The Higgs transverse momentum

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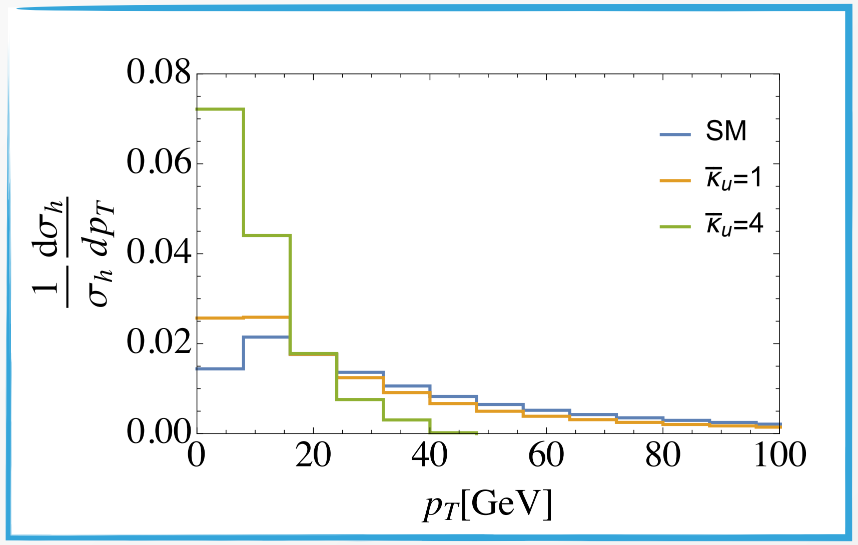
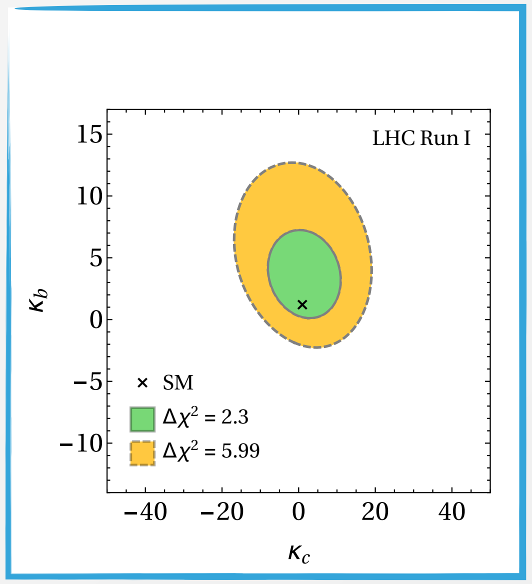
Why bother?

- ~40 inverse femtobarns collected in 2016
- Increase in statistics enables study of **differential distributions** in detail
- Transverse momentum distribution of the Higgs boson is sensitive to **new physics**

Light Yukawa

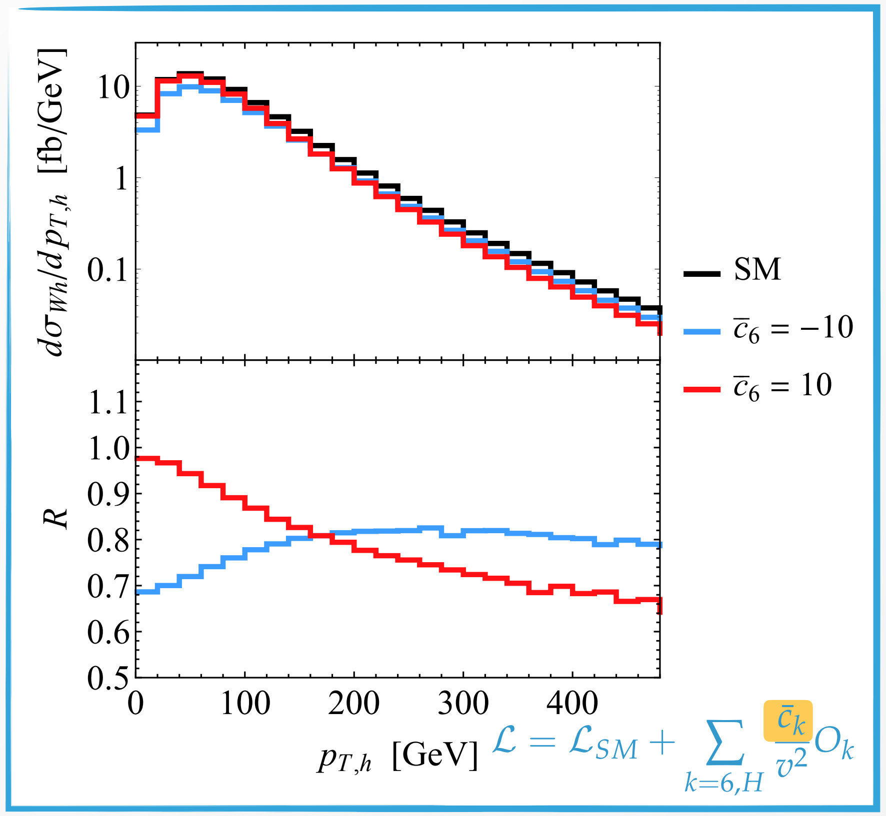


[Soreq et al, 1606.09621]

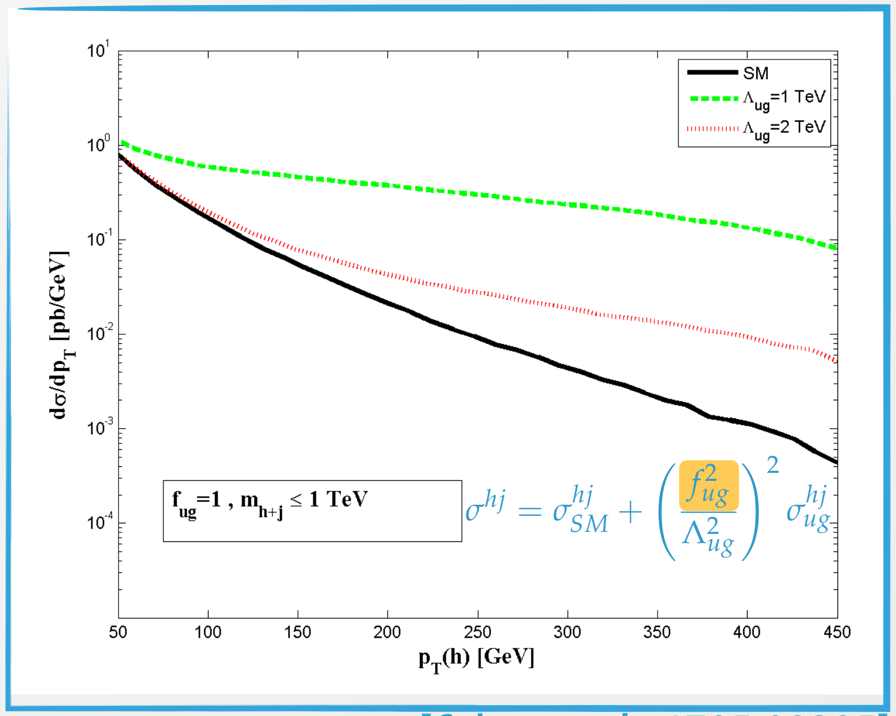


[Bishara et al.,1606.09253]

