Higgs mass prediction in supersymmetry with effective field theory techniques

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Supersymmetry

Still an attractive extension of the Standard Model!

Features:

- can solve the hierarchy problem (heavy BSM particles can give large corrections to the Higgs mass)
- gauge coupling unification at $\sim 10^{16}$ GeV (due to extra matter)
- possible connection to super-gravity models and string theory (E_6SSM, MRSSM)
- can explain deviation of $(g-2)_\mu$
- can stabilize the electroweak vacuum (see later)

Problem: LHC has not found any SUSY particles so far $\Rightarrow$ SUSY particles are probably heavy

Minimal supersymmetry (MSSM)

MSSM = supersymmetric extension of the 2HDM

Particle content (superfields):

$$
\begin{aligned}
\hat{Q} &: (\mathbf{3}, \mathbf{2}, \tfrac{1}{6}), & \hat{u}^c &: (\bar{\mathbf{3}}, \mathbf{1}, -\tfrac{2}{3}), & \hat{d}^c &: (\bar{\mathbf{3}}, \mathbf{1}, \tfrac{1}{3}), \\
\hat{L} &: (\mathbf{1}, \mathbf{2}, -\tfrac{1}{2}), & \hat{e}^c &: (\mathbf{1}, \mathbf{1}, 1), \\
\hat{H}_d &: (\mathbf{1}, \mathbf{2}, -\tfrac{1}{2}), & \hat{H}_u &: (\mathbf{1}, \mathbf{2}, \tfrac{1}{2}), \\
\tilde{B} &: (\mathbf{1}, \mathbf{1}, 0), & \tilde{W} &: (\mathbf{1}, \mathbf{3}, 0), & \tilde{g} &: (\mathbf{8}, \mathbf{1}, 0)
\end{aligned}
$$

Superpotential:

$$
\begin{aligned}
\mathcal{W}_{\text{MSSM}} = (Y_u)_{ij}\, \hat{Q}_i \cdot \hat{H}_u\, \hat{u}_j^c + (Y_d)_{ij}\, \hat{Q}_i \cdot \hat{H}_d\, \hat{d}_j^c + (Y_e)_{ij}\, \hat{L}_i \cdot \hat{H}_d\, \hat{e}_j^c \\
+ \mu \hat{H}_u \cdot \hat{H}_d
\end{aligned}
$$

Minimal supersymmetry (MSSM)

SUSY particles have not been observed ⇒ SUSY must be broken.
Here we consider soft breaking:

$$
\begin{aligned}
\mathcal{L}_{\text{MSSM}}^{\text{soft}} = &-\frac{1}{2}\Big[M_1 \bar{\tilde{B}}^0 \tilde{B}^0 + M_2 \bar{\tilde{W}} \tilde{W} + M_3 \bar{\tilde{g}} \tilde{g}\Big] \\
&- m_{H_u}^2 |H_u|^2 - m_{H_d}^2 |H_d|^2 - \Big[B\mu\, H_u \cdot H_d + \text{h.c.}\Big] \\
&- \Big[\tilde{Q}_i^\dagger (m_Q^2)_{ij} \tilde{Q}_j + \tilde{d}_{Ri}^\dagger (m_d^2)_{ij} \tilde{d}_{Rj} + \tilde{u}_{Ri}^\dagger (m_u^2)_{ij} \tilde{u}_{Rj} \\
&\qquad + \tilde{L}_i^\dagger (m_L^2)_{ij} \tilde{L}_j + \tilde{e}_{Ri}^\dagger (m_e^2)_{ij} \tilde{e}_{Rj}\Big] \\
&+ \Big[(T_u)_{ij} \tilde{Q}_i \cdot H_u \tilde{u}_{\mathrm{R}j}^\dagger + (T_d)_{ij} \tilde{Q}_i \cdot H_d \tilde{d}_{\mathrm{R}j}^\dagger + (T_e)_{ij} \tilde{L}_i \cdot H_d \tilde{e}_{\mathrm{R}j}^\dagger \\
&\qquad + \text{h.c.}\Big]
\end{aligned}
$$

Higgs sector of the MSSM

The two Higgs doublets develop VEVs as

$$H_d = \begin{pmatrix} \frac{v_d}{\sqrt{2}} + H_d^0 \\ H_d^- \end{pmatrix} \qquad H_u = \begin{pmatrix} H_u^+ \\ \frac{v_u}{\sqrt{2}} + H_u^0 \end{pmatrix}$$

$\Rightarrow$

$$m_Z^2 = \frac{1}{4}(g_Y^2 + g_2^2)v^2, \qquad m_W^2 = \frac{1}{4}g_2^2 v^2,$$

$$m_t = y_t v_u/\sqrt{2}, \qquad m_b = y_b v_d/\sqrt{2},$$

$$v^2 := v_u^2 + v_d^2, \qquad \tan\beta := v_u/v_d$$

Mixing of Higgs fields in the CP-conserving MSSM:

$$\left(\mathrm{Re}\, H_d^0, \mathrm{Re}\, H_u^0\right) \to (h, H)$$

$$\left(\mathrm{Im}\, H_d^0, \mathrm{Im}\, H_u^0\right) \to (G^0, A)$$

$$\left((H_d^-)^*, H_u^+\right) \to (G^+, H^+)$$

Considered SUSY scenarios

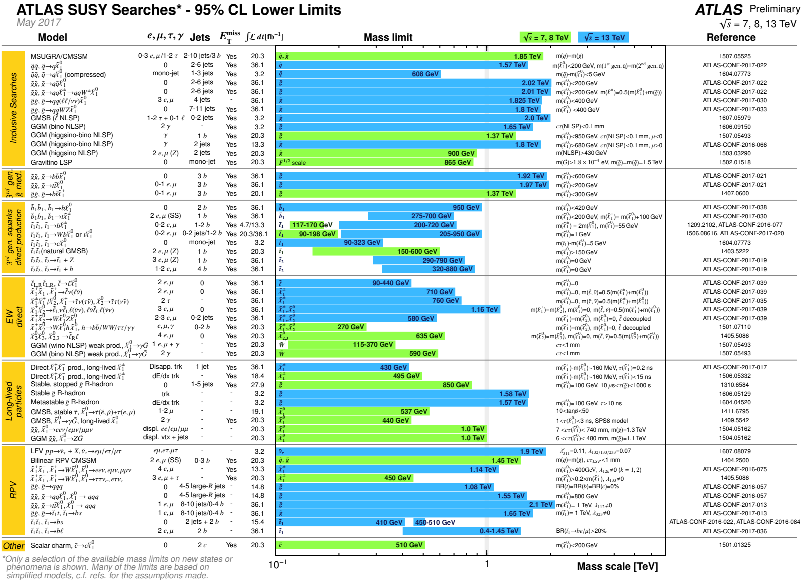
In the following I set for simplicity

$$
\begin{aligned}
&(m_f^2)_{ij}(M_S) = \delta_{ij} M_S^2, \quad (f = Q, u, d, L, e)\\
&M_i(M_S) = M_S, \quad (i = 1, 2, 3)\\
&\mu(M_S) = M_S,\\
&m_A^2(M_S) = \frac{B\mu(M_S)}{\sin\beta(M_S)\cos\beta(M_S)} = M_S^2,\\
&(X_f)_{ij} = (T_f)_{ij}/(Y_f)_{ij} - \begin{Bmatrix} \mu^* \tan\beta \\ \mu^* \cot\beta \end{Bmatrix} \quad \text{for} \quad \begin{cases} f = d, e,\\ f = u, \end{cases}
\end{aligned}
$$

Abbreviations:

$X_t := (X_u)_{33}$, $X_b := (X_d)_{33}$, $X_\tau := (X_e)_{33}$,
$s_\beta := \sin\beta$, $c_\beta := \cos\beta$, $t_\beta := \tan\beta$

Current limits on SUSY particle masses



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CP-even Higgs masses MSSM

$(\mathrm{Re}\,H_d^0, \mathrm{Re}\,H_u^0) \stackrel{\alpha}{\to} (h, H)$ with the mass matrix

$$M = \begin{pmatrix} m_A^2 s_\beta^2 + m_Z^2 c_\beta^2 & -(m_A^2 + m_Z^2) s_\beta c_\beta \\ \cdot & m_A^2 c_\beta^2 + m_Z^2 s_\beta^2 \end{pmatrix}$$

$\Rightarrow$

$$m_{h,H}^2 = \frac{1}{2}\left[m_Z^2 + m_A^2 \mp \sqrt{(m_Z^2 + m_A^2)^2 - 4 m_Z^2 m_A^2 c_{2\beta}^2}\right]$$

$\Rightarrow$

$$m_h^2 < \min(m_Z^2, m_A^2) c_{2\beta}^2$$

If $m_A \gg m_Z$:

$$m_h^2 \approx m_Z^2 c_{2\beta}^2 = \frac{1}{4}(g_Y^2 + g_2^2) v^2 c_{2\beta}^2$$

Higgs mass in the CP-conserving MSSM

If $m_A \gg m_Z$:

$$m_h^2 \approx m_Z^2 c_{2\beta}^2 = \frac{1}{4}(g_Y^2 + g_2^2)v^2 c_{2\beta}^2$$

In the MSSM the tree-level Higgs mass is restricted to be smaller than m_Z!

