The spectrum of electro-weak gauge bosons at N³LL+NNLO

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Gehrmann-De Ridder, Gehrmann, Glover, Huss, Monni, Re, LR, Walker, 190x.xxxx

NNLOJET



Transverse observables in colour-singl

Clean experimental and theoretical environment for p

Parameterized as

$$V(k) = \left(\frac{k_t}{M}\right)^a f(\phi)$$

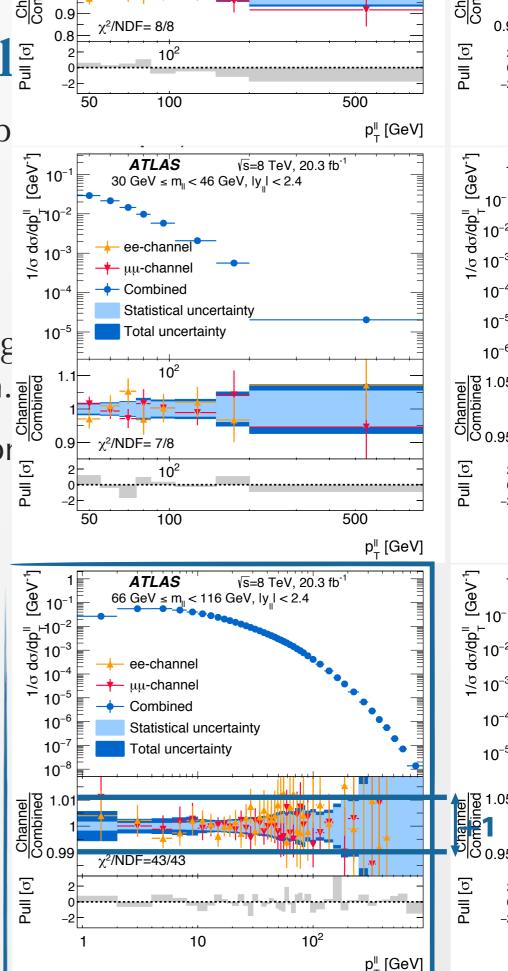
for a **single soft** QCD emission k **collinear** to incoming rapidity of radiation. $V \rightarrow 0$ for soft/collinear radiation.

Inclusive observables (e.g. transverse momentum p_t) proof the colour singlet

$$V(k_1, ...k_n) = V(k_1 + ... + k_n)$$

- negligible or no sensitivity to multi-parton interactions
- reduced sensitivity to non-perturbative effects
- measured extremely precisely at experiments

Very accurate theoretical predictions needed



Precision physics at the LHC: theoretical predictions

$$\sigma(s,Q^2) = \sum_{a,b} \int dx_1 dx_2 f_{a/h_1}(x_1,Q^2) f_{b/h_2}(x_2,Q^2) \hat{\sigma}_{ab\to X}(Q^2, x_1 x_2 s) + \mathcal{O}(\Lambda_{\text{QCD}}^p/Q^p)$$



Input parameters:

strong coupling PDFs

 a_s few percent uncertainty; f improvable

Non-perturbative effects

percent effect; not yet under control

Precision physics at the LHC: theoretical predictions

$$\sigma(s,Q^{2}) = \sum_{a,b} \int dx_{1} dx_{2} f_{a/h_{1}}(x_{1},Q^{2}) f_{b/h_{2}}(x_{2},Q^{2}) \hat{\sigma}_{ab \to X}(Q^{2},x_{1}x_{2}s) + \mathcal{O}(\Lambda_{\text{QCD}}^{p}/Q^{p})$$

$$\hat{\sigma} = \hat{\sigma}_{0}(1 + \alpha_{s}C_{1} + \alpha_{s}^{2}C_{2} + \alpha_{s}^{3}C_{3} + \dots)$$

$$\text{LO NLO NNLO N}^{3}\text{LO}$$

δ~1-5%

NNLO (or even N³LO)

 $\alpha_{\rm s} \sim 0.1$

All-order resummation

Cumulative cross section

$$\Sigma(v) = \int_0^v dV \frac{d\sigma}{dV} \sim \sigma_0 [1 + \alpha_s \# + \alpha_s^2 \# + \dots]$$

Fixed-order prediction: reliable for **inclusive enough** observables and in regions not marred by **soft/collinear radiation** ($v \rightarrow 0$)

Real and virtual contributions can become **highly unbalanced** in processes where the real radiation is strongly constrained by kinematics

Large logarithms appear at all order as a left-over of the real-virtual cancellation of IRC divergences

$$\ln \Sigma(v) = \sum_{n} \left\{ \mathcal{O}(\alpha_s^n L^{n+1}) + \mathcal{O}(\alpha_s^n L^n) + \mathcal{O}(\alpha_s^n L^{n-1}) + \ldots \right\}$$

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Fixed order predictions no longer reliable: all-order resummation of the perturbative series

Case study: transverse momentum p_t

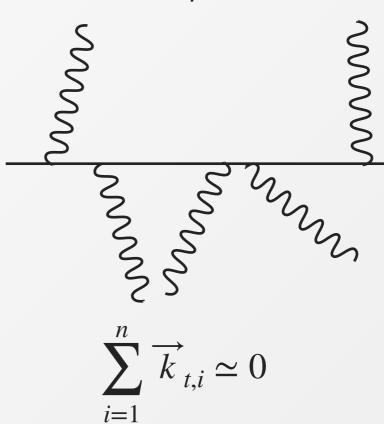
Resummation of transverse momentum is particularly delicate because p_t is a **vectorial quantity**

Two concurring mechanisms leading to a system with small p_t

$$p_t^2 \sim k_{t,i}^2 \ll M^2$$

cross section naturally suppressed as there is no phase space left for gluon emission (Sudakov limit)