Trilinear coupling



[Bizon et al.,1610.05771]



[Cohen et al., 1705.09295]

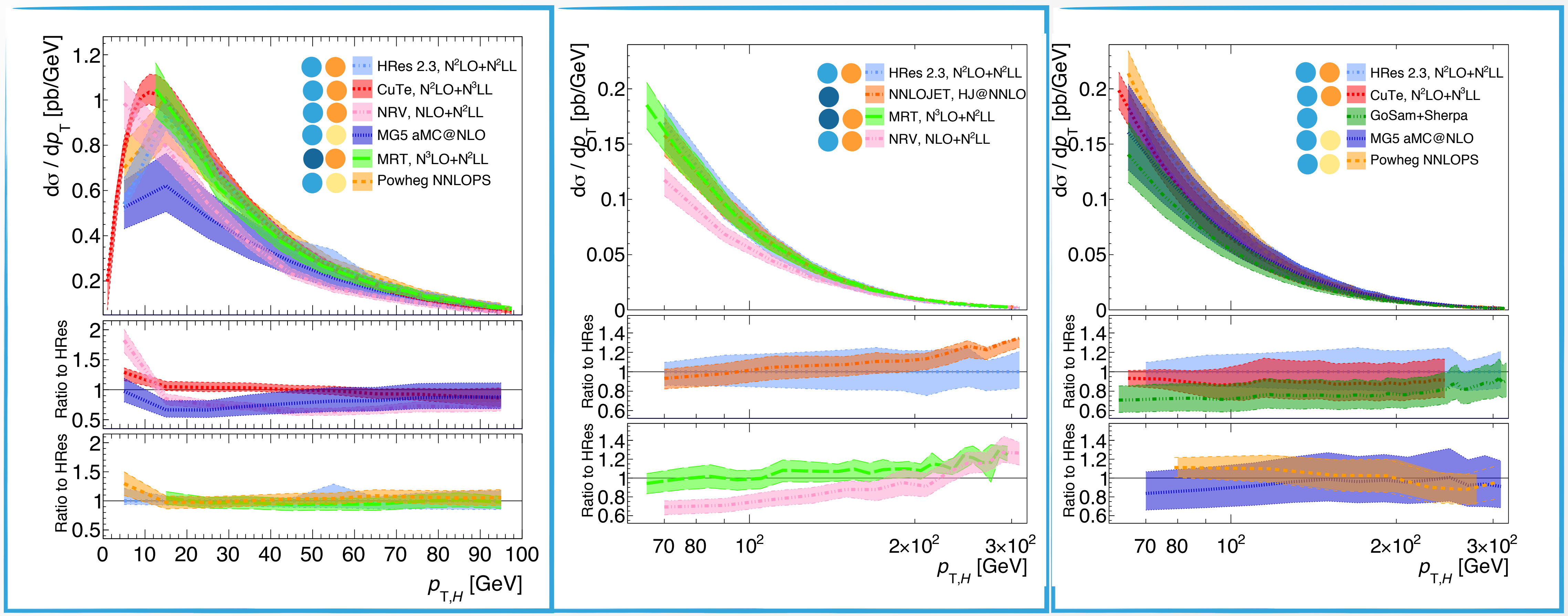
Theoretical prelude: Yellow Report 2016

α_s^3 α_s^4 α_s^5

formal FO accuracy

LL NLL NNLL N3LL

formal RES accuracy



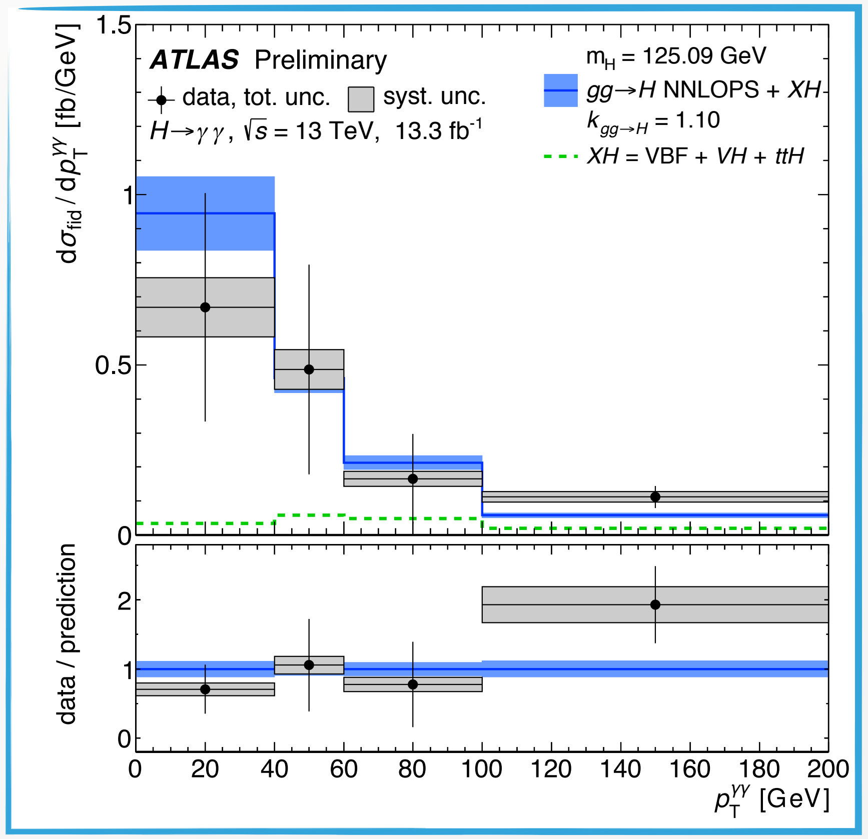
not all predictions include the same set of "uncertainties"

(all include QCD scale variations)

