

Leptonic Dirac CP Phase with Residual Symmetry & μ DAR

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SFG, Duane A. Dicus, Wayne W. Repko, PLB **702**, 220 (2011) [arXiv:1104.0602]

SFG, Duane A. Dicus, Wayne W. Repko, PRL **108**, 041801 (2012) [arXiv:1108.0964]

Andrew D. Hanlon, **SFG**, Wayne W. Repko, PLB **729**, 185-191 (2014) [arXiv:1308.6522]

SFG [arXiv:1406.1985]

Jarah Evslin, **SFG**, Kaoru Hagiwara, JHEP **1602** (2016) 137 [arXiv:1506.05023]

SFG, Pedro Pasquini, M. Tortola, J. W. F. Valle, PRD **95** (2017) No.3, 033005 [arXiv:1605.01670]

SFG, Alexei Smirnov, JHEP **1610** (2016) 138 [arXiv:1607.08513]

SFG [arXiv:1704.08518]

ν Oscillation Data

(for NH)	-1σ	Best Value	$+1\sigma$
$\Delta m_s^2 \equiv \Delta m_{12}^2$ (10^{-5}eV^2)	7.37	7.56	7.75
$ \Delta m_a^2 \equiv \Delta m_{13}^2 $ (10^{-3}eV^2)	2.51	2.55	2.59
$\sin^2 \theta_s$ ($\theta_s \equiv \theta_{12}$)	0.305 (33.5°)	0.321 (34.5°)	0.339 (35.6°)
$\sin^2 \theta_a$ ($\theta_a \equiv \theta_{23}$)	0.412 (39.9°)	0.430 (41.0°)	0.450 (42.1°)
$\sin^2 \theta_r$ ($\theta_r \equiv \theta_{13}$)	0.02080 (8.29°)	0.02155 (8.44°)	0.02245 (8.62°)
δ_D, δ_{Mi}	?, ??	?, ??	?, ??

Salas, Forero, Ternes, Tortola & Valle, arXiv:1708.01186

Evidence of μ - τ Symmetry

- Two small deviations (1σ level):

$$-3.5^\circ < \theta_a - 45^\circ < 5.8^\circ \quad 8.4^\circ < \theta_r < 9.2^\circ$$

with **Best Fit Value**: $\theta_a - 45^\circ = -3.9^\circ$ & $\theta_r = 8.8^\circ$.

- Zeroth Order Approximation:

$$\theta_a \approx 45^\circ, \quad \theta_r \approx 0^\circ.$$

\Rightarrow **CP & μ - τ Symmetric** Mass Matrix:

$$M_\nu^{(0)} = \begin{pmatrix} A & \mathbf{B} & \mathbf{B} \\ & \mathbf{C} & \mathbf{D} \\ & & \mathbf{C} \end{pmatrix}$$

Mohapatra & Nussinov [hep-ph/9809415], Lam [hep-ph/0104116]

- Mass Matrix M_ν invariant under **Transformation**:

$$G_\nu^T M_\nu G_\nu = M_\nu$$

- **Diagonalization**:

$$V_\nu^T M_\nu V_\nu = D_\nu$$

- **Rephasing**:

$$D_\nu = d_\nu^T D_\nu d_\nu$$

with $d_\nu^2 = I_3 \Rightarrow d_\nu = \text{diag}(\pm, \pm, \pm)$.

- Together

$$\begin{aligned} M_\nu &= G_\nu^T M_\nu G_\nu = \underline{G_\nu^T V_\nu^* D_\nu V_\nu^\dagger G_\nu} \\ &= \underline{V_\nu^* D_\nu V_\nu^\dagger} = \underline{V_\nu^* d_\nu^T D_\nu d_\nu V_\nu^\dagger} \end{aligned}$$

- **Consequence**: $V_\nu^\dagger G_\nu = d_\nu V_\nu^\dagger \Leftrightarrow G_\nu = V_\nu d_\nu V_\nu^\dagger$

- **For Leptons**: $\underline{F_\ell} = \underline{V_\ell d_\ell V_\ell^\dagger}$ with $d_\ell = \text{diag}(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3})$.

- **Two Nontrivial Independent** possibilities of \mathbf{d}_ν :

$$\mathbf{d}_\nu^{(1)} = \text{diag}(-1, 1, 1), \quad \mathbf{d}_\nu^{(2)} = \text{diag}(1, -1, 1), \quad \mathbf{d}_\nu^{(3)} = -\mathbf{d}_\nu^{(1)}\mathbf{d}_\nu^{(2)}.$$

- θ_s parameterized in terms of \mathbf{k} : $\tan \theta_s = \sqrt{2}/k$

$$V_\nu(k) = \begin{pmatrix} \frac{k}{\sqrt{2+k^2}} & \frac{\sqrt{2}}{\sqrt{2+k^2}} & 0 \\ \frac{1}{\sqrt{2+k^2}} & \frac{k}{\sqrt{2(2+k^2)}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2+k^2}} & \frac{k}{\sqrt{2(2+k^2)}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad \begin{array}{ll} \mathbf{k} = 2 & \theta_s = 35.3^\circ \text{ [TBM]} \\ \mathbf{k} = \frac{3}{\sqrt{2}} & \theta_s = 33.7^\circ \\ \mathbf{k} = \sqrt{6} & \theta_s = 30.0^\circ \end{array}$$

- **Two Independent Symmetry Transformations** $\mathbf{G}_i = \mathbf{V}_\nu \mathbf{d}_\nu^{(i)} \mathbf{V}_\nu^\dagger$

$$\mathbf{G}_1 = \frac{1}{2+k^2} \begin{pmatrix} 2-k^2 & 2k & 2k \\ & k^2 & -2 \\ & & k^2 \end{pmatrix}, \quad \mathbf{G}_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

