

# The calculation of threshold corrections in the Exceptional Supersymmetric Standard Model ( $E_6$ SSM)

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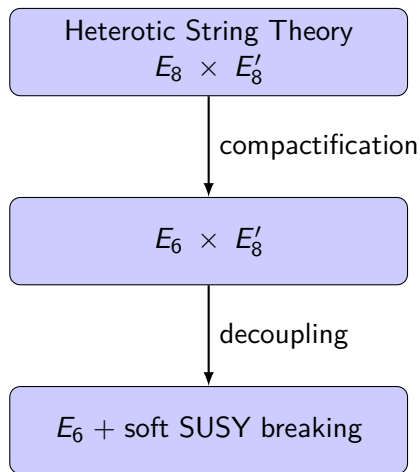
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- ① The Exceptional Supersymmetric Standard Model ( $E_6$ SSM)
  - Model motivation
  - Model definition
- ② What are threshold corrections and why do we need them?
- ③ Procedure to calculate threshold corrections
  - Matching conditions
  - Threshold relation for strong coupling  $g_3$
- ④ Results
- ⑤ Conclusions and Future plans

# Motivation of the model – from String Theory

[F. del Aguila, G. A. Blair, M. Daniel, G. G. Ross, Nucl.Phys.B272 (1986)]



# Motivation of the model – $\mu$ problem

MSSM superpotential:

$$\mathcal{W}_{\text{MSSM}} = \mu H_d H_u - h_{ij}^u (H_u Q_i) u_j^c - h_{ij}^d (H_d Q_i) d_j^c - h_{ij}^e (H_d L_i) e_j^c$$

In fact

- bilinear term  $\mu H_d H_u$  can be present before SUSY is broken
- $\rightarrow$  model definition at high scale  $M_X$  suggests  $\mu \approx M_X$
- But from EWSB conditions

$$\frac{1}{2} m_Z^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2$$

therefore  $\mu \sim m_Z$  to allow  $v = 174$  GeV

[D. J. H. Chung et Al. Phys.Rept.407 (2005)]

# Definition of the $E_6$ SSM – Gauge structure

[S. F. King, S. Moretti, R. Nevzorov, Phys.Rev.D73:035009 (2006)]

## Definition of the $E_6$ SSM

Supersymmetric gauge theory based on GUT gauge group  $E_6$ , which is broken at the GUT scale

$$E_6 \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_N$$

and  $U(1)_N$  broken above EW scale

$$\begin{aligned} SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_N \\ \longrightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \end{aligned}$$

# Definition of the $E_6$ SSM – Matter content

## Matter content

- 3 complete fundamental 27 representations  $(\mathbf{27})_i$  of the  $E_6$
- 2 Higgs like doublets  $H', \overline{H}'$  from  $(\mathbf{27})', (\overline{\mathbf{27}})'$

⇒ model is anomaly free

Decomposition of  $(\mathbf{27})_i$  under  $SU(5) \times U(1)_N$ :

$$(\mathbf{27})_i \rightarrow \underbrace{(\mathbf{10}, 1)_i + (\overline{\mathbf{5}}, 2)_i}_{Q_i, u_i^c, d_i^c, L_i, e_i^c} + \underbrace{(\overline{\mathbf{5}}, -3)_i + (\mathbf{5}, -2)_i}_{H_{1i}, H_{2i}, X_i, \overline{X}_i} + \underbrace{(\mathbf{1}, 5)_i}_{S_i} + \underbrace{(\mathbf{1}, 0)_i}_{n_i^c}$$

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$Q_i, u_i^c, d_i^c, L_i, e_i^c, n_i^c$	Standard Model matter
$S_i$	$U(1)_N$ singlet fields
$H_{1i}, H_{2i}$	Higgs like fields
$X_i, \overline{X}_i$	exotic colored matter
$H', \overline{H}'$	extra Higgs like doublets

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## Approximate the Superpotential:

- imposing a  $Z_2^{B/L}$  (analog to  $R$  parity) and (approximate)  $Z_2^H$  symmetry to evade rapid proton decay and FCNC
- integrate out:  $n_i^c$ ,  $H'$ ,  $\overline{H}'$
- keep dominant terms

