

Higgs mass prediction in supersymmetry with effective field theory techniques

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- ① Introduction to supersymmetry
- ② Higgs mass calculation in the MSSM
at fixed loop order
in an EFT
in a mixed approach
- ③ Uncertainty estimate
- ④ Summary

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Supersymmetry

Still an attractive extension of the Standard Model!

Features:

- can solve the hierarchy problem (heavy BSM particles can give large corrections to the Higgs mass)
- gauge coupling unification at $\sim 10^{16}$ GeV (due to extra matter)
- possible connection to super-gravity models and string theory (E_6 SSM, MRSSM)
- can explain deviation of $(g - 2)_\mu$
- can stabilize the electroweak vacuum (see later)

Problem: LHC has not found any SUSY particles so far \Rightarrow SUSY particles are probably heavy

Minimal supersymmetry (MSSM)

MSSM = supersymmetric extension of the 2HDM

Particle content (superfields):

$$\begin{aligned}\hat{Q} &: (\mathbf{3}, \mathbf{2}, \frac{1}{6}), & \hat{u}^c &: (\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3}), & \hat{d}^c &: (\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3}), \\ \hat{L} &: (\mathbf{1}, \mathbf{2}, -\frac{1}{2}), & \hat{e}^c &: (\mathbf{1}, \mathbf{1}, 1), \\ \hat{H}_d &: (\mathbf{1}, \mathbf{2}, -\frac{1}{2}), & \hat{H}_u &: (\mathbf{1}, \mathbf{2}, \frac{1}{2}), \\ \tilde{B} &: (\mathbf{1}, \mathbf{1}, 0), & \tilde{W} &: (\mathbf{1}, \mathbf{3}, 0), & \tilde{g} &: (\mathbf{8}, \mathbf{1}, 0)\end{aligned}$$

Superpotential:

$$\begin{aligned}\mathcal{W}_{\text{MSSM}} &= (Y_u)_{ij} \hat{Q}_i \cdot \hat{H}_u \hat{u}_j^c + (Y_d)_{ij} \hat{Q}_i \cdot \hat{H}_d \hat{d}_j^c + (Y_e)_{ij} \hat{L}_i \cdot \hat{H}_d \hat{e}_j^c \\ &+ \mu \hat{H}_u \cdot \hat{H}_d\end{aligned}$$

Minimal supersymmetry (MSSM)

SUSY particles have not been observed \Rightarrow SUSY must be broken.
Here we consider soft breaking:

$$\begin{aligned}\mathcal{L}_{\text{MSSM}}^{\text{soft}} = & -\frac{1}{2} \left[M_1 \bar{B}^0 \tilde{B}^0 + M_2 \bar{W} \tilde{W} + M_3 \bar{g} \tilde{g} \right] \\ & - m_{H_u}^2 |H_u|^2 - m_{H_d}^2 |H_d|^2 - \left[B\mu H_u \cdot H_d + \text{h.c.} \right] \\ & - \left[\tilde{Q}_i^\dagger (m_Q^2)_{ij} \tilde{Q}_j + \tilde{d}_{Ri}^\dagger (m_d^2)_{ij} \tilde{d}_{Rj} + \tilde{u}_{Ri}^\dagger (m_u^2)_{ij} \tilde{u}_{Rj} \right. \\ & \quad \left. + \tilde{L}_i^\dagger (m_L^2)_{ij} \tilde{L}_j + \tilde{e}_{Ri}^\dagger (m_e^2)_{ij} \tilde{e}_{Rj} \right] \\ & + \left[(T_u)_{ij} \tilde{Q}_i \cdot H_u \tilde{u}_{Rj}^\dagger + (T_d)_{ij} \tilde{Q}_i \cdot H_d \tilde{d}_{Rj}^\dagger + (T_e)_{ij} \tilde{L}_i \cdot H_d \tilde{e}_{Rj}^\dagger \right. \\ & \quad \left. + \text{h.c.} \right]\end{aligned}$$

Higgs sector of the MSSM

The two Higgs doublets develop VEVs as

$$H_d = \begin{pmatrix} \frac{v_d}{\sqrt{2}} + H_d^0 \\ H_d^- \end{pmatrix} \quad H_u = \begin{pmatrix} H_u^+ \\ \frac{v_u}{\sqrt{2}} + H_u^0 \end{pmatrix}$$

\Rightarrow

$$\begin{aligned} m_Z^2 &= \frac{1}{4}(g_Y^2 + g_2^2)v^2, & m_W^2 &= \frac{1}{4}g_2^2v^2, \\ m_t &= y_t v_u / \sqrt{2}, & m_b &= y_b v_d / \sqrt{2}, \\ v^2 &:= v_u^2 + v_d^2, & \tan \beta &:= v_u / v_d \end{aligned}$$

Mixing of Higgs fields in the CP-conserving MSSM:

$$\begin{aligned} (\text{Re } H_d^0, \text{Re } H_u^0) &\rightarrow (h, H) \\ (\text{Im } H_d^0, \text{Im } H_u^0) &\rightarrow (G^0, A) \\ ((H_d^-)^*, H_u^+) &\rightarrow (G^+, H^+) \end{aligned}$$

Considered SUSY scenarios

In the following I set for simplicity

$$(m_f^2)_{ij}(M_S) = \delta_{ij} M_S^2, \quad (f = Q, u, d, L, e)$$

$$M_i(M_S) = M_S, \quad (i = 1, 2, 3)$$

$$\mu(M_S) = M_S,$$

$$m_A^2(M_S) = \frac{B\mu(M_S)}{\sin \beta(M_S) \cos \beta(M_S)} = M_S^2,$$

$$(X_f)_{ij} = (T_f)_{ij} / (Y_f)_{ij} - \begin{cases} \mu^* \tan \beta \\ \mu^* \cot \beta \end{cases} \quad \text{for} \quad \begin{cases} f = d, e, \\ f = u, \end{cases}$$

Abbreviations:

$$X_t := (X_u)_{33}, \quad X_b := (X_d)_{33}, \quad X_\tau := (X_e)_{33},$$

$$s_\beta := \sin \beta, \quad c_\beta := \cos \beta, \quad t_\beta := \tan \beta$$

