The anomalous magnetic moment of the muon and supersymmetry

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Magnetic moment

$$\vec{m} = g \frac{Qe}{2m_\mu} \vec{s}$$

g . . . gyromagnetic ratio

$$a_\mu = \frac{g-2}{2}$$

a_μ . . . anomalous magnetic moment

Anomalous magnetic moment in QFT

In QFT a_μ can be extracted from the μ–$\bar{\mu}$–A^ρ 3-point function

$$\bar{u}(p')\Gamma_{\mu\bar{\mu}A^\rho}u(p) = e\bar{u}(p')\left[\gamma_\rho F_V(q^2) + (p+p')_\rho F_M(q^2) + \ldots\right]u(p)$$

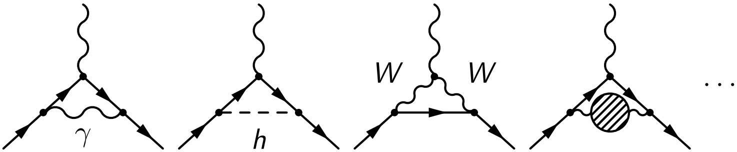
in the limit of vanishing photon momentum, $q^2 \to 0$:

$$g = 2\left[1 - 2m_\mu F_M(0)\right] \qquad \Rightarrow \qquad a_\mu = -2m_\mu F_M(0)$$

$a_\mu = 0$ at tree-level $\Rightarrow$ a_μ^{1L} is leading order!

SM contribution

$$\begin{aligned}
a_\mu^{\text{SM}} &= a_\mu^{\text{QED}} + a_\mu^{\text{weak}} + a_\mu^{\text{had}} \\
a_\mu^{\text{QED}} &= 11658471.895(0.008) \cdot 10^{-10} \\
a_\mu^{\text{weak}} &= 15.4(0.1) \cdot 10^{-10} \\
a_\mu^{\text{had}} &= (692.3(4.2)^{\text{LO}} + 0.7(2.6)^{\text{NLO}}) \cdot 10^{-10}{}_{[1010.4180,0901.0306]} \\
\Rightarrow \quad a_\mu^{\text{SM}} &= 11659180.3(0.1)(4.2)(2.6) \cdot 10^{-10}
\end{aligned}$$



Measurement

Spin precession in magnetic field $\vec{B}$ relative to momentum

$$\vec{\omega} = \frac{e}{m_\mu}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta} \times \vec{E}\right]$$

vanishes for "magic" momentum $p = 3.094\,\text{GeV}$.

$$a_\mu^{\text{exp}} = 11659208.9(6.3) \cdot 10^{-10}{}_{\text{[E821@BNL]}}$$

$\Rightarrow$

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = \begin{cases} (28.7 \pm 8.0) \cdot 10^{-10} & (3.6\sigma)_{\text{[1010.4180]}} \\ (26.1 \pm 8.0) \cdot 10^{-10} & (3.2\sigma)_{\text{[1105.3149]}} \end{cases}$$

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BSM contributions

generic expected scaling:

$$a_\mu^{\text{BSM}} \propto C \frac{m_\mu^2}{M_{\text{BSM}}^2}$$

Reason: F_M corresponds to chirality flip of muon

$$\bar{\mu}\mu = \mu_R \mu_L + \bar{\mu}_L \bar{\mu}_R$$

Terms in $\mathcal{L}$ corresponding to chirality flip $\propto m_\mu$ include:

μ_L ———•——— μ_R $\quad \propto m_\mu$

$\tilde{\mu}_L$ - - - - -•- - - - - $\tilde{\mu}_R$ $\quad \propto m_\mu \tan\beta$

Potential enhancement factors: $C \propto \tan\beta$, $\log\left(\frac{M_{\text{BSM}}}{m_\mu}\right)$