Exponential suppression



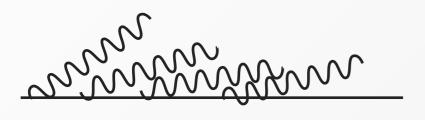
Large kinematic cancellations $p_t \sim 0$ far from the Sudakov limit

Power suppression

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Exponential suppression

Dominant at small p_t

Segmentarial Segments of the Segment of the Segment

$$\sum_{i=1}^{n} \overrightarrow{k}_{t,i} \simeq 0$$

Large kinematic cancellations

 $p_t \sim 0$ far from the Sudakov limit

Power suppression

Resummation in direct and in conjugate space

Phase-space constraints do not usually factorize in direct space

Resummation usually performed in impact-parameter (*b*) space where the two competing mechanisms are handled trough a **Fourier transform**. **Transverse-momentum conservation** is respected

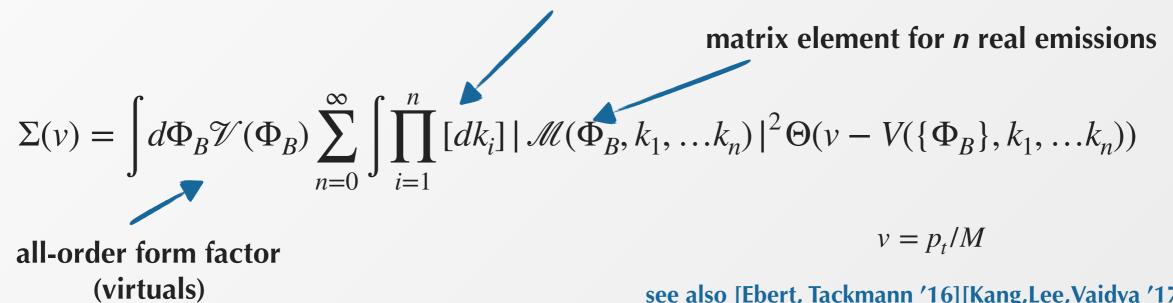
$$\delta\left(\overrightarrow{p}_{t} - \sum_{i=1}^{n} \overrightarrow{k}_{t,i}\right) = \int d^{2}b \frac{1}{4\pi^{2}} e^{i\overrightarrow{b} \cdot \overrightarrow{p}_{t}} \prod_{i=1}^{n} e^{-i\overrightarrow{b} \cdot \overrightarrow{k}_{t,i}}$$

[Parisi, Petronzio '78; Collins, Soper, Sterman '85]

Resummation in direct space: RadISH

[Monni, Re, Torrielli '16, Bizon, Monni, Re, LR, Torielli '17]

single-particle phase space



see also [Ebert, Tackmann '16] [Kang, Lee, Vaidya '17] for alternative approaches within SCET formalism

[Monni, Re, Torrielli '16, Bizon, Monni, Re, LR, Torielli '17]

1. Establish a **logarithmic counting** for the squared matrix element $|\mathcal{M}(\Phi_B, k_1, ...k_n)|^2$

Decompose the squared amplitude in terms of *n*-particle correlated blocks, denoted by $|\tilde{\mathcal{M}}(k_1,...,k_n)|^2$ $(|\tilde{\mathcal{M}}(k_1)|^2 = |\mathcal{M}(k_1)|^2)$

$$\sum_{n=0}^{\infty} |\mathcal{M}(\Phi_B, k_1, ..., k_n)|^2 = |\mathcal{M}_B(\Phi_B)^2$$
*expression valid for inclusive observables
$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \left\{ \prod_{i=1}^{n} \left(|\mathcal{M}(k_i)|^2 + \int [dk_a][dk_b] |\mathcal{M}(k_a, k_b)|^2 \delta^{(2)}(\overrightarrow{k}_{ta} + \overrightarrow{k}_{tb} - \overrightarrow{k}_{ti}) \delta(Y_{ab} - Y_i) \right\} \right\}$$

$$+ \int [dk_a][dk_b][dk_c] |\mathcal{M}(k_a, k_b, k_c)|^2 \delta^{(2)}(\overrightarrow{k}_{ta} + \overrightarrow{k}_{tb} + \overrightarrow{k}_{tc} - \overrightarrow{k}_{ti}) \delta(Y_{abc} - Y_i) + ... \right\}$$

$$\equiv |\mathcal{M}_B(\Phi_B)|^2 \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} |\mathcal{M}(k_i)|^2_{\text{inc}}$$

Upon integration over the phase space, the expansion can be put in a **one to one correspondence** with the logarithmic structure

Systematic recipe to include terms up to the desired logarithmic accuracy

[Monni, Re, Torrielli '16, Bizon, Monni, Re, LR, Torielli '17]

2. Exploit rIRC safety to single out the IRC singularities of the real matrix element and achieve the cancellation of the exponentiated divergences of virtual origin

Introduce a slicing parameter $\epsilon \ll 1$ such that all inclusive blocks with $k_{t,i} < \epsilon k_{t,1}$, with $k_{t,1}$ hardest emission, can be neglected in the computation of the observable

$$\Sigma(v) = \int d\Phi_{B} |\mathcal{M}_{B}(\Phi_{B})|^{2} \mathcal{V}(\Phi_{B})$$
 unresolved emissions
$$\times \int [dk_{1}] |\mathcal{M}(k_{1})|_{\mathrm{inc}}^{2} \left(\sum_{l=0}^{\infty} \frac{1}{l!} \int \prod_{j=2}^{l+1} [dk_{j}] |\mathcal{M}(k_{j})|_{\mathrm{inc}}^{2} \Theta(\epsilon V(k_{1}) - V(k_{j})) \right)$$

$$\times \left(\sum_{m=0}^{\infty} \frac{1}{m!} \int \prod_{i=2}^{m+1} [dk_{i}] |\mathcal{M}(k_{i})|_{\mathrm{inc}}^{2} \Theta(V(k_{i}) - \epsilon V(k_{1})) \Theta\left(v - V(\Phi_{B}, k_{1}, \dots, k_{m+1})\right) \right)$$

resolved emissions

Unresolved emission doesn't contribute to the evaluation of the observable: it can be exponentiated directly and employed to cancel the virtual divergences, giving rise to a Sudakov radiator

$$\mathcal{V}(\Phi_B) \exp \left\{ \int [dk] |\mathcal{M}(k)|_{\text{inc}}^2 \Theta(\epsilon V(k_1) - V(k)) \right\} \simeq e^{-R(\epsilon V(k_1))}$$