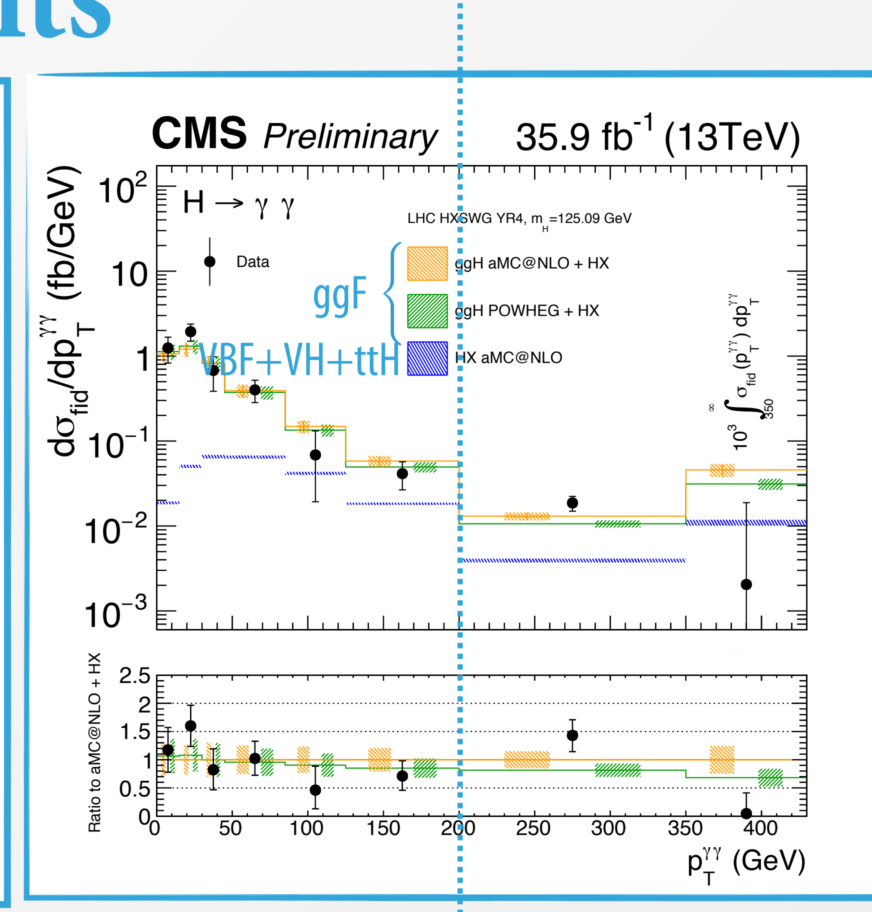
Experimental prelude: Run II results

H → γγ

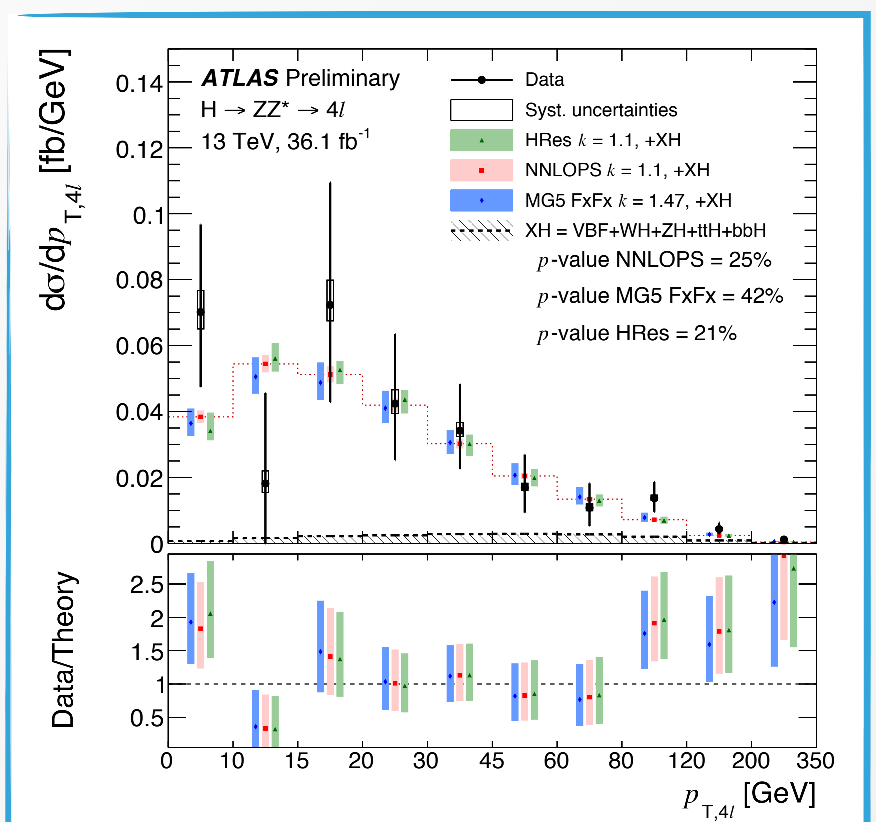
- precise reconstruction of the diphoton invariant mass
- Signal fitted in **each differential bin**
- Good agreement with Standard Model predictions