Q: How can $M_h \approx 125\,\text{GeV}$ be possible in the MSSM?

A: Large loop corrections are to be expected!

$$M_h^2 = m_h^2 + \Delta m_h^2 \qquad \Rightarrow \qquad \Delta m_h^2 \geq (85\,\text{GeV})^2$$

Current status of MSSM Higgs mass calculation

Usual approach: Spectrum generators calculate r.h.s. of

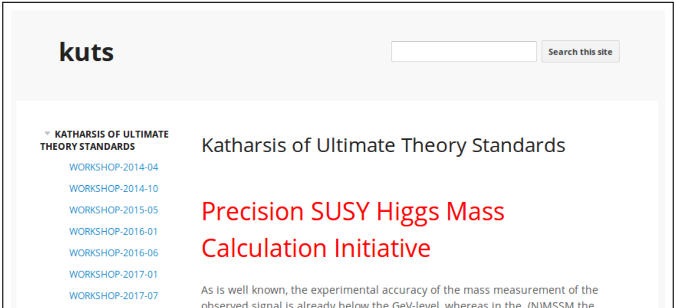
$$M_h^2 = m_h^2 + \Delta m_h^2$$

as a function of all SM and SUSY parameters. Because of large loop corrections Δm_h^2:

$$\Delta M_h^{\text{theo}} \gtrsim (1 \ldots 2)\,\text{GeV} \quad \text{at least!}$$

$$\Delta M_h^{\text{exp}} = 0.24\,\text{GeV}$$ [PDG-2017]

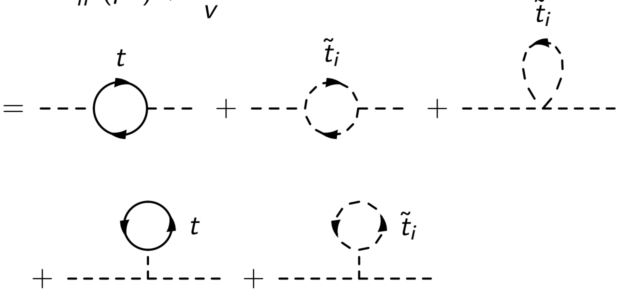
Theory calculation needs to improve! Current workshop series:



Higgs mass at 1-loop level

In the MSSM the following diagrams give the dominant contribution to M_h at the 1-loop level:

$$(\Delta m_h^2)^{1L} = -\Sigma_h^{1L}(p^2) + \frac{t_h^{1L}}{v}$$



$$\approx \frac{12 m_t^2 y_t^2}{(4\pi)^2} \left(\ln \frac{m_{\tilde{t}}^2}{m_t^2} + \frac{X_t^2}{m_{\tilde{t}}^2} - \frac{X_t^4}{12 m_{\tilde{t}}^4} \right) \quad \text{for} \quad m_{\tilde{t}_1} = m_{\tilde{t}_2}, p^2 = 0$$

Higgs mass at 1-loop level

$$(\Delta m_h^2)^{1L} \approx \frac{12 m_t^2 y_t^2}{(4\pi)^2} \left(\ln \frac{m_{\tilde{t}}^2}{m_t^2} + \frac{X_t^2}{m_{\tilde{t}}^2} - \frac{X_t^4}{12 m_{\tilde{t}}^4} \right) + O(p^2)$$

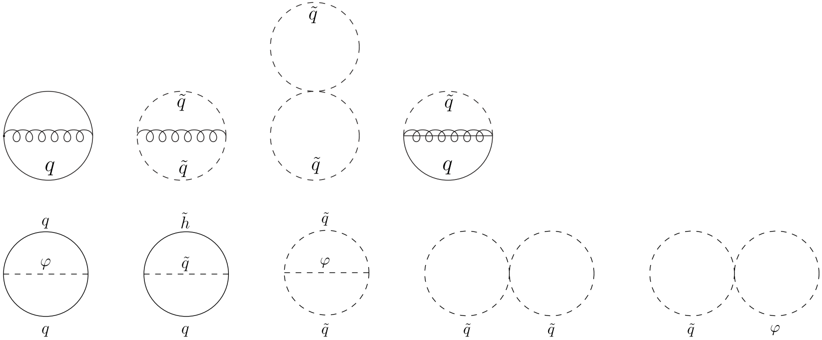
$X_t = A_t - \mu / t_\beta =$ stop mixing parameter

Observations:

- logarithmically enhanced by $m_{\tilde{t}}/m_t$
- maximal for $X_t \approx \sqrt{6} m_{\tilde{t}}$
- high sensitivity on m_t, due to prefactor $m_t^2 y_t^2 = 2 m_t^4 / v^2$
- ambiguity of definition of m_t: pole mass or $\overline{\text{DR}}$ mass?
  $M_t \approx 173.3\,\text{GeV}$, $m_t^{\overline{\text{DR}}} \approx 165\,\text{GeV}$
  $\Rightarrow$ huge theoretical uncertainty!
  $\Rightarrow$ 2-loop calculation needed to resolve this ambiguity
- to get $M_h \approx 125\,\text{GeV}$, $m_{\tilde{t}} \gtrsim 1\,\text{TeV}$ needed (see later)

Higgs mass at 2-loop level

Known contributions: $O(\alpha_s(\alpha_t + \alpha_b) + (\alpha_t + \alpha_b)^2 + \alpha_\tau^2)$ for $p^2 = 0$ [hep-ph/0105096,hep-ph/0112177]



Higgs mass at 2-loop level

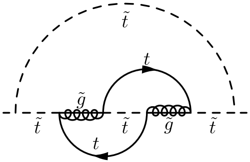
$$(\Delta m_h^2)^{2L} \approx \frac{m_t^2 y_t^4}{(4\pi)^4} \left(c_1 \ln^2 \frac{M_S^2}{m_t^2} + c_2 \ln \frac{M_S^2}{m_t^2} + c_3 \right) + \frac{m_t^2 y_t^2 g_3^2}{(4\pi)^4} \left(c_4 \ln^2 \frac{M_S^2}{m_t^2} + c_5 \ln \frac{M_S^2}{m_t^2} + c_6 \right)$$

Observations:

- logarithmically enhanced by M_S/m_t
- still high sensitivity on m_t
- ambiguity of definition of m_t is resolved
- ambiguity of definition of α_s: $\alpha_s^{\text{SM}}(M_Z)$, $\alpha_s^{\text{MSSM}}(M_S)$, ...?
  $\Rightarrow$ 3-loop calculation needed to resolve this ambiguity

Higgs mass at 3-loop level

Known contributions: $O(\alpha_t \alpha_s^2)$ for $p^2 = 0$ [arXiv:1005.5709]



$$(\Delta m_h^2)^{3L} \approx \frac{m_t^2 y_t^2 g_3^4}{(4\pi)^6} \left(c_7 \ln^3 \frac{M_S^2}{m_t^2} + c_8 \ln^2 \frac{M_S^2}{m_t^2} + c_9 \ln \frac{M_S^2}{m_t^2} + c_{10} \right)$$

Observations:

- logarithmically enhanced by M_S/m_t
- still high sensitivity on m_t
- ambiguity of definition of m_t is resolved
- ambiguity of definition of α_s is resolved

Summary of fixed loop order calculations

Typical order of magnitude of loop contributions (depends on parameter scenario):

$$\begin{aligned} M_h &= m_h + \Delta m_h^{1L} + \Delta m_h^{2L} + \Delta m_h^{3L} + \cdots \\ &\approx [91 + O(20\ldots30) + O(2\ldots4) + O(1\ldots2)]\,\text{GeV} \end{aligned}$$

Advantages:

- includes logarithmic, non-logarithmic and suppressed terms of the order $O(v^2/M_S^2)$ at fixed loop order
- precise prediction if $M_S \sim m_t$

Problem:

- large logarithmic corrections, if $M_S \gg m_t$
  $\Rightarrow$ slow convergence of perturbation series
  $\Rightarrow$ large theoretical uncertainty, (1–2 GeV, or more)
  $M_h^{\text{exp}} = (125.09 \pm 0.24)\,\text{GeV}$

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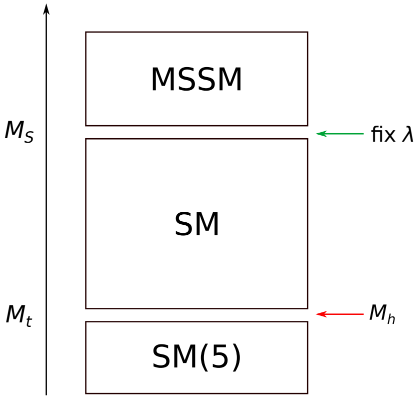
Higgs mass calculation in an EFT

Idea: Integrate out SUSY particles at M_S (expand in v^2/M_S^2)
$\Rightarrow \lambda(M_S)$ is fixed by the MSSM (Remember: in the SM $m_h^2 = \lambda v^2$)
$\Rightarrow$ effectively: separation of scales M_S and M_t.