[Monni, Re, Torrielli '16, Bizon, Monni, Re, LR, Torielli '17]

Final result at NLL

$$\frac{d\Sigma(v)}{d\Phi_{B}} = \int \frac{dk_{t,1}}{k_{t,1}} \int_{0}^{2\pi} \frac{d\phi_{1}}{2\pi} e^{-R(k_{t,1})} e^{R'(k_{t,1})} \mathcal{L}_{NLL}(k_{t,1}) R'(k_{t,1})$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^{1} \frac{d\zeta_{i}}{\zeta_{i}} \int_{0}^{2\pi} \frac{d\phi_{i}}{2\pi} R'(\zeta_{i}k_{t,1}) \Theta\left(v - V(\Phi_{B}, k_{1}, ..., k_{n+1})\right)$$

Parton luminosity at NLL reads

$$\mathcal{L}_{\text{NLL}}(k_{t,1}) = \sum_{c} \frac{d |M_B|_{c\bar{c}}^2}{d\Phi_B} f_c(x_1, k_{t,1}^2) f_{\bar{c}}(x_2, k_{t,1}^2)$$

At higher logarithmic accuracy, it includes **coefficient functions** and **hard-virtual** corrections

All ingredients to perform resummation at N³LL accuracy are now available [Catani et al. '11, '12][Gehrmann et al. '14][Li, Zhu '16, Vladimirov '16][Moch et al. '18, Lee et al. '19]

Fixed-order predictions now available at NNLO

[A. Gehrmann-De Ridder et al. '15, 16, '17] [Boughezal et al. '15, 16]

Matching to fixed order: multiplicative matching

Cumulative cross section should reduce to the fixed order at large v

$$\Sigma_{\rm matched}^{\rm mult}(v) \sim \Sigma_{\rm res}(v) \left[\frac{\Sigma_{\rm f.o.}(v)}{\Sigma_{\rm res}(v)} \right]_{\rm expanded}^{*}$$

- allows to include constant terms from NNLO (if N³LO total xs available)
- physical suppression at small v cures potential instabilities

*actual scheme slightly more involved, see backup slides

To ensure that resummation does not affect the hard region of the spectrum when the matching is performed we introduce **modified logarithms**

This corresponds to restrict the rapidity phase space at large k_t

$$\ln(Q/k_{t1}) \to \frac{1}{p} \ln \left(1 + \left(\frac{Q}{k_{t1}} \right)^p \right)$$

Q: perturbative resummation scale used to probe the size of subleading logarithmic corrections

p: arbitrary matching parameter

Theoretical predictions for Z and W observables at 13 TeV

Results obtained using the following fiducial cuts (agreed with ATLAS)

$$Z p_t^{\ell^{\pm}} > 25 \,\text{GeV}, \quad |\eta^{\ell^{\pm}}| < 2.5, \quad 66 \,\text{GeV} < M_{\ell\ell} < 116 \,\text{GeV}$$

$$W p_t^{\ell} > 25 \,\text{GeV}, \quad |\eta^{\ell}| < 2.5, \quad E_T^{\nu_{\ell}} > 25 \,\text{GeV}, \quad m_T > 50 \,\text{GeV}$$

using NNPDF3.1 with $\alpha_s(M_Z)$ =0.118 and setting the central scales to

$$\mu_R = \mu_F = M_T = \sqrt{M_{\ell\ell'}^2 + p_T^2}, \quad Q = \frac{M_{\ell\ell'}}{2}$$

5 flavour (massless) scheme: no HQ effects and no PDF thresholds

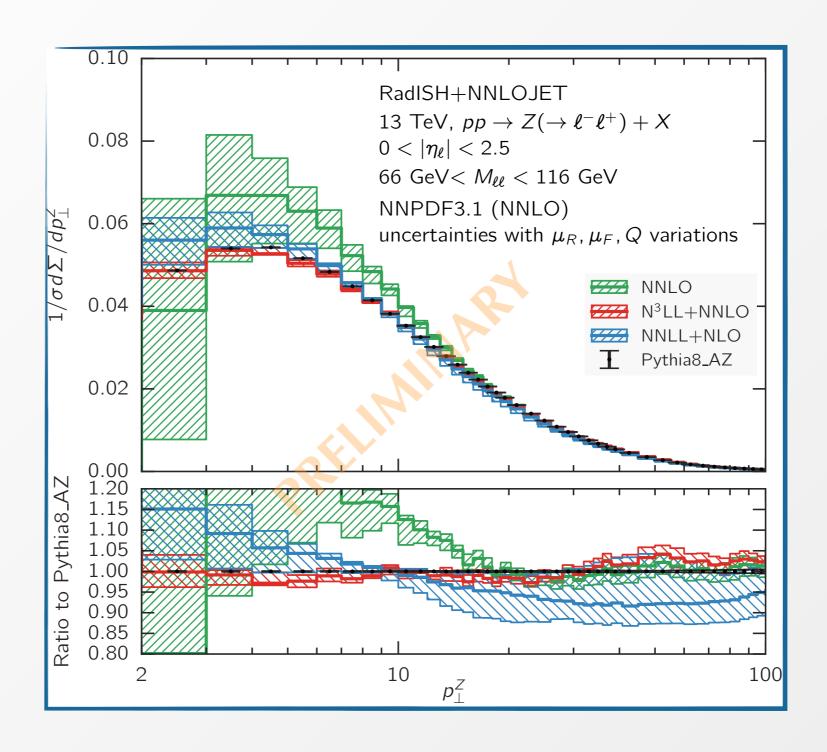
NB: quark thresholds now implemented in the resummation, not turned on in the following results