ATLAS-CONF-2016-067



CMS PAS HIG-17-015



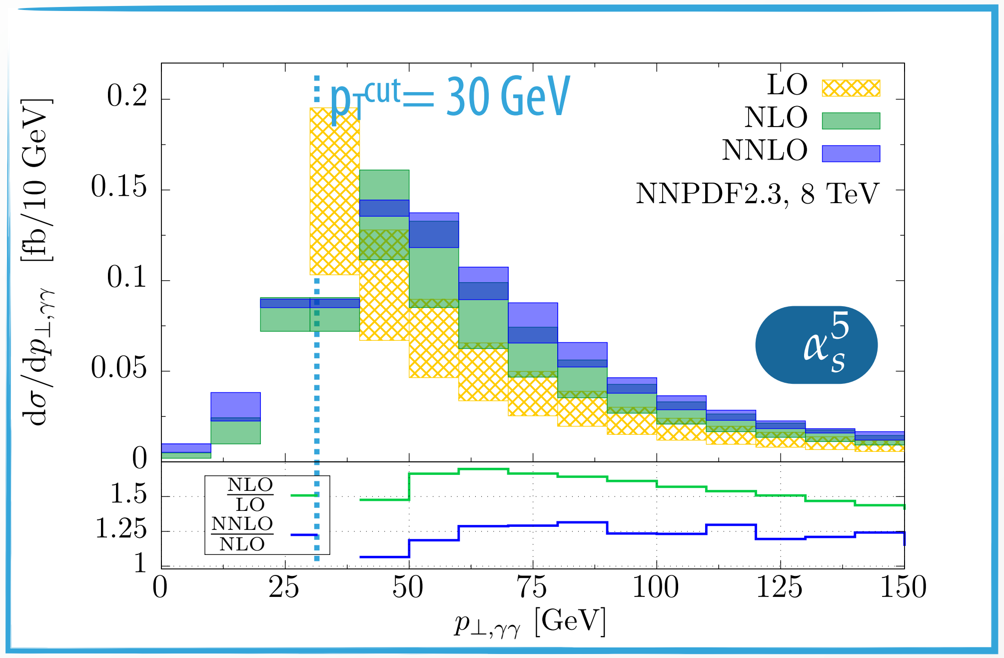
ATLAS-CONF-2017-032

H → 4l

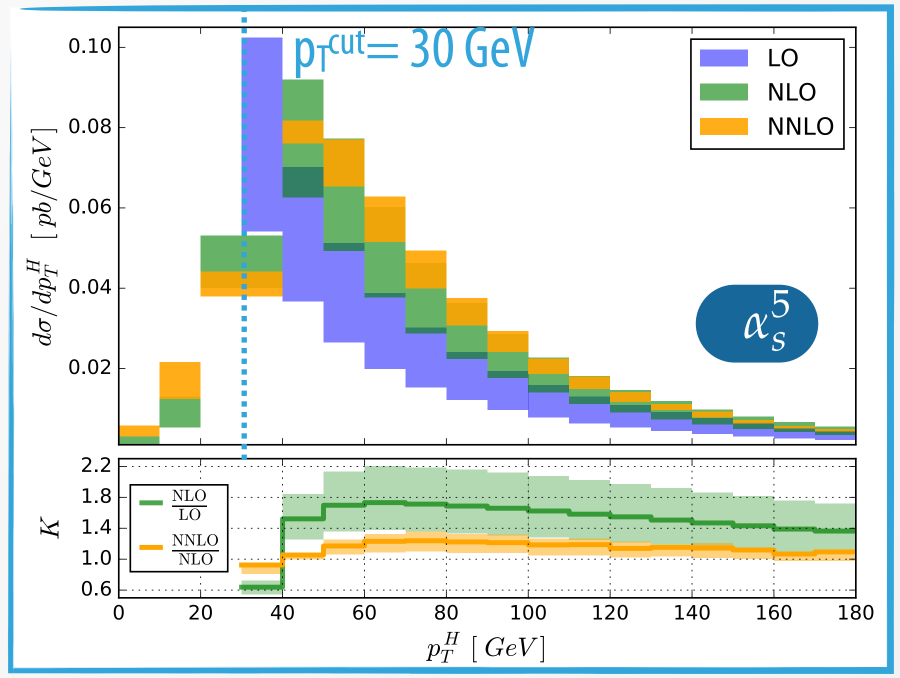
- measured cross sections at high slightly higher than the predictions
- Distribution is consistent with the (rescaled) SM predictions within the uncertainties

Fixed-order predictions: state-of-the-art

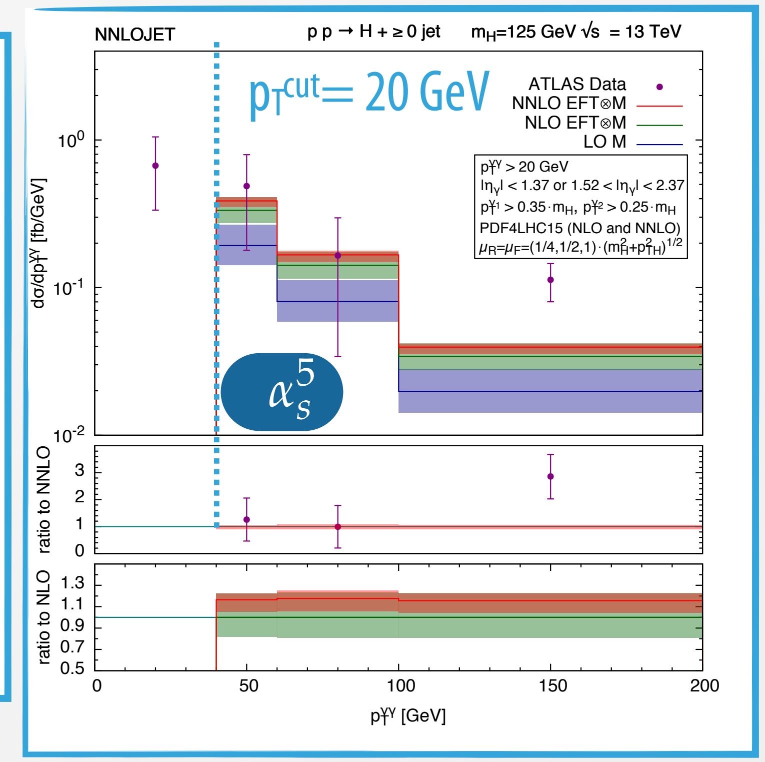
Fixed-order predictions available through NNLO QCD in the EFT
NNLO correction ~ 10-20%



[Boughezal et al. 1504.07922]
[Caola et al.1508.02684]



[Boughezal et al. 1505.03893]



[Chen et al. 1607.08817]

- **sector**- improved residue **subtraction** approach
- fiducial cross sections

- **jettiness subtraction**

- **antenna subtraction**
- comparison with ATLAS data

Fixed Scale Choice

$$\mu = m_H$$

Dynamical Scale Choice

$$\mu = \frac{1}{2}\sqrt{m_H^2 + (p_T^H)^2}$$

Resummation in the small-p_T region

Resummation

Fixed-order results are crucial to obtain reliable theoretical predictions away from the **soft** and **collinear** regions of the phase space

However, regions dominated by soft and collinear QCD radiation affected by **large logarithms**

$$\frac{1}{p_T}\alpha_s^n \ln^k(p_T/M), \qquad k \leq 2n-1$$

Perturbative series spoiled → All-order **resummation** of the logarithmically enhanced terms

Effects propagate away from the singularity, **resummation is necessary** to obtain a good control of the small-p_T region

$$\Sigma(v) = \int_0^v \frac{d\sigma}{dv'}dv' \sim e^{\alpha_s^n L^{n+1} + \alpha_s^n L^n + \alpha_s^n L^{n-1} + \ldots} \qquad v = p_T/M$$

LL NLL NNLL

Logarithmic counting commonly defined at the level of the logarithm of the integrated cross section

Zeros in the small-p_T region and b-space formulation

Two different mechanisms give a contribution in the small p_T region

- configurations where the transverse momenta of the radiated partons is small (**Sudakov limit**) — Exponential suppression — Sudakov peak region
- configurations where p_T tends to zero because of cancellations of non-zero transverse momenta of the emissions (**azimuthal cancellations**) — Power suppression $\Sigma \sim \mathcal{O}(p_T^2)$ — $p_T \to 0$ limit

Power-law scaling at very small p_T

For inclusive observables the vectorial nature of the cancellations can be handled via a **Fourier transform**

[Parisi, Petronzio '78; Collins, Soper, Sterman '85]

[Catani, Grazzini '11][Catani et al. '12,Gehrmann][Luebbert, Yang '14]

coefficient functions

$$\frac{d^2\Sigma(v)}{d\Phi_B dp_t} = \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \int b\, db\, p_t J_0(p_t b)\, \mathbf{f}^T(b_0/b)\, \mathbf{C}^{c_1;T}_{N_1}(\alpha_s(b_0/b)) H_{\rm CSS}(M)\, \mathbf{C}^{c_2}_{N_2}(\alpha_s(b_0/b))\, \mathbf{f}(b_0/b)$$

hard-virtual corrections

$$\times \exp\left\{-\sum_{\ell=1}^{2}\int_0^M \frac{dk_t}{k_t} \mathbf{R}'_{{\rm CSS},\ell}(k_t)\, \Theta\!\left(k_t - \frac{b_0}{b}\right)\right\}$$

$$R_{\rm CSS}(b) = \sum_{l=1}^{2}\int_{b0/b}^{M} \frac{dk_T}{k_T} R'_{{\rm CSS},l}(k_T) = \sum_{l=1}^{2}\int_{b_0/b}^{M} \frac{dk_T}{k_T}\left(A_{{\rm CSS},\ell}(\alpha_s(k_T)) \ln\frac{M^2}{k_T^2} + B_{{\rm CSS},\ell}(\alpha_s(k_T))\right)$$

anomalous dimensions

[Davies, Stirling '84] [De Florian, Grazzini '01] [Becher, Neubert '10][Li, Zhu '16][Vladimirov '16]

Momentum space

[Monni, Re, Torrielli, Phys.Rev.Lett. 116 (2016) no.24, 242001]
[Bizon, Monni, Re, LR, Torrielli, 1705.09127]
[Ebert, Tackmann 1611.08610] talk by Markus

Is it possible to obtain a formulation in momentum space?

