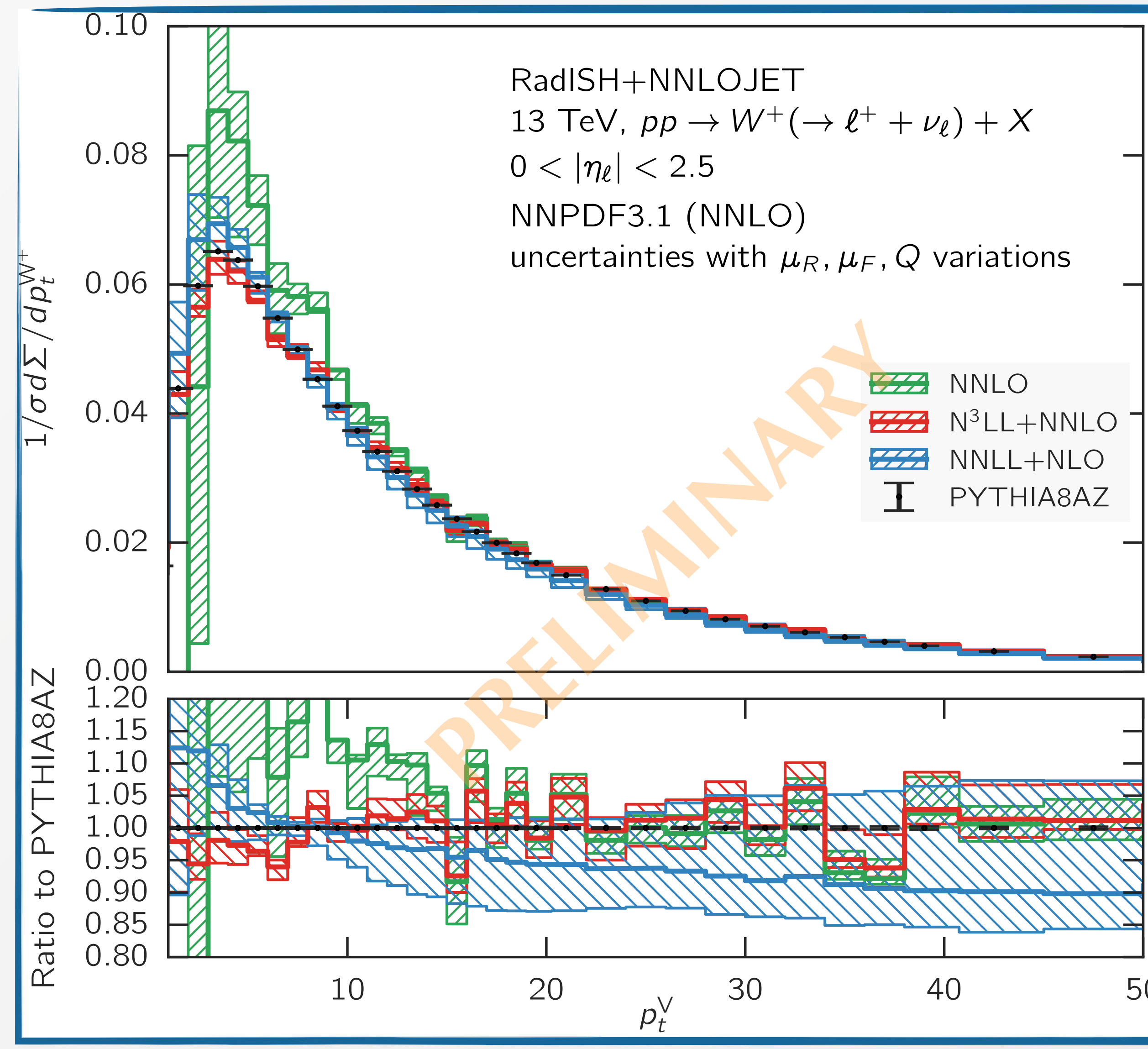


W uncertainties similar to the *Z* case

Results for the p^{W}_{t} distribution

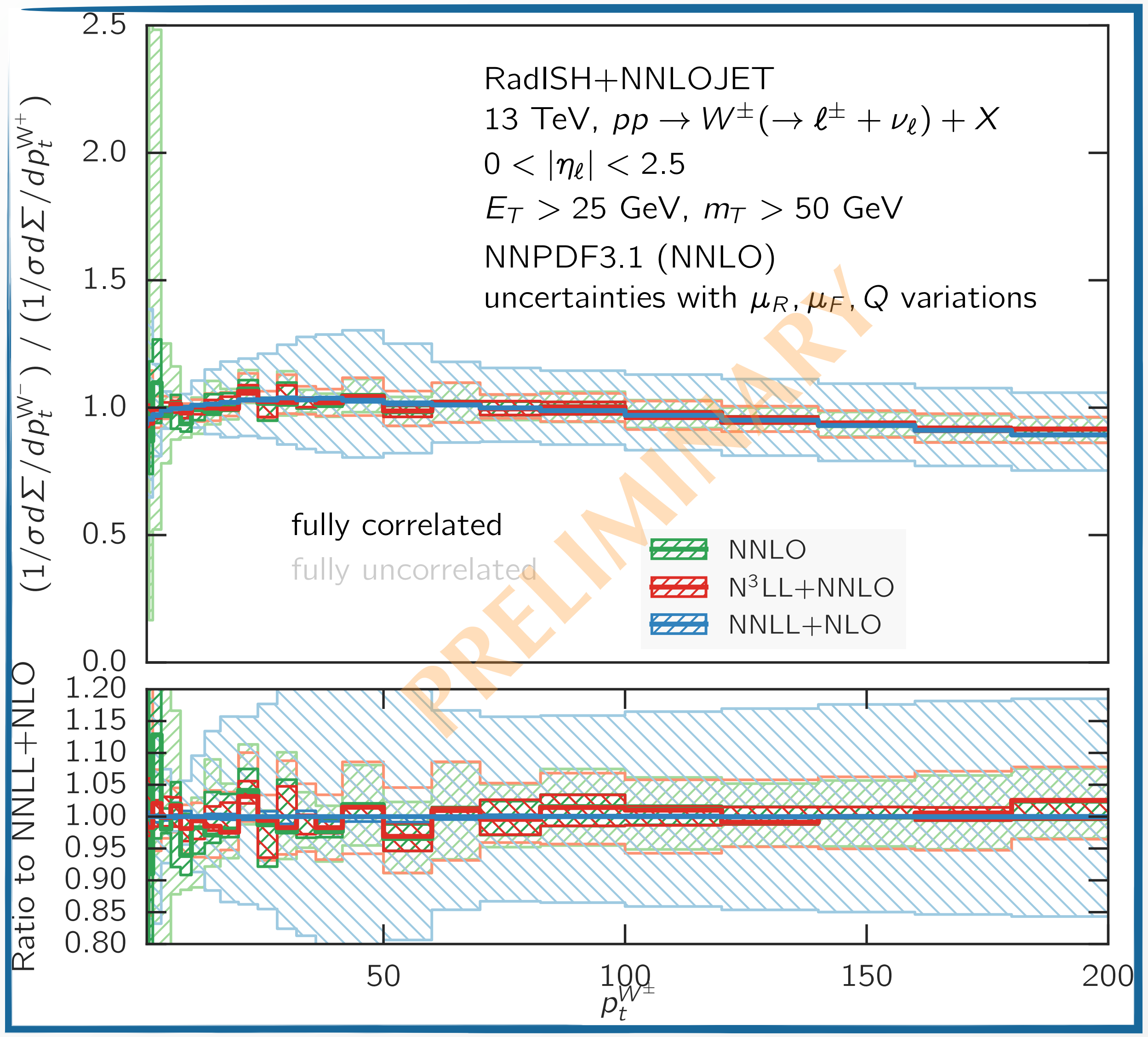