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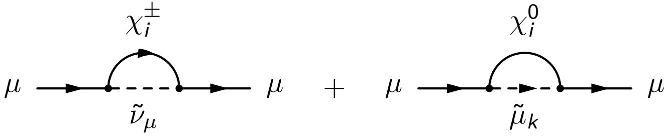
MSSM contribution – 1-loop



$$a_\mu^{\mathrm{MSSM,1L}} \approx 13 \cdot 10^{-10} \left(\frac{100\,\mathrm{GeV}}{M_{\mathrm{SUSY}}}\right)^2 \tan\beta \;\mathrm{sign}(\mu M_2)$$

Properties:

- enhanced by $\tan\beta$
- suppressed by $1/M_{\mathrm{SUSY}}^2$
- can explain deviation for e.g. $M_{\mathrm{SUSY}} = 500\,\mathrm{GeV}$, $\tan\beta = 50$

MSSM contribution – 2-loop

$$a_\mu^{\mathrm{2L}} = a_\mu^{\mathrm{2L(a)}} + a_\mu^{\mathrm{2L(b)}}$$

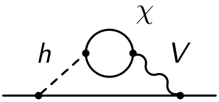
where

$a_\mu^{\mathrm{2L(a)}}$ = 2-loop correction to SM 1-loop diagram
$a_\mu^{\mathrm{2L(b)}}$ = 2-loop correction to MSSM 1-loop diagram

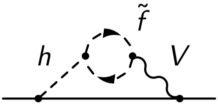
MSSM contribution – $a_\mu^{\text{2L(a)}}$

$$a_\mu^{\text{2L(a)}} = a_\mu^{\text{2L},\chi,} + a_\mu^{\text{2L},\tilde{f}} + a_\mu^{\text{2L},f} + a_\mu^{\text{2L,bos}}$$

$a_\mu^{\text{2L},\chi,} + a_\mu^{\text{2L},\tilde{f}} =$

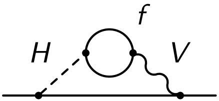


$+$



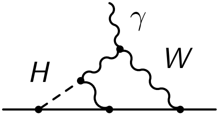
$= O(10 \cdot 10^{-10})$

$a_\mu^{\text{2L},f} =$



$< 10^{-10}$

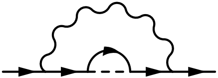
$a_\mu^{\text{2L,bos}} =$

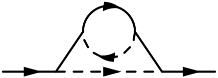


$< 10^{-10}$

MSSM contribution – $a_\mu^{\text{2L(b)}}$

$$a_\mu^{\text{2L(b)}} = a_\mu^{\text{2L},\gamma} + a_\mu^{\text{2L},f/\tilde{f}} + a_\mu^{\text{2L,rest}}$$

$a_\mu^{\text{2L},\gamma} =$  $\propto \log \frac{M_{\text{SUSY}}}{m_\mu} = -(0.07\ldots0.09)\,a_\mu^{\text{MSSM,1L}}$

$a_\mu^{\text{2L},f/\tilde{f}} =$  $\propto \log \frac{M_{\text{SUSY}}}{m_\mu} < 0.1\,a_\mu^{\text{MSSM,1L}}$

$a_\mu^{\text{2L,rest}} =$  $= O(2 \cdot 10^{-10})$

known

incomplete

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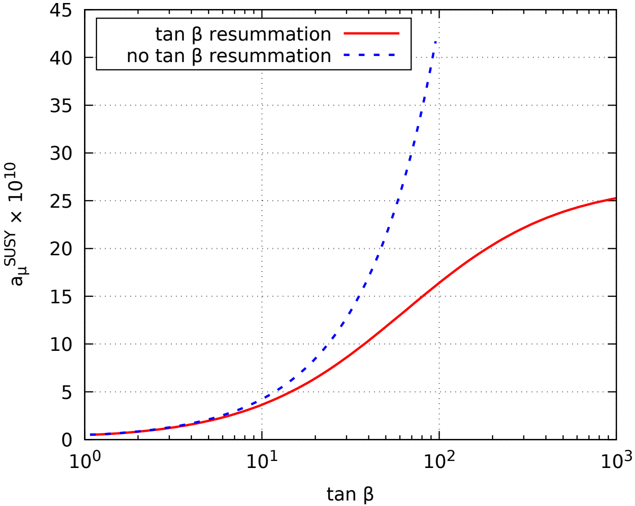
Comparision of public codes

