Multi-parton interactions at hadron colliders

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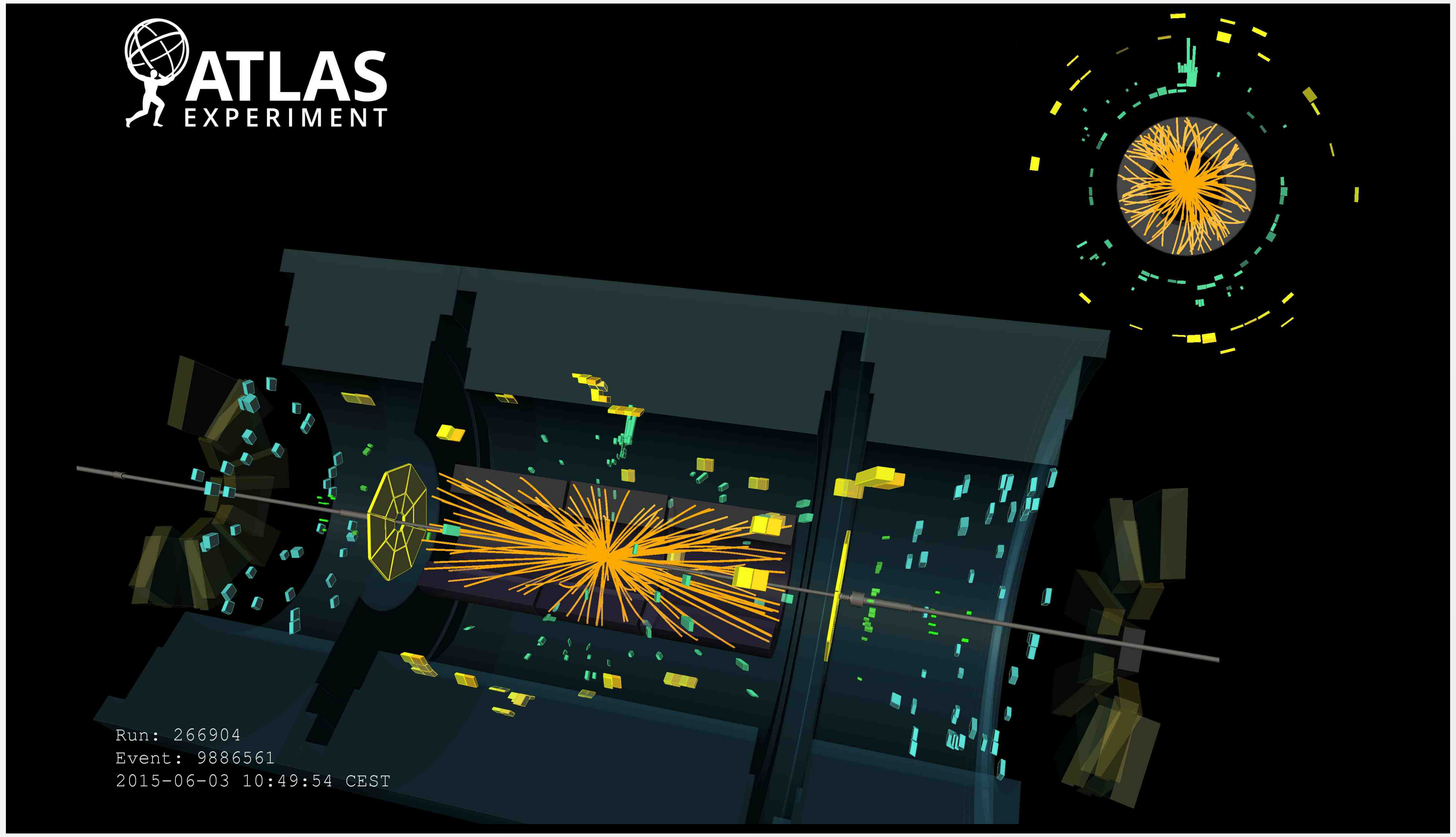
Multi-parton interactions

Inelastic hadron-hadron collisions are dominated by **soft** (low-p_T) interactions, also known as **minimum bias (MB)**

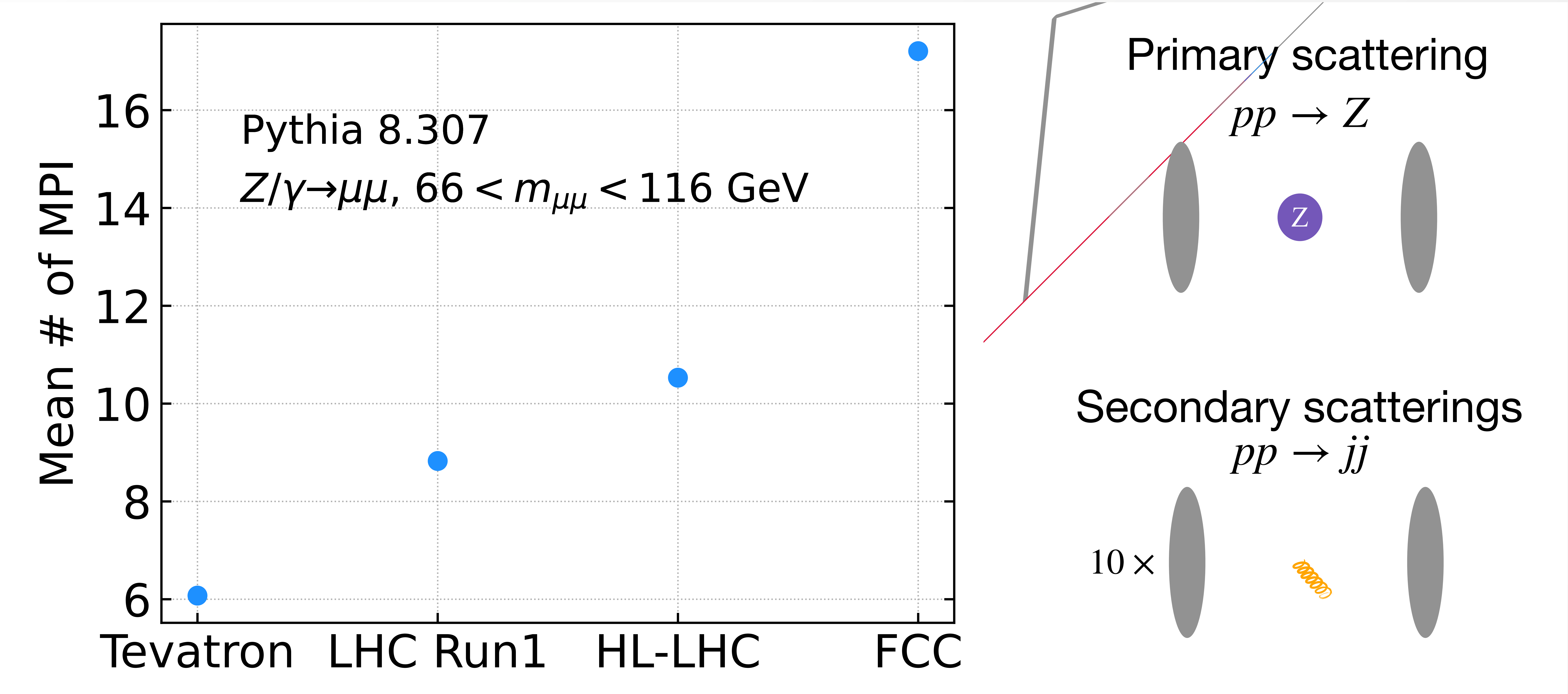
Typically associated with the **inelastic non-diffractive component** σ_{nd} of the total cross section σ_{tot}



Composite nature of hadrons implies that in such generic collisions **multi-parton interactions (MPI) are inevitable**

Underlying event at hadron colliders

This **remains true also when selecting events where a hard collisions takes place**: these collisions at the LHC can be interpreted as a hard scattering between partons accompanied by an "**underlying event" (UE)** of additional soft interactions



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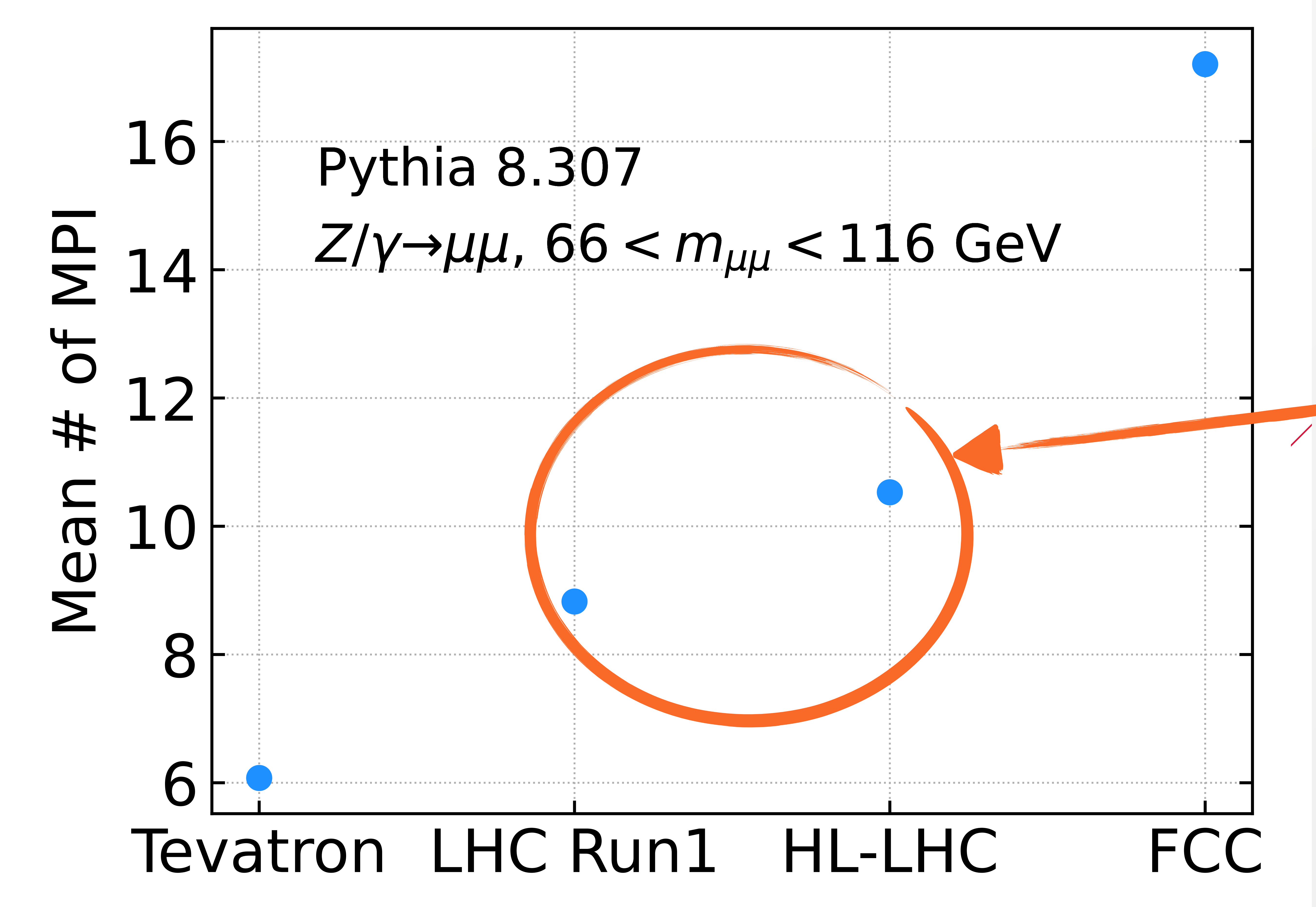

$\mathcal{O}(10)$ **additional parton-parton collisions** per Drell-Yan event at the LHC

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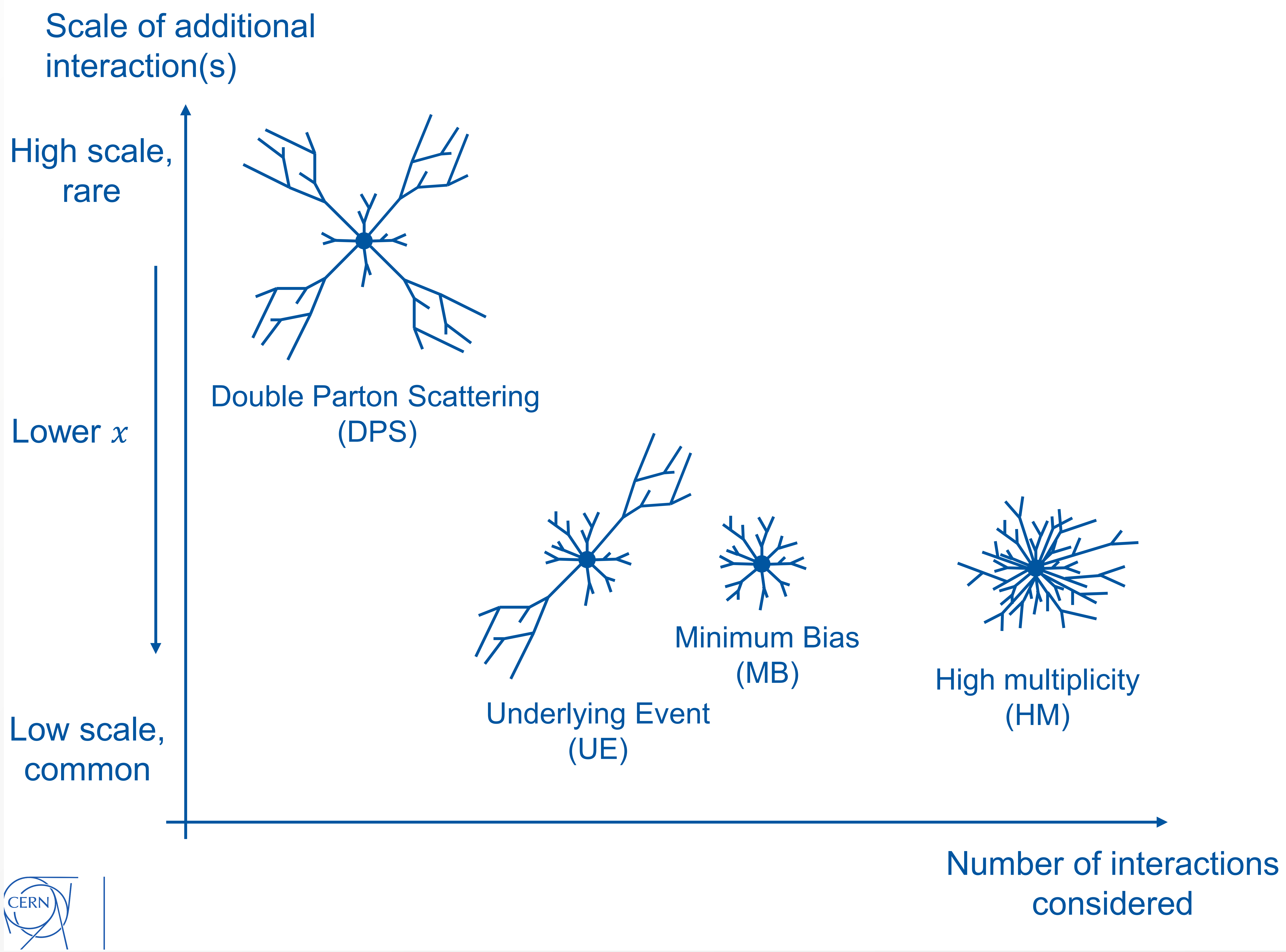
MPI ‘landscape’



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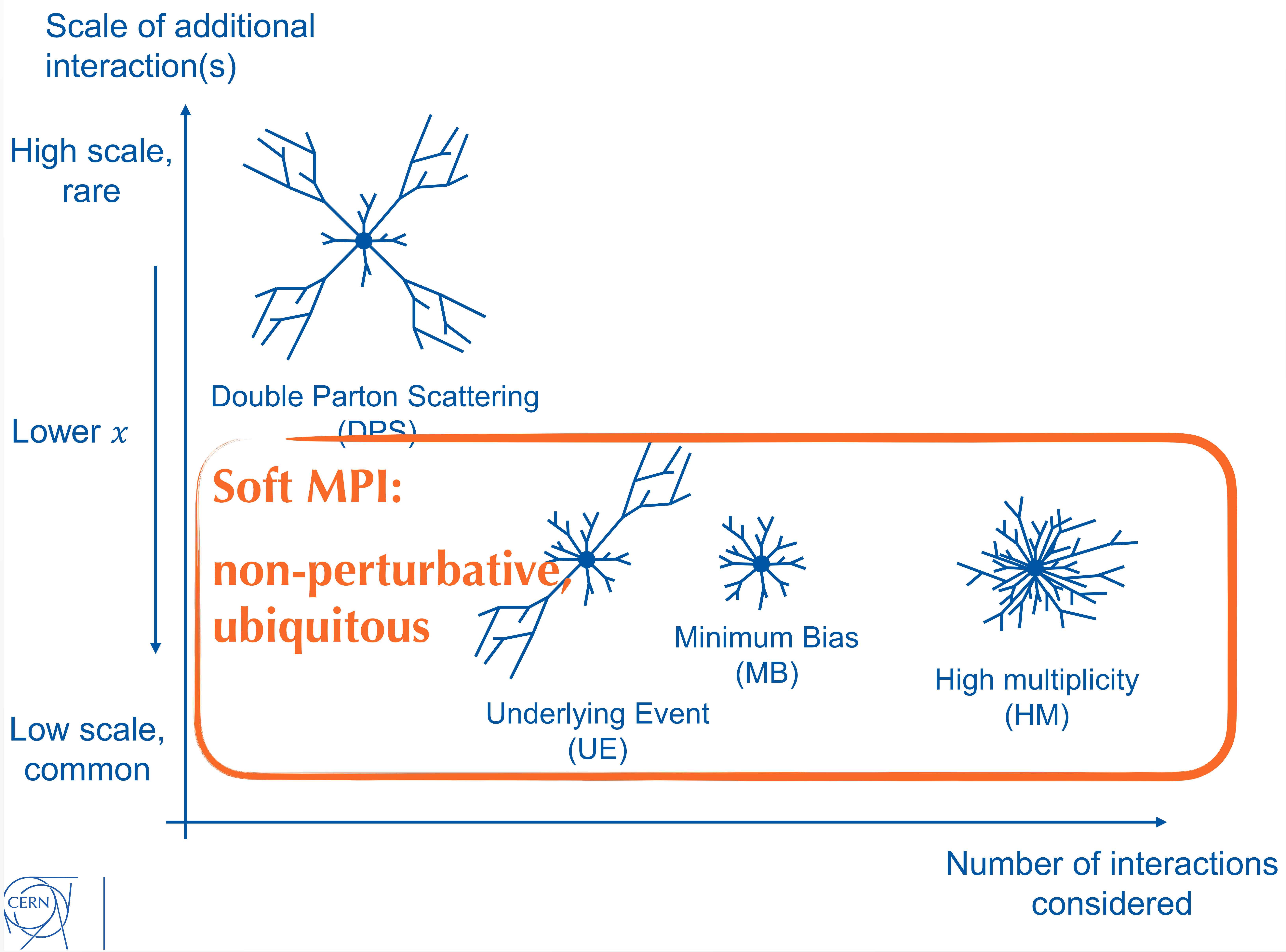

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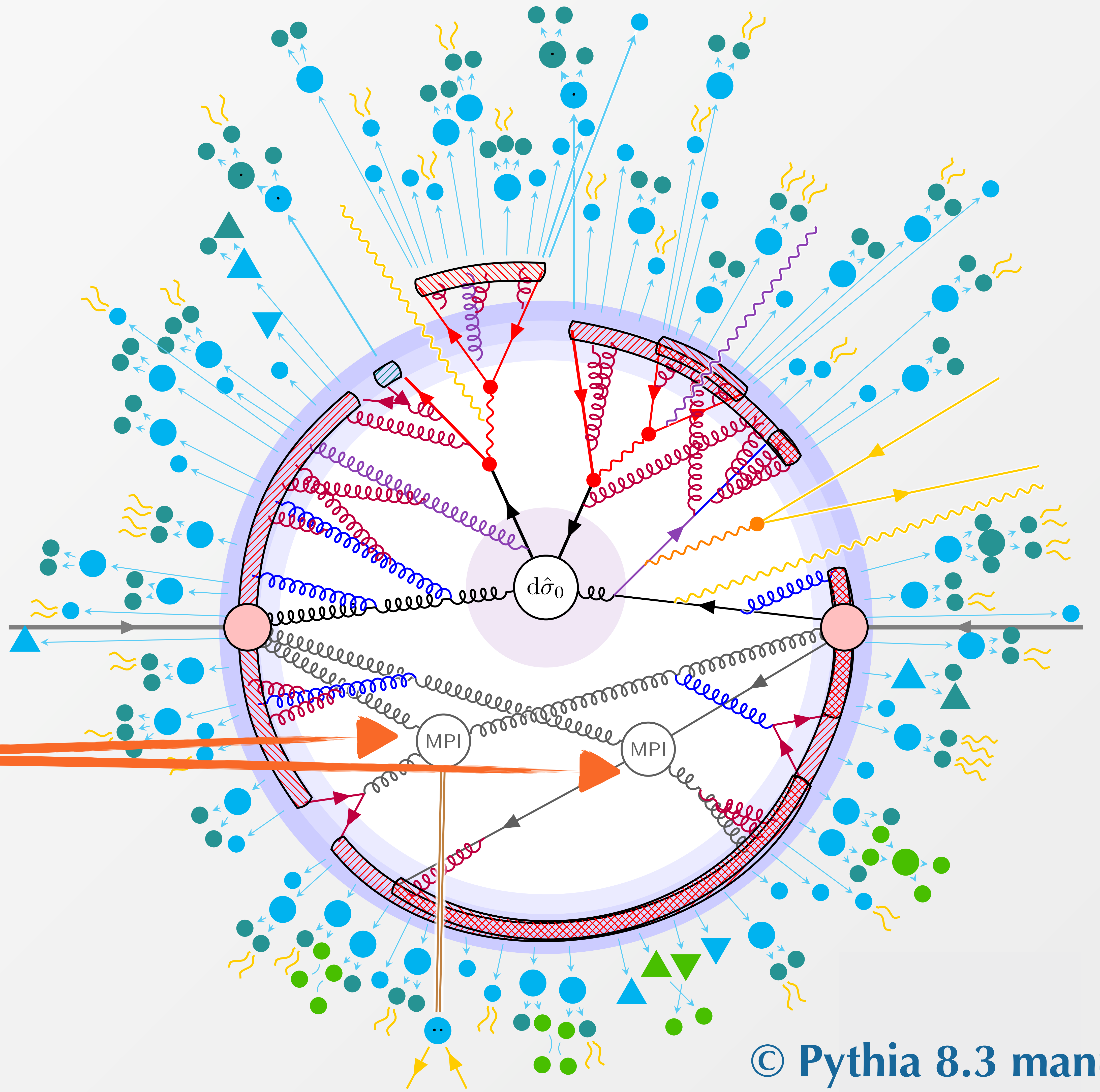
MPI modelling in MC event generators

Accurate modeling of additional (**soft**) scatters is **crucial to ensure proper interpretation of hard events at hadronic colliders**

A complete description of the MPI is challenging due to their **large number** and complex interplay and their **chiefly non-perturbative nature**

At present soft MPI are modelled in an approximate way by **general-purpose Monte Carlo (MC) event generators**



Basics of MPI modelling

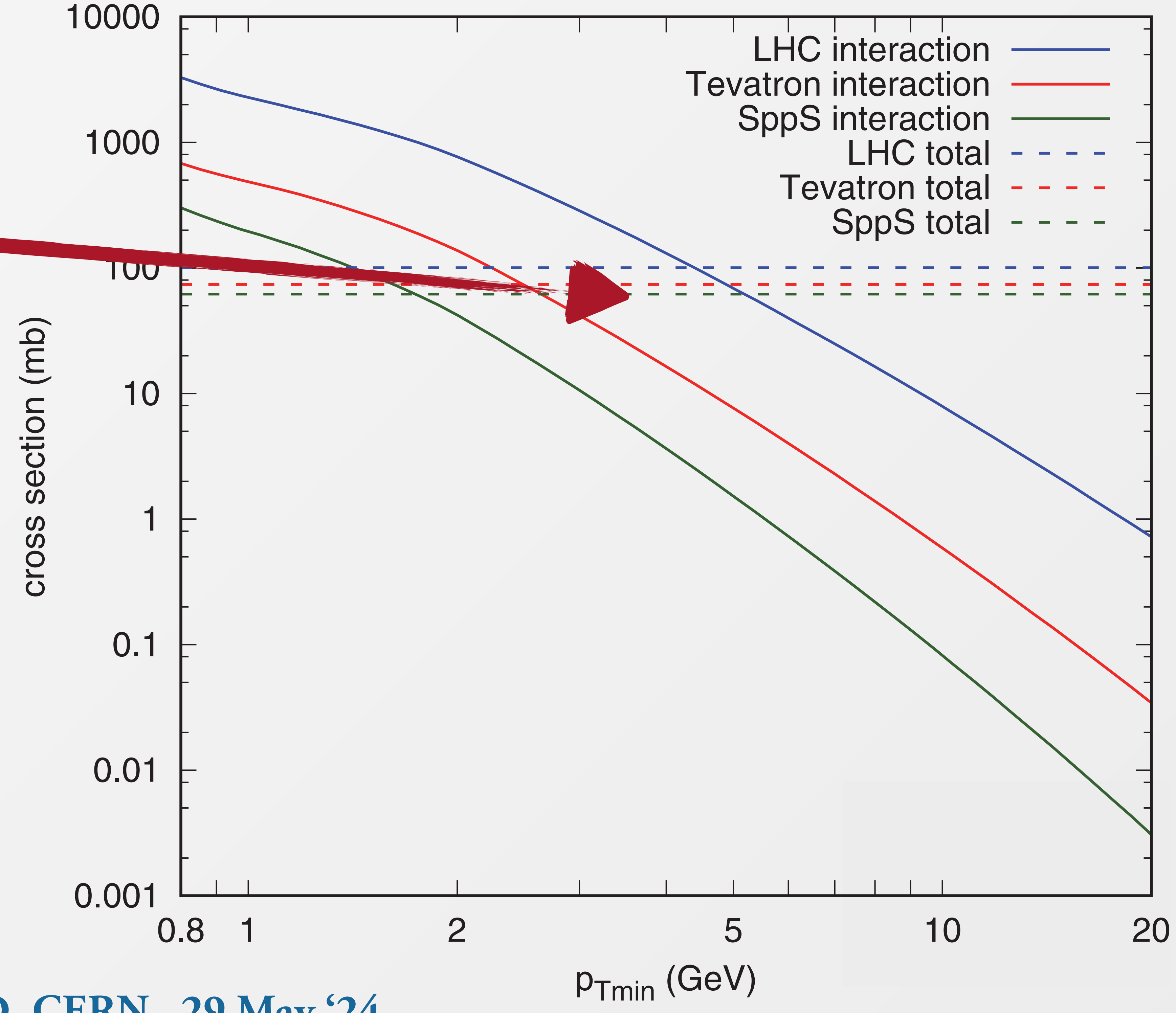
MC event generators such as Pythia, Herwig and Sherpa have refined and extended the modelling of MPI during the years, but all share the **same fundamental model** developed by T. Sjostrand and M. van Zijl in 1987 in the effort of improving the description of S$\bar{p}p$S data

Starting point of the model is the differential perturbative QCD $2 \to 2$ dijet cross section

$$\frac{d\sigma}{dp_T^2} = \sum_{i,j} \int f_i(x_1, Q^2) f_j(x_2, Q^2) \frac{d\hat{\sigma}_{ij}}{d\hat{t}} \delta(p_T^2 - \hat{t}\hat{u}/\hat{s}) dx_1 dx_2 d\hat{t}$$

The integrated cross section depends on the chosen $p_{T,\min}$ scale and ultimately **exceeds the total cross section** even when $p_{T,\min} \gg \Lambda_{QCD}$

$$\sigma_{\text{int}}(p_{T,\min}) = \int_{p_{T,\min}}^{\sqrt{s}/2} \frac{d\sigma}{dp_T} dp_T \sim \frac{1}{p_{T,\min}^2}$$