Scale uncertainties estimated by varying **renormalization** and **factorization** scale by a factor of two around their central value (**7 point variation**) and varying the **resummation** scale by a factor of 2 around its central value for factorization and renormalization scales set to their central value: **9 point envelope**

Matching parameter *p* set to 4 as a default

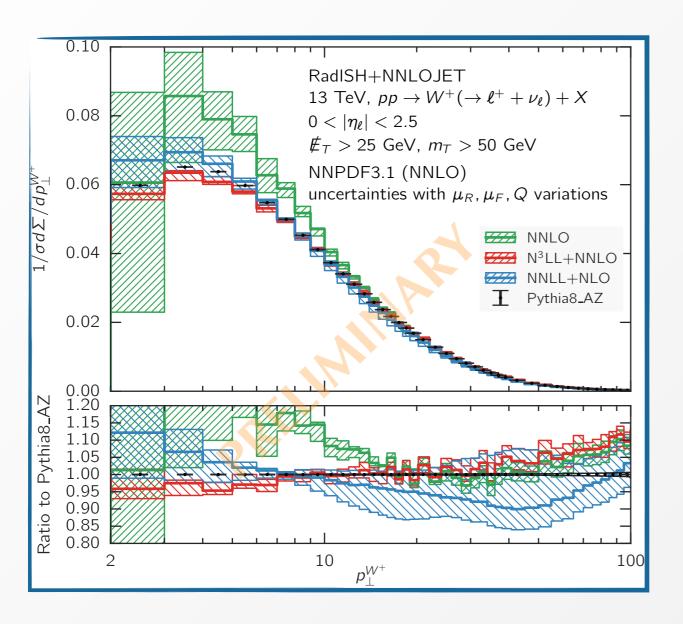
No non perturbative parameters included in the following

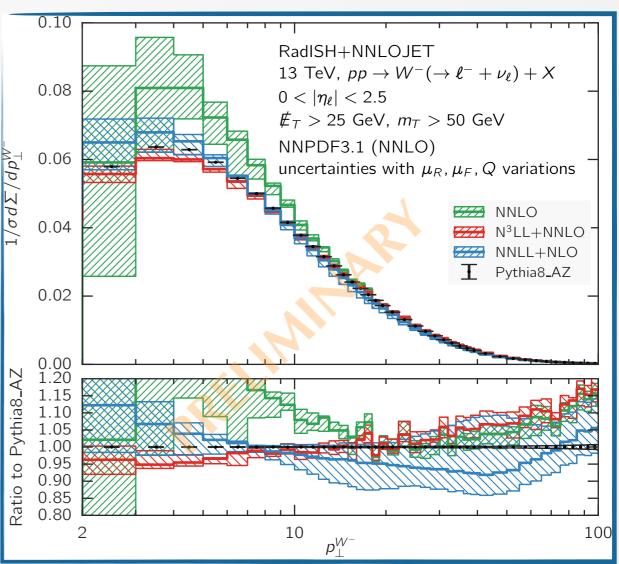
Results for the p_{\perp}^{Z} distribution



Thanks to Jan Kretzschmar for providing the Phytia8 AZ tune results

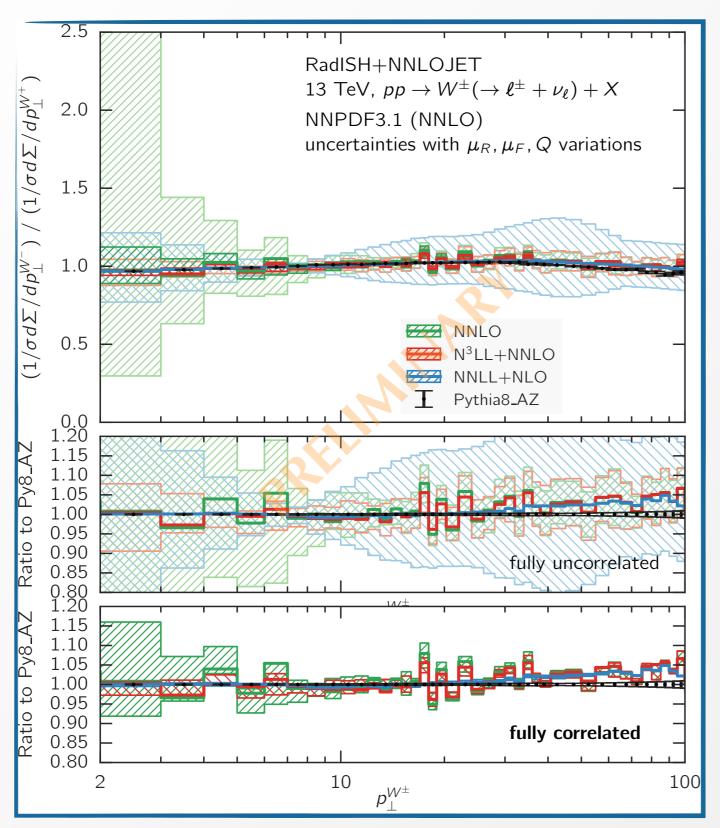
Results for the $p_{\perp}^{W^{\pm}}$ distribution





Discrepancies with MC still needs some further understanding (does the tune work out of the box at 13 TeV?)