EFT procedure

Match all renormalized n-point functions at $p^2 = v^2 = 0$, $Q = M_S$ of the SM and MSSM:

$$\partial_{p^2}^{(k)} \Gamma_{h,\dots,h}^{\text{MSSM},(n)} = \partial_{p^2}^{(k)} \Gamma_{h,\dots,h}^{\text{SM},(n)}$$

$\Rightarrow$

$$\lambda(M_S) = \frac{1}{4}\left[g_Y^2 + g_2^2\right] c_{2\beta}^2 + \Delta\lambda^{1L} + \Delta\lambda^{2L} + \cdots$$

RG running of $\lambda(M_S)$ to $Q = M_t$.
Calculation of M_h in the Standard Model:

$$(M_h^{\text{SM}})^2 = \lambda(M_t) v^2 + (\Delta m_h^2)^{1L} + (\Delta m_h^2)^{2L} + \cdots$$

EFT avoids large logarithmic corrections

1. Calculate λ at $Q = M_S$:

$$\lambda(Q) = \frac{1}{4}\left[g_Y^2 + g_2^2\right] c_{2\beta}^2 + \frac{12 m_t^2 y_t^2}{(4\pi)^2 v^2}\left[\ln\frac{M_S^2}{Q^2} + \frac{X_t^2}{M_S^2} - \frac{X_t^4}{12 M_S^4}\right] + \cdots$$

$\Rightarrow$ no large logs

2. RG running from $Q = M_S \to M_t$.
$\Rightarrow$ logs are resummed to all orders

3. Calculate M_h in the SM at $Q = M_t$:

$$(M_h^{\text{SM}})^2 = \lambda v^2 + \frac{12 m_t^2 y_t^2}{(4\pi)^2 v^2} \ln\frac{Q^2}{m_t^2} + \cdots$$

$\Rightarrow$ no large logs

Summary of EFT approach

Typical order of magnitude of loop contributions (depends on parameter scenario, here $X_t = 0$, $M_S = 20\,\text{TeV}$):

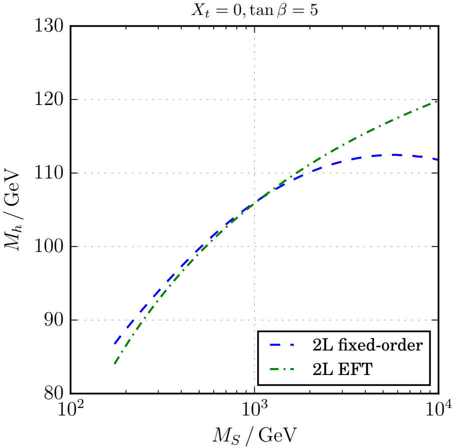
$$\begin{aligned}
M_h &= m_h + \Delta m_h^{1L} + \Delta m_h^{2L} + \cdots \\
&= \sqrt{\lambda(M_t)}v + \Delta m_h^{1L} + \Delta m_h^{2L} + \cdots \\
&\approx [O(124) + O(0.5\ldots1) + O(0.1\ldots0.2)]\,\text{GeV} \\
&= \sqrt{\lambda(M_S)}v + \text{logs} + \Delta m_h^{1L} + \Delta m_h^{2L} + \cdots \\
&\approx [O(84) + O(40) + O(0.5\ldots1) + O(0.1\ldots0.2)]\,\text{GeV}
\end{aligned}$$

Advantages:

- large logarithmic fixed order loop corrections are avoided
- large logarithms $\propto \ln(M_S/M_t)$ are resummed to all orders

Disadvantage: usually terms $O(v^2/M_S^2)$ are neglected
$\Rightarrow$ imprecise when $v \sim M_S$ $\Rightarrow$ then large theoretical uncertainty

Comparison of fixed-order and EFT approaches



Summary of fixed-order and EFT approaches

	low M_S $M_S \lesssim 2\,\text{TeV}$	high M_S $M_S \gtrsim 2\,\text{TeV}$
fixed-order	✓	✗
EFT	✗	✓
? mixed	✓	✓

Q: Can the fixed-order and EFT approaches be combined?

A: Yes! [arXiv:1312.4937, arXiv:1609.00371]

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Mixed fixed-order and EFT approaches

Goal: resum large logarithms **and** include suppressed $O(v^2/M_S^2)$ terms

Two known approaches:

- FeynHiggs [arXiv:1312.4937]: Replace logs from fixed-order calculation by resummed logs

$$M_h^2 = (M_h^2)_{\text{fixed-order}} - (M_h^2)_{\text{logs}} + (M_h^2)_{\text{resummed logs}}$$

- FlexibleEFTHiggs [arXiv:1609.00371]: Incorporate $O(v^2/M_S^2)$ terms into λ by using the matching condition

$$(M_h^2)_{\text{SM}} = (M_h^2)_{\text{MSSM}} \qquad \text{at 1L level at } Q = M_S$$

FlexibleEFTHiggs approach

Idea:

1. Determine $\lambda(M_S)$ from the condition

$$(M_h^2)_{\text{SM}} = (M_h^2)_{\text{MSSM}} \qquad 1\text{L}, \; Q = M_S$$

   No suppressed terms are neglected. $\Rightarrow \lambda$ contains all $O(v^2/M_S^2)$ suppressed terms

2. RG running of $\lambda(M_S)$ from $M_S \to M_t$
   Note: M_h is RG invariant.

3. Calculate M_h in the Standard Model at $Q = M_t$:

$$M_h^2 = \lambda(M_t) v^2 + (\Delta m_h^2)_{\text{SM}}^{1L}$$

   $\Rightarrow M_h$ contains all suppressed terms.

FlexibleEFTHiggs – EFT equivalence

Proof of equivalence: Start with matching condition:

$$(M_h^2)_{\mathrm{SM}} = (M_h^2)_{\mathrm{MSSM}} \qquad 1\mathrm{L},\ Q = M_S$$

$$\lambda v^2 + (\Delta m_h^2)^{1L}_{\mathrm{SM}} = (M_h^2)_{\mathrm{MSSM}}$$

$\Rightarrow$

$$\begin{aligned}\lambda(M_S) &= \frac{1}{v^2}\left[(M_h^2)_{\mathrm{MSSM}} - (\Delta m_h^2)^{1L}_{\mathrm{SM}}\right] \\ &= \frac{1}{v^2}\left[(m_h^2)_{\mathrm{MSSM}} + (\Delta m_h^2)^{1L}_{\mathrm{MSSM}} - (\Delta m_h^2)^{1L}_{\mathrm{SM}}\right]\end{aligned}$$

Now insert $(m_h^2)_{\mathrm{MSSM}}$ and $(\Delta m_h^2)^{1L}_{\mathrm{MSSM}}$ ...

FlexibleEFTHiggs – EFT equivalence

Inserting $(m_h^2)_{\text{MSSM}}$ and $(\Delta m_h^2)^{1L}_{\text{MSSM}}$ for $X_t = 0$:

$$\lambda(M_S) = \frac{1}{v^2}\Bigg[\frac{1}{4}(g_Y^2 + g_2^2)v^2 c_{2\beta}^2 + \frac{c_\alpha^2}{s_\beta^2}(\Delta m_h^2)^{1L}_{\text{SM}} - \frac{c_\alpha^2}{s_\beta^2}\frac{12(y_t^{\text{SM}})^2 m_t^2}{(4\pi)^2} B_0(m_h^2, M_S^2, M_S^2) - (\Delta m_h^2)^{1L}_{\text{SM}}\Bigg]$$

Now go to the decoupling limit $c_\alpha^2/s_\beta^2 \to 1$ ...