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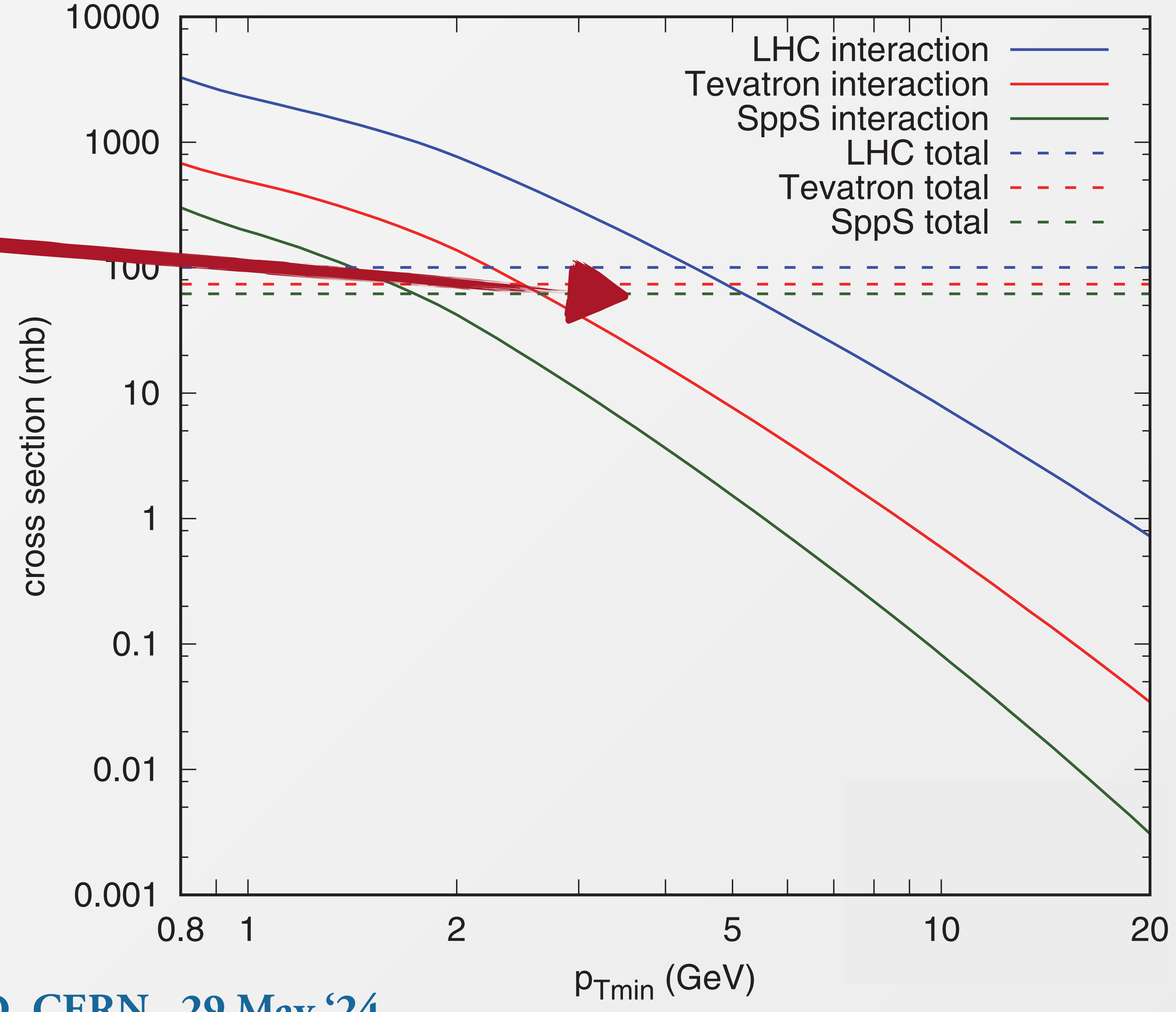


Interpretation: several parton-parton interactions per hadron-hadron interactions

$$\langle n_{\mathrm{MPI}}(p_{T,\min}) \rangle \simeq \frac{\sigma_{\mathrm{int}}(p_{T\min})}{\sigma_{\mathrm{tot}}}$$

Example: MPI modelling in Pythia

Assuming that each interaction is independent: $\langle n_{\mathrm{MPI}}(p_{T,\min})\rangle$ follows a Poissonian distribution

A naive approach does not take into account even basic correlations (e.g. energy–momentum conservation): the sum of interaction energies may exceed the total CM energy

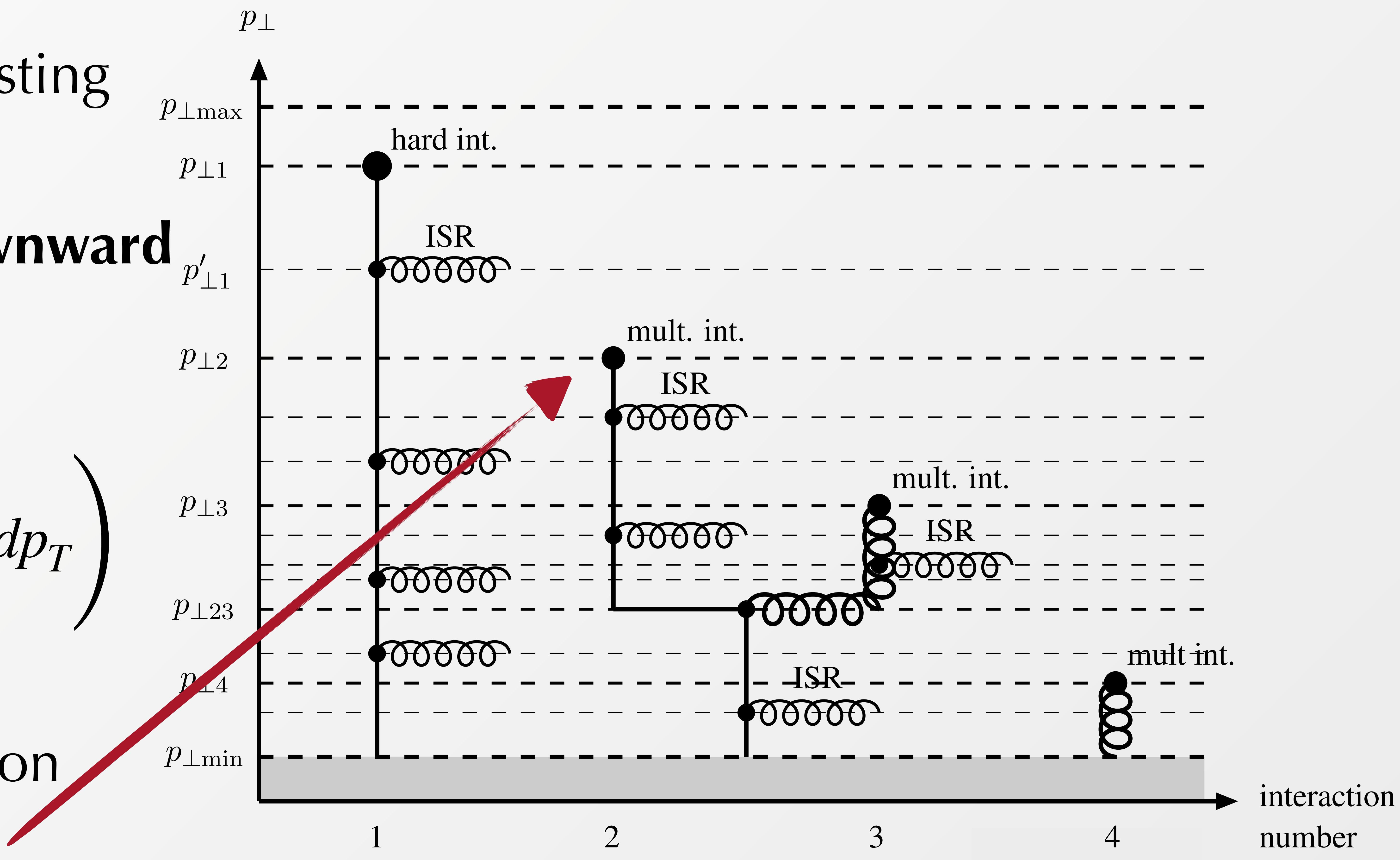
Solution to the problem comes by recasting MPI as a **Sudakov-style evolution**

Generation of MPI formulated as a **downward evolution in p_T** with probability

$$\frac{d\mathscr{P}}{dp_{T,i}} = \frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma}{dp_{T,i}} \exp\left(- \int_{p_{T,i}}^{p_{T,i-i}} \frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma}{dp_T} dp_T \right)$$

Interleave of MPI, ISR and FSR evolution in **one common sequence** in p_T

[Sjostrand, Skands 2005][Sjostrand, Cork 2011]

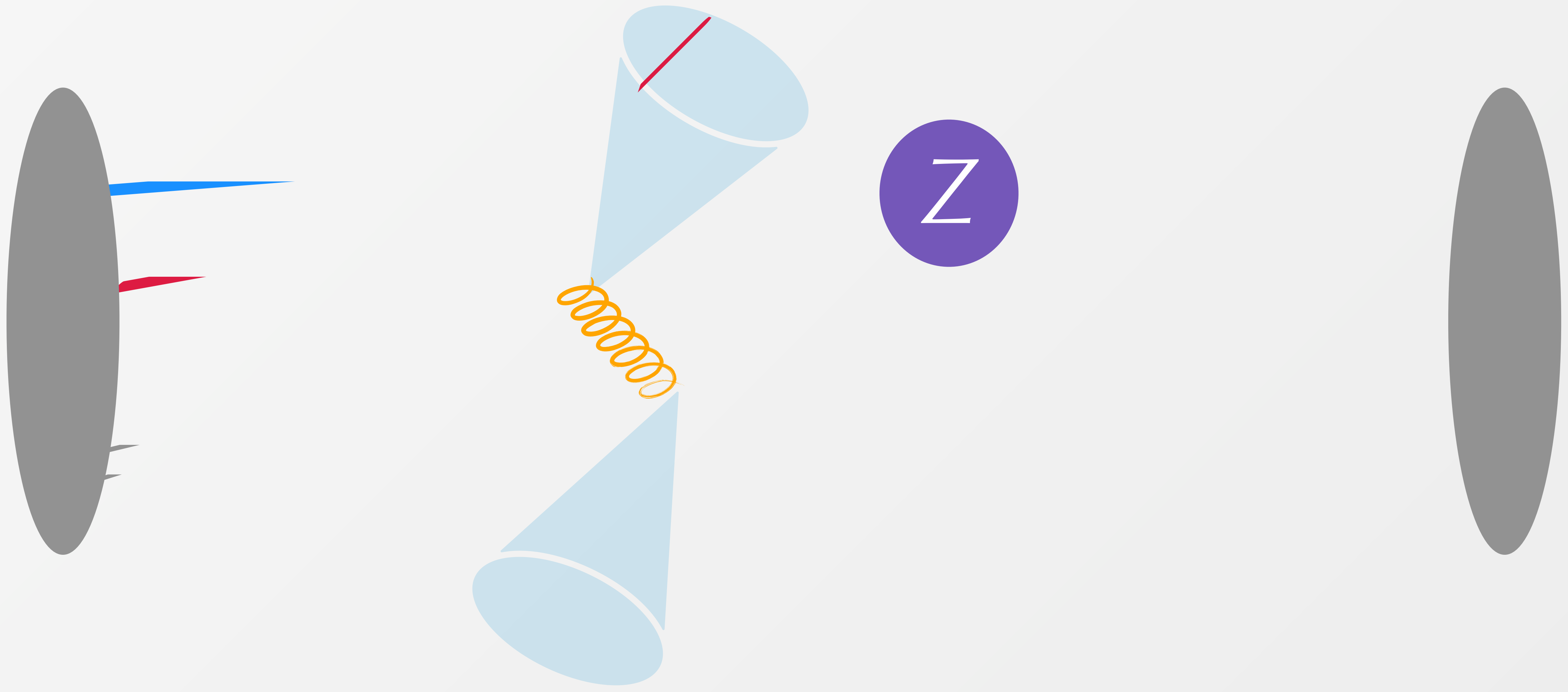


Soft MPI: summary

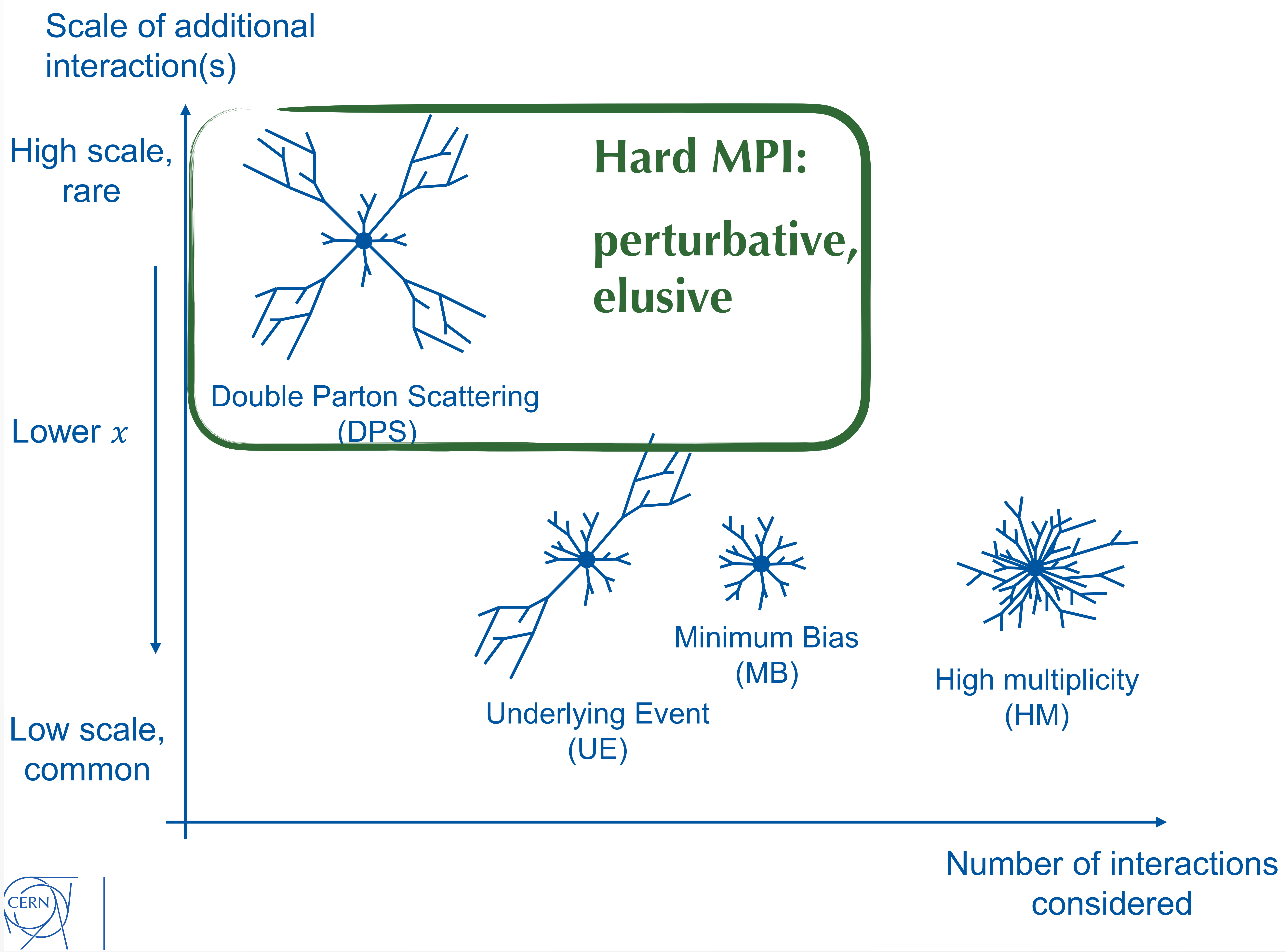
Monte Carlo models seek an unified description of hard jets, UE and MB

Although inspired by theoretical ideas, the development of MPI models is **rather decoupled from detailed theoretical calculations** due to the complexity of the environment with $\mathcal{O}(10)$ **soft** MPIs

An alternative approach to study MPI physics in a **much cleaner environment** is in events with two (or more) separate **hard processes** in an individual hadron-hadron collisions: double- or triple-parton scattering (DPS, TPS), more amenable to first-principle theoretical approaches



MPI ‘landscape’



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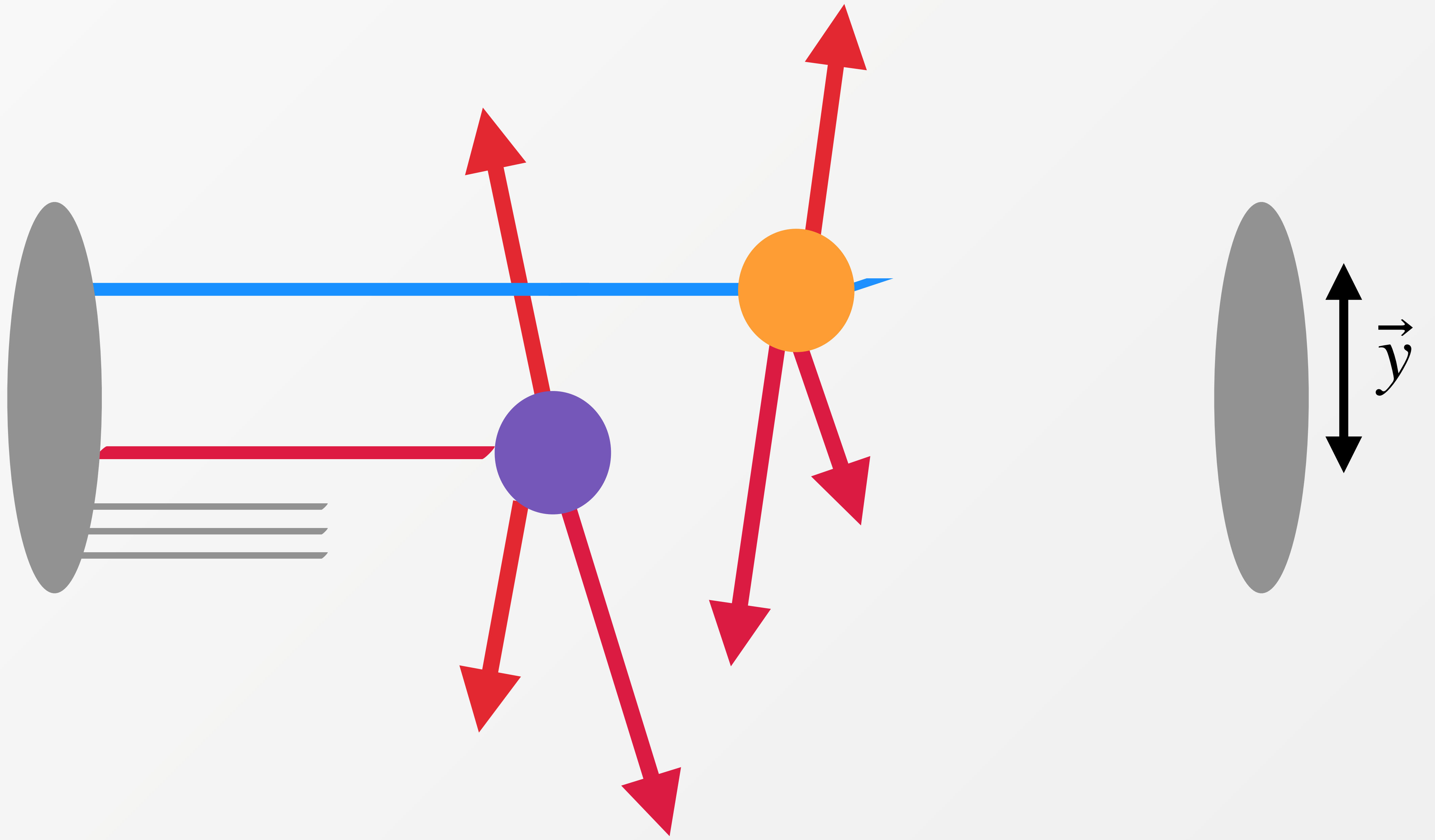
Double parton scattering: basic features



Two separate **hard interactions** in a **single** proton-proton collision

Postulated factorisation formula for integrated DPS cross section based on parton model / low order Feynman diagrams considerations

Parton level cross sections

$$\sigma^{A,B} = \frac{m}{2} \sum_{i,j,k,l} \int F_h^{ik}(x_1, x_2, \vec{y}, Q_A, Q_B) F_h^{jl}(x_1', x_2', \vec{y}, Q_A, Q_B) \times \hat{\sigma}_{ij}^{A}(x_1, x_1') \hat{\sigma}_{kl}^{B}(x_2, x_2')\, dx_1 dx_1' dx_2 dx_2' d^2\vec{y}$$

Collinear double-parton distributions

[Paver, Treleani 1982]
[Mekhfi, 1985]
[Blok, Dokshitzer, Frankfurt, Strikman 2011]
[Diehl, Ostermeier, Schaefer 2012]

Double parton scattering: simplified assumptions

If **correlations between partons** are ignored

$$F_h^{ik}(x_1, x_2, \vec{y}) = \int d^2\vec{b} D^i(x_1, \vec{b}) D^j(x_1, \vec{b} + \vec{y})$$

D: impact-parameter dependent PDFs

Justified at small x given the large population of partons

Often the further approximation $D^i(x_1, \vec{b}) = D^i(x_1) G(\vec{b})$ is made, leading to the **pocket formula**

$$\sigma_{AB} = \frac{\sigma_A \sigma_B}{\sigma_{\text{eff}}}$$

where $\sigma_{\text{eff}} \sim O(20\,\text{mb})$ is a normalisation factor roughly connected with area over which partons are concentrated in the proton.

Challenges in hard MPI studies

Distinguishing two contributions:

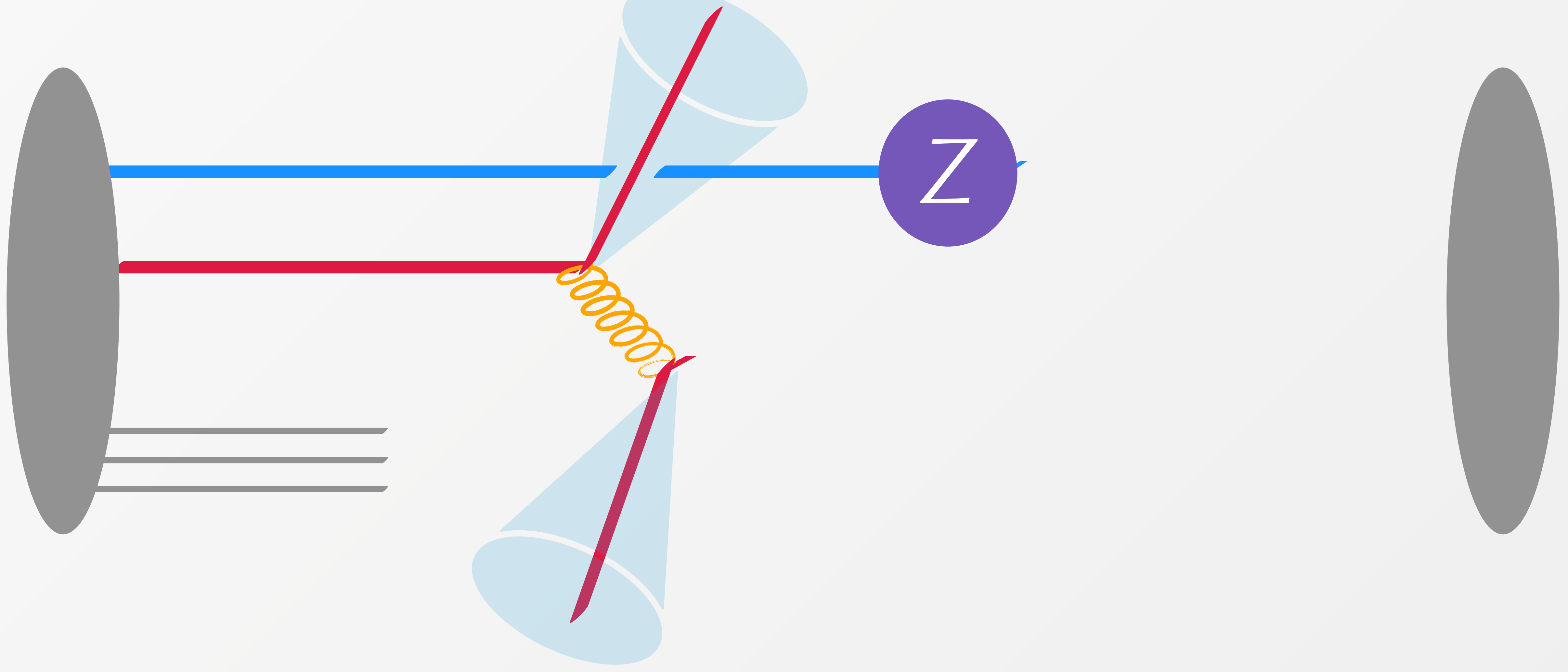
- two independent hard scatterings **(2HS)**
- a single hard scattering **(1HS)** with extra radiation

e.g. Z boson production: both contributions have experimental signature of Z boson ($\rightarrow$ 2 leptons) + jets

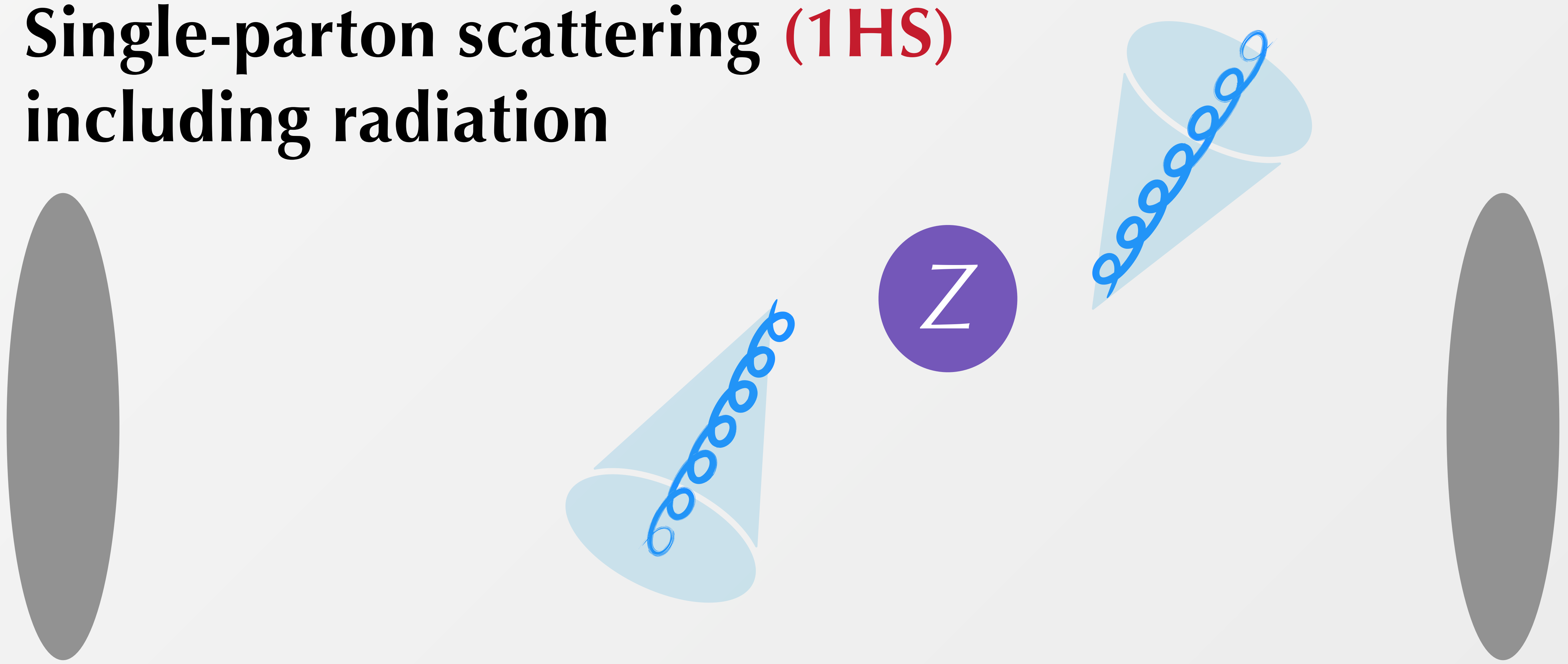
At the level of the total cross section, 2HS mechanism is power suppressed with respect to 1HS

$$\sigma_{2HS}/\sigma_{1HS} \sim \Lambda^2/Q^2$$

Double-parton scattering (2HS)

