

# Precise Higgs mass calculations in models beyond the Standard Model

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HEP Webinar  
Monash University

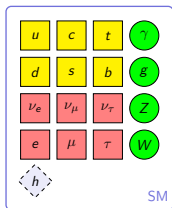
28/04/2020



# Contents

- ① Introduction
- ② BSM phenomenology with FlexibleSUSY
- ③ Higgs mass calculations in BSM models
  - Fixed order
  - Effective Field Theory
  - Hybrid
- ④  $x_t$  resummation
- ⑤ Summary and conclusions

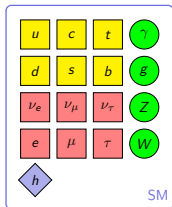
# Open questions before/after the Higgs discovery



## Open questions:

- Does the Higgs exist?
- What does DM consist of?
- What causes the deviation of  $(g - 2)_\mu$ ?
- Is the vacuum stable up to  $M_{\text{Pl}}$ ?
- Why is  $M_h = 125 \text{ GeV}$ ?
- Is there a solution to the hierarchy problem?
- Can QFT and gravity be unified?

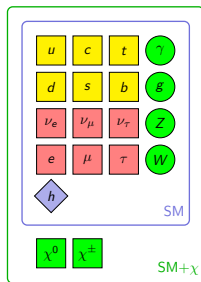
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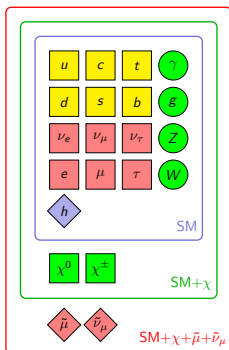
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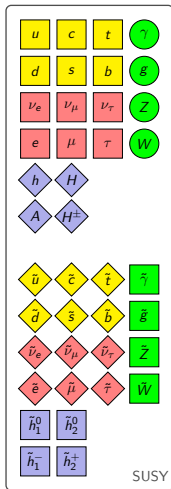
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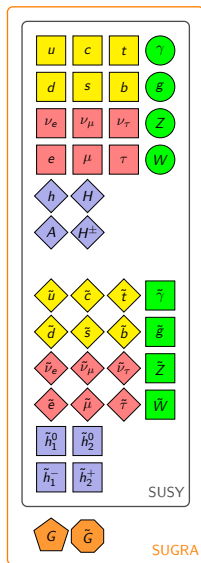
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# Many extensions of the Standard Modell must be studied



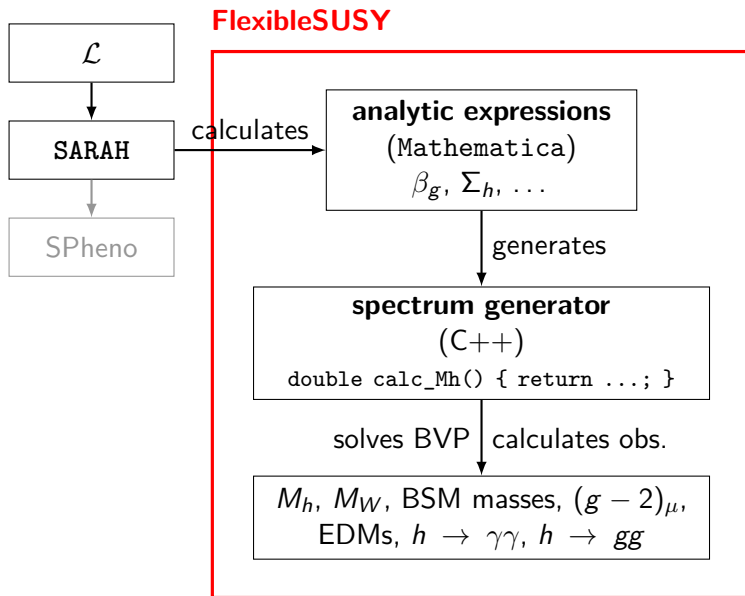
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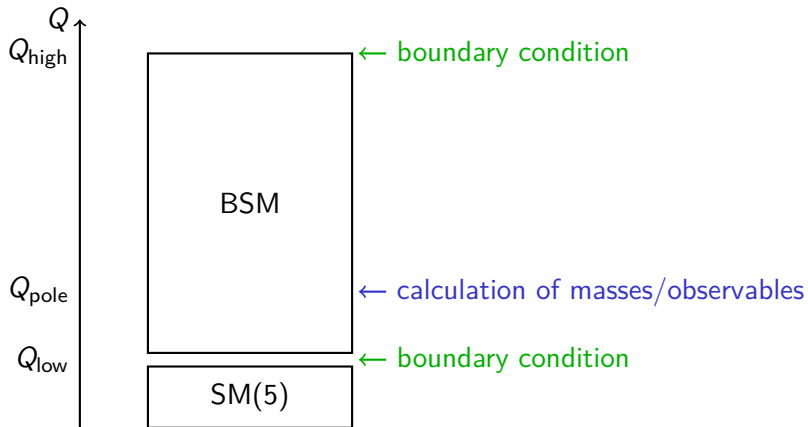
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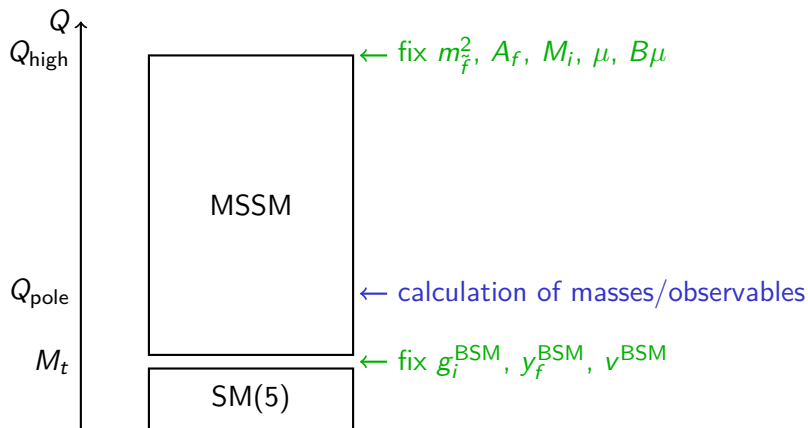
# FlexibleSUSY – a spectrum generator generator



# Spectrum generator setup in FlexibleSUSY



# Example spectrum generator: MSSM



# Features for all models (SUSY and non-SUSY)

## Observables

$M_h$ ,  $M_W$ , BSM masses,  
 $(g - 2)_\mu$ , EDMs,  
 $h \rightarrow \gamma\gamma$ ,  $h \rightarrow gg$