$\Rightarrow$

$$\begin{aligned} \mathcal{W}_{E_6\text{SSM}} \approx & h_t(H_u Q)t^c + h_b(H_d Q)b^c + h_\tau(H_d L)\tau^c \\ & + \lambda_i S_3(H_{1i}H_{2i}) + \kappa_i S_3(X_i\overline{X}_i) \\ & (i = 1, 2, 3) \end{aligned}$$

**Note:**  $\mu H_{1i}H_{2i}$  forbidden by  $U(1)_N$  gauge symmetry

# Constrained Exceptional Supersymmetric Standard Model

[P. Athron, S. F. King, D. J. Miller, S. Moretti, R. Nevzorov, Phys.Rev.D80:035009 (2009)]

Constrained model defined by mass universality at  $M_X$ :

$$\begin{aligned}\text{scalar masses} &= m_0, \\ \text{Gaugino masses} &= M_{1/2}, \\ \text{trilinear couplings} &= A\end{aligned}$$

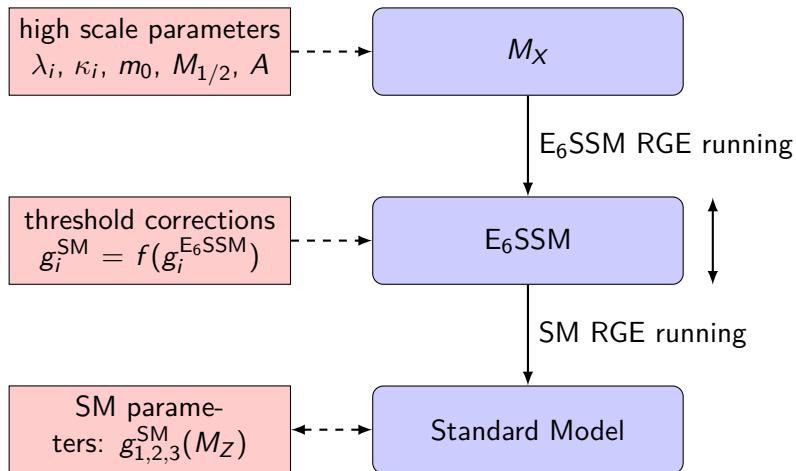
Input parameters for the constrained model:

$$\begin{aligned}\lambda_i(M_X), \kappa_i(M_X), m_0, M_{1/2}, A \\ \Leftrightarrow \lambda_i(M_X), \kappa_i(M_X), v, \tan \beta, \langle S_3 \rangle\end{aligned}$$

**Our aim:** More precise particle masses



# Why do we need threshold corrections?



**Most important:** Gauge coupling  $g_3$  of  $SU(3)_c$ , since  $\beta_3^{1L} = 0$

Lagrangian of full and effective theory:

$$\mathcal{L}^{\text{E}_6\text{SSM}} = \bar{q}(i\not{D} - m_q)q + \sum_{i=1}^2 \left( |D_\mu \tilde{q}_i|^2 - m_{\tilde{q}_i}^2 \tilde{q}_i^* \tilde{q}_i \right) + \dots$$

$$\mathcal{L}^{\text{SM}} = \hat{q}(i\hat{D} - \hat{m}_q)\hat{q} + \dots$$

where

$$D_\mu = \partial_\mu + ig_3 T^a A_\mu^a$$

$$\hat{D}_\mu = \partial_\mu + i\hat{g}_3 T^a \hat{A}_\mu^a$$

## Relative field renormalization

Impose relative field renormalization between fields in full and effective theory

$$\hat{A}_\mu^a = \left(1 + \frac{1}{2}K_A\right) A_\mu^a$$
$$\hat{q} = \left(1 + \frac{1}{2}K_q\right) q$$

# Matching procedure – Step 2

## Matching condition

All one particle irreducible greens functions  $\Gamma$  (and their derivatives) in the full theory shall be equal to these of the effective theory at renormalization scale  $\mu$  and momentum  $p = 0$ .