Current limits on SUSY particle masses

ATLAS SUSY Searches* - 95% CL Lower Limits

May 2017

ATLAS Preliminary

$\sqrt{s} = 7, 8, 13 \text{ TeV}$

Model	$\epsilon, \mu, \tau, \gamma$	Jets	E_T^{miss}	$[\mathcal{L} \text{ d}(\text{fb}^{-1})]$	Mass limit	$\sqrt{s} = 7, 8 \text{ TeV}$	$\sqrt{s} = 13 \text{ TeV}$	Reference
Inclusive Searches	MSUGRA/CMSSM	$0.3 \epsilon, \mu, \tau + 2 \epsilon$	2.10 jets/3 b	Yes	20.3	4.8	1.85 TeV	$m(\tilde{g})=m(\tilde{u})$ 1507.05525
	$\tilde{g}, \tilde{u}, \tilde{d}$	0	2 jets	Yes	36.1	1.57 TeV	$m(\tilde{g})=200 \text{ GeV}, m(\tilde{t}^*) \text{ gen. q}(m(\tilde{t}^*)^2 \text{ gen. q})$	ATLAS-CONF-2017-022
	$\tilde{g}, \tilde{u}, \tilde{d}$ (compressed)	mono-jet	1-3 jets	Yes	3.2	908 GeV	$m(\tilde{g})=m(\tilde{t}^*)=5 \text{ GeV}$	1604.07773
	$\tilde{g}, \tilde{u}, \tilde{d}$	0	2-6 jets	Yes	36.1	2.02 TeV	$m(\tilde{g})=200 \text{ GeV}$	ATLAS-CONF-2017-022
	$\tilde{g}, \tilde{u}, \tilde{d}$	0	2-6 jets	Yes	36.1	2.01 TeV	$m(\tilde{g})=200 \text{ GeV}, m(\tilde{t}^*)=0.5 m(\tilde{t}^*)+m(\tilde{g})$	ATLAS-CONF-2017-022
	$\tilde{g}, \tilde{u}, \tilde{d}$	3	4 jets	Yes	36.1	1.825 TeV	$m(\tilde{g})=400 \text{ GeV}$	ATLAS-CONF-2017-030
	$\tilde{g}, \tilde{u}, \tilde{d}$	0	7-11 jets	Yes	36.1	1.8 TeV	$m(\tilde{g})=400 \text{ GeV}$	ATLAS-CONF-2017-033
	GMSB (\tilde{g} NLSP)	$1.2 \epsilon + 0.1 \epsilon$	0-2 jets	Yes	3.2	2.0 TeV	$m(\tilde{g})=200 \text{ GeV}$	1607.05979
	GGM (bino NLSP)	2 γ	-	Yes	3.2	1.65 TeV	$\tau \rightarrow \tau \text{ NLSP} < 0.1 \text{ mm}$	1606.09150
	GGM (Higgsino-bino NLSP)	2 γ	1 jet	Yes	20.3	1.37 TeV	$m(\tilde{g})=950 \text{ GeV}, m(\text{NLSP}) < 0.1 \text{ mm}, \mu < 0$	1507.05493
1/2 gen. squarks direct	$\tilde{g}, \tilde{u}, \tilde{d}$	0	3 b	Yes	36.1	1.92 TeV	$m(\tilde{g})=800 \text{ GeV}$	ATLAS-CONF-2017-021
	$\tilde{g}, \tilde{u}, \tilde{d}$	0-1 ϵ, μ	3 b	Yes	36.1	1.97 TeV	$m(\tilde{g})=200 \text{ GeV}$	ATLAS-CONF-2017-021
	$\tilde{g}, \tilde{u}, \tilde{d}$	0-1 ϵ, μ	3 b	Yes	20.1	1.37 TeV	$m(\tilde{g})=300 \text{ GeV}$	1407.8600
	$\tilde{g}, \tilde{u}, \tilde{d}$	0	2 b	Yes	36.1	990 GeV	$m(\tilde{g})=420 \text{ GeV}$	ATLAS-CONF-2017-038
	$\tilde{g}, \tilde{u}, \tilde{d}$	2 ϵ, μ (SS)	1 b	Yes	36.1	275-700 GeV	$m(\tilde{g})=200 \text{ GeV}, m(\tilde{t}^*)=m(\tilde{b}^*)=100 \text{ GeV}$	ATLAS-CONF-2017-030
	$\tilde{g}, \tilde{u}, \tilde{d}$	0-2 ϵ, μ	1-2 b	Yes	4.71/3.13	117-170 GeV	$m(\tilde{g})=200 \text{ GeV}, m(\tilde{t}^*)=200-720 \text{ GeV}$	1329.2102, ATLAS-CONF-2016-077
	$\tilde{g}, \tilde{u}, \tilde{d}$	0-2 ϵ, μ	0-2 jets/1-2 b	Yes	20.3/36.1	90-196 GeV	$m(\tilde{g})=200 \text{ GeV}, m(\tilde{t}^*)=55 \text{ GeV}$	1506.08616, ATLAS-CONF-2017-020
	$\tilde{g}, \tilde{u}, \tilde{d}$	0	mono-jet	Yes	3.2	90-323 GeV	$m(\tilde{g})=10 \text{ GeV}$	1604.07773
	$\tilde{g}, \tilde{u}, \tilde{d}$ (natural GMSB)	2 ϵ, μ (Z)	1 b	Yes	20.3	150-600 GeV	$m(\tilde{g}), m(\tilde{t}^*) < 6 \text{ GeV}$	1403.5222
	$\tilde{g}, \tilde{u}, \tilde{d}$	3 ϵ, μ (Z)	1 b	Yes	36.1	250-750 GeV	$m(\tilde{g}) > 150 \text{ GeV}$	ATLAS-CONF-2017-019
1/2 gen. squarks indirect	$\tilde{g}, \tilde{u}, \tilde{d}$	1-2 ϵ, μ	4 b	Yes	36.1	320-850 GeV	$m(\tilde{g})=0 \text{ GeV}$	ATLAS-CONF-2017-019
	$\tilde{g}, \tilde{u}, \tilde{d}$	2 ϵ, μ	0	Yes	36.1	90-440 GeV	$m(\tilde{g})=0$	ATLAS-CONF-2017-039
	$\tilde{g}, \tilde{u}, \tilde{d}$	2 ϵ, μ	0	Yes	36.1	719 GeV	$m(\tilde{g})=0, m(\tilde{t}^*)=0.5 m(\tilde{t}^*)+m(\tilde{g}^*)$	ATLAS-CONF-2017-038
	$\tilde{g}, \tilde{u}, \tilde{d}$	2 ϵ, μ	0	Yes	36.1	760 GeV	$m(\tilde{g})=0, m(\tilde{t}^*)=0.5 m(\tilde{t}^*)+m(\tilde{g}^*)$	ATLAS-CONF-2017-035
	$\tilde{g}, \tilde{u}, \tilde{d}$	2 ϵ, μ	0	Yes	36.1	1.16 TeV	$m(\tilde{g})=0, m(\tilde{t}^*)=0.5 m(\tilde{t}^*)+m(\tilde{g}^*)$	ATLAS-CONF-2017-039
	$\tilde{g}, \tilde{u}, \tilde{d}$	3 ϵ, μ	0	Yes	36.1	580 GeV	$m(\tilde{g})=0, m(\tilde{t}^*)=0, \tilde{f}$ decoupled	ATLAS-CONF-2017-039
	$\tilde{g}, \tilde{u}, \tilde{d}$	2-3 ϵ, μ	0-2 jets	Yes	36.1	270 GeV	$m(\tilde{g})=0, m(\tilde{t}^*)=0, \tilde{f}$ decoupled	ATLAS-CONF-2017-039
	$\tilde{g}, \tilde{u}, \tilde{d}$	3 ϵ, μ	0	Yes	20.3	635 GeV	$m(\tilde{g})=0, m(\tilde{t}^*)=0, \tilde{f}$ decoupled	1405.5086
	GGM (wino NLSP) weak prod., $\tilde{g}, \tilde{u}, \tilde{d}$	1 $\epsilon, \mu + \gamma$	-	Yes	20.3	115-370 GeV	$m(\tilde{g})=0, m(\tilde{t}^*)=0, m(\tilde{b}^*)=0.5 m(\tilde{b}^*)+m(\tilde{g}^*)$	1507.05493
	GGM (bino NLSP) weak prod., $\tilde{g}, \tilde{u}, \tilde{d}$	2 γ	-	Yes	20.3	590 GeV	$\tau \rightarrow \tau \text{ NLSP} < 1 \text{ mm}$	1507.05493
Long-lived particles	Direct $\tilde{g}, \tilde{u}, \tilde{d}$ prod., long-lived $\tilde{g}, \tilde{u}, \tilde{d}$	Disapp. trk	1 jet	Yes	36.1	430 GeV	$m(\tilde{g})=m(\tilde{t}^*)=160 \text{ MeV}, \tau(\tilde{g}, \tilde{u}, \tilde{d}) > 2 \text{ ns}$	ATLAS-CONF-2017-017
	Direct $\tilde{g}, \tilde{u}, \tilde{d}$ prod., long-lived $\tilde{g}, \tilde{u}, \tilde{d}$	dE/dx trk	-	Yes	18.4	405 GeV	$m(\tilde{g})=m(\tilde{t}^*)=180 \text{ MeV}, \tau(\tilde{g}, \tilde{u}, \tilde{d}) > 15 \text{ ns}$	1506.09332
	Stable \tilde{g} R-hadron	0	1-5 jets	Yes	27.9	850 GeV	$m(\tilde{g}) > 100 \text{ GeV}, 10 \mu\text{s} < \tau < 1000 \text{ s}$	1310.6504
	Stable \tilde{g} R-hadron	trk	-	3.2	1.58 TeV	$m(\tilde{g}) > 100 \text{ GeV}, \tau > 10 \text{ ns}$	1606.05129	
	Metastable \tilde{g} R-hadron	dE/dx trk	-	3.2	1.57 TeV	$m(\tilde{g}) > 100 \text{ GeV}, \tau > 10 \text{ ns}$	1604.04520	
	GMSB, stable \tilde{g} , $\tilde{g}, \tilde{u}, \tilde{d}$	1-2 μ	-	Yes	19.1	537 GeV	$10 < \text{decay length} < 50$	1411.6795
	GMSB, $\tilde{g}, \tilde{u}, \tilde{d}$, long-lived $\tilde{g}, \tilde{u}, \tilde{d}$	2 γ	-	Yes	20.3	440 GeV	$1 < \tau(\tilde{g}, \tilde{u}, \tilde{d}) < 2 \text{ ns}, \text{SPS8 model}$	1409.5942
	GGM $\tilde{g}, \tilde{u}, \tilde{d}$	disp. vtx μ /pp	-	Yes	20.3	1.0 TeV	$7 < \tau(\tilde{g}, \tilde{u}, \tilde{d}) < 740 \text{ mm}, m(\tilde{g}, \tilde{u}, \tilde{d}) = 1.3 \text{ TeV}$	1504.05162
	GGM $\tilde{g}, \tilde{u}, \tilde{d}$	disp. vtx ϵ -jets	-	Yes	20.3	1.0 TeV	$6 < \tau(\tilde{g}, \tilde{u}, \tilde{d}) < 480 \text{ mm}, m(\tilde{g}, \tilde{u}, \tilde{d}) = 1.1 \text{ TeV}$	1504.05162
	RPV	LFV $\tilde{g}, \tilde{u}, \tilde{d}$, $X, Y, \tilde{g}, \tilde{u}, \tilde{d}$	$\mu\mu, \tau\mu, \tau\tau$	-	3.2	1.9 TeV	$A_{11}, A_{21}, A_{31} < 0.07$	1607.26079
Bilinear RPV CMSSM		2 ϵ, μ (SS)	0-3 b	Yes	20.3	1.45 TeV	$m(\tilde{g})=m(\tilde{t}^*), \tau_{\text{RPV}} < 1 \text{ mm}$	1404.2500
$\tilde{g}, \tilde{u}, \tilde{d}$		4 ϵ, μ	-	Yes	13.3	1.14 TeV	$m(\tilde{g}) > 400 \text{ GeV}, A_{13}, \mu < 0$	ATLAS-CONF-2016-075
$\tilde{g}, \tilde{u}, \tilde{d}$		3 $\epsilon, \mu + \tau$	-	Yes	20.3	450 GeV	$m(\tilde{g}) > 0.2 m(\tilde{t}^*), A_{13}, \mu < 0$	1405.5086
$\tilde{g}, \tilde{u}, \tilde{d}$		0	4.5 large- θ jets	-	14.8	1.08 TeV	$\text{BR}(\tilde{g} \rightarrow \tilde{g} \tilde{g}) = \text{BR}(\tilde{u} \rightarrow \tilde{u} \tilde{u}) = 0.75$	ATLAS-CONF-2016-057
$\tilde{g}, \tilde{u}, \tilde{d}$		1 ϵ, μ	8-10 jets/0-4 b	-	36.1	1.55 TeV	$m(\tilde{g})=800 \text{ GeV}$	ATLAS-CONF-2016-057
$\tilde{g}, \tilde{u}, \tilde{d}$		1 ϵ, μ	8-10 jets/0-4 b	-	36.1	2.1 TeV	$m(\tilde{g})=1 \text{ TeV}, A_{12}, \mu < 0$	ATLAS-CONF-2017-013
$\tilde{g}, \tilde{u}, \tilde{d}$		1 ϵ, μ	8-10 jets/0-4 b	-	36.1	1.65 TeV	$m(\tilde{g})=1 \text{ TeV}, A_{12}, \mu < 0$	ATLAS-CONF-2017-013
$\tilde{g}, \tilde{u}, \tilde{d}$		0	2 jets + 2 b	-	15.4	410 GeV	$\text{BR}(\tilde{g} \rightarrow \tilde{g} \tilde{g}) = 20\%$	ATLAS-CONF-2016-022, ATLAS-CONF-2016-084
$\tilde{g}, \tilde{u}, \tilde{d}$		2 ϵ, μ	2 b	-	36.1	0.4-1.45 TeV	$m(\tilde{g})=200 \text{ GeV}$	ATLAS-CONF-2017-036
Other	Scalar charm, $\tilde{g}, \tilde{u}, \tilde{d}$	0	2 ϵ	Yes	20.3	510 GeV	$m(\tilde{g})=200 \text{ GeV}$	1501.01325

*Only a selection of the available mass limits on new states or phenomena is shown. Many of the limits are based on simplified models, c.f. refs. for the assumptions made.