## Flexibility

multiple BVP solvers,  
user-defined BCs,  
modular C++ code,  
SLHA input/output,  
Mathematica input/output,  
SQLite output

## High precision

2L RGEs + 1L self energies (via SARAH) + 1L thresholds,  
NLL resummation for  $M_h$   
(FlexibleEFTHiggs)

## High speed

multi-threading,  
self-optimizing linear algebra,  
lazy evaluation

# Additional model-specific precision corrections

## MSSM

3L RGEs, 3L  $M_h$ , 2L  $M_{H,A,H^\pm}$ ,  
2L  $(g-2)_\mu$ , 2L thresholds

## NMSSM

2L  $M_h$ , 2L  $M_{H,A,H^\pm}$ , 2L  
thresholds

## Split-MSSM

3L  $M_h$ , 2L thresholds for  $y_t$ , 2L  
thresholds for  $\lambda$ ,  $\tilde{g}_{i\rho}$  the MSSM

## SM

4L RGEs, 4L  $M_h$ , 4L thresholds  
for  $\alpha_s$ ,  $y_t$ , 3L thresholds for  $\lambda$  to  
the MSSM (HSSUSY)

## THDM-II

2L thresholds for  $\lambda_i$  to the  
MSSM

## THDM-II + $\tilde{h}$ + $\tilde{g}$

1L thresholds for  $\lambda_i$  to the  
MSSM



# FlexibleSUSY resources



FlexibleSUSY reference manuals:

[\[1406.2319\]](#), [\[1710.03760\]](#)

Software download:

[flexiblesusy.hepforge.org](http://flexiblesusy.hepforge.org)

[github.com/FlexibleSUSY](https://github.com/FlexibleSUSY)

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# Prediction of the Higgs boson mass in supersymmetry

**SM:** Higgs potential (ad-hoc):

$$V(\phi) = \frac{\lambda}{8}\phi^4 - \frac{\mu^2}{2}\phi^2$$

$\Rightarrow$

$$m_h^2 = \lambda v^2$$

**MSSM:** Higgs potential (required by SUSY):

$$V(\phi) = \frac{1}{8} \frac{1}{4} (g_Y^2 + g_2^2) \cos^2(2\alpha) \phi^4 + \dots$$

$\Rightarrow$

$$\begin{aligned} m_h^2 &= \frac{1}{4} (g_Y^2 + g_2^2) \cos^2(2\alpha) v^2 \\ &= m_Z^2 \cos^2(2\alpha) \\ &\leq m_Z^2 \end{aligned}$$

## Problem: Is the predicted Higgs mass too small?

MSSM **predicts** at tree-level:

$$m_h^2 = m_Z^2 \cos^2 2\alpha \leq m_Z^2$$

But from experiment we know [PDG]:

$$M_h = (125.10 \pm 0.14) \text{ GeV}$$

$$M_Z = (91.1876 \pm 0.0021) \text{ GeV}$$

$\Rightarrow$  **large loop corrections** required!

$$M_h^2 = m_h^2 + \Delta m_h^2 \quad \Rightarrow \quad \Delta m_h^2 \geq (85 \text{ GeV})^2$$

# Use the Higgs mass to put constraints on a SUSY model

## Idea:

- 1 Calculate  $M_h$  as precisely as possible in the SUSY model:

$$M_h^2 = m_h^2 + \Delta m_h^2$$

- 2 Constrain the parameter space by requiring:

$$M_h \stackrel{!}{=} (125.10 \pm 0.14) \text{ GeV} \pm \Delta M_h^{\text{theo}}$$

**Problem:** because of large loop corrections  $\Delta m_h^2$ :

$$\Delta M_h^{\text{theo}} \sim 1 \text{ GeV} \quad \text{☹}$$

$$\Delta M_h^{\text{exp}} = 0.14 \text{ GeV} \quad [\text{PDG-2019}]$$

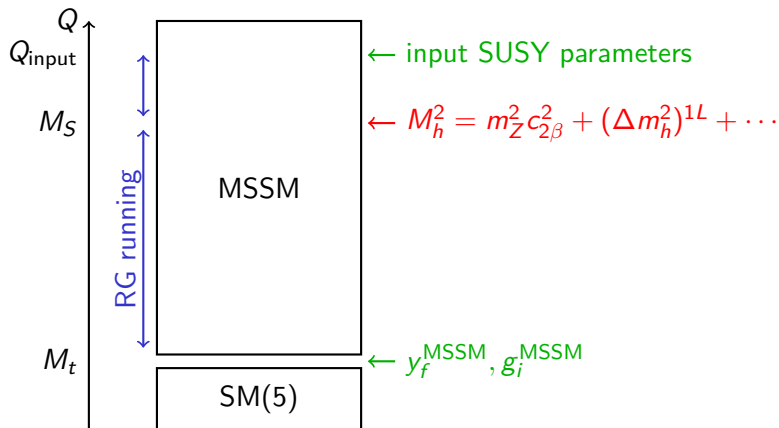
→ Theorists must increase their precision!

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# Fixed-order calculation

**Input:**  $\alpha_{em}(M_Z)$ ,  $\alpha_s(M_Z)$ ,  $G_F$ ,  $M_Z$ ,  $M_t$ ,  $m_b(m_b)$ ,  
SUSY parameters, ...



# Fixed-order calculation

Dominant contribution to  $M_h$  at the 1-loop level:

$$\begin{aligned}
 (\Delta m_h^2)^{1L} &= -\Sigma_h^{1L}(p^2) + \frac{t_h^{1L}}{v} \\
 &= \text{---} \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---} \\
 &\quad + \frac{1}{v} \left( \text{---} \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---} \right) \\
 &\approx \frac{6y_t^4 v^2}{(4\pi)^2} \left( \log \frac{M_S^2}{m_t^2} + \frac{X_t^2}{M_S^2} - \frac{X_t^4}{12M_S^4} \right) + \mathcal{O} \left( \frac{v^2}{M_S^2} \right)
 \end{aligned}$$



## Higgs mass at 1-loop level

$$(\Delta m_h^2)^{1L} \approx \frac{6y_t^4 v^2}{(4\pi)^2} \left( \log \frac{M_S^2}{m_t^2} + \frac{X_t^2}{M_S^2} - \frac{X_t^4}{12M_S^4} \right) + \mathcal{O} \left( \frac{v^2}{M_S^2} \right)$$