Program	$a_\mu^{\text{MSSM,1L}}$	$a_\mu^{2\text{L},\chi,}$	$a_\mu^{2\text{L},\tilde{f}}$	$a_\mu^{2\text{L},f}$	$a_\mu^{2\text{L},\text{bos}}$
GM2Calc	✓	✓ (approximated)	✓ (approximated)	✗	✗
FeynHiggs	✓	✓	✓	✓	✓
FlexibleSUSY	✓	✗	✗	✗	✗
SARAH/SPheno	✓	✗	✗	✗	✗

Program	$a_\mu^{2\text{L},\gamma}$	$a_\mu^{2\text{L},f/\tilde{f}}$	$a_\mu^{2\text{L},\text{rest}}$	t_β–res.	scheme
GM2Calc	✓	✓ (approximated)	✗	✓	OS/$\overline{\text{DR}}$
FeynHiggs	✓	✗	✗	✗	OS/$\overline{\text{DR}}$
FlexibleSUSY	✓ (approximated)	✗	✗	✗	$\overline{\text{DR}}$
SARAH/SPheno	✗	✗	✗	✗	$\overline{\text{DR}}$

✓ = full, ✓ = approximated, ✗ = missing

Impact of t_β resummation



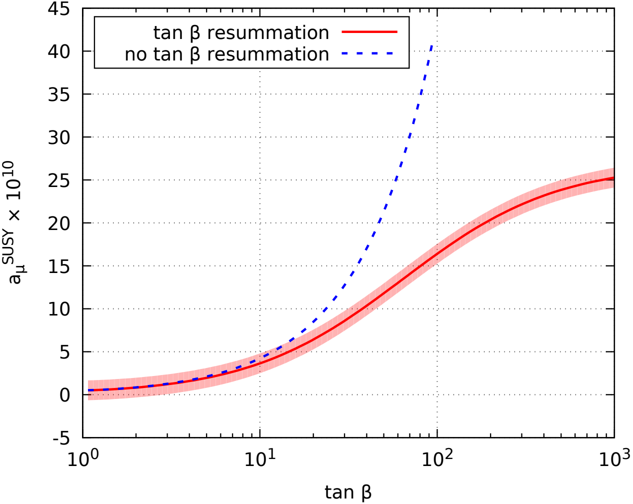
BM1 [arXiv:0808.1530]

Remaining theory uncertainty

GM2Calc (conservative):

$$
\begin{aligned}
\Delta a_\mu^{\mathrm{MSSM}} &= \Delta\left[a_\mu^{\mathrm{2L},\chi,} + a_\mu^{\mathrm{2L},\tilde{f}}\right] + \Delta\left[a_\mu^{\mathrm{2L},f} + a_\mu^{\mathrm{2L,bos}}\right] + \Delta\left[a_\mu^{\mathrm{2L,rest}}\right] \\
&= 0.3\left(|a_\mu^{(\chi\gamma H)}| + |a_\mu^{(\tilde{f}\gamma H)}|\right) + 0.3\cdot 10^{-10} + 2\cdot 10^{-10} \\
&= 0.3\left(|a_\mu^{(\chi\gamma H)}| + |a_\mu^{(\tilde{f}\gamma H)}|\right) + 2.3\cdot 10^{-10}
\end{aligned}
$$

Remaining theory uncertainty



BM1 [arXiv:0808.1530]

Sources of uncertainties in public codes

1-loop calculations suffer from:

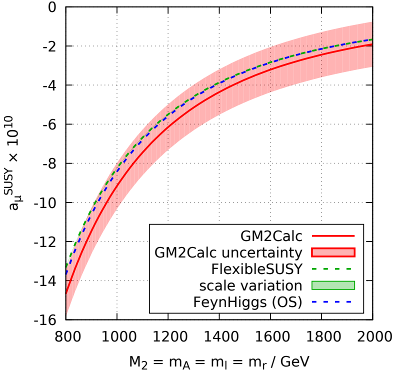
- uncertainty from choice of renormalization scheme for important parameters (α_{em}, $m_{\tilde{\mu}_i}$, ...), because 1L = LO
  $\rightarrow$ (partially) resolved at 2-loop level