10⁻¹ 1 Mass scale [TeV]

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CP-even Higgs masses MSSM

$(\text{Re } H_d^0, \text{Re } H_u^0) \xrightarrow{\alpha} (h, H)$ with the mass matrix

$$M = \begin{pmatrix} m_A^2 s_\beta^2 + m_Z^2 c_\beta^2 & -(m_A^2 + m_Z^2) s_\beta c_\beta \\ \cdot & m_A^2 c_\beta^2 + m_Z^2 s_\beta^2 \end{pmatrix}$$

\Rightarrow

$$m_{h,H}^2 = \frac{1}{2} \left[m_Z^2 + m_A^2 \mp \sqrt{(m_Z^2 + m_A^2)^2 - 4m_Z^2 m_A^2 c_{2\beta}^2} \right]$$

\Rightarrow

$$m_h^2 < \min(m_Z^2, m_A^2) c_{2\beta}^2$$

If $m_A \gg m_Z$:

$$m_h^2 \approx m_Z^2 c_{2\beta}^2 = \frac{1}{4} (g_Y^2 + g_2^2) v^2 c_{2\beta}^2$$

Higgs mass in the CP-conserving MSSM

If $m_A \gg m_Z$:

$$m_h^2 \approx m_Z^2 c_{2\beta}^2 = \frac{1}{4}(g_Y^2 + g_2^2)v^2 c_{2\beta}^2$$

In the MSSM the tree-level Higgs mass is restricted to be smaller than m_Z !

Q: How can $M_h \approx 125$ GeV be possible in the MSSM?

A: Large loop corrections are to be expected!

$$M_h^2 = m_h^2 + \Delta m_h^2 \quad \Rightarrow \quad \Delta m_h^2 \geq (85 \text{ GeV})^2$$

Current status of MSSM Higgs mass calculation

Usual approach: Spectrum generators calculate r.h.s. of

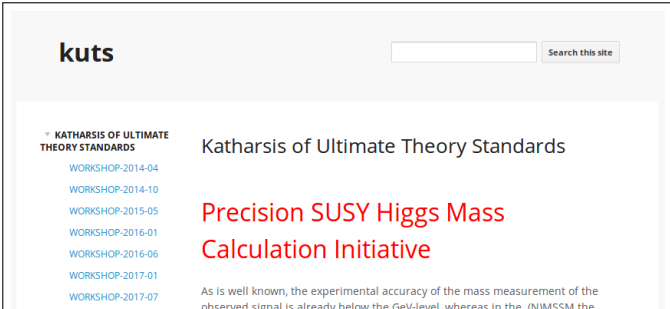
$$M_h^2 = m_h^2 + \Delta m_h^2$$

as a function of all SM and SUSY parameters. Because of large loop corrections Δm_h^2 :

$$\Delta M_h^{\text{theo}} \gtrsim (1 \dots 2) \text{ GeV} \quad \text{at least!}$$

$$\Delta M_h^{\text{exp}} = 0.24 \text{ GeV} \quad [\text{PDG-2017}]$$

Theory calculation needs to improve! Current workshop series:

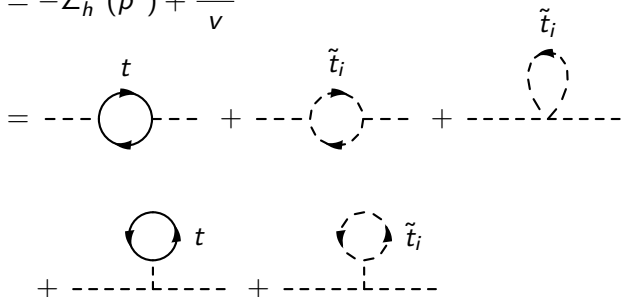


The screenshot shows the website for the Katharsis of Ultimate Theory Standards (KUTS). At the top left is the logo "kuts". To its right is a search bar with the text "Search this site". Below the logo, there is a list of workshop dates: "WORKSHOP-2014-04", "WORKSHOP-2014-10", "WORKSHOP-2015-05", "WORKSHOP-2016-01", "WORKSHOP-2016-06", "WORKSHOP-2017-01", and "WORKSHOP-2017-07". To the right of this list, the text "Katharsis of Ultimate Theory Standards" is displayed. Below that, the title "Precision SUSY Higgs Mass Calculation Initiative" is shown in red. At the bottom, a partial sentence is visible: "As is well known, the experimental accuracy of the mass measurement of the observed signal is already below the GeV-level, whereas in the (N)MSSM the

Higgs mass at 1-loop level

In the MSSM the following diagrams give the dominant contribution to M_h at the 1-loop level:

$$(\Delta m_h^2)^{1L} = -\Sigma_h^{1L}(p^2) + \frac{t_h^{1L}}{v}$$



$$\approx \frac{12m_t^2 y_t^2}{(4\pi)^2} \left(\ln \frac{m_{\tilde{t}}^2}{m_t^2} + \frac{X_t^2}{m_{\tilde{t}}^2} - \frac{X_t^4}{12m_{\tilde{t}}^4} \right) \quad \text{for } m_{\tilde{t}_1} = m_{\tilde{t}_2}, p^2 = 0$$

Higgs mass at 1-loop level

$$(\Delta m_h^2)^{1L} \approx \frac{12m_t^2 y_t^2}{(4\pi)^2} \left(\ln \frac{m_{\tilde{t}}^2}{m_t^2} + \frac{X_t^2}{m_{\tilde{t}}^2} - \frac{X_t^4}{12m_{\tilde{t}}^4} \right) + O(p^2)$$

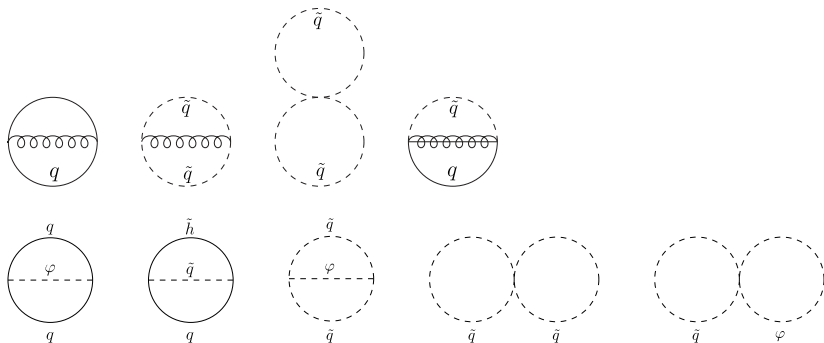
$X_t = A_t - \mu/t_\beta =$ stop mixing parameter

Observations:

- logarithmically enhanced by $m_{\tilde{t}}/m_t$
- maximal for $X_t \approx \sqrt{6}m_{\tilde{t}}$
- high sensitivity on m_t , due to prefactor $m_t^2 y_t^2 = 2m_t^4/v^2$
- ambiguity of definition of m_t : pole mass or $\overline{\text{DR}}$ mass?
 $M_t \approx 173.3 \text{ GeV}$, $m_t^{\overline{\text{DR}}} \approx 165 \text{ GeV}$
 \Rightarrow huge theoretical uncertainty!
 \Rightarrow 2-loop calculation needed to resolve this ambiguity
- to get $M_h \approx 125 \text{ GeV}$, $m_{\tilde{t}} \gtrsim 1 \text{ TeV}$ needed (see later)

Higgs mass at 2-loop level

Known contributions: $O(\alpha_s(\alpha_t + \alpha_b) + (\alpha_t + \alpha_b)^2 + \alpha_\tau^2)$ for $p^2 = 0$ [hep-ph/0105096, hep-ph/0112177]



Higgs mass at 2-loop level

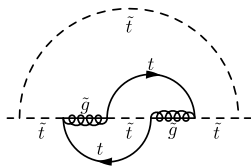
$$\begin{aligned}(\Delta m_h^2)^{2L} &\approx \frac{m_t^2 y_t^4}{(4\pi)^4} \left(c_1 \ln^2 \frac{M_S^2}{m_t^2} + c_2 \ln \frac{M_S^2}{m_t^2} + c_3 \right) \\ &+ \frac{m_t^2 y_t^2 g_3^2}{(4\pi)^4} \left(c_4 \ln^2 \frac{M_S^2}{m_t^2} + c_5 \ln \frac{M_S^2}{m_t^2} + c_6 \right)\end{aligned}$$

Observations:

- logarithmically enhanced by M_S/m_t
- still high sensitivity on m_t
- ambiguity of definition of m_t is resolved
- ambiguity of definition of α_s : $\alpha_s^{\text{SM}}(M_Z)$, $\alpha_s^{\text{MSSM}}(M_S)$, ... ?
⇒ 3-loop calculation needed to resolve this ambiguity

Higgs mass at 3-loop level

Known contributions: $O(\alpha_t \alpha_s^2)$ for $p^2 = 0$ [arXiv:1005.5709]



$$(\Delta m_h^2)^{3L} \approx \frac{m_t^2 y_t^2 g_3^4}{(4\pi)^6} \left(c_7 \ln^3 \frac{M_S^2}{m_t^2} + c_8 \ln^2 \frac{M_S^2}{m_t^2} + c_9 \ln \frac{M_S^2}{m_t^2} + c_{10} \right)$$

Observations:

- logarithmically enhanced by M_S/m_t
- still high sensitivity on m_t
- ambiguity of definition of m_t is resolved
- ambiguity of definition of α_s is resolved

Summary of fixed loop order calculations

Typical order of magnitude of loop contributions (depends on parameter scenario):

$$\begin{aligned} M_h &= m_h + \Delta m_h^{1L} + \Delta m_h^{2L} + \Delta m_h^{3L} + \dots \\ &\approx [91 + O(20 \dots 30) + O(2 \dots 4) + O(1 \dots 2)] \text{ GeV} \end{aligned}$$

Advantages:

- includes logarithmic, non-logarithmic and suppressed terms of the order $O(v^2/M_S^2)$ at fixed loop order
- precise prediction if $M_S \sim m_t$

Problem:

- large logarithmic corrections, if $M_S \gg m_t$
 - \Rightarrow slow convergence of perturbation series
 - \Rightarrow large theoretical uncertainty, (1–2 GeV, or more)

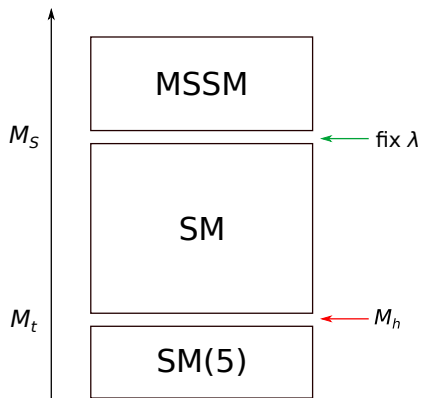
$$M_h^{\text{exp}} = (125.09 \pm 0.24) \text{ GeV}$$

Contents

- ① Introduction to supersymmetry
- ② Higgs mass calculation in the MSSM
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 - in an EFT**
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Higgs mass calculation in an EFT

- Idea:** Integrate out SUSY particles at M_S (expand in v^2/M_S^2)
 $\Rightarrow \lambda(M_S)$ is fixed by the MSSM (Remember: in the SM $m_h^2 = \lambda v^2$)
 \Rightarrow effectively: separation of scales M_S and M_t .