FlexibleEFTHiggs – EFT equivalence

In the decoupling limit $c_\alpha^2/s_\beta^2 \to 1$:

$$\begin{aligned}
\lambda(M_S) &= \frac{1}{4}(g_Y^2 + g_2^2)c_{2\beta}^2 - 12\frac{m_t^2 (y_t^{\text{SM}})^2}{(4\pi)^2 v^2} B_0(m_h^2, M_S^2, M_S^2) \\
&= \frac{1}{4}(g_Y^2 + g_2^2)c_{2\beta}^2 - 12\frac{m_t^2 (y_t^{\text{SM}})^2}{(4\pi)^2 v^2} \left[-\log\frac{M_S^2}{Q^2} + \frac{m_h^2}{6M_S^2} + O\Big(\frac{m_h^4}{M_S^4}\Big) \right] \\
&= \frac{1}{4}(g_Y^2 + g_2^2)c_{2\beta}^2 + 12\frac{m_t^2 (y_t^{\text{SM}})^2}{(4\pi)^2 v^2} \left[\log\frac{M_S^2}{Q^2} \right] + O\Big(\frac{v^2}{M_S^2}\Big) \\
&= \lambda^{\text{EFT,tree}} + \Delta\lambda^{\text{EFT},1L} + O\Big(\frac{v^2}{M_S^2}\Big)
\end{aligned}$$

In the decoupling limit $\lambda(M_S)$ in the FlexibleEFTHiggs approach is equivalent to the EFT approach at 1-loop, up to suppressed terms $O(v^2/M_S^2)$

Summary FlexibleEFTHiggs approach

$$(M_h^2)_{\text{SM}} = (M_h^2)_{\text{MSSM}} \qquad 1\text{L},\ Q = M_S$$

$\Rightarrow$

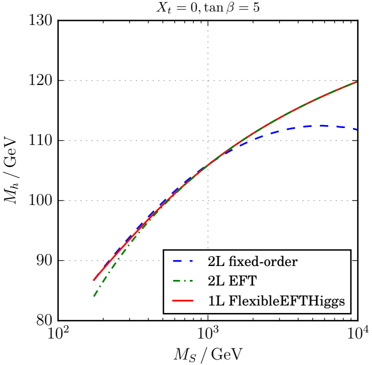
$$\lambda(M_S) = \lambda^{\text{EFT,tree}} + \lambda^{\text{EFT,}1L} + O(v^2/M_S^2)$$

Observations:

- large 1-loop logarithms cancel in matching condition
- for $v = p = 0$ FlexibleEFTHiggs is identical to a 1-loop EFT calculation
- all suppressed terms are incorporated in λ
- RG running resums (N)LL to all orders

$\Rightarrow$ FlexibleEFTHiggs leads to a correct Higgs mass prediction at the full 1-loop level (including suppressed terms) with additional (N)LL resummation.

Comparison of the three approaches



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Ways to estimate the theoretical uncertainty

Good ansatz: Change the calculation by higher orders beyond the calculational accuracy.

Examples:

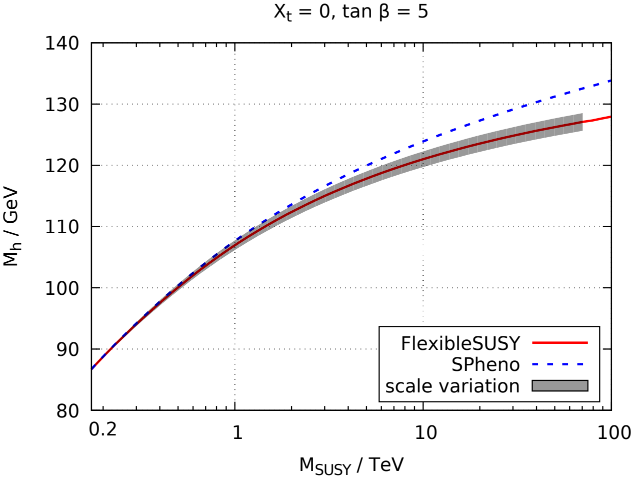
- variation of the unphysical renormalization scale(s)
- change m_t or α_s by higher orders
- re-parametrization of M_h

Potential pitfalls:

- some changes alone are an under-estimation of the uncertainty $\rightarrow$ a combination of multiple changes should be used
- potential over-estimation of the uncertainty when large cancellations occur at higher orders
- some changes are sensitive to the same kind of higher order contributions $\rightarrow$ potential "double counting"
- new higher order dependencies might be difficult to estimate
  Example: How would one estimate α_s dependence of M_h^{2L} when only M_h^{1L} is known?

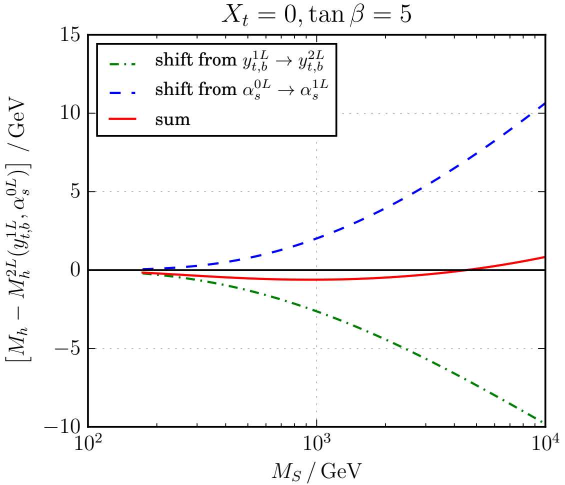
Example: under-estimation of the uncertainty

The variation of the renormalization scale alone might be an under-estimation of the uncertainty:



Example: over-estimation of the uncertainty

When large cancellations between the same kind of corrections from different sources occur, including only one source might lead to an over-estimation:



Higgs mass uncertainty estimate

fixed-order:

- $|M_h^{2L}(Q_\text{pole} = M_S/2) - M_h^{2L}(Q_\text{pole} = 2M_S)|$
- $|M_h^{2L}(y_t^{1L}) - M_h^{2L}(y_t^{2L})|$

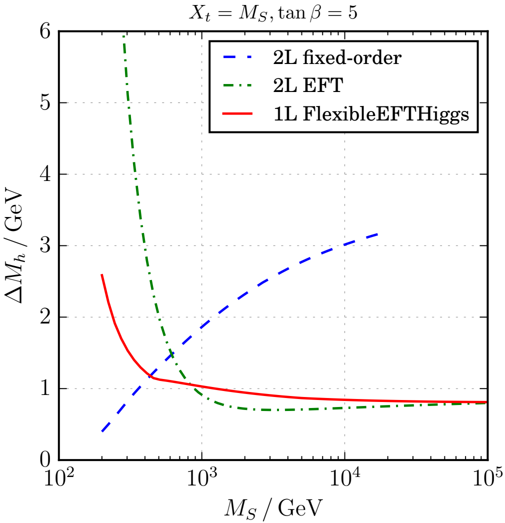
EFT (SUSYHD):

- $|M_h^{2L}(Q_\text{pole} = M_t/2) - M_h^{2L}(Q_\text{pole} = 2M_t)|$
- $|M_h^{2L}(y_t^{2L}) - M_h^{2L}(y_t^{3L})|$
- $|M_h^{2L}(Q_\text{match} = M_S/2) - M_h^{2L}(Q_\text{match} = 2M_S)|$
- $|M_h^{2L} - M_h^{2L}(\lambda \to \lambda(1 + v^2/M_S^2))|$

FlexibleEFTHiggs:

- $|M_h^{2L}(Q_\text{pole} = M_t/2) - M_h^{2L}(Q_\text{pole} = 2M_t)|$
- $|M_h^{2L}(y_t^{2L}) - M_h^{2L}(y_t^{3L})|$
- $|M_h^{2L}(Q_\text{match} = M_S/2) - M_h^{2L}(Q_\text{match} = 2M_S)|$

Higgs mass uncertainty estimate



Summary

Supersymmetry is still viable, but LHC continuously excludes light SUSY scenarios

Approaches to calculate M_h:

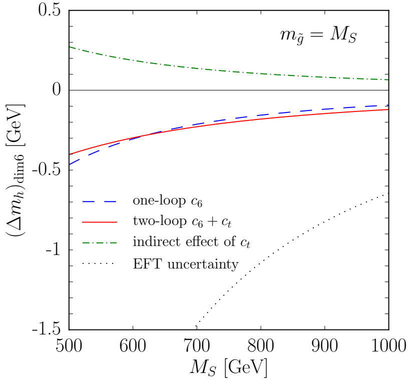
	low M_S $M_S \lesssim 2\,\text{TeV}$	high M_S $M_S \gtrsim 2\,\text{TeV}$
fixed-order	✓	✗
EFT	✗	✓
mixed	✓	✓