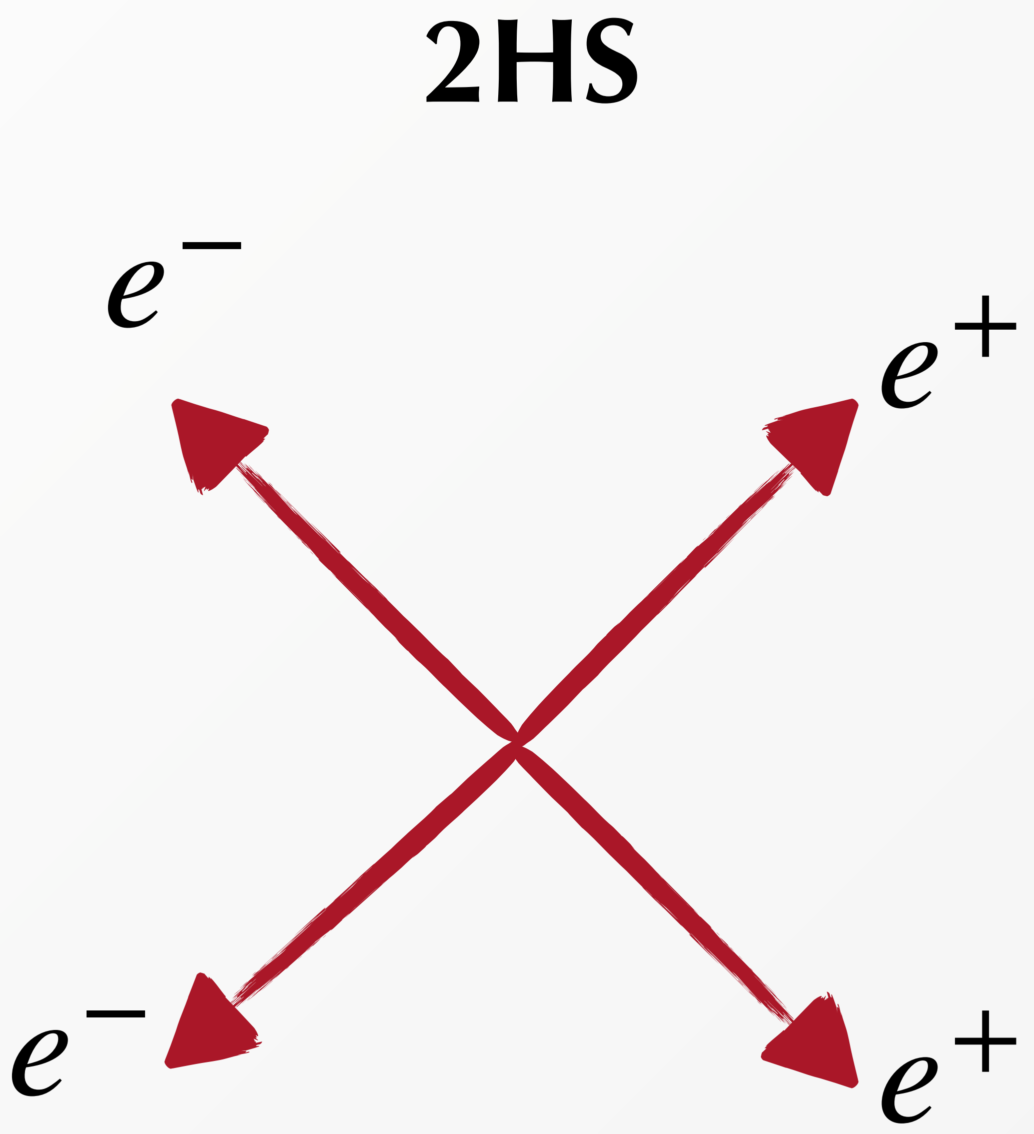


Background from Single-parton scattering (1HS) including radiation

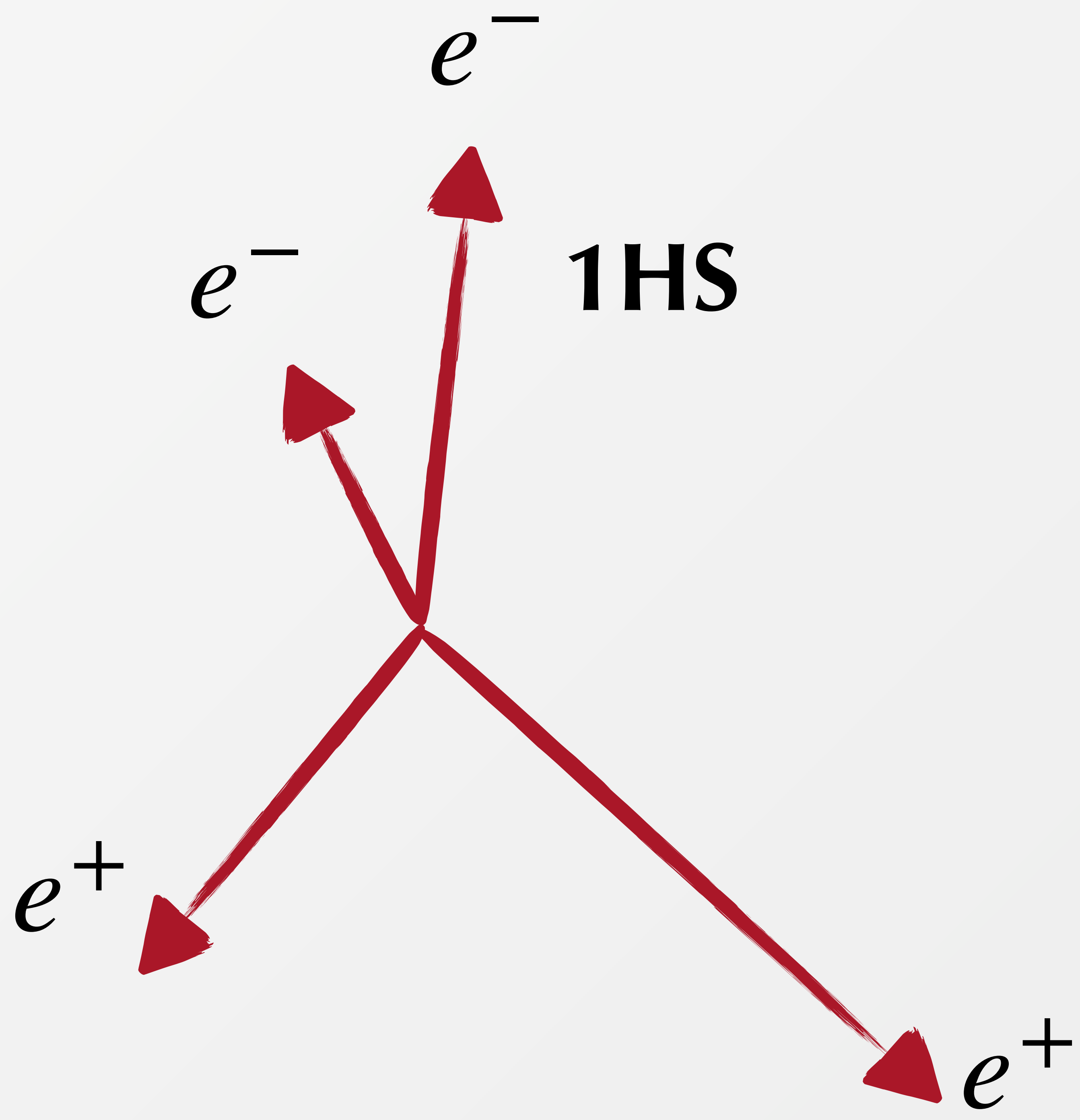


Strategy A

DPS populates the final state phase space in a different way from 1HS. In particular, for processes A and B it tends to populate the region of small $\vec{q}_A$, $\vec{q}_B$



$$pp \to e^+e^-e^+e^-$$

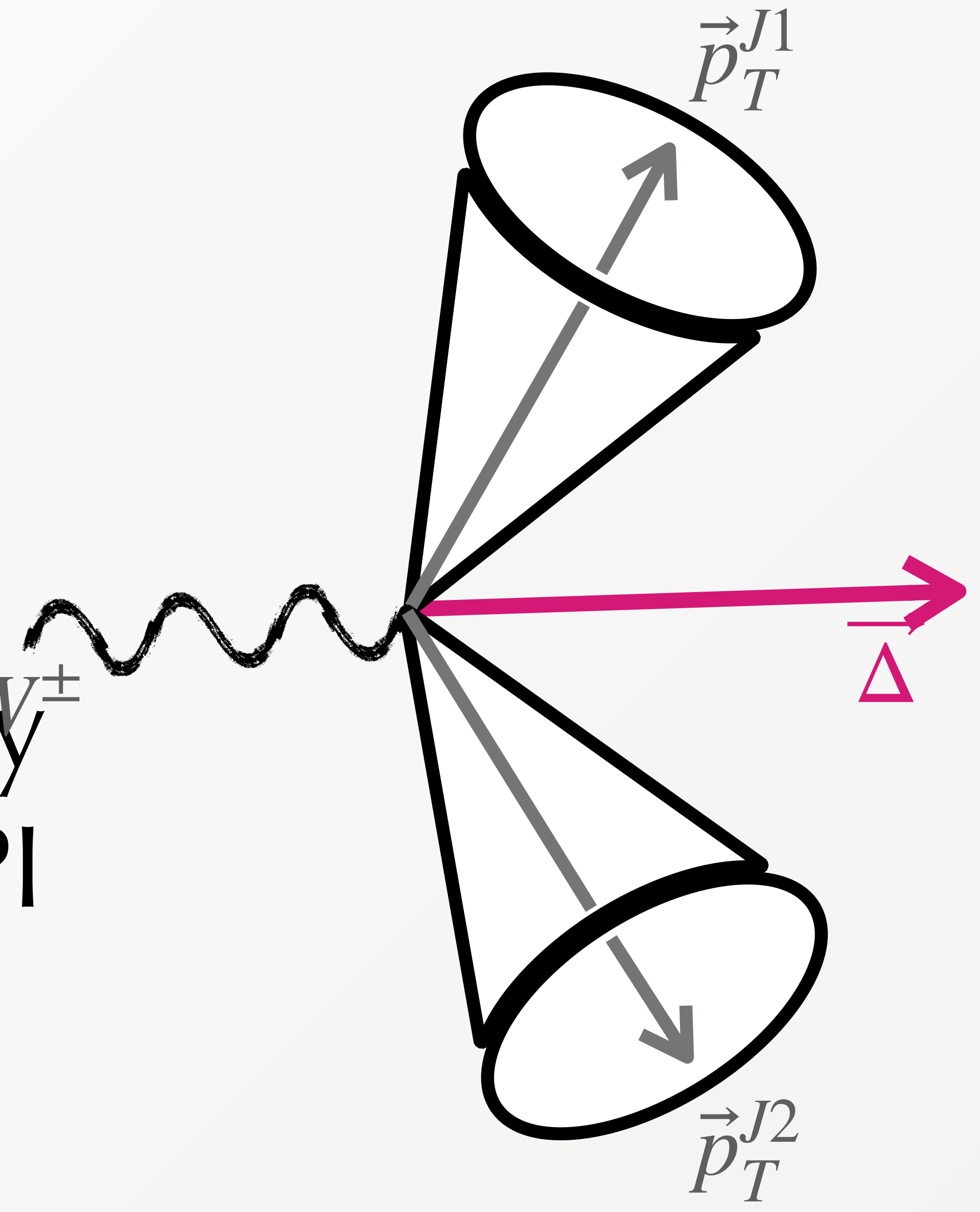


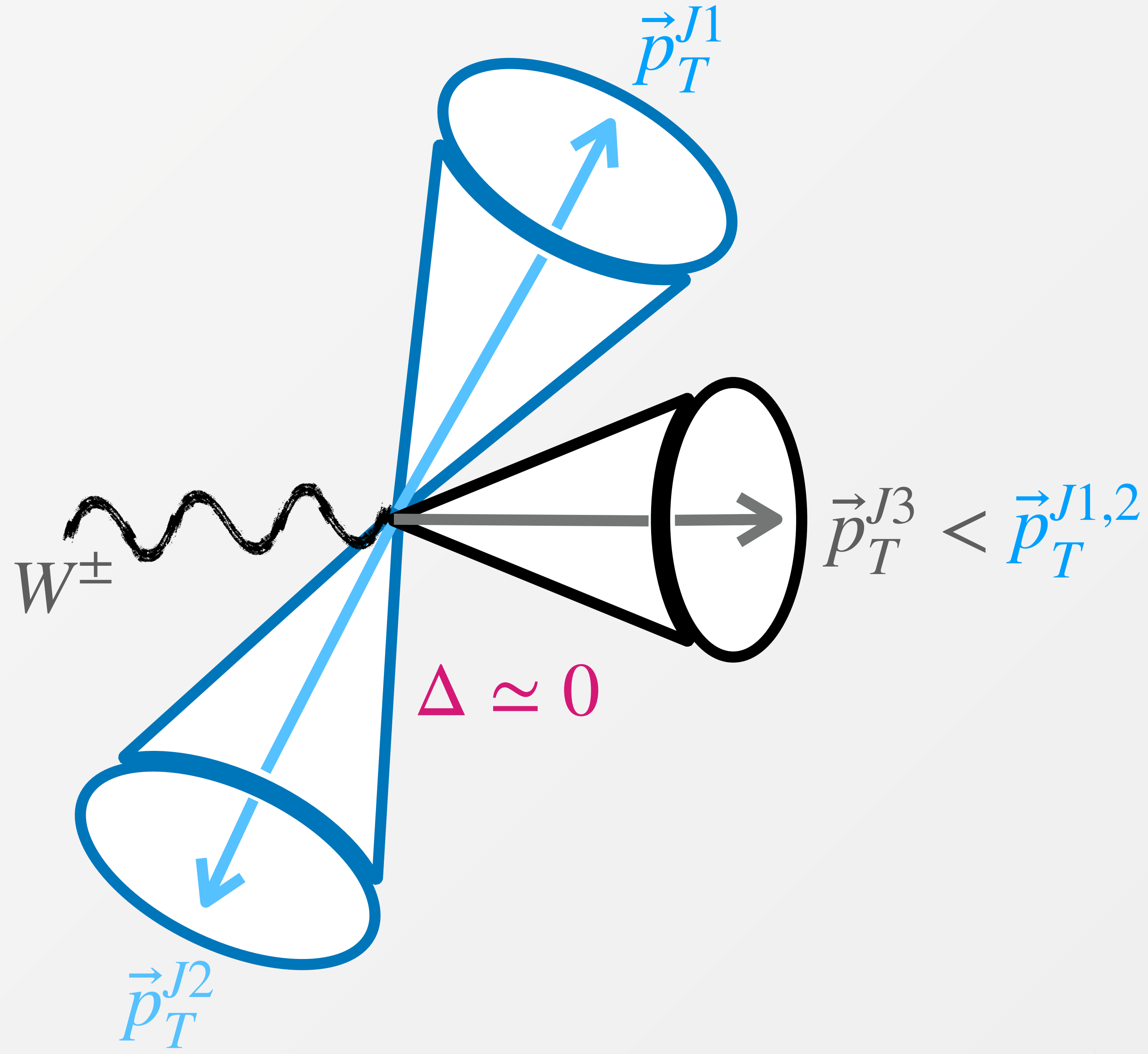
W+2-jets study

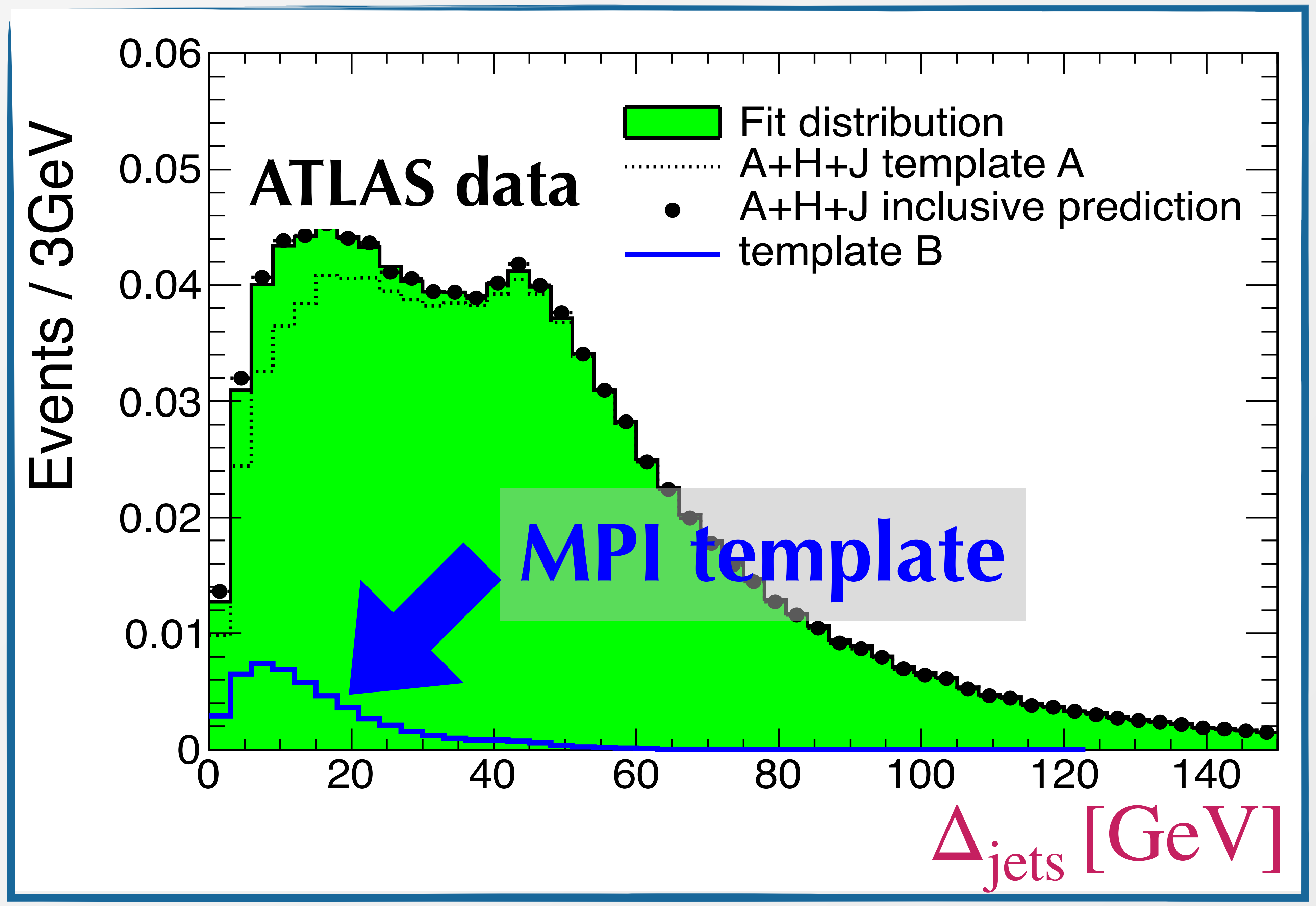
- E.g. ATLAS, $W \to \ell\nu$ + 2 jets 1301.6872
- Exploits fact that MPI jet-pair more likely to balance than radiation jet pair, so MPI should be enhanced for

$$\Delta_{\rm jets} = \left|\vec{p}_T^{\,\rm J1} + \vec{p}_T^{\,\rm J2}\right| \to 0$$

- That works to some extent, but relative MPI (2HS) fraction is moderate ($\lesssim$ 10%)
- Quantitative analysis requires very good understanding of radiation in single hard scattering (1HS)



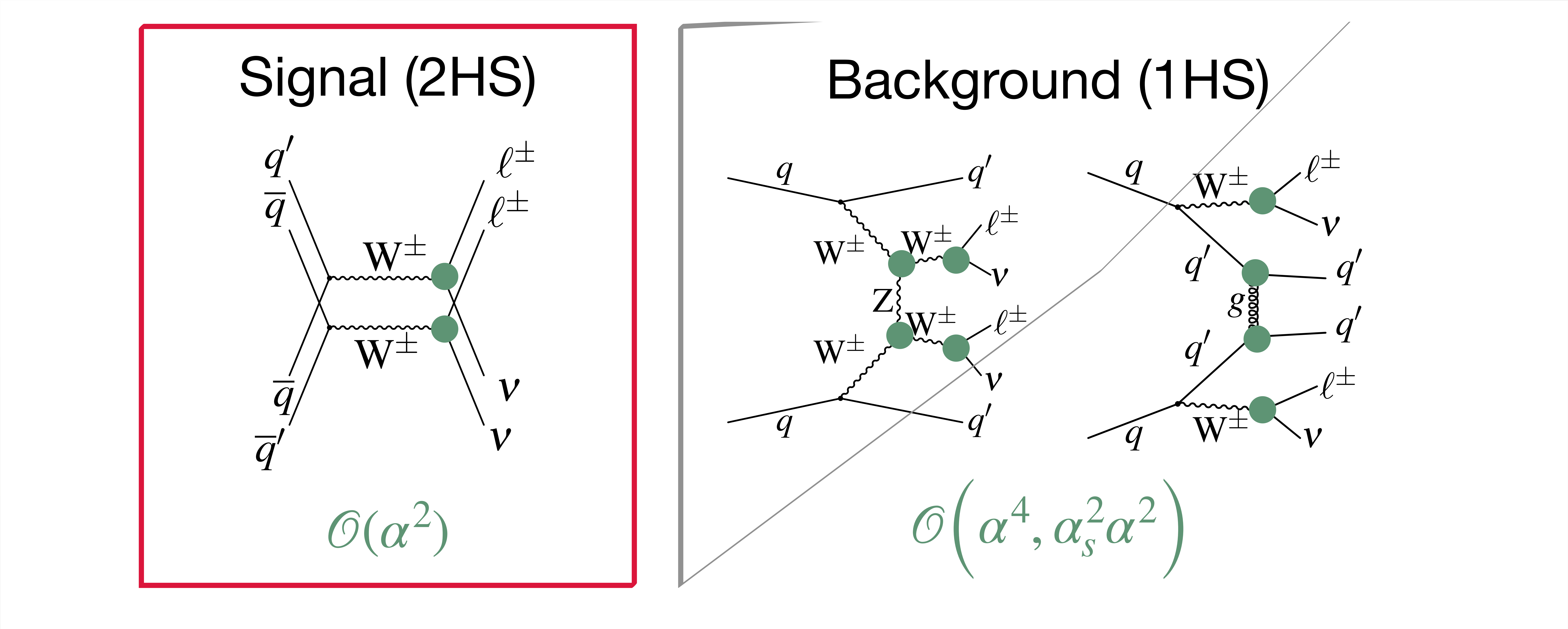




[ATLAS 1301.6872]

Strategy B

Identify processes where **1HS mechanism is suppressed** (e.g. small couplings)



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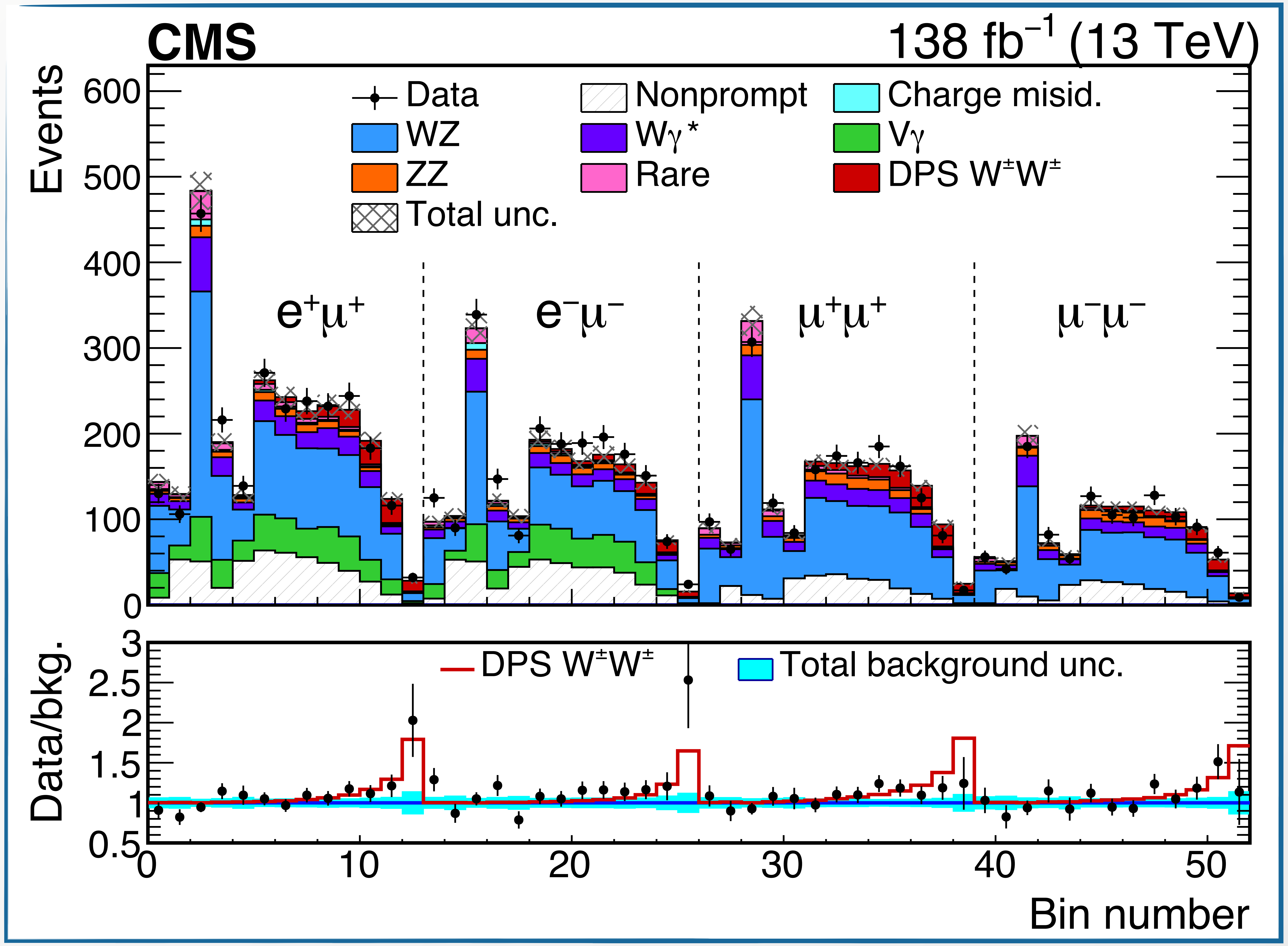
Avoid radiation issue: same-sign *WW*

- Here $W^{\pm}W^{\pm} \to$ same-sign leptons, CMS 2206.02681
- many other backgrounds: need for BDT makes it difficult to study MPI physics
- 6.2σ observation with full Run 2 dataset

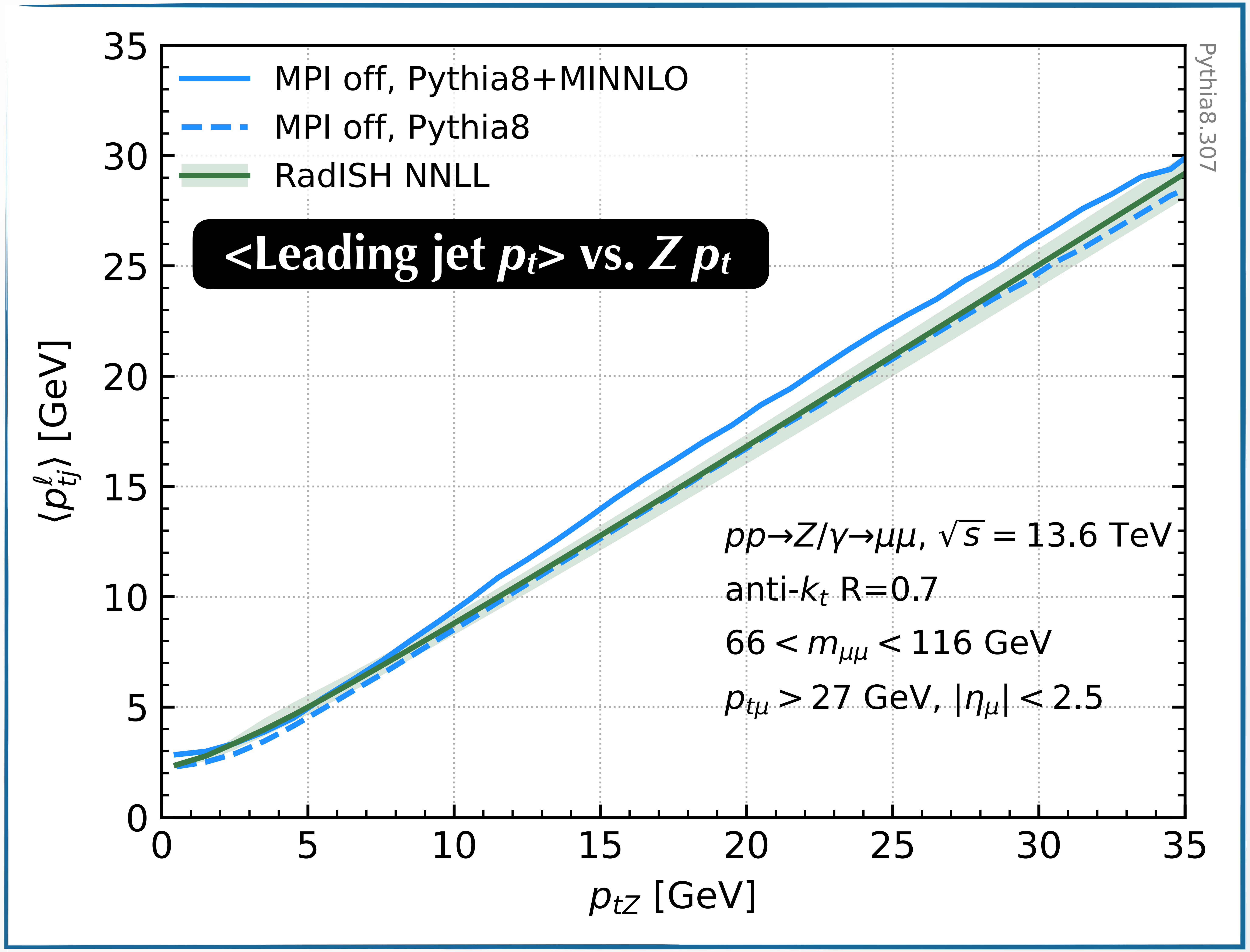
See also triple J/ψ production
[Shao, Zang 2019][CMS 2023]



[CMS 2206.02681]

Alternative route: can we study MPI in Z scattering?