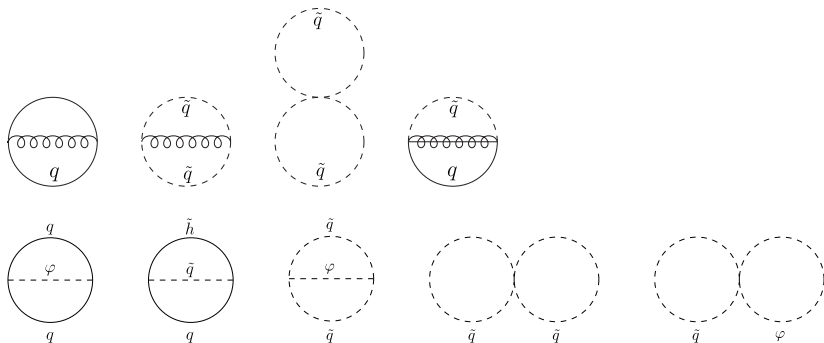
$X_t = A_t - \mu/t_\beta =$  stop mixing parameter,  $M_S = (m_Q)_{33} = (m_U)_{33}$

### Observations:

- logarithmically enhanced by  $\log(M_S/m_t)$
- maximal for  $X_t/M_S \approx \sqrt{6}$
- high sensitivity on  $m_t$ , due to prefactor  $y_t = \sqrt{2}m_t/v$
- ambiguity of definition of  $m_t$ : pole mass or  $\overline{\text{DR}}$  mass?  
 $M_t \approx 173.3 \text{ GeV}$ ,  $m_t^{\overline{\text{DR}}} \approx 165 \text{ GeV}$   
 $\Rightarrow$  huge theoretical uncertainty!  
 $\Rightarrow$  2-loop calculation needed to resolve this ambiguity
- $M_h \approx 125 \text{ GeV}$  requires  $M_S \gtrsim 5 \text{ TeV}$

# Higgs mass at 2-loop level

Known contributions:  $\mathcal{O}(\alpha_s(\alpha_t + \alpha_b) + (\alpha_t + \alpha_b)^2 + \alpha_\tau^2)$  for  $p^2 = 0$  [hep-ph/0105096, hep-ph/0112177]



## Higgs mass at 2-loop level

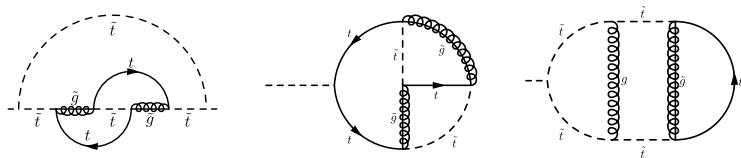
$$\begin{aligned}(\Delta m_h^2)^{2L} \approx & \frac{y_t^6 v^2}{(4\pi)^4} \left( c_1 \log^2 \frac{M_S^2}{m_t^2} + c_2 \log \frac{M_S^2}{m_t^2} + c_3 \right) \\ & + \frac{y_t^4 g_3^2 v^2}{(4\pi)^4} \left( c_4 \log^2 \frac{M_S^2}{m_t^2} + c_5 \log \frac{M_S^2}{m_t^2} + c_6 \right)\end{aligned}$$

### Observations:

- logarithmically enhanced by  $\log^2(M_S/m_t)$
- still high sensitivity on  $m_t$
- ambiguity of definition of  $m_t$  is resolved ✓
- ambiguity of definition of  $\alpha_s$ :  $\alpha_s^{\text{SM}}(M_Z)$ ,  $\alpha_s^{\text{MSSM}}(M_S)$ , ... ?  
⇒ 3-loop calculation needed to resolve this ambiguity

# Higgs mass at 3-loop level

Known contributions:  $\mathcal{O}(\alpha_t \alpha_s^2)$  for  $p^2 = 0$  [1005.5709,1708.05720]



$$(\Delta m_h^2)^{3L} \approx \frac{y_t^4 g_3^4 v^2}{(4\pi)^6} \left( c_7 \log^3 \frac{M_S^2}{m_t^2} + c_8 \log^2 \frac{M_S^2}{m_t^2} + c_9 \log \frac{M_S^2}{m_t^2} + c_{10} \right)$$

## Observations:

- logarithmically enhanced by  $\log^3(M_S/m_t)$
- still high sensitivity on  $m_t$
- ambiguity of definition of  $m_t$  is resolved ✓
- ambiguity of definition of  $\alpha_s$  is resolved ✓

# Summary of fixed loop order calculation

Typical order of magnitude of loop contributions (depends on parameter scenario):

$$\begin{aligned} M_h &= m_h + \Delta m_h^{1L} + \Delta m_h^{2L} + \Delta m_h^{3L} + \dots \\ &\approx [91 + \mathcal{O}(20 \dots 30) + \mathcal{O}(2 \dots 4) + \mathcal{O}(1 \dots 2)] \text{ GeV} \end{aligned}$$

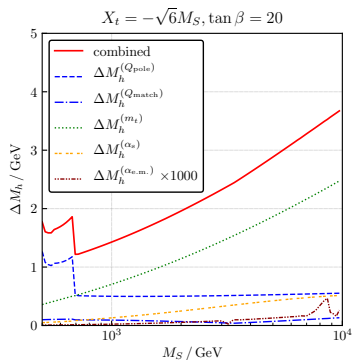
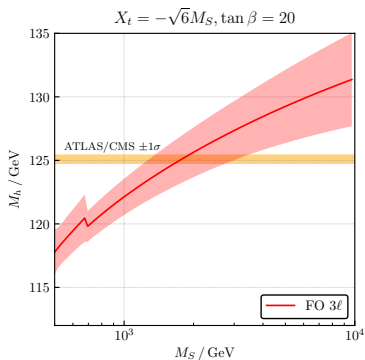
## Advantages:

- ✓ includes logarithmic, non-logarithmic and suppressed terms of  $\mathcal{O}((v^2/M_\xi^2)^\infty)$  at fixed loop order
- ✓ precise prediction if  $M_S \sim m_t$  (then  $\log(M_S/m_t) \approx 0$ )

## Disadvantages:

- ✗ large logarithmic corrections if  $M_S \gg m_t$ 
  - ⇒ slow convergence of perturbation series
  - ⇒ large theoretical uncertainty, (1–2 GeV, or more)

# Uncertainty estimate of the fixed-order $\overline{DR}'$ calculation



[1804.09410]