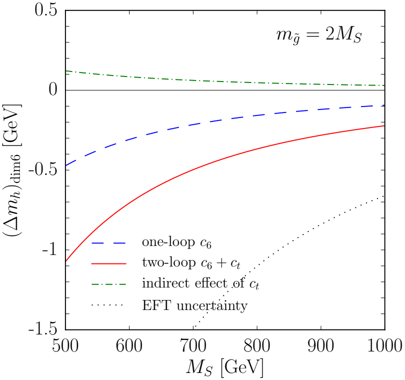
Uncertainty of M_h **in SUSY:**

- tricky to estimate and still ongoing effort!
- $\Delta M_h \gtrsim 1\text{–}2\,\text{GeV}$ at least, but continuously improves

Backup

Effect of higher-dimensional operators

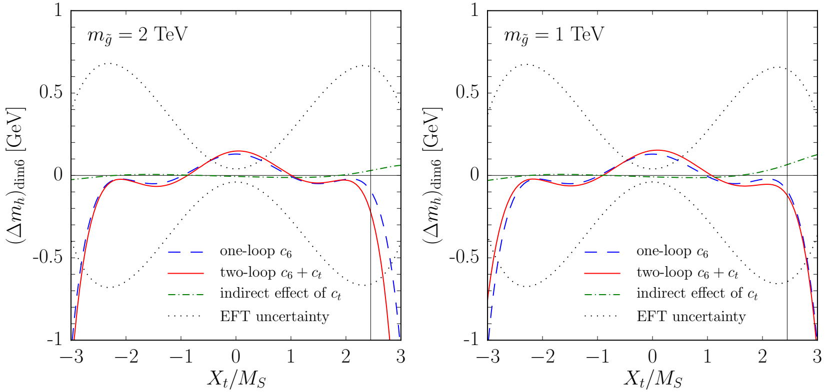




$\tan\beta = 20$, $X_t = \sqrt{6}\, M_S$

[1703.08166]

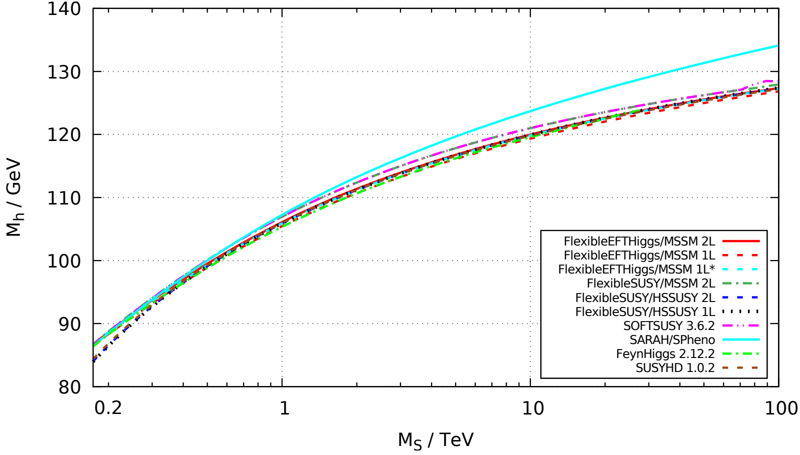
Effect of higher-dimensional operators



$M_S = 1\,\text{TeV}, \tan\beta = 20$

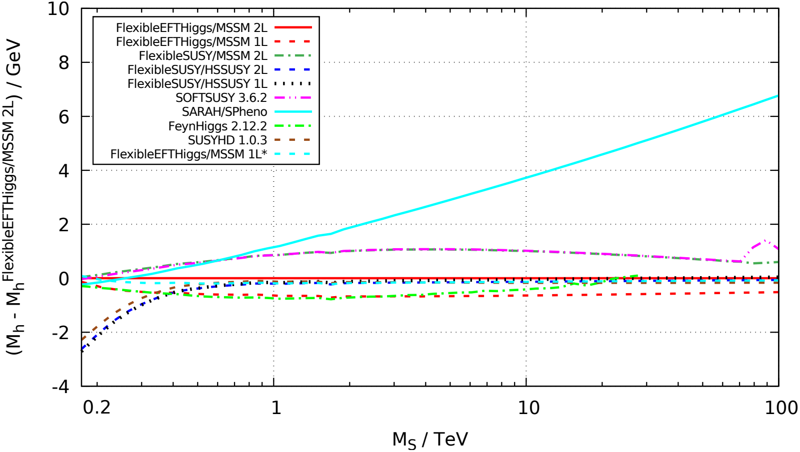
[1703.08166]

Numerical comparison



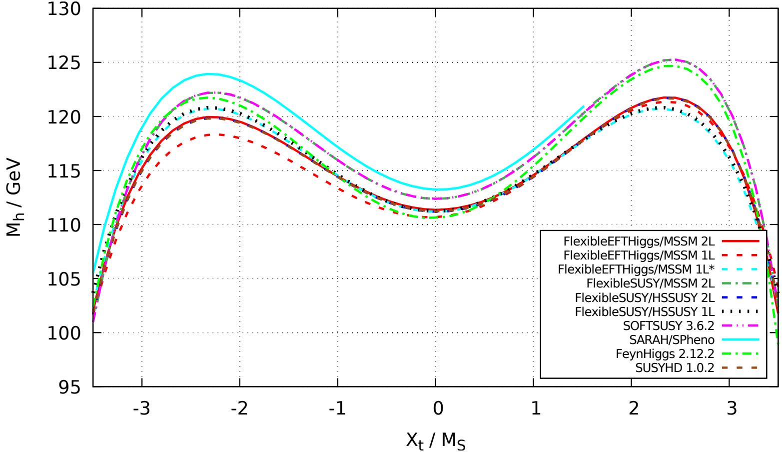
$\tan\beta = 5,\ X_{t,b,\tau} = 0$

Numerical comparison



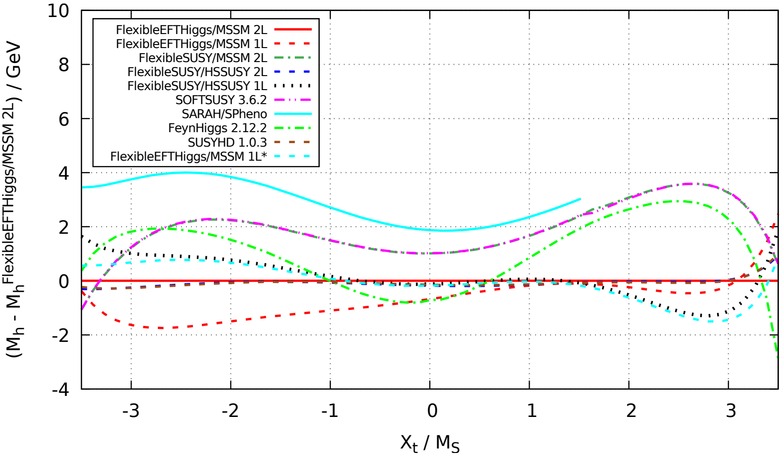
$\tan\beta = 5,\ X_{t,b,\tau} = 0$

Numerical comparison



$\tan\beta = 5,\ M_S = 2\,\text{TeV},\ X_{b,\tau} = 0$

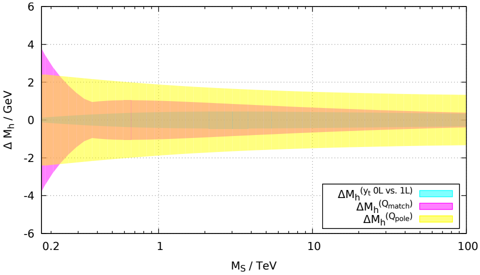
Numerical comparison

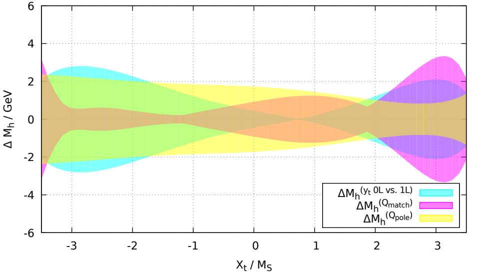


$\tan\beta = 5,\ M_S = 2\,\text{TeV},\ X_{b,\tau} = 0$

For large X_t deviation from HSSUSY-1L due to $p \neq 0 \neq v$.