[PRL 132 (2024) 4, 041901 Andersen, Monni, LR, Salam, Soto-Ontoso]



- Consider process with **MPI simulation turned off** (i.e. just 1HS)
- Look at avg. p_t of leading jet (p_{tj}^{ℓ}) as a function of Z p_t (p_{tZ})

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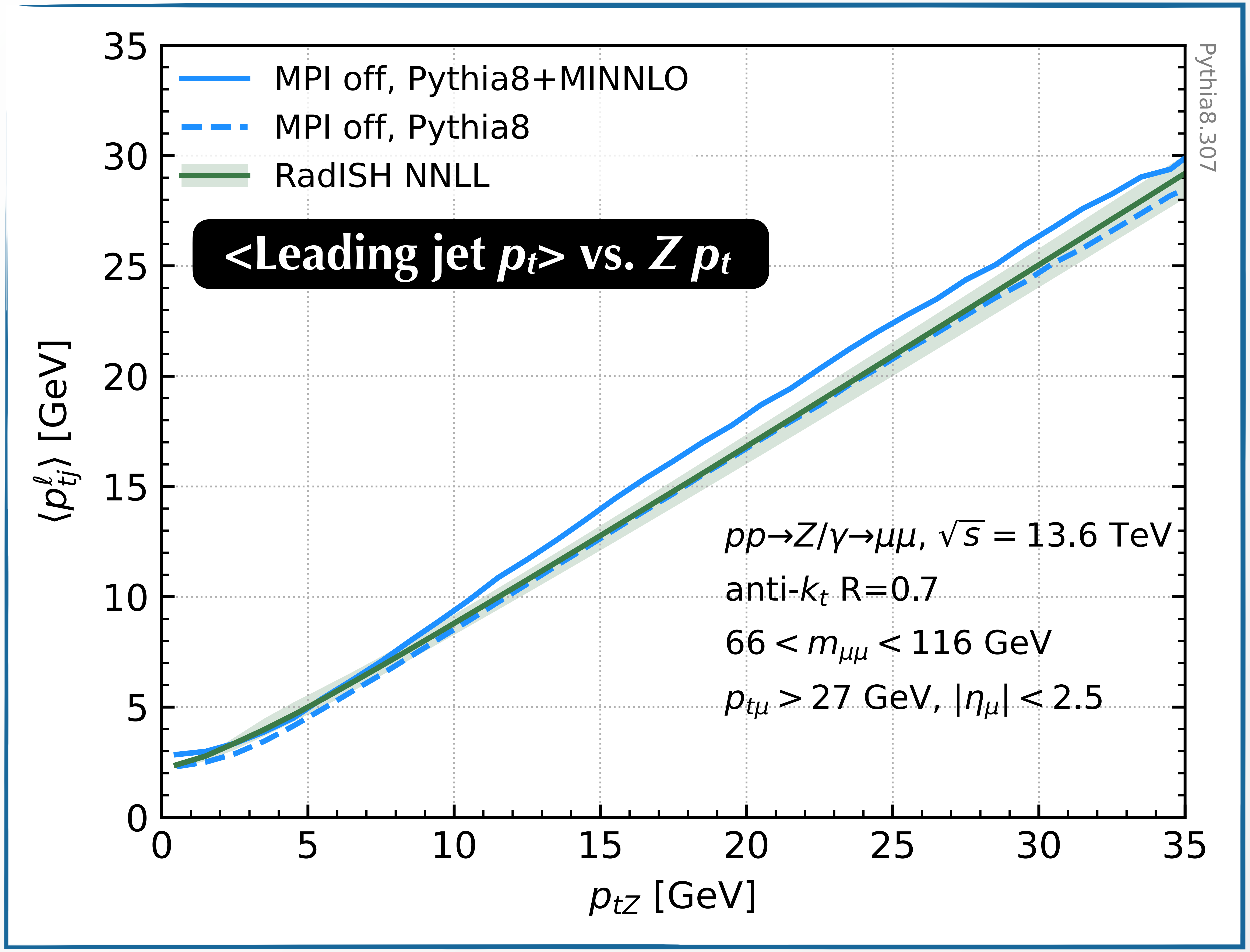

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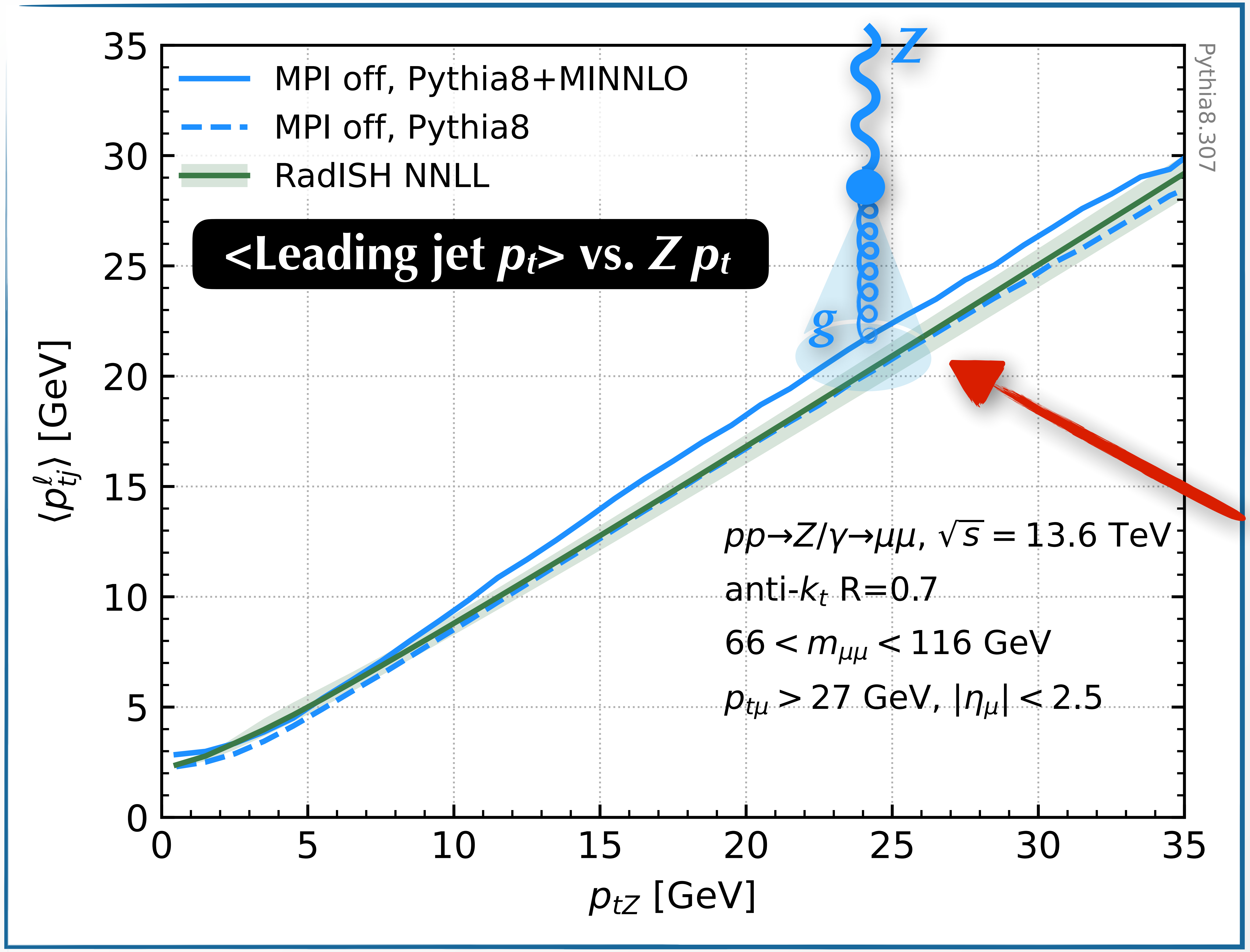

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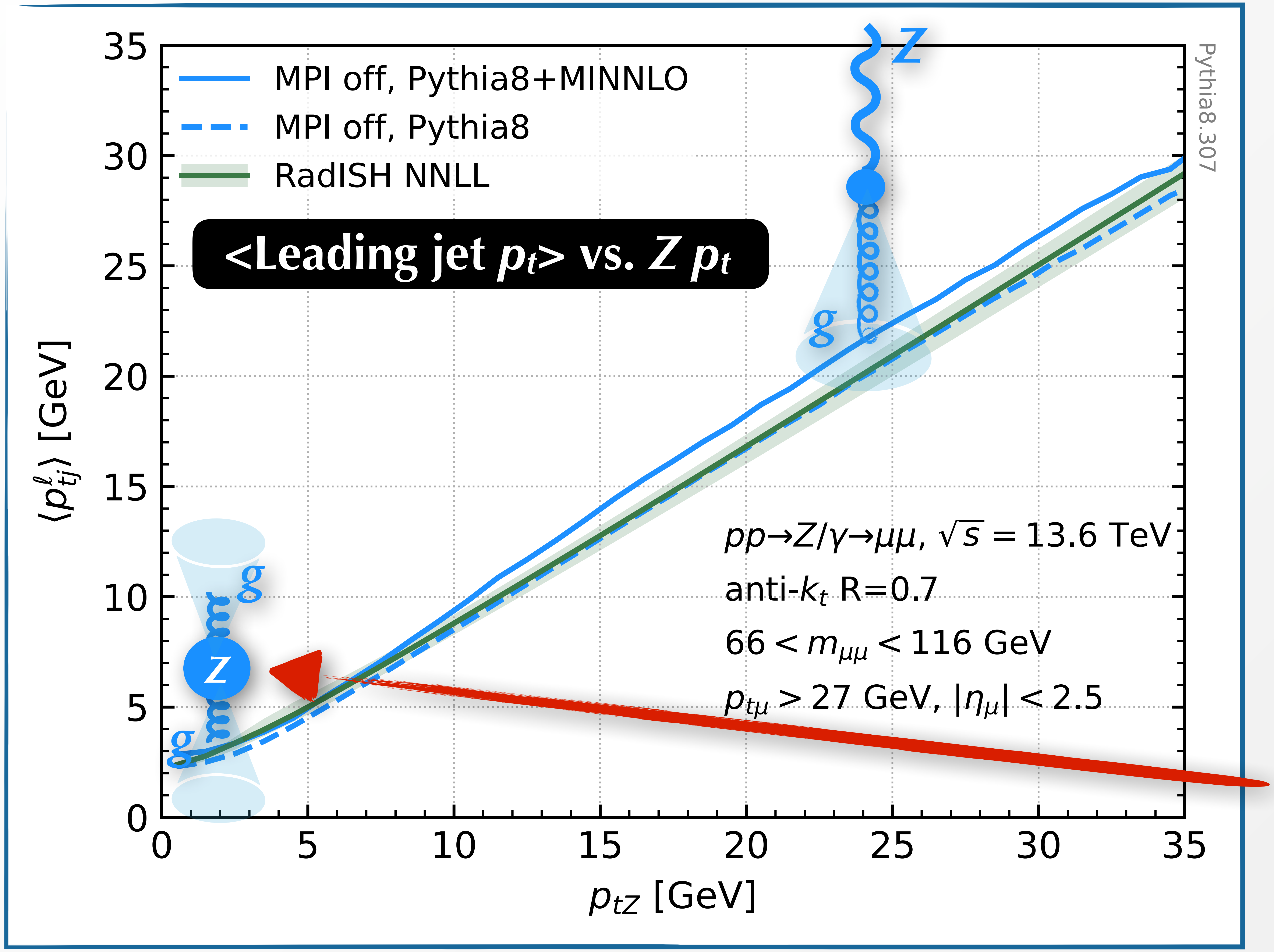

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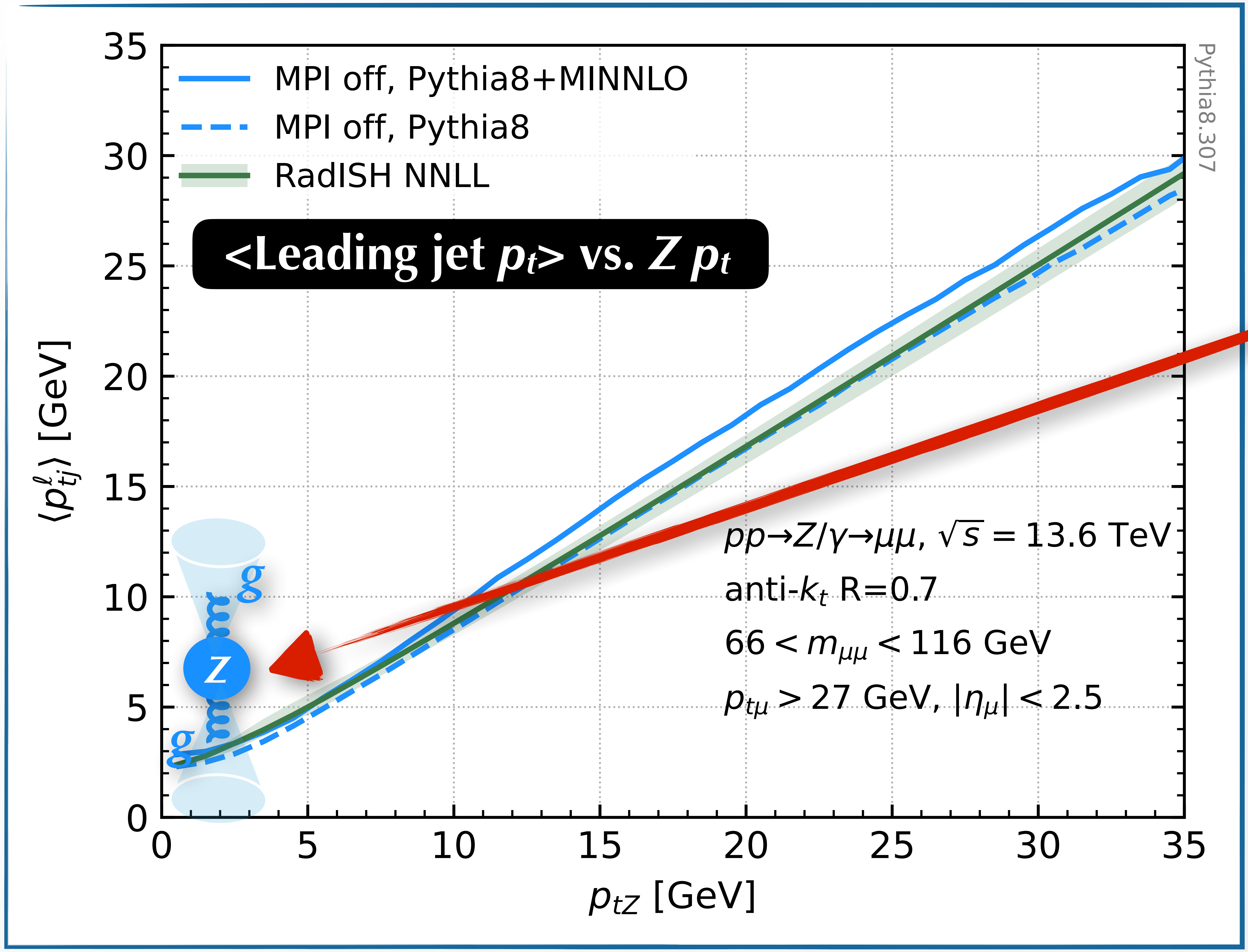
- **Most of p_{tZ} range:** almost perfect linear correlation, since **leading jet balances p_{tZ}**
- **For $p_{tZ} \to 0$:** $\langle p_{tj}^{\ell} \rangle$ saturates at about 2–3 GeV: **two (or more) soft jets balance each other**

Alternative route: can we study MPI in *Z* scattering?



Average leading jet p_T can be calculated from **resummation** in the limit $p_{tZ} \to 0$ [Monni, LR, Torrielli '19]

$$\langle p_{tj}^{\ell}\rangle_{p_{tZ}\to 0} \sim \Lambda_{\rm QCD}\left(\frac{m_Z}{\Lambda_{\rm QCD}}\right)^{\kappa \ln\frac{2+\kappa}{1+\kappa}}$$

$\sim 2-3\,\text{GeV}$

$$\kappa = \frac{2C_F}{\pi\beta_0}$$

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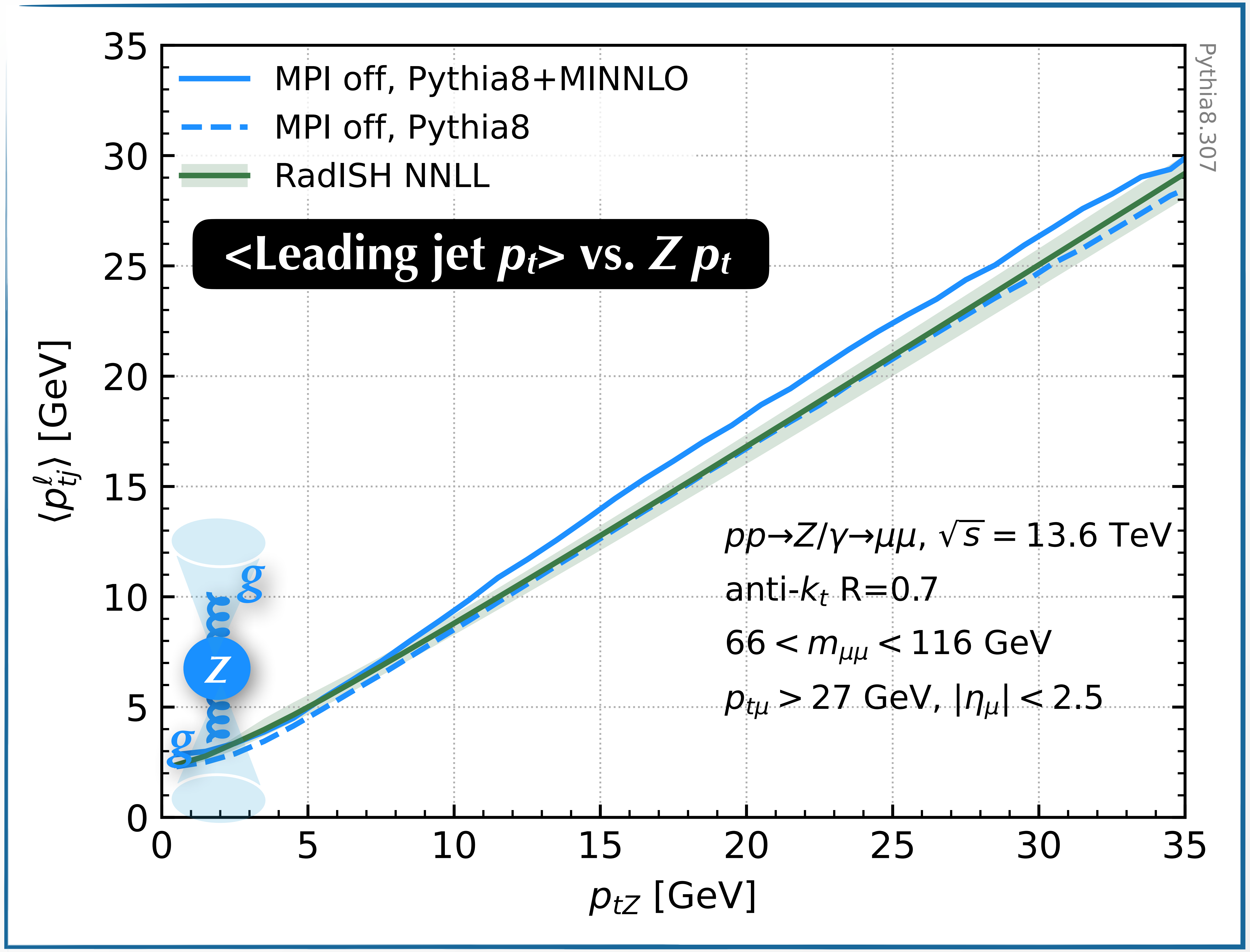

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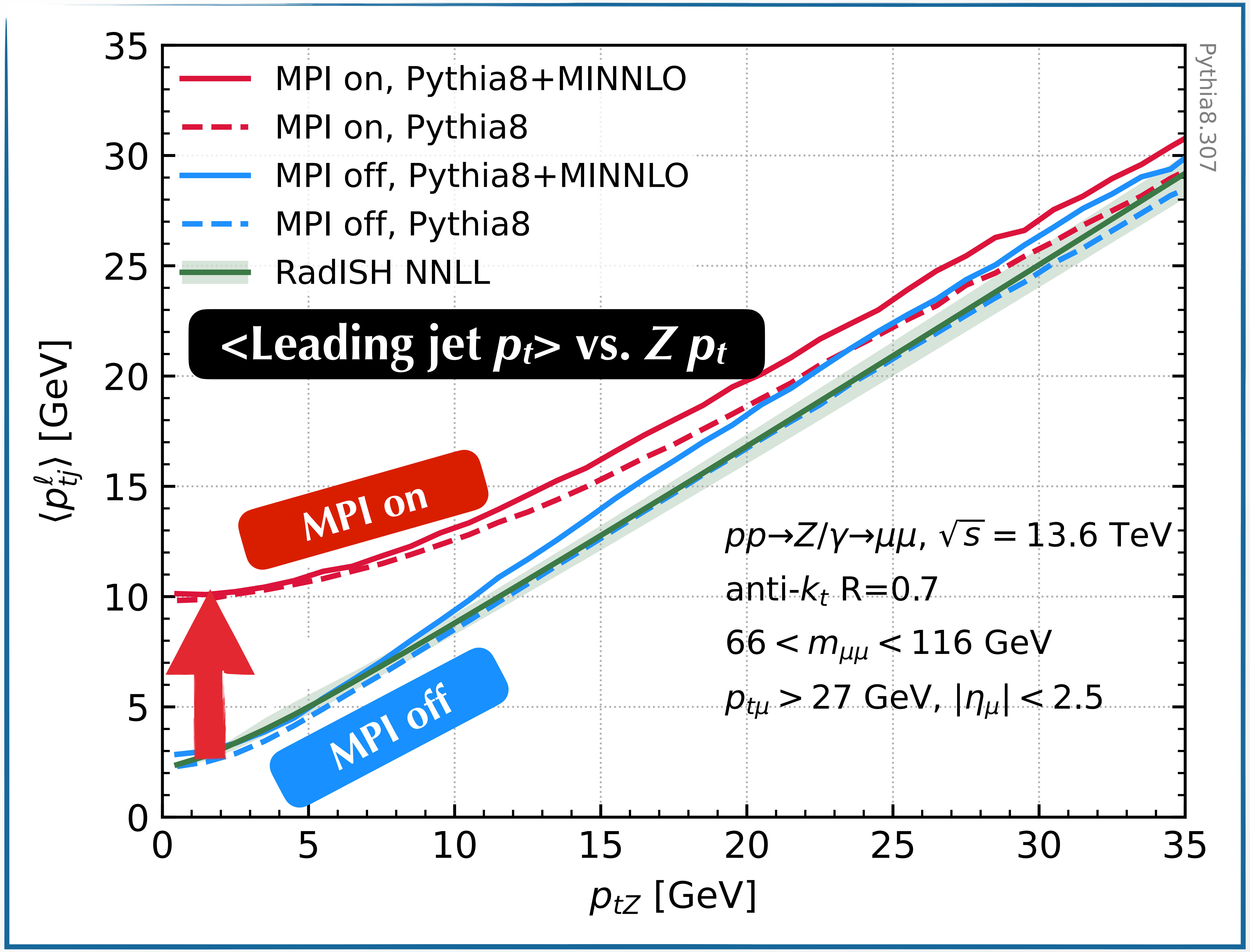
$\sim 2-3\,\mathrm{GeV}$ $\qquad \kappa = \frac{2C_F}{\pi\beta_0}$

By **constraining** p_{tZ} we can **forbid most radiation** above this characteristic 2–3 GeV scale

[classic Parisi-Petronzio '79]

Alternative route: can we study MPI in Z scattering?



What happens when turning on MPI?