Preliminary results - low statistics

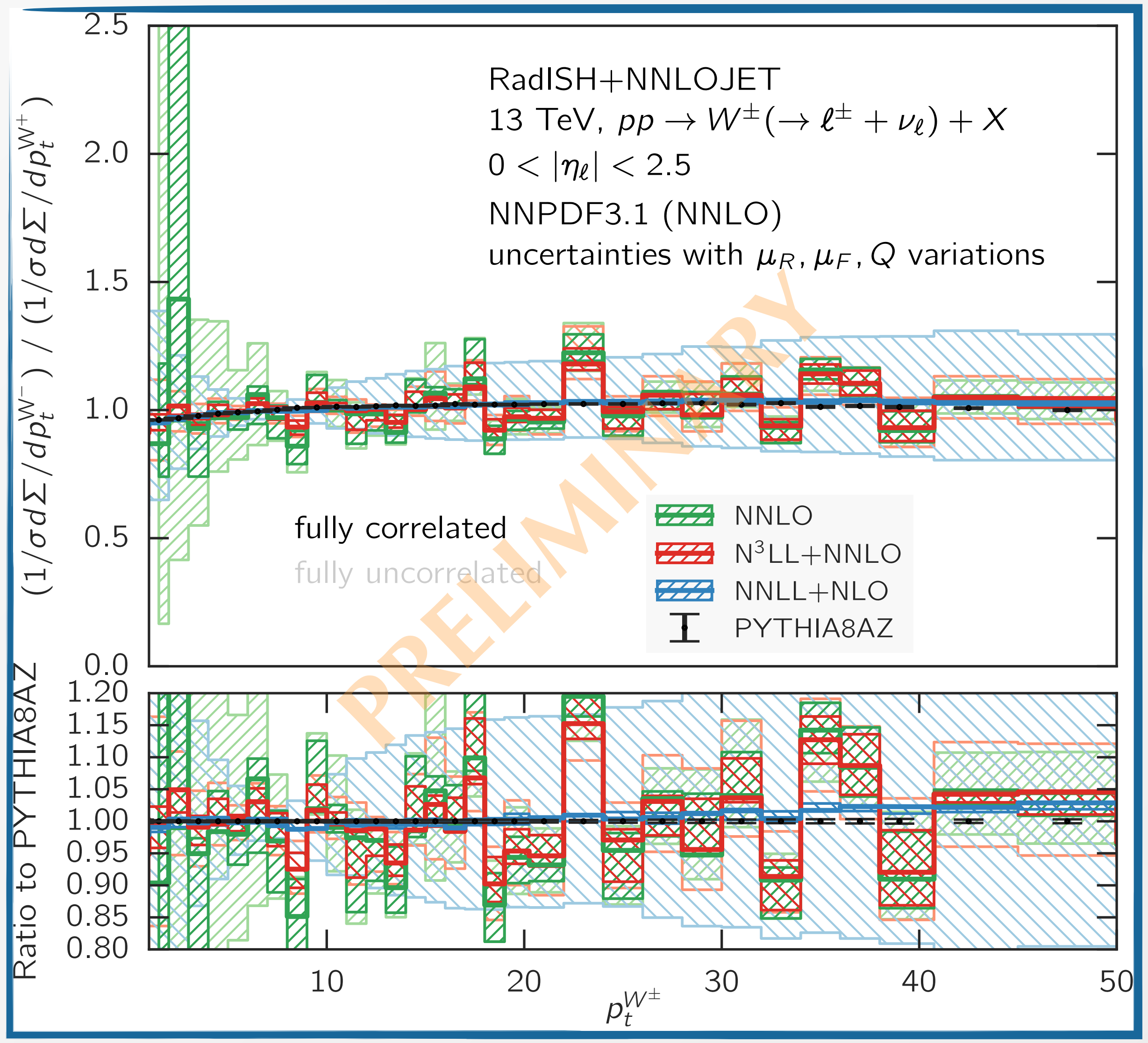




Results for the $p^{W^-}_t / p^{W^+}_t$ distribution

Preliminary results - low statistics