EFT procedure

Match all renormalized n -point functions at $p^2 = v^2 = 0$, $Q = M_S$ of the SM and MSSM:

$$\partial_{p^2}^{(k)} \Gamma_{h,\dots,h}^{\text{MSSM},(n)} = \partial_{p^2}^{(k)} \Gamma_{h,\dots,h}^{\text{SM},(n)}$$

\Rightarrow

$$\lambda(M_S) = \frac{1}{4} \left[g_Y^2 + g_2^2 \right] c_{2\beta}^2 + \Delta\lambda^{1L} + \Delta\lambda^{2L} + \dots$$

RG running of $\lambda(M_S)$ to $Q = M_t$.

Calculation of M_h in the Standard Model:

$$(M_h^{\text{SM}})^2 = \lambda(M_t)v^2 + (\Delta m_h^2)^{1L} + (\Delta m_h^2)^{2L} + \dots$$

EFT avoids large logarithmic corrections

- 1 Calculate λ at $Q = M_S$:

$$\lambda(Q) = \frac{1}{4} [g_Y^2 + g_2^2] c_{2\beta}^2 + \frac{12m_t^2 y_t^2}{(4\pi)^2 v^2} \left[\ln \frac{M_S^2}{Q^2} + \frac{X_t^2}{M_S^2} - \frac{X_t^4}{12M_S^4} \right] + \dots$$

\Rightarrow no large logs

- 2 RG running from $Q = M_S \rightarrow M_t$.

\Rightarrow logs are resummed to all orders

- 3 Calculate M_h in the SM at $Q = M_t$:

$$(M_h^{\text{SM}})^2 = \lambda v^2 + \frac{12m_t^2 y_t^2}{(4\pi)^2 v^2} \ln \frac{Q^2}{m_t^2} + \dots$$

\Rightarrow no large logs

Summary of EFT approach

Typical order of magnitude of loop contributions (depends on parameter scenario, here $X_t = 0$, $M_S = 20$ TeV):

$$\begin{aligned}M_h &= m_h + \Delta m_h^{1L} + \Delta m_h^{2L} + \dots \\&= \sqrt{\lambda(M_t)}v + \Delta m_h^{1L} + \Delta m_h^{2L} + \dots \\&\approx [O(124) + O(0.5 \dots 1) + O(0.1 \dots 0.2)] \text{ GeV} \\&= \sqrt{\lambda(M_S)}v + \text{logs} + \Delta m_h^{1L} + \Delta m_h^{2L} + \dots \\&\approx [O(84) + O(40) + O(0.5 \dots 1) + O(0.1 \dots 0.2)] \text{ GeV}\end{aligned}$$

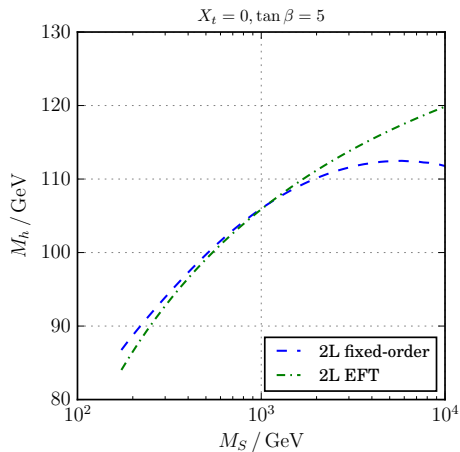
Advantages:

- large logarithmic fixed order loop corrections are avoided
- large logarithms $\propto \ln(M_S/M_t)$ are resummed to all orders

Disadvantage: usually terms $O(v^2/M_S^2)$ are neglected

\Rightarrow imprecise when $v \sim M_S \Rightarrow$ then large theoretical uncertainty

Comparison of fixed-order and EFT approaches



Summary of fixed-order and EFT approaches

	low M_S $M_S \lesssim 2 \text{ TeV}$	high M_S $M_S \gtrsim 2 \text{ TeV}$
fixed-order	✓	✗
EFT	✗	✓
? mixed	✓	✓

Q: Can the fixed-order and EFT approaches be combined?

A: Yes! [arXiv:1312.4937, arXiv:1609.00371]

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Mixed fixed-order and EFT approaches

Goal: resum large logarithms **and** include suppressed $O(v^2/M_S^2)$ terms

Two known approaches:

- FeynHiggs [arXiv:1312.4937]: Replace logs from fixed-order calculation by resummed logs

$$M_h^2 = (M_h^2)_{\text{fixed-order}} - (M_h^2)_{\text{logs}} + (M_h^2)_{\text{resummed logs}}$$

- FlexibleEFTHiggs [arXiv:1609.00371]: Incorporate $O(v^2/M_S^2)$ terms into λ by using the matching condition

$$(M_h^2)_{\text{SM}} = (M_h^2)_{\text{MSSM}} \quad \text{at 1L level at } Q = M_S$$

FlexibleEFTHiggs approach

Idea:

- 1 Determine $\lambda(M_S)$ from the condition

$$(M_h^2)_{\text{SM}} = (M_h^2)_{\text{MSSM}} \quad 1L, Q = M_S$$

No suppressed terms are neglected. $\Rightarrow \lambda$ contains all $O(v^2/M_S^2)$ suppressed terms

- 2 RG running of $\lambda(M_S)$ from $M_S \rightarrow M_t$

Note: M_h is RG invariant.

- 3 Calculate M_h in the Standard Model at $Q = M_t$:

$$M_h^2 = \lambda(M_t)v^2 + (\Delta m_h^2)_{\text{SM}}^{1L}$$

$\Rightarrow M_h$ contains all suppressed terms.

FlexibleEFTHiggs – EFT equivalence

Proof of equivalence: Start with matching condition:

$$\begin{aligned}(M_h^2)_{\text{SM}} &= (M_h^2)_{\text{MSSM}} & 1L, Q = M_S \\ \lambda v^2 + (\Delta m_h^2)_{\text{SM}}^{1L} &= (M_h^2)_{\text{MSSM}}\end{aligned}$$

\Rightarrow

$$\begin{aligned}\lambda(M_S) &= \frac{1}{v^2} \left[(M_h^2)_{\text{MSSM}} - (\Delta m_h^2)_{\text{SM}}^{1L} \right] \\ &= \frac{1}{v^2} \left[(m_h^2)_{\text{MSSM}} + (\Delta m_h^2)_{\text{MSSM}}^{1L} - (\Delta m_h^2)_{\text{SM}}^{1L} \right]\end{aligned}$$

Now insert $(m_h^2)_{\text{MSSM}}$ and $(\Delta m_h^2)_{\text{MSSM}}^{1L} \dots$

FlexibleEFT Higgs – EFT equivalence

Inserting $(m_h^2)_{\text{MSSM}}$ and $(\Delta m_h^2)_{\text{MSSM}}^{1L}$ for $X_t = 0$:

$$\lambda(M_S) = \frac{1}{v^2} \left[\frac{1}{4} (g_Y^2 + g_2^2) v^2 c_{2\beta}^2 \right. \\ \left. + \frac{c_\alpha^2}{s_\beta^2} (\Delta m_h^2)_{\text{SM}}^{1L} - \frac{c_\alpha^2}{s_\beta^2} \frac{12 (y_t^{\text{SM}})^2 m_t^2}{(4\pi)^2} B_0(m_h^2, M_S^2, M_S^2) \right. \\ \left. - (\Delta m_h^2)_{\text{SM}}^{1L} \right]$$

Now go to the decoupling limit $c_\alpha^2/s_\beta^2 \rightarrow 1 \dots$

FlexibleEFTHiggs – EFT equivalence

In the decoupling limit $c_\alpha^2/s_\beta^2 \rightarrow 1$:

$$\begin{aligned}\lambda(M_S) &= \frac{1}{4}(g_Y^2 + g_2^2)c_{2\beta}^2 - 12 \frac{m_t^2 (y_t^{\text{SM}})^2}{(4\pi)^2 v^2} B_0(m_h^2, M_S^2, M_S^2) \\ &= \frac{1}{4}(g_Y^2 + g_2^2)c_{2\beta}^2 - 12 \frac{m_t^2 (y_t^{\text{SM}})^2}{(4\pi)^2 v^2} \left[-\log \frac{M_S^2}{Q^2} + \frac{m_h^2}{6M_S^2} + O\left(\frac{m_h^4}{M_S^4}\right) \right] \\ &= \frac{1}{4}(g_Y^2 + g_2^2)c_{2\beta}^2 + 12 \frac{m_t^2 (y_t^{\text{SM}})^2}{(4\pi)^2 v^2} \left[\log \frac{M_S^2}{Q^2} \right] + O\left(\frac{v^2}{M_S^2}\right) \\ &= \lambda^{\text{EFT,tree}} + \Delta\lambda^{\text{EFT,1L}} + O\left(\frac{v^2}{M_S^2}\right)\end{aligned}$$

In the decoupling limit $\lambda(M_S)$ in the FlexibleEFTHiggs approach is equivalent to the EFT approach at 1-loop, up to suppressed terms $O(v^2/M_S^2)$