- $\mathbb{Z}_2^S (\times \overline{\mathbb{Z}}_2^S) \times \mathbb{Z}_2^{\mu T} \equiv \mathcal{G} = \{\mathbf{E}, \mathbf{G}_1, \mathbf{G}_2 (\equiv \mathbf{G}_1 \mathbf{G}_3), \mathbf{G}_3\}$

- Full Symmetries:

$\mathcal{H} \equiv \mathcal{G} \times \mathcal{F}$	\mathcal{G}	\mathcal{F}
S_4	$\mathbb{Z}_2^S \times \mathbb{Z}_2^{\mu T}$	$\mathbb{Z}_3 \equiv \{I, F, F^2\}$
$\{G_1, G_3, F\}$	$G_1(G_2), G_3$	$F \equiv \text{diag}(1, \omega, \omega^2)$

Bottom-Up \uparrow

\downarrow Top-Down

See also Smirnov et. al., 1204.0445, 1212.2149, 1510.00344

- Residual Symmetries:

$$\nu_i: \mathcal{G} \equiv \mathbb{Z}_2^S(\overline{\mathbb{Z}}_2^S) \times \mathbb{Z}_2^{\mu T} \quad \text{for} \quad d_\nu^i = \text{diag}(\pm 1, \pm 1, \pm 1)$$

$$\ell_i: \mathcal{F} \in U(1) \times U(1) \quad \text{for} \quad d_\ell^i = \text{diag}(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3})$$

- Lepton's Representation:

$$\begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} \sim \mathbf{3}, \quad \begin{matrix} e_R \sim \mathbf{1} \\ \mu_R \sim \mathbf{1}' \\ \tau_R \sim \mathbf{1}'' \end{matrix}, \quad \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix} \sim \mathbf{3}.$$

- A_4 invariant Lagrangian:

$$\begin{aligned} \mathcal{L}_\ell &= \mathbf{y}_1 \bar{e}_R (\mathbf{1} \varphi_1^\dagger e_L + \mathbf{1} \varphi_2^\dagger \tau_L + \mathbf{1} \varphi_3^\dagger \tau_L) \\ &+ \mathbf{y}_2 \bar{\mu}_R (\omega \varphi_1^\dagger e_L + \mathbf{1} \varphi_2^\dagger \tau_L + \omega^2 \varphi_3^\dagger \tau_L) \\ &+ \mathbf{y}_3 \bar{\tau}_R (\omega^2 \varphi_1^\dagger e_L + \mathbf{1} \varphi_2^\dagger \tau_L + \omega \varphi_3^\dagger \tau_L). \end{aligned}$$

- Mass term with $\langle \varphi_i \rangle = v_i$:

$$\mathcal{L}_\ell = \begin{pmatrix} \bar{e}_R & \bar{\mu}_R & \bar{\tau}_R \end{pmatrix} \begin{pmatrix} \mathbf{y}_1 & & \\ & \mathbf{y}_2 & \\ & & \mathbf{y}_3 \end{pmatrix} \begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \omega & \mathbf{1} & \omega^2 \\ \omega^2 & \mathbf{1} & \omega \end{pmatrix} \begin{pmatrix} \mathbf{v}_1 & & \\ & \mathbf{v}_2 & \\ & & \mathbf{v}_3 \end{pmatrix} \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}.$$

$$\mathbf{v}_1 = \mathbf{v}_2 = \mathbf{v}_3 = \mathbf{v} \Rightarrow U_{\ell,R} = I, \mathbf{U}_{\ell,L}(\omega), m_{\ell,i} = \mathbf{y}_i \mathbf{v}.$$

$$\mathbf{y}_1 = \mathbf{y}_2 = \mathbf{y}_3 = \mathbf{y} \Rightarrow U_{\ell,L} = I, \mathbf{U}_{\ell,R}(\omega), m_{\ell,i} = \mathbf{y} \mathbf{v}_i.$$

Residual Symmetry as Effective Theory

- Full symmetry **HAS TO** be **Broken!**
 - Fermion needs to acquire mass.
 - Non-trivial mixing $V_{\text{PMNS}} = V_{\ell}^{\dagger} V_{\nu}$
- If mixing is **TRUELY determined by symmetry**, it has to be **residual symmetry**
 - VEVs
 - Yukawa couplings
- **Residual Symmetry** as **Custodial Symmetry**
 - **Gauge symmetry has to be broken.** Otherwise, no mixing.
 - **Weak mixing angle** is a function of gauge couplings, which **cannot be dictated by gauge symmetry** (and VEV).
 - **Weak mixing angle is related to** the physical observables, the **gauge boson masses**, by **custodial symmetry**.

Partial Residual Symmetry \mathbb{Z}_2^s or $\overline{\mathbb{Z}}_2^s$

- Although $\mathbb{Z}_2^{\mu\tau}$, represented by \mathbf{G}_3 , is Broken!
- **No particular reason** for \mathbb{Z}_2^s or $\overline{\mathbb{Z}}_2^s$ to be Broken!

$$\mathbb{Z}_2^s : \mathbf{G}_1(\mathbf{k}) = \frac{1}{2+k^2} \begin{pmatrix} 2-k^2 & 2k & 2k \\ & k^2 & -2 \\ & & k^2 \end{pmatrix},$$
$$\overline{\mathbb{Z}}_2^s : \mathbf{G}_2(\mathbf{k}) = \frac{1}{2+k^2} \begin{pmatrix} 2-k^2 & 2k & 2k \\ & -2 & k^2 \\ & & -2 \end{pmatrix}.$$