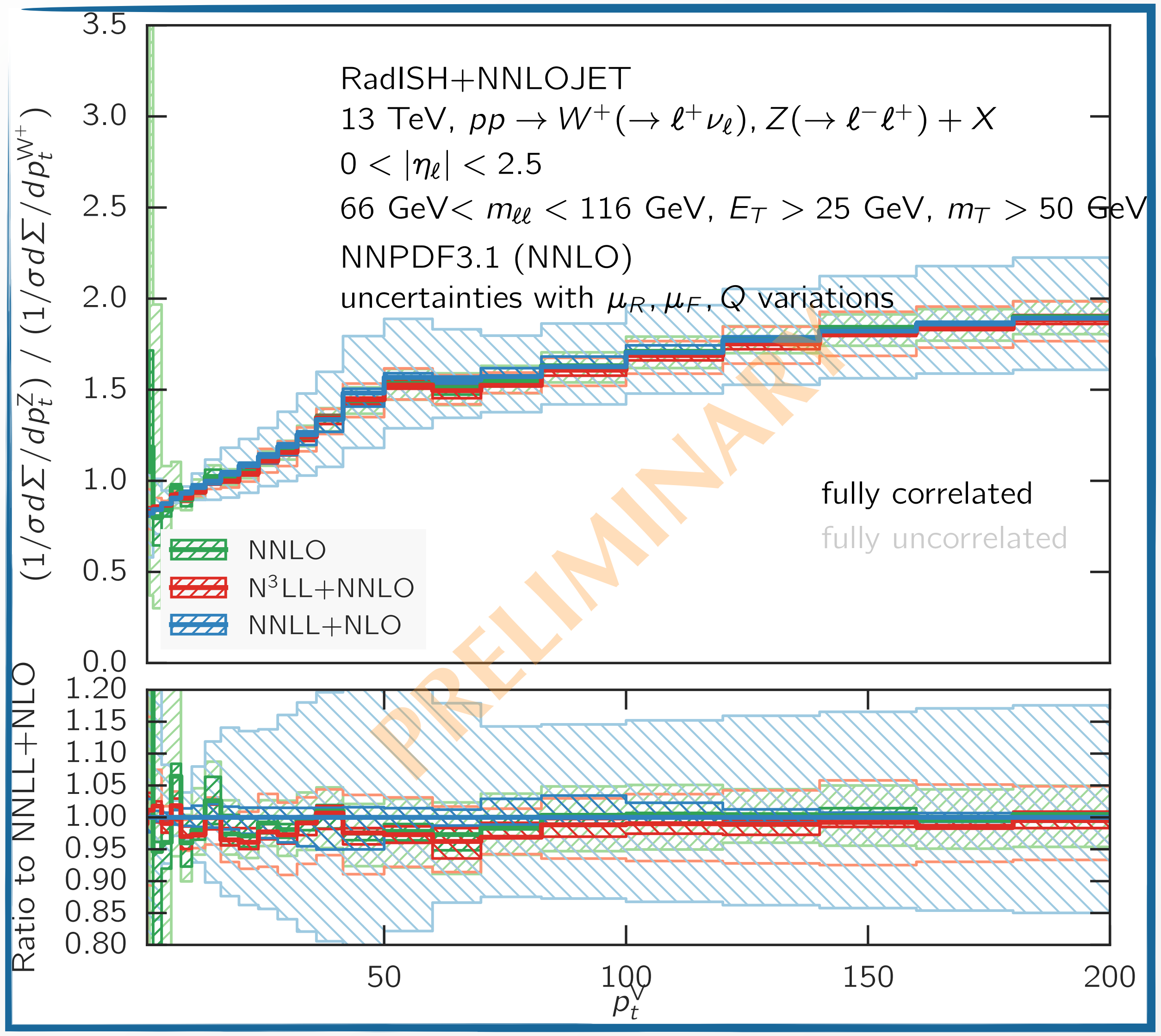


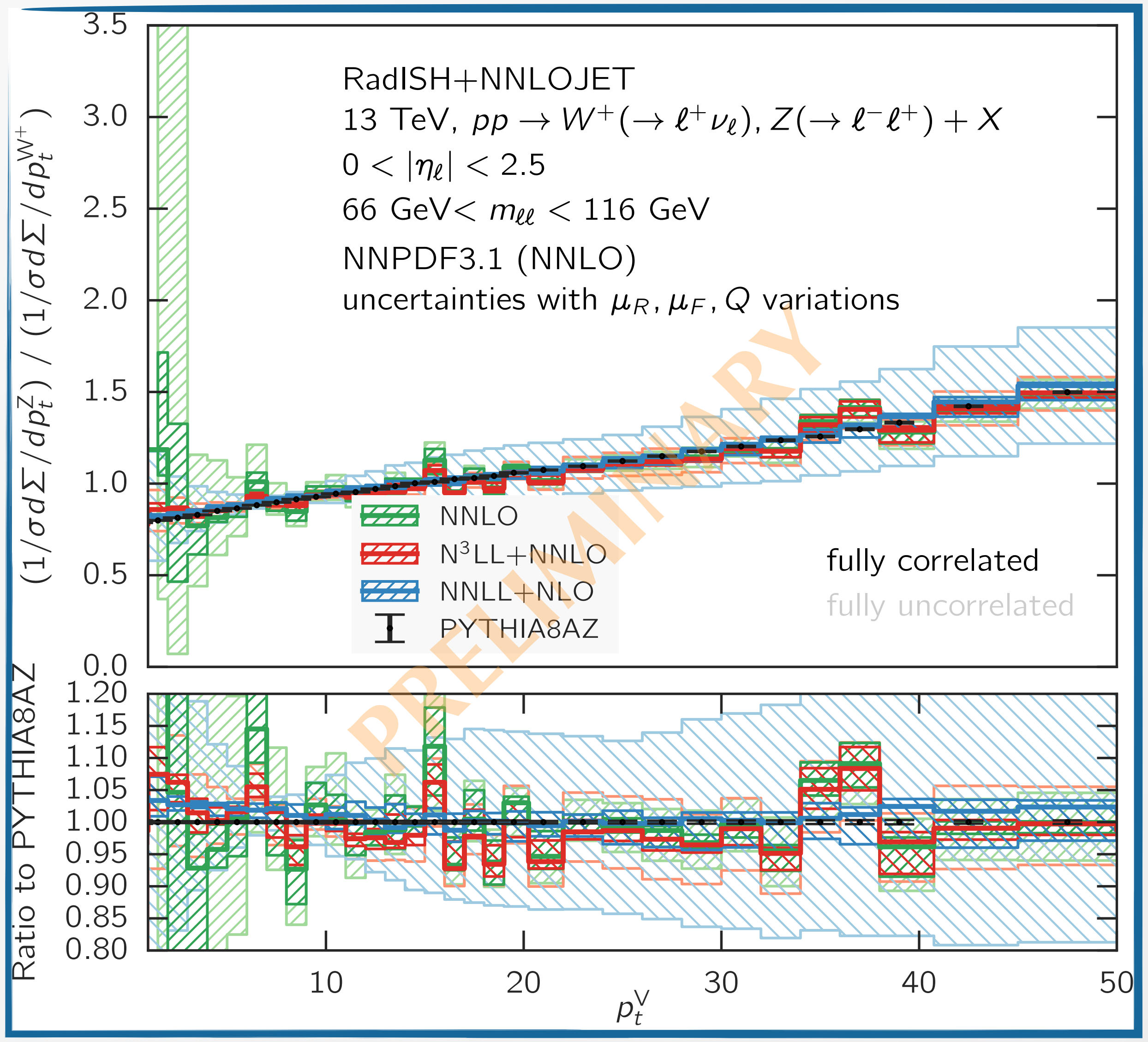
Thanks to Jan Kretzschmar for providing the PYTHIA8 AZ tune results

