Renormalization of vacuum expectation values in spontaneously broken gauge theories

Alexander Voigt

Technische Universität Dresden
Institut für Kern- und Teilchenphysik

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In collaboration with Marcus Sperling and Dominik Stöckinger

[Sperling et al. 13]

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Motivation

Theories with spontaneously broken gauge symmetry very successful!

$$\phi \to \phi + v$$

Need to renormalize:

$$\phi \to \sqrt{Z}\phi, \quad v \to v + \delta v$$

Theorists need:

- δv in loop calculations
- β_v in spectrum generators (Softsusy, SPheno, FlexibleSUSY etc.)

Current status of β_v calculations

Current status of $\beta_v^{\overline{\text{DR}}/\overline{\text{MS}}}$:

Model	$\beta_v^{(1)}$	$\beta_v^{(2)}$
MSSM	✓ [Chankowski Nucl.Phys. B423]	✓ [Yamada 94] $O(g^2 Y^2)$
E_6SSM	✓ [Athron et al. 12]	✗
∀ gauge theory	?	✗
∀ SUSY model	?	✗

We calculate:

$\delta v^{(1,2)}$ and $\beta_v^{(1,2)}$ in a general and supersymmetric gauge theory with R_ξ gauge fixing [Sperling et al. 13]

Renormalization of v

Spontaneously broken gauge theory:

$$\phi \to \phi + v$$

Most generic renormalization transformation:

$$(\phi + v) \to \sqrt{Z}\phi + v + \delta v$$

$$\text{or} \quad (\phi + v) \to \sqrt{Z}(\phi + v + \delta\bar{v})$$

$\delta\bar{v}$ characterizes to what extent v renormalizes differently from ϕ.

With $\sqrt{Z} = 1 + \frac{1}{2}\delta Z \Rightarrow$

$$\delta v = \frac{1}{2}\delta Z v + \delta\bar{v}$$

Influence of global gauge invariance

When does $\delta\bar{v}$ appear?

$$\text{global gauge invariance} \quad \Rightarrow \quad \delta\bar{v} = 0$$
$$\text{no global gauge invariance} \quad \Rightarrow \quad \delta\bar{v} \neq 0$$

R_ξ gauge fixing:

$$\mathcal{L}_{\text{fix,gh}} = s\left[\bar{c}^A \left(F^A + \xi B^A/2\right)\right]$$
$$F^A = \partial^\mu V_\mu^A + ig\xi v_a T_{ab}^A \phi_b$$

R_ξ breaks global gauge invariance for $\xi \neq 0 \Rightarrow \delta\bar{v} \neq 0$.

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Background field method – Introducing $\hat{\phi}$

Problem: R_ξ breaks global gauge invariance for $\xi \neq 0 \Rightarrow \delta\bar{v} \neq 0$.

Trick: Keep global gauge invariance in intermediate calculation!

[Kraus,Sibold 95]

Introduce background field $\hat{\phi}$ and shift $\hat{v}$

$$\phi \to \phi_{\text{eff}} := \phi + \hat{\phi} + \hat{v}$$

where $\hat{\phi} + \hat{v}$ has same gauge transformation as ϕ.
Modified R_ξ gauge fixing:

$$F^A = \partial^\mu V_\mu^A + ig\xi(\hat{\phi} + \hat{v})_a T_{ab}^A \phi_b$$

$\Rightarrow$ global gauge invariance! $\Rightarrow \delta\bar{v} = 0$

Background field method – Lagrangian

Modified BRS transformations:

$$s\phi_{\text{eff}} = -igT^A c^A \phi_{\text{eff}}, \quad s\hat{\phi} = \hat{q}, \quad s\hat{q} = 0, \quad s\phi = s\phi_{\text{eff}} - \hat{q}$$