Not possible to find a closed analytic expression in direct space which is both a) free of logarithmically subleading corrections and b) free of singularities at finite p_T values [Frixione, Nason, Ridolfi '98]

Why? A naive logarithmic counting at small p_T is not sensible, as one loses the correct power-suppressed scaling if only logarithms are retained: it's not possible to reproduce a power behaviour with logs of p_T/M

(logarithms of b do not correspond to logarithms of p_T)

Necessary to establish a well defined logarithmic counting in momentum space in order to reproduce the correct behaviour of the observable at small p_T

Since b-space formulation works well, why should one bother so much for a single observable?

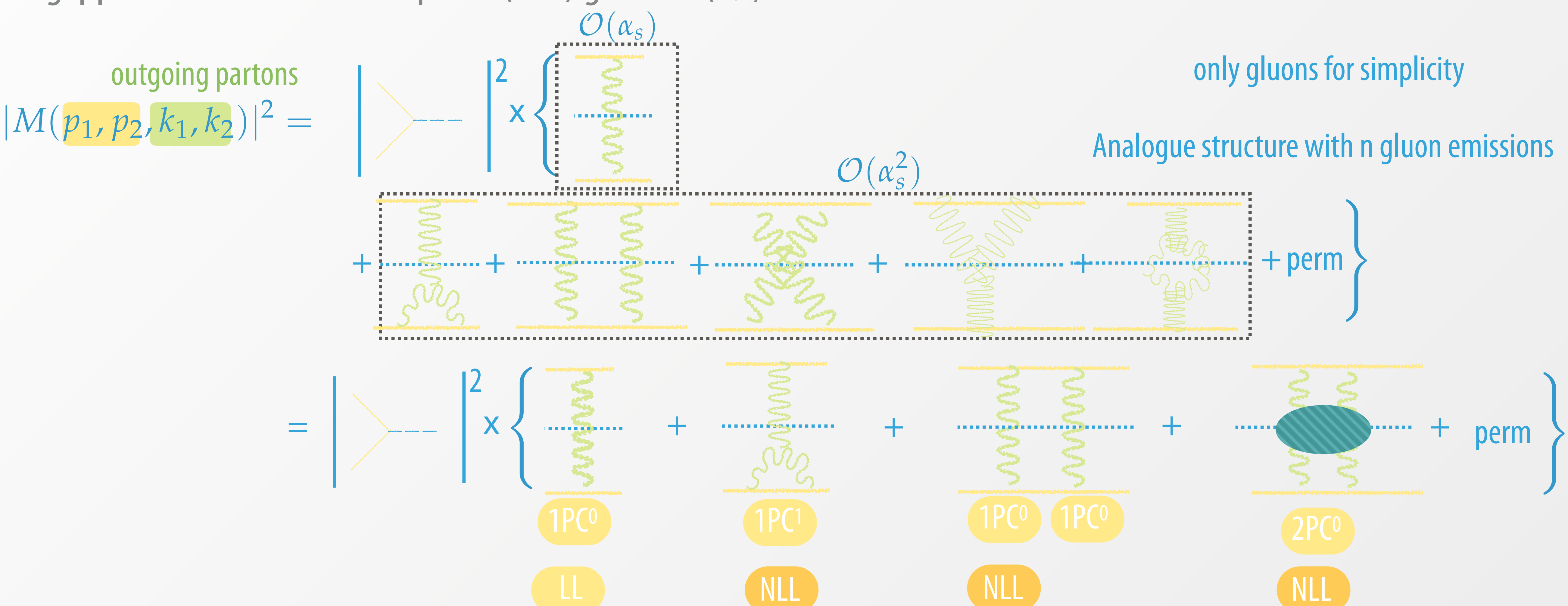
- No need to have a factorization theorem established (more **observable independent** than b-space formulation)
- Important to understand the dynamics of the radiation to improve generators
- What we learn will have a broader application range, possible generalisation beyond the simple inclusive-observable case
- Possibility to perform **joint resummation** of observables
- As a byproduct, the result in momentum space can be implemented in a code fully differential in the Born phase space (easy to introduce cuts, dynamical scales, etc)

Logarithmic counting

[Monni, Re, Torrielli, Phys.Rev.Lett. 116 (2016) no.24, 242001]
[Bizon, Monni, Re, LR, Torrielli, 1705.09127]

Necessary to establish a **well defined logarithmic counting**: possibile to do that by decomposing the squared amplitude in terms of n-particle correlated blocks (**nPC**)

e.g. pp → H + emission of up to 2 (soft) gluons O(α_s^2)



Logarithmic counting defined in terms of **nPC blocks** (owing to rIRC safety of the observable)

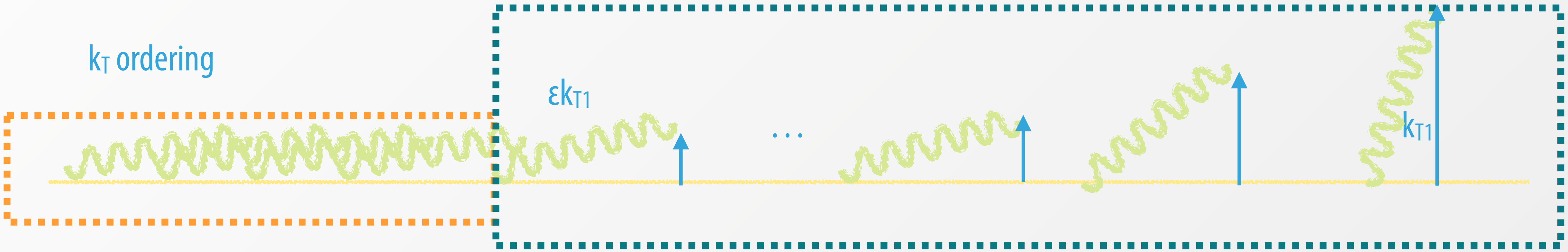
Resolved and unresolved emissions

For inclusive observables (such as Higgs p_T) $V(p_1, p_2, k_1, \ldots, k_n) = V(p_1, p_2, k_1 + \cdots + k_n)$

$$|M(p_1, p_2, k_1, \ldots, k_n)|^2 = |M_B(p_1, p_2)|^2 \times \frac{1}{n!} \left\{ \prod_{i=1}^{n} \left(|M(k_i)|^2 + \int [dk_a][dk_b] |\tilde{M}(k_a, k_b)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} - \vec{k}_{ti}) \delta(Y_{ab} - Y_i) + \int [dk_a][dk_b][dk_c] |\tilde{M}(k_a, k_b, k_c)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} + \vec{k}_{tc} - \vec{k}_{ti}) \delta(Y_{abc} - Y_i) + \ldots \right) \right\}$$

1PC 2PC 3PC

Introduction of a **resolution scale** εk_{T1}



unresolved emission

resolved emission

can be integrated inclusively to cancel the divergences of the virtuals (rIRC): exponential factor

$$e^{-R(\varepsilon k_{t1})}$$

Sudakov form factor

ε dependence cancels against the resolved real corrections

treated exclusively: for inclusive observables can be parametrised exactly as a Sudakov **unintegrated** in k_t and azimuthal angle