Summary FlexibleEFTHiggs approach

$$(M_h^2)_{\text{SM}} = (M_h^2)_{\text{MSSM}} \quad 1\text{L}, Q = M_S$$

⇒

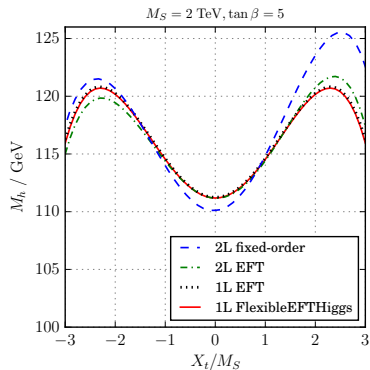
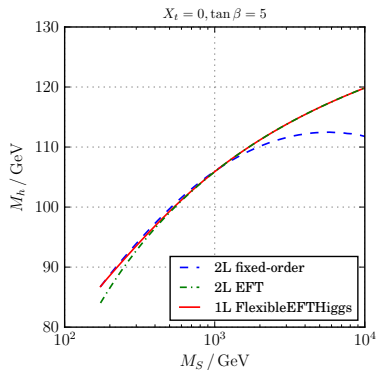
$$\lambda(M_S) = \lambda^{\text{EFT,tree}} + \lambda^{\text{EFT,1L}} + O(v^2/M_S^2)$$

Observations:

- large 1-loop logarithms cancel in matching condition
- for $v = p = 0$ FlexibleEFTHiggs is identical to a 1-loop EFT calculation
- all suppressed terms are incorporated in λ
- RG running resums (N)LL to all orders

⇒ FlexibleEFTHiggs leads to a correct Higgs mass prediction at the full 1-loop level (including suppressed terms) with additional (N)LL resummation.

Comparison of the three approaches



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Ways to estimate the theoretical uncertainty

Good ansatz: Change the calculation by higher orders beyond the calculational accuracy.

Examples:

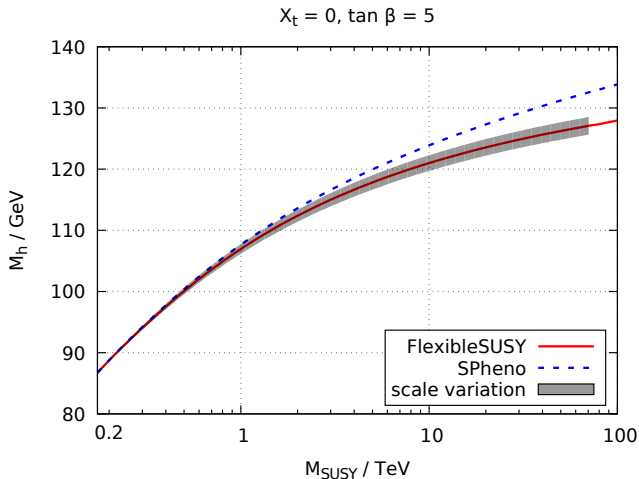
- variation of the unphysical renormalization scale(s)
- change m_t or α_s by higher orders
- re-parametrization of M_h

Potential pitfalls:

- some changes alone are an under-estimation of the uncertainty
→ a combination of multiple changes should be used
- potential over-estimation of the uncertainty when large cancellations occur at higher orders
- some changes are sensitive to the same kind of higher order contributions → potential “double counting”
- new higher order dependencies might be difficult to estimate
Example: How would one estimate α_s dependence of M_h^{2L} when only M_h^{1L} is known?

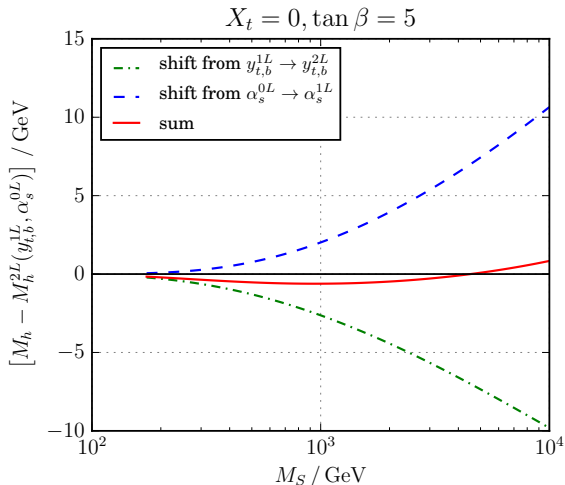
Example: under-estimation of the uncertainty

The variation of the renormalization scale alone might be an under-estimation of the uncertainty:



Example: over-estimation of the uncertainty

When large cancellations between the same kind of corrections from different sources occur, including only one source might lead to an over-estimation:



Higgs mass uncertainty estimate

fixed-order:

- $|M_h^{2L}(Q_{\text{pole}} = M_S/2) - M_h^{2L}(Q_{\text{pole}} = 2M_S)|$
- $|M_h^{2L}(y_t^{1L}) - M_h^{2L}(y_t^{2L})|$

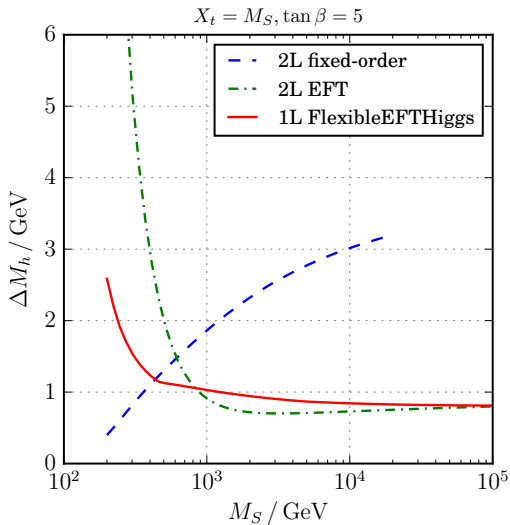
EFT (SUSYHD):

- $|M_h^{2L}(Q_{\text{pole}} = M_t/2) - M_h^{2L}(Q_{\text{pole}} = 2M_t)|$
- $|M_h^{2L}(y_t^{2L}) - M_h^{2L}(y_t^{3L})|$
- $|M_h^{2L}(Q_{\text{match}} = M_S/2) - M_h^{2L}(Q_{\text{match}} = 2M_S)|$
- $|M_h^{2L} - M_h^{2L}(\lambda \rightarrow \lambda(1 + v^2/M_S^2))|$

FlexibleEFTHiggs:

- $|M_h^{2L}(Q_{\text{pole}} = M_t/2) - M_h^{2L}(Q_{\text{pole}} = 2M_t)|$
- $|M_h^{2L}(y_t^{2L}) - M_h^{2L}(y_t^{3L})|$
- $|M_h^{2L}(Q_{\text{match}} = M_S/2) - M_h^{2L}(Q_{\text{match}} = 2M_S)|$

Higgs mass uncertainty estimate



Summary

Supersymmetry is still viable, but LHC continuously excludes light SUSY scenarios

Approaches to calculate M_h :

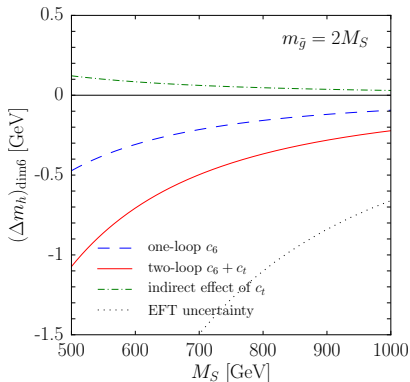
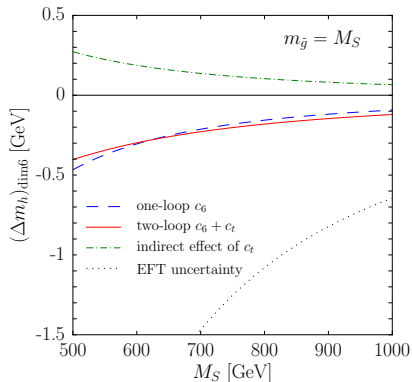
	low M_S $M_S \lesssim 2 \text{ TeV}$	high M_S $M_S \gtrsim 2 \text{ TeV}$
fixed-order	✓	✗
EFT	✗	✓
mixed	✓	✓

Uncertainty of M_h in SUSY:

- tricky to estimate and still ongoing effort!
- $\Delta M_h \gtrsim 1\text{--}2 \text{ GeV}$ at least, but continuously improves

Backup

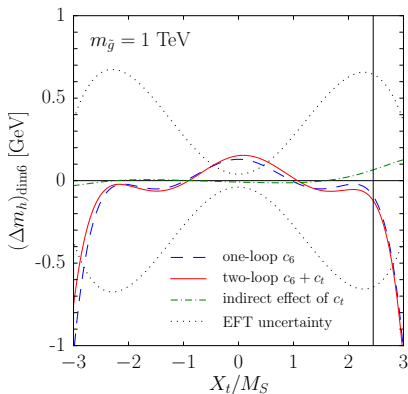
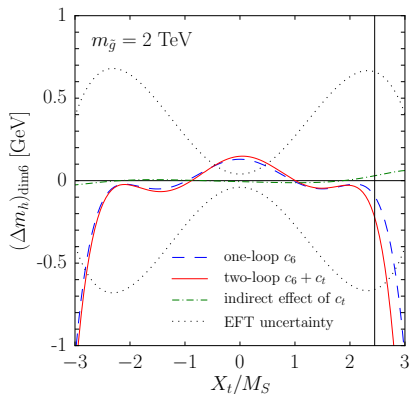
Effect of higher-dimensional operators



$$\tan \beta = 20, X_t = \sqrt{6} M_S$$

[1703.08166]