# Additional problem: no SUSY particles observed so far

ATLAS SUSY Searches\* - 95% CL Lower Limits

July 2019

ATLAS Preliminary

$\sqrt{s} = 13$  TeV

Model	Signature	$\mathcal{L} dt [fb^{-1}]$	Mass limit	Reference		
Inclusive Searches	$\tilde{g}, \tilde{q} \rightarrow \tilde{q} \tilde{q}^*$ mono-jet	2-6 jets	$E_{T}^{miss}$ 36.1		$m(\tilde{t}_1) > 100$ GeV $m(\tilde{g}) > 95$ GeV	
		1-3 jets	$E_{T}^{miss}$ 36.1		$m(\tilde{t}_1) > 200$ GeV $m(\tilde{q}) > 200$ GeV	
	$\tilde{b}, \tilde{b} \rightarrow \tilde{q} \tilde{q}^*$	2-6 jets	$E_{T}^{miss}$ 36.1		$m(\tilde{t}_1) > 200$ GeV $m(\tilde{b}_1) > 200$ GeV	
		3-6 jets	4 jets 36.1 $\nu_e, \mu_e$ 2 jets $E_{T}^{miss}$ 36.1		$m(\tilde{t}_1) > 350$ GeV $m(\tilde{b}_1) > 350$ GeV	
	$\tilde{b}, \tilde{b} \rightarrow \tilde{q} \tilde{q}^* \tilde{WZ} \tilde{t}_1^*$	0-1 $\nu_e, \mu$	7-11 jets $E_{T}^{miss}$ 36.1		$m(\tilde{t}_1) > 350$ GeV	
		SS $\nu_e, \mu$	6 jets $E_{T}^{miss}$ 139		$m(\tilde{t}_1) > 350$ GeV	
	$\tilde{b}, \tilde{b} \rightarrow \tilde{q} \tilde{q}^*$	0-1 $\nu_e, \mu$	3-6 jets $E_{T}^{miss}$ 79.8		$m(\tilde{t}_1) > 400$ GeV $m(\tilde{b}_1) > 200$ GeV	
		SS $\nu_e, \mu$	6 jets $E_{T}^{miss}$ 139		$m(\tilde{t}_1) > 200$ GeV $m(\tilde{b}_1) > 300$ GeV	
	3 <sup>rd</sup> gen. quarks and production	$\tilde{b}_1, \tilde{b}_1 \rightarrow \tilde{b}_1 \tilde{b}_1^* \tilde{H}_1^*$	Multiple	36.1		$m(\tilde{t}_1) > 300$ GeV, $BR(\tilde{t}_1^* \tilde{t}_1) = 1$ $m(\tilde{b}_1) > 300$ GeV, $BR(\tilde{b}_1^* \tilde{b}_1) > 0.5$ $m(\tilde{t}_1) > 200$ GeV, $m(\tilde{b}_1) > 300$ GeV, $BR(\tilde{t}_1^* \tilde{t}_1) = 1$
			Multiple	36.1		$m(\tilde{t}_1) > 300$ GeV, $BR(\tilde{t}_1^* \tilde{t}_1) = 1$ $m(\tilde{b}_1) > 300$ GeV, $BR(\tilde{b}_1^* \tilde{b}_1) = 1$
$\tilde{b}_1, \tilde{b}_1 \rightarrow \tilde{b}_1 \tilde{b}_1^* \rightarrow \tilde{b} \tilde{b}^*$		0-1 $\nu_e, \mu$	6-8 jets $E_{T}^{miss}$ 139		$m(\tilde{t}_1) > 100$ GeV $m(\tilde{b}_1) > 150$ GeV, $m(\tilde{t}_1) > 100$ GeV	
		0-2 $\nu_e, \mu$	0-2 jets/1-2 $b$ jets $E_{T}^{miss}$ 36.1		$m(\tilde{t}_1) > 1$ GeV	
$\tilde{b}_1, \tilde{b}_1 \rightarrow \tilde{W} \tilde{b}_1^* \rightarrow \tilde{W} \tilde{b}_1^*$		1 $\nu_e, \mu$	3 jets/1 $b$ jet $E_{T}^{miss}$ 139		$m(\tilde{t}_1) > 400$ GeV $m(\tilde{b}_1) > 800$ GeV	
		1 $\tau + 1 \nu_e, \mu, \tau$	2 jets/1 $b$ jet $E_{T}^{miss}$ 36.1		$m(\tilde{t}_1) > 0$ GeV	
$\tilde{b}_1, \tilde{b}_1 \rightarrow \tilde{t}_1 \tilde{t}_1^* \nu_e, \tau \rightarrow \nu_e \tilde{t}_1^*$		0-1 $\nu_e, \mu$	2 $c$ jets $E_{T}^{miss}$ 36.1		$m(\tilde{t}_1) > 50$ GeV $m(\tilde{b}_1) > 50$ GeV	
		0-1 $\nu_e, \mu$	mono-jet $E_{T}^{miss}$ 36.1		$m(\tilde{t}_1) > 150$ GeV	
$\tilde{b}_1, \tilde{b}_1 \rightarrow \tilde{t}_1 \tilde{t}_1^* + Z$		1-2 $\nu_e, \mu$	4 $b$ jets $E_{T}^{miss}$ 36.1		$m(\tilde{t}_1) > 80$ GeV, $m(\tilde{b}_1) > 180$ GeV	
		3 $\nu_e, \mu$	1 $b$ jet $E_{T}^{miss}$ 139		$m(\tilde{t}_1) > 300$ GeV, $m(\tilde{b}_1) > 40$ GeV	
EW direct	$\tilde{t}_1^* \tilde{t}_1^*$ via WZ	2-3 $\nu_e, \mu$	$E_{T}^{miss}$ 36.1		$m(\tilde{t}_1) > 0$ $m(\tilde{t}_1) > 35$ GeV	
		$\nu_e, \mu, \tau$	$\geq 1$ jet $E_{T}^{miss}$ 139		$m(\tilde{t}_1) > 0$	
	$\tilde{t}_1^* \tilde{t}_1^*$ via WW	2 $\nu_e, \mu$	$E_{T}^{miss}$ 139		$m(\tilde{t}_1) > 0$	
		0-1 $\nu_e, \mu$	2 $b/\tau$ jets $E_{T}^{miss}$ 139		$m(\tilde{t}_1) > 70$ GeV	
	$\tilde{t}_1^* \tilde{t}_1^*$ via $\tilde{t}_1 \tilde{t}_1^*$	2 $\nu_e, \mu$	$E_{T}^{miss}$ 139		$m(\tilde{t}_1) > 0$	
		$\tau, \tau \rightarrow \nu_e \tilde{t}_1^*$	2 $\tau$ jets $E_{T}^{miss}$ 139		$m(\tilde{t}_1) > 0$	
	$\tilde{b}_1, \tilde{b}_1 \rightarrow \tilde{t}_1 \tilde{t}_1^* \nu_e$	2 $\nu_e, \mu$	0 jets $E_{T}^{miss}$ 139		$m(\tilde{t}_1) > 0$	
		2 $\nu_e, \mu$	$\geq 1$ jet $E_{T}^{miss}$ 139		$m(\tilde{t}_1) > 10$ GeV	
	$\tilde{H}, \tilde{H} \rightarrow \tilde{H} Z \tilde{G}$	0-1 $\nu_e, \mu$	$\geq 3$ $b$ jets $E_{T}^{miss}$ 36.1		$m(\tilde{t}_1) > 100$ GeV $BR(\tilde{t}_1^* \tilde{t}_1) = 1$	
		4 $\nu_e, \mu$	0 jets $E_{T}^{miss}$ 36.1		$BR(\tilde{t}_1^* \tilde{t}_1) > 20\%$	
Direct $\tilde{t}_1^* \tilde{t}_1^*$ prod., long-lived $\tilde{t}_1^*$	Disapp. 1/6	1 jet $E_{T}^{miss}$ 36.1		Pure Wino Pure Higgsino		
				$m(\tilde{t}_1) > 100$ GeV		
				$m(\tilde{t}_1) > 100$ GeV		
Stable $\tilde{g}$ R-hadron	Multiple	36.1		$m(\tilde{g}) > 100$ GeV		
	Metastable $\tilde{g}$ R-hadron, $\tilde{g} \rightarrow \tilde{q} \tilde{q}^*$	Multiple		36.1	$m(\tilde{g}) > 100$ GeV	
				36.1	$m(\tilde{g}) > 100$ GeV	
LFV $\tilde{g} \rightarrow \tilde{g} + X$ , $\tilde{t}_1 \rightarrow \tilde{q} \nu_e / \tilde{t}_1 \rightarrow \tilde{q} \tau$	$\tilde{t}_1^* \tilde{t}_1^* \rightarrow W \tilde{Z} \tilde{t}_1^* \nu_e$	4 $\nu_e, \mu$	0 jets $E_{T}^{miss}$ 36.1		$X_{11} = 0.11, X_{22} = 0.00$	
		4-5 large-R jets	Multiple		36.1	$m(\tilde{t}_1) > 100$ GeV
	$\tilde{b}_1, \tilde{b}_1 \rightarrow \tilde{t}_1 \tilde{t}_1^* \nu_e$	Multiple	36.1		Large $\tilde{t}_1$	
		Multiple	36.1		$m(\tilde{t}_1) > 200$ GeV, bro- $\tilde{b}_e$	
	$\tilde{t}_1, \tilde{t}_1 \rightarrow \tilde{t}_1 \tilde{t}_1^* \nu_e$	Multiple	36.1		$m(\tilde{t}_1) > 200$ GeV, bro- $\tilde{b}_e$	
		Multiple	36.1		$m(\tilde{t}_1) > 200$ GeV, bro- $\tilde{b}_e$	
	$\tilde{t}_1, \tilde{t}_1 \rightarrow \tilde{b}_1 \tilde{b}_1^*$	2 $\nu_e, \mu$	3 $b$ jets 36.1		$m(\tilde{t}_1) > 0$	
		1 $\mu$	DV 136		$m(\tilde{t}_1) > 0$	
					$m(\tilde{t}_1) > 0$	