Momentum space formulation

need some care in the treatment of the hard-collinear emissions

Result can be expressed as

$$\frac{d\Sigma(v)}{d\Phi_B} = \int_{\mathcal{C}_1} \frac{dN_1}{2\pi i} \int_{\mathcal{C}_2} \frac{dN_2}{2\pi i} x_1^{-N_1} x_2^{-N_2} \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \mathbf{f}^T_{N_1}(\mu_0)\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v)\mathbf{f}_{N_2}(\mu_0)$$

DGLAP anomalous dimensions

RG evolution of coefficient functions

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0))H(\mu_R)\mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi}$$

$$\times e^{-\mathbf{R}(\epsilon k_{t1})} \exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t}\frac{\alpha_s(k_t)}{\pi}\mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$

$$\sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi}\mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$

$$\times \sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=2}^{n+1}\int_\epsilon^1 \frac{d\zeta_i}{\zeta_i}\int_0^{2\pi}\frac{d\phi_i}{2\pi}\sum_{\ell_i=1}^{2}\left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi}\mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right)$$

$$\times\,\Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1})\right)$$

Result valid for all inclusive observables (e.g. p_T, φ^*)

$$V(k) = d_l g_l(\phi)\frac{k_T}{M}$$

unresolved emission + virtual corrections

resolved emission

Formulation **equivalent to b-space** result (up to a scheme change in the anomalous dimensions)

$$\frac{d^2\Sigma(v)}{d\Phi_B dp_t} = \sum_{c_1,c_2}\frac{d|M_B|^2_{c_1c_2}}{d\Phi_B}\int b\,db\,p_t J_0(p_t b)\,\mathbf{f}^T(b_0/b)\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(b_0/b))H(M)\mathbf{C}^{c_2}_{N_2}(\alpha_s(b_0/b))\mathbf{f}(b_0/b)$$

$$\times\exp\left\{-\sum_{\ell=1}^{2}\int_0^M\frac{dk_t}{k_t}\mathbf{R}'_\ell(k_t)\left(1-J_0(bk_t)\right)\right\}$$

$$(1-J_0(bk_t)) \simeq \Theta(k_t - \frac{b_0}{b}) + \frac{\zeta_3}{12}\frac{\partial^3}{\partial\ln(Mb/b_0)^3}\Theta(k_t - \frac{b_0}{b})$$

Resummation in momentum space

Formulation in Mellin space already implementable. However, it is convenient to perform the evaluation entirely in momentum space

In previous formula, resummation of logarithms of $k_{T,i}/M$

subleading logarithms in p_T

free of singularity at low p_T values (power-law scaling)

$k_{Ti}/k_{T1} \sim O(1)$
(everywhere in the resolved phase space, due to rIRC safety)

Integrands can be expanded about $k_{Ti} \sim k_{T1}$ to the desired accuracy: more efficient

Sudakov region: $k_{T1} \sim p_T$

$\ln(M/p_T)$ resummed at the desired accuracy

+ additional subleading terms that **cannot be neglected**

azimuthal region: $k_{Ti} \sim k_{T1}$

correct description of the kinematics after expansion $k_{Ti} \sim k_{T1}$

correct scaling of the cumulant $\mathbf{O(p_T^2)}$

Result at NLL accuracy

we expanded around k_{T1}

The divergences cancel with the terms contained in the resolved real radiation

$$= e^{-R'(k_{T1})\ln\frac{1}{\epsilon}} \qquad R' = \frac{d}{d\ln(M/k_{t1})}R$$

$$\frac{d\Sigma(v)}{d\Phi_B} = \int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}\partial_L\left(-e^{-R(k_{t1})}\mathcal{L}_{\rm NLL}(k_{t1})\right)\epsilon^{R'(k_{t1})}\sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=2}^{n+1}\int_{\epsilon}^{1}\frac{d\zeta_i}{\zeta_i}\int_0^{2\pi}\frac{d\phi_i}{2\pi}R'(k_{t1})\Theta(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1}))$$

resolved emission

$$\zeta_i = k_{ti}/k_{t1}$$

parton luminosity at NLL reads

$$\mathcal{L}_{\rm NLL}(k_{t1}) = \sum_{c,c'}\frac{d|M_B|^2_{cc'}}{d\Phi_B}f_c(k_{t1},x_1)\,f_{c'}(k_{t1},x_2)$$

At higher logarithmic accuracy, it includes coefficient functions and hard-virtual corrections

This formula can be evaluated by means of fast Monte Carlo methods

RadISH (Radiation off Initial State Hadrons)

Result at N3LL accuracy

$$
\begin{aligned}
\frac{d\Sigma(v)}{d\Phi_B} &= \int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}\partial_L\left(-e^{-R(k_{t1})}\mathcal{L}_{\mathrm{N^3LL}}(k_{t1})\right)\int d\mathcal{Z}[\{R',k_i\}]\Theta\left(v-V(\{\tilde{p}\},k_1,\dots,k_{n+1})\right)\\
&+\int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}e^{-R(k_{t1})}\int d\mathcal{Z}[\{R',k_i\}]\int_0^1\frac{d\zeta_s}{\zeta_s}\frac{d\phi_s}{2\pi}\Bigg\{\left(R'(k_{t1})\mathcal{L}_{\mathrm{NNLL}}(k_{t1})-\partial_L\mathcal{L}_{\mathrm{NNLL}}(k_{t1})\right)\\
&\times\left(R''(k_{t1})\ln\frac{1}{\zeta_s}+\frac{1}{2}R'''(k_{t1})\ln^2\frac{1}{\zeta_s}\right)-R'(k_{t1})\left(\partial_L\mathcal{L}_{\mathrm{NNLL}}(k_{t1})-2\frac{\beta_0}{\pi}\alpha_s^2(k_{t1})\hat{P}^{(0)}\otimes\mathcal{L}_{\mathrm{NLL}}(k_{t1})\ln\frac{1}{\zeta_s}\right)\\
&+\frac{\alpha_s^2(k_{t1})}{\pi^2}\hat{P}^{(0)}\otimes\hat{P}^{(0)}\otimes\mathcal{L}_{\mathrm{NLL}}(k_{t1})\Bigg\}\left\{\Theta\left(v-V(\{\tilde{p}\},k_1,\dots,k_{n+1},k_s)\right)-\Theta\left(v-V(\{\tilde{p}\},k_1,\dots,k_{n+1})\right)\right\}\\
&+\frac{1}{2}\int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}e^{-R(k_{t1})}\int d\mathcal{Z}[\{R',k_i\}]\int_0^1\frac{d\zeta_{s1}}{\zeta_{s1}}\frac{d\phi_{s1}}{2\pi}\int_0^1\frac{d\zeta_{s2}}{\zeta_{s2}}\frac{d\phi_{s2}}{2\pi}R'(k_{t1})\\
&\times\Bigg\{\mathcal{L}_{\mathrm{NLL}}(k_{t1})\left(R''(k_{t1})\right)^2\ln\frac{1}{\zeta_{s1}}\ln\frac{1}{\zeta_{s2}}-\partial_L\mathcal{L}_{\mathrm{NLL}}(k_{t1})R''(k_{t1})\left(\ln\frac{1}{\zeta_{s1}}+\ln\frac{1}{\zeta_{s2}}\right)\\
&+\frac{\alpha_s^2(k_{t1})}{\pi^2}\hat{P}^{(0)}\otimes\hat{P}^{(0)}\otimes\mathcal{L}_{\mathrm{NLL}}(k_{t1})\Bigg\}\\
&\times\Big\{\Theta\left(v-V(\{\tilde{p}\},k_1,\dots,k_{n+1},k_{s1},k_{s2})\right)-\Theta\left(v-V(\{\tilde{p}\},k_1,\dots,k_{n+1},k_{s1})\right)-\\
&\Theta\left(v-V(\{\tilde{p}\},k_1,\dots,k_{n+1},k_{s2})\right)+\Theta\left(v-V(\{\tilde{p}\},k_1,\dots,k_{n+1})\right)\Big\}+\mathcal{O}\left(\alpha_s^n\ln^{2n-6}\frac{1}{v}\right), \qquad (3.18)
\end{aligned}
$$