$$\Gamma_{q,\bar{q}}^{E_6SSM} = \Gamma_{q,\bar{q}}^{SM} \quad \Rightarrow \quad K_q = \left. \frac{\partial}{\partial p} \Gamma_{q,\bar{q}}^{E_6SSM,heavy} \right|_{p=0}$$

$$\Gamma_{A_\mu^a, A_\nu^b}^{E_6SSM} = \Gamma_{A_\mu^a, A_\nu^b}^{SM} \quad \Rightarrow \quad K_A = - \left. \frac{\partial}{\partial p^2} \Gamma_{A_\mu^a, A_\nu^b, T}^{E_6SSM,heavy} \right|_{p=0}$$

$$\Gamma_{A_\mu^a, q, \bar{q}}^{E_6SSM} = \Gamma_{A_\mu^a, q, \bar{q}}^{SM} \quad \Rightarrow \quad -g_3 \gamma^\mu T^a K_1 = \left. \Gamma_{A_\mu^a, q, \bar{q}}^{E_6SSM,heavy} \right|_{p=0}$$

$$\hat{g}_3 = g_3 \left( 1 + K_1 - K_q - \frac{1}{2} K_A \right)$$

# Matching procedure – Contributions

**For example:** Contributions from squarks  $\tilde{q}_1, \tilde{q}_2$ :

$$i\Gamma_{q,\bar{q}}^{E_6SSM,heavy} = \sum_{i=1}^2 \text{Diagram 1}$$

The diagram shows a loop of a squark  $\tilde{q}_i$  (dashed line) and a gluon  $\tilde{g}$  (wavy line). An incoming quark line enters from the left and an outgoing quark line exits to the right, both connected to the loop.

$$i\Gamma_{A_\mu^a, A_\nu^b}^{E_6SSM,heavy} = \sum_{i=1}^2 \text{Diagram 2} + \text{Diagram 3}$$

The first diagram shows a loop of a squark  $\tilde{q}_i$  (dashed line) with two incoming gluon lines (wavy) from the left and two outgoing gluon lines (wavy) to the right. The second diagram shows a loop of a gluon  $\tilde{g}$  (wavy line) with two incoming gluon lines (wavy) from the left and two outgoing gluon lines (wavy) to the right.

$$i\Gamma_{A_\mu^a, q, \bar{q}}^{E_6SSM,heavy} = \sum_{i=1}^2 \text{Diagram 4} + \text{Diagram 5}$$

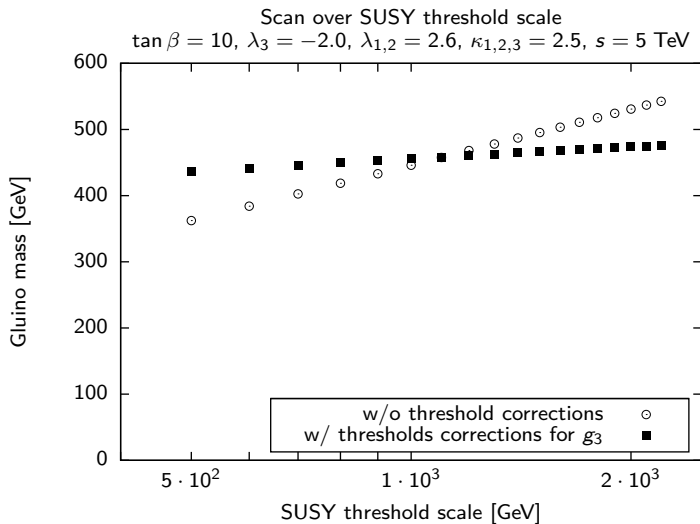
The first diagram shows a vertex where a gluon  $\tilde{g}$  (wavy line) splits into two quarks  $q$  and  $\bar{q}$  (solid lines). A squark  $\tilde{q}_i$  (dashed line) is attached to the vertex. The second diagram shows a vertex where a gluon  $\tilde{g}$  (wavy line) splits into two quarks  $q$  and  $\bar{q}$  (solid lines). A squark  $\tilde{q}_i$  (dashed line) is attached to the vertex.

Result when all non-Standard Model particles are integrated out:

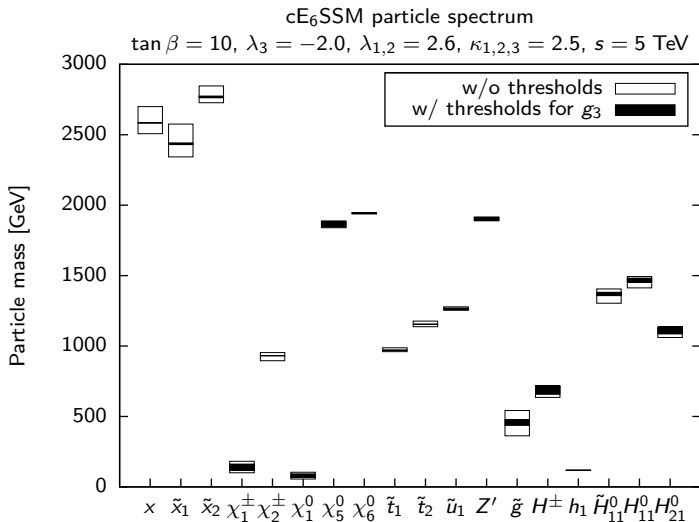
[J. L. Hall, Nucl.Phys.B178 (1981)]