- for $p_{tZ} \to 0$, leading jet p_t is now ~10 GeV instead of 2–3 GeV
- Why? Because there is almost always an MPI jet that is much harder than the soft jets from Z-process cfr. current models in Pythia/Herwig/Sherpa which simulate MPI as semi-hard scatterings

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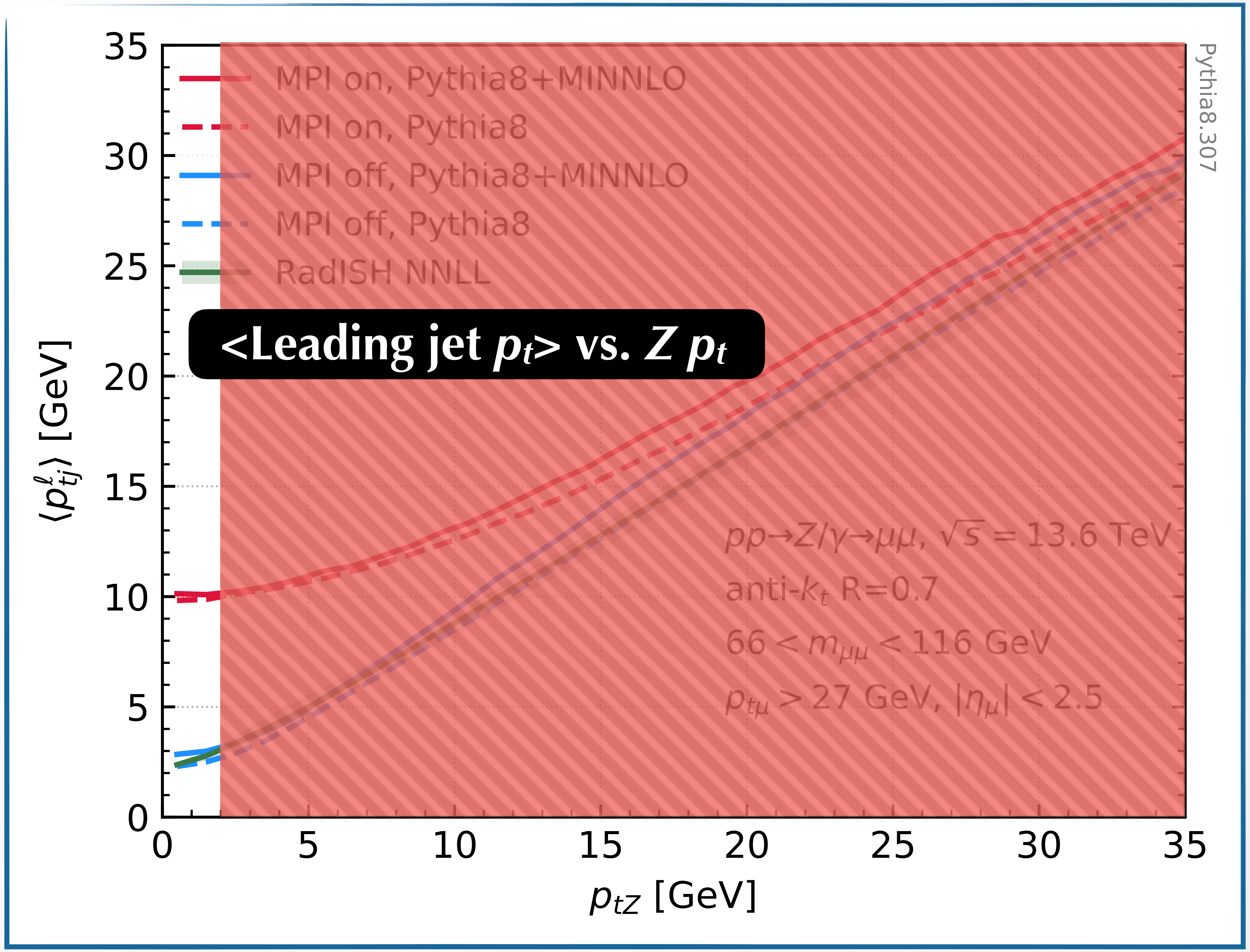

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Suggests we should study MPI with **help of a tight cut on p_{tZ}**

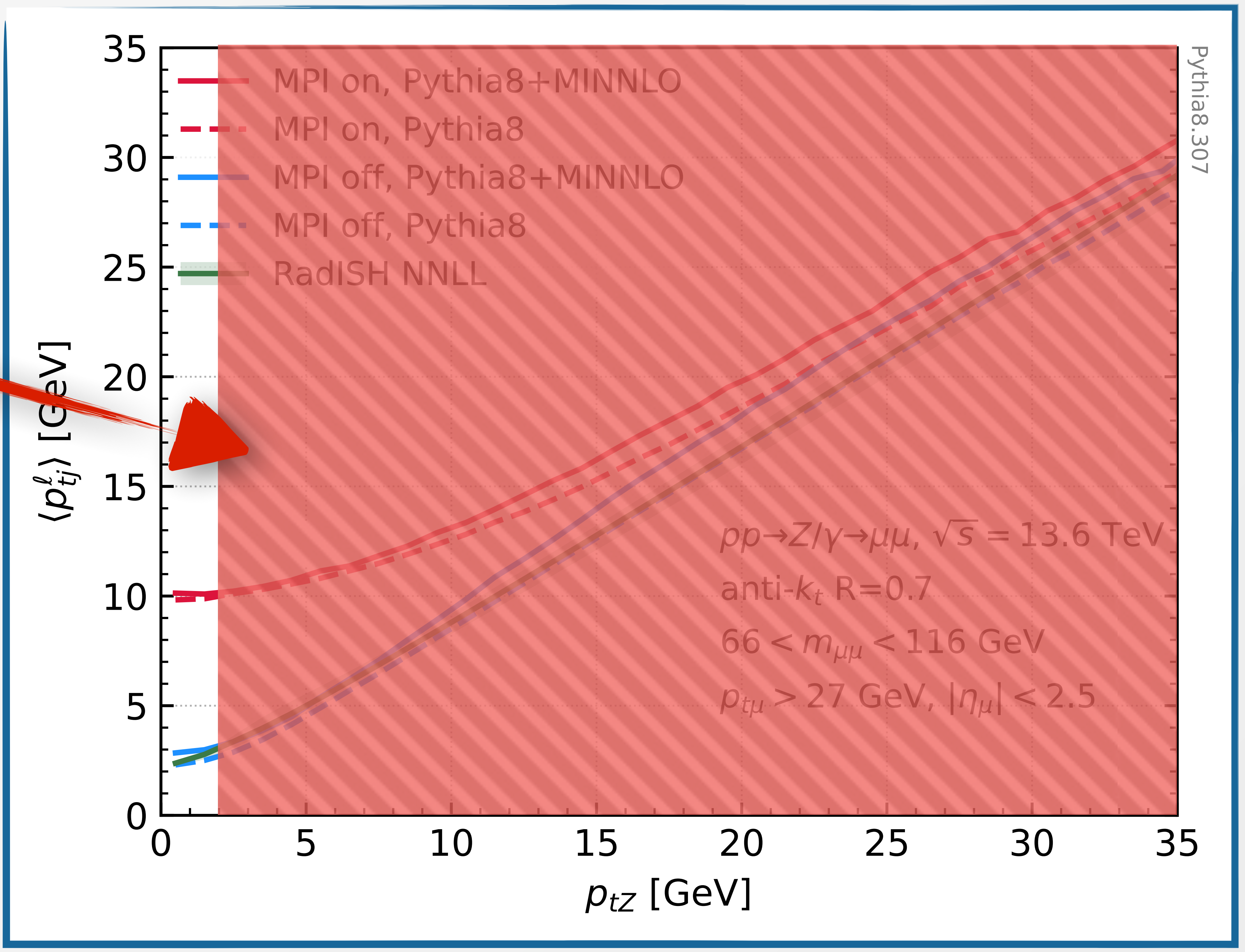
Our study: establish what cut to use, explore opportunities that open up

Need **balance** between

- **maximising statistics** (favours loose cut on Z)
- **minimising radiation** from Z hard system (favours tight cut on Z)

From $\langle p_{tj}^{\ell} \rangle$ vs. p_{tZ} plot optimum requirement is $p_{tZ} \lesssim 2\,\mathrm{GeV}$

- Smaller cut does not reduce scale of soft radiation from Z process and lower stats
- Higher cut increases average p_T of radiation
- Feasible given current experimental resolution



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$p_{tZ} < 2$ GeV cut retains 4 – 5% of Z-pole Drell-Yan events

For $Z \to \mu^+\mu^-$ residual cross section is ~40pb

~12 million events for 300fb^{-1} in Run 3

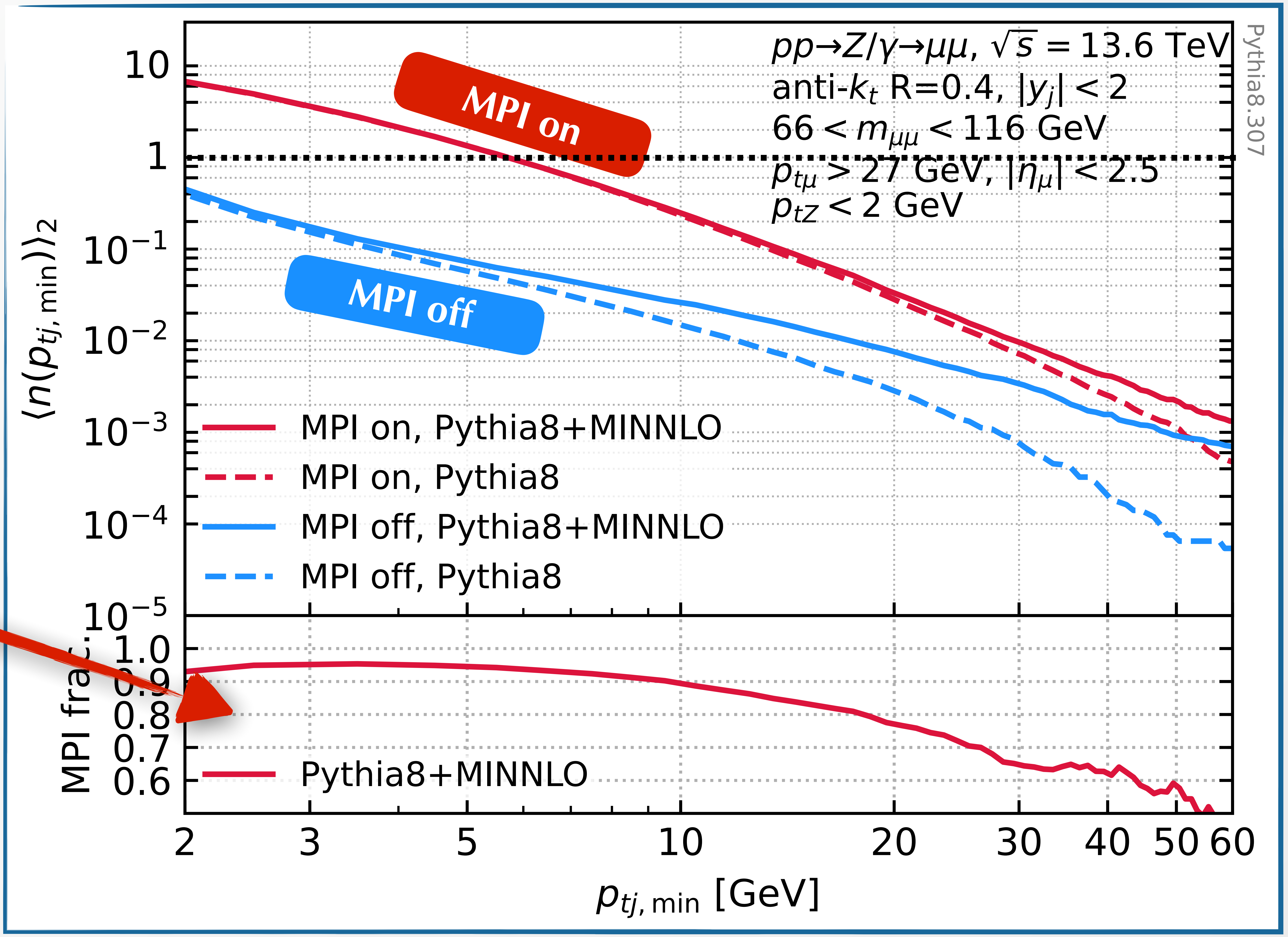
Simplest observable: cumulative inclusive jet spectrum for $p_{tZ} < 2$ GeV

Linear sum (for small jet radius) of

- **cumulative jet spectrum from 1HS process**
- **cumulative jet spectrum from any additional hard scatters (dominant!)**

MPI purity remains significant also at relatively high values of $p_{tj,\text{min}}$

$p_{tj,\text{min}}$	MPI purity
10 GeV	90%
20 GeV	78%
40 GeV	60%



Connection with "pocket formula" (σ effective)

$\langle n(p_{tj,\min})\rangle_{C_Z}$ = average number of jets above $p_{tj,\min}$ for a given cut C_Z on p_{tZ}

$$\langle n(p_{tj,\min})\rangle_{C_Z} = \frac{1}{\sigma(p_{tZ} < C_Z)} \int_{p_{tj,\min}} dp_{tj} \frac{d\sigma_{\text{jet}}(p_{tZ} < C_Z)}{dp_{tj}}$$

Pure MPI part extracted by subtracting no-MPI calculation (thanks to linearity)

$$\langle n(p_{tj,\min})\rangle_{C_Z}^{\text{pure-MPI}} \equiv \langle n(p_{tj,\min})\rangle_{C_Z} - \langle n(p_{tj,\min})\rangle_{C_Z}^{\text{no-MPI}}$$

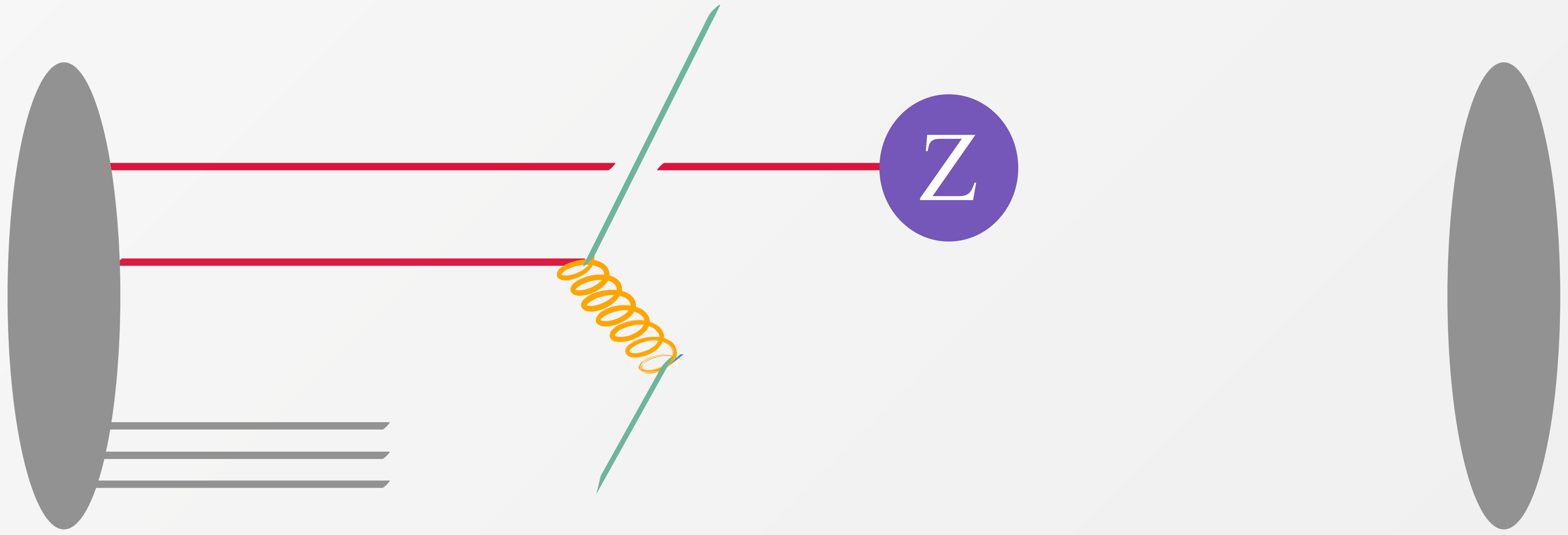
In σ_{eff} picture, pure-MPI part can be connected with jet rate in min-bias events (i.e. no Z)

NB: can be directly measured on data, identical systematics (e.g. with charge-track jets at low p_{tj})

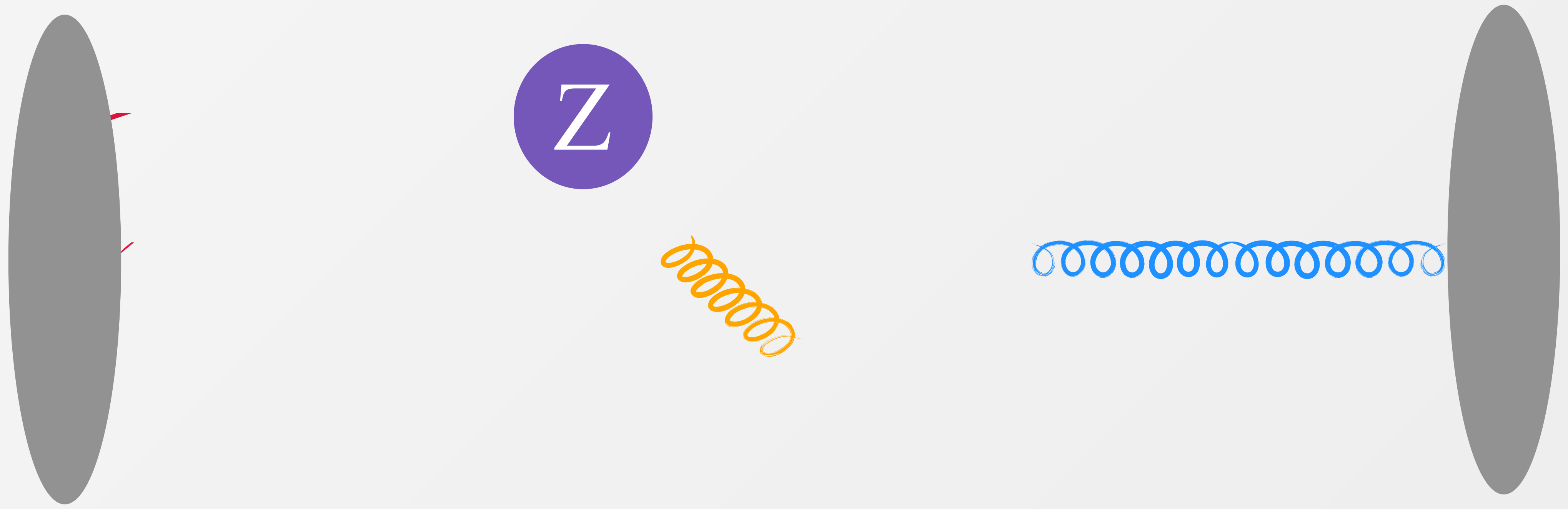
$$\langle n(p_{tj,\min})\rangle_{C_Z}^{\text{pure-MPI}} \simeq \frac{1}{\sigma_{\text{eff}}} \int_{p_{tj,\min}} dp_{tj} \frac{d\sigma_{\text{jet}}^{\text{min-bias}}}{dp_{tj}}$$

Beyond the pocket formula

- Pocket formula is based on independent scatterings, with some effective transverse size over which partons are spread



- But we expect some partons to come from splitting of common parents,“**perturbative interconnection**”
- Such splittings tend to give more p_t to the partons → higher p_{tZ}



- **We should see an change of MPI jet rate if we relax the p_{tZ} cut**

Interconnection studies: Diehl & Schafer 1102.3081; Blok, Dokshitzer, Frankfurt & Strikman 1106.5533; Diehl, Gaunt & Schönwald, 1702.06486

Can one see effect of perturbative interconnection?

Measure cumulative jet rate with two p_{tZ} cuts:

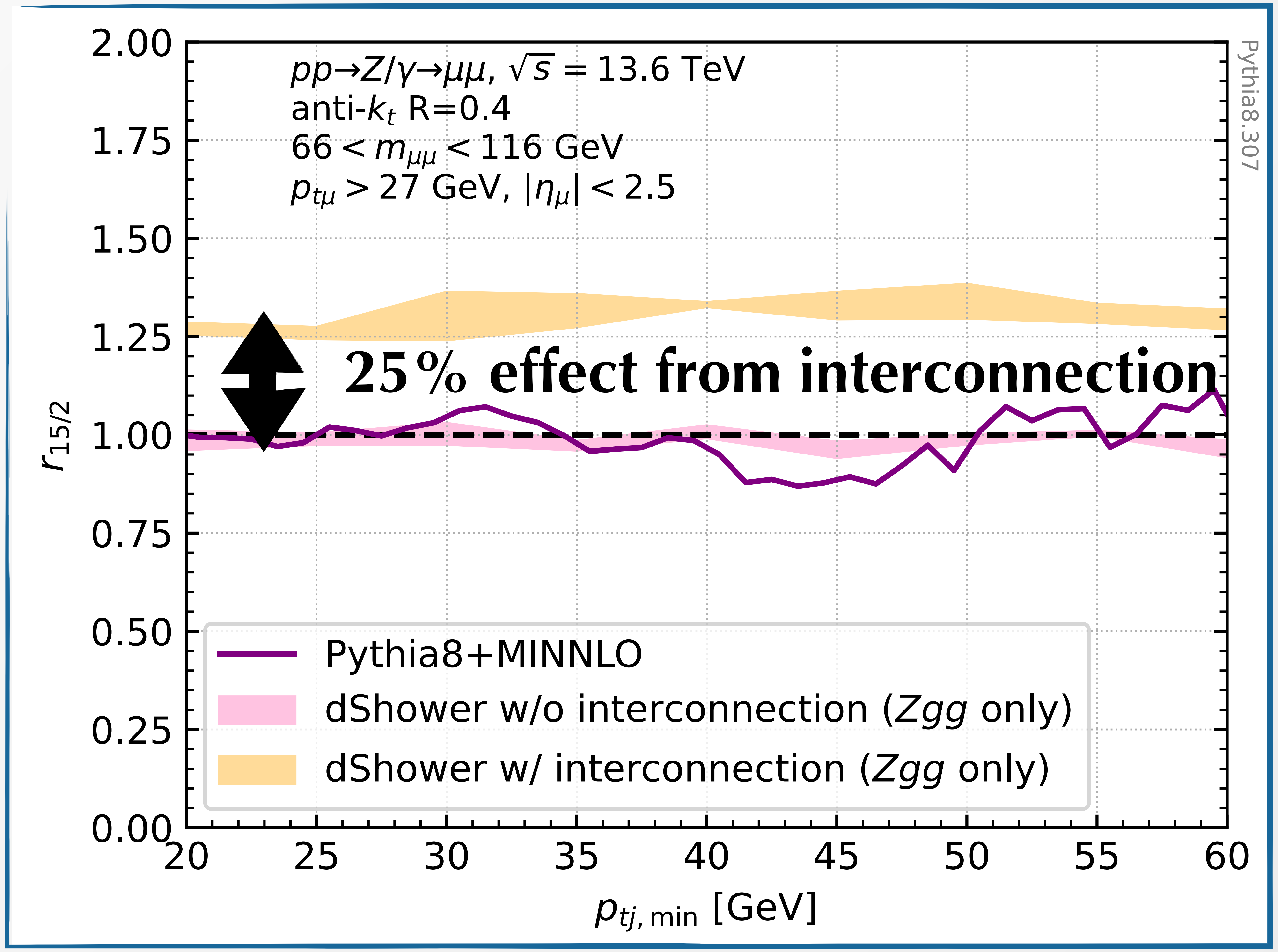
- tight (2 GeV)
- loose (15 GeV)

Take ratio of pure-MPI jet rates

$$r_{15/2} = \frac{\langle n(p_{tj,\min})\rangle_{15}^{\text{pure-MPI}}}{\langle n(p_{tj,\min})\rangle_{2}^{\text{pure-MPI}}}$$

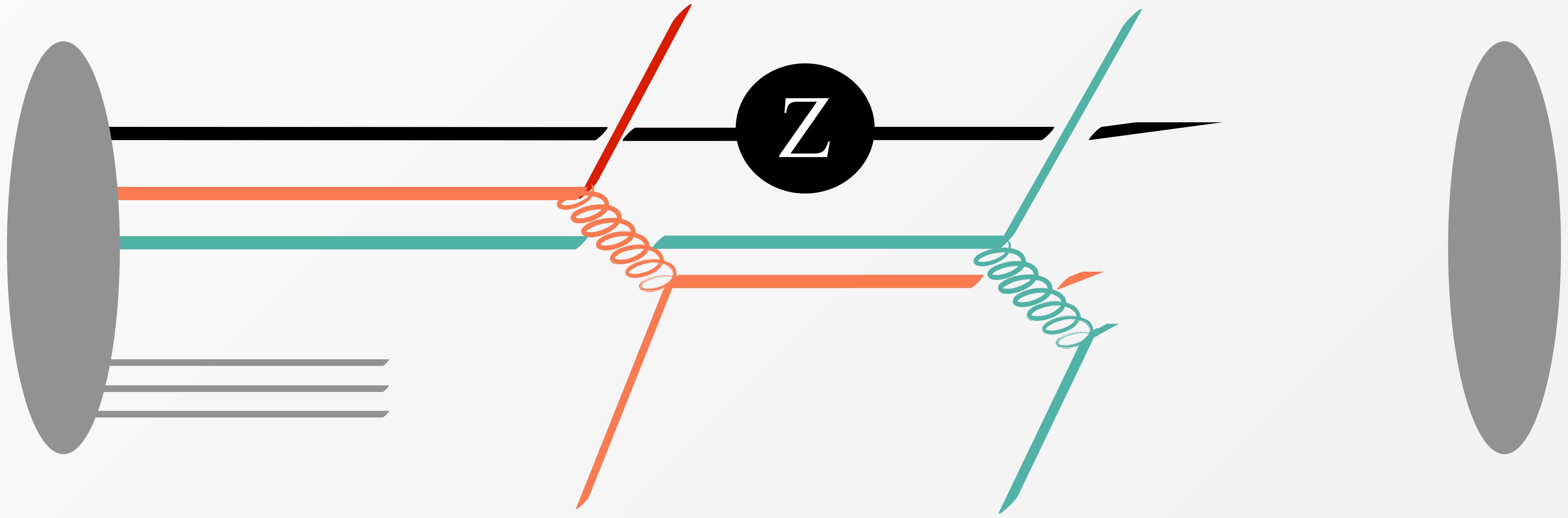
Compare to

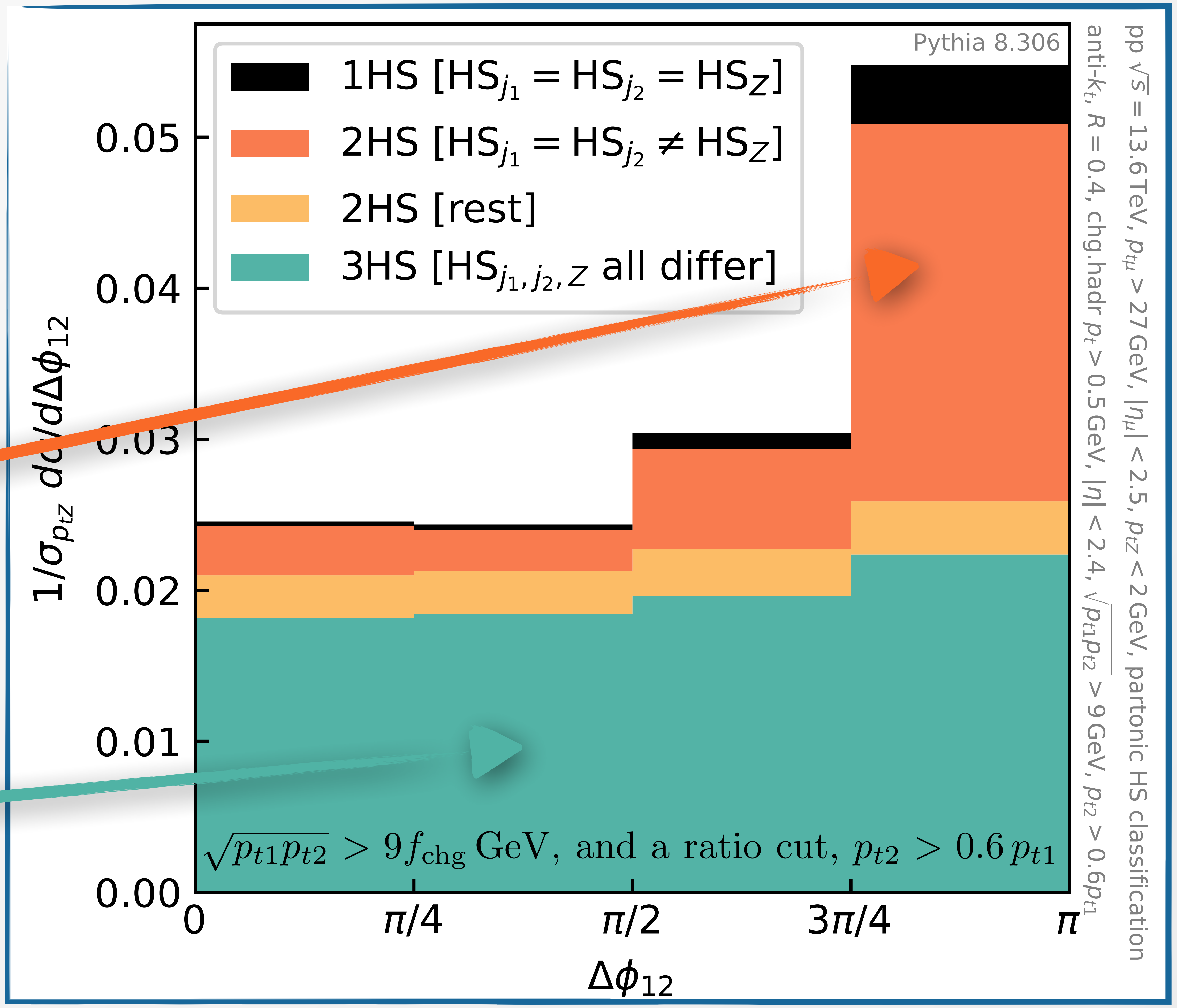
- **Pythia (+MiNNLO)**: no interconnection (expect $r = 1$)
- **dShower**: with option of interconnection
  [Cabouat, Gaunt, Ostrolenk, 1906.04669; Cabouat, Gaunt, 2008.01442]