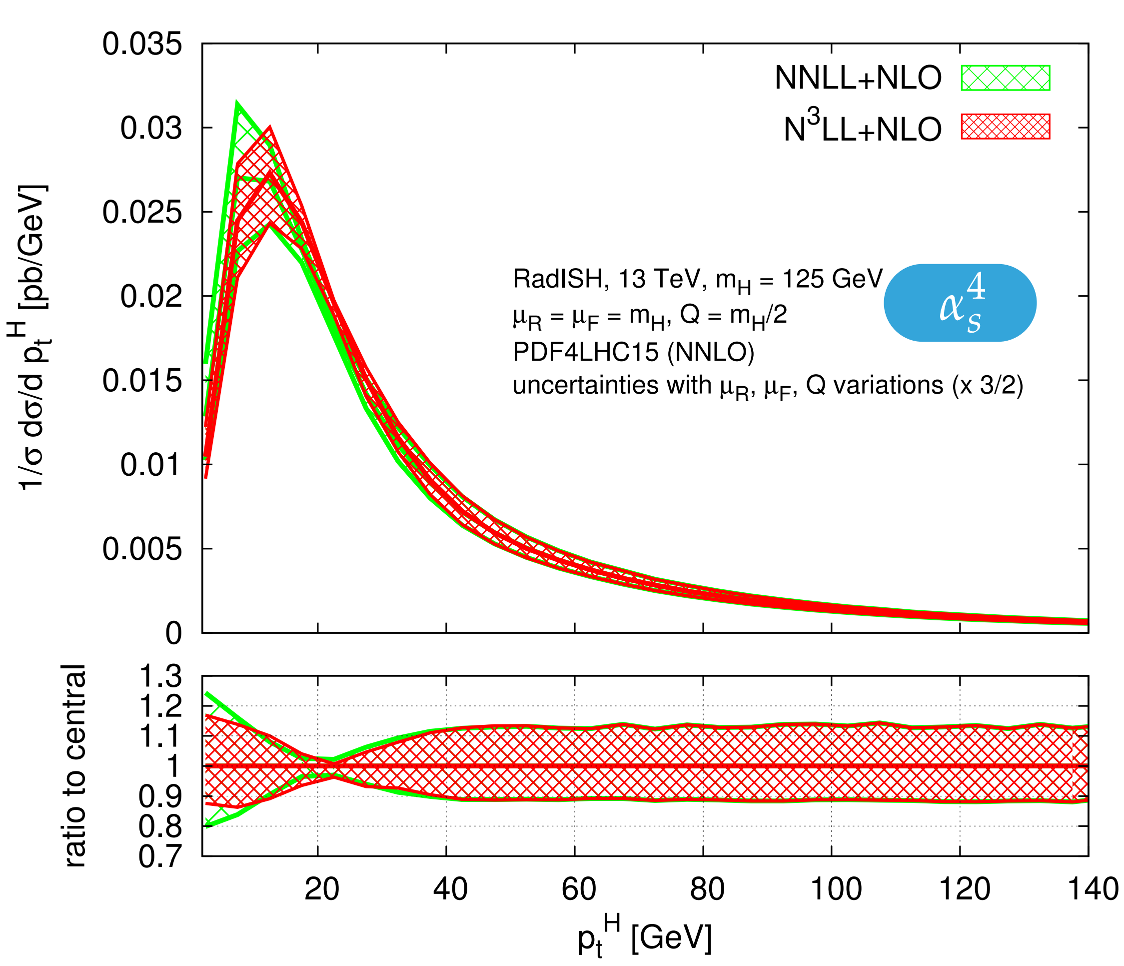
Checks and remarks

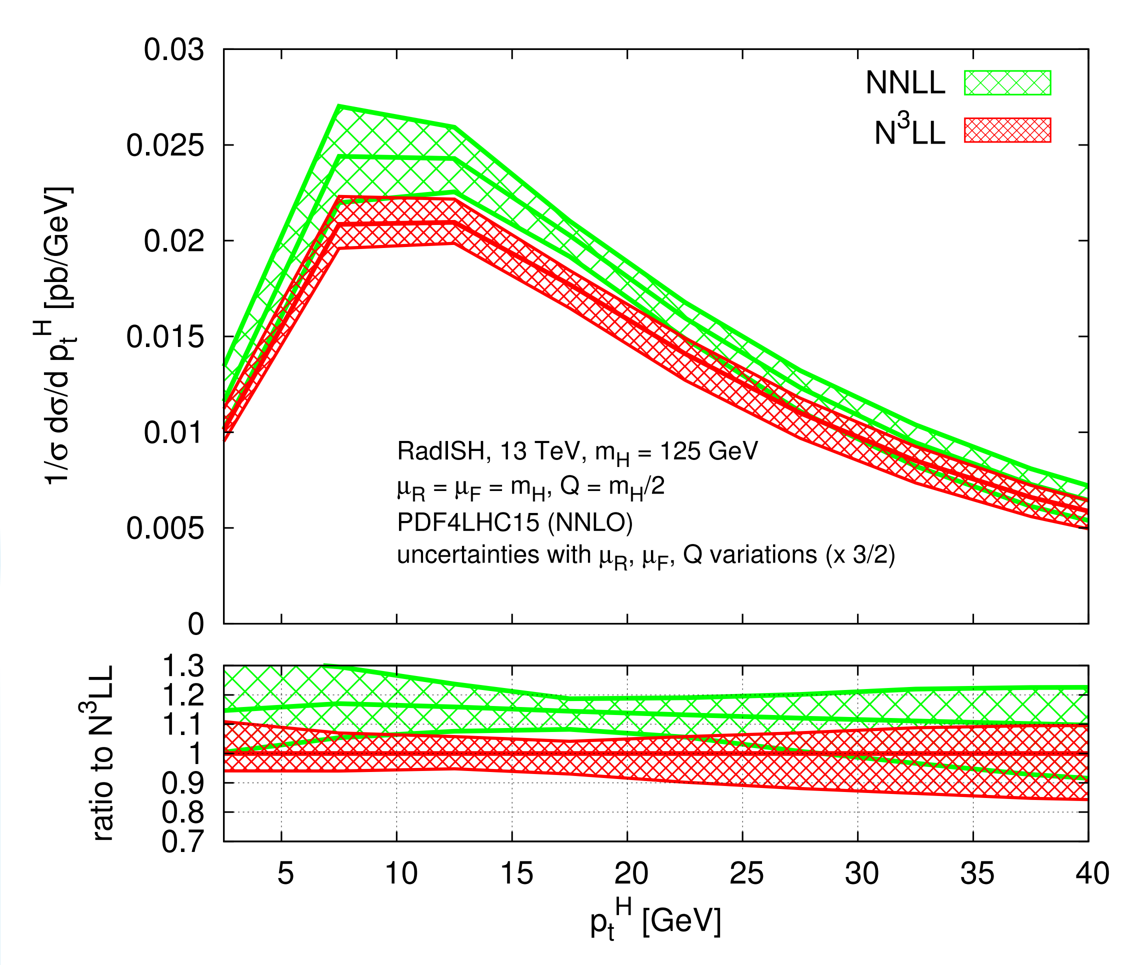
- **b-space** formulation **reproduced analytically** at the resummed level
- **correct scaling** at small p_T computed analytically
- **numerical checks** down to very low p_T against b-space codes (`HqT`, `CuTe`) [Grazzini et al.][Becher et al.]
- check that the FO expansion of the final expression in momentum space up to $O(\alpha^5)$ yields the corresponding expansion in b-space (CSS)
- expansion checked against `MCFM` up to $O(\alpha^4)$ [Campbell et al.]