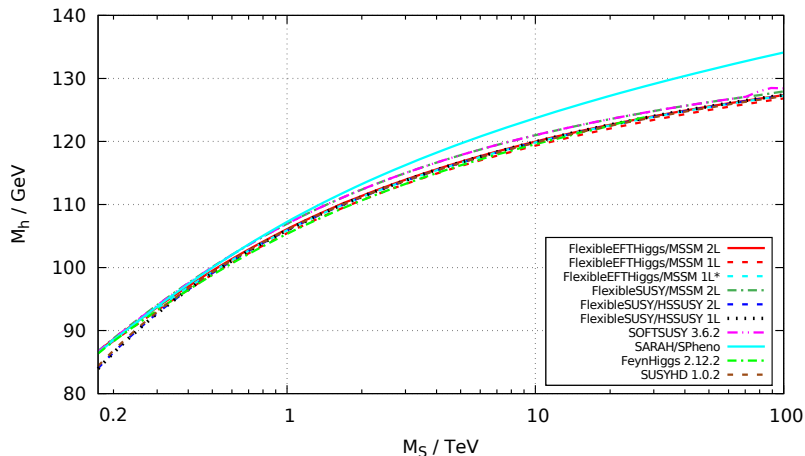
Effect of higher-dimensional operators



$M_S = 1 \text{ TeV}, \tan \beta = 20$

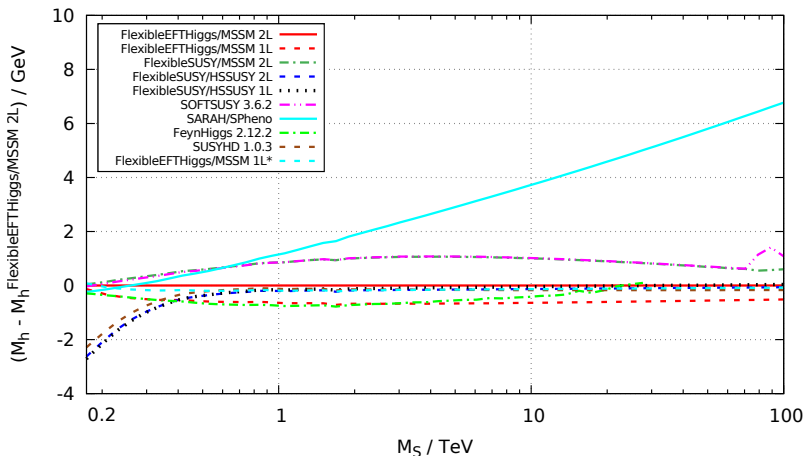
[1703.08166]

Numerical comparison



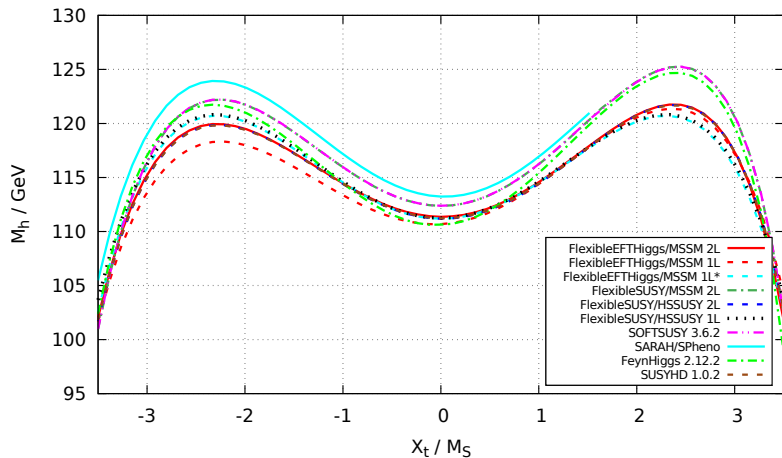
$$\tan \beta = 5, X_{t,b,\tau} = 0$$

Numerical comparison



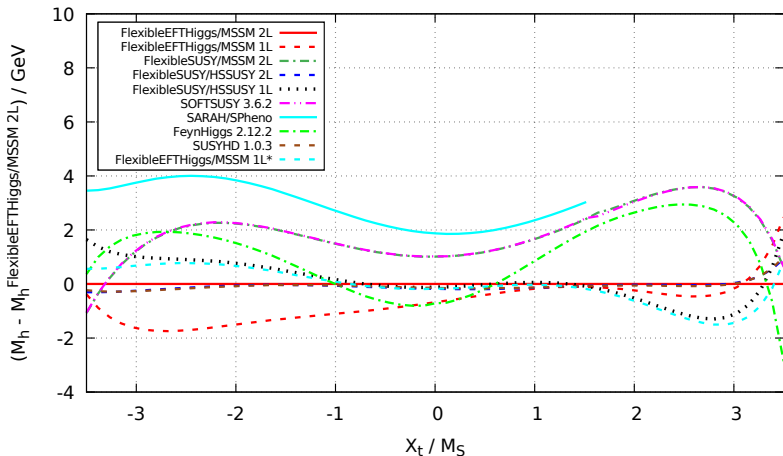
$$\tan \beta = 5, X_{t,b,\tau} = 0$$

Numerical comparison



$$\tan \beta = 5, M_S = 2 \text{ TeV}, X_{b,\tau} = 0$$

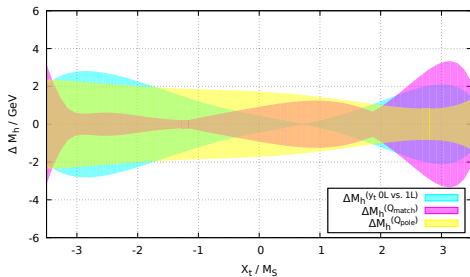
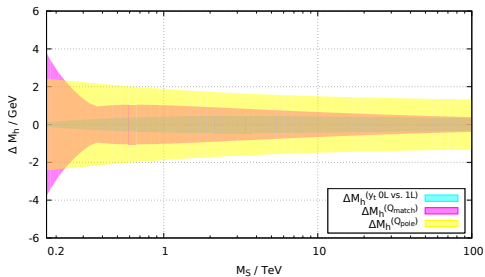
Numerical comparison



$\tan \beta = 5, M_S = 2 \text{ TeV}, X_{b,\tau} = 0$

For large X_t deviation from HSSUSY-1L due to $p \neq 0 \neq v$.

Uncertainty estimation of original FlexibleEFTHiggs-1L



Incorrect 2L logs in original FlexibleEFTHiggs-1L

Matching condition:

$$\lambda \leftarrow \frac{1}{v^2} \left[(m_h^{\text{SM}})^2 + (M_h^{\text{MSSM}})^2 - (M_h^{\text{SM}})^2 \right]$$

Expansion of momentum iteration up to 1L level:

$$\lambda = \frac{1}{v^2} \left[(m_h^{\text{MSSM}})^2 + \Delta m_{h,\text{MSSM}}^2 - \Delta m_{h,\text{SM}}^2 + \mathcal{O}(\hbar^2) \right]$$

with

$$\begin{aligned} \Delta m_{h,\text{MSSM}}^2 &= -\Sigma_{\text{MSSM}}^{1L} + t_{\text{MSSM}}^{1L}/v_{\text{MSSM}} \\ \Delta m_{h,\text{SM}}^2 &= -\Sigma_{\text{SM}}^{1L} + t_{\text{SM}}^{1L}/v_{\text{SM}} \end{aligned}$$

Incorrect 2L logs in original FlexibleEFTHiggs-1L

Problem: $y_t^{\text{MSSM}} = y_t^{\text{SM}}/s_\beta[1 + O(\hbar)]$

\Rightarrow

$$\begin{aligned}\Delta m_{h,\text{MSSM}}^2 - \Delta m_{h,\text{SM}}^2 &\propto \hbar \left[(y_t^{\text{MSSM}} s_\beta)^4 \log \frac{m_t}{M_S} - (y_t^{\text{SM}})^4 \log \frac{m_t}{M_S} \right] \\ &= \hbar \left[0 + \propto \hbar y_t^4 \log \frac{m_t}{M_S} + O(\hbar^2) \right] \\ &= O(\hbar^2 y_t^4 \log \frac{m_t}{M_S})\end{aligned}$$

\Rightarrow

incorrect 2L logs remain in FlexibleEFTHiggs-1L

Summary original FlexibleEFTHiggs-1L

Advantages:

- ✓ easily automatizable
- ✓ correctly resums LL
- ✓ all non-log terms correct at 1L, including all terms $O(v^n/M_S^n)$

Disadvantage:

- ✗ incorrect 2L logs $O(\hbar^2 \log(m_t/M_S))$

Improved FlexibleEFTHiggs-1L

strict handling of loop orders in matching condition

$$(M_h^{\text{SM}})^2 = \lambda v^2 - \Sigma_h^{\text{SM}}((m_h^{\text{MSSM}})^2) + \frac{t_h^{\text{SM}}}{v}$$
$$(M_h^{\text{MSSM}})^2 = \text{EV of } \left[M_\phi^{(1)} - \Sigma_\phi^{\text{MSSM}}((m_h^{\text{MSSM}})^2) + \tilde{t}_\phi^{\text{MSSM}} \right]$$

with

$M_\phi^{(1)}$ = tree-level mass matrix w/ 1L parameters

$\Sigma_\phi^{\text{MSSM}}$ = 1L self-energy w/ 0L parameters

$t_{\phi_i}^{\text{MSSM}}$ = 1L tadpole w/ 0L parameters

m_h^{MSSM} = tree-level mass w/ 0L parameters

$(\tilde{t}_\phi^{\text{MSSM}})_i = t_{\phi_i}^{\text{MSSM}} / v_i$

Improved FlexibleEFTHiggs-1L

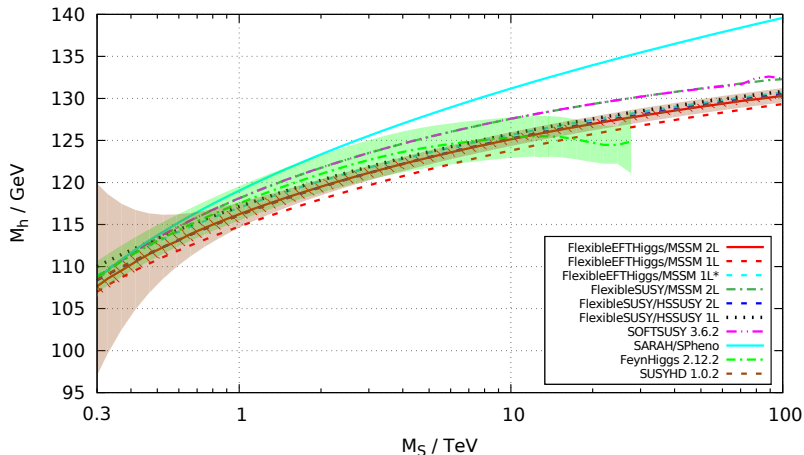
Advantages:

- ✓ easily automatizable
- ✓ correctly resums LL + NLL
- ✓ all non-log terms correct at 1L, including all terms $O(v^n/M_S^n)$

Disadvantage:

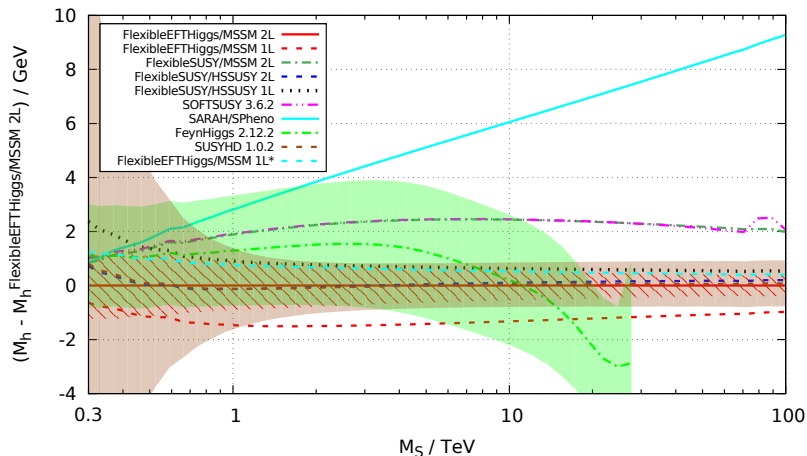
- non-logarithmic 2L terms arise at M_S
- ✗ difficult to add 2L corrections

Numerical comparison



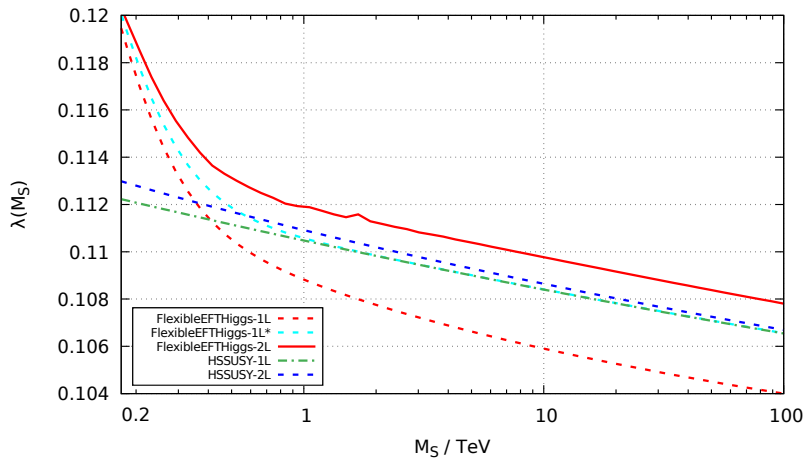
$$\tan \beta = 5, X_t = -2M_S, X_{b,\tau} = 0$$

Numerical comparison



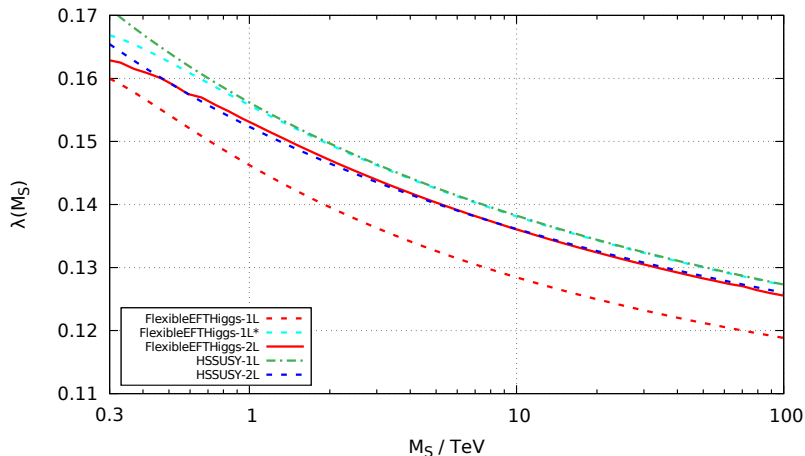
$$\tan \beta = 5, X_t = -2M_S, X_{b,\tau} = 0$$

Numerical comparison



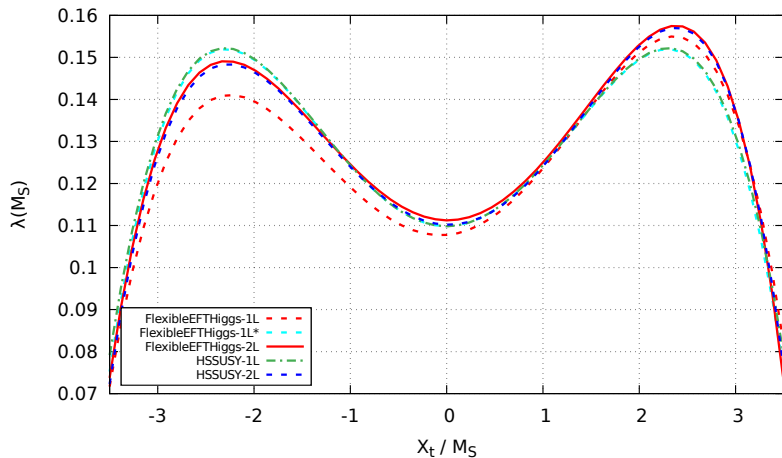
$$\tan \beta = 5, X_{t,b,\tau} = 0$$

Numerical comparison



$$\tan \beta = 5, X_t = -2M_S, X_{b,\tau} = 0$$

Numerical comparison



$$\tan \beta = 5, M_S = 2 \text{ TeV}, X_{b,\tau} = 0$$

Equivalence of pure EFT and FlexibleEFTHiggs

Equivalence pure EFT and FlexibleEFT Higgs $O(\hbar y_t^4)$

$$M_h^{\text{SM}} \stackrel{!}{=} M_h^{\text{MSSM}} \quad \text{at} \quad Q = M_S, 1L$$

where

$$(M_h^{\text{SM}})^2 = \lambda v^2 - \Sigma_h^{\text{SM}} + t_h^{\text{SM}}/v$$

$$t_h^{\text{SM}}/v = -6(y_t^{\text{SM}})^2 A_0(m_t)/(4\pi)^2$$

and [neglecting stop mass mixing $O(m_t X_t/M_S^2)$]

$$(M_h^{\text{MSSM}})^2 = \frac{1}{4}(g_Y^2 + g_2^2)(v_u^2 + v_d^2)c_{2\beta}^2 - \Sigma_h^{\text{MSSM}} + t_h^{\text{MSSM}}/v$$

$$\Sigma_h^{\text{MSSM}} = \Sigma_h^{\text{SM}} \frac{c_\alpha^2}{s_\beta^2} + 3 \frac{(y_t^{\text{SM}})^2}{(4\pi)^2} \frac{c_\alpha^2}{s_\beta^2} \left\{ A_0(m_{Q_3}) + A_0(m_{U_3}) \right.$$

$$\left. + 2m_t [B_0(m_{Q_3}, m_{Q_3}) + B_0(m_{U_3}, m_{U_3})] \right\}$$

$$t_h^{\text{MSSM}}/v = -3 \frac{(y_t^{\text{SM}})^2}{(4\pi)^2} \frac{c_\alpha^2}{s_\beta^2} \left[2A_0(m_t) - A_0(m_{Q_3}) - A_0(m_{U_3}) \right]$$

Equivalence pure EFT and FlexibleEFT Higgs $O(\hbar y_t^4)$

in SM limit $\frac{c_\alpha^2}{s_\beta^2} \rightarrow 1$
 \Rightarrow

$$\begin{aligned}\lambda &= \frac{1}{4}(g_Y^2 + g_2^2)c_{2\beta}^2 \\ &\quad - 3\frac{(y_t^{\text{SM}})^4}{(4\pi)^2} \left[B_0(p^2, m_{Q_3}, m_{Q_3}) + B_0(p^2, m_{U_3}, m_{U_3}) \right] \\ &= \frac{1}{4}(g_Y^2 + g_2^2)c_{2\beta}^2 \\ &\quad - 3\frac{(y_t^{\text{SM}})^4}{(4\pi)^2} \left[-\log \frac{m_{Q_3}^2}{Q^2} + \frac{p^2}{6m_{Q_3}^2} + O\left(\frac{p^4}{m_{Q_3}^4}\right) \right. \\ &\quad \quad \left. -\log \frac{m_{U_3}^2}{Q^2} + \frac{p^2}{6m_{U_3}^2} + O\left(\frac{p^4}{m_{U_3}^4}\right) \right] \\ &= [\text{Bagnaschi et. al. 2014}] + O\left(\frac{p^2}{m_{Q_3}^2}\right) + O\left(\frac{p^2}{m_{U_3}^2}\right)\end{aligned}$$

Determination of MSSM parameters

Determination of MSSM parameters

Fixed by observables:

Input			Output
$\alpha_{\text{em}}^{\text{SM}(5)}(M_Z)$	\rightarrow	$\alpha_{\text{em}}^{\text{MSSM}}(M_Z)$	\rightarrow $g_1^{\text{MSSM}}(M_Z)$
G_F	\rightarrow	$\sin \theta_W^{\text{MSSM}}(M_Z)$	\rightarrow $g_2^{\text{MSSM}}(M_Z)$
$\alpha_s^{\text{SM}(5)}(M_Z)$	\rightarrow		\rightarrow $g_3^{\text{MSSM}}(M_Z)$
M_Z	\rightarrow	$m_Z^{\text{MSSM}}(M_Z)$	\rightarrow $v^{\text{MSSM}}(M_Z)$
M_t	\rightarrow	$m_t^{\text{MSSM}}(M_Z)$	\rightarrow $y_t^{\text{MSSM}}(M_Z)$
$m_b^{\text{SM}(5)}(m_b)$	\rightarrow	$m_b^{\text{MSSM}}(M_Z)$	\rightarrow $y_b^{\text{MSSM}}(M_Z)$
M_τ	\rightarrow	$m_\tau^{\text{MSSM}}(M_Z)$	\rightarrow $y_\tau^{\text{MSSM}}(M_Z)$