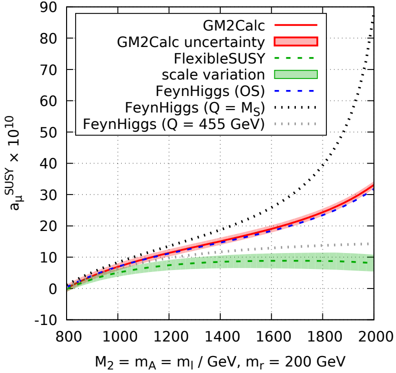
$\overline{\text{DR}}$ calculations suffer from:

- renormalization scale uncertainty, because 1L = LO
  $\rightarrow$ (partially) resolved at 2-loop level
- potentially large 2-loop corrections from quadratically enhanced smuon self-energy contributions
  $\rightarrow$ avoided in the OS scheme

$\overline{\text{DR}}$–OS conversion can suffer from large corrections!

$\overline{\text{DR}}$–OS conversion





BM4' from [1309.0980] with $\overline{\text{DR}}$ input parameters $t_\beta(Q) = 50$, $\mu(Q) = -160$, $M_1(Q) = 140$, $A_f(Q) = 0$ at $Q = 454.7\,\text{GeV}$
Note: scale variation = **lower bound** of uncertainty

Summary and conclusions

Facts:

- 3–4σ deviation between a_μ^{exp} and a_μ^{SM}
  $\to$ might be due to by BSM physics!
- $a_\mu^{\text{BSM}} \propto C \frac{m_\mu^2}{M_{\text{BSM}}^2}$
  - large for **low** BSM masses
  - can be enhanced by model-dependent factors
    $C \propto \tan\beta, \log(\frac{M_{\text{BSM}}}{m_\mu}), \dots$

Recommendations:

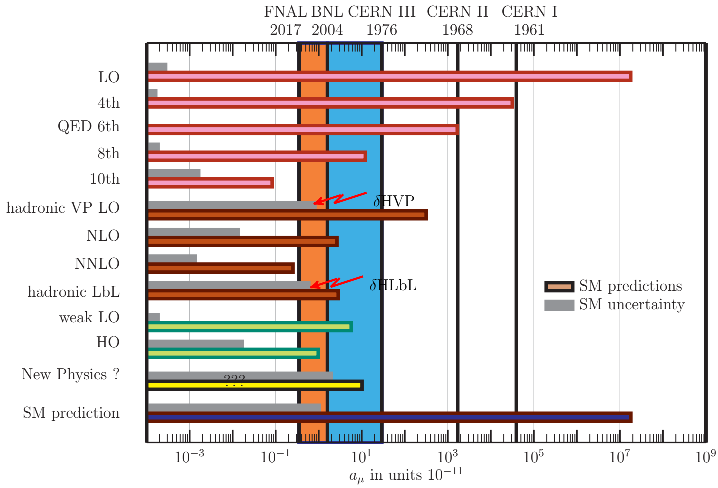
- prefer 2-loop calculations over 1-loop calculations
  (1-loop = LO, with many ambiguities)
- prefer on-shell calculations over $\overline{\text{DR}}$ calculations in the MSSM
  (to avoid large corrections to the smuon mass)
- avoid $\overline{\text{DR}}$–OS parameter conversions, if large corrections present

Backup

$\tan\beta$ resummation

$$[a_\mu]_{\tan\beta\text{–res.}} = [a_\mu]_{y_f \to \tilde{y}_f}\,,$$
$$\tilde{y}_f = \frac{y_f}{1+\Delta_f}(f = \mu, \tau, b)$$
$$\Delta_f = \tan\beta\text{-enhanced contributions to } f \text{ self energy}$$

Contributions to a_μ



[arXiv:1705.00263]