Beyond 2HS

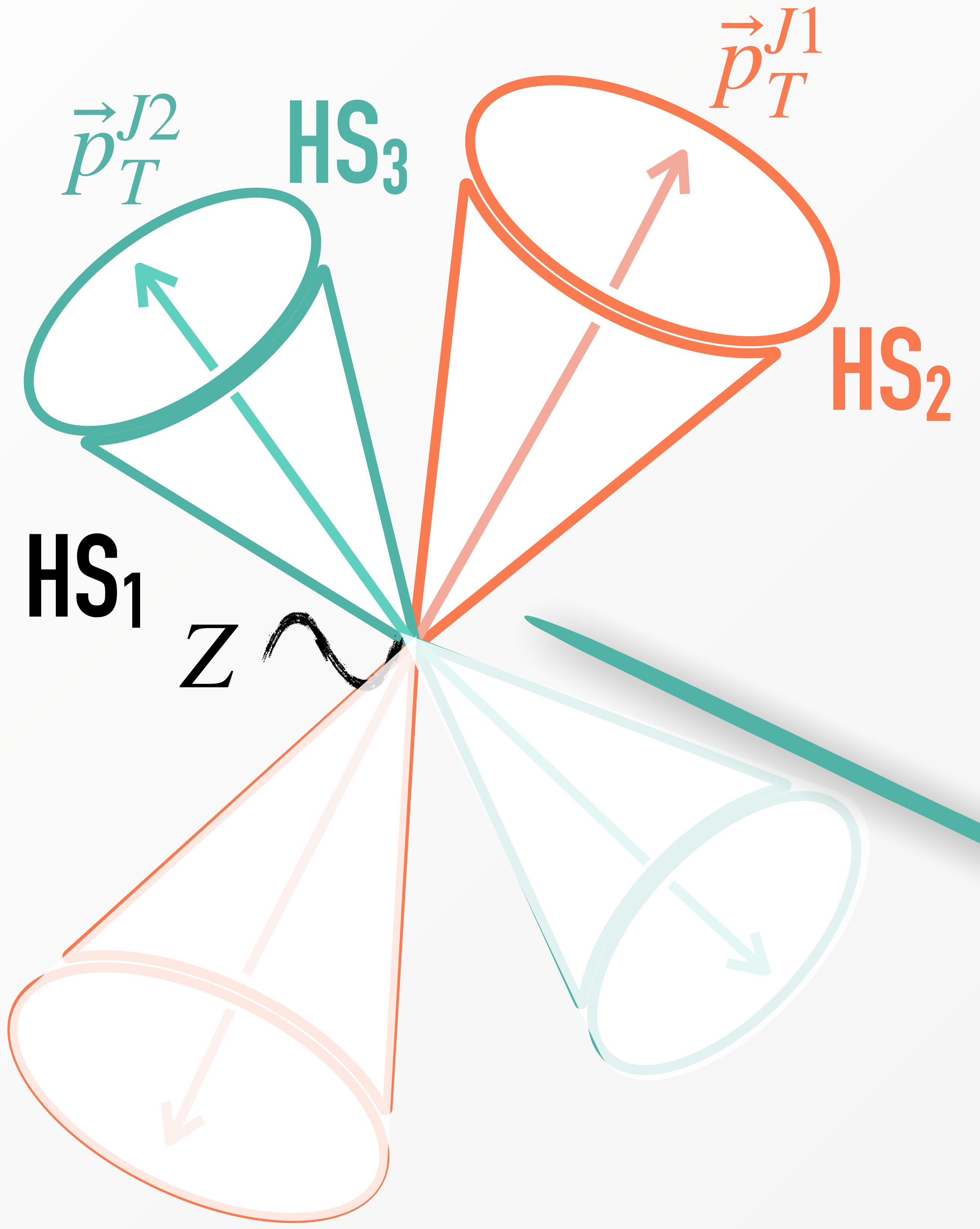
- Only measurements of 3HS are in J/ψ production, which is a difficult process to interpret even with just 1HS!
- Instead, put tight $p_{tZ} < 2\,\text{GeV}$ cut and look at $\Delta\phi$ between two leading charged-track jets, with low p_{tj} cuts (~ 5 GeV on charged-track sum)
- gives clear **2HS** peak at $|\Delta\phi| \simeq \pi$
- gives distribution ~independent of $|\Delta\phi|$, when the Z and the 2 jets each come from different hard scatters **(total of 3HS)**

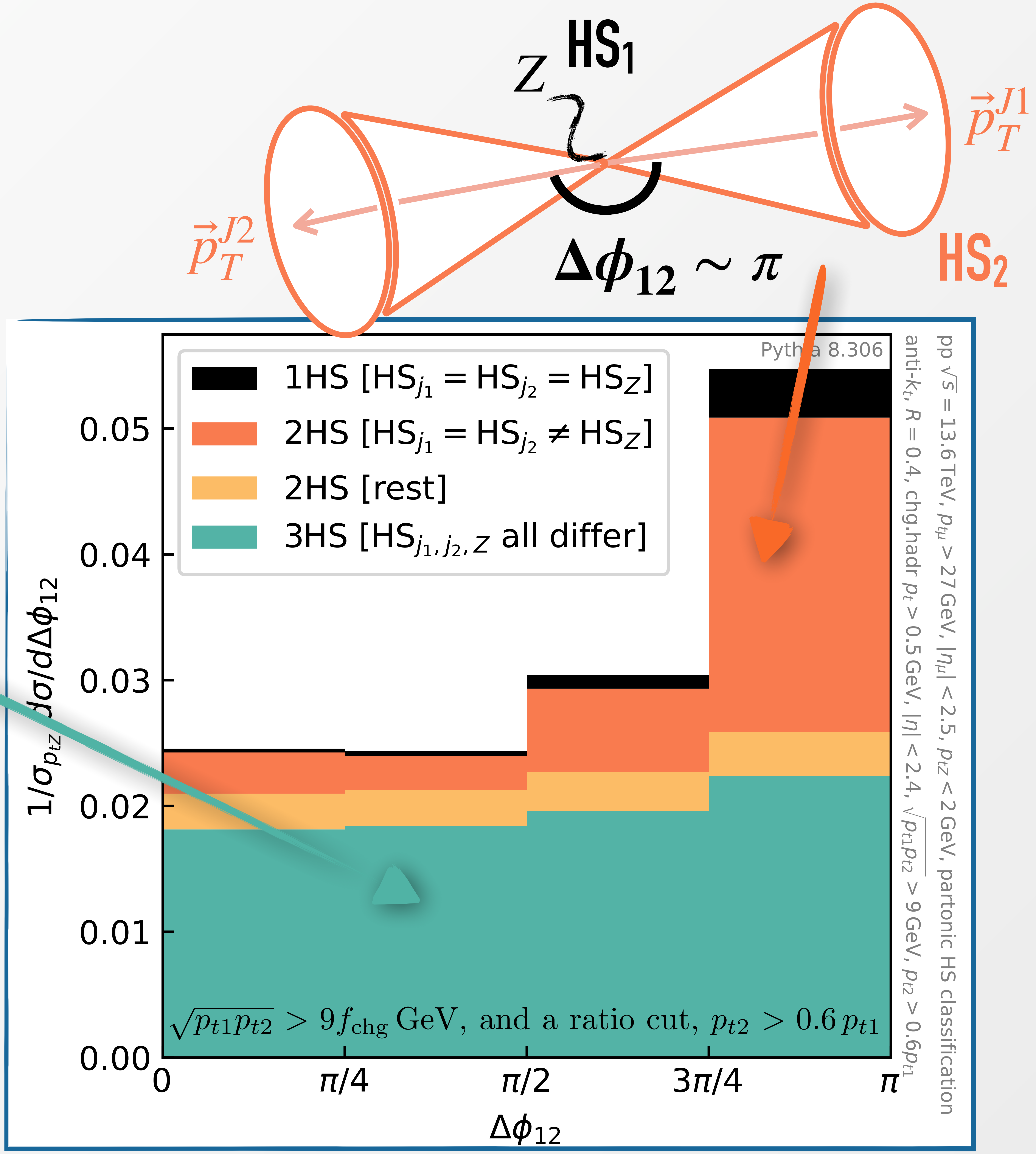




Beyond 2HS

$0 < \Delta\phi_{12} < \pi$





More challenging: repeating analysis examining $|\Delta\phi_{34}|$ to access **4HS contribution**

Conclusion

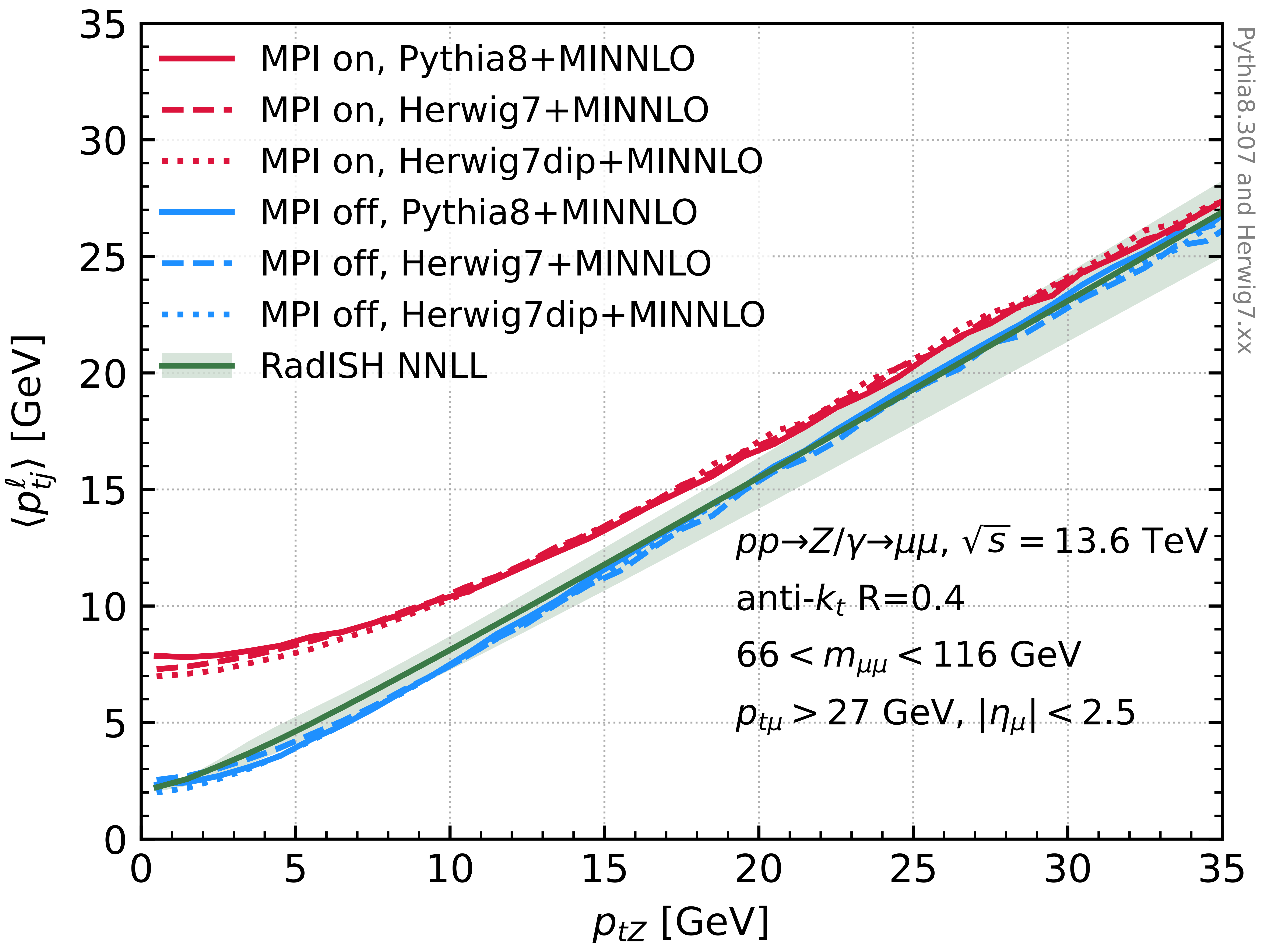
- MPI occur in almost all high-energy collisions at hadron colliders but their quantitative study remains challenging
- Modelling of (soft) MPI is an essential component of all major simulation tools
- Process with one (or more) hard extra interaction, 2HS/3HS etc, is interesting as a signal, and as a background to rare processes
- The insight gained by studying hard MPI may improve the modelling of soft MPI for MB/UE, since soft MPI models are conceived as extensions of the one used for harder MPI
- Study of Drell-Yan events with tight cut on p_{tZ} opens door to numerous new MPI studies (high-purity 2HS samples, interconnections, 3HS studies)

Overall

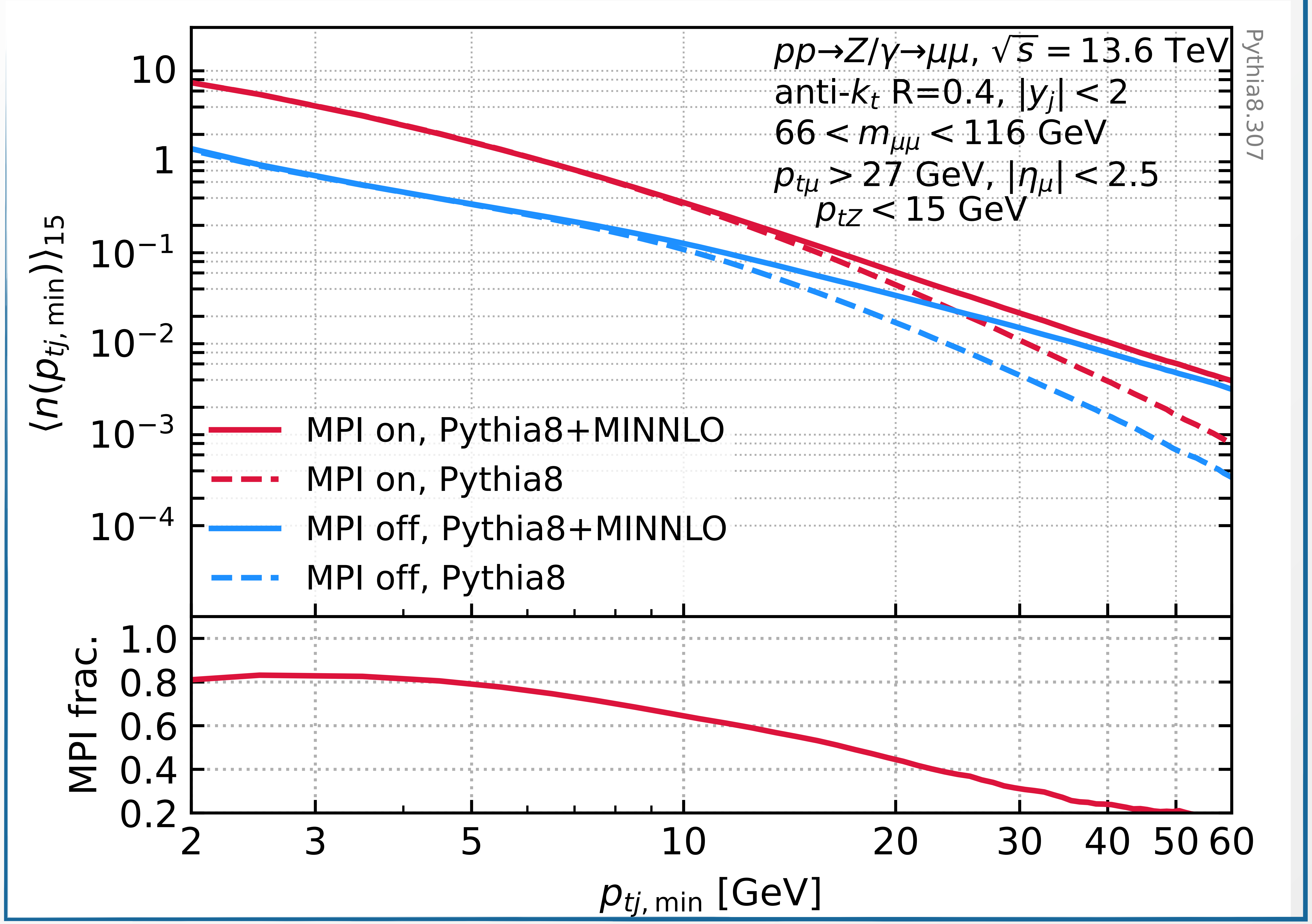
potential for significant impact on conceptual and quantitative understanding of MPI

Backup

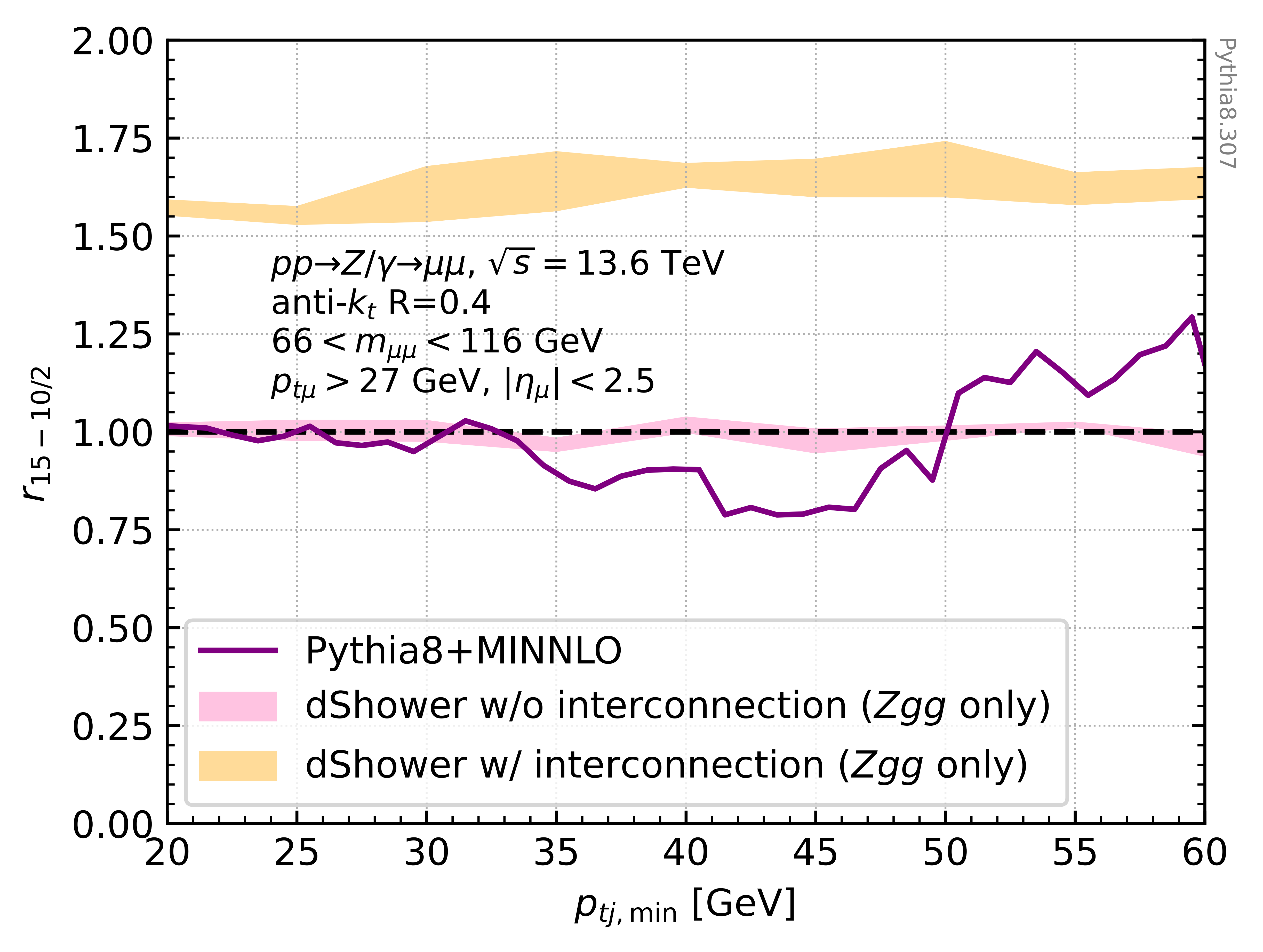
HERWIG results

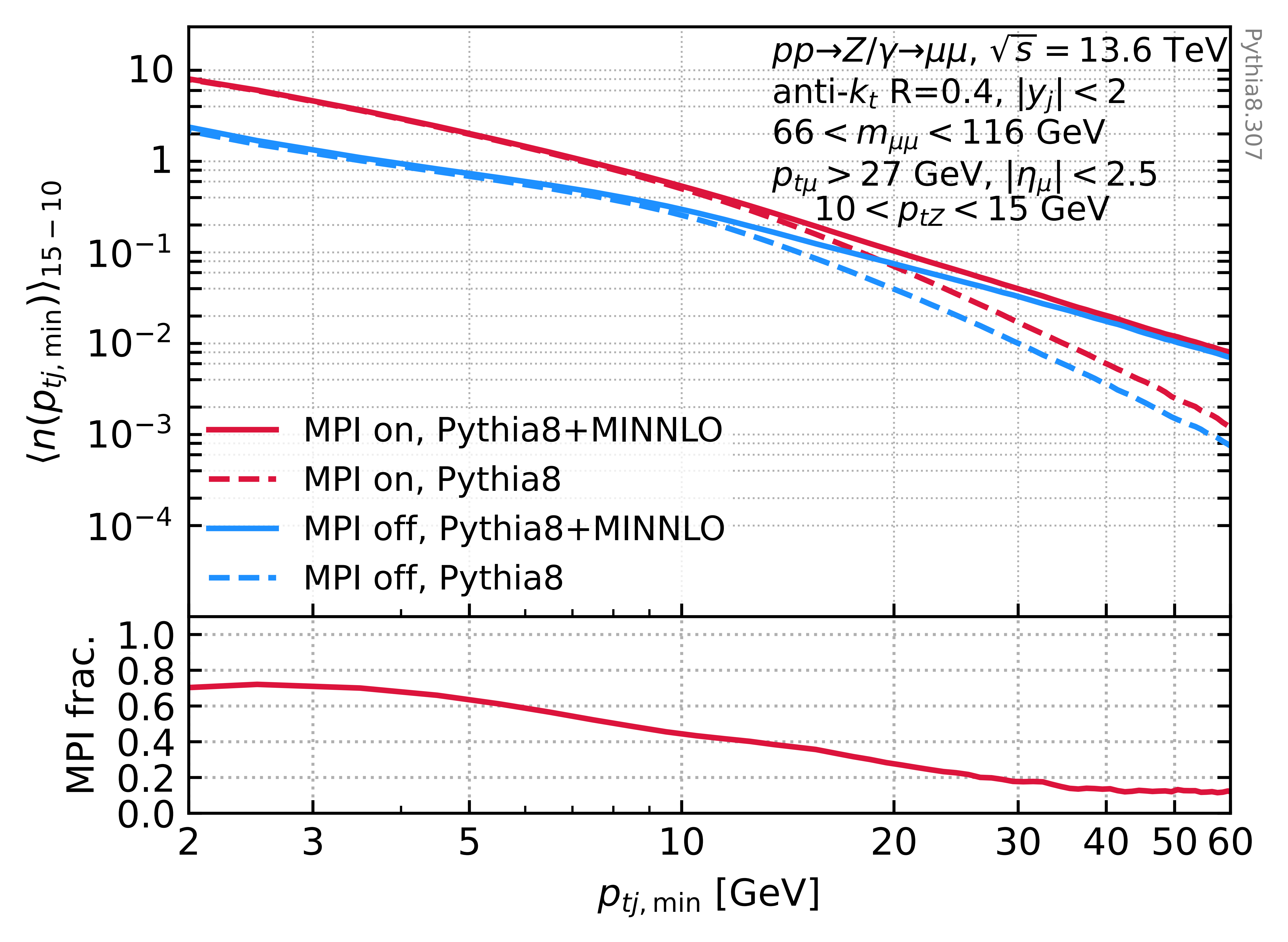


MPI purity with 15 GeV cut on p_{tZ}

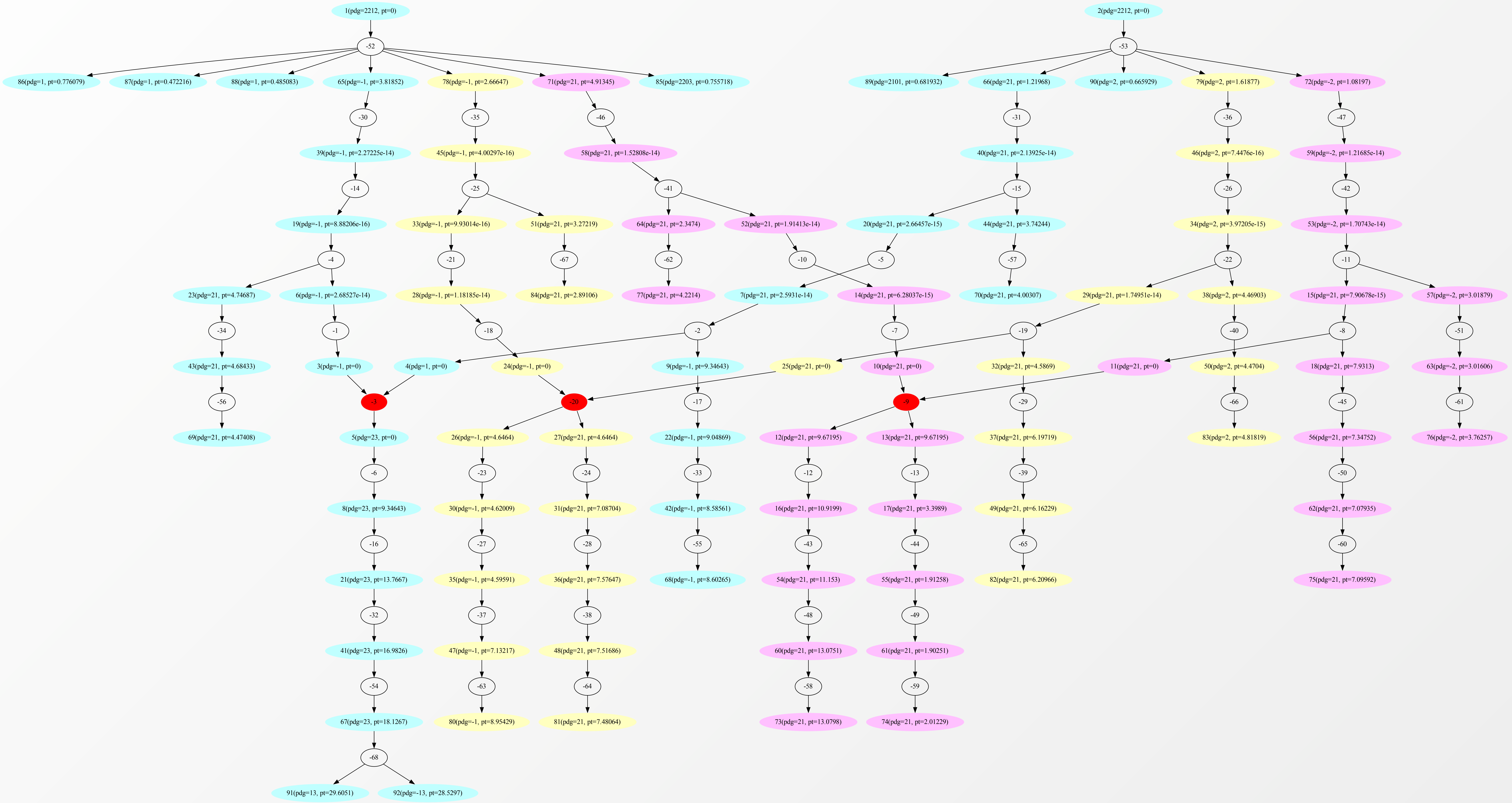


10 < p_{tZ} < 15 GeV for the loose sample: increases interconnection, reduces purity

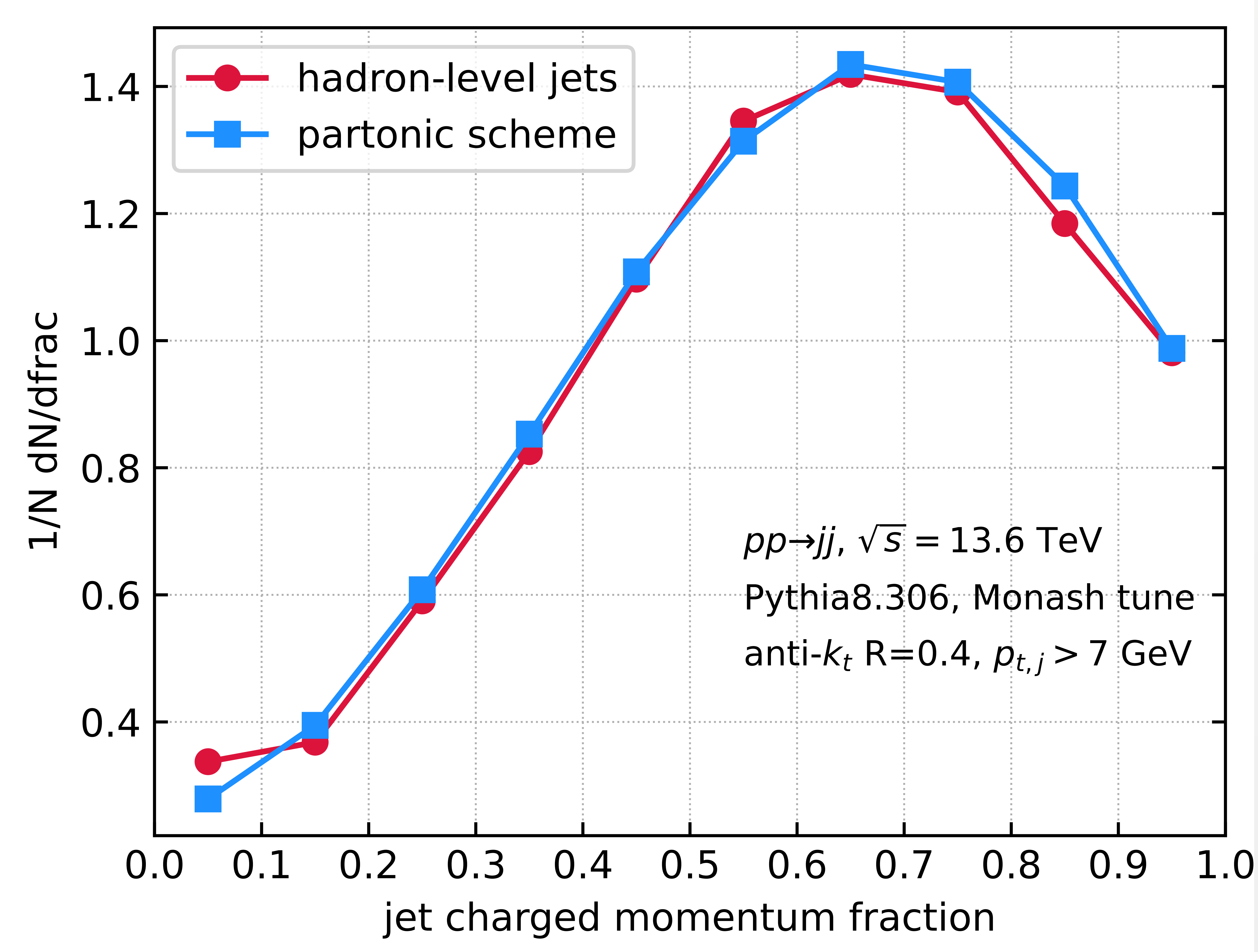




Extracting partonic hard-scattering classification from Pythia (via HepMC)