Fixed by 2 EWSB conditions: $m_{H_u}^2, m_{H_d}^2$

Free parameters: $\tan \beta, \mu, B\mu, m_{\tilde{f},ij}^2, M_i, T_{ij}^f$

Determination of SM parameters

Fixed by observables:

Input		Output
$\alpha_{\text{em}}^{\text{SM}(5)}(M_Z)$	$\rightarrow \alpha_{\text{em}}^{\text{SM}}(M_Z)$	$\rightarrow g_1^{\text{SM}}(M_Z)$
G_F	$\rightarrow \sin \theta_W^{\text{SM}}(M_Z)$	$\rightarrow g_2^{\text{SM}}(M_Z)$
$\alpha_s^{\text{SM}(5)}(M_Z)$		$\rightarrow g_3^{\text{SM}}(M_Z)$
M_Z	$\rightarrow m_Z^{\text{SM}}(M_Z)$	$\rightarrow v^{\text{SM}}(M_Z)$
M_t	$\rightarrow m_t^{\text{SM}}(M_Z)$	$\rightarrow y_t^{\text{SM}}(M_Z)$
$m_b^{\text{SM}(5)}(m_b)$	$\rightarrow m_b^{\text{SM}}(M_Z)$	$\rightarrow y_b^{\text{SM}}(M_Z)$
M_τ	$\rightarrow m_\tau^{\text{SM}}(M_Z)$	$\rightarrow y_\tau^{\text{SM}}(M_Z)$

Fixed by 1 EWSB condition: μ^2

Free parameter: λ

Determination of $g_3^{\text{MSSM}}(M_S)$

$$\alpha_s^{\text{MSSM}}(M_S) = \frac{\alpha_s^{\text{SM}}(M_S)}{1 - \Delta\alpha_s(M_S)}$$

with

$$\Delta\alpha_s(Q) = \frac{\alpha_s}{2\pi} \left[\frac{1}{2} - \sum_{\text{SUSY particle } f} T_f \log \frac{m_f}{Q} \right]$$

\Rightarrow

$$g_3^{\text{MSSM}}(M_S) = \sqrt{4\pi\alpha_s^{\text{MSSM}}(M_S)}$$

Determination of $v_i^{\text{MSSM}}(M_S)$

$$M_Z^{\text{SM}} = M_Z^{\text{MSSM}}$$

\Rightarrow

$$(m_Z^{\text{MSSM}}(M_S))^2 = (M_Z^{\text{SM}})^2 + \Pi_Z^{\text{MSSM},1L}(Q = M_S)$$

$$(M_Z^{\text{SM}})^2 = \frac{1}{4} \left[(g_Y^{\text{SM}})^2 + (g_2^{\text{SM}})^2 \right] (v^{\text{SM}})^2 - \Pi_Z^{\text{SM},1L}(Q = M_S)$$

\Rightarrow

$$v^{\text{MSSM}}(M_S) = \frac{2m_Z^{\text{MSSM}}(M_S)}{\sqrt{(g_Y^{\text{MSSM}})^2 + (g_2^{\text{MSSM}})^2}}$$

\Rightarrow

$$v_u^{\text{MSSM}}(M_S) = v^{\text{MSSM}}(M_S) \sin \beta(M_S)$$

$$v_d^{\text{MSSM}}(M_S) = v^{\text{MSSM}}(M_S) \cos \beta(M_S)$$

Determination of $y_i^{\text{MSSM}}(M_S)$

$$M_f^{\text{SM}} = M_f^{\text{MSSM}}$$

\Rightarrow

$$m_f^{\text{MSSM}}(M_S) = M_f^{\text{SM}} + \Sigma_f^{\text{MSSM},1L}(Q = M_S)$$

$$M_f^{\text{SM}} = \frac{\sqrt{2}m_f^{\text{SM}}}{v_i^{\text{SM}}} - \Sigma_f^{\text{SM},1L}(Q = M_S)$$

\Rightarrow

$$y_f^{\text{MSSM}}(M_S) = \frac{\sqrt{2}m_f^{\text{MSSM}}(M_S)}{v_i^{\text{MSSM}}(M_S)}$$

Determination of SM parameters

Determination of $g_3^{\text{SM}}(M_Z)$

Input: $\alpha_s^{\text{SM}(5)}(M_Z) = 0.1185$

→

$$\alpha_s^{\text{SM}}(M_Z) = \frac{\alpha_s^{\text{SM}(5)}(M_Z)}{1 - \Delta\alpha_s(M_Z)}$$

with

$$\Delta\alpha_s(Q) = \frac{\alpha_s}{2\pi} \left[-\frac{2}{3} \log \frac{m_t}{Q} \right]$$

⇒

$$g_3^{\text{SM}}(M_Z) = \sqrt{4\pi\alpha_s^{\text{SM}}(M_Z)}$$

Determination of $y_t^{\text{SM}}(M_Z)$

$$y_t^{\text{SM}}(M_Z) = \frac{\sqrt{2} m_t^{\text{SM}}(M_Z)}{v(M_Z)}$$

where

$$m_t^{\text{SM}}(Q) = M_t + \text{Re} \Sigma_t^S(M_Z) + M_t \left[\text{Re} \Sigma_t^L(M_Z) \right. \\ \left. + \text{Re} \Sigma_t^R(M_Z) + \Delta m_t^{1L, \text{gluon}} + \Delta m_t^{2L, \text{gluon}} \right]$$

$$\Delta m_t^{1L, \text{gluon}} = -\frac{g_3^2}{12\pi^2} \left[4 - 3 \log \left(\frac{m_t^2}{Q^2} \right) \right]$$

$$\Delta m_t^{2L, \text{gluon}} = \left(\Delta m_t^{1L, \text{gluon}} \right)^2 \\ - \frac{g_3^4}{4608\pi^4} \left[396 \log^2 \left(\frac{m_t^2}{Q^2} \right) - 1452 \log \left(\frac{m_t^2}{Q^2} \right) \right. \\ \left. - 48\zeta(3) + 2053 + 16\pi^2(1 + \log 4) \right]$$

Determination of v^{SM}

The VEV v^{SM} is calculated from the running Z mass at $Q = M_Z$:

$$v^{\text{SM}}(M_Z) = \frac{2m_Z^{\text{SM}}(M_Z)}{\sqrt{g_Y^2 + g_2^2}}$$

$$m_Z^{\text{SM}}(M_Z) = \sqrt{M_Z^2 + \Pi_Z^{1L}(p^2 = M_Z^2, Q = M_Z)}$$

v^{SM} evolves under RG running according to
[Sperling, Stöckinger, AV, 2013, 2014]

Comparison full model vs. EFT approach

Q: Why is FlexibleSUSY/MSSM so close to the EFT approaches
and SPheno so far off?

Calculation of $y_t^{\text{MSSM}}(M_Z)$

A: Different treatment of 2-loop corrections to $y_t^{\text{MSSM}}(M_Z)$:

FlexibleSUSY:

$$m_t = M_t + \text{Re} \left[\tilde{\Sigma}_t^{(1),S}(M_t) \right] + M_t \text{Re} \left[\tilde{\Sigma}_t^{(1),L}(M_t) + \tilde{\Sigma}_t^{(1),R}(M_t) \right] \\ + M_t \left[\tilde{\Sigma}_t^{(1),\text{qcd}}(m_t) + \left(\tilde{\Sigma}_t^{(1),\text{qcd}}(m_t) \right)^2 + \tilde{\Sigma}_t^{(2),\text{qcd}}(m_t) \right]$$

SPheno:

$$m_t = M_t + \text{Re} \left[\tilde{\Sigma}_t^{(1),S}(m_t) \right] + m_t \text{Re} \left[\tilde{\Sigma}_t^{(1),L}(m_t) + \tilde{\Sigma}_t^{(1),R}(m_t) \right] \\ + m_t \left[\tilde{\Sigma}_t^{(1),\text{qcd}}(m_t) + \tilde{\Sigma}_t^{(2),\text{qcd}}(m_t) \right]$$

Calculation of $y_t^{\text{MSSM}}(M_Z)$

\Rightarrow

$$\tilde{y}_t^{\text{FlexibleSUSY}} = y_t + t^2 \kappa^2 \left(\frac{184}{9} g_3^4 y_t - 24 g_3^2 y_t^3 + \frac{9}{8} y_t^5 \right) + \dots$$

$$\tilde{y}_t^{\text{SPheno}} = y_t + t^2 \kappa^2 \left(\frac{248}{9} g_3^4 y_t - 16 g_3^2 y_t^3 + \frac{27}{8} y_t^5 \right) + \dots$$

with

$$y_t \equiv y_t^{\text{SM}}(M_S),$$

$$g_3 \equiv g_3^{\text{SM}}(M_S),$$

$$\tilde{y}_t \equiv y_t^{\text{MSSM}}(M_S),$$

$$\tilde{g}_3 \equiv g_3^{\text{MSSM}}(M_S),$$

$$t \equiv \log \frac{M_S}{M_t},$$

$$\kappa \equiv \frac{1}{(4\pi)^2}$$

Calculation of $y_t^{\text{MSSM}}(M_Z)$

$$(M_h^2)^{\text{EFT}} = m_h^2 + v^2 y_t^4 \left[12t\kappa + 12t^2\kappa^2 (16g_3^2 - 9y_t^2) + 4t^3\kappa^3 (736g_3^4 - 672g_3^2 y_t^2 + 90y_t^4) + \dots \right],$$

$$(M_h^2)^{\text{FlexibleSUSY}} = m_h^2 + v^2 y_t^4 \left[12t\kappa + 12t^2\kappa^2 (16g_3^2 - 9y_t^2) + 4t^3\kappa^3 \left(\frac{736g_3^4}{3} - 288g_3^2 y_t^2 + \frac{27y_t^4}{2} \right) + \dots \right],$$

$$(M_h^2)^{\text{SPHeno}} = m_h^2 + v^2 y_t^4 \left[12t\kappa + 12t^2\kappa^2 (16g_3^2 - 9y_t^2) + 4t^3\kappa^3 \left(\frac{992g_3^4}{3} - 192g_3^2 y_t^2 + \frac{81y_t^4}{2} \right) + \dots \right].$$