- \mathbb{Z}_2^s & $\overline{\mathbb{Z}}_2^s$ are **Dependent**

$$\mathbf{G}_1(\mathbf{k}) = \mathbf{G}_2(\mathbf{k})\mathbf{G}_3$$

- **DIFFERENT Consequences!!!**

Correlation between Physical Observables

$$\mathbf{G}_\nu = \mathbf{V}_\nu \mathbf{d}_\nu \mathbf{V}_\nu^\dagger$$

$$\overline{\mathbb{Z}}_2^s (G_1)$$

$$\overline{\mathbb{Z}}_2^s (G_2)$$

$$\cos \delta_D = \frac{(s_s^2 - c_s^2 s_r^2)(c_a^2 - s_a^2)}{4c_a s_a c_s s_s s_r}$$

$$\cos \delta_D = \frac{(s_s^2 s_r^2 - c_s^2)(c_a^2 - s_a^2)}{4c_a s_a c_s s_s s_r}$$

$$\sin \theta_r = \pm \left[\pm \sqrt{c_D^2 + \cot^2 2\theta_a - c_D} \right] \tan 2\theta_a (\tan \theta_s)^{\pm 1}$$

$$\frac{\delta_r}{\delta_a} = -\frac{\tan \theta_s}{\cos \delta_D}$$

$$\frac{\delta_r}{\delta_a} = +\frac{\cot \theta_s}{\cos \delta_D}$$

Model-Independence

$$\frac{\delta_r}{\delta_a} = -\frac{\tan \theta_s}{\cos \delta_D} \quad \cos \delta_D = \frac{(s_s^2 - c_s^2 s_r^2)(c_a^2 - s_a^2)}{4c_a s_a c_s s_s s_r},$$

- Minimal Seesaw [SFG, He, Yin, JCAP2010]

$$\frac{\delta_r}{\delta_a} = -\frac{\tan \theta_s}{\cos \delta_D}$$

Common origin of soft μ - τ & CP breaking

- Trimaximal Mixing ($k = 2$) @ A_4/S_4 [King, Luhn, 1107.5332]

$$\sqrt{2}s_a - 1 = -\frac{1}{\sqrt{2}}s_r \cos \delta_D$$

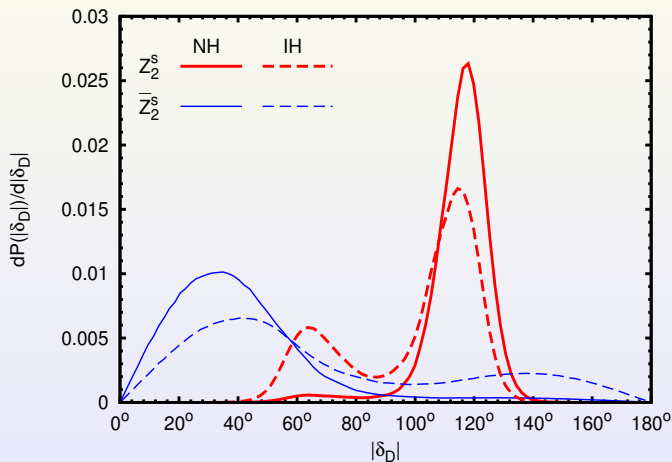
- Unrealistic Bimaximal Mixing [Lam, 1105.5166]

$$\left\langle s_s, s_a, s_r e^{i\delta_D} \right\rangle = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}} e^{-\frac{i}{2}\pi} \right\rangle$$

Prediction of Large δ_D

$$\cos \delta_D = \frac{(s_s^2 - c_s^2 s_r^2)(c_a^2 - s_a^2)}{4c_a s_a c_s s_s s_r}$$

$$\cos \delta_D = \frac{(s_s^2 s_r^2 - c_s^2)(c_a^2 - s_a^2)}{4c_a s_a c_s s_s s_r}$$



1σ Indication for $\delta_D = -74^\circ (-110^\circ)$ [Schwetz et.al. 1108.1376]

Precision Measurement of CP δ_D is Needed!

$$\mathbf{G}_\nu = \mathbf{V}_\nu \mathbf{d}_\nu \mathbf{V}_\nu^\dagger$$

$$\mathbb{Z}_2^s (G_1)$$

$$\overline{\mathbb{Z}}_2^s (G_2)$$

$$\cos \delta_D = \frac{(s_s^2 - c_s^2 s_r^2)(c_a^2 - s_a^2)}{4c_a s_a c_s s_s s_r}$$

$$\cos \delta_D = \frac{(s_s^2 s_r^2 - c_s^2)(c_a^2 - s_a^2)}{4c_a s_a c_s s_s s_r}$$

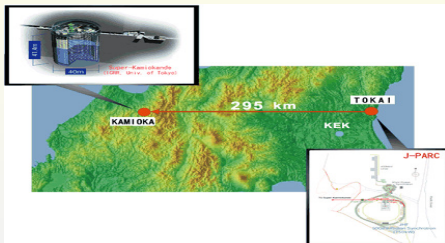
(for NH)	-1σ	Best Value	$+1\sigma$
$\sin^2 \theta_s (\theta_s \equiv \theta_{12})$	0.305 (33.5°)	0.321 (34.5°)	0.339 (35.6°)
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δ_D	?	?	?

Why neutrino mass & oscillation?

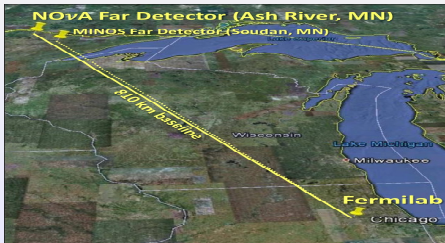
- Higgs boson for electroweak symmetry breaking & mass.
- Chiral symmetry breaking for mass.
- **The world seems not affected by the tiny neutrino mass!**
 - Neutrino mass \Rightarrow Mixing
 - 3 Neutrino \Rightarrow possible CP violation
 - CP violation \Rightarrow Leptogenesis
 - Leptogenesis \Rightarrow **Matter-Antimatter Asymmetry**
 - There is something left in the Universe.
 - Baryogenesis from quark mixing is not enough.