Results for the $p_{\perp}^{W^-}/p_{\perp}^{W^+}$ distribution



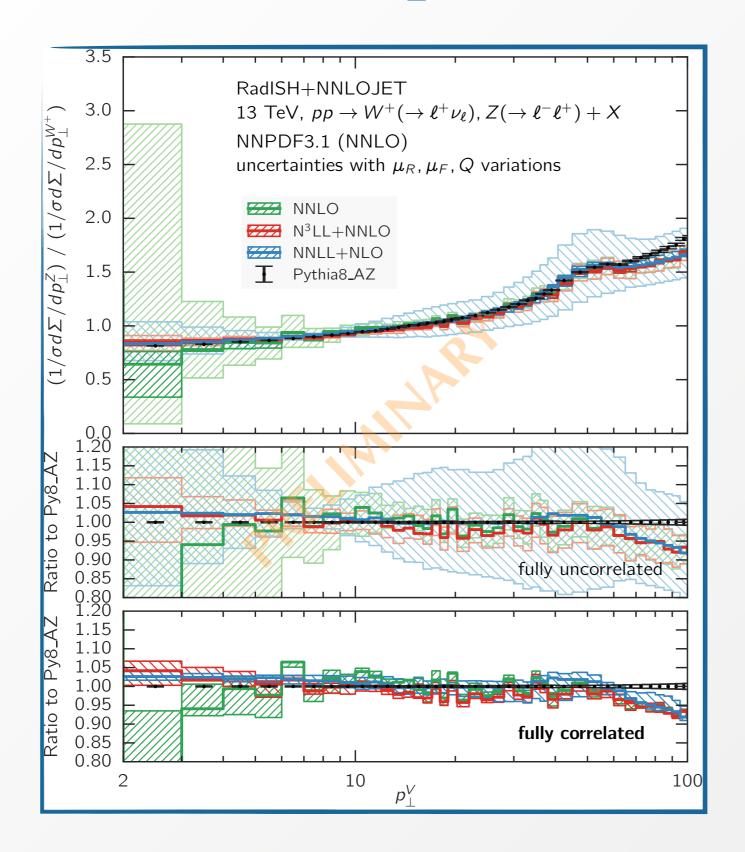
N³LL+NNLO and NNLL+NLO description of the spectrum almost identical: virtually no change if theoretical accuracy changes

This may suggest that **fully correlated uncertainties** are appropriate (massless QCD description)

Non necessarily the case for extra sources of uncertainty (mass effects, EW corrections...)

Thanks to Jan Kretzschmar for providing the Phytia8 AZ tune results

Results for the $p_{\perp}^{Z}/p_{\perp}^{W^{+}}$ distribution



N³LL+NNLO and NNLL+NLO description of the spectrum almost identical: virtually no change if theoretical accuracy changes

This supports the use of **fully correlated uncertainties** (massless QCD description)

Non necessarily the case for extra sources of uncertainty (mass effects, EW corrections...)

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Benchmarking among different resummation codes

At this level of accuracy, it becomes increasingly important to compare different resummed result with the **same formal accuracy** to quantify the effect of **subleading** differences and to assess the so-called '**algorithmic**' uncertainties (e.g. matching, modified logarithms, scale variations, etc)

The roadmap for benchmark exercise is now (almost) final

Step 1

Setup:

- a) On-shell (Q=MZ) at sqrts = 13 TeV (integrated over y)
- b) five generations, all massless
- c) PDF choice: NNPDF31_nnlo_as_0118 with evolution from LHAPDF. Evolve from alpha_s(mZ)=0.118 as relevant for each resummation order.
- d) Include 2 levels of resummation results: 1. canonical logarithms (as much as possible, ln(QbT/b0), ln(qT/Q)) in b (if possible) and qT space, 2. Nominal logarithms in qT space. Summarized in Frank's slide 4.
- e) Order of resummation: LL and NLL('). Other points can be added in parallel (NNLL('),N3LL (if possible)) for step 2 (see 1_c above).
- f) Binning in qT and in b_T space for b0/bT (if possible), $\{1,2,3,\ldots,39,40,45,\ldots,95,100\}$. Also include the log binning for qT: log10 qT $\{-1,-0.9,\ldots,1.9,2\}$.

Results:

- g) With Z couplings to all five generations
- h) Limit the Z couplings to the first generation, and then add also the second generation
- i) Q=MZ and two fixed y values: 0 and 2.4. To be done for both g) and h) above