Study of correlation of the uncertainty necessary

Results for the $p^Z{}_t / p^W{}_t$ distribution

Preliminary results - low statistics





Study of correlation of the uncertainty necessary

Recapitulation

- No sign of NP at the LHC so far - necessary to perform detailed theory/experimental comparisons, to look for deviations from SM. Perturbation theory must be pushed at its limit
- New formalism formulated in **direct space** for all-order resummation up to **N^3LL accuracy** for inclusive, transverse observables.
- Preliminary results at NNLO+N^3LL for *W* and *Z* differential distributions. Encouraging good agreement with the PYTHIA8 AZ tune results in the **fiducial distributions**, with **uncertainties at the few percent level**.
- Preliminary results on the W^+/W^- ratios and Z/W ratios. Study of correlation of the uncertainty necessary.

Backup

Transverse observable resummation with RadISH

Result at NLL accuracy can be written as

$$\Sigma(v) = \sigma^{(0)} \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(\epsilon v_1)} R'\left(v_1\right) \qquad v_i = V(k_i), \quad \zeta_i = v_i/v_1$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^{1} \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'\left(\zeta_i v_1\right) \Theta\left(v - V(\Phi_B, k_1, \ldots, k_{n+1})\right)$$

Formula can be evaluated with Monte Carlo method; dependence on ϵ vanishes exactly and result is finite in four dimensions

It contains **subleading effect** which in the original CAESAR approach are disposed of by expanding R and R' around v

$$R(\epsilon v_1) = R(v) + \frac{dR(v)}{d\ln(1/v)} \ln\frac{v}{\epsilon v_1} + \mathcal{O}\left(\ln^2 \frac{v}{\epsilon v_1}\right)$$

$$R'\left(v_i\right) = R'(v) + \mathcal{O}\left(\ln\frac{v}{v_i}\right)$$

Not possible! valid only if the ratio v_i/v remains of order one in the whole emission phase space, but for observables which feature kinematic cancellations there are configurations with $v_i \gg v$. **Subleading effects necessary**

Transverse observable resummation with RadISH

Result at NLL accuracy can be written as

$$\Sigma(v) = \sigma^{(0)} \int \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(\epsilon k_{t1})} R'\left(k_{t1}\right) \qquad \zeta_i = k_{ti}/k_{t1}$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^{1} \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'\left(\zeta_i k_{t1}\right) \Theta\left(v - V(\Phi_B, k_{t1}, \ldots, k_{tn+1})\right)$$

Formula can be evaluated with Monte Carlo method; dependence on ϵ vanishes exactly and result is finite in four dimensions

Convenient to perform an expansion around k_{t1} (more efficient and simpler implementation)

$$R(\epsilon k_{t1}) = R(k_{t1}) + \frac{dR(k_{t1})}{d\ln(1/k_{t1})} \ln\frac{1}{\epsilon} + \mathcal{O}\left(\ln^2\frac{1}{\epsilon}\right)$$

$$R'\left(k_{ti}\right) = R'(k_{t1}) + \mathcal{O}\left(\ln\frac{k_{t1}}{k_{ti}}\right)$$

Subleading effects retained: no divergence at small v, power-like behaviour respected

Logarithmic accuracy defined in terms of $\ln(M/k_{t1})$

Result formally equivalent to the *b*-space formulation

Matching to fixed order: multiplicative matching

Cumulative cross section should reduce to the fixed order at large v

$$\Sigma^{\text{mult}}_{\text{matched}}(v) = \Sigma_{\text{res}}(v)\left[\frac{\Sigma_{\text{f.o.}}(v)}{\Sigma_{\text{res}}(v)}\right]_{\text{expanded}}$$

- allows to include constant terms from NNLO
- physical suppression at small v cures potential instabilities

To ensure that resummation does not affect the hard region of the spectrum when the matching is performed we introduce **modified logarithms**
This corresponds to restrict the rapidity phase space at large k_t

$$\ln(Q/k_{t1}) \to \frac{1}{p}\ln\left(1+\left(\frac{Q}{k_{t1}}\right)^p\right)$$

Q : perturbative **resummation scale** used to probe the size of subleading logarithmic corrections

p : arbitrary matching parameter

Matching improved by normalizing to the **asymptotic value to** avoid **spurious** $\mathcal{O}(\alpha_s^4)$ **contributions**