CP Measurement @ Accelerator Exps

- T2K



- NO ν A



- DUNE, T2KII/T2HK/T2KK/T2KO, MOMENT/ADS-CI, Super-PINGU

The Dirac CP Phase δ_D @ Accelerator Exp

- To leading order in $\alpha = \frac{\delta M_{21}^2}{|\delta M_{31}^2|} \sim 3\%$, the oscillation probability relevant to measuring δ_D @ T2(H)K,

$$P_{\nu_\mu \rightarrow \nu_e} \approx 4s_a^2 c_r^2 s_r^2 \sin^2 \phi_{31} - 8c_a s_a c_r^2 s_r c_s s_s \sin \phi_{21} \sin \phi_{31} [\cos \delta_D \cos \phi_{31} \pm \sin \delta_D \sin \phi_{31}]$$

$$P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_e}$$

for ν & $\bar{\nu}$, respectively. $[\phi_{ij} \equiv \frac{\delta m_{ij}^2 L}{4E_\nu}]$

- $\nu_\mu \rightarrow \nu_\mu$ Exps measure $\sin^2(2\theta_a)$ precisely, but not $\sin^2 \theta_a$.
- Run both ν & $\bar{\nu}$ modes @ first peak $[\phi_{31} = \frac{\pi}{2}, \phi_{21} = \alpha \frac{\pi}{2}]$,

$$P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_e} + P_{\nu_\mu \rightarrow \nu_e} = 2s_a^2 c_r^2 s_r^2,$$

$$P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_e} - P_{\nu_\mu \rightarrow \nu_e} = \alpha \pi \sin(2\theta_s) \sin(2\theta_r) \sin(2\theta_a) \cos \theta_r \sin \delta_D.$$

The Dirac CP Phase δ_D @ Accelerator Exp

Accelerator experiment, such as **T2(H)K**, uses off-axis beam to compare ν_e & $\bar{\nu}_e$ appearance @ the oscillation maximum.

- **Disadvantages:**

- **Efficiency:**

- Proton accelerators produce ν more efficiently than $\bar{\nu}$ ($\sigma_\nu > \sigma_{\bar{\nu}}$).
- The $\bar{\nu}$ mode needs more beam time [**$T_{\bar{\nu}} : T_\nu = 2 : 1$**].
- Undercut statistics \Rightarrow Difficult to reduce the uncertainty.

- **Degeneracy:**

- Only **$\sin \delta_D$** appears in $P_{\nu_\mu \rightarrow \nu_e}$ & $P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_e}$.
- Cannot distinguish δ_D from $\pi - \delta_D$.

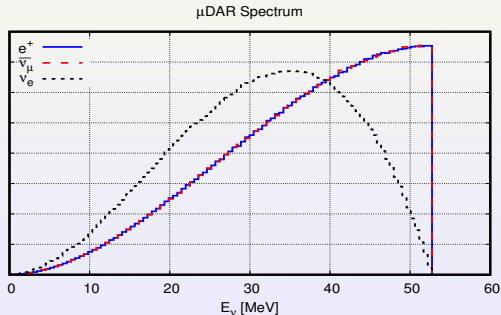
- **CP Uncertainty** $\frac{\partial P_{\mu e}}{\partial \delta_D} \propto \cos \delta_D \Rightarrow \Delta(\delta_D) \propto$ **$1 / \cos \delta_D$** .

- **Solution:**

Measure $\bar{\nu}$ mode with μ^+ decay @ rest (μ DAR)

μ DAR $\bar{\nu}$ Oscillation Experiments

- A cyclotron produces 800 MeV proton beam @ fixed target.
- Produce π^\pm which stops &
 - π^- is absorbed,
 - π^+ decays @ rest: $\pi^+ \rightarrow \mu^+ + \nu_\mu$.
- μ^+ stops & decays @ rest: $\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$.

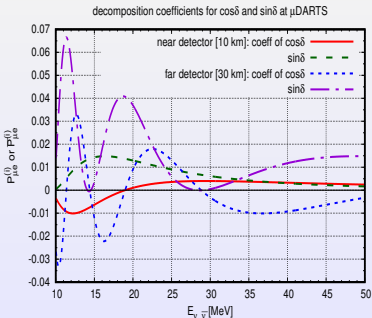
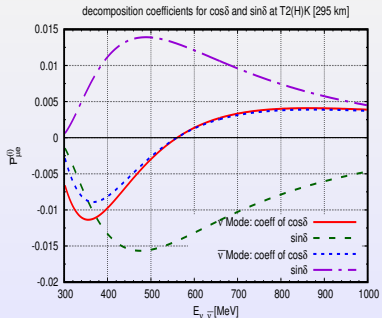


- $\bar{\nu}_\mu$ travel in all directions, oscillating as they go.
- A detector measures the $\bar{\nu}_e$ from $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ **oscillation**.

Accelerator + μ DAR Experiments

Combining $\nu_\mu \rightarrow \nu_e$ @ accelerator [narrow peak @ 550 MeV] & $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ @ μ DAR [wide peak \sim 45 MeV] solves the 3 problems:

- **Efficiency:**
 - $\bar{\nu}$ @ high intensity, μ DAR is plentiful enough.
 - Accelerator Exps can devote all run time to the ν mode. With same run time, the statistical uncertainty drops by $\sqrt{3}$.
- **Degeneracy:** (**decomposition in propagation basis** [1309.3176])



DAE δ ALUS Project

- It's the **FIRST** proposal along this line:
 - **3** μ DAR with **3** high-intensity cyclotron complexes.
 - **1** detector.
 - Different baselines: **1.5, 8 & 20** km to break degeneracies.
- **Disadvantages:**
 - The scattering lepton from IBD @ low energy is **isotropic**.
 - **Cannot** distinguish $\bar{\nu}_e$ from different sources
 - Baseline **cannot be measured**.
 - Cyclotrons **cannot** run simultaneously (20~25% duty factor).
 - **Large** statistical uncertainty.
 - **Higher intensity** is necessary.
 - **Expensive** & Technically **challenging**.

