PQM Lectures - B.C. Allanach.

- Please ask questions and challenge deliberate mistakes!
- Paul Adrian Maurice Dirac's Principles of QM.

0 Introduction — Recall the features of elementary (IB) QM:

- wave-part^e duality: light quanta, photon etc. behave like waves sometimes, part^s at other.
- this means we need a w.f. $\psi(x)$ for a part^e: prob. density $|\psi(x)|^2$. Probability intrinsic in QM.
- observables are operators on w.f.s: $[\hat{O}_1, \hat{O}_2] \neq 0$ limits simultaneous measurement.
- Schrö eq^n specifies dynamics + En.-levels.
- Can understand H atom with it.

Course aims: · Reformulate QM in more powerful, flexible / useful form: Dirac formalism (goes beyond w.f.s), allowing
- simpler analysis of known problems eg oscillator, but also clearest way to understand spin/other quantum #s.
- symmetries (eg translations, rotations) + conservation laws
- identical part^s (special states in QM).
- fundamental framework for quantising other more general systems, eg e.m. field (+ other forces - Part III SM)
- we won't dwell on applications, but try to keep track of what the mathematics is for.

Assume QM IB but NOT E/M beyond Coulomb's law + intuitive ideas about magnetism and only IA dynamics

PLAN

1	Dirac formalism — will then be your hero!	[5 lectures]
2	Harmonic oscillator	[2]
3	Pictures + quantisation	[2]
4	Composite systems + identical part^es	[3]
5	Perturbation theory	[2]
6	Angular momentum	[4]
7	Transformations + symmetries	[3]
8	Time dependent pert. theory	[3]
9	Quantum basics	[2]

1a States + Operators — A quantum system is described, at any instant, by a state $|\psi\rangle$ which belongs to a $\mathbb{C}$ vector space V. ie if $|\psi\rangle, |\varphi\rangle$ are poss. states, so is $\alpha|\psi\rangle + \beta|\varphi\rangle$. $\forall\ \alpha, \beta \in \mathbb{C}$.

"Physically this is a superpos^n principle leading to wave-like behaviour (interference).

But state is NOT a wavefn^n. $\exists$ dual or conjugate states $\langle\varphi|$ which belong to the dual space V^*. By def^n "states + duals can be paired to give a $\mathbb{C}$ number"

$$\langle\varphi|\ ,\ |\psi\rangle \longrightarrow \langle\varphi|\psi\rangle \quad \text{map: } V^* \times V \to \mathbb{C}$$

'bra' 'ket' 'bra(c)ket'

with $\langle\varphi|(\alpha_1|\psi_1\rangle + \alpha_2|\psi_2\rangle) = \alpha_1\langle\varphi|\psi_1\rangle + \alpha_2\langle\varphi|\psi_2\rangle$ and then

$(\beta_1\langle\varphi_1| + \beta_2\langle\varphi_2|)|\psi\rangle = \beta_1\langle\varphi_1|\psi\rangle + \beta_2\langle\varphi_2|\psi\rangle$.

"The space of states V (and the dual V^*) comes with an inner-product which can be described as a 1-1 correspondence between states + duals"

$$V \leftrightarrow V^* \quad \text{def}^n$$

$$|\psi\rangle \leftrightarrow \langle\psi| = (|\psi\rangle)^+$$

$$\alpha|\psi\rangle + \beta|\varphi\rangle \leftrightarrow \alpha^*\langle\psi| + \beta^*\langle\varphi|$$

Inner product: $V \times V \to \mathbb{C}$

$$|\varphi\rangle, |\psi\rangle \longmapsto \langle\varphi|\psi\rangle = (|\varphi\rangle)^+|\psi\rangle.$$

and is assumed to obey $\langle\varphi|\psi\rangle = \langle\psi|\varphi\rangle^*$ (ie as hermitian).

$\Rightarrow \langle\psi|\psi\rangle = \||\psi\rangle\|^2 \in$ real ≥ 0 and is $0 \Leftrightarrow |\psi\rangle = 0$

NB: If we know $\langle\varphi|\psi\rangle\ \forall\ |\varphi\rangle$, then $|\psi\rangle$ is determined uniquely - will see later.

Physical content of a state is unchanged by $|\psi\rangle \to \alpha|\psi\rangle$ ($\alpha \neq 0$), since we will normalise $\||\psi\rangle\|^2 = 1$, but can fix α so $|\psi\rangle \to e^{i\theta}|\psi\rangle$ NO significance, but relative phases $\alpha|\psi\rangle + \beta|\varphi\rangle$ can be.