Matching to fixed order

nb: Cusp anomalous dimension at order α^4 currently unknown set to zero

- Pure N^3LL correction amounts to 10-15% (partly induced by the inclusion of the two-loop coefficient functions)
- Residual scale dependence (μ_R,μ_F,Q) ~10%

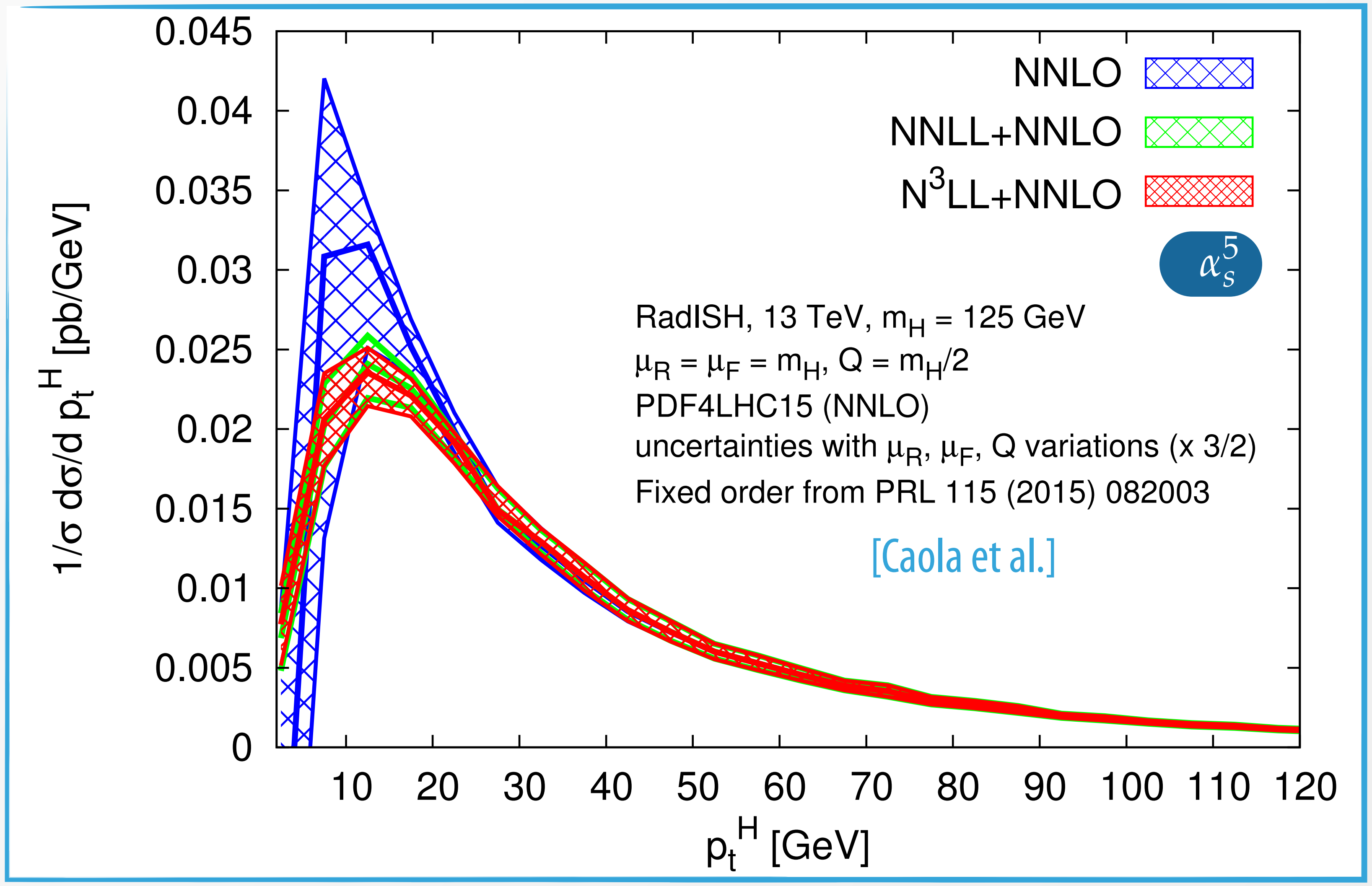




- When matched at NLO, N^3LL correction is *O*(10%) near the peak of the distribution; somewhat larger at small p_T
- Scale ~~uncertainties~~ variations almost halved below 10 GeV, unchanged for larger p_T

Matching to fixed order

- When matched to NNLO, the N^3LL correction is a few % at the peak, and O(10%) at smaller values of p_T
- Rather moderate reduction of scale dependence at N^3LL+NNLO. Need for very stable NNLO distributions below 15 GeV to appreciate reduction. Further runs ongoing
- Mass effects corrections necessary to improve further (see Claudio later)



- Integral of the matched curves yields the N^3LO total cross section [Anastasiou et al.]
- Constant terms at N^3LO recovered thanks to a **multiplicative scheme matching**

Conclusions Part 1

- New formalism for all-order resummation up to **N^3LL accuracy** for inclusive, transverse observables.
- Method formulated in **momentum space**, does not rely on any specific factorization theorem
- Formally equivalent to the standard b-space formalism
- Method allows for an **efficient implementation in a computer code**. Code `RadISH` can process any colour singlet with arbitrary cuts in the Born phase space. Public release soon.
- Extension to more general transverse observables possible thanks to the universality of the Sudakov radiator

$$V(k) = d_l g_l(\phi) \left(\frac{k_T}{M}\right)^a$$

Phenomenological results for the Higgs p_T spectrum:

- N^3LL+NLO correction to the NNLL+NLO spectrum is O(10%) at the peak and below; reduction of scale dependence below the peak.
- N^3LL+NNLO correction to NNLL+NNLO is a few % at the peak and ~10% level below. Moderate reduction of scale dependence, which is now ~10% for the whole spectrum at small p_T

Resummation in the high-p_T region

Mass effects