New Proposals

1 μ DAR source + 2 detectors

Advantages:

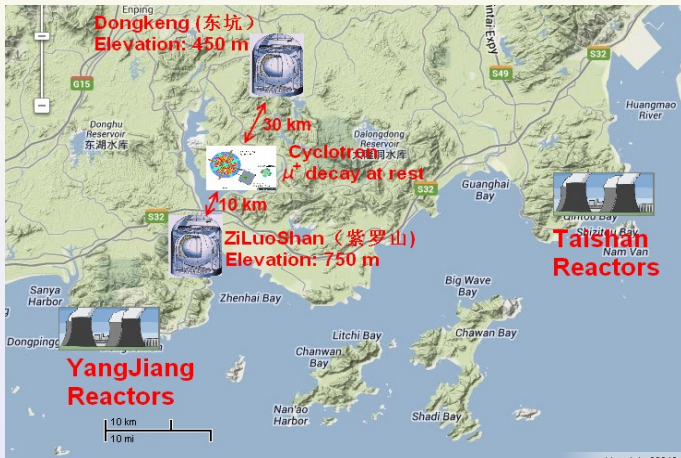
- Full (**100%**) duty factor!
- **Lower** intensity: $\sim 9\text{mA}$ [$\sim 4\times$ lower than DAE δ ALUS]
- Not far beyond the current state-of-art technology of cyclotron [2.2mA @ Paul Scherrer Institute]
- MUCH **cheaper** & technically **easier**.
 - Only one cyclotron.
 - Lower intensity.

Disadvantage?

- A second detector!
 - μ DAR with Two Scintillators (μ DARTS) [1401.3977]
 - Tokai 'N Toyama to(2) Kamioka (TNT2K) [1506.05023]

μ DARTS – JUNO/RENO-50

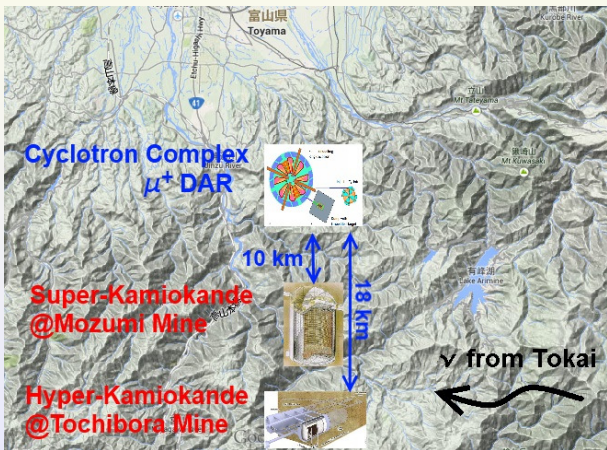
- **Two detectors** are suggested to overcome the **unknown energy response**. [Ciuffoli et al., PRD 2014; 1307.7419, 1401.3977]



- China Atomic Energy Center has a proposal for cyclotron

TNT2K

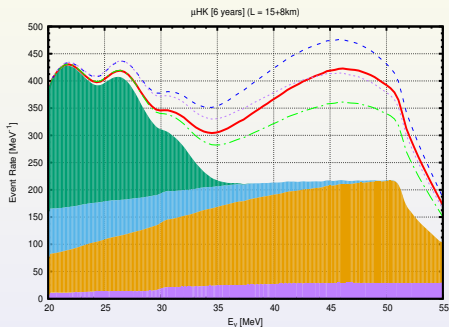
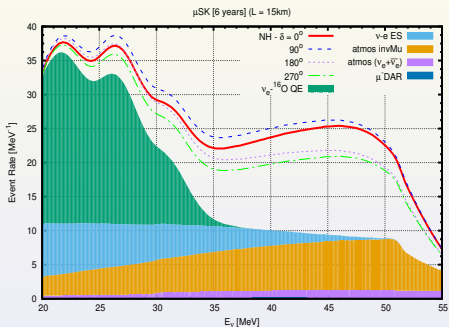
- $T2(H)K + \mu SK + \mu HK$



- μ DAR is also useful for **material**, **medicine** industries in Toyama

Event Shape @ TNT2K

Evslin, Ge & Hagiwara [1506.05023]

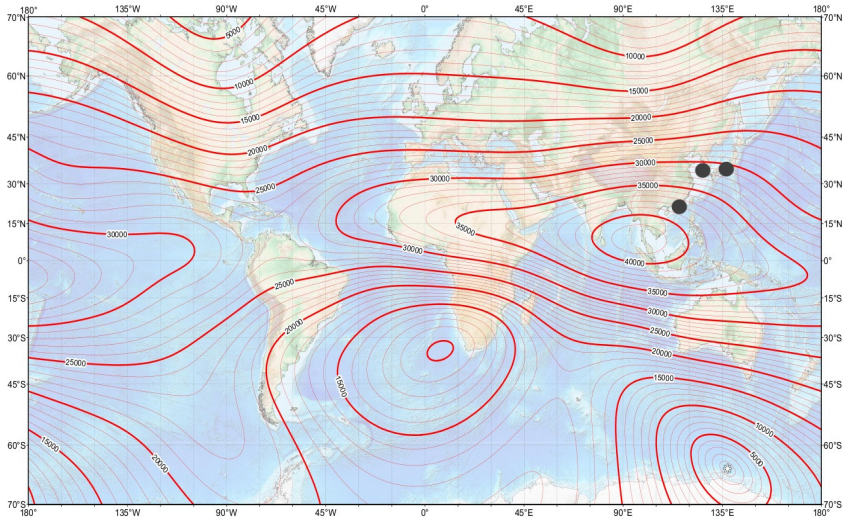


Expected μ DAR IBD signal from 6 yrs of running @ SK (15km) & HK (23km) with NH.