$$g_3^{\text{E}_6\text{SSM},\overline{\text{DR}}} = g_3^{\text{SM},\overline{\text{MS}}} + \frac{g_3^3}{(4\pi)^2} \left\{ \frac{1}{2} - 2 \log \left( \frac{m_{\tilde{g}}}{\mu} \right) - \frac{1}{6} \sum_{\tilde{q}} \log \left( \frac{m_{\tilde{q}}}{\mu} \right) \right. \\ \left. - \frac{2}{3} \sum_x \log \left( \frac{m_x}{\mu} \right) - \frac{1}{6} \sum_{\tilde{x}} \log \left( \frac{m_{\tilde{x}}}{\mu} \right) \right\}$$

# Matching scale dependence

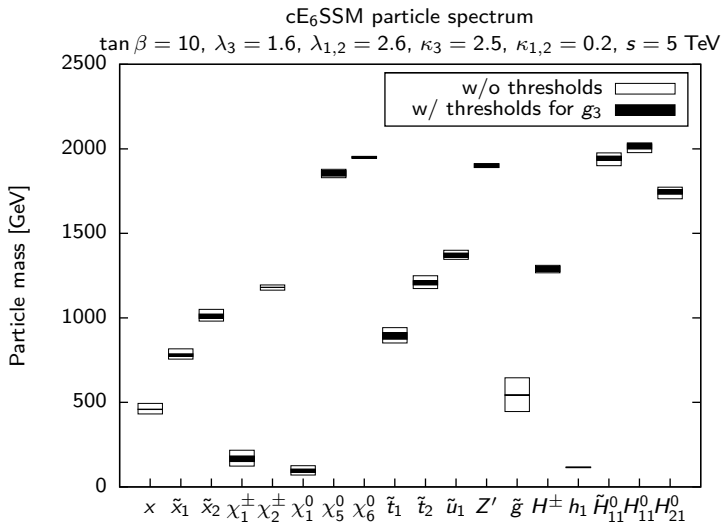


# Matching scale dependence

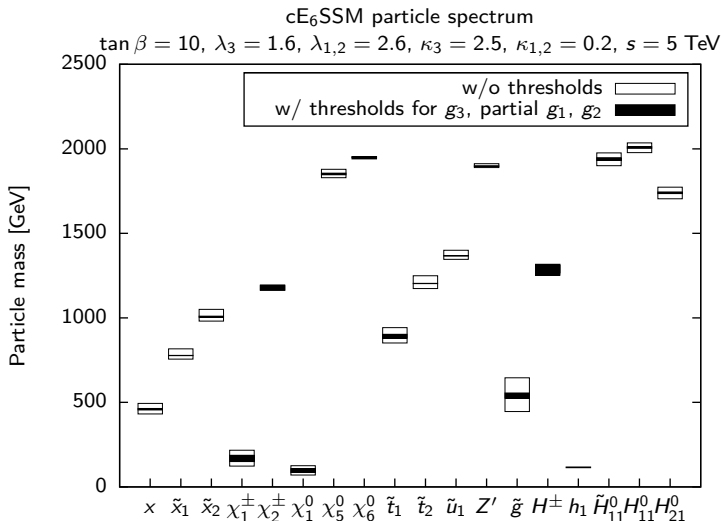




# Matching scale dependence



# Matching scale dependence



## Conclusions:

- $(c)E_6SSM$  is an interesting, well motivated model
- First study of threshold effects in  $cE_6SSM$
- Very split spectrum  $\rightarrow$  threshold corrections important
- threshold corrections reduce dependency of masses upon matching scale

## Future plans:

Increase the precision of the particle spectrum prediction

- complete  $E_6SSM$  threshold corrections for  $g_1, g_2$  (partly done already)
- calculate  $E_6SSM$  threshold corrections for Yukawa couplings
- calculate 2-loop scalar masses
- add shifts to poles masses