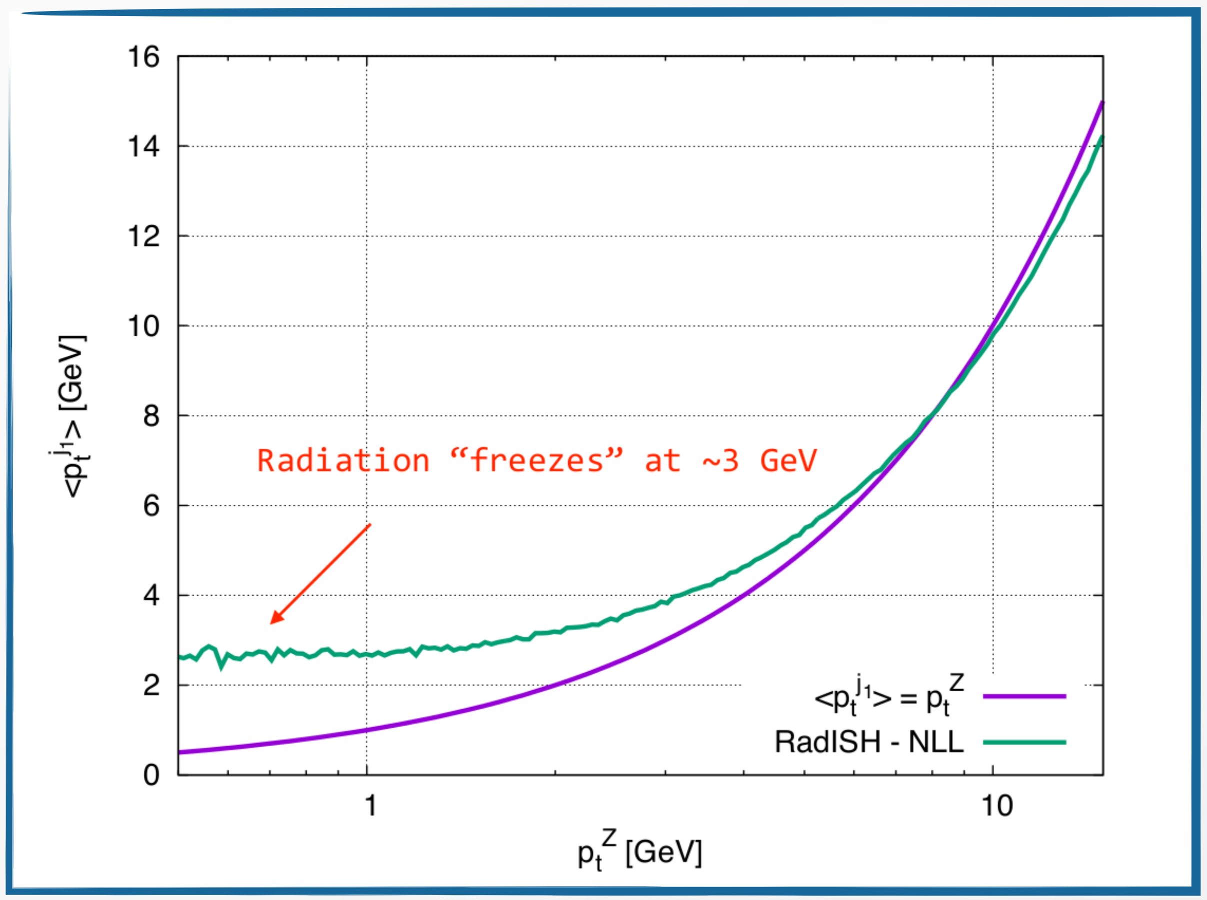
$$\Sigma^{\text{matched}}_{\text{mult}}(v) = \frac{\Sigma^{\text{res}}(v)}{\Sigma^{\text{res}}_{\text{asym.}}}\left[\Sigma^{\text{res}}_{\text{asym.}}\frac{\Sigma^{\text{f.o.}}(v)}{\Sigma^{\text{exp}}(v)}\right]_{\text{expanded}} \qquad \Sigma^{\text{res}}_{\text{asym.}} = \int_{\text{with cuts}} d\Phi_B \left(\lim_{L\to 0}\mathcal{L}_{\text{N}^k\text{LL}}\right)$$

The Landau pole and the small-p_t limit

Running coupling $\alpha_s(k_{t1}{}^2)$ and Sudakov radiator hit Landau pole at

$$\alpha_s(\mu_R^2)\beta_0 \ln Q/k_{t1} = \frac{1}{2} \qquad k_{t1} \sim 0.01\,\text{GeV}, \quad \mu_R = Q = m_Z$$

Only real cutoff in the calculation: emission probability is set to zero below this scale and parton densities are frozen.



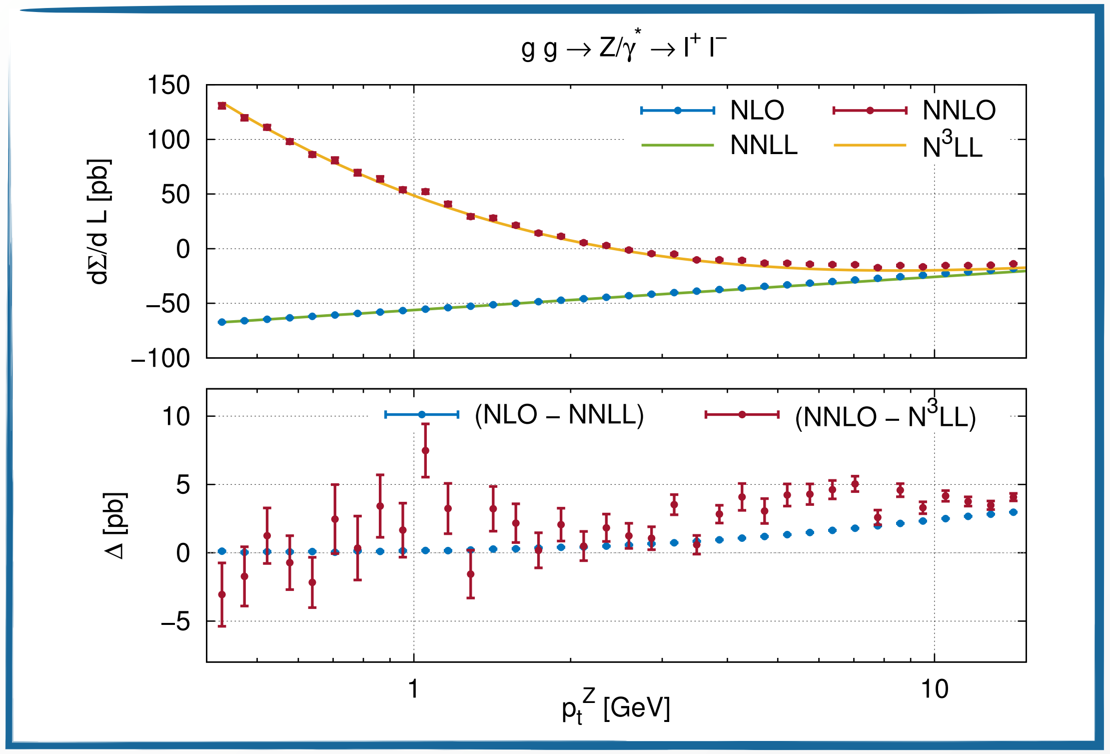
At small p_t the large azimuthal cancellations dominate over the Sudakov suppression: the cutoff is never an issue in practice