Simulated by [NuPro](http://nupro.hepforge.org/), <http://nupro.hepforge.org/>

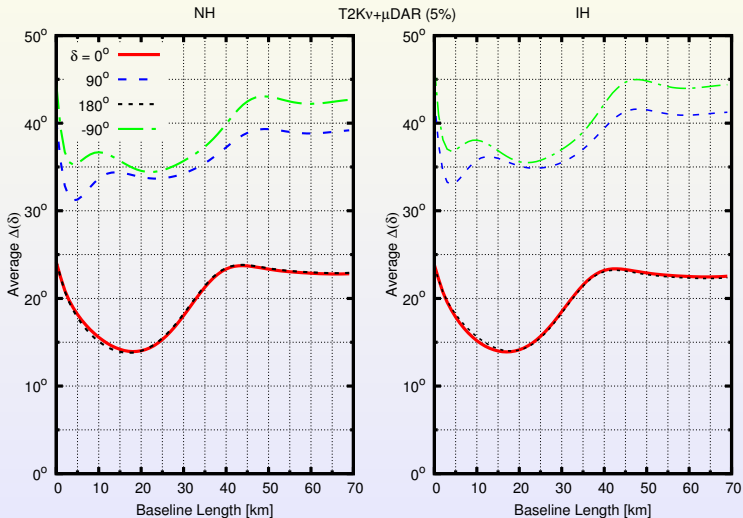
Lowest Atmospheric Neutrino Background

US/UK World Magnetic Model -- Epoch 2010.0
Main Field Horizontal Intensity (H)



δ_D Precision @ TNT2K

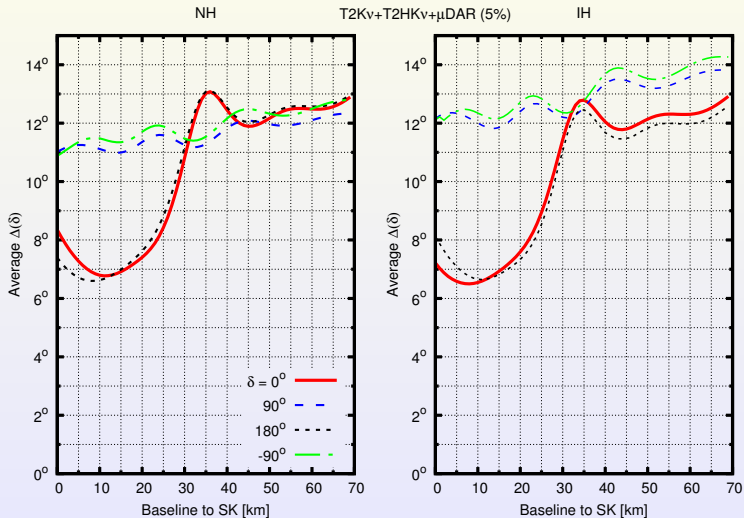
Evslin, Ge & Hagiwara [1506.05023]



Simulated by NuPro, <http://nupro.hepforge.org/>

δ_D Precision @ TNT2K

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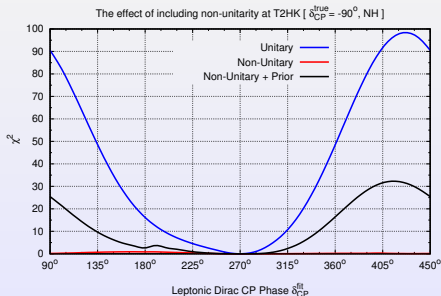
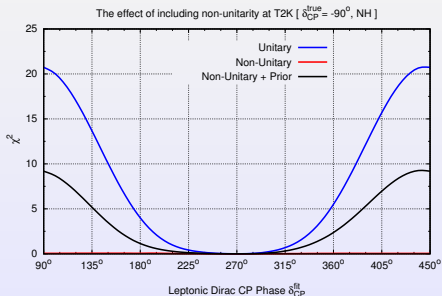
Simulated by [NuPro](http://nupro.hepforge.org/), <http://nupro.hepforge.org/>

Non-Unitarity Mixing (NUM)

Ge, Pasquini, Tortola & Valle [1605.01670]

$$N = N^{NP} U = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ |\alpha_{21}| e^{i\phi} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} U.$$

$$P_{\mu e}^{NP} = \alpha_{11}^2 \left\{ \alpha_{22}^2 \left[c_a^2 |S'_{12}|^2 + s_a^2 |S'_{13}|^2 + 2c_a s_a (\cos \delta_D \mathbb{R} - \sin \delta_D \mathbb{I}) (S'_{12} S'_{13}^*) \right] + |\alpha_{21}|^2 P_{ee} \right. \\ \left. + 2\alpha_{22} |\alpha_{21}| \left[c_a (c_\phi \mathbb{R} - s_\phi \mathbb{I}) (S'_{11} S'_{12}^*) + s_a (c_{\phi+\delta_D} \mathbb{R} - s_{\phi+\delta_D} \mathbb{I}) (S'_{11} S'_{13}^*) \right] \right\}.$$