\*Only a selection of the available mass limits on new states or phenomena is shown. Many of the limits are based on simplified models, c.f. refs. for the assumptions made.

10<sup>-1</sup> 1 Mass scale [TeV]

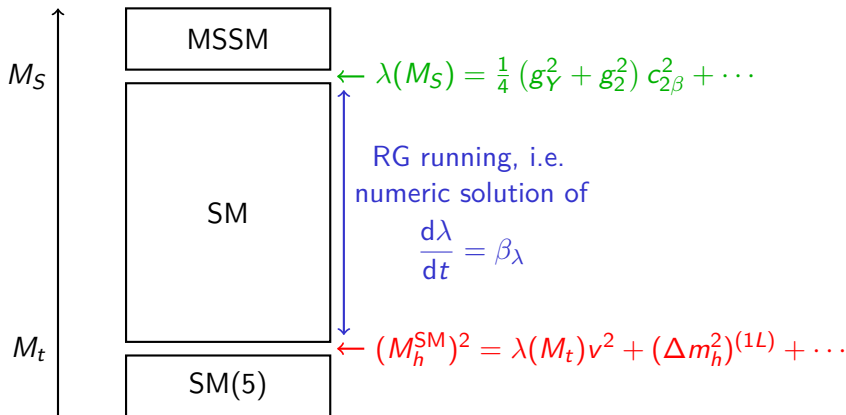
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# Higgs mass calculation in an Effective Field Theory

**Idea:** Decouple SUSY particles at  $M_S$  (expand in  $v^2/M_S^2$ )  
 $\Rightarrow \lambda(M_S)$  is fixed by the MSSM



## Resummation of large logarithms in an EFT

System of coupled DEQs ( $t = \log(Q)$ ,  $\kappa = 1/(4\pi)^2$ ):

$$\frac{d\lambda}{dt} = \beta_\lambda \approx -12\kappa y_t^4, \quad \frac{dy_t}{dt} \approx -8\kappa y_t g_3^2, \quad \frac{dg_3}{dt} \approx -7\kappa g_3^3$$

Solution ( $L = \log(M_S/M_t)$ ):

$$\lambda(M_t) = \lambda(M_S) - \frac{2y_t^4}{3g_3^2} \left[ \left(1 + 14g_3^2 \kappa L\right)^{-9/7} - 1 \right]$$

Insert in  $M_h^2 = \lambda(M_t)v^2$ , with  $\lambda(M_S) = m_Z^2 c_{2\beta}^2/v^2$ :

$$\begin{aligned} M_h^2 &= m_Z^2 c_{2\beta}^2 - \frac{2y_t^4 v^2}{3g_3^2} \left[ \left(1 + 14g_3^2 \kappa L\right)^{-9/7} - 1 \right] \\ &= m_Z^2 c_{2\beta}^2 + 12y_t^4 v^2 \left[ \kappa L - 16g_3^2 \kappa^2 L^2 + \frac{736}{3} g_3^4 \kappa^3 L^3 + \mathcal{O}(\kappa^4 L^4) \right] \end{aligned}$$

$\Rightarrow M_h^2$  contains **infinite series** of  $(\kappa L)^n$  terms in an EFT

# Summary of EFT approach

Typical order of magnitude of loop contributions (depends on parameter scenario, here  $X_t = 0$ ,  $M_S = 20$  TeV):

$$M_h = \sqrt{\lambda(M_t)} v + \Delta m_h^{1L} + \Delta m_h^{2L} + \Delta m_h^{3L} + \dots \\ \approx [\mathcal{O}(124) + \mathcal{O}(0.5 \dots 1) + \mathcal{O}(0.1 \dots 0.2) + \mathcal{O}(0.02 \dots 0.04)] \text{ GeV}$$