Modified Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\text{inv}}\big|_{\phi\to\phi_{\text{eff}}} + \mathcal{L}_{\text{fix,gh}} + \mathcal{L}_{\text{ext}}$$

$$\begin{aligned}
\mathcal{L}_{\text{inv}} &= -\frac{1}{4}F^A_{\mu\nu}F^{A\mu\nu} + \frac{1}{2}\left(D_\mu\phi\right)_a\left(D^\mu\phi\right)_a + i\psi^\alpha_p\sigma^\mu_{\alpha\dot\alpha}\left(D^\dagger_\mu\bar\psi^{\dot\alpha}\right)_p \\
&\quad - \frac{1}{2!}m^2_{ab}\phi_a\phi_b - \frac{1}{3!}h_{abc}\phi_a\phi_b\phi_c - \frac{1}{4!}\lambda_{abcd}\phi_a\phi_b\phi_c\phi_d \\
&\quad - \frac{1}{2}\left[\left(m_f\right)_{pq}\psi^\alpha_p\psi_{q\alpha} + \text{h.c.}\right] - \frac{1}{2}\left[Y^a_{pq}\psi^\alpha_p\psi_{q\alpha}\phi_a + \text{h.c.}\right] \\
\mathcal{L}_{\text{fix,gh}} &= s\left[\bar{c}^A\left(F^A + \xi B^A/2\right)\right] \\
\mathcal{L}_{\text{ext}} &= K_{\phi_a}s\phi_a + K_{V^A_\mu}sV^A_\mu + K_{c^A}sc^A + \left[K_{\psi_p}s\psi_p + \text{h.c.}\right]
\end{aligned}$$

Background field method – Renormalization of v

Modified renormalization transformation:

$$\phi \to \sqrt{Z}\phi$$

$$(\hat{\phi} + \hat{v}) \to \sqrt{Z}\sqrt{\hat{Z}}(\hat{\phi} + \hat{v})$$

$$\hat{q} \to \sqrt{Z}\sqrt{\hat{Z}}\hat{q}$$

$$\Rightarrow \phi_{\text{eff}} \to \sqrt{Z}\left(\phi + \sqrt{\hat{Z}}\left(\hat{\phi} + \hat{v}\right)\right)$$

Standard approach:

$$(\phi + v) \to \sqrt{Z}\phi + v + \delta v$$

For $\hat{\phi} = 0 \Rightarrow$

$$\boxed{\delta v = \left(\sqrt{Z}\sqrt{\hat{Z}} - 1\right) v = \frac{1}{2}\left(\delta Z + \delta\hat{Z}\right) v + O(\hbar^2)}$$

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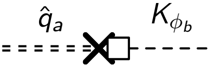
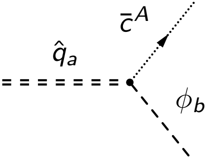
Calculation of $\delta\hat{Z}$ – 1 Loop I

$$
\begin{aligned}
\mathcal{L}_{\text{ext}} &= K_\phi\, s\phi + \cdots \\
&= -K_\phi\, igT^A c^A \phi - K_\phi\, \hat{q} + \cdots \\
\mathcal{L}_{\text{fix,gh}} &= s\left[\bar{c}^A \left(F^A + \xi B^A/2\right)\right]
\end{aligned}
$$

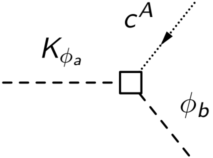
$\Rightarrow$



$= -\frac{i}{2}\delta\hat{Z}^{(1)}_{ba}$

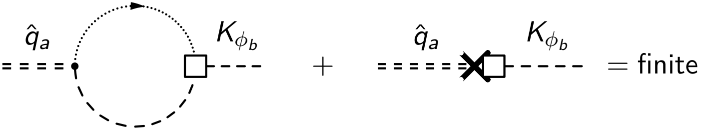


$= \xi g T^A_{ab}$,



$= g T^A_{ab}$

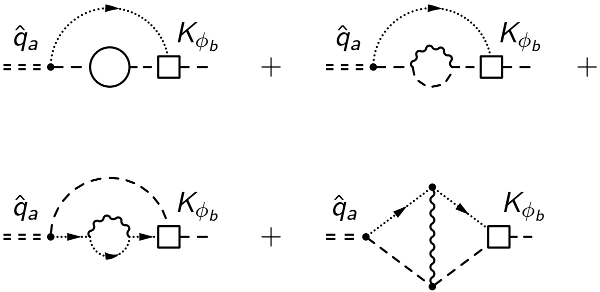
Calculation of $\delta\hat{Z}$ – 1 Loop II