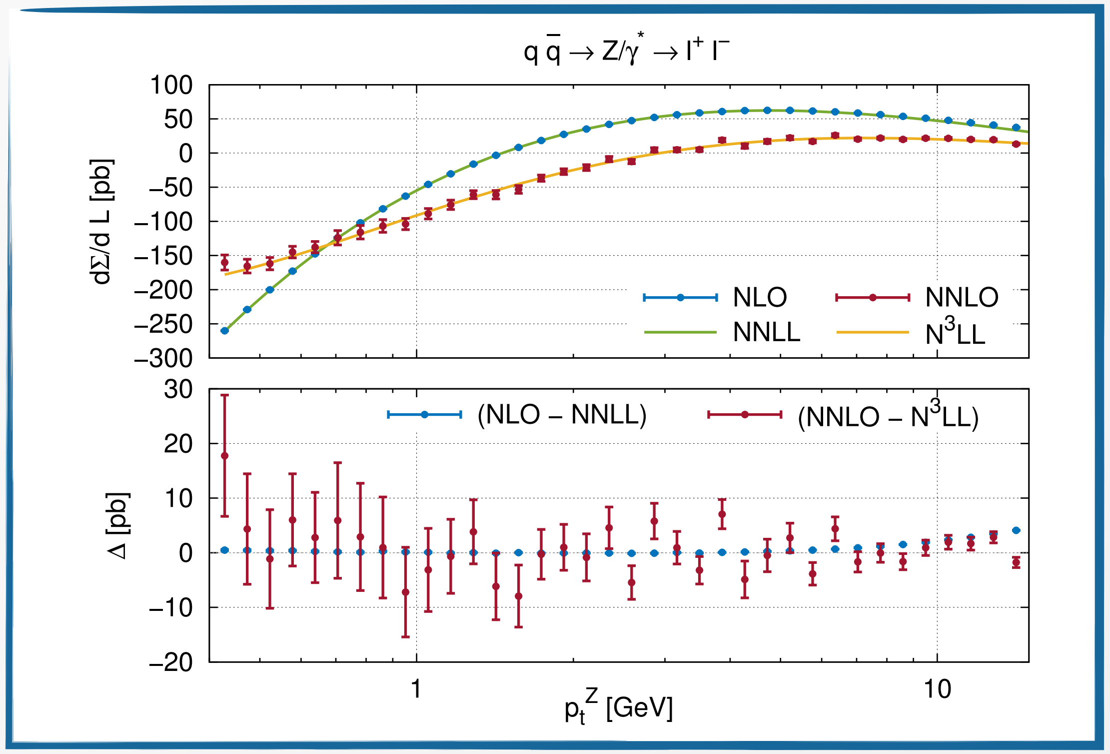
$$\frac{d^2\Sigma(v)}{dp_t d\Phi_B} \simeq 2\sigma^{(0)}(\Phi_B)p_t \left(\frac{\Lambda^2_{\text{QCD}}}{M^2}\right)^{\frac{16}{25}\ln\frac{41}{16}}$$

No NP parameters included in the following

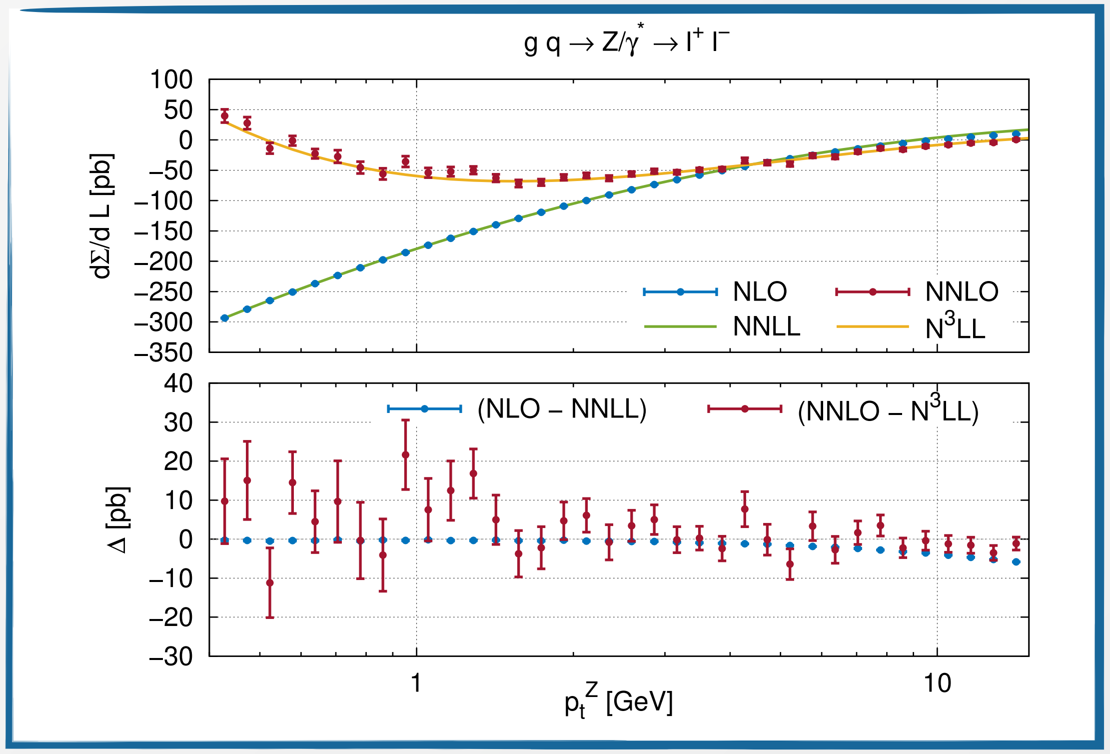
Fixed order vs. resummation

[Bizon, Chen et al. 1805.05916]