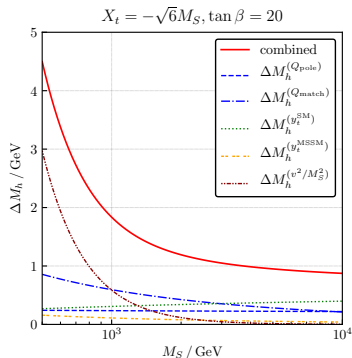
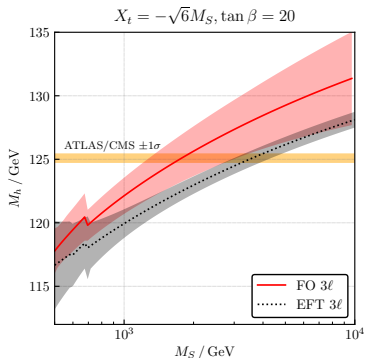
## Advantages:

- ✓ large corrections of  $\log^n(M_S/M_t)$  at *fixed order*  $n$  are avoided
- ✓ large logarithms  $\propto \log^\infty(M_S/M_t)$  are resummed to *all orders*

## Disadvantage:

- ✗ Terms of  $\mathcal{O}(v^2/M_S^2)$  are (typically) neglected  
⇒ imprecise when  $M_S \sim v$  (relevant?)  
⇒ large theoretical uncertainty when  $M_S \sim v$

# Comparison of fixed-order and EFT calculation



$$\Delta M_h^{(FO)} \stackrel{!}{=} \Delta M_h^{(EFT)}$$

$\Rightarrow M_S^{\text{equal}} \sim 1 \text{ TeV}$  for small/large  $\tan \beta$  and/or  $X_t$

## Summary of fixed-order and EFT approaches

	low $M_S$ $M_S \lesssim 1 \text{ TeV}$	high $M_S$ $M_S \gtrsim 1 \text{ TeV}$
fixed-order	✓	✗
EFT	✗	✓
? hybrid	✓	✓

Q: Can the fixed-order and EFT approaches be combined?

A: Yes! [1312.4937, 1609.00371, 1710.03760, 1910.03595, 2003.04639]

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# Hybrid calculation – FeynHiggs approach

**Goal:** resum large logarithms **and** include suppressed  $\mathcal{O}(v^2/M_\Sigma^2)$  terms

**Idea I:** (“FeynHiggs approach” [1312.4937, 1706.00346, 1805.00867, 1910.03595])  
Replace logs from fixed-order calculation by resummed logs:

$$M_h^2 = (M_h^2)_{\text{fixed-order}} - (M_h^2)_{\text{logs}} + (M_h^2)_{\text{resummed logs}}$$

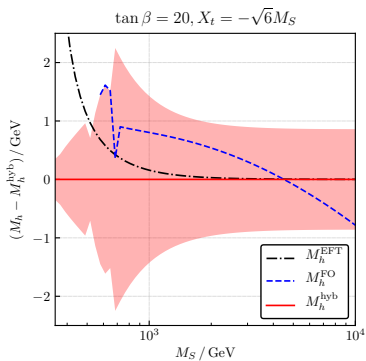
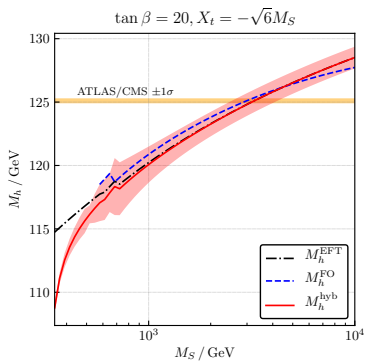
## Advantages:

- ✓ approach applicable to any BSM model
- ✓ any EFT can be used

## Disadvantages:

- ✗ requires knowledge of fixed-order and EFT expressions
- ✗ care must be taken to avoid double counting

# FeynHiggs approach in FlexibleSUSY at 3-loop



[1910.03595]



# Hybrid calculation – FlexibleEFTHiggs

**Idea II:** (“FlexibleEFTHiggs” [1609.00371, 1710.03760, 2003.04639])

Incorporate all  $\mathcal{O}((v^2/M_S^2)^\infty)$  terms into  $\lambda$  by using the matching condition for the FO calculations:

$$(M_h^2)_{\text{SM}} \stackrel{!}{=} (M_h^2)_{\text{BSM}} \quad \text{at } Q = M_S$$
$$\lambda(M_S)v^2 + (\Delta m_h^2)_{\text{SM}} = (M_h^2)_{\text{BSM}}$$

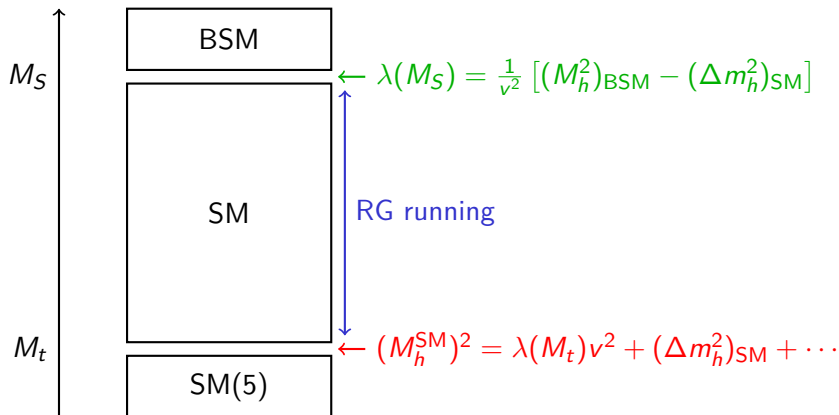
$\Rightarrow$

$$\lambda(M_S) = \frac{1}{v^2} \left[ (M_h^2)_{\text{BSM}} - (\Delta m_h^2)_{\text{SM}} \right]$$

Continue as in the EFT calculation ...