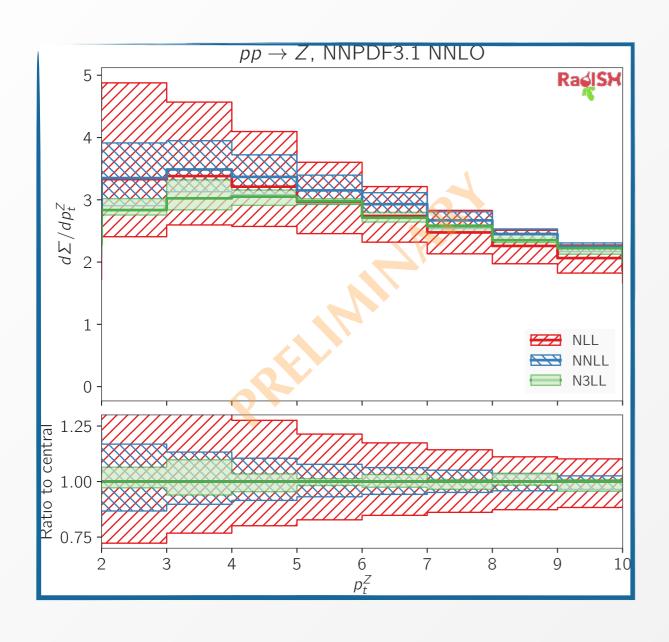
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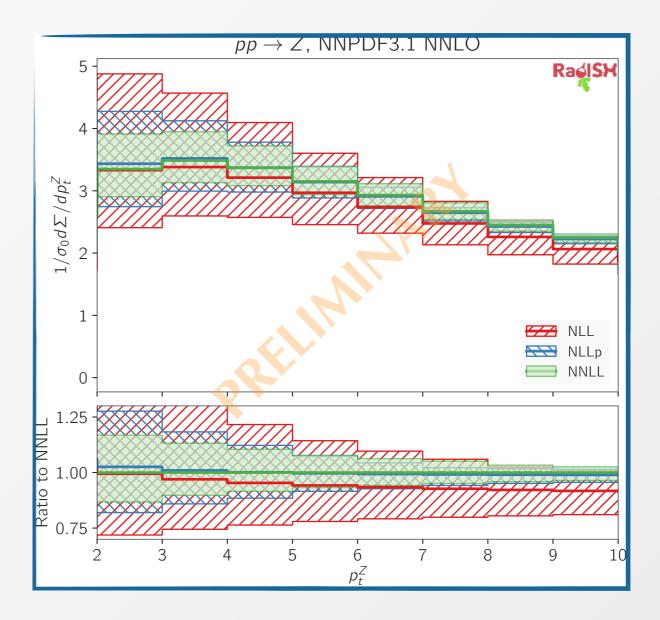
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We plan to start with the following:

```
Step 1
Setup:
a) On-shell (Q=MZ) at sqrts = 13 TeV (integrated over y)
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Benchmarking among different resummation codes





Recapitulation

- No sign of NP at the LHC so far necessary to perform detailed theory/ experimental comparisons, to look for deviations from SM. Perturbation theory must be pushed to its limit
- New formalism formulated in direct space for all-order resummation up to N³LL accuracy for inclusive, transverse observables.
- Preliminary results at NNLO+N³LL for *W* and *Z* differential distributions with **uncertainties at the few percent level**. Some discrepancies with the Pythia8 AZ tune results to be understood.
- Preliminary results on the W+/W- ratios and Z/W ratios. Uncertainty <5-10% even if the error is taken as fully uncorrelated. The excellent convergence of the perturbative corrections supports **fully correlated** scale variations, with uncertainties **at the 1-2% level**.

Backup

Result at NLL accuracy can be written as

$$\Sigma(v) = \sigma^{(0)} \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(\epsilon v_1)} R'(v_1) \qquad v_i = V(k_i), \quad \zeta_i = v_i / v_1$$

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'(\zeta_i v_1) \Theta\left(v - V(\Phi_B, k_1, ..., k_{n+1})\right)$$

Formula can be evaluated with Monte Carlo method; dependence on ϵ vanishes exactly and result is finite in four dimensions

It contains **subleading effect** which in the original CAESAR approach are disposed of by expanding *R* and *R'* around *v*

$$R(\epsilon v_1) = R(v) + \frac{dR(v)}{d \ln(1/v)} \ln \frac{v}{\epsilon v_1} + \mathcal{O}\left(\ln^2 \frac{v}{\epsilon v_1}\right)$$

$$R'(v_i) = R'(v) + \mathcal{O}\left(\ln \frac{v}{v_i}\right)$$

Not possible! valid only if the ratio v_i/v remains of order one in the whole emission phase space, but for observables which feature kinematic cancellations there are configurations with $v_i \gg v$. **Subleading effects necessary**

[Monni, Re, Torrielli '16, Bizon, Monni, Re, LR, Torielli '17]