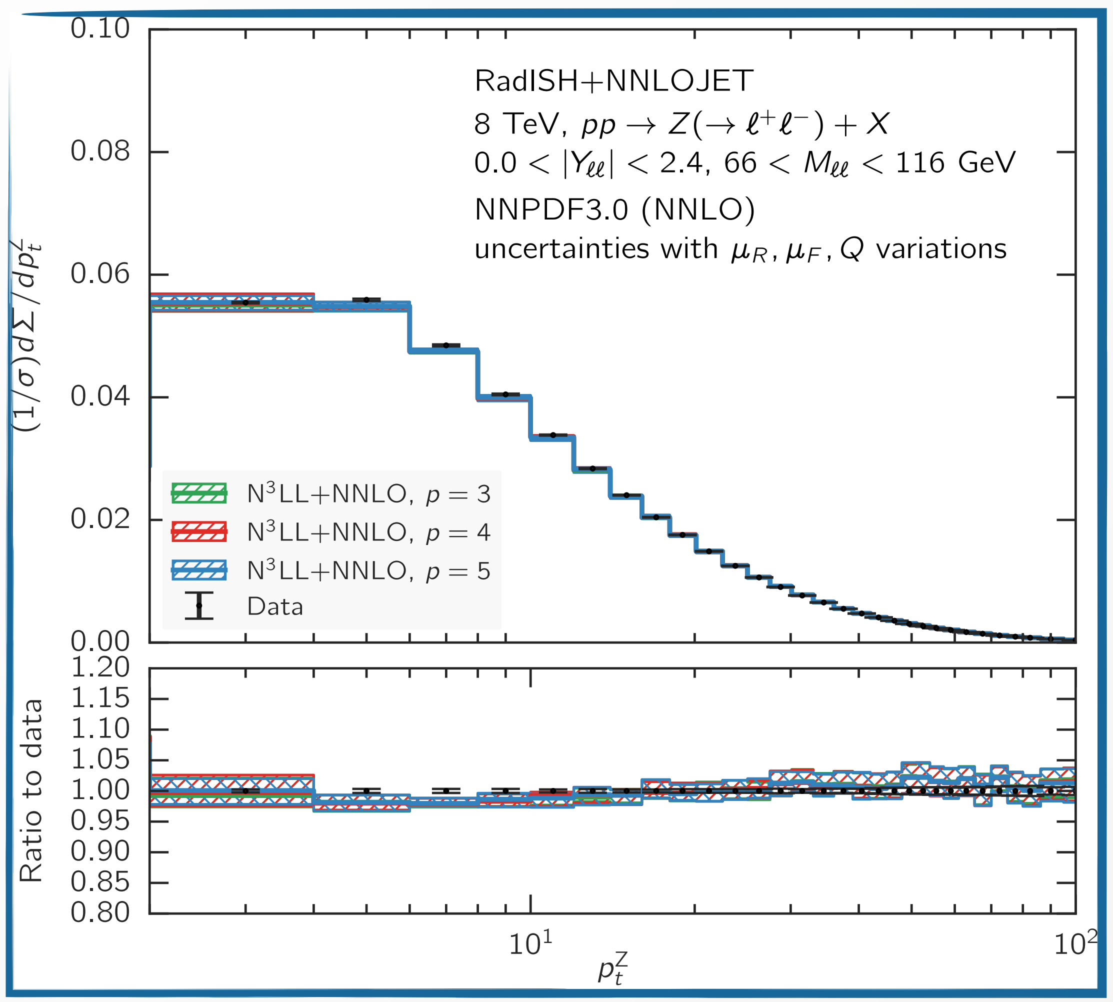


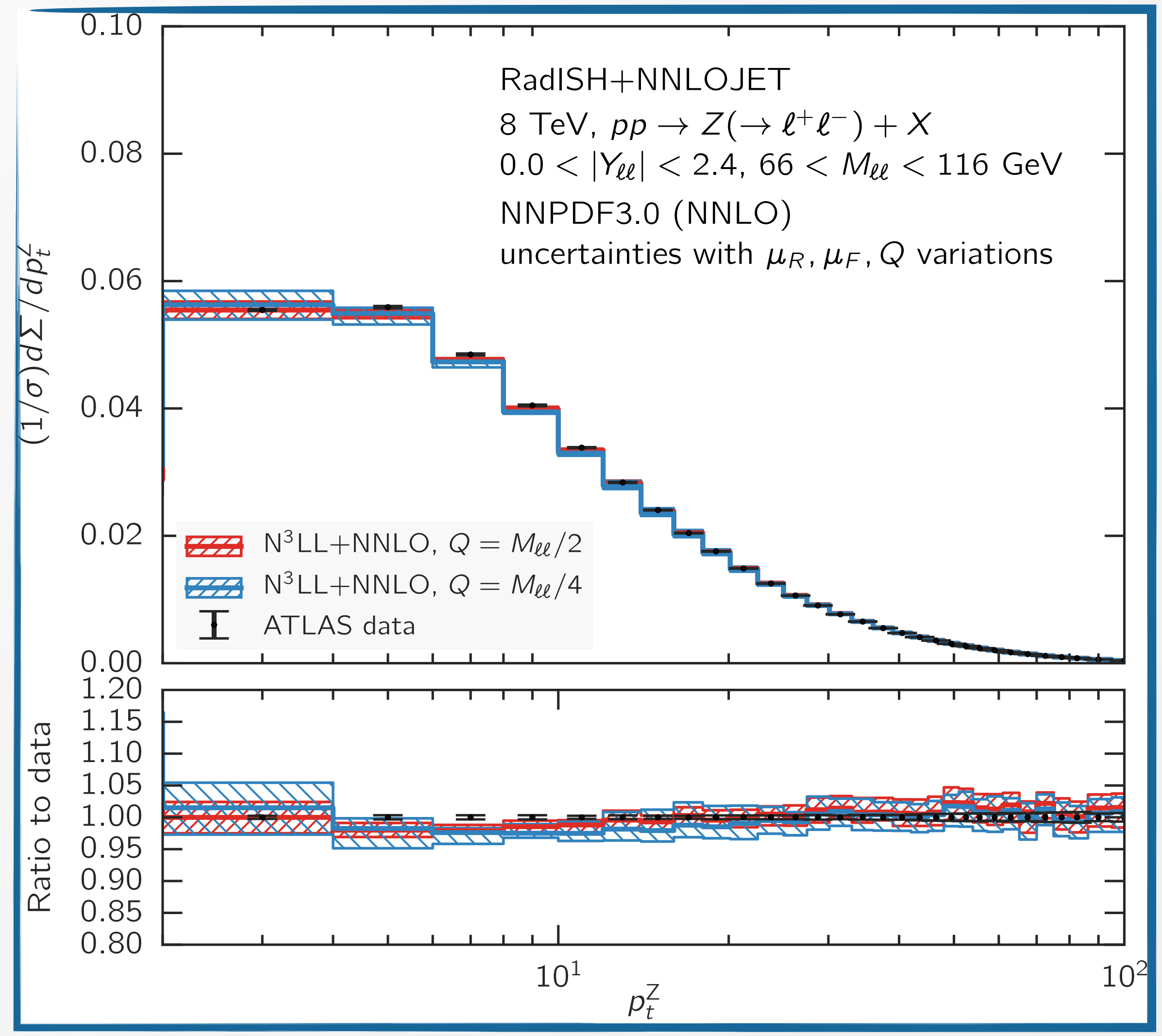
- Very good agreement with the fixed order at small p_t
- Very strong validation of both calculations
- Fixed an implementational error in the fixed order computation