# Hybrid calculation – FlexibleEFTHiggs

Continue as in the EFT calculation:



# Proof: FlexibleEFTHiggs $\rightarrow$ EFT for $v^2 \ll M_S^2$

Consider the matching condition:

$$\lambda(M_S) = \frac{1}{v^2} \left[ (M_h^2)_{\text{MSSM}} - (\Delta m_h^2)_{\text{SM}} \right]$$

where at  $\mathcal{O}(y_t^4)$ :

$$(\Delta m_h^2)_{\text{SM}} = -\Sigma_h^{\text{SM}} - 6\kappa y_t^2 A_0(m_t)$$

$$(M_h^2)_{\text{MSSM}} = m_Z^2 c_{2\alpha}^2 + \left[ -\Sigma_h^{\text{SM}} - 6\kappa y_t^2 A_0(m_t) \right] \frac{c_\alpha^2}{s_\beta^2} \\ - 6\kappa y_t^4 v^2 \frac{c_\alpha^2}{s_\beta^2} B_0(M_S, M_S)$$

Proof: FlexibleEFT Higgs  $\rightarrow$  EFT for  $v^2 \ll M_S^2$

$$\begin{aligned} \lambda(M_S) &= \frac{1}{v^2} \left[ (M_h^2)_{\text{MSSM}} - (\Delta m_h^2)_{\text{SM}} \right] \\ &= \frac{m_Z^2}{v^2} c_{2\alpha}^2 + \frac{1}{v^2} \left[ -\Sigma_h^{\text{SM}} - 6\kappa y_t^2 A_0(m_t) \right] \left( \frac{c_\alpha^2}{s_\beta^2} - 1 \right) \\ &\quad - 6\kappa y_t^4 \frac{c_\alpha^2}{s_\beta^2} B_0(M_S, M_S) \end{aligned}$$

In the limit  $v^2 \ll M_S^2$ :  $c_\alpha^2 \rightarrow s_\beta^2$ ,  $c_{2\alpha}^2 \rightarrow c_{2\beta}^2 \Rightarrow$

$$\begin{aligned} \lambda(M_S) &= \frac{1}{4} (g_Y^2 + g_2^2) c_{2\beta}^2 - 6\kappa y_t^4 B_0(M_S, M_S) \\ &= \frac{1}{4} (g_Y^2 + g_2^2) c_{2\beta}^2 - 6\kappa y_t^4 \left[ -\log \frac{M_S^2}{Q^2} + \frac{p^2}{6M_S^2} + \mathcal{O}\left(\frac{p^4}{M_S^4}\right) \right] \end{aligned}$$

# Summary of FlexibleEFTHiggs hybrid calculation

Matching condition:

$$(M_h^2)_{\text{SM}} \stackrel{!}{=} (M_h^2)_{\text{BSM}}$$

## Advantages:

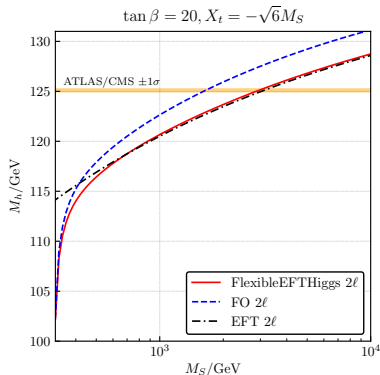
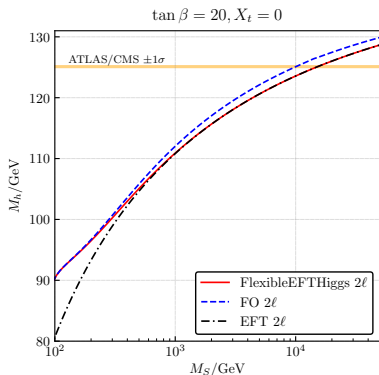
- ✓ includes all power-suppressed terms at  $n$ -loop order  
 $(v^2/M_S^2)^\infty [c + \log^n(M_S/Q) + \log^n(M_S/m_t)]$
- ✓ resums large non-suppressed logarithms  $c \log^\infty(M_S/m_t)$
- ✓ approach applicable to any BSM model
- ✓ only 1- and 2-point fixed-order expressions required  $\rightarrow$  easy to automate (e.g. SARAH/FlexibleSUSY)

## Disadvantages:

- ✗ difficult to extend to other EFTs beyond the SM (2HDM, ...)
- tricky to reach 2-loop accuracy  
(requires careful treatment of parameter matching)

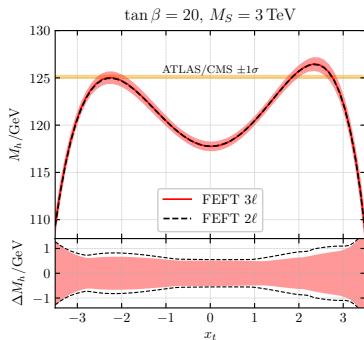
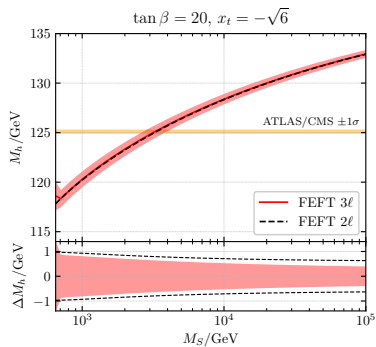
# Interpolation behaviour of FlexibleEFTHiggs in the MSSM

Interpolation behaviour between FO and EFT calculation:



[plots along the lines of 2003.04639]

# Hybrid calculation – FlexibleEFTHiggs



[2003.04639]

Theory uncertainty  $\Delta M_h \lesssim 1 \text{ GeV}$  in relevant degenerate SUSY scenarios.

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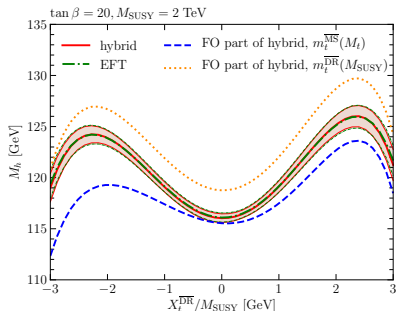
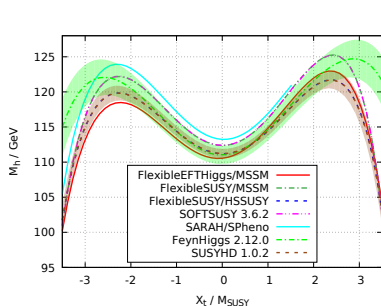


## Further improvements: $x_t$ resummation

Large stop mixing,  $x_t \equiv X_t/M_S \approx \pm\sqrt{6}$ , is an attractive scenario for low-scale supersymmetry.

However: “With large stop mixing comes large uncertainty.”

— P. Slavich



[1609.00371, 1912.10002]

## Recap: $\tan \beta$ resummation in $m_b$

Relation between running  $m_b$  in the MSSM and  $m_b^{\text{EFT}}$  in the EFT:

$$m_b^{\text{EFT}} = m_b(1 + \Delta m_b)$$

where  $\Delta m_b$  is expressed in terms of MSSM parameters.

### Theorem (hep-ph/9912516)

*There are no contributions to  $\Delta m_b$  of  $\mathcal{O}((\alpha_s \tan \beta)^n)$  for  $n \geq 2$ .*

$\Rightarrow$  All higher-order MSSM contributions in  $m_b$  of  $\mathcal{O}((\alpha_s \tan \beta)^n)$  can be resummed by writing:

$$m_b = \frac{m_b^{\text{EFT}}}{1 + \Delta m_b} = m_b^{\text{EFT}} \sum_{n=0}^{\infty} (-\Delta m_b)^n$$

## $x_t$ resummation

Relation between running  $y_t$  in the MSSM and  $y_t^{\text{EFT}}$  in the EFT:

$$y_t^{\text{EFT}} = y_t s_\beta + \Delta y_t$$

where  $\Delta y_t$  is expressed in terms of MSSM parameters.