Uncertainty estimation of original FlexibleEFTHiggs-1L





Incorrect 2L logs in original FlexibleEFTHiggs-1L

Matching condition:

$$\lambda \leftarrow \frac{1}{v^2}\left[(m_h^{\text{SM}})^2 + (M_h^{\text{MSSM}})^2 - (M_h^{\text{SM}})^2\right]$$

Expansion of momentum iteration up to 1L level:

$$\lambda = \frac{1}{v^2}\left[(m_h^{\text{MSSM}})^2 + \Delta m^2_{h,\text{MSSM}} - \Delta m^2_{h,\text{SM}} + O(\hbar^2)\right]$$

with

$$\begin{aligned}\Delta m^2_{h,\text{MSSM}} &= -\Sigma^{1L}_{\text{MSSM}} + t^{1L}_{\text{MSSM}}/v_{\text{MSSM}} \\ \Delta m^2_{h,\text{SM}} &= -\Sigma^{1L}_{\text{SM}} + t^{1L}_{\text{SM}}/v_{\text{SM}}\end{aligned}$$

Incorrect 2L logs in original FlexibleEFTHiggs-1L

Problem: $y_t^{\mathrm{MSSM}} = y_t^{\mathrm{SM}}/s_\beta[1 + O(\hbar)]$
$\Rightarrow$

$$\begin{aligned}
\Delta m^2_{h,\mathrm{MSSM}} - \Delta m^2_{h,\mathrm{SM}} &\propto \hbar\left[(y_t^{\mathrm{MSSM}} s_\beta)^4 \log\frac{m_t}{M_S} - (y_t^{\mathrm{SM}})^4 \log\frac{m_t}{M_S}\right] \\
&= \hbar\left[0 + \propto \hbar y_t^4 \log\frac{m_t}{M_S} + O(\hbar^2)\right] \\
&= O(\hbar^2 y_t^4 \log\frac{m_t}{M_S})
\end{aligned}$$

$\Rightarrow$
incorrect 2L logs remain in FlexibleEFTHiggs-1L

Summary original FlexibleEFTHiggs-1L

Advantages:

- ✓ easily automatizable
- ✓ correctly resums LL
- ✓ all non-log terms correct at 1L, including all terms $O(v^n/M_S^n)$

Disadvantage:

- ✗ incorrect 2L logs $O(\hbar^2 \log(m_t/M_S))$

Improved FlexibleEFTHiggs-1L

strict handling of loop orders in matching condition

$$(M_h^{\text{SM}})^2 = \lambda v^2 - \Sigma_h^{\text{SM}}((m_h^{\text{MSSM}})^2) + \frac{t_h^{\text{SM}}}{v}$$

$$(M_h^{\text{MSSM}})^2 = \text{EV of } \left[M_\phi^{(1)} - \Sigma_\phi^{\text{MSSM}}((m_h^{\text{MSSM}})^2) + \tilde{t}_\phi^{\text{MSSM}}\right]$$

with

$$M_\phi^{(1)} = \text{tree-level mass matrix w/ 1L parameters}$$
$$\Sigma_\phi^{\text{MSSM}} = \text{1L self-energy w/ 0L parameters}$$
$$t_{\phi_i}^{\text{MSSM}} = \text{1L tadpole w/ 0L parameters}$$
$$m_h^{\text{MSSM}} = \text{tree-level mass w/ 0L parameters}$$
$$(\tilde{t}_\phi^{\text{MSSM}})_i = t_{\phi_i}^{\text{MSSM}}/v_i$$

Improved FlexibleEFTHiggs-1L

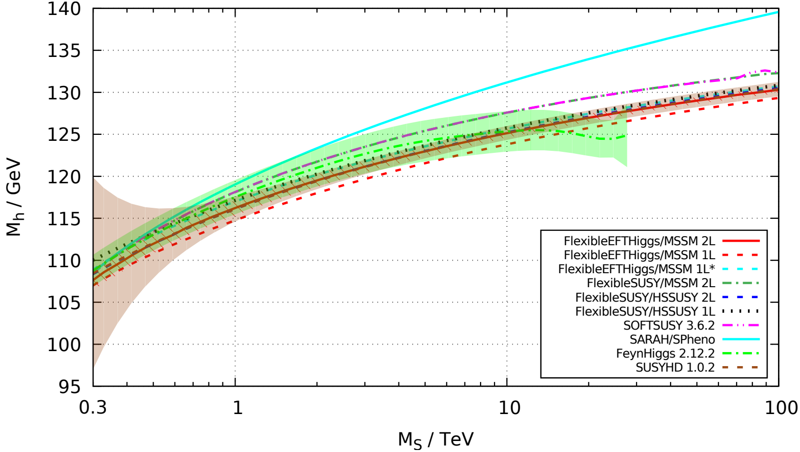
Advantages:

- ✓ easily automatizable
- ✓ correctly resums LL + NLL
- ✓ all non-log terms correct at 1L, including all terms $O(v^n/M_S^n)$

Disadvantage:

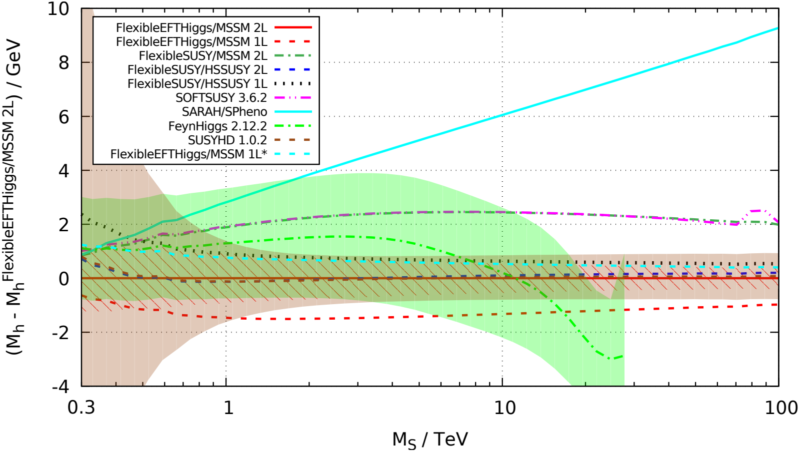
- – non-logarithmic 2L terms arise at M_S
- ✗ difficult to add 2L corrections

Numerical comparison



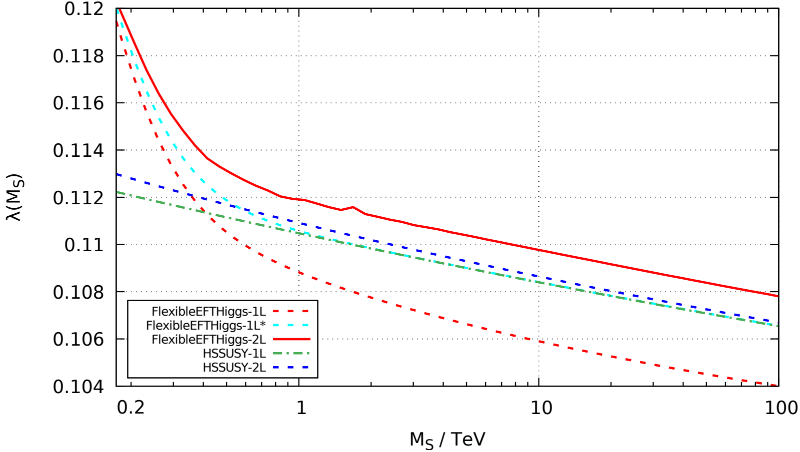
$\tan\beta = 5,\ X_t = -2M_S,\ X_{b,\tau} = 0$

Numerical comparison



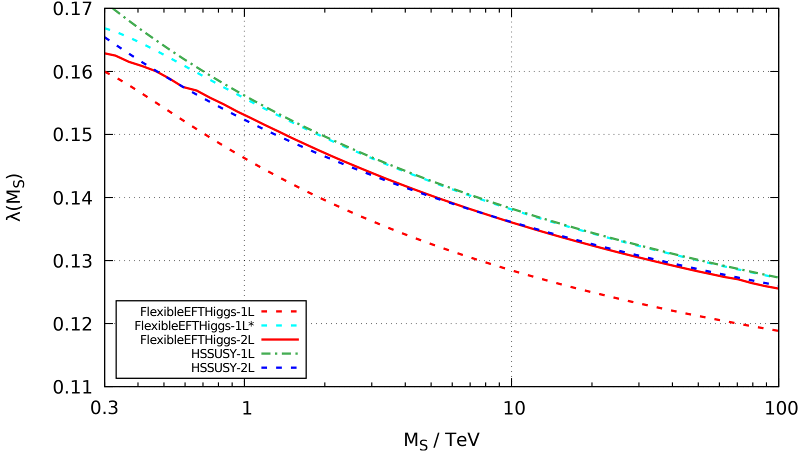
$\tan\beta = 5,\ X_t = -2M_S,\ X_{b,\tau} = 0$