$\Rightarrow$

$$(4\pi)^2 \delta\hat{Z}^{(1)} = 2g^2 \xi C^2(S) \frac{1}{\epsilon}$$

Calculation of $\delta\hat{Z}$ – 2 Loop



$\Rightarrow$

$$(4\pi)^4 \delta\hat{Z}^{(2)} = g^2 \xi C^2(\mathrm{S}) Y^2(\mathrm{S}) \left(\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\right) + O(g^4)$$

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Results – β_v

$$\delta v = \left(\sqrt{Z}\sqrt{\hat{Z}} - 1\right) v$$

$$\gamma(S) = \left(\mu\partial_\mu \sqrt{Z}^{-1}\right)\sqrt{Z}\,, \qquad \hat{\gamma}(S) = \left(\mu\partial_\mu \sqrt{\hat{Z}}^{-1}\right)\sqrt{\hat{Z}}$$

$\Rightarrow$

$$\beta_v^{(n)} = \left[\gamma^{(n)}(S) + \hat{\gamma}^{(n)}(S)\right] v$$

$$\hat{\gamma}^{(1)}(\mathrm{S}) = \frac{\xi}{(4\pi)^2} 2g^2 C^2(\mathrm{S})$$

$$\hat{\gamma}^{(2)}(\mathrm{S}) = \frac{\xi}{(4\pi)^4}\Bigg\{ g^4 \left[2\left(1+\xi\right) C^2(\mathrm{S})C^2(\mathrm{S}) + \frac{7-\xi}{2} C_2(\mathrm{G})C^2(\mathrm{S})\right] - 2g^2 C^2(\mathrm{S}) Y^2(\mathrm{S}) \Bigg\}$$

$$\tan\beta = \frac{v_u}{v_d} \quad \Rightarrow \quad \frac{\beta_{\tan\beta}}{\tan\beta} = \gamma_{uu} - \gamma_{dd} + \hat{\gamma}_{uu} - \hat{\gamma}_{dd}$$

MSSM:

$$\frac{\beta^{(1)}_{\tan\beta}}{\tan\beta} = \gamma^{(1)}_{uu} - \gamma^{(1)}_{dd} \qquad \leftarrow \text{cancellation of } \hat{\gamma} \text{ terms}$$

$$\frac{\beta^{(2)}_{\tan\beta}}{\tan\beta} = \gamma^{(2)}_{uu} - \gamma^{(2)}_{dd} + \frac{\xi}{(4\pi)^2}\left(\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2\right)\frac{\beta^{(1)}_{\tan\beta}}{\tan\beta}$$

[Yamada 94]

E$_6$SSM: extra $U(1)_N$ gauge symmetry

$$\frac{\beta^{(1)}_{\tan\beta}}{\tan\beta} = \gamma^{(1)}_{uu} - \gamma^{(1)}_{dd} + \frac{\xi}{(4\pi)^2}2g_N^2\left[\left(\frac{N_{H_u}}{2}\right)^2 - \left(\frac{N_{H_d}}{2}\right)^2\right]$$

[Athron et al. 12]

Conclusions

- In R_ξ gauge fixings (with $\xi \neq 0$), v renormalizes differently from ϕ

$$\phi \to \sqrt{Z}(\phi + v + \delta\bar{v})$$

- difference $\delta\bar{v}$ can be interpreted as field renormalization $\delta\hat{Z}$ of a background field $\hat{\phi}$

$$\delta\bar{v} = \frac{1}{2}\delta\hat{Z}v$$

- We've calculated $\delta\hat{Z}^{(1,2)}$, $\delta v^{(1,2)}$, $\beta_v^{(1,2)}$, in the $\overline{\mathrm{DR}}$ scheme for arbitrary ξ in a general gauge theory [Sperling et al. 13]