NUM vs Seesaw Mechanism

- Heavy neutrinos

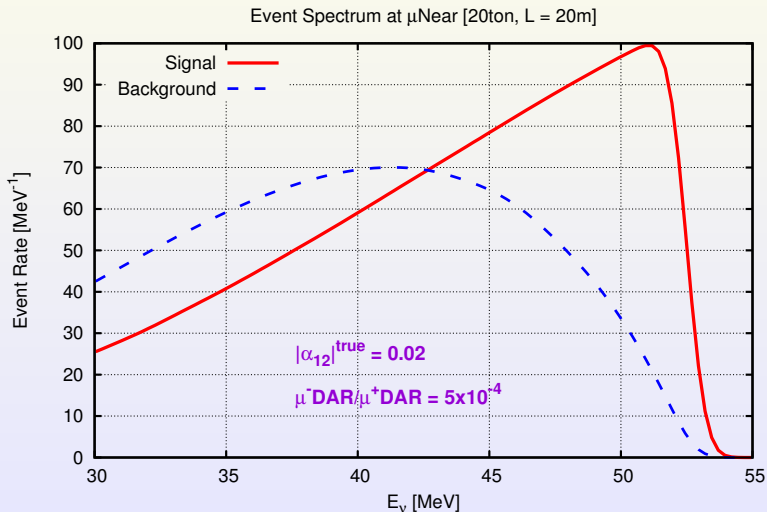
$$\bar{\nu} M_D \mathcal{N} + h.c. + \bar{\mathcal{N}} M_N \mathcal{N} = \begin{pmatrix} \bar{\nu} & \bar{\mathcal{N}} \end{pmatrix} \begin{pmatrix} 0 & M_D \\ M_D^T & M_N \end{pmatrix} \begin{pmatrix} \nu \\ \mathcal{N} \end{pmatrix}$$

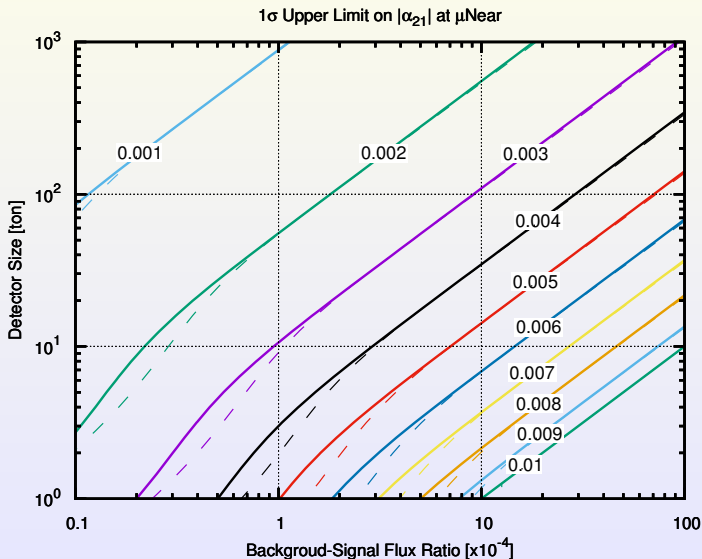
- Seesaw Mechanism

$$M_\nu = -M_D M_N^{-1} M_D^T, \quad \nu' = \nu + M_D M_N^{-1} \mathcal{N}$$

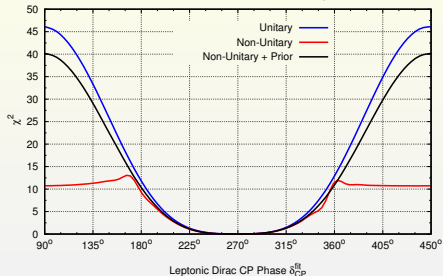


$$P_{\mu e}^{NP}(L \rightarrow 0) = \alpha_{11}^2 |\alpha_{21}|^2 P_{ee} \approx \alpha_{11}^2 |\alpha_{21}|^2 \approx |\alpha_{21}|^2$$

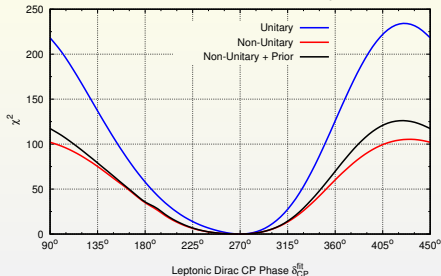




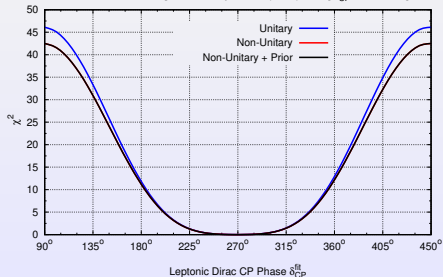
The effect of including non-unitarity at T2K+ μ SK [$\delta_{CP}^{true} = -90^\circ$, NH]



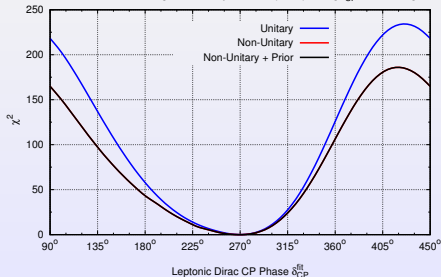
The effect of including non-unitarity at T2HK+ μ HK [$\delta_{CP}^{true} = -90^\circ$, NH]



The effect of including non-unitarity at T2K+ μ SK+ μ Near [$\delta_{CP}^{true} = -90^\circ$, NH]

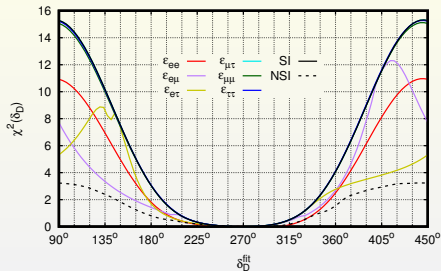
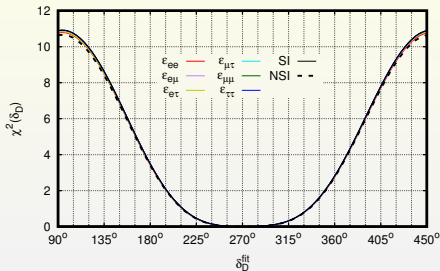
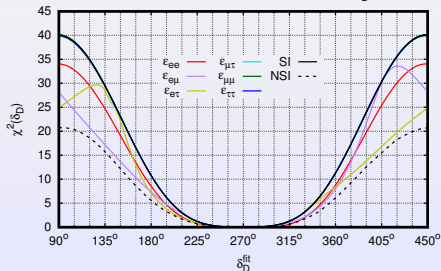
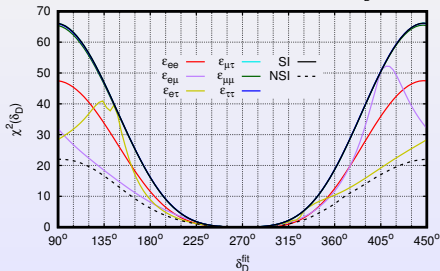