### Theorem (Kwasnitza, Stöckinger (2003.04639))

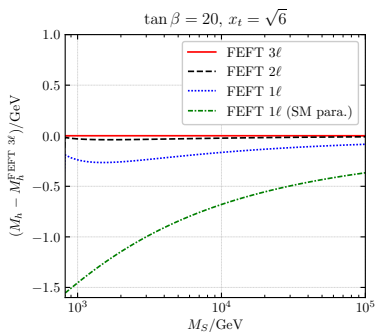
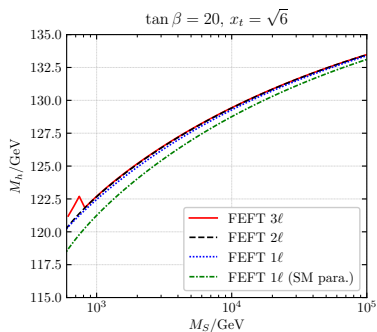
*There are no unsuppressed contributions to  $\Delta y_t$  of  $\mathcal{O}(\alpha_s^n x_t^{>1})$  for  $n \geq 1$ .*

$\Rightarrow$  All higher-order MSSM contributions in  $y_t$  of  $\mathcal{O}(\alpha_s^n x_t^{>1})$  can be resummed by writing:

$$y_t = \frac{y_t^{\text{EFT}}}{1 + \Delta y_t / (y_t s_\beta)}$$

## $x_t$ resummation

$\Rightarrow$  by expressing  $\lambda(M_S)$  in terms of **MSSM parameters** ( $y_t$ ,  $m_b$ , ...), certain higher-order contributions can be resummed to all orders



[2003.04639]

# Summary and Conclusions

**Precise prediction of the Higgs boson mass** in supersymmetric models allows to constrain the model parameter space

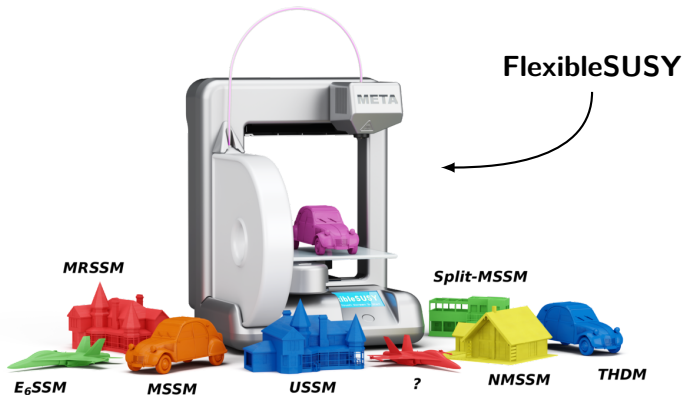
**Effective field theory** and resummation methods necessary for a precise prediction (resummation of large logarithms and  $x_t^\infty$  corrections)

⇒

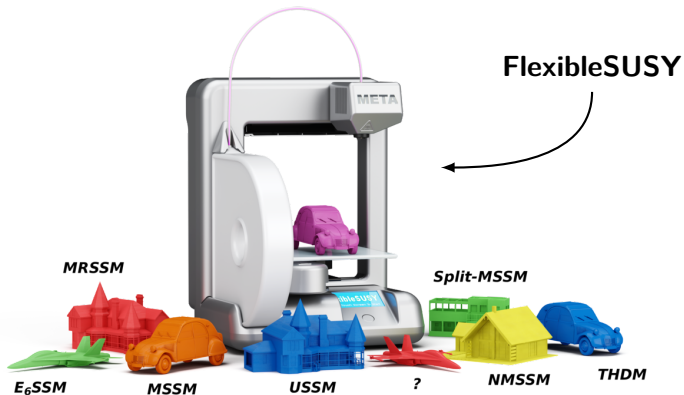
**Current precision** in the MSSM in relevant parameter space:  
 $\Delta M_h \lesssim 1 \text{ GeV}$

**Stop quarks must be heavy**,  $M_S \gtrsim 2 \text{ TeV}$ , in the MSSM for correct prediction of  $M_h \approx 125 \text{ GeV}$

Large zoo of SUSY models  
⇒ automation important!



Thank you for your attention!



# Backup



# Status of Higgs mass calculations

	FO	EFT	hybrid
MSSM			
FeynHiggs	2L	2L	2L
FlexibleSUSY	3L	3L	3L <sup>‡</sup>
SARAH	2L	2L	2L*
SOFTSUSY	3L	–	–
SPheno	2L	–	2L*
generic			
FlexibleSUSY	1L	–	1L <sup>†</sup>
SARAH/SPheno	2L	1L	2L*

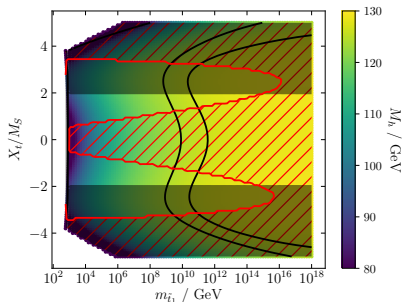
<sup>‡</sup>: in preparation, including  $x_t$  resummation

\*: NNLO + LL resummation

<sup>†</sup>: NLO + NLL resummation

# Maximum stop masses in the MSSM

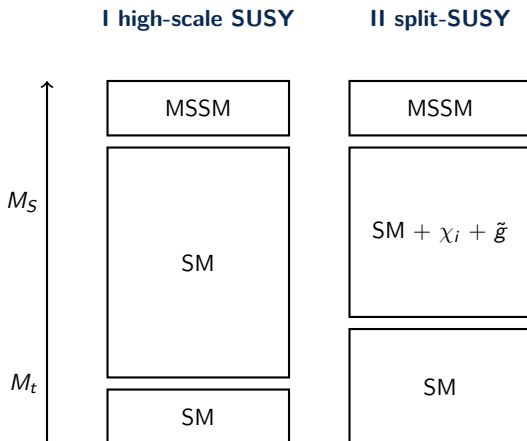
Maximum stop mass scenario with  $\tan \beta = 1$ :



Dark region: charge and color breaking minima

Hatched region: unbounded Higgs potential,  $\lambda(M_S) < 0$

# Scenarios with 1 light Higgs doublet



# Scenarios with 2 light/intermediate Higgs doublets