Numerical comparison



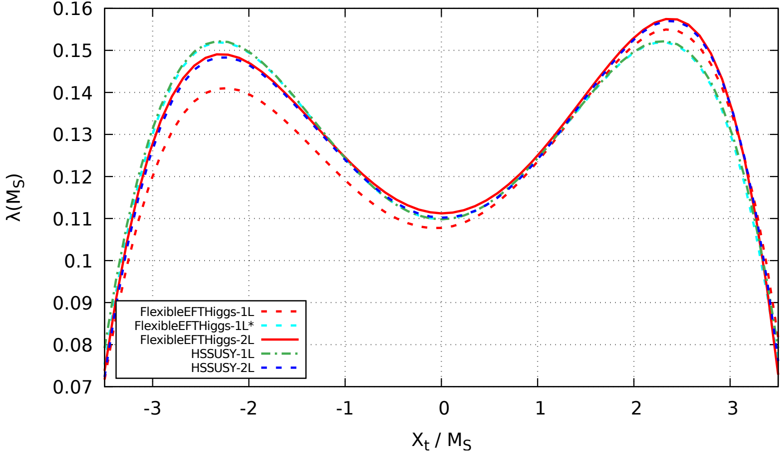
$\tan\beta = 5,\ X_{t,b,\tau} = 0$

Numerical comparison



$\tan\beta = 5,\ X_t = -2M_S,\ X_{b,\tau} = 0$

Numerical comparison



$\tan\beta = 5,\ M_S = 2\,\text{TeV},\ X_{b,\tau} = 0$

Equivalence of pure EFT and FlexibleEFTHiggs

Equivalence pure EFT and FlexibleEFTHiggs $O(\hbar y_t^4)$

$$M_h^{\text{SM}} \stackrel{!}{=} M_h^{\text{MSSM}} \quad \text{at} \quad Q = M_S, 1L$$

where

$$(M_h^{\text{SM}})^2 = \lambda v^2 - \Sigma_h^{\text{SM}} + t_h^{\text{SM}}/v$$

$$t_h^{\text{SM}}/v = -6(y_t^{\text{SM}})^2 A_0(m_t)/(4\pi)^2$$

and [neglecting stop mass mixing $O(m_t X_t/M_S^2)$]

$$(M_h^{\text{MSSM}})^2 = \frac{1}{4}(g_Y^2 + g_2^2)(v_u^2 + v_d^2)c_{2\beta}^2 - \Sigma_h^{\text{MSSM}} + t_h^{\text{MSSM}}/v$$

$$\Sigma_h^{\text{MSSM}} = \Sigma_h^{\text{SM}} \frac{c_\alpha^2}{s_\beta^2} + 3\frac{(y_t^{\text{SM}})^2}{(4\pi)^2}\frac{c_\alpha^2}{s_\beta^2}\Big\{A_0(m_{Q_3}) + A_0(m_{U_3}) + 2m_t\big[B_0(m_{Q_3}, m_{Q_3}) + B_0(m_{U_3}, m_{U_3})\big]\Big\}$$

$$t_h^{\text{MSSM}}/v = -3\frac{(y_t^{\text{SM}})^2}{(4\pi)^2}\frac{c_\alpha^2}{s_\beta^2}\Big[2A_0(m_t) - A_0(m_{Q_3}) - A_0(m_{U_3})\Big]$$

Equivalence pure EFT and FlexibleEFTHiggs $O(\hbar y_t^4)$

in SM limit $\frac{c_\alpha^2}{s_\beta^2} \to 1$
$\Rightarrow$

$$
\begin{aligned}
\lambda &= \frac{1}{4}(g_Y^2 + g_2^2)c_{2\beta}^2 \\
&\quad - 3\frac{(y_t^{\text{SM}})^4}{(4\pi)^2}\Big[B_0(p^2, m_{Q_3}, m_{Q_3}) + B_0(p^2, m_{U_3}, m_{U_3})\Big] \\
&= \frac{1}{4}(g_Y^2 + g_2^2)c_{2\beta}^2 \\
&\quad - 3\frac{(y_t^{\text{SM}})^4}{(4\pi)^2}\Big[- \log\frac{m_{Q_3}^2}{Q^2} + \frac{p^2}{6m_{Q_3}^2} + O\Big(\frac{p^4}{m_{Q_3}^4}\Big) \\
&\qquad\qquad\qquad - \log\frac{m_{U_3}^2}{Q^2} + \frac{p^2}{6m_{U_3}^2} + O\Big(\frac{p^4}{m_{U_3}^4}\Big)\Big] \\
&= [\text{Bagnaschi et. al. 2014}] + O\Big(\frac{p^2}{m_{Q_3}^2}\Big) + O\Big(\frac{p^2}{m_{U_3}^2}\Big)
\end{aligned}
$$

Determination of MSSM parameters

Determination of MSSM parameters

Fixed by observables:

Input				Output
$\alpha_{\text{em}}^{\text{SM(5)}}(M_Z)$	$\rightarrow$	$\alpha_{\text{em}}^{\text{MSSM}}(M_Z)$	$\rightarrow$	$g_1^{\text{MSSM}}(M_Z)$
G_F	$\rightarrow$	$\sin\theta_W^{\text{MSSM}}(M_Z)$	$\rightarrow$	$g_2^{\text{MSSM}}(M_Z)$
$\alpha_{\text{s}}^{\text{SM(5)}}(M_Z)$			$\rightarrow$	$g_3^{\text{MSSM}}(M_Z)$
M_Z	$\rightarrow$	$m_Z^{\text{MSSM}}(M_Z)$	$\rightarrow$	$v^{\text{MSSM}}(M_Z)$
M_t	$\rightarrow$	$m_t^{\text{MSSM}}(M_Z)$	$\rightarrow$	$y_t^{\text{MSSM}}(M_Z)$
$m_b^{\text{SM(5)}}(m_b)$	$\rightarrow$	$m_b^{\text{MSSM}}(M_Z)$	$\rightarrow$	$y_b^{\text{MSSM}}(M_Z)$
M_τ	$\rightarrow$	$m_\tau^{\text{MSSM}}(M_Z)$	$\rightarrow$	$y_\tau^{\text{MSSM}}(M_Z)$

Fixed by 2 EWSB conditions: $m_{H_u}^2$, $m_{H_d}^2$

Free parameters: $\tan\beta$, μ, $B\mu$, $m_{\tilde{f},ij}^2$, M_i, T_{ij}^f

Determination of SM parameters

Fixed by observables:

Input				Output
$\alpha_{\text{em}}^{\text{SM}(5)}(M_Z)$	$\rightarrow$	$\alpha_{\text{em}}^{\text{SM}}(M_Z)$	$\rightarrow$	$g_1^{\text{SM}}(M_Z)$
G_F	$\rightarrow$	$\sin\theta_W^{\text{SM}}(M_Z)$	$\rightarrow$	$g_2^{\text{SM}}(M_Z)$
$\alpha_{\text{s}}^{\text{SM}(5)}(M_Z)$			$\rightarrow$	$g_3^{\text{SM}}(M_Z)$
M_Z	$\rightarrow$	$m_Z^{\text{SM}}(M_Z)$	$\rightarrow$	$v^{\text{SM}}(M_Z)$
M_t	$\rightarrow$	$m_t^{\text{SM}}(M_Z)$	$\rightarrow$	$y_t^{\text{SM}}(M_Z)$
$m_b^{\text{SM}(5)}(m_b)$	$\rightarrow$	$m_b^{\text{SM}}(M_Z)$	$\rightarrow$	$y_b^{\text{SM}}(M_Z)$
M_τ	$\rightarrow$	$m_\tau^{\text{SM}}(M_Z)$	$\rightarrow$	$y_\tau^{\text{SM}}(M_Z)$

Fixed by 1 EWSB condition: μ^2

Free parameter: λ

Determination of $g_3^{\mathsf{MSSM}}(M_S)$

$$\alpha_s^{\mathsf{MSSM}}(M_S) = \frac{\alpha_s^{\mathsf{SM}}(M_S)}{1 - \Delta\alpha_s(M_S)}$$

with

$$\Delta\alpha_s(Q) = \frac{\alpha_s}{2\pi}\left[\frac{1}{2} - \sum_{\text{SUSY particle } f} T_f \log\frac{m_f}{Q}\right]$$