The effect of including non-unitarity at T2HK+ μ HK+ μ Near [$\delta_{CP}^{true} = -90^\circ$, NH]



$$\mathcal{H} \equiv \frac{1}{2\mathbf{E}_\nu} U \begin{pmatrix} 0 & & \\ & \Delta m_s^2 & \\ & & \Delta m_a^2 \end{pmatrix} U^\dagger + V_{cc} \begin{pmatrix} 1 + \epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{e\mu}^* & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon_{e\tau}^* & \epsilon_{\mu\tau}^* & \epsilon_{\tau\tau} \end{pmatrix}$$

- Standard Interaction – V_{cc} (also V_{nc})
- Non-Standard Interaction – $\epsilon_{\alpha\beta}$
 - Diagonal $\epsilon_{\alpha\alpha}$ are real
 - Off-diagonal $\epsilon_{\alpha\neq\beta}$ are complex
 - Both can fake CP
- Z' in LMA-Dark model with $L_\mu - L_\tau$ gauged as $U(1)$
 - $M_{Z'} \sim \mathcal{O}(10)\text{MeV}$
 - $g_{Z'} \sim 10^{-5}$

The effect of NSI on the CP sensitivity at T2K [$\delta_D^{\text{true}} = -90^\circ$]

 The effect of NSI on the CP sensitivity at μ SK [$\delta_D^{\text{true}} = -90^\circ$]

 The effect of NSI on the CP sensitivity at T2K+ μ SK [$\delta_D^{\text{true}} = -90^\circ$]

 The effect of NSI on the CP sensitivity at ν T2K+ μ SK [$\delta_D^{\text{true}} = -90^\circ$]


CP Sensitivities

SFG & Smirnov [arXiv:1607.08513]

$\delta_D = -90^\circ$ vs 0° Event Numbers	T2K		μ SK		T2K+ μ SK		ν T2K+ μ SK	
	$114\nu + 56\bar{\nu}$		$212\bar{\nu}$		$57\nu + 268\bar{\nu}$		$342\nu + 212\bar{\nu}$	
χ^2 for SI & NSI	4.08	1.54	2.81	2.75	11.3	6.10	18.7	7.59
χ^2	2.50	–	2.75	–	8.77	–	12.5	–
$\epsilon_{ee}^{\text{bf}}$	0.69	0.57	0.48	0.47	0.81	0.63	1.07	0.70
χ^2	4.06	–	2.81	–	11.3	–	18.6	–
$\epsilon_{\mu\mu}^{\text{bf}}$	-0.02	-0.01	-0.02	-0.01	-0.02	-0.01	-0.02	-0.01
χ^2	4.11	–	2.81	–	11.3	–	18.7	–
$\epsilon_{\tau\tau}^{\text{bf}}$	0.02	0.01	0.02	0.01	0.02	0.01	0.01	0.01
χ^2	2.68	–	2.81	–	8.01	–	11.7	–
$\epsilon_{e\mu}^{\text{bf}}$	0.12	0.07	0.01	0.01	0.19	0.11	0.23	0.10
χ^2	2.77	–	2.81	–	8.42	–	10.4	–
$\epsilon_{e\tau}^{\text{bf}}$	0.25	0.14	0.01	0.01	0.37	0.21	0.51	0.30
χ^2	4.08	–	2.81	–	11.3	–	18.7	–
$\epsilon_{\mu\tau}^{\text{bf}}$	0	0	0	0	0	0	0	0

Summary – Theory

- **Horizontal Symmetry**

- $\mathbb{Z}_2^{\mu\tau}$ [G_3 v.s. $d_\nu^{(3)}$]
- \mathbb{Z}_2^s [G_1 v.s. $d_\nu^{(1)}$] & $\overline{\mathbb{Z}}_2^s$ [G_2 v.s. $d_\nu^{(2)}$]

- **Residual Symmetry** as **Custodial Symmetry**

- Full symmetry has to be broken. Otherwise, no mixing.
- Mixing angles dictated by residual symmetry.
- Physical observables, mixing angles & CP phase, are correlated.

$$\cos \delta_D = \frac{(s_s^2 - c_s^2 s_r^2)(c_a^2 - s_a^2)}{4c_a s_a c_s s_s s_r} \quad \cos \delta_D = \frac{(s_s^2 s_r^2 - c_s^2)(c_a^2 - s_a^2)}{4c_a s_a c_s s_s s_r}$$

- **Phenomenological consequences**

- Nonzero θ_r
- Large δ_D
- Non-maximal θ_a
- Distinguishing \mathbb{Z}_2^s & $\overline{\mathbb{Z}}_2^s$ with $\text{NO}\nu\text{A}$, T2K, μDAR

Summary – Experiment

- **Better CP measurement than T2K, T2KII, T2HK, T2KK**
 - Much larger event numbers
 - Much better CP sensitivity around maximal CP
 - Solve degeneracy between δ_D & $\pi - \delta_D$
 - Guarantee CP sensitivity against NUM
 - Guarantee CP sensitivity against NSI
- **Better configuration than DAE δ ALUS**
 - Only one cyclotron
 - 100% duty factor
 - Much lower flux intensity
 - Much easier
 - Much cheaper
 - Single near detector

Thank You!