Validation of simple parton → charged hadron conversion for hard-scatter classification

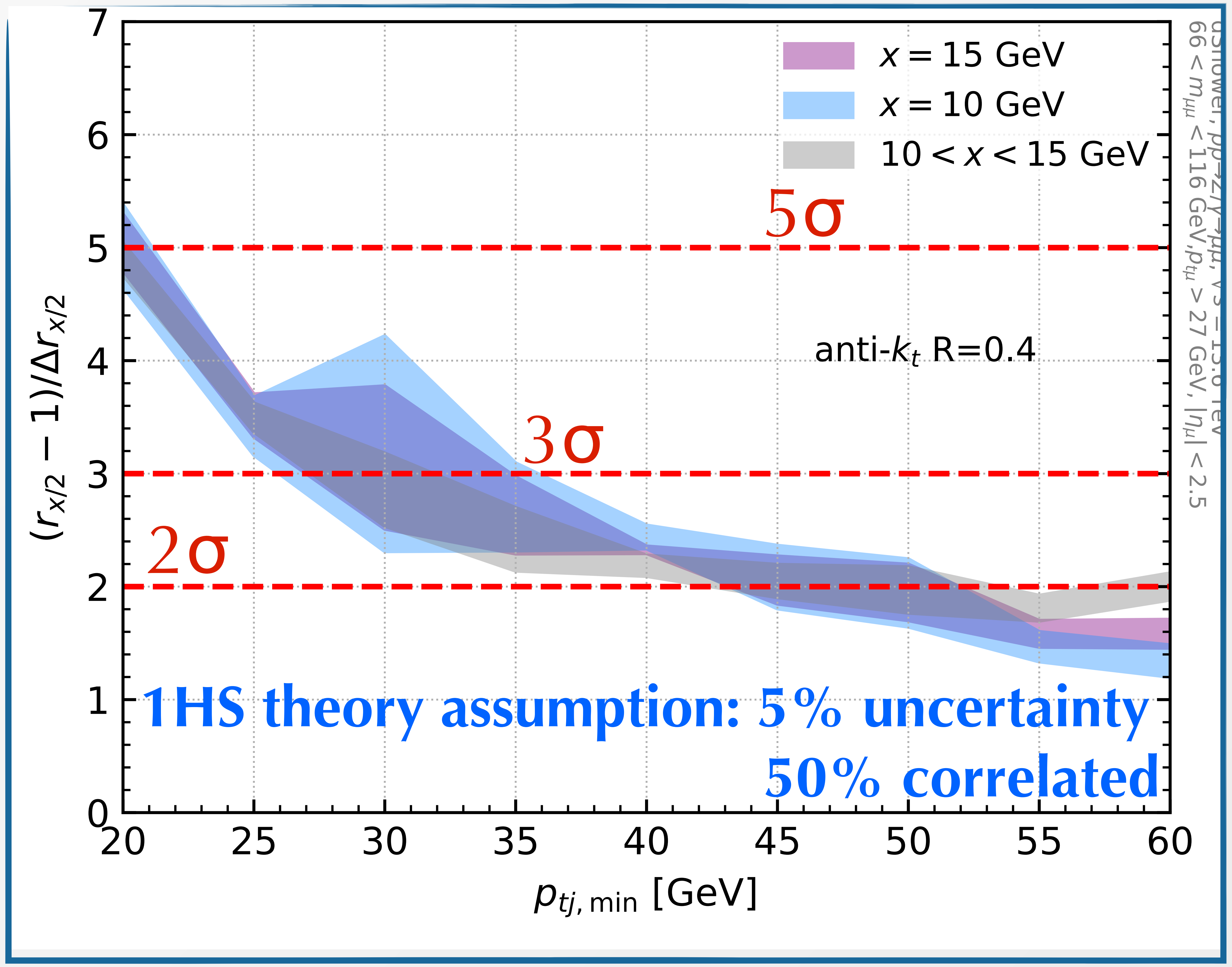


Interplay of significance with MPI purity

Significance of signal of perturbative interconnection in simulation, for dShower-sized effect vs. $p_{tj,\min}$ depends on assumptions for sizes of **theory uncertainties** on 1HS subtraction + **their correlation** between the two p_{tZ} cuts

- **Just barely feasible?**
- motivates NNLO (matched) *Z+2j* calculations to reduce current theory uncertainty (10-20%)

significance of signal of perturbative interconnection



Interplay of significance with MPI purity in different scenarios

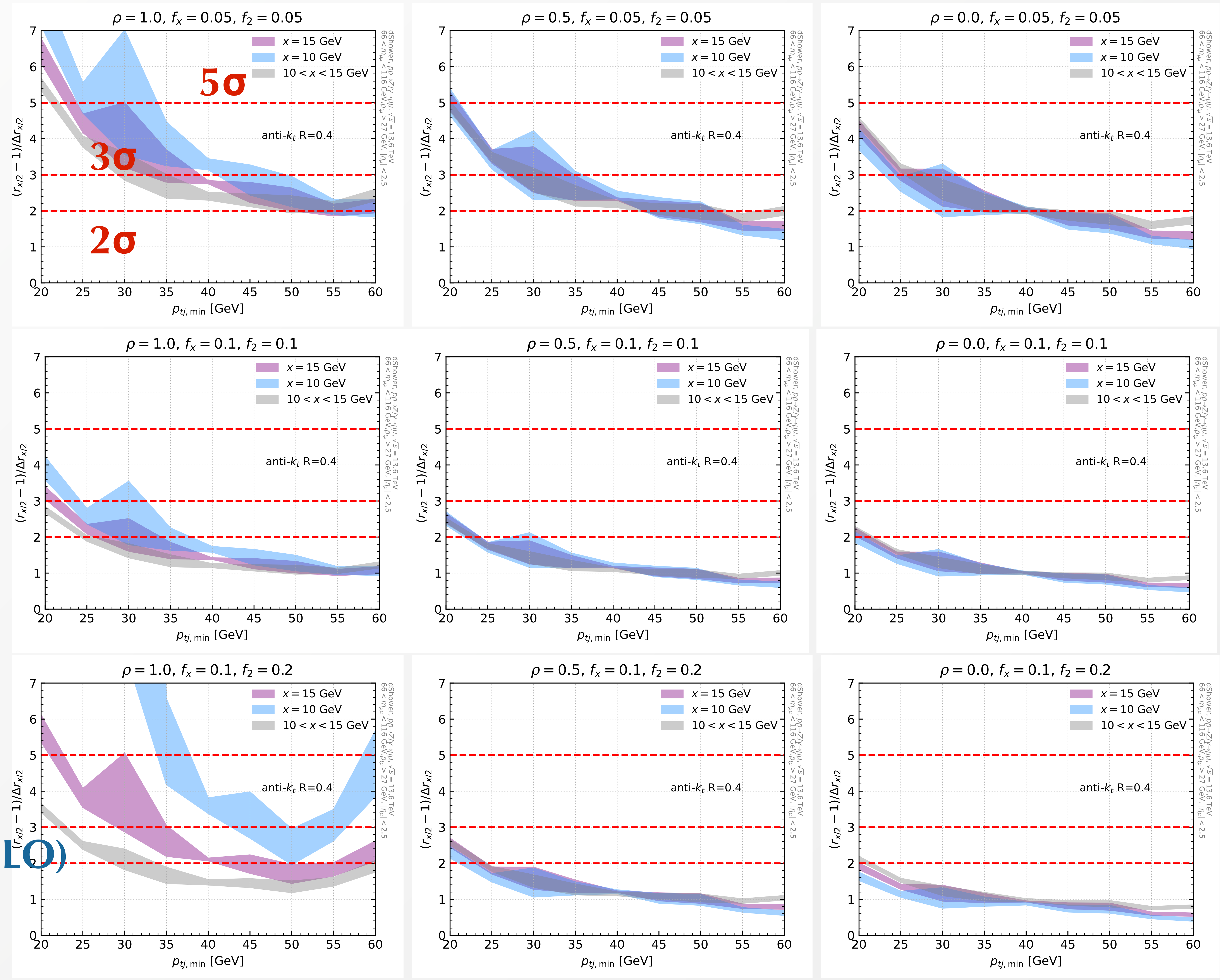
1HS Th. uncert. → 100% correlated | 50% correlated | uncorrelated

↓

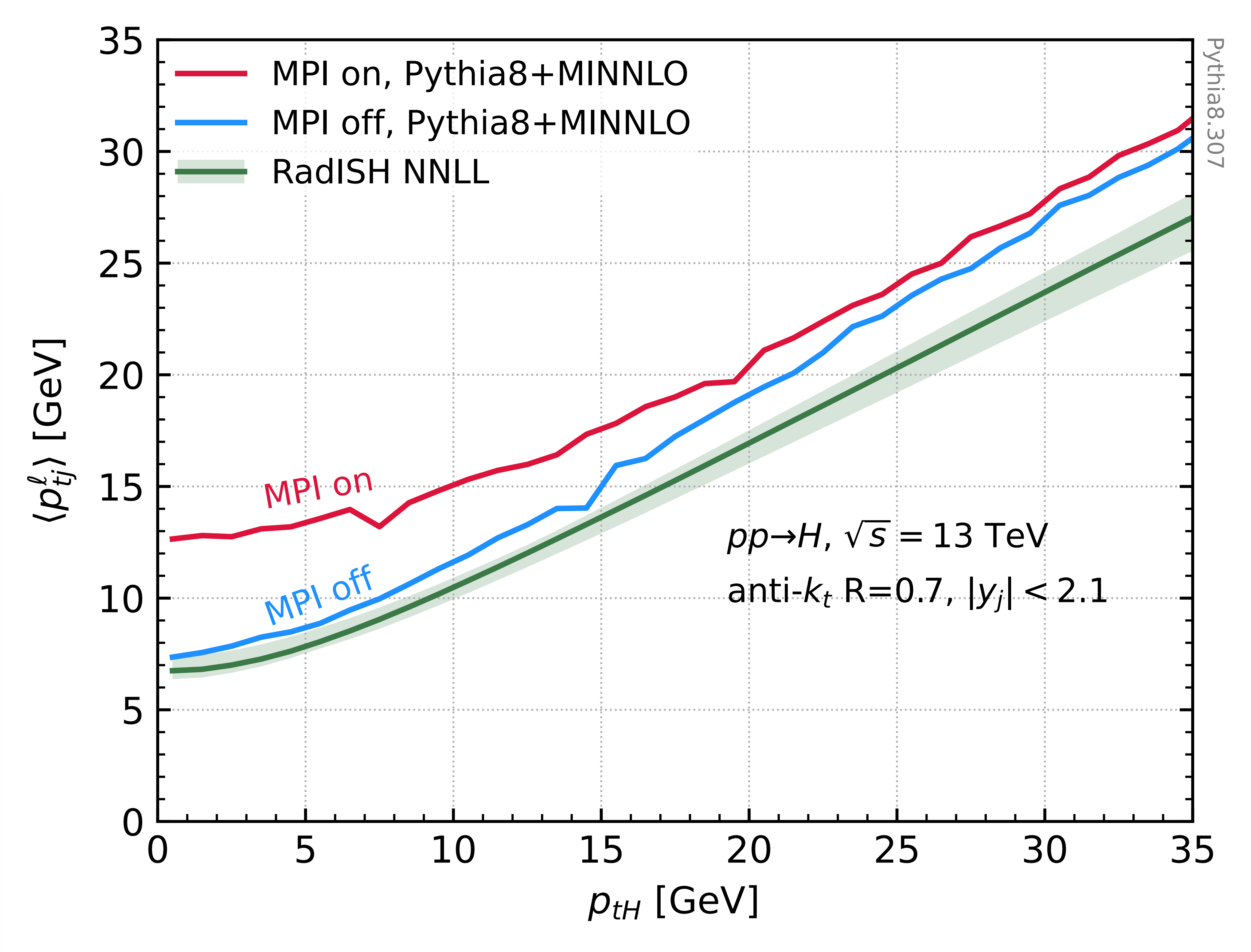
5%

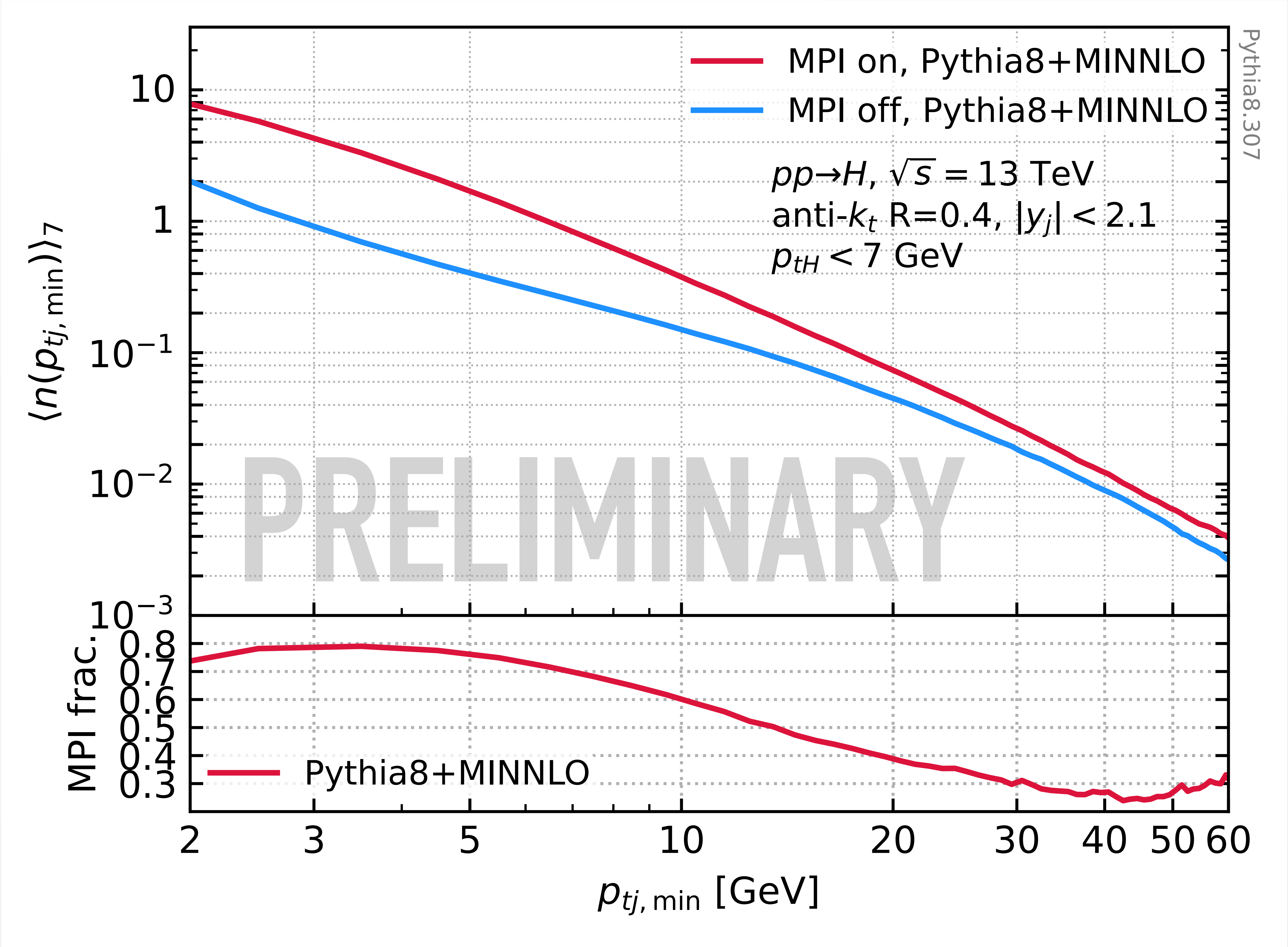
10%

10% / 20% (=MINNLO)



Higgs production (gg channel said to have smaller σeff, mainly from J/ψ)





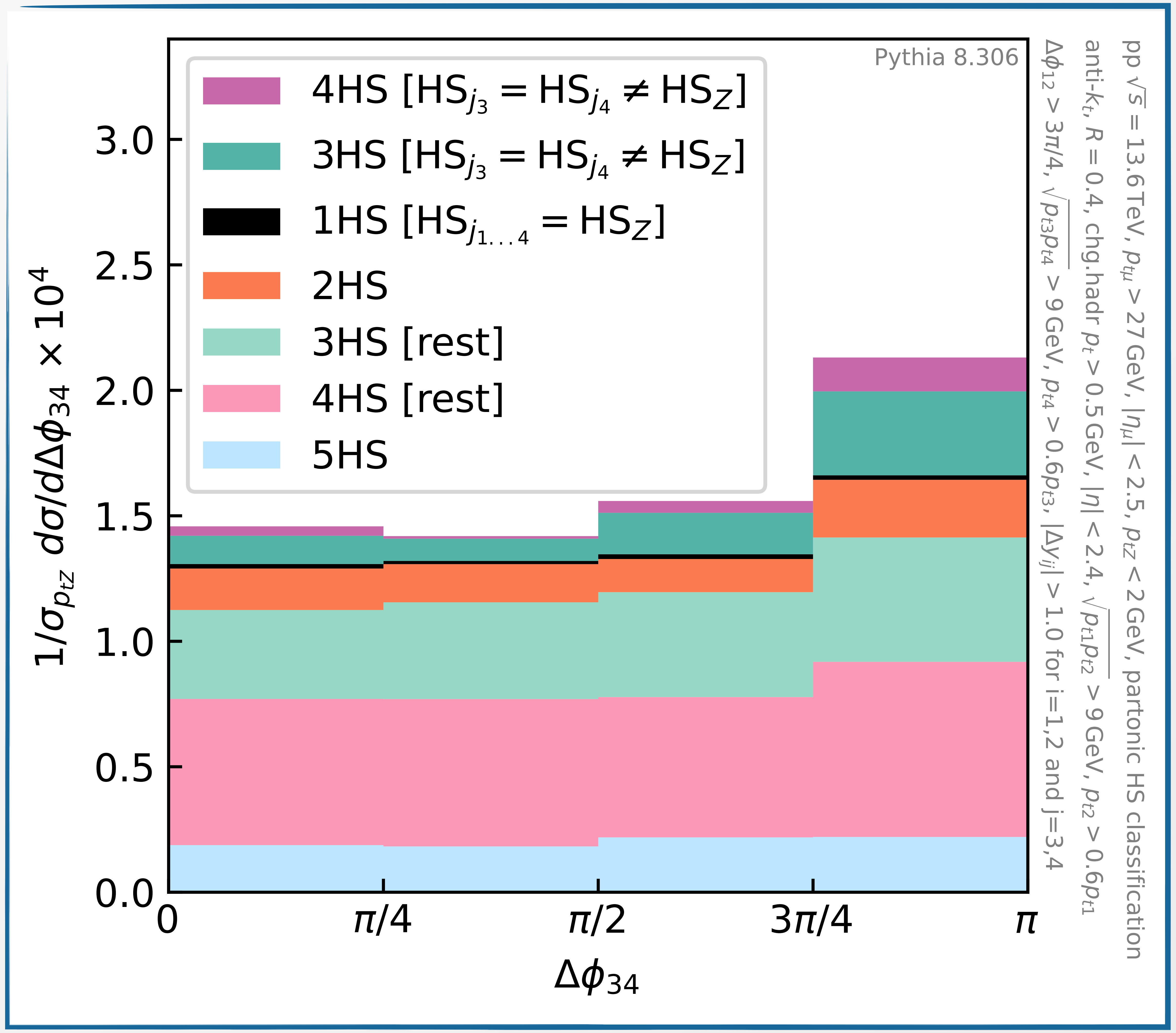
Optimal cut is $p_{tH} \lesssim 7$ [GeV]

~10% of events H events pass this cut

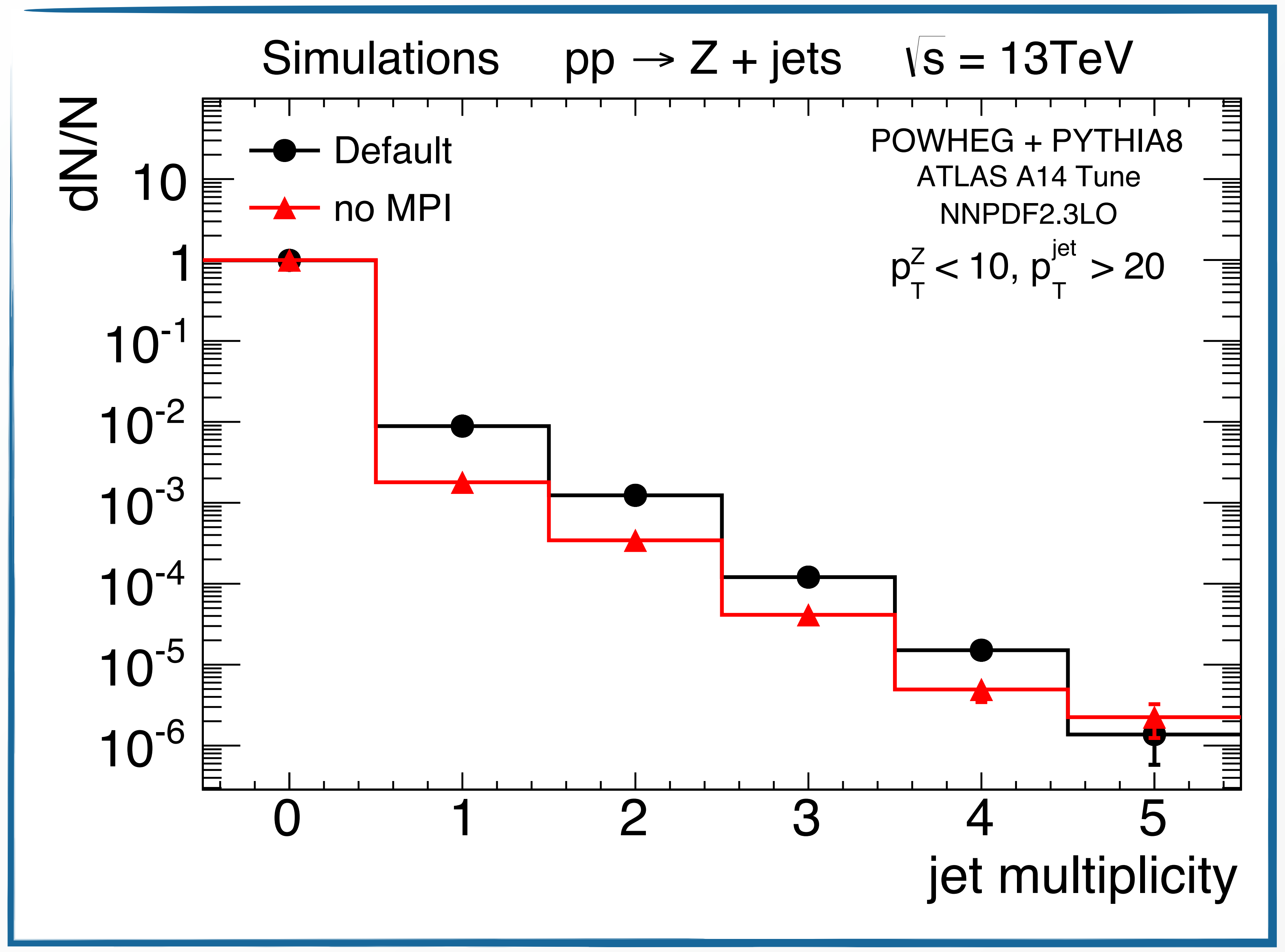
(with p_{tH} cut, full run 2+3 stats in $H \to ZZ^* \to 4\ell$ c. 50–100 events)

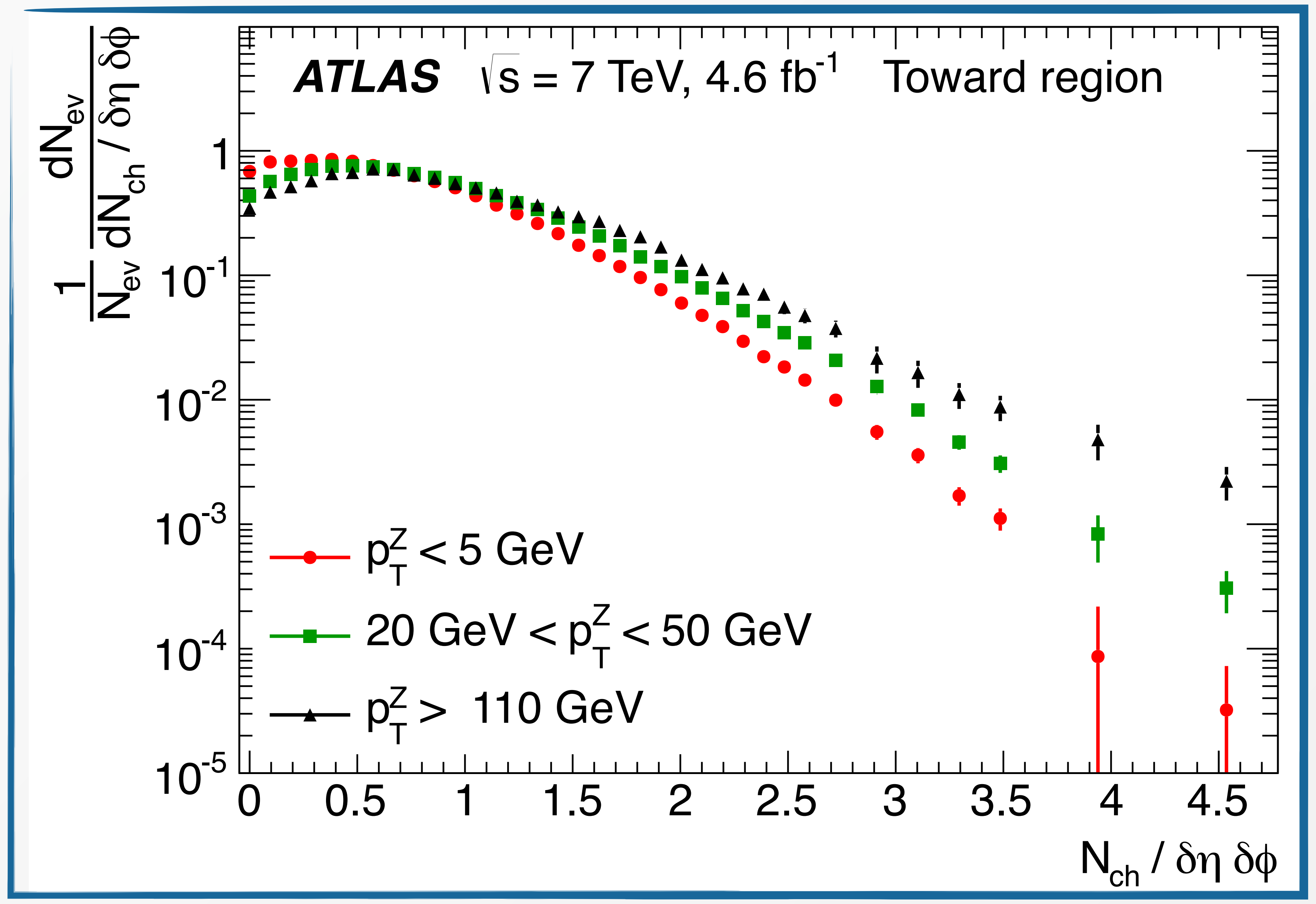
Beyond 3HS?