$\Rightarrow$

$$g_3^{\mathsf{MSSM}}(M_S) = \sqrt{4\pi\alpha_s^{\mathsf{MSSM}}(M_S)}$$

Determination of $v_i^{\text{MSSM}}(M_S)$

$$M_Z^{\text{SM}} = M_Z^{\text{MSSM}}$$

$\Rightarrow$

$$\begin{aligned}(m_Z^{\text{MSSM}}(M_S))^2 &= (M_Z^{\text{SM}})^2 + \Pi_Z^{\text{MSSM},1L}(Q = M_S) \\ (M_Z^{\text{SM}})^2 &= \frac{1}{4}\left[(g_Y^{\text{SM}})^2 + (g_2^{\text{SM}})^2\right](v^{\text{SM}})^2 - \Pi_Z^{\text{SM},1L}(Q = M_S)\end{aligned}$$

$\Rightarrow$

$$v^{\text{MSSM}}(M_S) = \frac{2m_Z^{\text{MSSM}}(M_S)}{\sqrt{(g_Y^{\text{MSSM}})^2 + (g_2^{\text{MSSM}})^2}}$$

$\Rightarrow$

$$\begin{aligned}v_u^{\text{MSSM}}(M_S) &= v^{\text{MSSM}}(M_S)\sin\beta(M_S) \\ v_d^{\text{MSSM}}(M_S) &= v^{\text{MSSM}}(M_S)\cos\beta(M_S)\end{aligned}$$

Determination of $y_i^{\text{MSSM}}(M_S)$

$$M_f^{\text{SM}} = M_f^{\text{MSSM}}$$

$\Rightarrow$

$$m_f^{\text{MSSM}}(M_S) = M_f^{\text{SM}} + \Sigma_f^{\text{MSSM},1L}(Q = M_S)$$
$$M_f^{\text{SM}} = \frac{\sqrt{2} m_f^{\text{SM}}}{v_i^{\text{SM}}} - \Sigma_f^{\text{SM},1L}(Q = M_S)$$

$\Rightarrow$

$$y_f^{\text{MSSM}}(M_S) = \frac{\sqrt{2} m_f^{\text{MSSM}}(M_S)}{v_i^{\text{MSSM}}(M_S)}$$

Determination of SM parameters

Determination of $g_3^{\mathsf{SM}}(M_Z)$

Input: $\alpha_s^{\mathsf{SM}(5)}(M_Z) = 0.1185$

$\rightarrow$

$$\alpha_s^{\mathsf{SM}}(M_Z) = \frac{\alpha_s^{\mathsf{SM}(5)}(M_Z)}{1 - \Delta\alpha_s(M_Z)}$$

with

$$\Delta\alpha_s(Q) = \frac{\alpha_s}{2\pi}\left[-\frac{2}{3}\log\frac{m_t}{Q}\right]$$

$\Rightarrow$

$$g_3^{\mathsf{SM}}(M_Z) = \sqrt{4\pi\alpha_s^{\mathsf{SM}}(M_Z)}$$

Determination of $y_t^{\mathrm{SM}}(M_Z)$

$$y_t^{\mathrm{SM}}(M_Z) = \frac{\sqrt{2}\, m_t^{\mathrm{SM}}(M_Z)}{v(M_Z)}$$

where

$$m_t^{\mathrm{SM}}(Q) = M_t + \mathrm{Re}\,\Sigma_t^S(M_Z) + M_t\Big[\mathrm{Re}\,\Sigma_t^L(M_Z) + \mathrm{Re}\,\Sigma_t^R(M_Z) + \Delta m_t^{1L,\mathrm{gluon}} + \Delta m_t^{2L,\mathrm{gluon}}\Big]$$

$$\Delta m_t^{1L,\mathrm{gluon}} = -\frac{g_3^2}{12\pi^2}\left[4 - 3\log\left(\frac{m_t^2}{Q^2}\right)\right]$$

$$\Delta m_t^{2L,\mathrm{gluon}} = \left(\Delta m_t^{1L,\mathrm{gluon}}\right)^2 - \frac{g_3^4}{4608\pi^4}\left[396\log^2\left(\frac{m_t^2}{Q^2}\right) - 1452\log\left(\frac{m_t^2}{Q^2}\right) - 48\zeta(3) + 2053 + 16\pi^2(1+\log 4)\right]$$

Determination of v^{SM}

The VEV v^{SM} is calculated from the running Z mass at $Q = M_Z$:

$$v^{\text{SM}}(M_Z) = \frac{2 m_Z^{\text{SM}}(M_Z)}{\sqrt{g_Y^2 + g_2^2}}$$
$$m_Z^{\text{SM}}(M_Z) = \sqrt{M_Z^2 + \Pi_Z^{1L}(p^2 = M_Z^2, Q = M_Z)}$$

v^{SM} evolves under RG running according to
[Sperling, Stöckinger, AV, 2013, 2014]

Comparison full model vs. EFT approach

Q: Why is FlexibleSUSY/MSSM so close to the EFT approaches and SPheno so far off?

Calculation of $y_t^{\text{MSSM}}(M_Z)$

A: Different treatment of 2-loop corrections to $y_t^{\text{MSSM}}(M_Z)$:

FlexibleSUSY:

$$m_t = M_t + \text{Re}\left[\widetilde{\Sigma}_t^{(1),S}(M_t)\right] + M_t \, \text{Re}\left[\widetilde{\Sigma}_t^{(1),L}(M_t) + \widetilde{\Sigma}_t^{(1),R}(M_t)\right] + M_t\left[\widetilde{\Sigma}_t^{(1),\text{qcd}}(m_t) + \left(\widetilde{\Sigma}_t^{(1),\text{qcd}}(m_t)\right)^2 + \widetilde{\Sigma}_t^{(2),\text{qcd}}(m_t)\right]$$

SPheno:

$$m_t = M_t + \text{Re}\left[\widetilde{\Sigma}_t^{(1),S}(m_t)\right] + m_t \, \text{Re}\left[\widetilde{\Sigma}_t^{(1),L}(m_t) + \widetilde{\Sigma}_t^{(1),R}(m_t)\right] + m_t\left[\widetilde{\Sigma}_t^{(1),\text{qcd}}(m_t) + \widetilde{\Sigma}_t^{(2),\text{qcd}}(m_t)\right]$$

Calculation of $y_t^{\text{MSSM}}(M_Z)$

$\Rightarrow$

$$\tilde{y}_t^{\text{FlexibleSUSY}} = y_t + t^2\kappa^2\left(\frac{184}{9}g_3^4 y_t - 24 g_3^2 y_t^3 + \frac{9}{8}y_t^5\right) + \ldots$$

$$\tilde{y}_t^{\text{SPheno}} = y_t + t^2\kappa^2\left(\frac{248}{9}g_3^4 y_t - 16 g_3^2 y_t^3 + \frac{27}{8}y_t^5\right) + \ldots$$

with

$$y_t \equiv y_t^{\text{SM}}(M_S), \qquad g_3 \equiv g_3^{\text{SM}}(M_S),$$

$$\tilde{y}_t \equiv y_t^{\text{MSSM}}(M_S), \qquad \tilde{g}_3 \equiv g_3^{\text{MSSM}}(M_S),$$

$$t \equiv \log\frac{M_S}{M_t}, \qquad \kappa \equiv \frac{1}{(4\pi)^2}$$

Calculation of $y_t^{\text{MSSM}}(M_Z)$

$$
\begin{aligned}
(M_h^2)^{\text{EFT}} &= m_h^2 + v^2 y_t^4 \Big[12 t\kappa + 12 t^2 \kappa^2 \left(16 g_3^2 - 9 y_t^2\right) \\
&\quad + 4 t^3 \kappa^3 \left(736 g_3^4 - 672 g_3^2 y_t^2 + 90 y_t^4\right) + \dots \Big], \\
(M_h^2)^{\text{FlexibleSUSY}} &= m_h^2 + v^2 y_t^4 \Big[12 t\kappa + 12 t^2 \kappa^2 \left(16 g_3^2 - 9 y_t^2\right) \\
&\quad + 4 t^3 \kappa^3 \left(\frac{736 g_3^4}{3} - 288 g_3^2 y_t^2 + \frac{27 y_t^4}{2}\right) + \dots \Big], \\
(M_h^2)^{\text{SPheno}} &= m_h^2 + v^2 y_t^4 \Big[12 t\kappa + 12 t^2 \kappa^2 \left(16 g_3^2 - 9 y_t^2\right) \\
&\quad + 4 t^3 \kappa^3 \left(\frac{992 g_3^4}{3} - 192 g_3^2 y_t^2 + \frac{81 y_t^4}{2}\right) + \dots \Big].
\end{aligned}
$$