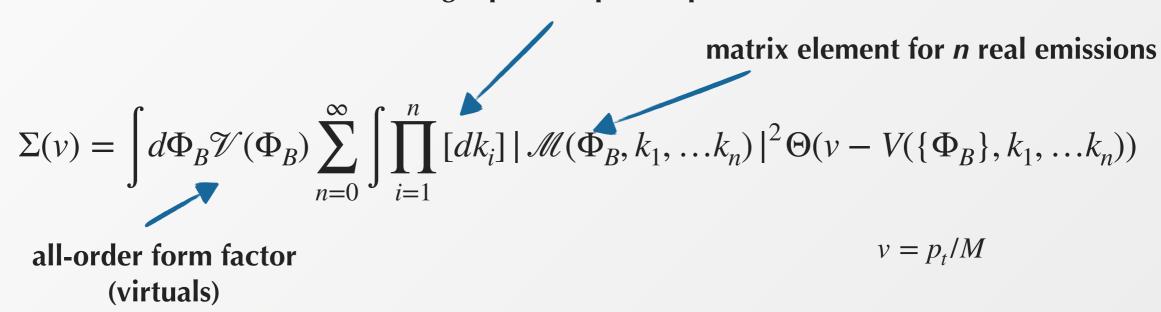
Translate the resummability of the observable into properties of the observable in the presence of multiple radiation: recursive infrared and collinear (rIRC) safety

[Banfi, Salam, Zanderighi '01, '03, '04]

Existence of a **resolution scale** q_0 , **independent of the observable**, such that emissions below q_0 (**unresolved**) do not contribute significantly to the observable's value.

Starting point: all-order cumulative cross section

single-particle phase space



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$$\Sigma(v) = \sigma^{(0)} \int \frac{dk_{t1}}{k_{t1}} \int_{0}^{2\pi} \frac{d\phi_{1}}{2\pi} e^{-R(\epsilon k_{t1})} R'(k_{t1}) \qquad \qquad \zeta_{i} = k_{ti}/k_{t1}$$

$$\times \sum_{v=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^{1} \frac{d\zeta_{i}}{\zeta_{i}} \int_{0}^{2\pi} \frac{d\phi_{i}}{2\pi} R'(\zeta_{i}k_{t1}) \Theta\left(v - V(\Phi_{B}, k_{t1}, ..., k_{tn+1})\right)$$

Formula can be evaluated with Monte Carlo method; dependence on ϵ vanishes exactly and result is finite in four dimensions

Convenient to perform an expansion around k_{t1} (more efficient and simpler implementation)

$$R(\epsilon k_{t1}) = R(k_{t1}) + \frac{dR(k_{t1})}{d\ln(1/k_{t1})} \ln\frac{1}{\epsilon} + \mathcal{O}\left(\ln^2\frac{1}{\epsilon}\right)$$

$$R'(k_{ti}) = R'(k_{t1}) + \mathcal{O}\left(\ln\frac{k_{t1}}{k_{ti}}\right)$$

Subleading effects retained: no divergence at small *v*, power-like behaviour respected

Logarithmic accuracy defined in terms of $ln(M/k_{t1})$

Result formally equivalent to the b-space formulation

Matching to fixed order: multiplicative matching

Cumulative cross section should reduce to the fixed order at large *v*

$$\Sigma_{\text{matched}}^{\text{mult}}(v) = \Sigma_{\text{res}}(v) \left[\frac{\Sigma_{\text{f.o.}}(v)}{\Sigma_{\text{res}}(v)} \right]_{\text{expanded}}$$

- allows to include constant terms from NNLO
- trom NNLOphysical suppression at small v cures potential instabilities

To ensure that resummation does not affect the hard region of the spectrum when the matching is performed we introduce modified logarithms This corresponds to restrict the rapidity phase space at large k_t