Resummation and matching uncertainties

[data from ATLAS 1512.02192]
[Bizon, Chen et al. 1805.05916]





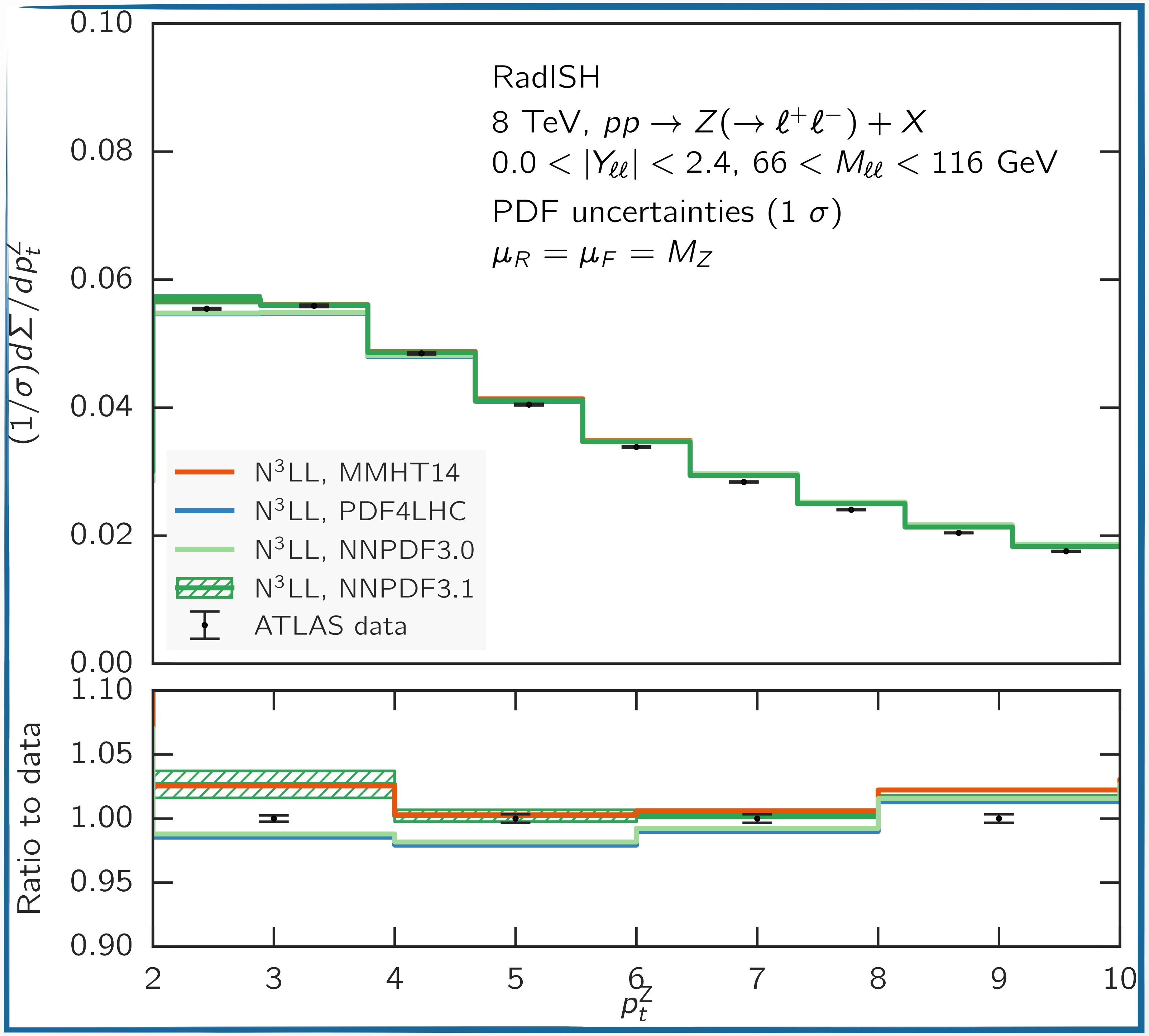
- Matching uncertainties at the sub percent level

- Predictions stable wrt variation of central value of the resummation scale

PDF uncertainties

[data from ATLAS 1512.02192]

Beware of different PDFs and central scales



- Uncertainty with state-of-the-art PDFs at the 1-2% level
- Spectrum gets slightly harder than NNPDF3.0 (used in our current studies)