- Select four leading jets
- Pair them up (first two, next two)
- Require first two to be back-to-back
- Require $|\Delta y| > 1$ rapidity separations between first two and next two
- examine $|\Delta\phi_{34}|$
- see small peak around $|\Delta\phi_{34}| = \pi$ (3HS)
- **continuum includes substantial 4HS contribution!**



Past MPI studies with cuts on p_{tZ}

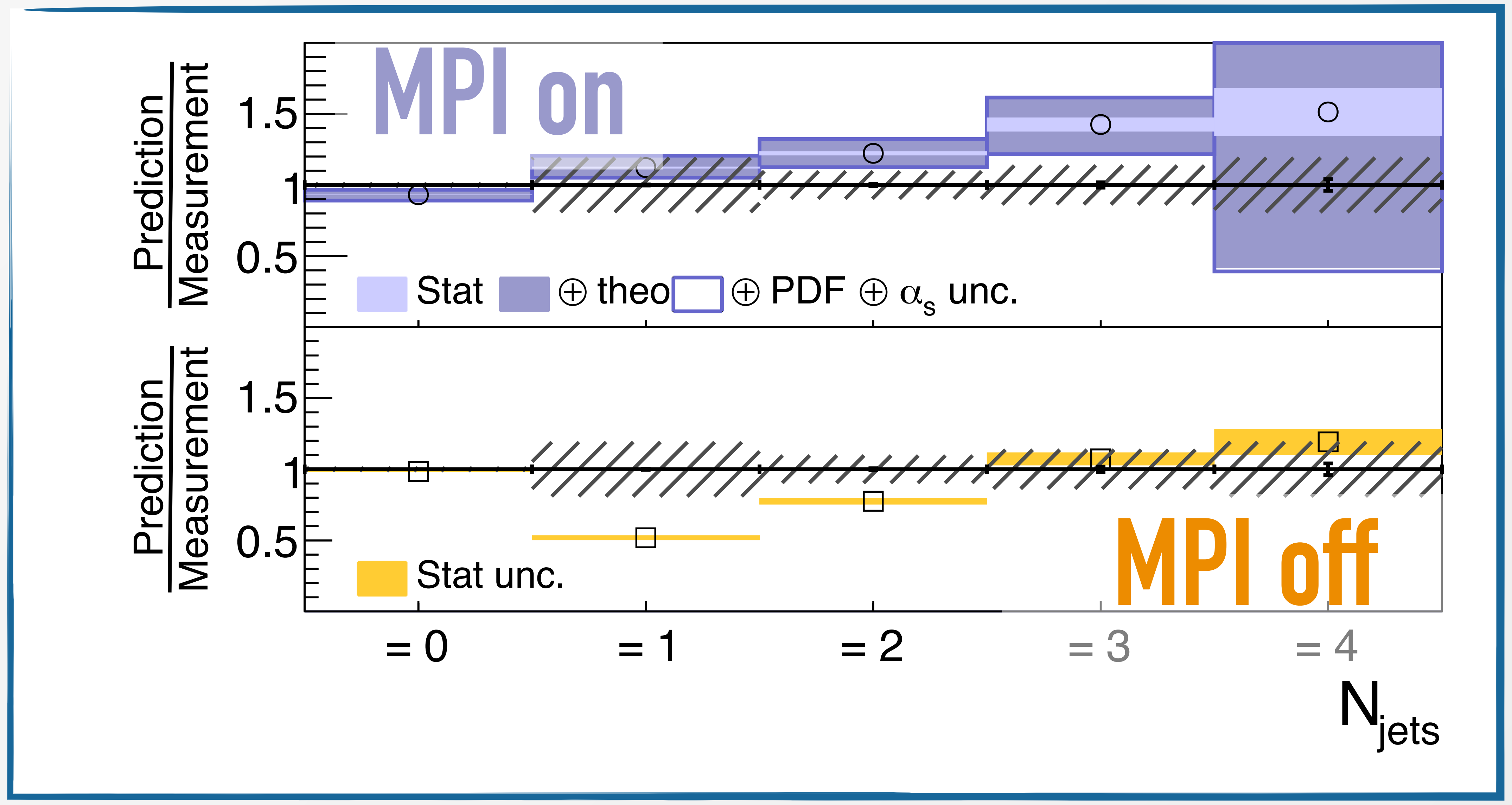




ATLAS 1409.3433 mostly an underlying-event study, used $p_{tZ} < 5\,\mathrm{GeV}$

Bansal, Bansal, Kumar, Singh 1602.05392 suggested MPI studies with $p_{tZ} < 10\,\mathrm{GeV}$ for improved MPI purity

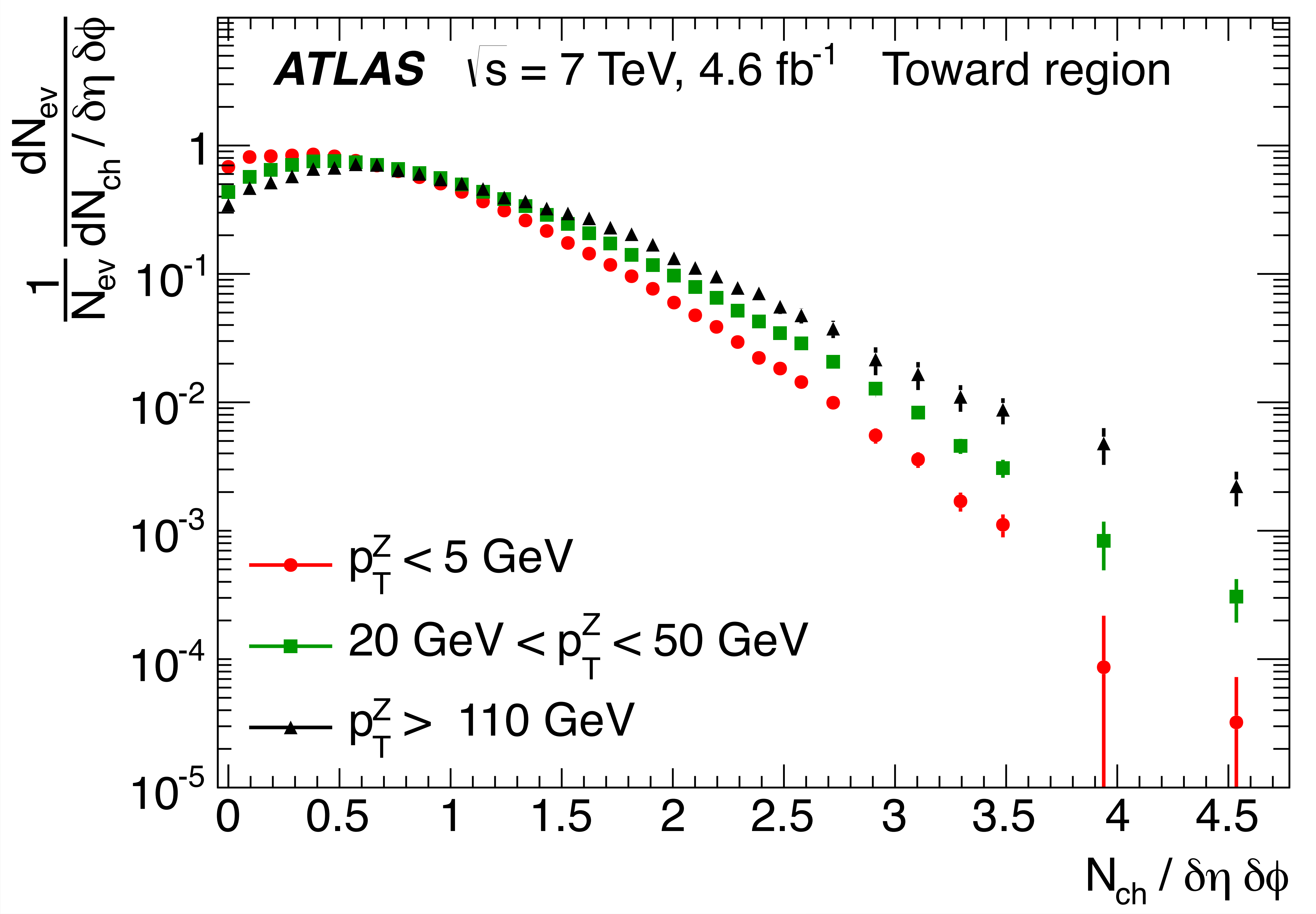
See also Alioli, Bauer, Guns, Tackmann, 1605.07192



CMS 2210.16139: results with $p_{tZ} < 10\,\mathrm{GeV}$, confirming some MPI enhancement

ATLAS 1409.3433

- mostly a UE study
- uses $p_T^Z < 5\,\mathrm{GeV}$



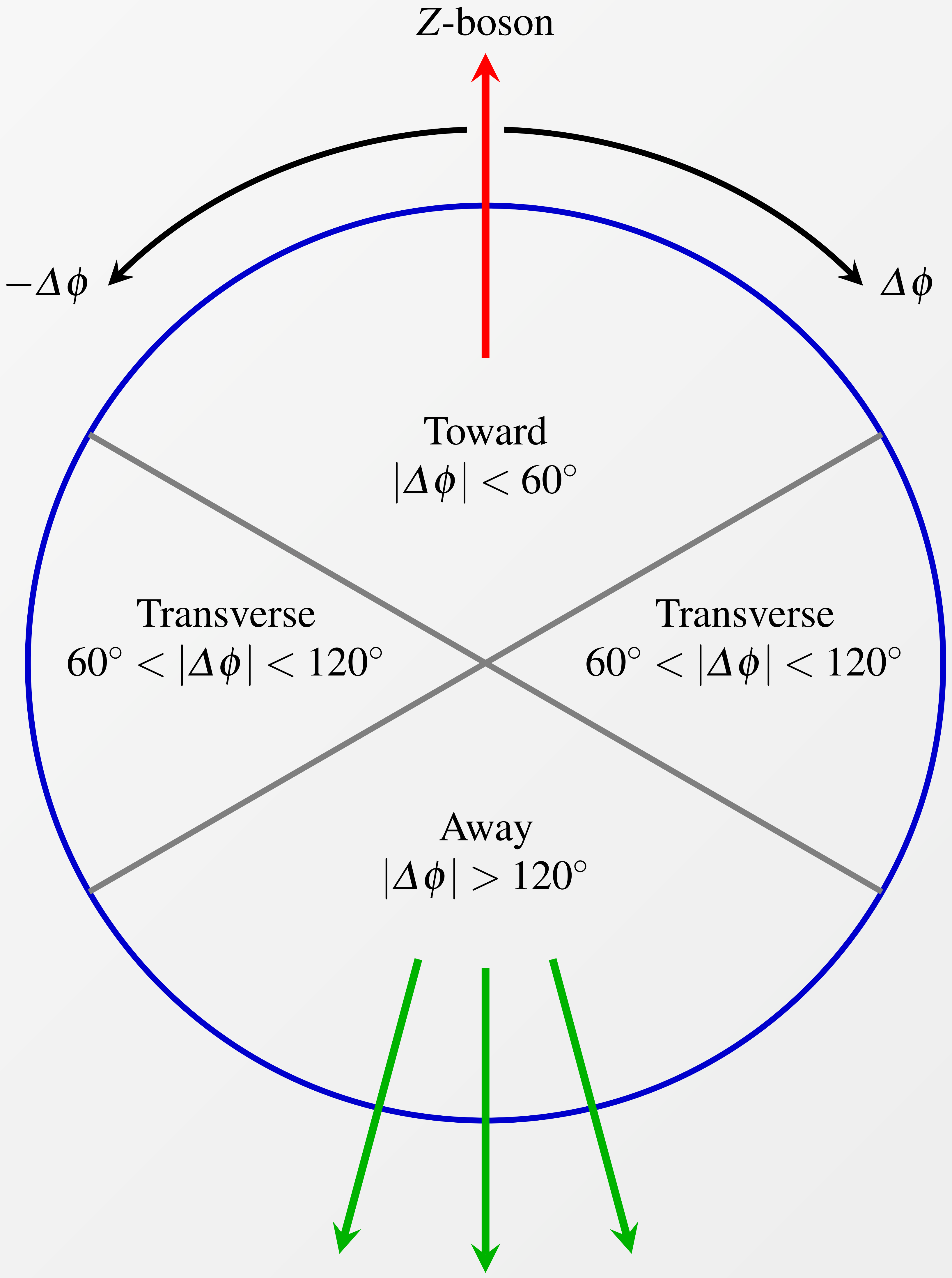
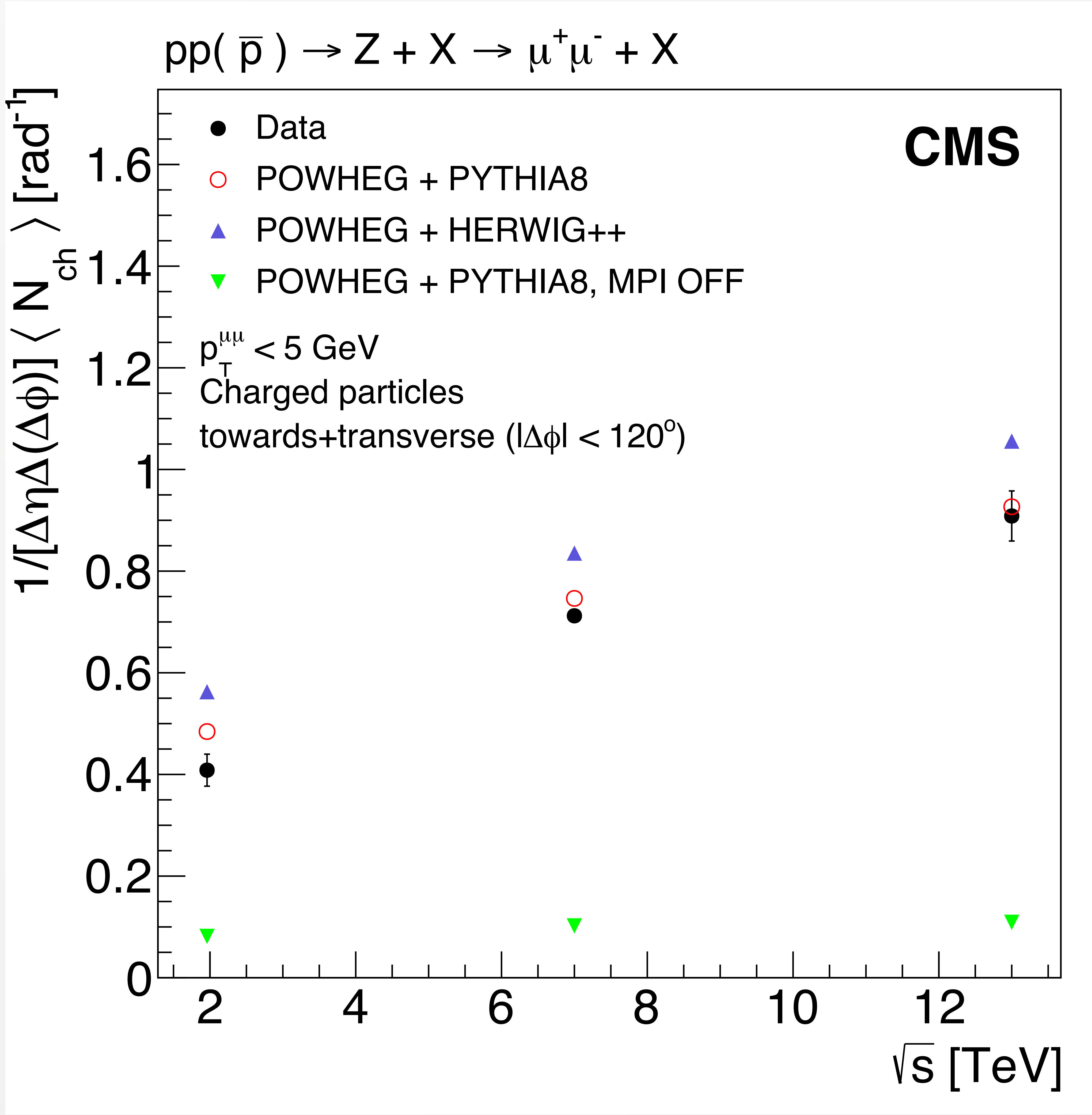


Fig. 1 Definition of UE regions as a function of the azimuthal angle with respect to the Z-boson.

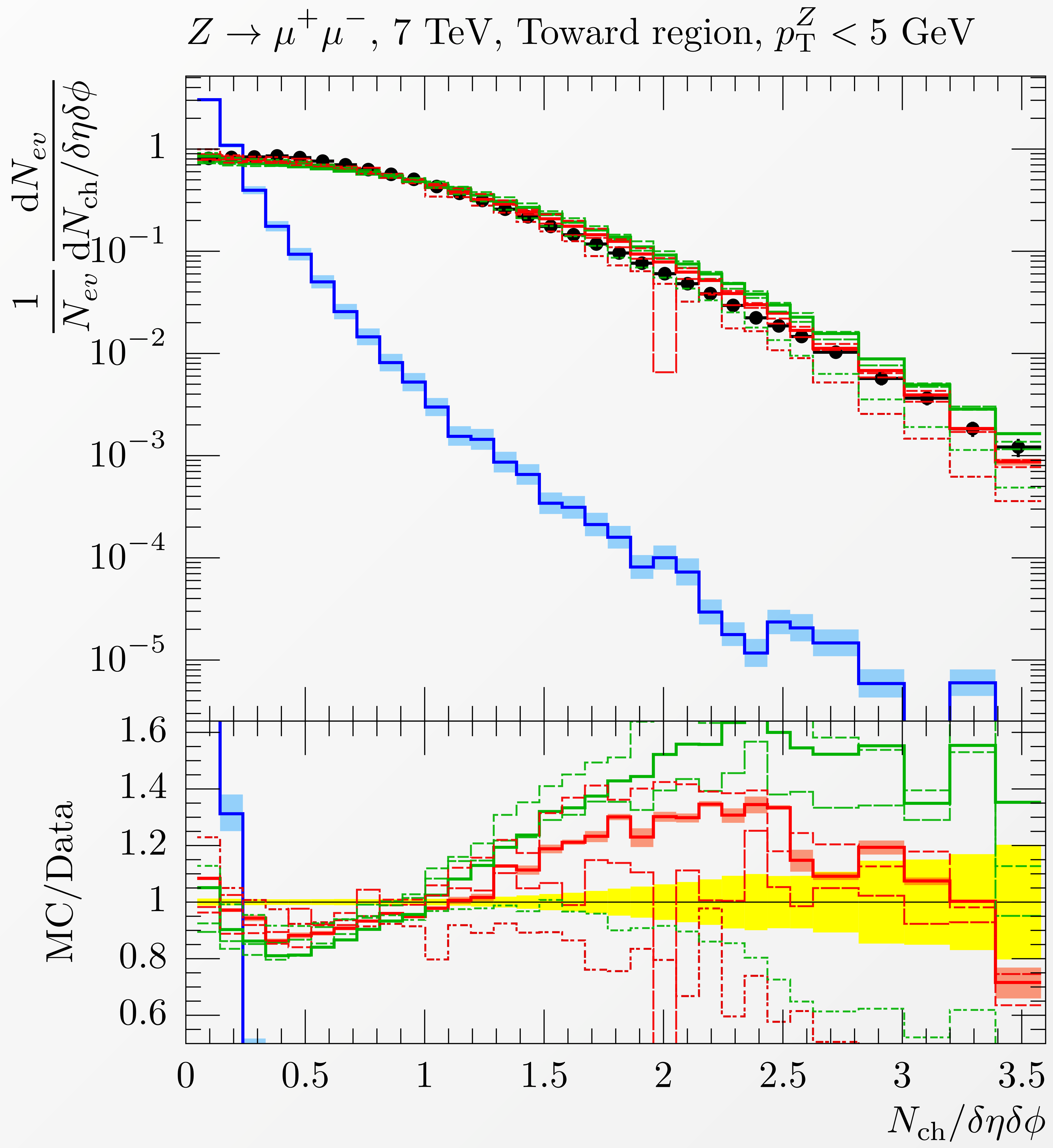
CMS 1711.04299

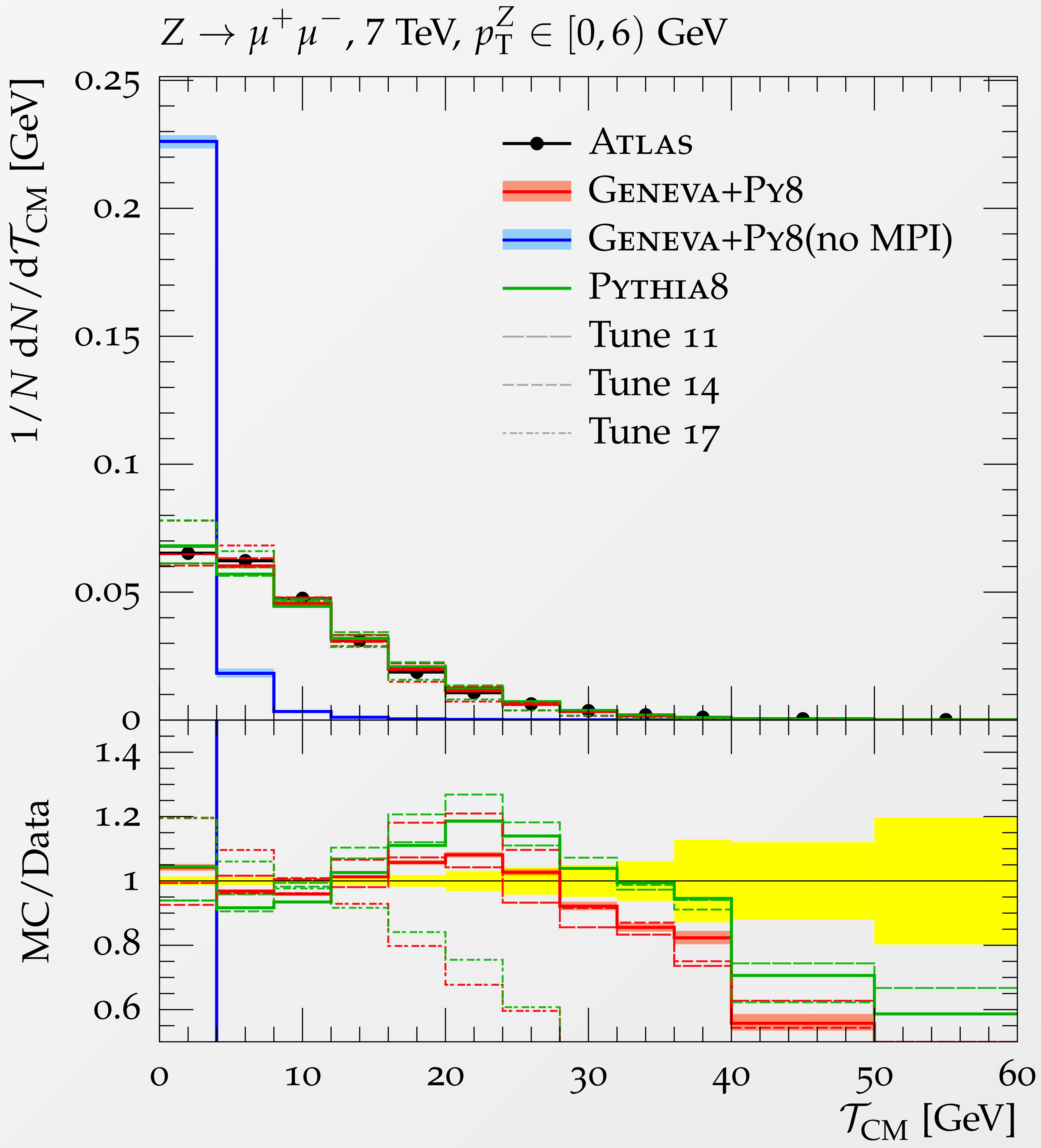
- mostly a UE study
- uses $p_T^Z < 5\,\mathrm{GeV}$



Alioli, Bauer, Guns, Tackmann, 1605.07192

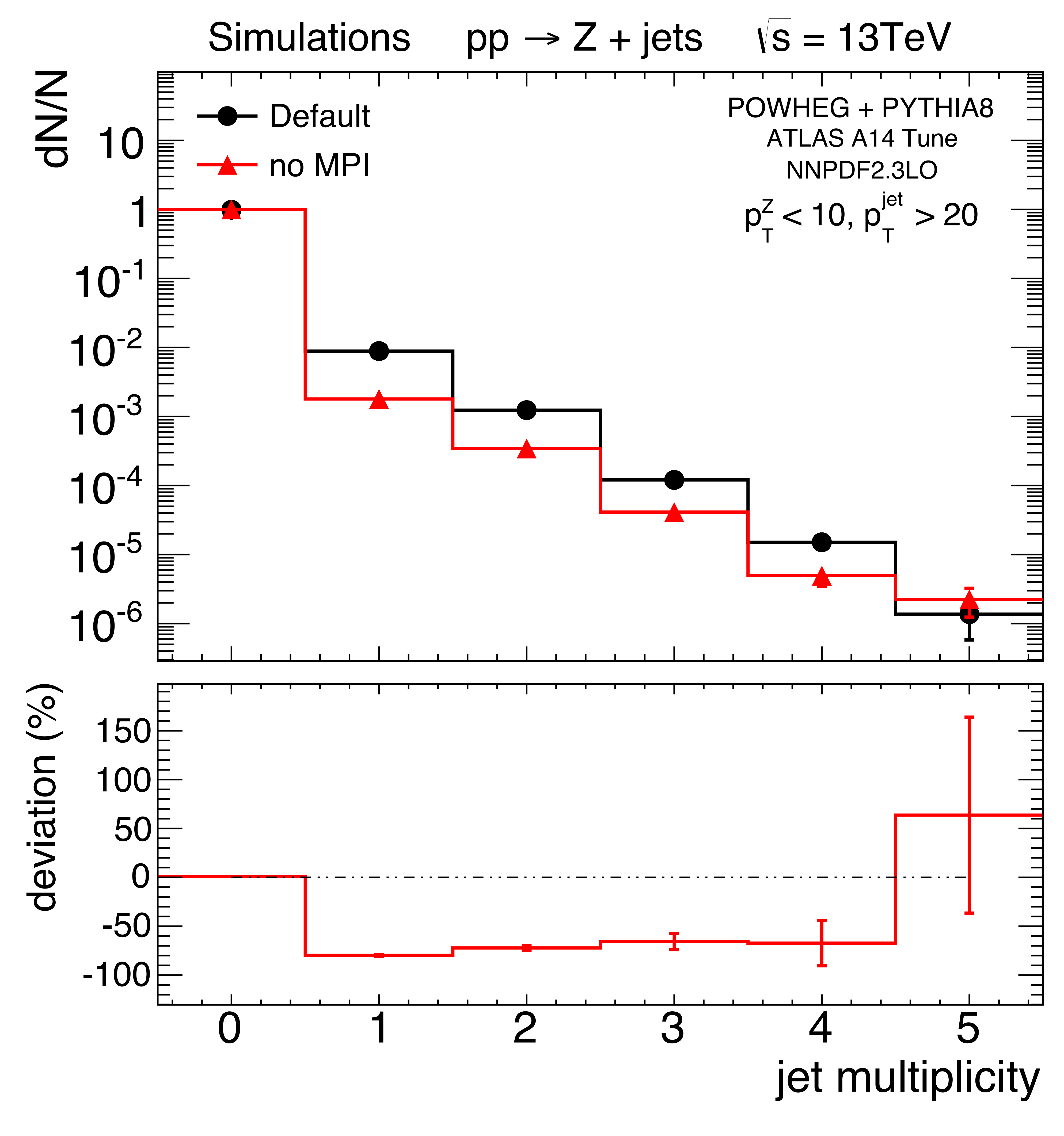
- explores $p_T^Z < 5\,\text{GeV}$
- mainly a “UE” study





Bansal, Bansal, Kumar, Singh 1602.05392

- explores $p_T^Z < 10\,\text{GeV}$ as central part of their study
- explores various jet cuts, including $p_T^{\text{jet}} > 5\,\text{GeV}$



CMS 2210.16139

- includes $p_T^Z < 10\,\text{GeV}$ bin, with 25-50% MPI contribution for jets with $p_T^J > 30\,\text{GeV}$
- includes $\Delta\phi_{j_1 j_2'}$ though high p_T^J cut means only 2HS