The reason that the e'vals of $\underline{J}^2$ aren't obvious : does it commute with J_3?

$$\underline{J}^2 = (\underline{J}^{(1)} + \underline{J}^{(2)})^2 = \underline{J}^{(1)2} + \underline{J}^{(2)2} + 2\underline{J}^{(1)}\cdot\underline{J}^{(2)} \quad — ①$$

Strategy: write RHS in terms of $J_\pm^{(i)}, J_3^{(i)}$ so we know the action on basis states.

$$J_+^{(1)} J_-^{(2)} = (J_1^{(1)} + iJ_2^{(1)}) \cdot (J_1^{(2)} - iJ_2^{(2)})$$
$$= J_1^{(1)}J_1^{(2)} + J_2^{(1)}J_2^{(2)} + i\,(J_2^{(1)}J_1^{(2)} - J_1^{(1)}J_2^{(2)})$$

$J_-^{(1)} J_+^{(2)}$ is the same except for $(1) \leftrightarrow (2)$.

~~Sum~~ 2 ~~eqns~~. eq$^{\underline{n}}$s $\Rightarrow 2\underline{J}^{(1)}\cdot\underline{J}^{(2)} = J_+^{(1)}J_-^{(2)} + J_-^{(1)}J_+^{(2)} + 2J_3^{(1)}J_3^{(2)}$.

Subst $\to$ ①,

$$\underline{J}^2 = \underline{J}^{(1)2} + \underline{J}^{(2)2} + J_+^{(1)}J_-^{(2)} + J_-^{(1)}J_+^{(2)} + 2J_3^{(1)}J_3^{(2)}.$$

All terms manifestly commute with J_3 except for 3^{rd} + 4^{th} on RHS:

use $[J_\pm^{(i)}, J_3^{(j)}] = \mp J_\pm^{(i)}\delta_{ij}$

$$\Rightarrow [\underline{J}^2, J_3^{(1)} + J_3^{(2)}] = [J_+^{(1)}, J_3^{(1)}]J_-^{(2)} + J_+^{(1)}[J_-^{(2)}, J_3^{(2)}] + (1 \leftrightarrow 2)$$
$$= -J_+^{(1)}J_-^{(2)} + J_+^{(1)}J_-^{(2)} + (1 \leftrightarrow 2) = 0.$$

Hence $[\underline{J}^2, J_3] = 0$, as required. You could use $\underline{J}^2$ to verify the J values of the combined states we're finding.