$$\ln(Q/k_{t1}) \to \frac{1}{p} \ln \left(1 + \left(\frac{Q}{k_{t1}} \right)^p \right)$$

Q: perturbative resummation scale used to probe the size of subleading logarithmic corrections

p: arbitrary matching parameter

Matching improved by normalizing to the **asymptotic value to** avoid **spurious** $\mathcal{O}(\alpha_s^4)$ contributions

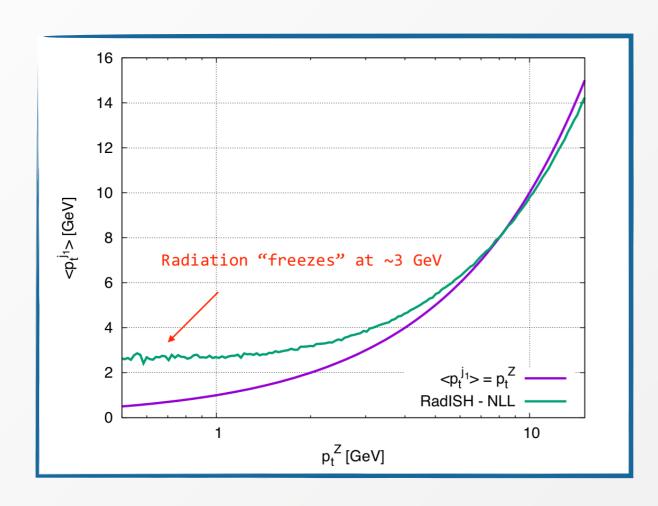
$$\Sigma_{\text{mult}}^{\text{matched}}(v) = \frac{\Sigma^{\text{res}}(v)}{\Sigma_{\text{asym.}}^{\text{res}}} \left[\Sigma_{\text{asym.}}^{\text{res}} \frac{\Sigma^{\text{f.o.}}(v)}{\Sigma^{\text{exp}}(v)} \right]_{\text{expanded}} \qquad \Sigma_{\text{asym.}}^{\text{res}} = \int_{\text{with cuts}} d\Phi_B \quad \left(\lim_{L \to 0} \mathcal{L}_{\text{N}^k \text{LL}} \right)$$

The Landau pole and the small-p_t limit

Running coupling $\alpha_s(k_{t1}^2)$ and Sudakov radiator hit Landau pole at

$$\alpha_s(\mu_R^2)\beta_0 \ln Q/k_{t1} = \frac{1}{2}$$
 $k_{t1} \sim 0.01 \,\text{GeV}, \quad \mu_R = Q = m_Z$

Only real cutoff in the calculation: emission probability is set to zero below this scale and parton densities are frozen.

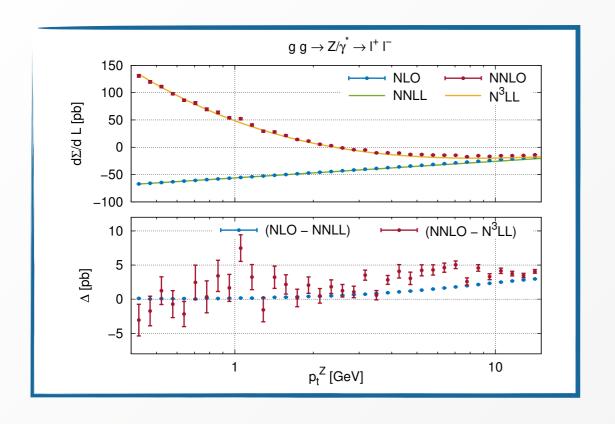


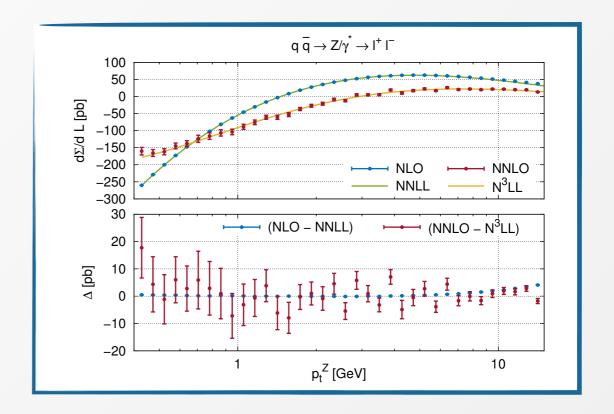
At small p_t the large azimuthal cancellations dominate over the Sudakov suppression: the cutoff is never an issue in practice

$$\frac{d^2\Sigma(v)}{dp_t d\Phi_B} \simeq 2\sigma^{(0)}(\Phi_B)p_t \left(\frac{\Lambda_{\rm QCD}^2}{M^2}\right)^{\frac{16}{25}\ln\frac{41}{16}}$$

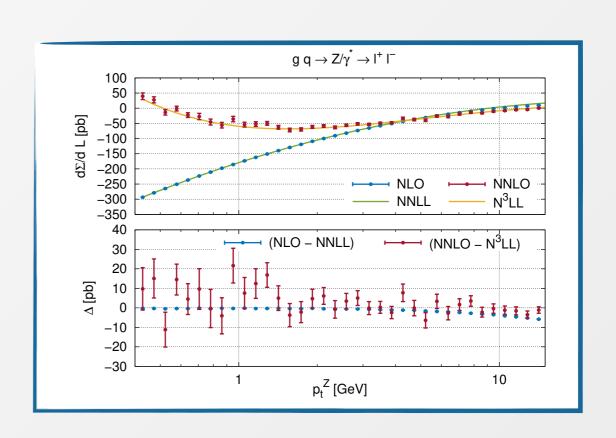
No NP parameters included in the following

Fixed order vs. resummation

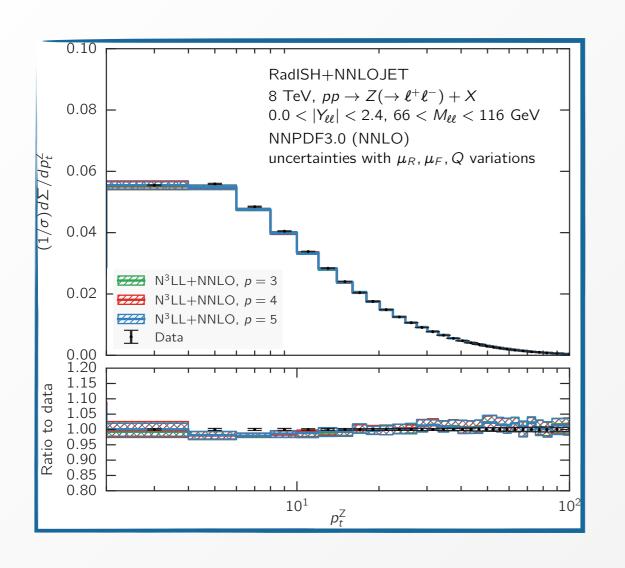


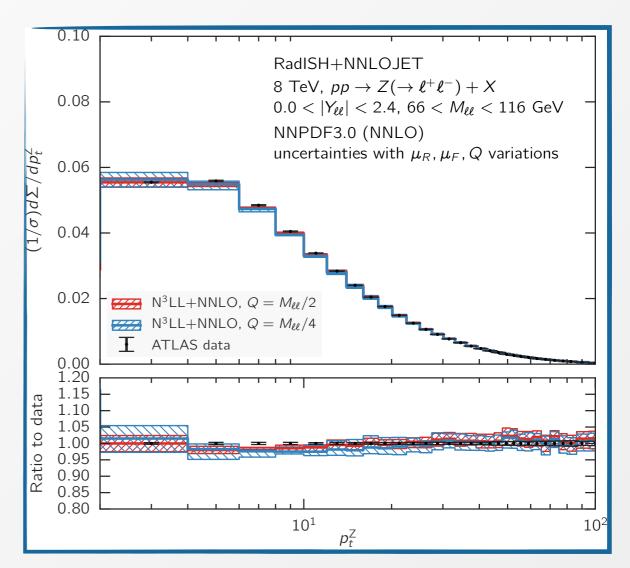


- Very good agreement with the fixed order at small p_t
- Very strong validation of both calculations
- Fixed an implementational error in the fixed order computation



Resummation and matching uncertainties

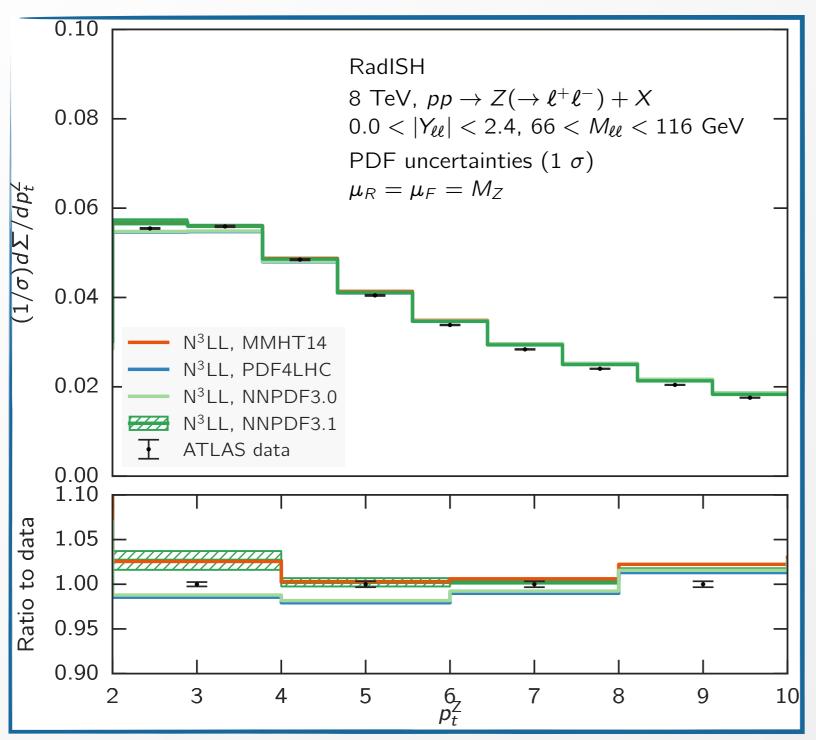




 Matching uncertainties at the sub percent level Predictions stable wrt variation of central value of the resummation scale

PDF uncertainties

Beware of different PDFs and central scales



- Uncertainty with state-of-theart PDFs at the 1-2% level
- Spectrum gets slightly harder than NNPDF3.0 (used in our current studies)